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Money in Search Equilibrium, in Competitive Equilibrium, and in Competitive Search Equilibrium

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We compare three market structures for monetary economies: bargaining (search equilibrium); price taking (competitive equilibrium); and price posting (competitive search equilibrium). We also extend work on the microfoundations of money by allowing a general matching technology and entry. We study how equilibrium and the effects of policy depend on market structure. Under bargaining, trade and entry are both inefficient, and inflation implies first-order welfare losses. Under price taking, the Friedman rule solves the first inefficiency but not the second, and inflation may actually improve welfare. Under posting, the Friedman rule yields the first best, and inflation implies second-order welfare losses.

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1. INTRODUCTION

We compare three models of monetary exchange that differ in terms of their assumed market structures. In the first, at least some activity takes place in highly decentralized markets where anonymous buyers and sellers match randomly and bargain bilaterally over the terms of trade. Following the literature, we call this *search equilibrium*. In the second model there are still frictions, and in particular agents are still anonymous, but they meet in large markets where prices are taken as given. We call this *competitive equilibrium*. In the third model, we assume there are different submarkets with posted prices and buyers and sellers can direct their search across these submarkets, although within each submarket there are again frictions. We call this *competitive search equilibrium*. In each case, we provide results on existence and on uniqueness or multiplicity of equilibrium. We also analyze efficiency and optimal monetary policy.

The underlying framework is related to recent search-theoretic models of money following Lagos and Wright (2002) – hereafter referred to as LW. The key assumption in this framework is that, in addition to the activity that takes place in the more or less decentralized markets described above, agents also have periodic access to centralized competitive markets. The existence of the decentralized markets, and in particular the assumption that agents are anonymous, generates an essential role for money (Kocherlakota (1998), Wallace (2001)). The existence of the centralized markets greatly simplifies the analysis, because when combined with the assumption that preferences are quasi-linear, it implies that all agents of a given type will carry the same amount of money into the decentralized market. This renders the distribution of money holdings in this market simple, which makes the model very easy to analyze, compared to similar models with no centralized markets like Green and Zhou (1998), Molico (1999), Camera and Corbae (1999), or Zhou (1999).[2]

Intuitively, the simplification comes from the fact that quasi-linearity eliminates wealth effects on the demand for money, and hence eliminates dispersion in money holdings based on trading histories. While we believe that the wealth/distribution effects from which we are abstracting are interesting, as discussed in Levine (1991) and Molico (1999) e.g., the goal here is to focus on other effects that have not been analyzed previously.[3] To introduce these new effects we extend existing monetary models by adding a generalized matching technology and a free entry condition. These extensions can be thought of as being adapted from labor market models like those discussed in Pissarides (2000). Their role here is to allow us to discuss the effects of inflation on the extensive margin (the number of trades) as well as the intensive margin (the amount exchanged per trade), and to discuss "search externalities" – i.e. the dependence of the amount of trade on the composition of the market.

[2]Shi (1997) provides a different approach that also delivers simple distributions; see LW for a comparison. Note also that, as emphasized by Zhou (1999) and Kamiya and Shimizu (2003), a problem with some of the models mentioned above is that they have a continuum of stationary equilibria. This is not true in the LW model, even if we generalize preferences away from quasi-linearity, for the following reason. In the models in Zhou or Kamiya and Shimizu, there are equilibria where agents value money only in integer multiples of p; i.e. the value of having m dollars, $V(m)$, is a step function with jumps as at $p, 2p, 3p$... Given this, nothing actually pins down p. With periodic competitive markets as in LW, however, $V(m)$ must be strictly increasing, and such equilibria do not arise.

[3]Moreover, although our specification ignores wealth effects, recent work suggests the results are robust in the following sense. Khan, Thomas and Wright (2004) solve numerically a version of the model in LW with a more general class of utility functions, and show that as the wealth effects get small, both the distribution of money holdings and the welfare cost of inflation converge to the results derived analytically for quasi-linear specification.

Our main result is to show that the different market structures have very different implications for the nature of equilibrium and for the effects of policy. In search equilibrium (bargaining), we prove that the quantity traded and entry are both inefficient. In this model inflation implies a first-order welfare loss, and although the Friedman Rule is the optimal policy, it cannot correct the inefficiencies on the intensive and extensive margins. In competitive equilibrium (price taking), the Friedman Rule gives efficiency along the intensive margin but not the extensive margin. In this model the effects of policy are ambiguous, and inflation in excess of the Friedman Rule may be desirable. In competitive search equilibrium (posting), the Friedman Rule achieves the first best. In this model inflation reduces welfare but the effect is second order. We think these results are interesting because they help to sort out which results in recent monetary theory are due to features of the environment – preferences, information etc. – and which are due to the assumed market structure – bargaining, posting etc.

The three market structures have previously been used, of course, in different contexts. In the context of monetary economics, dating back to Shi (1995) and Trejos and Wright (1995) most search-based models use bargaining. Competitive pricing is used in, say, overlapping generations models by Wallace (1980) and turnpike models by Townsend (1980), but it has not been used in monetary models with search-type frictions. Price posting and directed search have been used in several monetary models (see Section 5 for references), but not in combination, and it is the combination that is critical for the concept of competitive search equilibrium. In terms of the literature on labor markets, one way to understand the three market structures is the following. Our bargaining model is monetary economics' analog to the Mortensen and Pissarides (1994) search model; our price-taking model is the analog to the Lucas and Prescott (1974) search model; and our price-posting model with directed search is the analog to Moen (1997) and Shimer (1996).

From a different perspective, consider Diamond (1984), who introduced a cash-in-advance constraint in the Diamond (1982) model because he wanted to discuss "Money in Search Equilibrium." Although his approach to bargaining was primitive at best, perhaps a bigger problem was that money is imposed exogenously via the cash-in-advance constraint. Later, Kiyotaki and Wright (1991) showed that in a very similar environment a role for money can be derived endogenously. Kocherlakota (1998) clarified exactly what makes money essential in those environments: a double coincidence problem, imperfect enforcement, and anonymity. It seems natural to look for a physical environment that incorporates these features, but also allows one to consider alternative market structures. Here, in addition to being able to discuss what Diamond wanted we can also analyze "Money in Competitive Equilibrium" and "Money in Competitive Search Equilibrium."

The rest of the paper is organized as follows. Section 2 presents the basic assumptions and discusses efficiency. Section 3-5 analyze equilibrium in the models with bargaining, price taking, and price posting. Section 6 concludes by summarizing the results.

2. THE BASIC MODEL

Time is discrete and continues forever. As in LW, each period is divided into two subperiods, called day and night, where economic activity will differ. During the day there will be a frictionless centralized market, while at night there will be explicit frictions and trade will be more or less decentralized, depending on which model (market structure) we consider. There is a continuum of agents divided into two types that differ in terms of when they produce and consume. We find it convenient to call them *buyers* and *sellers*. The sets of buyers and sellers are denoted $\mathcal{B}$ and $\mathcal{S}$, respectively. The difference is the following: while all agents produce and consume during the day, at night buyers want to consume but cannot produce while sellers are able to produce but do not want to consume. This generates a temporal double coincidence problem at night. Combined with the assumption that agents are anonymous, which precludes credit in the decentralized night market, this generates an essential role for money.[4]

It is well know by now that several different models can generate a similar role for money, including a variety of specifications with many goods and specialization in tastes and technologies (e.g. Kiyotaki and Wright (1989, 1993)). We choose to work with a single consumption good and a temporal double coincidence problem for the following reason. In the typical search-based model, any agent engaged in decentralized trade may end up either buying or selling, depending on who they meet, while here sellers can only sell and buyers can only buy in the night market. Differentiating types ex ante allows us to introduce an entry decision on one side of the decentralized market, and thereby allows us to capture extensive margin effects in a very simple way. Thus, the measure of $\mathcal{B}$ is normalized to 1 and all buyers participate in the night market at no cost, while only a subset $\overline{\mathcal{S}}_t \subseteq \mathcal{S}$ with measure n_t of sellers enter the night market at each date t, and we will

[4]Essential in this context means we can achieve allocations with money that we could not achieve without it; again see Kocherlakota (1998) and Wallace (2001).

consider both the case where n_t is exogenous and where sellers may or may not choose to enter at cost k.[5]

Money in the model is perfectly divisible, and agents can hold any non-negative amount. The quantity of money per buyer grows at constant rate γ: $M_{t+1} = \gamma M_t$. New money is injected, or withdrawn if $\gamma < 1$, by lump-sum transfers in the centralized market. For simplicity we assume these transfers go only to buyers, but this is not essential for the results (i.e., things are basically the same if we also give transfers to sellers, as long as they are lump sum in the sense that they do not depend on behavior, and in particular, on their entry decisions). Also, we restrict attention to policies where $\gamma \geq \beta$, where β is a discount rate to be discussed below, since it can easily be checked that for $\gamma < \beta$ there is no equilibrium. Furthermore, when $\gamma = \beta$ – which is the Friedman Rule – we only consider equilibria obtained by taking the limit $\gamma \to \beta$. In general, at date t the distribution of (real) money holdings across buyers is F_t^b and the distribution across sellers is F_t^s.

The instantaneous utility of a buyer at date t is

$$U_t^b = v(x_t) - y_t + \beta_d u(q_t), \tag{1}$$

where x_t is the quantity consumed and y_t the quantity produced during the day, q_t is consumption at night, and β_d is a discount factor between day and the night. There is also a discount factor between night and the next day,

[5] When we allow entry, we assume the set $\mathcal{S}$ is large enough that n_t is never constrained. Also, as in standard search models with constant returns, we are only interested in the ratio of buyers and sellers and not the overall size of the market; this is why we have entry by one side only. There are alternatives to entry that can be used to endogenize the extensive margin. For example, Rocheteau and Wright (2004) consider a fixed total number of agents that can choose whether to be buyers or sellers. In a related model Lagos and Rocheteau (2003) keep the buyer-seller ratio fixed but introduce endogenous search intensity.

β_n, and we let $\beta = \beta_d\beta_n < 1$.[6] Lifetime utility for a buyer is $\sum_{t=0}^{\infty}\beta^t U_t^b$. We assume $u(0) = 0$, $u'(0) = \infty$, $u'(q) > 0$, and $u''(q) < 0$. Also, $v'(x) > 0$, $v''(x) < 0$ for all x, and there exists $x^* > 0$ such that $v'(x^*) = 1$. Without loss of generality, we normalize $v(x^*) - x^* = 0$. Similarly, the instantaneous utility of a seller is

$$U_t^s = v(x_t) - y_t - \beta_d c(q_t), \tag{2}$$

where x_t is consumption and y_t production during the day, and q_t is production at night. Lifetime utility for a seller is $\sum_{t=0}^{\infty}\beta^t U_t^s$. We assume $c(0) = c'(0) = 0$, $c'(q) > 0$ and $c''(q) > 0$. Also we assume $c(q) = u(q)$ for some $q > 0$, and let q^* denote the solution to $u'(q^*) = c'(q^*)$.

In the centralized day market the price of goods is normalized to 1 at each date t, while the relative price of money is denoted ϕ_t. In the decentralized night market, details in terms of prices will differ across the models studied below, but there will always be some friction in the following sense: each period t only a subset $\widetilde{\mathcal{B}}_t \subseteq \mathcal{B}$ of buyers and a subset $\widetilde{\mathcal{S}}_t \subseteq \overline{\mathcal{S}}_t \subseteq \mathcal{S}$ of sellers who participate get to trade in this market. Agents in $\widetilde{\mathcal{B}}_t \cup \widetilde{\mathcal{S}}_t$ may either trade bilaterally or multilaterally in the models discussed below. The measure of $\widetilde{\mathcal{B}}_t$ is $\alpha_b(n_t)$ and the measure of $\widetilde{\mathcal{S}}_t$ is $n_t\alpha_s(n_t)$, and we assume agents in these sets are chosen at random; hence the probabilities of trading for buyers and sellers at night are $\alpha_b(n_t)$ and $\alpha_s(n_t)$, respectively. This allows for "search externalities" in the sense that trading probabilities may depend on the ratio of sellers to buyers, n_t. Typically, unless otherwise indicated we assume $\alpha_b(n) = \alpha(n)$ and $\alpha_s(n) = \alpha(n)/n$, which means the

[6] A special case is when agents do not discount between one subperiod and the next, i.e. either $\beta_n = 1$ or $\beta_d = 1$, as in LW. Another special case is when $\beta_d = \beta_n$, so that the two subperiods can be thought of as even and odd dates, as in the version of the model studied by Williamson (2004).

same number of buyers and sellers trade in the decentralized market, but we also discuss some cases where we relax this. We also assume $\alpha'(n) > 0$, $\alpha''(n) < 0$, $\alpha(n) \leq \min\{1, n\}$, $\alpha(0) = 0$, $\alpha'(0) = 1$ and $\alpha(\infty) = 1$.[7]

We now consider efficiency, which of course can be discussed independently of the assumed market structure and prices. Consider a social planner who chooses each period the measure n_t of sellers in the night market, as well as an allocation $A_t = [q_t^b(i), q_t^s(i), x_t(i), y_t(i)]$ specifying consumption and production during the day $[x_t(i), y_t(i)]$ for all $i \in \mathcal{B} \cup \mathcal{S}$, consumption at night $q_t^b(i)$ for all $i \in \widetilde{\mathcal{B}}_t$, and production at night $q_t^s(i)$ for all $i \in \widetilde{\mathcal{S}}_t$. The planner is constrained by the frictions in the environment, in the sense that he cannot actually choose $\widetilde{\mathcal{B}}_t$ and $\widetilde{\mathcal{S}}_t$, but only n_t, and then these sets are determined at random in such a way that $\widetilde{\mathcal{B}}_t$ has measure $\alpha_b(n_t)$ and $\widetilde{\mathcal{S}}_t$ has measure $n_t \alpha_s(n_t)$. In the case where the same number of buyers and sellers trade in the decentralized market, $\widetilde{\mathcal{B}}_t$ and $\widetilde{\mathcal{S}}_t$ both have measure $\alpha(n_t)$.

Given quasi-linear utility, we only consider the case where the planner weights all agents equally. Thus, let

$$\begin{aligned} \mathcal{W}_t &= \int_{\mathcal{B} \cup \mathcal{S}} \{v[x_t(i)] - y_t(i)\} \, di \\ &\quad + \beta_d \int_{\widetilde{\mathcal{B}}_t} u[q_t^b(i)] di - \beta_d \int_{\widetilde{\mathcal{S}}_t} c[q_t^s(i)] di - \beta_d k n_t. \end{aligned} \tag{3}$$

The planner wants to maximize $\sum_{t=0}^{\infty} \beta^t \mathcal{W}_t$. Feasibility requires several things. First, we obviously must have $\int_{\mathcal{B} \cup \mathcal{S}} x_t(i) di \leq \int_{\mathcal{B} \cup \mathcal{S}} y_t(i) di$. Similarly we must have $\int_{\widetilde{\mathcal{B}}_t} q_t^b(i) di \leq \int_{\widetilde{\mathcal{S}}_t} q_t^s(i) di$, but in addition, in a model where

[7] The function $\alpha(n)$ can be given several interpretations; for now one can think of it as the standard specification coming from a constant returns to scale matching technology. Thus, if $\mu(n_b, n_s)$ is the number of meetings when there are n_b buyers and n_s sellers, constant returns implies the arrival rate for a representative buyer is $\alpha_b = \mu(n_b, n_s)/n_b = \alpha(n_s/n_b)$. See Petrongolo and Pissarides (2001) for an extensive discussion of the matching function.

agents trade bilaterally at night, we have the stronger requirement $q_t^b(i) \leq q_t^s(j)$ for each trading pair (i,j). An *efficient outcome* is defined as paths for n_t and A_t that maximize $\sum_{t=0}^{\infty} \beta^t \mathcal{W}_t$ subject to these restrictions.

PROPOSITION 1: An efficient outcome is stationary and satisfies: $x(i) = x^*$ for all agents in the day market; $q^b(i) = q^b$ and $q^s(i) = q^s$ for all $i \in \widetilde{\mathcal{B}}$ and all $j \in \widetilde{\mathcal{S}}$, where $u'(q^b) = c'(q^s)$ and $\alpha_b(n)q^b = n\alpha_s(n)q^s$; and

$$\alpha_b'(n)\left[u(q^b) - q^b c'(q^s)\right] + [\alpha_s(n) + n\alpha_s'(n)][q^s c'(q^s) - c(q^s)] = k. \qquad (4)$$

In the case where $\alpha_b(n) = n\alpha_s(n) = \alpha(n)$, this implies $q^b(i) = q^s(j) = q^*$ and $n = n^*$, where

$$\alpha'(n^*)\,[u(q^*) - c(q^*)] = k. \qquad (5)$$

PROOF: Note first that the planner's problem is equivalent to a sequence of static problems. Maximizing $\mathcal{W}_t$ at each date leads to first order conditions that imply $v'[x(i)] = 1$, and therefore $x(i) = x^*$ for all i in the day market, and $u'[q^b(i)] = c'[q^s(j)] = \lambda/\beta_d$ for all i and j that trade in the night market where λ is the Lagrange multiplier associated with the feasibility constraint $\int_{\widetilde{\mathcal{B}}} q^b(i)di = \int_{\widetilde{\mathcal{S}}} q^s(i)di$. From the feasibility constraint, since $\widetilde{\mathcal{B}}$ and $\widetilde{\mathcal{S}}$ have measures $\alpha_b(n)$ and $n\alpha_s(n)$, respectively, we have $\alpha_b(n)q^b = n\alpha_s(n)q^s$. Given this, the first order condition for n is (4). Finally, (5) is derived from (4) by using $q^s = q^b$ and $\alpha_s(n) + n\alpha_s'(n) = \alpha'(n)$. *Q.E.D.*

As long as $x = x^*$, which will turn out to be true in every equilibrium considered below, and ignoring constants, for any n and q welfare per period can be measured by $\alpha(n)\,[u(q) - c(q)] - kn$. From this it is clear that the efficient q is the one that solves $u'(q^*) = c'(q^*)$, and as seen in (5) the efficient n is the one that makes a seller's marginal contribution to the trading process $\alpha'(n)$ times the surplus $u(q^*) - c(q^*)$ equal to the participation cost k.

3. SEARCH EQUILIBRIUM (BARGAINING)

In this section we study the market structure used in much of the recent literature on the microfoundations of money, where buyers and sellers trade bilaterally and bargain over the terms of trade. In this model, one can think of the event that a buyer gets to trade as the event that he *meets* a seller, and vice-versa, as in standard matching models. In the following we define the real value of an amount of money m_t in the hands of an agent at date t by $z_t = \phi_t m_t$. Here we focus on steady-state equilibria, where aggregate real variables, including the aggregate real money supply $Z_t = \phi_t M_t$, are constant. Therefore, in steady-state equilibrium, we have $\phi_{t+1}/\phi_t = 1/\gamma$ because $M_{t+1}/M_t = \gamma$.

If a buyer with real balances z_b meets a seller with z_s, let $d = d(z_b, z_s)$ and $q = q(z_b, z_s)$ denote the real dollars and units of the good that are traded. Let $V^b(z_b)$ and $W^b(z_b)$ be the value functions for a buyer with z_b in the night market and day market, respectively, and let $V^s(z_s)$ and $W^s(z_s)$ be the value functions for sellers. Bellman's equation for a buyer in the decentralized night market is

$$\begin{aligned} V^b(z_b) &= \alpha(n) \int \left\{ u\left[q(z_b, z_s)\right] + \beta_n W^b \left[\frac{z_b - d\left(z_b, z_s\right)}{\gamma} \right] \right\} dF^s(z_s) \\ &\qquad + \left[1 - \alpha(n)\right] \beta_n W^b \left(\frac{z_b}{\gamma} \right). \end{aligned} \tag{6}$$

In words, with probability $\alpha(n)$ he meets a seller who has a random z_s, at which point he consumes $q(z_b, z_s)$ and starts the next day with real balances $\left[z_b - d(z_b, z_s)\right]/\gamma$; and with probability $1 - \alpha(n)$ he does not trade and starts the next day with z_b/γ.[8] Similarly, for sellers,

$$\begin{aligned} V^s(z_s) &= \frac{\alpha(n)}{n} \int \left\{ -c\left[q\left(z_b, z_s\right)\right] + \beta_n W^s \left[\frac{z_s + d\left(z_b, z_s\right)}{\gamma} \right] \right\} dF^b(z_b) \\ &\quad + \left[1 - \frac{\alpha(n)}{n}\right] \beta_n W^s \left(\frac{z_s}{\gamma}\right) - k. \end{aligned} \tag{7}$$

Comparing (6) and (7), notice only sellers pay the participation cost k.

In the centralized day market a buyer's problem is

$$W^b(z_b) = \max_{\hat{z},x,y} \left\{ v(x) - y + \beta_d V^b(\hat{z}) \right\} \tag{8}$$

$$\text{subject to } \hat{z} + x = z_b + T + y, \tag{9}$$

where T is his real transfer and $\hat{z}$ is the real balances he takes into that period's decentralized market.[9] Similarly, for any seller who enters the night market,

$$W^s(z_s) = \max_{\hat{z},x,y} \left\{ v(x) - y + \beta_d V^s(\hat{z}) \right\} \tag{10}$$

$$\text{subject to } \hat{z} + x = z_s + y. \tag{11}$$

LEMMA 1: For all agents in the centralized market, $\hat{z}$ is independent of z. Also, $W^b(z_b) = z_b + W^b(0)$ and $W^s(z_s) = z_s + W^s(0)$ are linear.

PROOF: Consider a buyer. Substituting (9) into (8), we have

$$W^b(z_b) = z_b + T + \max_{\hat{z},x} \left\{ v(x) - x - \hat{z} + \beta_d V^b(\hat{z}) \right\}. \tag{12}$$

The rest is obvious. *Q.E.D.*

[8] If a buyer holds $z_t = m_t \phi_t$ at the end of period t, his real balances at the beginning of period $t+1$ are $z_{t+1} = m_t \phi_{t+1} = z_t \phi_{t+1}/\phi_t = z_t/\gamma$.

[9] We are ignoring non-negativity constraints. For variables other than y, these will be satisfied under the usual conditions. For y, we simply look for equilibria with the property that $y > 0$ for all agents, but one can impose conditions on primitives to guarantee this is valid, as in LW. It is important that $y \geq 0$ is not binding for a key result proved below – the result that all agents of a given type choose the same $\hat{z}$, independent of z.

Assuming V^b is differentiable (it will be) the first order condition for $\hat{z}$ from (12) is

$$-1 + \beta_d V_z^b(\hat{z}) \leq 0, \quad = 0 \text{ if } \hat{z} > 0, \tag{13}$$

and if V^b is strictly concave over the relevant range (it will be under weak conditions) there is a unique solution to (13) and all buyers choose the same $\hat{z}$.[10] Similarly all sellers choose the same $\hat{z}$. To say more, we need to discuss the terms of trade in the decentralized market.

Consider a meeting between a buyer with z_b and a seller with z_s at night. To determine $q(z_b, z_s)$ and $d(z_b, z_s)$ in this model we use the generalized Nash bargaining solution, where $\theta \in (0, 1]$ is the bargaining power of a buyer and threat points are given by continuation values. Thus, the payoffs of the buyer and seller are $u(q) + \beta_n W^b[(z_b - d)/\gamma]$ and $-c(q) + \beta_n W^s[(z_s + d)/\gamma]$, and the threat points are $\beta_n W^b(z_b/\gamma)$ and $\beta_n W^s(z_s/\gamma)$. Linearity of $W^b(z_b)$ and $W^s(z_s)$ implies $W^b[(z_b - d)/\gamma] - W^b[z_b/\gamma] = -d/\gamma$ and $W^s[(z_s + d)/\gamma] - W^s[z_s/\gamma] = d/\gamma$. Hence the generalized Nash bargaining solution reduces to

$$\max_{q,d} \left[u(q) - \frac{\beta_n}{\gamma} d\right]^\theta \left[-c(q) + \frac{\beta_n}{\gamma} d\right]^{1-\theta} \tag{14}$$

where d is subject to the resource constraint $d \leq z_b$.

It is immediate that the solution to (14) is independent of z_s. Moreover, (q, d) depends on z_b if and only if the constraint $d \leq z_b$ binds. If it does not bind, the first order conditions from (14) are

$$u'(q) = c'(q), \tag{15}$$

$$\frac{\beta_n}{\gamma} d = (1-\theta)u(q) + \theta c(q) \tag{16}$$

[10] LW provide details on the existence, differentiability, and strict concavity of the value functions for their version of the model, and the same arguments apply here. We will discuss how things change in the other models considered below.

which imply $q = q^*$ and $d = z^* = [\theta c(q^*) + (1-\theta)u(q^*)]\,\gamma/\beta_n$. If the constraint does bind, then q solves the first order condition from (14) with $d = z_b$, which we can write as

$$\frac{\beta_n}{\gamma} z_b = g(q,\theta) = \frac{\theta u'(q)c(q) + (1-\theta)c'(q)u(q)}{\theta u'(q) + (1-\theta)c'(q)}. \tag{17}$$

For future reference, note that the function $g(q,\theta)$ defined in (17) satisfies $g_q(q,\theta) > 0$ for all $q < q^*$.

This fully describes decentralized trade under bargaining. To return to the determination of $\hat{z}$, we now make the following assumption:

ASSUMPTION 1: (i) $\lim_{q\to 0} u'(q)/g_q(q,\theta) = \infty$; (ii) for all $q < q^*$, $u'(q)/g_q(q,\theta)$ is strictly decreasing.

Part (i) is a standard Inada condition. Part (ii) implies equilibrium will be unique when n is exogenous, and is made so that we will know any multiplicity of equilibria that occurs when n is endogenous is due to free entry.[11] We have the following.

LEMMA 2: Sellers set $\hat{z} = 0$. Buyers set $\hat{z} = \gamma g(q,\theta)/\beta_n$, where g is defined in (17) and q is the unique positive solution to

$$\frac{\gamma - \beta}{\beta\alpha(n)} + 1 = \frac{u'(q)}{g_q(q,\theta)}. \tag{18}$$

PROOF: Consider first sellers. From the bargaining solution, $\partial q/\partial z_s = \partial d/\partial z_s = 0$ for all z_s. Hence, the first order condition for $\hat{z}$ for a seller is

$$-1 + \beta_d V_z^s = -1 + \beta/\gamma \leq 0, \quad = 0 \text{ if } \hat{z} > 0.$$

[11] LW establish that a sufficient condition for (ii) is that either θ is not too small or u' is log-concave and c is linear. Some such condition is required because under bargaining q is generally a nonlinear function of z_b and it depends on u'''. As we will see, this is not a problem in the models in the later sections.

Since as we said above we only consider either the case $\gamma > \beta$, or the case $\gamma = \beta$ but equilibrium is the limit as $\gamma \to \beta$ from above, the solution is $\hat{z} = 0$.

Now consider buyers. From (15)-(17), if $z_b > z^*$ then $\partial q/\partial z_b = \partial d/\partial z_b = 0$, and if $z_b < z^*$ then $\partial q/\partial z_b = \beta_n/\gamma g_q(q,\theta)$ and $\partial d/\partial z_b = 1$. From (6), if $z_b > z^*$ then $V_z^b(z_b) = \beta_n/\gamma$ and if $z_b < z^*$ then

$$V_z^b(z_b) = \frac{\beta_n}{\gamma}\left\{\alpha(n)\left[\frac{u'(q)}{g_q(q,\theta)} - 1\right] + 1\right\}. \tag{19}$$

Given $\gamma > \beta$, $-\hat{z} + \beta_d V^b(\hat{z})$ is strictly decreasing for all $\hat{z} > z^*$. Also, one can show that as $z \to z^*$ from below, we have $\lim u'[q(z)]/g_q[q(z),\theta] \leq 1$. This establishes that as $z \to z^*$ from below $-1 + \beta_d V_z^b(z) < 0$, and so the optimizing choice is $\hat{z} < z^*$. Inserting (19) into the first order condition $1 = \beta_d V_z^b(\hat{z})$ and rearranging we get (18). Assumption 1 guarantees that it has a unique positive solution. *Q.E.D.*

Having discussed z and q we now move to n. We consider two alternatives: either it is exogenous at $n = \bar{n}$, or it is endogenous and determined by free entry.

LEMMA 3: Free entry implies

$$\frac{\alpha(n)}{n}\left[-c\left(q\right) + \frac{\beta_n}{\gamma}d\right] = k. \tag{20}$$

PROOF: First note that a seller who does not enter gets a payoff each day of $v(x^*) - x^* = 0$. Since sellers do not bring money to the decentralized market, $W^s(z) = z + \beta_d \max\left[V^s(0), 0\right] = z$. Free entry implies $0 = V^s(0) = \frac{\alpha(n)}{n}\left\{-c\left[q\left(z_b, 0\right)\right] + \beta_n d\left(z_b, 0\right)/\gamma\right\} - k$, which reduces to (20). *Q.E.D.*

By (20), the participation cost is equal to the probability of trading multiplied by the seller's surplus. It can be rewritten using (17) as

$$\frac{\alpha(n)}{n}\frac{(1-\theta)c'(q)}{\theta u'(q)+(1-\theta)c'(q)}\left[u(q)-c(q)\right]=k. \tag{21}$$

As a necessary condition for $n > 0$ we impose

ASSUMPTION 2: $k < \frac{(1-\theta)c'(\tilde{q})}{\theta u'(\tilde{q})+(1-\theta)c'(\tilde{q})}\left[u(\tilde{q})-c(\tilde{q})\right]$,

where $\tilde{q}$ is the solution to (18) when $\gamma = \beta$; notice that $\tilde{q} = q^*$ if $\theta = 1$ while $\tilde{q} < q^*$ otherwise. Given $k > 0$, naturally Assumption 2 requires $\theta < 1$.

We now define equilibrium formally for this model. In all definitions in this paper, when we say equilibrium we mean a steady-state monetary equilibrium with $q, n > 0$.[12]

DEFINITION 1: (i) With $n = \bar{n}$, *search equilibrium* is a list $(q, z) \in \mathbb{R}^2_+$ satisfying (17) and (18). (ii) With free entry, *search equilibrium* is a list $(q, z, n) \in \mathbb{R}^3_+$ satisfying (17), (18) and (21).

Note that equilibrium has a recursive structure: with n fixed q is determined by (18), and with free entry (q, n) is determined by (18) and (21), but in either case we can solve for $z = \gamma g(q, \theta)/\beta_n$ after we find q. Hence, we concentrate on q and n in what follows.

PROPOSITION 2: (i) Assume $n = \bar{n}$. Search equilibrium exists and is unique. Furthermore, $\partial q/\partial\gamma < 0$. (ii) Assume free-entry. There is a $\bar{\gamma} > \beta$ such that equilibrium exists if and only if $\gamma \leq \bar{\gamma}$. For all $\gamma \in (\beta, \bar{\gamma})$ equilibrium is generically not unique. At the equilibrium with the highest q, $\partial q/\partial\gamma < 0$ and $\partial n/\partial\gamma < 0$. When $\gamma = \beta$ there exists a unique equilibrium.

PROOF: (i) If $n = \bar{n}$ then equilibrium is simply a $q > 0$ solving (18), which exists uniquely by Assumption 1. It is easy to check $\partial q/\partial\gamma < 0$.

[12] Nonstationary equilibria for a version of the model with n fixed are analyzed in Lagos and Wright (2003).

(ii) Now consider free entry. Let $\bar{q}$ be the value of q that solves (21) when $n = 0$, and notice $n > 0$ if and only if $q > \bar{q}$. A necessary condition for equilibrium to exist is $\bar{q} < \tilde{q}$ which holds by Assumption 2. For all $q \in [\bar{q}, \tilde{q}]$, (21) can be written $n = n(q)$ with $n'(q) = \alpha(n)[g_q(q,\theta) - c'(q)]/k[1-\eta(n)] > 0$, $n(\bar{q}) = 0$ and $n(\tilde{q}) > 0$. Let $\Gamma(q;\gamma)$ be defined by

$$\Gamma(q;\gamma) = \beta\alpha\left[n(q)\right]\left[\frac{u'(q)}{g_q(q,\theta)} - 1\right] - (\gamma - \beta).$$

From Definition 1, an equilibrium exists if and only if there is a $q \in (\bar{q}, \tilde{q}]$ such that $\Gamma(q;\gamma) = 0$.

Consider first the limiting case $\gamma = \beta$. Then the unique $q \in (\bar{q}, \tilde{q}]$ such that $\Gamma(q;\beta) = 0$ is $q = \tilde{q}$. Consider next $\gamma > \beta$. Then $\Gamma(\bar{q};\gamma) = \Gamma(\tilde{q};\gamma) = \beta - \gamma < 0$. As $\Gamma(q;\gamma)$ is continuous, if equilibrium exists it is generically not unique. Furthermore, $\Gamma(q;\gamma)$ is decreasing in γ, $\Gamma(q;\beta) > 0$ for all $q \in (\bar{q}, \tilde{q})$, and for large values of γ, $\Gamma(q;\gamma) < 0$ for all $q \in (\bar{q}, \tilde{q}]$. Therefore, there is a $\bar{\gamma} > \beta$ such that equilibrium exists if and only if $\gamma \leq \bar{\gamma}$. Finally, $\partial q/\partial\gamma = 1/\Gamma_q$ and $\Gamma_q < 0$ at the equilibrium with the highest q, which means $\partial q/\partial\gamma < 0$, and, from (21), $\partial n/\partial\gamma < 0$. *Q.E.D.*

In the case with n endogenous, equilibrium obtains at the intersection of two curves in (n, q) space, the q-curve defined by (18) and the n-curve defined by (21). See Figure 1. Both curves are upward-sloping, and as γ increases the q-curve rotates downward. For $\gamma > \beta$, at both $n = 0$ and $n = n(\tilde{q})$ the n-curve is above the q-curve, so equilibria are generically not unique. It is clear that this multiplicity requires a participation decision since when n is exogenous Assumption 1 guarantees uniqueness, but note that it does not require increasing returns, as in the typical nonmonetary search model. The reason is that there is a strategic interaction here between entry by sellers and money demand by buyers.[13] However, in the limit as $\gamma \to \beta$ the q-curve becomes horizontal at $\tilde{q}$ for all $n > 0$, and we get uniqueness as the equilibrium with low (q, n) coalesces with the origin.

INSERT FIGURE 1 ABOUT HERE

We now analyze efficiency and the effects of inflation.

PROPOSITION 3: (i) Assume $n = \bar{n}$. The optimal monetary policy is $\gamma = \beta$ and it yields the efficient outcome if and only if $\theta = 1$. (ii) Assume free entry. Equilibria with higher q and n yield higher $\mathcal{W}$. In the best equilibrium, the optimal monetary policy is $\gamma = \beta$, but it can never achieve the efficient outcome.

PROOF: With $n = \bar{n}$, $\mathcal{W} = \beta_d \alpha(\bar{n}) [u(q) - c(q)]$ is maximized at $q = q^*$. From (18), $q = q^*$ if and only if $\gamma = \beta$ and $\theta = 1$. Under free entry $\alpha(n) [g(q, \theta) - c(q)] = nk$ and $\mathcal{W} = \beta_d \alpha(n)[u(q) - c(q)] - \beta_d nk = \beta_d \alpha(n) [u(q) - g(q, \theta)]$. Since $\alpha(n)$ is increasing in n and $u(q) - g(q, \theta)$ is increasing in q for all $q \in [0, \tilde{q}]$, equilibria with higher q and n imply higher $\mathcal{W}$. Since $\partial q / \partial \gamma < 0$ and $\partial n / \partial \gamma < 0$ at the best equilibrium, $\partial \mathcal{W} / \partial \gamma < 0$, and the best policy is $\gamma = \beta$. At $\gamma = \beta$, we have $q = q^*$ if and only if $\theta = 1$. But $\theta = 1$ implies $n = 0$, so there is no way to achieve $q = q^*$ and $n > 0$. *Q.E.D.*

For all $\theta < 1$ and all γ, q is too low due to a holdup problem that reduces the demand for money: when a buyer brings cash to the decentralized market he is making an investment, but when $\theta < 1$ he is not getting the full return on his investment. This reduces the equilibrium value of q below the efficient

[13] A similar point has been made by Johri (1999), who shows that a version of Diamond (1982) with constant returns can have multiple equilibria once money is introduced in a sensible way.

level. Consider Figure 2, which plots the total surplus from decentralized trade $S(q) = u(q) - c(q)$, as well as the buyer's share

$$S^b(q) = \frac{\theta u'(q)}{\theta u'(q) + (1-\theta)c'(q)} \left[u(q) - c(q)\right], \tag{22}$$

as functions of q.[14] The curve $S^b(q)$ reaches a maximum at $q = \tilde{q} \leq q^*$, with the inequality strict if $\theta < 1$. A buyer will never bring more money than needed to buy the quantity that maximizes $S^b(q)$. If there is an opportunity cost of holding money, which there is when $\gamma > \beta$, he will in fact prefer to buy less than $\tilde{q}$. Hence, we have $q < q^*$ whenever $\gamma > \beta$.

INSERT FIGURE 2 ABOUT HERE

In the case where n is endogenous, inflation affects both individual real balances and the frequency of trade. Comparison of (5) and (21) implies that, for any given q, n is efficient if and only if

$$\frac{(1-\theta)c'(q)}{\theta u'(q) + (1-\theta)c'(q)} = \eta\left(n\right), \tag{23}$$

where $\eta\left(n\right) = n\alpha'(n)/\alpha(n)$ measures sellers' contribution to buyers' probability of trade. This is the familiar Hosios (1990) condition: entry is efficient if and only if agents' share of the surplus from trade equals $\eta\left(n\right)$. It is possible for n to be either too high or too low in equilibrium, and if it is too high, inflation actually increases welfare along the extensive margin (basically, by driving out some sellers). Still, the negative effect on the intensive margin always dominates any positive effect on the extensive margin. We will discuss this further in the next section.

[14] To derive(22), insert $\beta_n z/\gamma = g(q, \theta)$ from (17) into $S^b(q) = u(q) - \beta_n z/\gamma$ and simplify. The seller's share is defined similarly.

4. COMPETITIVE EQUILIBRIUM (PRICE TAKING)

Considering a competitive market at night, with a Walrasian auctioneer, may make the decentralized market less decentralized but it does not make money inessential as long as we maintain the double coincidence problem and anonymity (Levine (1991) and Temzelides and Yu (2003) make a similar point). Also, we can still capture search-type frictions by assuming that, although there is a competitive market at night, not all agents get in. Following the notation in the previous section, the probabilities of getting an opportunity to trade – which now means getting into the market – for buyers and sellers are $\alpha_b(n)$ and $\alpha_s(n)$.[15] Note that entry by sellers in this model means entry into the group $\overline{\mathcal{S}}$ trying to get into the night market; of these only $\widetilde{\mathcal{S}} \subseteq \overline{\mathcal{S}}$ succeed. For those who do, after seeing the (real) price of night goods p, each buyer chooses demand q^b and each seller chooses supply q^s. Goods trade against money for exactly the same reason they did in the previous section: the double coincidence problem and anonymity.[16]

The value function of a buyer at night is now

$$\begin{aligned} V^b(z_b) &= \alpha_b(n) \max_{q^b} \left\{ u(q^b) + \beta_n W^b\left(\frac{z_b - pq^b}{\gamma}\right) \right\} \\ &\quad + [1-\alpha_b(n)]\, \beta_n W^b\left(\frac{z_b}{\gamma}\right), \end{aligned} \tag{24}$$

where the maximization is subject to the budget constraint $pq^b \leq z_b$, and $W^b(z_b)$ still satisfies (8) from the previous section. Similarly, for sellers, we

[15] Since competitive equilibrium does not require bilateral trade, for now we adopt a general specification for $\alpha_b(n)$ and $\alpha_s(n)$; later, we will specialize to $\alpha_b(n) = n\alpha_s(n) = \alpha(n)$ in order to make the different models comparable.

[16] The assumption that not all agents get into the night market is simply a convenient way to introduce search-type frictions into an otherwise Walrasian model; it can be thought of as a generalized version of Lucas and Prescott (1974) and Alvarez and Veracierto (1999).

have

$$V^s(z_s) = \alpha_s(n)\max_{q^s}\left\{-c(q^s) + \beta_n W^s\left(\frac{z_s + pq^s}{\gamma}\right)\right\} + [1-\alpha_s(n)]\,\beta_n W^s\left(\frac{z_s}{\gamma}\right) - k, \tag{25}$$

where $W^s(z_s)$ is the same as in the previous section.

LEMMA 4: Sellers set $\hat{z} = 0$. Buyers set $\hat{z} = pq^b$ where q^b is the unique positive solution to

$$\frac{\gamma - \beta}{\beta\alpha_b(n)} + 1 = \frac{\gamma}{\beta_n p}u'\left(q^b\right). \tag{26}$$

PROOF: For sellers, the reasoning is similar to the proof of Lemma 2. Now consider buyers. From (24),

$$V_z^b(z) = \begin{cases} \alpha_b(n)u'\left(\frac{z}{p}\right)\frac{1}{p} + [1-\alpha_b(n)]\frac{\beta_n}{\gamma} & z < z^* \\ \frac{\beta_n}{\gamma} & z > z^* \end{cases}$$

where z^* satisfies $u'(z^*/p) = \beta_n p/\gamma$. To establish the concavity of $V^b(z)$, note that $V_{zz}^b = \alpha_b u''/(p)^2 < 0$ for all $z < z^*$, $V_{zz}^b = 0$ for all $z > z^*$, and $V_z^b(z)$ is continuous at z^*. Furthermore, $-1 + \beta_d V_z^b(z)$ is strictly decreasing in z for all $z \in [0, z^*]$, $-1 + \beta_d V_z^b(0) = \infty$ and $-1 + \beta_d V_z^b(z) = -1 + \beta/\gamma$ for all $z \geq z^*$. Consequently, for all $\gamma > \beta$ there is a unique $z \in (0, z^*)$ satisfying $1 = \beta_d V_z^b(z)$. Finally, (26) results from substituting for $V_z^b(z)$ in $1 = \beta_d V_z^b(z)$. *Q.E.D.*

From Lemma 4, each of the $\alpha_b(n)$ buyers who get in to the market at night demand q^b. Similarly, each of the $n\alpha_s(n)$ sellers who get in supply q^s. To clear the market we require

$$n\alpha_s(n)q^s = \alpha_b(n)q^b. \tag{27}$$

From (25) $c'(q^s) = \beta_n p/\gamma$, which with $q^b = z/p$ implies

$$\frac{\beta_n}{\gamma}z = q^b c'\left(q^s\right), \tag{28}$$

and hence

$$\frac{u'(q^b)}{c'(q^s)} = 1 + \frac{\gamma - \beta}{\beta\alpha_b(n)}. \tag{29}$$

If n is endogenous, the free entry condition is analogous to (20) where $\beta_n d/\gamma$ is replaced by $\beta_n p q^s/\gamma = c'(q^s)q^s$,

$$\alpha_s(n)\left[q^s c'(q^s) - c(q^s)\right] = k. \tag{30}$$

DEFINITION 2: (i) With $n = \bar{n}$, a *competitive equilibrium* is a list $(q^b, q^s, z) \in \mathbb{R}^3_+$ satisfying (27), (28) and (29). (ii) With free entry, a *competitive equilibrium* is a list $(q^b, q^s, z, n) \in \mathbb{R}^4_+$ satisfying (27), (28), (29) and (30).

As we said above, we do not necessarily assume that the same number of buyers and sellers get into the night market. However, if we do make this assumption then one can imagine exchange being bilateral in this model, as it is in the other models we discuss, even though prices are determined in a competitive market. This assumption means $\alpha_b(n) = n\alpha_s(n) = \alpha(n)$, and then the market clearing condition (27) is simply $q^b = q^s = q$, where q is given by (29). This makes it easier to compare the different models, since, e.g., (29) is analogous to (18) in the previous section (indeed they coincide if and only if $\theta = 1$). Also, note that things are again recursive: we can first determine q and then $z = \gamma q c'(q)/\beta_n$.

DEFINITION 2': Consider the special case where $\alpha_b(n) = n\alpha_s(n) = \alpha(n)$, and therefore $q^b = q^s = q$. (i) With $n = \bar{n}$, a *competitive equilibrium* is a list $(q, z) \in \mathbb{R}^2_+$ satisfying (28) and (29). (ii) With free entry, a *competitive equilibrium* is a list $(q, z, n) \in \mathbb{R}^3_+$ satisfying (28), (29) and (30).

The following restriction is necessary for $n > 0$.

ASSUMPTION 2': $k < q^* c'(q^*) - c(q^*)$.

PROPOSITION 4: (i) Assume $n = \bar{n}$. Competitive equilibrium exists and is unique. Furthermore, $\partial q / \partial \gamma < 0$. (ii) Assume free-entry. There exists $\bar{\gamma} > \beta$ such that equilibrium exists if and only if $\gamma \leq \bar{\gamma}$. For all $\gamma \in (\beta, \bar{\gamma})$ equilibrium is generically not unique. At the equilibrium with the highest q, $\partial q / \partial \gamma < 0$ and $\partial n / \partial \gamma < 0$. When $\gamma = \beta$ there exists a unique equilibrium.

PROOF: The argument is essentially the same as the proof of Proposition 2 and therefore is omitted. *Q.E.D.*

Let n_β denote the equilibrium value of n at $\gamma = \beta$; since $\gamma = \beta$ implies $q = q^*$, this means n_β solves $\alpha(n_\beta) \left[q^* c' (q^*) - c (q^*)\right] = n_\beta k$.

PROPOSITION 5: (i) Assume $n = \bar{n}$. The optimal policy is $\gamma = \beta$ and it yields the efficient outcome. (ii) Assume free-entry. Equilibrium is efficient if and only if $\gamma = \beta$ and

$$\eta(n_\beta) = \frac{q^* c'(q^*) - c(q^*)}{u(q^*) - c(q^*)}. \tag{31}$$

In the equilibrium with highest q and n, optimal policy involves $\gamma > \beta$ if and only if $\eta(n_\beta) < \frac{q^* c'(q^*) - c(q^*)}{u(q^*) - c(q^*)}$.

PROOF: From (29) we have $q = q^*$ if and only if $\gamma = \beta$. Comparing (30) with (5), we see that $n_\beta = n^*$ if and only if (31) holds. Differentiating $\mathcal{W} = \beta_d \alpha(n) \left[u(q) - c(q)\right] - \beta_d kn$ and substituting for k from (30), we have

$$\frac{d\mathcal{W}}{d\gamma} = \beta_d \frac{\alpha(n_\beta)}{n_\beta} \left[u(q^*) - c(q^*)\right] \left\{ \eta(n_\beta) - \left[\frac{q^* c'(q^*) - c(q^*)}{u(q^*) - c(q^*)} \right] \right\} \frac{dn}{d\gamma} \tag{32}$$

where the derivatives are evaluated at the limit as $\gamma \to \beta$ from above. From (29) and (30),

$$\frac{dn}{d\gamma} = \frac{q^* c''(q^*) c'(q^*)}{\beta \left[u''(q^*) - c''(q^*)\right] \left[1 - \eta(n_\beta)\right] k} < 0.$$

As long as $\eta(n_\beta) < \frac{q^* c'(q^*) - c(q^*)}{u(q^*) - c(q^*)}$, we have $d\mathcal{W}/d\gamma > 0$. *Q.E.D.*

If $n = \bar{n}$ the Friedman Rule $\gamma = \beta$ yields full efficiency. This is in accordance with many models in monetary economics, although not the one in the previous section, where the Friedman Rule was the optimal policy but could not achieve full efficiency unless $\theta = 1$. The reason $\gamma = \beta$ implies efficiency here is that the holdup problem with money demand disappears under competitive pricing. If n is endogenous, in addition to $\gamma = \beta$, for efficiency we also need (31) to be satisfied. Hence, full efficiency is achieved if and only if the Friedman Rule and the Hosios condition both hold. There is no reason to expect the Hosios condition to hold, in general, since (31) relates the elasticity of the matching function to properties of preferences. Therefore n is typically inefficient, and may be either too high or too low. In particular, when the number of sellers in the economy is too high at $\gamma = \beta$, a deviation from the Friedman rule is welfare improving.

It is uncommon for a deviation from the Friedman Rule to be optimal. The intuition for our result is as follows. In general, when sellers decide to enter they impose, in the jargon of the literature, a "congestion" effect on other sellers and a "thick market" effect on buyers. If the former effect dominates – and it certainly will for some specifications – then n is too high, and inflation helps because it reduces sellers' incentive to enter. Inflation also reduces q, and this hurts along the intensive margin, but it has only a second-order effect in the neighborhood of the Friedman rule since q is close to q^* in the neighborhood of the Friedman rule. It is important to emphasize that this result is different from the bargaining model, where in general $q < q^*$ for $\theta \neq 1$, and hence the negative effect of inflation along the intensive margin has a first order effect.[17]

[17] There are a few related results in the literature based on "search externalities," in-

It may not be surprising that a single policy instrument γ cannot sort out both the intensive and the extensive margins. If taxes and transfers were available that could be made contingent on agents' types and actions, say, one could presumably do better in trying to correct for inefficient entry by sellers. When such transfers cannot be implemented, however, inflation is a natural instrument to target "congestion" since it reduces agents' incentives to participate in the market. We have shown that one cannot necessarily choose γ to get efficiency on both margins. One should not be too cavalier, however, about thinking that it is unsurprising that a single instrument cannot achieve efficiency on both margins – in the model of the next section, it turns out that it can.[18]

5. COMPETITIVE SEARCH EQUILIBRIUM (POSTING)

The concept of competitive search equilibrium is based on the idea that some agents can post a price (or, more generally, a contract) that specifies the terms at which buyers and sellers commit to trade. Buyers and sellers in the market observe posted prices and choose where to go, although again there may be frictions. In some versions, frictions manifest themselves by

cluding Li (1995) and Berentsen, Rocheteau and Shi (2001), but those results are not especially robust. In general, even with "search externalities" it is often the case that the Friedman Rule is optimal – this was certainly true in the previous section. It seems that one has to somehow get around the holdup problem for the potentially desirable extensive margin effects to dominate the bad intensive margin effects. Li assumes indivisible goods and indivisible money, which certainly does the trick, and Berentsen et al. invoke a special bargaining solution. Here we avoid the holdup problem because we have competitive price taking.

[18] We close this section by mentioning that the welfare results are robust if we relax the assumption that equal numbers of buyers and sellers get into the night market, and indeed it is even easier to construct examples. Assume, for instance, that all buyers get in with probability one while sellers get in with probability $\alpha_s(n)$ where $\alpha_s'(n) < 0$. Then (4) and (30) imply that n is necessarily too high at $\gamma = \beta$, and inflation above the Friedman rule necessarily improves welfare, by reducing n.

more or fewer buyers showing up at a seller's location that he has capacity to serve (Burdett, Shi and Wright (2001)), while in other versions agents get to choose a location with a given price but still have to search for trading partners at that location (Moen (1997)). In either case, there is (partially) directed search, and this generates competition among price setters.[19]

We adopt the interpretation of competitive search equilibrium that assumes there are agents called *market makers* who can open submarkets where they post the terms of trade (q, d).[20] Agents can direct their search in the sense that they can go to any submarket they like, but within any submarket there is random bilateral matching. Given knowledge of (q, d) across submarkets, and expectations about where other agents go, which determines the arrival rates across submarkets, each buyer or seller decides where to go, and in equilibrium expectations must be rational. When designing a submarket, a market maker takes into account the relationship between the posted (q, d) and the numbers of buyers and sellers who show up, summarized by the ratio n. In equilibrium the set of submarkets is *complete* in the sense that there is no submarket that could be opened that makes some buyers and sellers better off.

The timing of events in a period is as follows. At the beginning of each day, market makers announce the submarkets to be open that night,

[19] Corbae, Temzelides and Wright (2003) show that directed search models still have an essential role for money, but do not consider price posting, and the notion of competitive search equilibrium requires the combination of the two. For monetary models with posting and undirected search, see the references in Curtis and Wright (2004).

[20] One can think of market makers as profit-maximizing agents who charge submarket participants an entry fee, which we assume must be independent of agents' types; this fee will be 0 in equilibrium because the cost of opening a submarket is negligible. See Mortensen and Wright (2002). One can also interpret the model as having buyers or sellers themselves post (q, d) in order to maximize their expected utility. See Faig and Huangfu (2004) for a recent discussion and some extensions.

as described by (q, d), and this implies an expected n in each submarket. Agents then trade in the centralized market during the day and readjust their real balances, exactly as before, and go to submarkets of their choosing at night in a way consistent with expectations. In the submarkets at night agents trade goods and money bilaterally, like in search equilibrium, except they do not bargain – they are bound by the posted terms of trade (q, d). Obviously, this builds in a certain amount of commitment; this is a defining feature of the competitive search equilibrium concept. We let Ω denote the set of open submarkets, with generic element $\omega = (q, d, n)$ listing the terms of trade and the seller-buyer ratio.

For a buyer at night,

$$\begin{aligned} V^b(z_b) &= \max_{\omega} \left\{ \alpha(n) \mathbf{1}\left(z_b \geq d\right) \left[u(q) + \beta_n W^b \left(\frac{z^b - d}{\gamma} \right) \right] \right. \\ &\quad \left. + \left[1 - \alpha(n) \mathbf{1}\left(z_b \geq d\right) \right] \beta_n W^b \left(\frac{z^b}{\gamma} \right) \right\}, \end{aligned} \tag{33}$$

where $\mathbf{1}\left(z_b \geq d\right)$ is the indicator function that is equal to one if $z_b \geq d$ and zero otherwise. Thus, a buyer chooses ω among the set of open submarkets, and then he gets to trade if he meets a seller and has enough money to meet the posted price, $z_b \geq d$. For a seller at night,

$$\begin{aligned} V^s(z_s) &= \max_{\omega} \left\{ \frac{\alpha(n)}{n} \left[-c(q) + \beta_n W^s \left(\frac{z^s + d}{\gamma} \right) \right] \right. \\ &\quad \left. + \left[1 - \frac{\alpha(n)}{n} \right] \beta_n W^s \left(\frac{z^s}{\gamma} \right) \right\} - k. \end{aligned} \tag{34}$$

The functions W^b and W^s are exactly as in the previous sections.

As before, the seller's choice of real balances in the day market is $\hat{z} = 0$. The set of open submarkets is complete if there is no submarket that could beat existing submarkets, in the sense of making some buyers better off without making sellers worse off (it would obviously be equivalent to consider the dual). A submarket with a posted (q, d) will attract a measure n of sellers per buyer, where n satisfies

$$\frac{\alpha(n)}{n}\left[-c(q)+\frac{\beta_n}{\gamma}d\right]=J \tag{35}$$

if $-c(q)+\beta_n d/\gamma \geq J$, and $n=0$ otherwise, where J is the equilibrium expected utility of a seller at night. This constraint says that sellers will only show up if they get the J prevailing in the market; if $-c(q)+\beta_n d/\gamma \geq J$ then n will adjust to bring (35) into equality and if $-c(q)+\beta_n d/\gamma < J$ then sellers would not show up even if they could trade with probability 1.

Completeness means market makers choose (q, d, n) to maximize $W^b(z_b)$ subject to the constraint in (35). From (8) and (33), it is easy to check that $d = \hat{z}$ (i.e. the buyer will bring in just enough money to meet the posted d in the submarket of his choice) so that the problem can be reformulated as

$$\max_{q,n,\hat{z}}\left\{z_b+T-\left(1-\frac{\beta}{\gamma}\right)\hat{z}+\beta_d\alpha(n)\left[u(q)-\frac{\beta_n}{\gamma}\hat{z}\right]+\beta W^b(0)\right\} \tag{36}$$

subject to the same constraint. Ignoring constants in the objective function, this is equivalent to

$$\max_{q,z,n}\left\{\alpha(n)\left[u(q)-\frac{\beta_n}{\gamma}z\right]-\left(\frac{\gamma-\beta}{\beta}\right)\frac{\beta_n}{\gamma}z\right\} \tag{37}$$

subject to the same constraint. Letting $N(J)$ denote the set of solutions for n, we have the following results.

LEMMA 5: $N(J)$ is non-empty and upper hemi-continuous, and any selection from $N(J)$ is decreasing in J. For all $n \in N(J)$ such that $n > 0$, the corresponding q in that submarket satisfies

$$\frac{\gamma-\beta}{\beta\alpha(n)}+1=\frac{u'(q)}{c'(q)}. \tag{38}$$

PROOF: The objective function in (37) is continuous and, with no loss in generality, (q, z, α) can be restricted to the compact set

$$\Delta = \{(q, z, \alpha) : \alpha \in [0, 1], q \in [0, q^*], c(q) \leq \beta_n z/\gamma \leq u(q)\}.$$

Given this, the constraint (35) can be rewritten as $(q, z, \alpha) \in \Gamma(J)$ where $\Gamma(J)$ is a continuous and compact-valued correspondence. By virtue of the Theorem of the Maximum, the correspondence that gives the set of solutions for α is non-empty and upper hemi-continuous. Since $\alpha(n)$ is a bijection, $N(J)$ is non-empty and upper hemi-continuous.

We now show that any selection from $N(J)$ is decreasing in J. Consider $J^1 > J^0 > 0$ and denote by (q_i, z_i, n_i) a solution to (37) when $J = J^i$, for $i = 0, 1$. First, it is easy to check that if $n_0 = 0$ then $n_1 = 0$. Now consider the case where a solution to (37) is interior, $n > 0$. Substituting $\beta_n z/\gamma = Jn/\alpha(n) + c(q)$ from the constraint into (37), write the problem as $\max_{(n,q)} \Psi(n, q; J)$ where

$$\Psi(n, q; J) = \alpha(n)[u(q) - c(q)] - nJ - \left(\frac{\gamma - \beta}{\beta}\right)\left[\frac{n}{\alpha(n)}J + c(q)\right]. \quad (39)$$

Then $\Psi(n_0, q_0; J^0) \geq \Psi(n_1, q_1; J^0)$ and $\Psi(n_1, q_1; J^1) \geq \Psi(n_0, q_0; J^1)$, which implies

$$\left\{\left[\left(\frac{\gamma - \beta}{\beta}\right)\frac{n_1}{\alpha(n_1)} + n_1\right] - \left[\left(\frac{\gamma - \beta}{\beta}\right)\frac{n_0}{\alpha(n_0)} + n_0\right]\right\}(J^1 - J^0) \leq 0. \quad (40)$$

Since $n/\alpha(n)$ is strictly increasing in n, this implies $n_1 \leq n_0$. To show the inequality is strict, take the first-order conditions for q and n. These imply (38) and

$$\alpha'(n)[u(q) - c(q)] = J\left\{1 + [1 - \eta(n)]\left(\frac{\gamma - \beta}{\alpha(n)\beta}\right)\right\} \quad (41)$$

From (38), if $n_1 = n_0$ then $q_1 = q_0$, which is inconsistent with (41). *Q.E.D.*

We are ready to formally define competitive search equilibrium. This definition is slightly more involved than the ones in the previous sections because there may be multiple submarkets open in equilibrium and we have to keep track of where (to which submarket) agents go. The measure of buyers on a given submarket ω is denoted b. Also, we restrict our attention to equilibria where the set of open submarkets is countable.

DEFINITION 3: A *competitive search equilibrium* is a set of open submarkets Ω, for each $\omega \in \Omega$ a list $(q_\omega, z_\omega, n_\omega, b_\omega) \in \mathbb{R}_+^4$, and a $J \geq 0$ such that: (a) given J, for all $\omega \in \Omega$, $(q_\omega, z_\omega, n_\omega)$ maximizes (37) subject to the constraint that (35) holds if $n_\omega > 0$; (b) $\sum_\omega b_\omega = 1$; and if $n = \bar{n}$ then (c1) $\sum_\omega b_\omega n_\omega = \bar{n}$, or if we have free entry then (c2) $J = k$.

As in the previous models, we need some restriction on k to have $n > 0$.

ASSUMPTION 2"**:** $k < u(q^*) - c(q^*)$.

Now we have the following results.

PROPOSITION 6: (i) Assume $n = \bar{n}$. Competitive search equilibrium exists and J is uniquely determined. (ii) Assume free-entry. There is a $\bar{\gamma} > \beta$ such that equilibrium exists if and only if $\gamma \leq \bar{\gamma}$. For all $\gamma \in [\beta, \bar{\gamma}]$ equilibrium is generically unique.

PROOF: (i) Let $\tilde{N}(J)$ denote the convex hull of $N(J)$. The equilibrium conditions $\sum_{\omega \in \Omega} b_\omega n_\omega = \bar{n}$ and $\sum_{\omega \in \Omega} b_\omega = 1$, where $n_\omega \in N(J)$ for all ω, imply $\bar{n} \in \tilde{N}(J)$. We now describe $\tilde{N}(J)$ in detail, and depict it in Figure 3. If $J = 0$ then the market maker's problem becomes $\max_{(q,n)}\{\alpha(n)[u(q) - c(q)] - (\gamma - \beta)c(q)/\beta\}$ which implies $\tilde{N}(0) = \{\infty\}$. If $J > u(q^*) - c(q^*)$ then there is no $n > 0$ that satisfies (41) and therefore $\tilde{N}(J) = \{0\}$. Furthermore, it can be checked that $\tilde{N}(J)$ is convex-valued and upper hemi-continuous.

By virtue of Lemma 5 any selection from $\tilde{N}(J)$ is strictly decreasing in J for all $n > 0$. Therefore, there exists a unique $J \leq [u(q^*) - c(q^*)]$ such that $\bar{n} \in \tilde{N}(J)$.

(ii) Let $\mathcal{V}(k, \gamma)$ denote the value function defined by (37). For all $\gamma \geq \beta$ such that $\mathcal{V}(k, \gamma) > 0$, the solution to (37) is such that $n > 0$ and equilibrium exists. From the Theorem of the Maximum, $\mathcal{V}(k, \gamma)$ is continuous. Furthermore, it can easily be checked that $\mathcal{V}(k, \gamma)$ is decreasing in γ, and strictly decreasing when $n > 0$. From Assumption 2", $\mathcal{V}(k, \beta) = \max_{(q,n)} \{\alpha(n) [u(q) - c(q)] - nk\} > 0$. From (35), $\beta_n z/\gamma \geq k$, which implies that there is no interior solution to (37) for large enough values of γ. Consequently, there exists a threshold $\bar{\gamma} > \beta$ such that equilibrium exists if and only if $\gamma \in [\beta, \bar{\gamma}]$. Finally, given that $\alpha(n)$ is in the compact set $[0, 1]$ and strictly decreasing with J, there is at most a countable number of values for J such that $\tilde{N}(J)$ is not a singleton. *Q.E.D.*

INSERT FIGURE 3 ABOUT HERE

The curve $\tilde{N}(J)$ in Figure 3 can be interpreted as aggregate demand for sellers by market makers; it is the convex hull of the correspondence giving the value(s) of n solving the market maker's problem taking as given the price of sellers, J. It is downward sloping as the demand for sellers decreases with J. Without entry, J adjusts so that $\tilde{N}(J) = \bar{n}$; with entry, we have $J = k$ and the number of sellers adjusts. In Proposition 6 we show that equilibrium without entry always exists and J is uniquely determined. With entry, equilibrium exists assuming γ is not too high, and if it exists equilibrium is generically unique. The existence result is similar to what we found in previous models, but uniqueness here contrasts with the multiplicity found under bargaining and price taking. Intuitively, it reflects the fact that market makers effectively internalize any strategic complementarity between money demand and entry.

Assuming the solution to (37) is unique, all open submarkets must have the same (q, z, n). Eliminating J from the constraint and using (38), we can write (41) as

$$\frac{\beta_n}{\gamma} z = g\left[q, 1 - \eta(n)\right], \tag{42}$$

where g is defined in (17). Interestingly enough, (42) is the first order condition from the generalized Nash problem where the seller's bargaining power is $\eta(n)$; hence, in competitive search equilibrium the terms of trade *endogenously* satisfy the Hosios condition.

PROPOSITION 7: With either $n = \bar{n}$ or free-entry, the optimal policy is $\gamma = \beta$ and it implies equilibrium is unique and efficient.

PROOF: From (38), $q = q^*$ if and only if $\gamma = \beta$. From (35) and (42), the free-entry condition is

$$\frac{\alpha(n)}{n} \left\{ g\left[q, 1 - \eta(n)\right] - c(q) \right\} = k. \tag{43}$$

When $q = q^*$, (43) yields

$$\alpha'(n) \left[u(q^*) - c(q^*)\right] = k, \tag{44}$$

where we have used $g\left[q^*, 1 - \eta(n)\right] - c(q^*) = \eta(n)\left[u(q^*) - c(q^*)\right]$ from (17). Comparing (44) with (5), equilibrium is fully efficient if and only if $\gamma = \beta$. *Q.E.D.*

If $n = \bar{n}$ then equilibrium is efficient at the Friedman rule, basically because there is no holdup problem. A close examination of (37) suggests that competitive search equilibrium is equivalent to having buyers and sellers contract (commit to the terms of trade) before matching, which of course gets around the holdup problem. Hence competitive and competitive search equilibrium both yield efficiency along the intensive margin. When n is endogenous, competitive search equilibrium does more, because the Hosios condition arises endogenously; i.e. the extensive margin is also efficient at $\gamma = \beta$ because market makers internalize the effects of n on arrival rates.

6. CONCLUSION

We considered three different market structures for monetary economies: search equilibrium (bargaining), competitive equilibrium (price taking), and competitive search equilibrium (price posting with directed search). We found that efficiency and the effects of policy depend crucially on the market structure. Table 1 shows the efficiency properties of the different models at the Friedman rule. Regarding the intensive margin, $\gamma = \beta$ implies $q = q^*$ in competitive equilibrium and competitive search equilibrium, but $q < q^*$ in search equilibrium if $\theta < 1$. Regarding the extensive margin, n is generically inefficient in search equilibrium and competitive equilibrium, because these mechanisms do not generally internalize the effects of entry; efficient n requires the Hosios condition. In competitive search equilibrium the relevant condition holds endogenously, so n as well as q are efficient at the Friedman rule.

TABLE 1

	SE	CE	CSE
Intensive margin	$q < q^*$ if $\theta < 1$	$q = q^*$	$q = q^*$
Extensive margin	$n \gtrless n^*$	$n \gtrless n^*$	$n = n^*$

Table 2 shows the welfare effect of inflation for $\gamma \approx \beta$. With n exogenous, inflation has only a second-order effect in competitive and competitive search equilibrium, due the envelope theorem: $\mathcal{W}$ is maximized and $\partial\mathcal{W}/\partial\gamma = 0$ at $\gamma = \beta$. In the case of search equilibrium with $\theta < 1$, however, the envelope theorem does not apply: $\mathcal{W}$ is still maximized at $\gamma = \beta$, but $\partial\mathcal{W}/\partial\gamma < 0$ because we are at a corner solution ($\gamma = \beta$ is the minimum inflation rate consistent with equilibrium). Inflation has a first order effect in this case. With n endogenous, inflation decreases $\mathcal{W}$ in search equilibrium, but has an ambiguous effect in competitive equilibrium; it is possible to have $\partial\mathcal{W}/\partial\gamma > 0$. Finally, in competitive search equilibrium the envelope theorem applies to both q and n when n is endogenous, and so inflation has only a second-order effect on welfare near the Friedman Rule.

TABLE 2

	SE	CE	CSE
n exogenous	$\frac{\partial\mathcal{W}}{\partial\gamma} < 0$ if $\theta < 1$	$\frac{\partial\mathcal{W}}{\partial\gamma} \approx 0$	$\frac{\partial\mathcal{W}}{\partial\gamma} \approx 0$
n endogenous	$\frac{\partial\mathcal{W}}{\partial\gamma} < 0$	$\frac{\partial\mathcal{W}}{\partial\gamma} \gtrless 0$	$\frac{\partial\mathcal{W}}{\partial\gamma} \approx 0$

One can ask about the quantitative implications of the results. In Rocheteau and Wright (2004) we numerically study the models analyzed here by calibrating to standard data, and asking *how much* the welfare effects of inflation depend on the market structure.[21] The findings are as follows. In competitive search equilibrium our estimated welfare cost is very similar to previous estimates, such as those in Lucas (2000): going from 10% to 0% inflation is worth between 0.67% and 1.1% of consumption, depending on details of the calibration. In search equilibrium, the estimated cost can be

[21] We actually work with a slightly different framework in that paper, where instead of assuming a fixed number of buyers and free entry by sellers we let each agent choose whether to be a buyer or a seller.

between 3% and 5%, considerably bigger than what is found in most of the literature. In competitive equilibrium, the cost is somewhere between the other two models; while it is possible for positive inflation to be optimal this was not the case at the calibrated parameter values. While by no means definitive, we think these results are suggestive, and that it is worth pursuing further quantitative work on these models.

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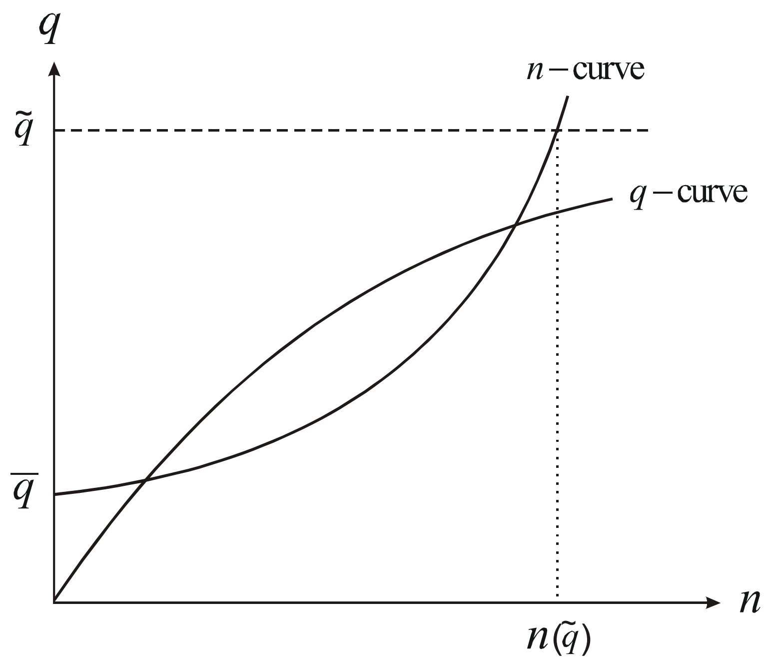

FIGURE 1–Search equilibrium

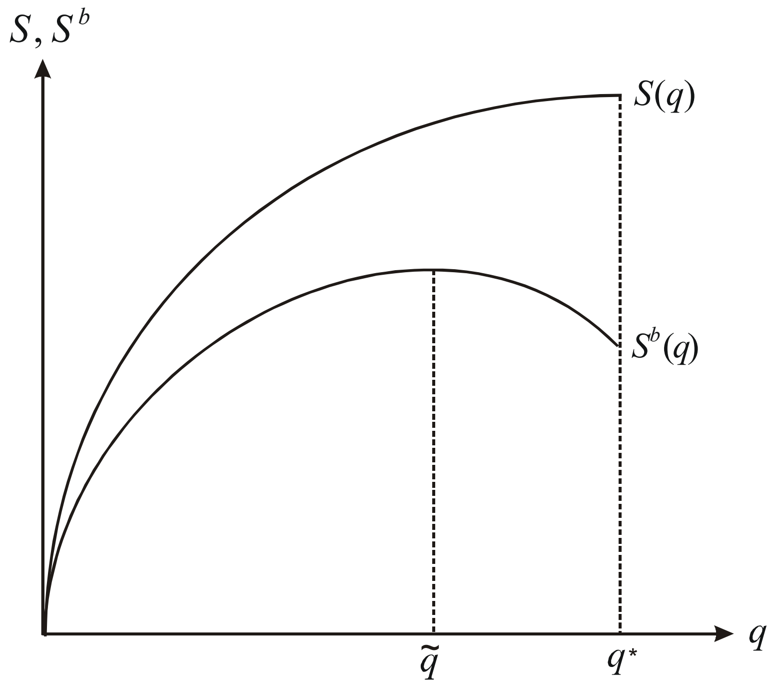

FIGURE 2–Surpluses

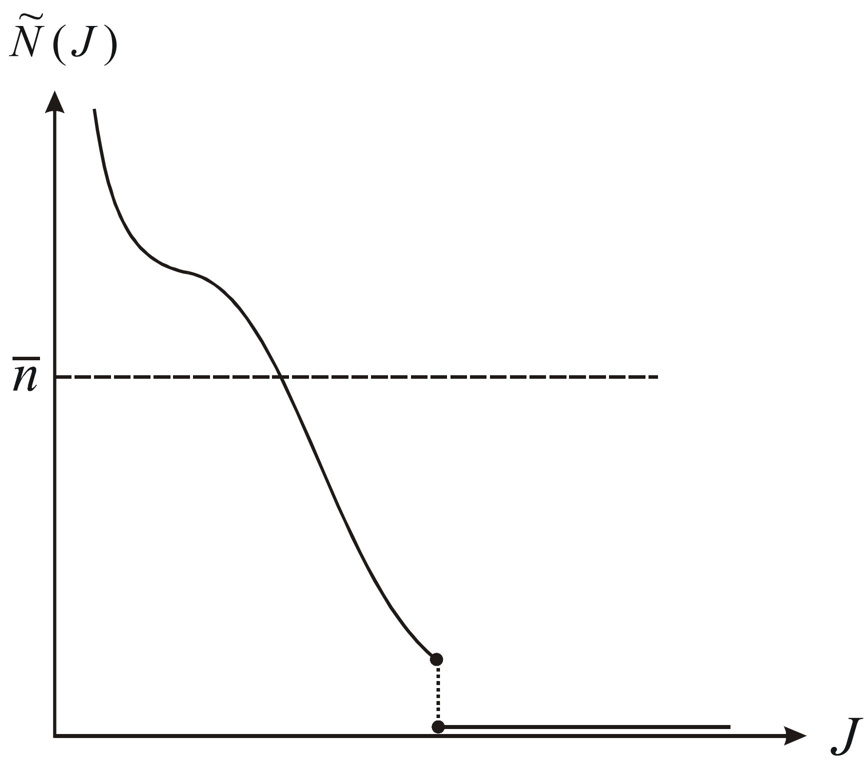

FIGURE 3 (a)–Competitive search equilibrium with $n = \bar{n}$.

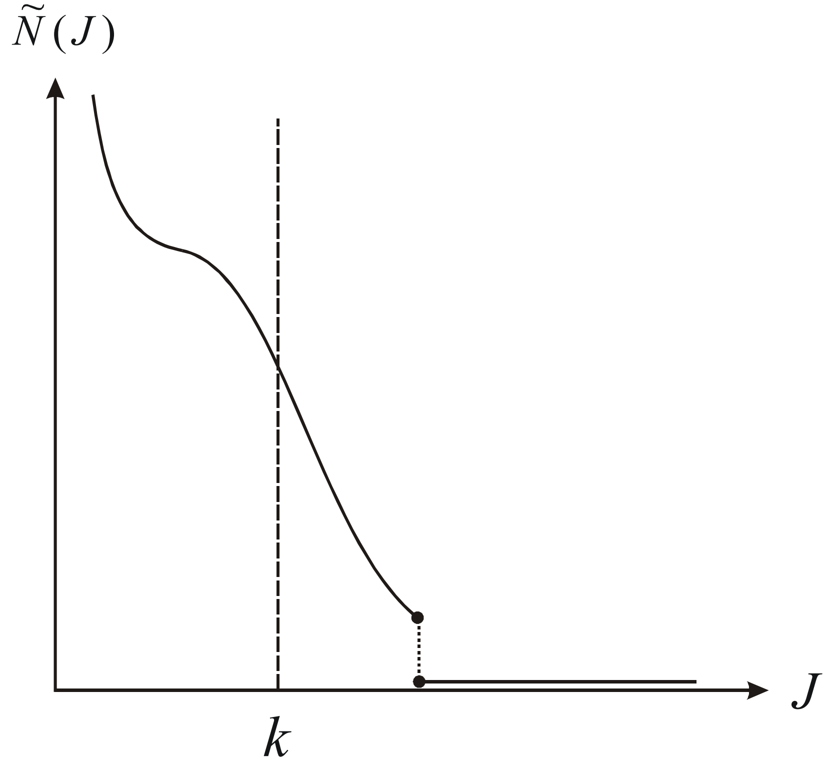

FIGURE 3 (b)–Competitive search equilibrium with free entry.

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