Signaling in a Model of Currency Circulation under Private Information*

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Abstract

How does the imperfect recognizability of commodity money affect its production, terms of trade, and circulation? This paper develops a model where coins of different intrinsic values are minted to be used as a medium of exchange, and are subject to a private information problem. Our model offers a new perspective over existing analyses by allowing owners of coins to signal the quality of their asset holdings through the offer they make. The implications for velocity, output and welfare are examined.

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How does imperfect recognizability of a coin's intrinsic value affect its production, terms of trade, and circulation? This is, indeed, an old question in monetary economics. A partial, hundreds-year-old answer—which goes under the name of Gresham's Law—is that high value coins will be driven out of circulation by coins of lower value. But, as pointed out by Velde, Weber and Wright (1999), henceforth VWW, "Despite [...] being one of the most generally accepted and frequently cited propositions in economics, ... [the] existing theoretical analyses of Gresham's Law are lacking." This paper develops a model where coins of different intrinsic values or *qualities* are minted to be used as a medium of exchange, and are subject to recognizability problems. Our model and analysis offer a fresh perspective on issues relating to currency circulation under private information. A major innovation in the analysis over the work of VWW is that owners of coins have the ability to signal their quality to potential trading partners. Because of this, we will uncover new types of equilibria where signaling allows coins holders to separate themselves, and we will show that the (pooling) equilibria identified in VWW cease to exist for taste and technology specifications that are typically adopted in models of modern monetary theory.[1]

We consider a search-theoretic environment where individuals trade in bilateral meetings and where a double-coincidence-of-wants problem generates a need for a medium of exchange. Prior to trading, heterogenous buyers must decide on the kinds of coins to mint. This decision problem is made interesting by the fact that coins are imperfectly recognizable: Borrowing from Williamson and Wright (1994), we assume that a seller will sometimes be able to perfectly assess the quality of a buyer's coin, and other times will not. To simplify buyers' minting decisions and buyers and sellers bargaining strategies, we assume, as do VWW, that coins are indivisible and individuals can hold at most one coin. Absent this assumption, buyers would have to decide on a *portfolio* of coins to mint, and the bargaining problem—and hence the buyer's minting decision—would be complicated by the seller's inability to recognize the quality of the buyer's portfolio of coins. We do, however, choose the minting technologies so that the indivisibility and

[1] The search-theoretic literature on currency circulation under private information includes Cuadras-Morato (1994), Li (1995), Haegler (1997) and Burdett, Trejos, and Wright (2001).

unit upper bound restrictions are not binding in a complete information version of the model. A major departure from VWW is to allow agents to use lotteries to trade indivisible coins in bilateral matches. Intuitively, the existence of lotteries makes indivisible coins "divisible" and, as a result, allows buyers to signal information about the quality of their coins: without lotteries, buyers have no mechanism to communicate the quality of the coins they hold.

We establish a simple condition on fundamentals under which high quality coins are traded at different terms of trades compared to low quality coins in all matches. In particular, and in contrast to VWW, the prices of coins are different even in matches where the quality of coins is not recognizable. At first, it may seem surprising that buyers with coins whose differences are completely indistinguishable to the seller can fully separate themselves. But this result hinges on the structure of the bargaining game, which is analogous to a standard signaling game. Buyers—who are the informed party—make take-it-or-leave-it offers to sellers—who are the uninformed party—that convey information about the quality of their coins. Buyers with high quality coins are able to distinguish themselves from buyers with low quality coins by offering to trade for less output at a lower price. This separating offer can be chosen in a way that a buyer with a low quality coin would not want to imitate. In fact, if one finds the Cho and Kreps (1987) intuitive criterion persuasive, then the *only* kind of equilibria that can exist when buyers mint different quality coins are ones where they fully separate themselves, i.e., there cannot be any pooling equilibria.

Equilibria where both low and high quality coins are minted exist provided that the recognizability problem is not too severe, or the discrepancy between the qualities of the two coins is sufficiently large. In such equilibria, the probability that high quality coins change hands in *uninformed* meetings and the quantity of output they buy are both lower than those for low quality coins. So the informational asymmetry reduces the liquidity of high quality coins. Interestingly, the terms of trade in uninformed matches are independent of the extent of the recognizability problem, i.e., how often the quality of a coin can be accurately assessed by the seller. However, they do depend on the relative qualities of the available coins. When the recognizability problem becomes too severe, then all buyers will mint only low quality coins.

We evaluate the implications of coins' recognizability for velocity, output and welfare. In equilibria where low and high quality coins circulate, aggregate output and welfare increase with recognizability. We show that a decrease in the recognizability of coins that triggers a transition from a "two coin" equilibrium to a "one coin" equilibrium always reduces welfare.

One can interpret our results in terms of counterfeiting by considering the case where the value of the low quality coin approaches zero: the low quality coin is a pure counterfeit. In this case, high quality coins are traded only when their quality is recognized, and there will be no trade in meetings where the seller cannot recognize the quality of the buyer's coin. If the recognizability problem becomes sufficiently severe, heavy coins will not be minted and the entire economy shuts down. This result is reminiscent to the "threat of counterfeiting" in Nosal and Wallace (2006).

The rest of the paper is organized as follows: Section 2 presents the physical environment of our model. Section 3 characterizes the benchmark economy with full information. Section 4 studies how imperfect information affects agents' minting decisions. Section 5 focusses on the consequences for output, welfare and velocity. Section 6 concludes. The proofs to all propositions and lemmas are in an appendix.

1 The model

There are two periods, $t = 1$ and $t = 2$.[2] There is a continuum of agents of measure two, divided evenly between buyers and sellers. The set of buyers, denoted $\mathcal{B}$, is itself divided into two subgroups: a set $\mathcal{B}_H$ of buyers with measure λ have a high marginal utility of consumption and a set $\mathcal{B}_L$ of buyers with measure $1-\lambda$ have a low marginal utility of consumption. The set of sellers is denoted by $\mathcal{S}$. There are three kinds of goods: a general good that can be produced and consumed by all agents, a special good that is only consumed by buyers and only produced by sellers, and coins. Both general and special goods are perfectly divisible and perishable. In

[2] Since there are no state variables that link the different periods, there is no loss in generality by considering only two periods. Note also that our model is essentially a commodity-money version of the Lagos and Wright (2005) framework.

contrast, coins are perfectly durable and indivisible.

General goods can be turned into coins according to a perfectly reversible technology. For simplicity, we suppose that there is no transaction cost associated with the minting and melting processes.[3] The minting technology allows for two types of coins: high intrinsic value or *high quality* coins are made of z_h units of general goods and low intrinsic value or *low quality* coins are made of $z_\ell < z_h$ units of general goods. For tractability purposes, we will assume that agents cannot carry more than one coin across periods. This indivisibility assumption allows us to abstract from the portfolio decision that an agent would have to make in period 1—i.e., the number of high and low quality coins to mint. This assumption also simplifies the bargaining problem in date 2. We would like to emphasize, however, that the intrinsic values of the coins will be chosen so that the indivisibility assumption is non-binding when information is symmetric.

The sequence of events is as follows: At $t = 1$, agents can produce the general good and have access to the minting technology. At $t = 2$, buyers and sellers are matched pairwise and at random, where the special goods are produced in bilateral matches by sellers. Since there is the same number of buyers and sellers, we assume that each buyer is randomly assigned to a seller. At the end of the second period, bilateral matches are dissolved, agents melt any coin that they possess, and consumption takes place.

The utility function of a buyer, $\mathcal{U}^b(\varepsilon)$, is given by

$$\mathcal{U}^b(\varepsilon) = c_1 + \beta\left[\varepsilon u(q) + c_2\right], \tag{1.1}$$

where c_t is the net consumption of general goods in period t (if $c_t < 0$ agents consume less than they produce), q is the quantity of special goods consumed, $\beta \in (0,1)$ is the discount factor across periods, and ε is a preference parameter where $\varepsilon = \varepsilon_L$ for buyers in $\mathcal{B}_L$ and $\varepsilon = \varepsilon_H > \varepsilon_L$ for buyers in $\mathcal{B}_H$. Note that general goods enter linearly in the utility function. We denote $r = \beta^{-1} - 1$ as the agents' rate of time preference. The function $u(q)$ is continuously differentiable, strictly increasing, and concave, and it satisfies $u(0) = 0$ and $u'(0) = \infty$.

[3] For a model where minting and melting are costly processes, see Sargent and Wallace (1983).

The utility function of a seller, $\mathcal{U}^s$, is given by

$$\mathcal{U}^s = c_1 + \beta \left[-\psi(q) + c_2 \right]. \tag{1.2}$$

The "cost" function, $\psi(q)$, is continuously differentiable, strictly increasing, and convex. Furthermore, $\psi(0) = 0$, $\psi'(0) = 0$, and there exists a $q_\varepsilon^* < \infty$ such that $\varepsilon u'(q_\varepsilon^*) = \psi'(q_\varepsilon^*)$. Output q_ε^* corresponds to the level of production that maximizes the total surplus in a match between a seller and a buyer of type ε.

The terms of trade in pairwise matches are determined by bargaining. Since we want the buyer to have the ability to signal the quality of the coin that he possesses, we will adopt the standard bargaining assumption for signaling games, which is that buyers make take-it-or-leave-it offers.

2 Equilibrium with complete information

In this section, we choose the intrinsic values of the low and high quality coins so that the unit upper bound and indivisibility constraints are not binding when there is no information problem. Hence, the first-best allocation can be implemented when information is complete, even though agents can carry at most one coin.

We first describe the determination of the terms of trade in bilateral meetings. We assume that agents in a match have access to a randomization device that allows them to bargain over lotteries. Lotteries open up signaling possibilities for the buyer.[4] Since goods are divisible, agents only randomize over the transfer of the coin. Denote (q, p) as the terms of trade, where $q \in \mathbb{R}_+$ is the quantity of the special good produced by the seller and consumed by the buyer, and $p \in [0, 1]$ is the probability that the buyer gives his coin to the seller. Suppose that the buyer holds a coin of quality z. Since buyers make take-it-or-leave-it offers, the terms of trade, (q, p), are given by the solution to the following problem,

$$\max_{q, p \leq 1} \varepsilon u(q) + (1 - p)\, z \quad \text{s.t.} \quad -\psi(q) + pz \geq 0. \tag{2.1}$$

[4] We allow lotteries because they lead to a Pareto improvement in bilateral matches when coins are indivisible. We will also see that lotteries play an important role in the presence of incomplete information. Lotteries in search models of money were introduced by Berentsen, Molico, and Wright (2002).

The buyer chooses (q,p) in order to maximize his expected surplus subject to the seller's participation constraint. The expected surplus of the buyer is the utility of consuming the special good, $\varepsilon u(q)$ plus the utility of consuming the general good embodied in the coin, z, times the probability that the coin is not delivered to the seller.[5] The seller's participation constraint has a similar interpretation. The solution to (2.1) is

$$q(\varepsilon, z) = \begin{cases} q_\varepsilon^* & \text{if } \psi(q_\varepsilon^*) \leq z \\ \psi^{-1}(z) & \text{otherwise} \end{cases}, \tag{2.2}$$

$$p(\varepsilon, z) = \begin{cases} \psi(q_\varepsilon^*)/z & \text{if } \psi(q_\varepsilon^*) \leq z \\ 1 & \text{otherwise} \end{cases}. \tag{2.3}$$

If the buyer's coin has insufficient value to purchase the efficient level of output, i.e., if $z < z_\varepsilon^* \equiv \psi(q_\varepsilon^*)$, then he gives his coin to the seller with probability one in exchange for as much output as his coin can purchase, which is equal to $\psi^{-1}(z)$; if, on the other hand, the buyer's coin has a value that exceeds the efficient level of output, i.e., $z \geq z_\varepsilon^* \equiv \psi(q_\varepsilon^*)$, then he will give the coin to the seller with probability $\psi(q_\varepsilon^*)/z$ in exchange for q_ε^* units of output.

From (1.1), (2.2) and (2.3), the expected utility of a buyer in period 1 satisfies

$$\max_{z \in \{0, z_\ell, z_h\}} -z + \beta \left\{ \varepsilon u \left[q(\varepsilon, z) \right] + \left[1 - p(\varepsilon, z) \right] z \right\}. \tag{2.4}$$

According to (2.4), a buyer chooses which coin to mint, if any. The disutility of minting a coin of quality z is z, where $z \in \{0, z_\ell, z_h\}$. In period 2, the buyer consumes q units of special goods, where q depends on the quality z of his coin and the buyer hands over his coin to the seller with probability p, where again p depends on z. Using the constraint in (2.1) with an equality, (2.4) can be simplified as

$$\max_{z \in \{0, z_\ell, z_h\}} \left\{ -rz + \varepsilon u[q(\varepsilon, z)] - \psi[q(\varepsilon, z)] \right\}. \tag{2.5}$$

The buyer chooses a coin that maximizes his surplus in a bilateral match, $\varepsilon u - \psi$, net of the (opportunity) cost of holding a coin, rz. It is straightforward to show that the sellers do not mint coins in period 1; hence, $z = 0$ for sellers.

[5] Note that $\max_{q, p \leq 1} \varepsilon u(q) + (1-p)\, z = \max_{q, p \leq 1} \varepsilon u(q) - pz$.

We are now in a position to define and characterize the equilibrium of the economy under complete information.

Definition 1 *An equilibrium is a list* $\left\{(z^i)_{i\in\mathcal{B}_\ell},(z^j)_{j\in\mathcal{B}_h}\right\}$ *such that* $z^i\in\{0,z_\ell,z_h\}$ *is solution to (2.5) with* $\varepsilon=\varepsilon_L$ *for all* $i\in\mathcal{B}_L$*, and* $z^j\in\{0,z_\ell,z_h\}$ *is solution to (2.5) with* $\varepsilon=\varepsilon_H$ *for all* $j\in\mathcal{B}_H$*.*

Proposition 1 *An equilibrium exists and it is generically unique.*

Let $\tilde{z}_\varepsilon$ denote an ε-buyer's optimal quality of coin, or his *ideal coin*, assuming there is no restriction on the intrinsic value of the coin that can be minted. It satisfies

$$\frac{\varepsilon u'[\psi^{-1}(\tilde{z}_\varepsilon)]}{\psi'[\psi^{-1}(\tilde{z}_\varepsilon)]}=1+r. \tag{2.6}$$

The quality of the ideal coin decreases with the rate of time preference, r, which is the opportunity cost of holding a coin.[6]

We will assume throughout the remainder of the paper that the minting technology is such that the quality of the coins is the ideal quality for each type of buyer. In the absence of asymmetries of information, the equilibrium outcome is characterized by all low (marginal-utility) buyers minting low quality coins and all high (marginal-utility) buyers minting high quality coins at date 1; sellers do not mint any coins. At date 2, high buyers make the take-it-or-leave it offer $\left(\psi^{-1}\left(z_h\right),1\right)$ to the seller, which the seller accepts; low buyers make the offer $\left(\psi^{-1}\left(z_\ell\right),1\right)$, which the seller accepts. At the end of period 2, the seller consumes either z_h or z_ℓ, depending upon whether he produced for the high or low buyer. For convenience, we will denote $\psi^{-1}\left(z_h\right)\equiv q_h$ and $\psi^{-1}\left(z_\ell\right)\equiv q_\ell$.

We will also make the following assumption on primitives:

Assumption 1. $\varepsilon_L/\tilde{z}_\ell>\varepsilon_H/\tilde{z}_h$.

The condition $\varepsilon_L/\tilde{z}_\ell>\varepsilon_H/\tilde{z}_h$ is satisfied for standard specifications for utility and cost functions (e.g., $u(q)=q^a$, with $a\in(0,1)$ and $\psi(q)=q$). This condition also gives rise to the

[6] Note that the equation for $\tilde{z}$ is the same as the equation for the choice of real balances in the Lagos–Wright model.

possibility of signaling when lotteries are introduced. If assumption 1 does not hold, then only pooling equilibria exist and signaling does not occur. (We discuss the pooling outcomes, which are qualitatively similar to those in VWW, in Dutu, Nosal and Rocheteau (2005)).

3 Equilibrium with asymmetric information

We now consider how equilibrium outcomes are affected when information about the quality of coins is imperfect. We capture the notion of imperfect information by borrowing the information structure used in VWW and Williamson and Wright (1994). In any match, the seller receives a common-knowledge signal regarding the quality of the coin held by the buyer. With probability $\theta \in (0,1)$, the signal is informative and the quality of the coin is revealed to the seller; with probability $1-\theta$, the signal is uninformative. The parameter θ captures the extent of the informational asymmetries. We also assume that the buyer's preference parameter, ε, is private information.

In matches where the seller is uninformed, the take-it-or-leave-it bargaining game has the structure of a signaling game. The buyer makes an offer $(q,p) \in \mathbb{R}_+ \times [0,1]$, and the seller uses this offer to update his prior belief about the quality of the coin held by the buyer.[7] Let $\lambda(q,p) \in [0,1]$ represent the updated belief of a seller that the coin held by the buyer is of high quality conditional on the offer (q,p). If (q,p) corresponds to an equilibrium offer, then $\lambda(q,p)$ is derived from the seller's prior belief, according to Bayes' rule. If (q,p) is an out-of-equilibrium offer, then Bayes' rule cannot be applied and the seller's belief is arbitrary.

In an *uninformed* match, the buyer who holds a coin of quality z makes the offer (q,p) that solves the problem

$$\max_{q\geq 0, p\in[0,1]} \varepsilon u(q) - pz \tag{3.1}$$

$$\text{s.t.} \quad -\psi(q) + p\left\{\lambda(q,p)z_h + [1-\lambda(q,p)]z_\ell\right\} \geq 0. \tag{3.2}$$

The buyer chooses an offer (q,p) that maximizes his surplus from the trade (3.1), subject to the

[7] In contrast to a standard signaling game, the type of the buyer in our bargaining game is (ε, z), which is endogenous because buyers choose the weight of the coin they hold.

seller's participation constraint (3.2). From (3.2), the buyer takes into account that his offer will affect the seller's belief regarding the quality of his coin. We will restrict our attention to equilibria such that whenever (3.2) holds with equality, the buyer's offer will be accepted with probability one.

Let $[q^u(\varepsilon,z), p^u(\varepsilon,z)]$ denote the offer made by an ε–type buyer holding a coin of quality z in an uninformed match. The buyer's choice of a coin, which modifies (2.5) in the obvious way, is now given by

$$\max_{z\in\{0,z_\ell,z_h\}} \left\{-rz+\theta\left\{\varepsilon u[q(\varepsilon,z)]-\psi[q(\varepsilon,z)]\right\}+(1-\theta)\left[\varepsilon u\left[q^u(\varepsilon,z)\right]-p^u(\varepsilon,z)z\right]\right\}, \tag{3.3}$$

where $\varepsilon=\varepsilon_H$ for high buyers and $\varepsilon=\varepsilon_L$ for low buyers. We will restrict our attention to equilibria where all buyers of a given type (ε,z) make the same offer. Because information may be imperfect in the bargaining games, the previous definition of an equilibrium must be modified.

Definition 2 *An equilibrium is a list* $\{(z^i)_{i\in\mathcal{B}_L},\ (z^j)_{j\in\mathcal{B}_H},\ (q(\varepsilon,z),p(\varepsilon,z),q^u(\varepsilon,z,\ p^u(\varepsilon,z);$ $(\varepsilon,z)\in\{\varepsilon_H,\varepsilon_L\}\times\{z_h,z_\ell\}),\ \lambda(q,p)\}$ *such that:*

1. z^i *is solution to (3.3) with* $\varepsilon=\varepsilon_L$ *for all* $i\in\mathcal{B}_L$*, and* z^j *is solution to (3.3) with* $\varepsilon=\varepsilon_H$ *for all* $j\in\mathcal{B}_H$*.*

2. $[q(\varepsilon,z),p(\varepsilon,z)]$ *is given by (2.2)–(2.3) for all* $(\varepsilon,z)\in\{\varepsilon_H,\varepsilon_L\}\times\{z_h,z_\ell\}$*.*

3. $[q^u(\varepsilon,z),p^u(\varepsilon,z)]$ *is solution to (3.1)–(3.2) for all* $(\varepsilon,z)\in\{\varepsilon_H,\varepsilon_L\}\times\{z_h,z_\ell\}$*.*

4. *The belief system* $\lambda(q,p)$ *is deduced from Bayes' rule whenever possible.*

A crucial element of the above definition is the belief system $\lambda(q,p)$. Below, we will put more structure on these beliefs by adopting a particular refinement. Before turning to the refinement, we can establish that the introduction of imperfect information does not affect the strategy of a low buyer. In particular,

Lemma 1 *In any equilibrium, a low buyer always mints a low quality coin.*

Intuitively, if imperfect information causes the value of a coin to deviate from its intrinsic value, then the low quality coin will tend to be overvalued and the high quality coin will tend to be undervalued. As well, the low quality coin has the ideal intrinsic value for low buyers in informed matches. These observations imply that a low buyer will never have an incentive to mint a high quality coin. Since in all equilibria the low buyer mints a low quality coin, a characterization of an equilibrium requires, among other things, that we determine whether a high buyer mints a high or low quality coin.

When bargaining with a seller, a buyer can attempt to signal the quality of his coin. But signaling raises the thorny issue of how a seller should interpret an offer that is not supposed to occur in equilibrium. In this regard, we restrict sellers' out-of-equilibrium beliefs to be consistent with the Cho–Kreps equilibrium refinement. The intuition behind this refinement is as follows: Suppose that a seller receives the out-of-equilibrium offer $(\hat{q}, \hat{p})$. If offer $(\hat{q}, \hat{p})$ reduces the utility of an ε buyer who holds a coin of quality z compared to his equilibrium payoff, then, according to the Cho–Kreps criterion, the seller should assign a probability equal to zero that this offer came from an ε buyer holding a coin of quality z.

We first demonstrate that assumption 1 is necessary for a separating equilibrium to exist. We then go on to explain the terms of trade associated with a separating equilibrium and, finally, the equilibrium minting strategies for buyers that support a separating equilibrium.

Proposition 2 *Consider an equilibrium where high buyers hold heavy coins. Under assumption 1, the outcome in uninformed matches is separating.*

The condition $\varepsilon_L/z_\ell > \varepsilon_H/z_h$ indicates that in order to obtain the same increase in consumption, a low buyer with a low quality coin is willing to give up his coin with a higher probability than a high buyer with a high quality coin. When this condition is satisfied, it is *not* possible to have a pooling equilibrium. If a pooling allocation is proposed as an equilibrium, then a high buyer with a high quality coin could instead offer to trade his coin for a lower quantity of output and with a lower probability, in a way that makes him better off compared to the proposed equilibrium and, at the same time, makes the low buyer with the low quality coin worse off. This point is illustrated in figure 3.1. We denote $U^b_{L\ell} = \varepsilon_L u(q) - pz_\ell$ as the surplus that a low buyer holding a low quality coin receives if the terms of trade are (q, p), and $U^b_{Hh} = \varepsilon_H u(q) - pz_h$ as the surplus that a high buyer holding a high quality coin receives. Consider a proposed pooling equilibrium where all buyers offer (q^u, p^u). Suppose that a high buyer defects from a proposed equilibrium play and instead offers $(\hat{q}^u, \hat{p}^u)$, which lies to the right of U^b_{Hh} and to the left of $U^b_{L\ell}$ in figure 2. According to the Cho–Kreps refinement, the seller should interpret this out-of-equilibrium offer as coming from a high buyer with a high quality coin. The seller will accept this offer because it provides him with a positive surplus, i.e., allocation $(\hat{q}^u, \hat{p}^u)$ lies above his reservation indifference curve U^s_h, defined by $-\psi(q) + pz_h = 0$. Therefore, if the two coins coexist, they will be traded at different terms of trade in uninformed matches.

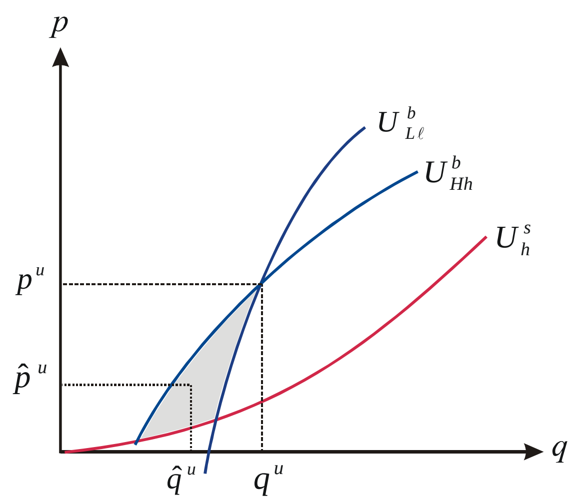

Figure 3.1: Ruling out pooling equilibria ($\varepsilon_L/z_\ell > \varepsilon_H/z_h$).

We now turn to the determination of the terms of trade. We will first characterize the set of equilibrium offers where the high buyer mints a high quality coin at date 1. In informed meetings, a buyer holding the high quality coin will make the offer $(q_h, 1)$, and a buyer holding the low quality coin will make the offer $(q_\ell, 1)$. The lemmas below describe what happens in uninformed meetings.

Lemma 2 *In any equilibrium where high-type buyers hold high quality coins, the low buyer will mint a low quality coin and will make the full information offer,*

$$[q^u(\varepsilon_L, z_\ell), p^u(\varepsilon_L, z_\ell)] = (q_\ell, 1). \tag{3.4}$$

The high buyer's offer maximizes his surplus

$$[q^u(\varepsilon_H, z_h), p^u(\varepsilon_H, z_h)] = \arg\max_{q, p \leq 1} \varepsilon_H u(q) - p z_h \tag{3.5}$$

subject to the seller accepting and the low buyer not mimicking the offer, i.e.,

$$-\psi(q) + p z_h \geq 0 \tag{3.6}$$

$$\varepsilon_L u(q_\ell) - z_\ell \geq \varepsilon_L u(q) - p z_\ell, \tag{3.7}$$

respectively. Furthermore, if buyers defect from their equilibrium minting strategies, their date 2 offers are

$$\begin{aligned} [q^u(\varepsilon_H, z_\ell), p^u(\varepsilon_H, z_\ell)] &= (q_\ell, 1), \\ [q^u(\varepsilon_L, z_h), p^u(\varepsilon_L, z_h)] &= [q^u(\varepsilon_H, z_h), p^u(\varepsilon_H, z_h)]. \end{aligned}$$

The equilibrium offer by a low buyer holding a low quality coin, $(q_\ell, 1)$, is represented in figure 3.2 at the intersection of the participation constraint of the seller who believes that the buyer is holding a low quality coin, $U_\ell^s = 0$, and $p = 1$. The indifference curve of a low buyer holding a low quality coin that goes through this point, $U_{L\ell}^b = \varepsilon_L u(q_\ell) - z_\ell$, represents the equilibrium surplus of the low buyer. Given that the equilibrium is separating—i.e., by their offers, buyers essentially reveal their type and the coin that they are holding—the low buyer holding the low quality coin can do no better than he could in an informed match.

In contrast, signaling is costly for the high buyer holding a high quality coin, and his payoff is lower than what it would be in an informed match. The best offer that a high buyer holding a high quality coin can propose, (3.5), must (i) satisfy the participation constraint of seller who believes that the buyer is holding a high quality coin, (3.6), and (ii) not be imitated by the low buyer holding a low quality coin, (3.7). The seller's participation constraint with an equality is depicted by U_h^s in figure 3.2, and the low buyer's incentive-compatibility constraint with an equality is given by $U_{L\ell}^b$. It can be seen from figure 3.2 that the solution to (3.5)–(3.7) is at the intersection of the seller's participation constraint, U_h^s, and the low buyer's incentive-compatibility constraint, $U_{L\ell}^b$. Notice that $q^u(\varepsilon_H, z_h) < q^u(\varepsilon_L, z_\ell) = q_\ell$ and $p^u(\varepsilon_H, z_h) < p^u(\varepsilon_L, z_\ell) = 1$. In order to signal the quality of his coin, the high buyer proposes an offer with lower output and a lower probability to deliver his coin, compared to the low buyer's offer. It can also be noticed from the incentive-compatibility condition (3.7) that the probability according to which a high quality coin changes hands decreases with the intrinsic value of the low coin, z_ℓ.

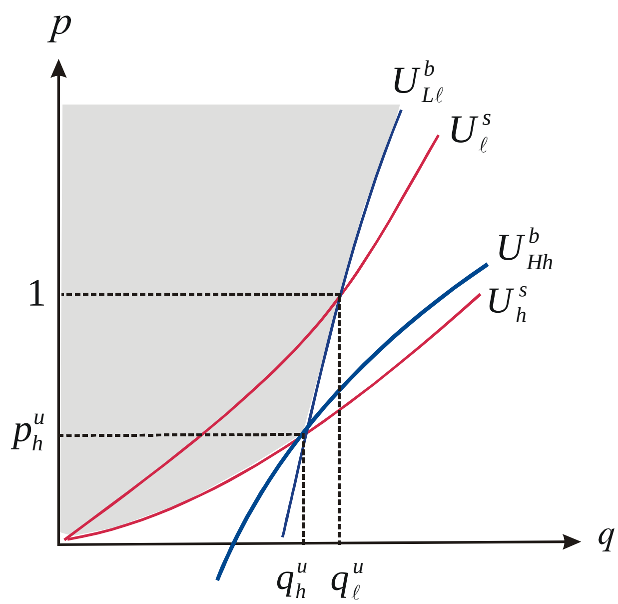

Figure 3.2: Separating offer.

Let's now turn to the buyers' date 1 minting strategies. Will a high buyer have an incentive to mint the heavy coin? From (3.3), the high buyer will mint a heavy coin if

$$-rz_h + \theta\Delta_{Hh} + (1-\theta)\,\Delta_{Hh}^u \geq -rz_\ell + \Delta_{H\ell}, \tag{3.8}$$

where $\Delta_{ji} = \varepsilon_j u[q(\varepsilon_j, z_i)] - p(\varepsilon_j, z_i)z_i$ is the surplus of a buyer of type (j, i) in an informed match with $i \in \{\ell, h\}$ and $j \in \{L, H\}$. Similarly, Δ_{ji}^u is the surplus of a buyer of type (j, i) in an uninformed match, $\Delta_{ji}^u = \varepsilon_j u[q^u(\varepsilon_j, z_i)] - p^u(\varepsilon_j, z_i) z_i$. Condition (3.8) can be re-expressed as

$$\theta \geq \theta_c \equiv \frac{r\,(z_h - z_\ell) + \Delta_{H\ell} - \Delta_{Hh}^u}{\Delta_{Hh} - \Delta_{Hh}^u}. \tag{3.9}$$

It can be demonstrated that $0 < \theta_c < 1$.[8] As long as the information problem is not too severe, high buyers will mint high quality coins. The benefits from trading with high quality coins in informed matches outweigh the costs associated with signaling for high buyers in uninformed matches.

Now let's turn to single-currency equilibria in which high buyers mint low quality coins. In that case, the following lemma describes the equilibrium offers that buyers make:

Lemma 3 *In any equilibrium where high buyers mint low quality coins,*

$$[q^u(\varepsilon_H, z_\ell), p^u(\varepsilon_H, z_\ell)] = [q^u(\varepsilon_L, z_\ell), p^u(\varepsilon_L, z_\ell)] = (q_\ell, 1)\,.$$

Furthermore, if buyers defect from equilibrium play and mint high quality coins, their (out-of-equilibrium) offers satisfy $[q^u(\varepsilon_H, z_h), p^u(\varepsilon_H, z_h)] = [q^u(\varepsilon_L, z_h), p^u(\varepsilon_L, z_h)]$, *where* $[q^u(\varepsilon_H, z_h), p^u(\varepsilon_H, z_h)]$ *is given by the solution to (3.5)–(3.7).*

Intuitively, the above equilibrium offer extracts all of the surplus from the seller, given that the buyer—who is either high or low—is holding a low quality coin. A single-currency equilibrium will exist only if the high-type buyer has an incentive to hold the low quality coin, and this will happen only when θ is smaller than the threshold θ_c. One can also show that there is no single-currency equilibrium for $\theta > \theta_c$.[9] The following proposition summarizes the above discussion.

[8] From (3.7), it can be checked that

$$\Delta_{Hh}^u = \Delta_{H\ell} - \left(\frac{\varepsilon_H}{\varepsilon_L} - 1\right) z_\ell - p^u(\varepsilon_H, z_h)\left(z_h - \frac{\varepsilon_H}{\varepsilon_L} z_\ell\right),$$

i.e., the surplus of a high-type buyer with a heavy coin in an uninformed meeting is lower than that of a high-type buyer with a light coin. This guarantees that $\theta_c > 0$.

[9] See our working paper for details, Dutu, Nosal and Rocheteau (2005).

Proposition 3 *There exists a threshold θ_c such that: (i) If $\theta > \theta_c$, then there can only exist a separating equilibrium, in which both high and low quality coins circulate; (ii) If $\theta \leq \theta_c$, then there can only exist a single currency (pooling) equilibrium where all buyers mint light coins.*

Proposition 3 shows that even though high buyers can separate themselves from low buyers, there is a threshold for θ below which high quality coins are driven out of circulation. The reason a high buyer would choose to mint a low quality coin is that when the information problem is severe, the holder of a high quality coin incurs a large signaling cost by reducing his average consumption in the second period. It is better for the high buyer to avoid these signaling costs by minting and holding a low quality coin.

When assumption 1 does not hold, i.e., when $\varepsilon_L/z_\ell < \varepsilon_H/z_h$, it is not possible to have a separating equilibrium. Because a pooling equilibrium is qualitatively equivalent to an equilibrium where lotteries are not allowed, which have already been described in various papers (e.g., VWW and Burdett, Trejos, and Wright (2001)), we do not study this case here.[10]

4 Recognizability and welfare

We now consider various positive and normative aspects associated with the recognizability of coins. We examine how recognizability affects output, welfare and the velocity of currency. We define the velocity of coin $i \in \{\ell, h\}$, v_i, as the average probability that the coin changes hands in a bilateral match, i.e.,

$$v_i = \theta p_i + (1-\theta) p_i^u, \tag{4.1}$$

where p_i and p_i^u are the probabilities that the coin of quality z_i changes hands in informed and uninformed matches, respectively. We measure aggregate output, Y, as the sum of the quantities traded in bilateral matches, i.e.,

$$Y = \int \left[\theta q(\varepsilon, z) + (1-\theta) q^u(\varepsilon, z)\right] dF(\varepsilon, z), \tag{4.2}$$

[10] This case is examined in our working paper, Dutu, Nosal and Rocheteau (2005).

where $F(\varepsilon, z)$ is the distribution of buyers' types $(\varepsilon, z) \in \{\varepsilon_L, \varepsilon_H\} \times \{z_\ell, z_h\}$. Finally, social welfare, W, is the sum of the utilities of all agents in the economy, i.e.,

$$W = \int \mathcal{U}^b(\varepsilon, z) dF(\varepsilon, z) + \mathcal{U}^s, \tag{4.3}$$

where $\mathcal{U}^s$ is the expected utility of a seller and $\mathcal{U}^b(\varepsilon, z)$ is the expected utility of an ε buyer who mints a coin of quality z, and $\mathcal{U}^b(\varepsilon, z)$, satisfies

$$\beta^{-1}\mathcal{U}^b(\varepsilon, z) = -rz + \theta \left[\varepsilon u \left[q(\varepsilon, z)\right] - p(\varepsilon, z)z\right] + (1-\theta)\left[\varepsilon u \left[q^u(\varepsilon, z)\right] - p^u(\varepsilon, z)z\right].$$

Note that $\mathcal{U}^s = 0$ for all the equilibria we have considered.

We first describe the effects of a change in recognizability on the dual- and single-currency equilibria previously studied. We then investigate the effects of a change in recognizability that triggers a transition from a dual-currency equilibrium to a single-currency equilibrium on output, welfare, and velocity. Finally, we discuss how our model could be interpreted as a model of counterfeiting.

Consider first a dual-currency equilibrium. In such an equilibrium, low quality coins are traded with probability one in all matches, i.e., $v_\ell = 1$, and a low quality coin buys q_ℓ units of output. In contrast, from (3.6) and (3.7), high quality coins are traded with probability less than one in uninformed matches, $p^u(\varepsilon_H, z_h) < 1$, and the velocity of heavy coins is given by

$$v_h = \theta + (1-\theta)p^u(\varepsilon_H, z_h). \tag{4.4}$$

From (3.6) and (3.7), the terms of trade are determined by the incentive-compatibility condition for low buyers and the individual-rationality condition for sellers. From this, $p^u(\varepsilon_H, z_h)$ is independent of the fraction of informed matches, θ. Therefore, the velocity of money increases with the level of recognizability, θ, because high quality coins have a higher velocity in informed matches.

The higher velocity associated with greater recognizability translates into higher aggregate output and higher welfare. To see this, note from (4.2) that aggregate output is

$$Y = \lambda\left[\theta q_h + (1-\theta)q^u(\varepsilon_H, z_h)\right] + (1-\lambda)q_\ell. \tag{4.5}$$

According to (4.5), high buyers, who represent a fraction λ of all buyers, consume q_h in informed matches and $q^u(\varepsilon_H, z_h)$ in uninformed matches; low buyers consume q_ℓ in all matches. Because $q^u(\varepsilon_H, z_h) < q_h$, aggregate output will increase as coins become more recognizable, i.e., as θ increases.

From (4.3), society's welfare is given by

$$W = \lambda \{-z_h + \beta\theta\varepsilon_H u(q_h) + \beta(1-\theta)[\varepsilon_H u[q^u(\varepsilon_H, z_h)] + [1 - p^u(\varepsilon_H, z_h)] z_h]\} \\ +(1-\lambda)[-z_\ell + \beta\varepsilon_L u(q_\ell)]. \tag{4.6}$$

Equation (4.6) has the following interpretation: A high buyer produces z_h units of output in the first period in order to mint a high quality coin. In the second period, he consumes q_h and trades his coin with probability one in informed matches; he consumes $q^u(\varepsilon_H, z_h)$ and trades his coin with probability $p^u(\varepsilon_H, z_h)$ in uninformed matches. A low buyer produces z_ℓ units of output in the first period in order to mint a low quality coin and always consumes q_ℓ in the second period. Whereas the expected utility of a low buyer is independent of θ, the expected utility of a high buyer increases with θ because $\varepsilon_H u(q_h) - z_h > \varepsilon_H u[q^u(\varepsilon_H, z_h)] - p^u(\varepsilon_H, z_h)z_h$. Hence, social welfare increases with the recognizability of coins because, in order to separate themselves from buyers holding low quality coins, buyers with high quality coins trade with a lower probability and buy less output in uninformed matches. Consequently, as the recognizability of coins improves, both output and welfare increase.

These results are summarized in

Proposition 4 *Consider an equilibrium with dual currency circulation. Output, welfare, and the velocity of high quality coins all increase with θ.*

We know from propositions 3 that a reduction in the recognizability of coins can trigger a change in the high buyer's minting strategy. Specifically, if $\theta < \theta_c$ then all buyers mint light coins, and heavy coins are driven out circulation. We now want to assess the welfare consequences of a transition from a dual-currency to a single-currency equilibrium.

Low buyers trade z_ℓ for q_ℓ in both the single- and dual-currency equilibria. Hence, the welfare of low buyers does not depend upon θ. The welfare of high buyers is, however, minimized when $\theta < \theta_c$. To see this, recall that high buyers always have the option of minting low quality coins in the first period of their lives. If they choose to mint high quality coins, as they do when $\theta > \theta_c$, their welfare must be higher than what they would obtain by minting low quality coins.

Proposition 5 *A decrease in θ that triggers a transition from a dual-currency equilibrium to a single-currency equilibrium is welfare-worsening in a Pareto sense.*

A decrease in coins' recognizability reduces welfare when it drives high quality coins out of circulation. High quality coins, which are useful to high buyers, may no longer be used if the asymmetries of information are sufficiently severe.[11] Propositions 4 and 5 hint at two aspects of Gresham's Law: (i) good coins are traded less often (proposition 4), and (ii) at the minting stage, buyers switch to minting the lower intrinsic value coin when recognizability problems become severe, (proposition 5).

4.1 Counterfeiting

Propositions 4 and 5 have shown that coins' imperfect recognizability imposes welfare costs on society. One can make this point in a rather dramatic way by focusing on a limiting case, where the intrinsic value of the low quality coin approaches zero. One can interpret the situation where $z_\ell \to 0$ as one of counterfeiting, where the high quality coin is the "genuine currency" and the low quality coin is the counterfeit. The assumption that the intrinsic value of the low quality coin is almost zero captures the idea that the marginal cost of producing counterfeit currency is close to zero.[12]

The following proposition characterizes the terms of trade in uninformed matches:

[11] It should be noted that the above proposition does not imply that a dual coin arrangement is necessarily better than an arrangement with a single coin. Indeed, the experiment that we have considered consists in taking the denomination structure $\{z_\ell, z_h\}$ that is ideal in the absence of an information problem and seeing how, *given this denomination structure*, the recognizability of coins affects welfare. Designing an *optimal* denomination structure in the presence of asymmetric information is left for future investigation.

[12] For a models of counterfeiting with fiat currencies, see Kulti (1996), Green and Weber (1996), and Nosal and Wallace (2006).

Proposition 6 *As $z_\ell \to 0$, both $q^u(\varepsilon_H, z_h)$ and $q^u(\varepsilon_H, z_\ell)$ approach 0.*

According to Proposition 6, the quantities produced in uninformed matches tend to zero as the intrinsic value of low quality coins tends to zero. In other words, if the low quality coin is almost costless to produce, trade in uninformed meetings shuts down. The intuition for this result is simple. The buyer with a genuine coin who wishes to separate himself from a buyer with a counterfeit coin looks for an offer that the buyer with a counterfeit coin does not want to imitate. However, in a separating equilibrium, the utility of a buyer with a counterfeit coin is almost zero. Therefore, the only offer that a buyer with genuine coin can make is such that q^u is close to 0.

Proposition 6 has dramatic implications for the way the economy works. If $\theta > \theta_c$, then buyers with high marginal utility of consumption mint high quality coins and trade only if they are in informed meetings. As θ decreases, the number of meetings in which trades take place falls. As θ falls below the threshold θ_c, high buyers have no incentive to mint high quality coins because the probability that they can use them in a bilateral match is too small. As a consequence, the entire economy shuts down.

5 Conclusion

In this paper we have studied the effects of the imperfect recognizability of coins on output, welfare and the velocity of money. We have developed a simple model in which heterogenous buyers can trade with two different coins, a low intrinsic value coin and a high intrinsic value one. The terms of trade in bilateral matches are determined by take-it-or-leave-it offers by buyers, and we have allowed the use of lotteries to overcome the indivisibility of coins. We have characterized the different types of equilibria that can emerge in the presence of asymmetric information. If is it very difficult to distinguish between low and high quality coins, then the equilibrium will be characterized by a single coin, the low quality one. This outcome has a Gresham's-law flavor to it. If the recognizability problem is not too severe, then both high and low quality coins will circulate. Under a simple condition on fundamentals, the equilibrium is separating in the sense that low and high quality coins are always traded according to different terms. In such equilibria, velocity, output, and welfare increase with the recognizability of coins. The separating equilibrium also has a Gresham's-law flavor to it in that the velocity of the high quality coin is less than that of the low quality coin. Furthermore, as the intrinsic value of the low quality coin tends to zero, so does the quantity traded in uninformed matches. Therefore, economic activity shuts down when agents can counterfeit good coins at a negligible cost.

References

[1] Berentsen, Aleksander, Miguel Molico, and Randall Wright (2002). "Indivisibilities, Lotteries and Monetary Exchange," Journal of Economic Theory 107, 70–94.

[2] Burdett, Kenneth, Alberto Trejos, and Randall Wright (2001). "Cigarette Money," Journal of Economic Theory 99, 117–142.

[3] Cho, In-Koo and David Kreps (1987). "Signaling Games and Stable Equilibria," Quarterly Journal of Economics 102, 179–221.

[4] Cuadras-Morató, Xavier (1994). "Commodity Money in the Presence of Goods of Heterogenous Quality," Economic Theory 4, 579–591.

[5] Dutu, Richard, Ed Nosal and Guillaume Rocheteau (2005). "On the Recognizability of Money," Federal Reserve Bank of Cleveland Working Paper 05-12.

[6] Green, Ed and Weber, Warren (1996). "Will the New $100 Bill Decrease Counterfeiting?" Federal Reserve Bank of Minneapolis Quarterly Review 20, 3–10.

[7] Haegler, Urs (1997). "Fiat Money and Quality Uncertainty," Economica 64, 547–565.

[8] Kulti, Klaus (1996). "A Monetary Economy with Counterfeiting," Journal of Economics 63, 175–186.

[9] Lagos, Ricardo and Randall Wright (2005). "A Unified Framework for Monetary Theory and Policy Analysis," Journal of Political Economy 113, 463–484.

[10] Li, Yiting (1995). "Commodity Money under Private Information", Journal of Monetary Economics 36, 573–592.

[11] Nosal, Ed and Neil Wallace (2006). "A Model of (the Threat) of Counterfeiting," Journal of Monetary Economics, In Press.

[12] Sargent, Thomas and Neil Wallace (1983). “A Model of Commodity Money,” Journal of Monetary Economics 12, 163–187.

[13] Velde, François, Warren Weber, and Randall Wright (1999). “A Model of Commodity Money, with Applications to Gresham’s Law and the Debasement Puzzle”, Review of Economic Dynamics 2, 291–323.

[14] Williamson, Steven and Randall Wright (1994). “Barter and Monetary Exchange under Private Information,” American Economic Review 84, 104–123.

Appendix

Proof to Proposition 1

The function in (2.5) is maximized over a finite set. Therefore, a solution exists. Denote

$$\tilde{z}_{\varepsilon} = \max_{z \in \mathbb{R}_+} \left\{ -rz + \varepsilon u[q(\varepsilon, z)] - \psi[q(\varepsilon, z)] \right\}.$$

The function in (2.5) is strictly increasing for all $z \in (0, \tilde{z}_{\varepsilon})$ and is strictly decreasing for all $z > \tilde{z}_{\varepsilon}$. Therefore, the function in (2.5) cannot take the same value for more than two distinct values for z. So the solution to (2.5) is unique except for a set of parameter values of measure 0, in which case the problem in (2.5) admits two solutions.

Proof of Lemma 1

Step 1: From (3.2), a low buyer with a low quality coin weakly prefers to trade with uninformed sellers than with informed sellers. This means that

$$\varepsilon_L u \left[q^u(\varepsilon_L, z_\ell) \right] - p^u(\varepsilon_L, z_\ell) z_\ell \geq \varepsilon_L u \left(q_\ell \right) - z_\ell.$$

Consequently,

$$\begin{aligned} -rz_\ell + \theta \left\{ \varepsilon_L u \left(q_\ell \right) - z_\ell \right\} + (1-\theta) \left\{ \varepsilon_L u \left[q^u(\varepsilon_L, z_\ell) \right] - p^u(\varepsilon_L, z_\ell) z_\ell \right\} \geq \\ -rz_\ell + \varepsilon_L u \left(q_\ell \right) - z_\ell. \end{aligned} \tag{5.1}$$

Step 2: If information is complete, a low buyer prefers to hold a low quality coin than a heavy coin. This means that

$$-rz_\ell + \varepsilon_L u \left(q_\ell \right) - z_\ell > -rz_h + \varepsilon_L u \left(q_h \right) - z_h. \tag{5.2}$$

Step 3: From (3.2), a low buyer holding a heavy coin weakly prefers to trade with an informed seller than with an uninformed one. It implies

$$\begin{aligned} -rz_h + \varepsilon_L u \left(q_h \right) - z_h \geq \\ -rz_h + \theta \left\{ \varepsilon_L u \left(q_h \right) - z_h \right\} + (1-\theta) \left\{ \varepsilon_L u \left[q^u(\varepsilon_L, z_h) \right] - p^u(\varepsilon_L, z_h) z_h \right\}. \end{aligned} \tag{5.3}$$

From (5.1)–(5.3), we deduce that low-type buyers strictly prefer to hold low quality coins.

Proof of Proposition 2

We show that if $\varepsilon_L/z_\ell > \varepsilon_H/z_h$, then in any equilibrium where high buyers hold heavy coins, $[q^u(\varepsilon_H, z_h), p^u(\varepsilon_H, z_h)] \neq [q^u(\varepsilon_L, z_\ell), p^u(\varepsilon_L, z_\ell)]$. This implies that there exists no pooling equilibrium. The proof is diagrammatic. In figure A.1, we denote U^b_{ij} as the locus of points in (q, p)-space that generates the same surplus from the bargaining game for an ε_i buyer, where $i \in \{H, L\}$, holding a coin of quality z_j, where $j \in \{h, \ell\}$. The equation for this indifference curve is

$$U^b_{ij} = \varepsilon_i u(q) - pz_j, \qquad \forall (i, j) \in \{H, L\} \times \{h, \ell\}.$$

Similarly, U^s_j denotes the indifference curve for a seller who believes that the buyer he is matched with holds a coin of quality z_j, where $j \in \{h, \ell\}$. The equation for this indifference curve is

$$U^s_j = -\psi(q) + pz_j, \qquad \forall j \in \{h, \ell\}.$$

Consider an equilibrium where high buyers hold heavy coins and suppose that, contrary to the claim made above, $[q^u(\varepsilon_H, z_h), p^u(\varepsilon_H, z_h)] = [q^u(\varepsilon_L, z_\ell), p^u(\varepsilon_L, z_\ell)] = (q^u, p^u)$ in an uninformed meeting. We represent the equilibrium utility levels of a low buyer holding low quality coins by $U^b_{L\ell}$ and a high buyer holding heavy coin by U^b_{Hh}, when both buyers make the offer (q^u, p^u) in figure A.1. (We also depict the utility that a high buyer can expect to receive if he deviates from the proposed equilibrium of holding a heavy coin and, instead, chooses to hold a low quality coin and offers (q^u, p^u); this is indicated by the dotted indifference curve labeled $U^b_{H\ell}$.)

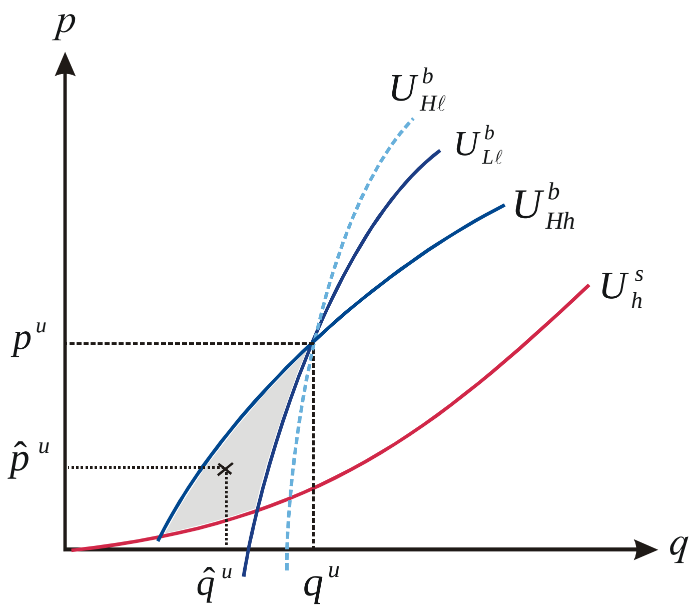

Figure A.1. No pooling equilibrium.

Because $\varepsilon_L/z_\ell > \varepsilon_H/z_h$, $U^b_{L\ell}$ is steeper than U^b_{Hh}. Consider now the out-of-equilibrium offer $(\hat{q}^u, \hat{p}^u)$ made by some buyer (see figure A.1). If such an offer were accepted, it would reduce the utility of *any* buyer holding a light coin compared to the utility associated with offer (q^u, p^u); i.e., offer $(\hat{q}^u, \hat{p}^u)$ is located to the left of the indifference curves $U^b_{L\ell}$ and $U^b_{H\ell}$. However, offer $(\hat{q}^u, \hat{p}^u)$ would increase the utility of a high buyer holding a heavy coin compared to the utility associated with the proposed equilibrium offer (q^u, p^u); i.e., offer $(\hat{q}^u, \hat{p}^u)$ is located to the right of the indifference curve U^b_{Hh}. Therefore, according to the Cho–Kreps criterion, the seller should believe that offer $(\hat{q}^u, \hat{p}^u)$ comes from a high buyer holding a heavy coin. Finally, the offer $(\hat{q}^u, \hat{p}^u)$ provides the seller with a payoff that is greater than zero. To see this, note first that (q^u, p^u) is an acceptable offer given the seller's initial belief, λ, that the high buyer is holding a heavy coin. Therefore, (q^u, p^u) is located above the zero payoff indifference curve of the seller who believes that the buyer is holding a heavy coin, denoted by U^s_h in figure A.1. The deviating offer $(\hat{q}^u, \hat{p}^u)$ is also chosen to be located above the indifference curve of the seller who believes that the buyer is holding a heavy coin, so that the seller will accept the offer. Hence, it is not possible to have an equilibrium in which the high buyer with a heavy coin and the low buyer with a light coin make the same offer.

Proof of Lemma 2

Consider first the equilibrium offer in an uninformed match of a low buyer holding a light coins, $[q^u(\varepsilon_L, z_\ell), p^u(\varepsilon_L, z_\ell)]$. Since from proposition 2 a buyer reveals his type through his offer, $\lambda\,[q^u(\varepsilon_L, z_\ell), p^u(\varepsilon_L, z_\ell)] = 0$. Therefore, a low buyer holding a light coin can do no better than making an offer that assumes that the seller can observe the coin he is holding, i.e., $[q^u(\varepsilon_L, z_\ell), p^u(\varepsilon_L, z_\ell)] = (q_\ell, 1)$.

Let us turn to the equilibrium offer in an uninformed match of a high buyer holding a heavy coin, $[q^u(\varepsilon_H, z_h), p^u(\varepsilon_H, z_h)]$. Since the equilibrium is separating, $\lambda[q^u(\varepsilon_H, z_h),\, p^u(\varepsilon_H, z_h)] = 1$. An offer cannot violate (3.6), otherwise, it would be rejected by a seller; nor can it violate (3.7); otherwise, low buyers would have an incentive to deviate from their equilibrium offer. If the equilibrium offer did not maximize the utility of a high buyer holding a heavy coin in (3.5) subject to (3.6) and (3.7), then one could construct a profitable deviation, as in the proof of proposition 2.

Consider next the offer of a high buyer who deviates in the first period by minting a light coin, $[q^u(\varepsilon_H, z_\ell), p^u(\varepsilon_H, z_\ell)]$. Any acceptable offer must satisfy (3.7); otherwise, the low buyer holding the light coin would have a profitable deviation. Since the indifference curve $U^b_{H\ell}$ is steeper than indifference curve $U^b_{L\ell}$, a high buyer holding the light coin can do no better than offering $[q^u(\varepsilon_L, z_\ell), 1]$, as in figure 3.2. Hence, a high buyer has no incentive to mint a light coin.

Finally, consider next the offer of a low buyer who deviates in the first period by minting a heavy coin, $[q^u(\varepsilon_L, z_h), p^u(\varepsilon_L, z_h)]$. Since the indifference curve U^b_{Hh} is steeper than indifference curve U^b_{Lh}, a low buyer holding a heavy coin cannot do better than offering $[q^u(\varepsilon_H, z_h), p^u(\varepsilon_H, z_h)]$ because otherwise the participation constraint of the seller would be violated, see figure 3.2. Hence, a low buyer has no incentive to mint a heavy coin.

Proof of Lemma 3

In equilibrium, all buyers hold light coins. Consequently, buyers cannot do better than the offer they would make in an informed match, namely, $(q_\ell, 1)$.

Consider next the offer of a high buyer who deviates and mints a heavy coin. This offer, $[q^u(\varepsilon_H, z_h), p^u(\varepsilon_H, z_h)]$, must satisfy (3.7), so that a low buyer with a light coin has no incentive to deviate from his equilibrium offer, and it must also satisfy the seller's participation constraint (3.6). In figure A.3, the offer $[q^u(\varepsilon_H, z_h), p^u(\varepsilon_H, z_h)]$, denoted (q_h^u, p_h^u), must be located on or to the left of $U_{L\ell}^b$. As well, high buyer with a light coin must not have an incentive to deviate from his equilibrium offer by proposing $[q^u(\varepsilon_H, z_h), p^u(\varepsilon_H, z_h)]$. In figure A.3, the offer must be located on or to the left of $U_{H\ell}^b$. Also, any such offer must satisfy the participation constraint of the seller under the belief that he faces a buyer with a heavy coin. In figure A.3, the offer must be located above the curve U_h^s.

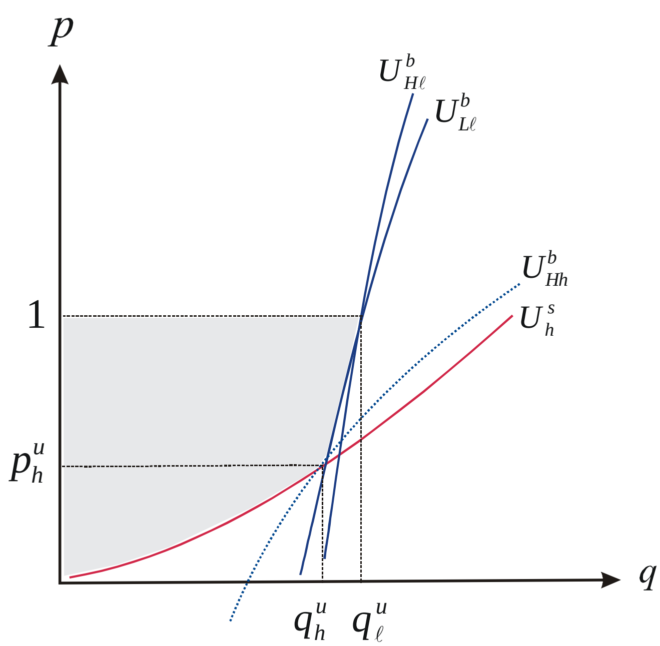

Figure A.3. Single currency equilibrium.

One can use the same reasoning as in lemma 2 to show that the offer $[q^u(\varepsilon_L, z_h), p^u(\varepsilon_L, z_h)]$ satisfies (3.6)–(3.7), and the best deviating offer is given by (q_h^u, p_h^u) in figure A.3. Hence, the high buyer will have no incentive to mint the heavy coin. The same reasoning applies to a low buyer who deviates and mints a heavy coin, i.e., the best deviating offer is given by (q_h^u, p_h^u).

Proof to Proposition 6

From (3.7), we have $q^u(\varepsilon_L, z_\ell) \geq q^u(\varepsilon_H, z_h)$. But $q^u(\varepsilon_L, z_\ell) = q_\ell \to 0$ as z_ℓ approaches 0.