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INTERVENTION AND THE RISK PREMIUM IN FOREIGN EXCHANGE RATES

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ABSTRACT

The shift from a fixed-exchange-rate regime to a flexible regime, in which central-bank exchange-market intervention has been highly visible, has renewed interest in studying the effects of intervention. In separate work started by Engle (1982), new techniques have been developed to analyze risk premia in asset returns and particularly in exchange rates. We utilize a framework developed by Hodrick (1989) to show how central-bank intervention can affect both the level of exchange rates and the risk premium. We assume specific forms for preferences and for the stochastic processes of the exogenous variables and show how the risk premium is related to the conditional variances of intervention and the other exogenous processes. This approach differs from previous analyses of intervention by explicitly relating intervention to the risk premium. This lays the groundwork for future tests of the theory's implications for the intervention/risk premium relationship.

I. Introduction

Central-bank intervention in exchange markets has increased markedly since 1985, renewing interest among economists in understanding the effects of this activity. Although the current regime is ostensibly one in which rates are permitted to float, central banks commonly intervene to influence the level of exchange rates as well as to reduce the rates' volatility. Continued intervention is based on the belief that such actions indeed have the desired effect.

A more general interest in discerning the effects of intervention results from the potential significance of this activity as a policy instrument. If sterilized intervention (intervention that has no impact on monetary policy) can influence exchange rates, then policymakers have a third instrument (in addition to monetary and fiscal policy) with which to achieve their targets.

Determining the effectiveness of intervention also has implications for other policies. If bonds that differ only in currency denomination are perfect substitutes for one another, then intervention may be ineffective. However, this may imply that fiscal policy would be ineffective in a small, open economy with floating exchange rates (Siebert [1989]).

Intervention may influence the risk premium in exchange rates as well as the level of exchange rates. Although reducing exchange-rate volatility is a somewhat different objective than influencing the level of exchange rates, intervention for this purpose may indirectly influence the level of exchange rates, because changes in volatility may influence the risk premium that investors require in their return on foreign exchange.

Most recent studies of exchange-rate determination give the risk premium a prominent role. This can be traced partly to the failure of earlier theories that did not explicitly consider risk. The presence of a risk premium can explain a divergence of the rates of return between domestic and foreign assets, measured in the same currency (that is, a violation of uncovered interest parity).

As a result of such findings, we now have theories to explain how such a risk premium could arise. In addition, largely as a result of the work of Engle (for example [1982]), new techniques are now available to analyze time variation in conditional variances. Conditional variances may be closely tied to perceptions of future volatility and, thus, risk.

II. Channels of Influence in Central-Bank Intervention

To understand the mechanics of a typical spot-market intervention, consider a transaction designed to offset a dollar depreciation. In this case, the Federal Reserve would purchase dollars for marks on the spot market from a commercial bank. This would typically give the Federal Reserve two business days for delivery of marks. To finance the transaction, the Federal Reserve would sell mark securities held in accounts with the Bundesbank. The Bundesbank would act as the agent for the Federal Reserve, establishing an account for the U.S. central bank with the proceeds of the security transactions. The Federal Reserve would then settle the spot transaction with the commercial bank by drawing on its account with the

Bundesbank. The net effect is to decrease U.S. reserves and the monetary base.

Then, in order to sterilize the intervention (that is, offset its impact on reserves), the Federal Reserve may sell the equivalent amount of U.S. government securities, leaving as the only net effect of the two transactions a change in the Federal Reserve's and the private sector's portfolios of domestic and foreign assets. If the initial transaction is not sterilized, then it is equivalent to an open market operation. Since the impact of open market operations is presumably better understood than the impact of intervention, most studies of intervention focus on sterilized interventions.

Sterilized intervention could matter if the currency composition of debt influenced the exchange rate. In the portfolio-balance approach, exchange rates are determined by expected nominal rates of return on debt of different currency denominations. If investors care about portfolio risk and expected rates of return, and if bonds of different denominations are imperfect substitutes, then shifts in asset supplies will alter portfolio risk and induce changes in rates of return and in the exchange rate. This was the predominant approach to analyzing the effects of intervention in the 1970s.

Even if foreign and domestic assets are imperfect substitutes, intervention may not matter under Ricardian equivalence (see Obstfeld [1982]). In that case, agents do not regard the government bond holdings as part of net wealth, and fully capitalize future tax effects, neutralizing the impact of intervention. Backus and Kehoe (1988) emphasize the key role played by the government budget constraint in analyses of intervention. If other government policies are changed, then the impact of the overall operation depends on the structure of the economy and on the exact nature of the policy change. However, under Ricardian equivalence, exchange rates are unaffected by intervention if lump-sum taxes are levied on the representative consumer.

Another channel through which intervention may matter is its effect on expectations of economic conditions or policies. In particular, intervention may provide a credible signal of changes in future monetary and/or fiscal policies. Exactly why intervention would be chosen as the signal is unclear. However, once the central bank has intervened, it may stand to lose money by not following through on the expected policy. For example, if the U.S. central bank purchases dollar-denominated bonds and sells foreign currency bonds to signal its intention to allow the price of dollars to rise, it has an incentive to increase the price of dollars and thus the value of its holdings. Recent research analyzing other possible incentive effects of central-bank intervention is summarized by Obstfeld (1989a).

III. Does Intervention Matter?

Most empirical studies conclude that intervention does not influence exchange rates. Many of these studies indirectly examine the influence of intervention by testing the hypothesis of perfect substitutability of bonds that differ in currency denomination. The usual technique is to regress either exchange rates or the difference between the rates of return on foreign and domestic bonds (the covered-interest parity condition) on measures such as relative supplies of debt denominated in different currencies. Numerous studies, summarized by Weber (1986) and Henderson (1984), include asset supplies as explanatory variables and find evidence against imperfect substitutability. On the other hand, Danker et al. (1985), Loopesko (1984), and Johnson (1988) find evidence for imperfect substitutability. However, little of the variation in the dependent variable can be explained by relative debt supplies. This, in turn, implies that intervention is not likely to have much impact, since it is small relative to the debt aggregates.

The previous discussion of the role of the government budget constraint and the tenuous link between perfect substitutability and the effects of intervention should make us cautious in interpreting these results. Without having specified and controlled for possible effects operating through the budget constraint, these empirical studies may be misspecified.

Recent investigations have implied a role for intervention as a signal. Domingues (1988) finds that U.S. intervention has played a role in signaling changes in monetary policy, but that the effectiveness of intervention depends on the credibility of the monetary policy. When actual and announced monetary policies are inconsistent, intervention may be used to send a false signal to the market. Thus, intervention should be considered part of overall monetary policy. Humpage (1988) finds that intervention has an initial, one-time impact if it is supported by consistent statements of changes in monetary and fiscal policy and by coordinated action of central banks.

There is some evidence that Canadian central-bank intervention has systematically reduced short-run exchange-rate fluctuations (Pippenger and Phillips [1973]). However, this conclusion is disputed by Sweeney (1981).

IV. Risk in Exchange Rates

Evidence

A wide variety of evidence suggests that there is a risk premium component to exchange rates (see Hodrick [1987]). Violation of the uncovered-interest parity condition (expected profits to forward speculation should be zero) and the poor out-of-sample predictive performance of log-linear exchange-rate models relying on first moments suggest a risk premium. However, evidence of a risk premium has been synonymous with the failure of previous theories of exchange-rate determination. Not all investigators are convinced that a risk premium exists (for example, Froot and Frankel [1989]). Expectational errors may explain the above anomalies. Tests of the parity condition involve the joint hypothesis of market efficiency, perfect substitution, and capital mobility. Such considerations further complicate interpretation of the results.

Many empirical investigations into the risk premium in foreign-exchange rates model risk with time variation in conditional variance using Autoregressive Conditional Heteroscedasticity (ARCH). Useful discussions of this literature are found in Hodrick (1987) and Frankel (1989). Pagan and Hong (1988) and Nelson (1987) question the appropriateness of the

ARCH formulation. Other investigators (for example, Lyons [1988]) extract variances implied by options-pricing formulas and find time variation in "risk." However, the significance of the magnitude and time variation in the risk premium is unclear.

Theory

Exchange rates have been at various times viewed as the relative prices of currencies, the relative prices of domestic versus foreign goods, and the relative price of assets denominated in different currencies. However, as Dornbusch (1985) states, "...it becomes readily apparent that in most instances real, monetary, and financial considerations interact in the determination of exchange rates."

In models of the risk premium that incorporate optimization and equilibrium behavior under uncertainty, the risk premium will depend on the risk preferences of the consumers, on other parameters of the model, and on the stochastic properties of exogenous variables such as money. Lucas (1982) and Siebert (1989) present contrasting theoretical approaches to the determination of exchange rates in general equilibrium under uncertainty.

Tests of theoretical models of the risk premium are growing in number. In international capital asset pricing models of mean-variance optimizing consumers, time variation in risk should be related to time variation in the covariance matrix of asset returns. Examples of this approach are Engel and Rodrigues (1987), Giovannini and Jorion (1989), and Mark (1988). Hodrick (1989), Cumby (1988), and Obstfeld (1989b) test consumption-based asset pricing models in which the risk premium is related to time variation in the stochastic processes of the exogenous variables, including money. Both approaches have had limited success in explaining risk premia.

The role of intervention in explaining foreign exchange risk is largely unexplored. One reason may be that early investigations focused on the ability of debt variables to explain the deviation from interest-rate parity, with that deviation being a measure of risk. However, there is evidence that the volatility of exchange rates has varied across monetary policy regimes (Lastrapes [1989]) and that the impact of intervention is related to monetary policy (Domingues [1988] and Humpage [1988]).

V. The Model

The theoretical model we present provides testable hypotheses about the influence of intervention on the risk premium in foreign exchange rates. The consumption-based asset pricing model of Hodrick (1989) is modified for this task. In his model, the risk premium in the exchange rate is a function of the conditional variances of money, government's share in production, and production itself. Simplifying assumptions about preferences and about the stochastic properties of exogenous variables are necessary in order to derive closed-form solutions indicating the relations among the exchange rate, the risk premium, and the first and second moments of the exogenous processes.[1] Without such assumptions, it is difficult to say much about the likely impacts of intervention on the risk premium.[2]

Our model differs from Hodrick's mainly by including intervention. In Hodrick's model, consumers and governments each face cash-in-advance (CIA) constraints, and the total stock of each currency is split between private and governmental holdings. We model intervention in terms of **governments'** holding of foreign currencies. Intervention is actually variation in the stock held, influencing the amount of currency available for private or **government** consumption. In Hodrick's model, the variability, as well as the level, of private money influences exchange rates and the risk premium. Thus, in our model, the level, as well as the variability, of intervention influences the rate and its risk premium. In effect, knowledge of the stochastic process describing intervention improves the ability of monetary aggregates to predict exchange rates.

Endowments and Timing

Two countries, indicated with subscripts 1 and 2, each produce one good, which is also the endowment of each country. The realizations of the two exogenous, nonstorable goods are denoted Y_{1t} and Y_{2t}. We assume that the goods markets are open at the start of the period and that asset markets are open at the end. It is convenient to think of each household as comprised of two agents, one that takes the accumulated cash out for shopping, and another that subsequently enters the asset market to purchase cash, bonds, and equities.

Information about the state of the world (detailed below) becomes available at the start of the period. The government and the private shopper enter the goods market with available cash balances. The government's cash balance can be augmented through new currency issue and is also influenced by intervention. Any remaining cash balances, in addition to the gross returns on bonds and stocks, become available to the consumer for the subsequent asset markets. Lump-sum taxes or transfers are also levied in the second half of the period.

Government

Each government purchases some of the endowment of its own country, collects lump-sum taxes, supplies state-contingent claims to its own currency, prints its own currency, and intervenes in the foreign exchange market by purchasing some of the foreign currency. The real quantity of government i's purchases of good i at time t is G_{it}. Because consumers do not value government spending, variation in G_{it} affects the amount of the endowment available for consumption. τ_{it} is the lump-sum tax levied by government i in the asset market. $B_{it+1}(x_{t+1})$ is the amount of money i that government i promises to pay if state x_{t+1} occurs. Its currency i value at time t in state

x_t is $n_i(x_{t+1},x_t)$. The gross growth rate of money i over period t, M_{it+1}/M_{it}, is denoted Ω_{it}. The outstanding amount of money i and the amount held by the foreign government at the end of period t-1 are denoted M_{it} and M^F_{it}, respectively. Nominal government purchases of endowment i in the time t goods market are constrained by the government's holding of currency i cash balances at the start of period t, $M^G_{i,t}$, plus any additional currency i to be supplied.

In Hodrick (1989), the additional amount represents the amount printed by government i and supplied in the asset market. Here, however, governments purchase foreign currency and do not spend it. So, the additional amount of currency i to be made available is the amount printed net of the increase in foreign holdings of the currency. This CIA constraint can be expressed as

$$(1)\quad P_{it}G_{it} \leq M^G_{it} + (M_{it+1} - M_{it}) - (M^F_{it+1} - M^F_{it}).$$

The holdings of the foreign currency have no effect other than to reduce the amount of currency available to purchase foreign goods. For simplicity, we ignore any effect of govemment earnings on foreign reserves.[3]

Expression (2) is the government budget constraint.

$$(2)\quad G_{it} = \tau_{it} + \frac{\int n_i(x_{t+1},x_t)B_{it+1}(x_{t+1})\,dx_{t+1} - B_{it}(x_t)}{P_{it}} + \frac{(M_{it+1} - M_{it})}{P_{it}}\ ,\quad i=1,2,$$

where P_{it} is price in currency i of the good/endowment of country i.

Agents' Preferences and Constraints

Following Hodrick (1989), we assume that all agents' preferences are homothetic and, thus, that there is a representative consumer in each country. Preferences and initial wealth levels of the two consumers are assumed to be identical and each consumer is taxed equally by the two countries. Each representative consumer maximizes expected lifetime utility as in

$$(3)\quad E_0 \sum_{t=0}^{\infty} \beta^t U(C_{1t}, C_{2t}),\ 0 < \beta < 1,$$

by choosing C_{1t} and C_{2t} and by making her savings decisions.

The consumer in each country faces two constraints: a CIA constraint and a budget constraint. The CIA constraint, expressed in real terms, shows that purchases of good i are constrained to be no greater than the amount of currency i held by the consumer when she enters the goods market:

$$(4)\quad C_{1t} \leq M^p{}_{1t}\Pi_{1t},$$

$$(5)\quad \Theta_t C_{2t} \leq M^p{}_{2t}\Pi_{2t}.$$

Here $\Pi_{1t} = 1/P_{1t}$ is the good one purchasing power of currency one, and $\Pi_{2t} = S_t/P_{1t}$ is the good one purchasing power of currency two. S_t is the exchange rate of currency one per unit of currency two, and $\Theta_t \equiv S_t P_{2t}/P_{1t}$ is a "real terms of trade," although goods cannot be exchanged directly in the model. Note also that monies cannot be exchanged directly in the goods markets. [4]

Purchases of assets are constrained by the agent's wealth at the time she enters the asset market, after having made her consumption choices, net of taxes. Wealth includes unspent monies, realizations on previous purchases of state-contingent bonds, and realizations on equity shares. Agents in each country can buy and trade titles to the endowments of each country. The number of titles to the endowment of country i purchased in the asset market at time t is denoted Z_{it+1}. The associated currency one price is denoted Q_{it}. For convenience, we assume that there is just one share of the endowment for each country. The period t budget constraint, identical to Hodrick (1989), is reproduced here:

$$(6)\quad \Pi_{1t}M^{p}_{1t+1} + \Pi_{2t}M^{p}_{2t+1} + \Pi_{1t}\int n_1(x_{t+1},x_t)B^{p}_{1t+1}(x_{t+1})dx_{t+1} + \Pi_{2t}\int n_2 x(x_{t+1},x_t)B^{p}_{2t+1}(x_{t+1})dx_{t+1}$$

$$+ \psi_{1t}Z_{1t+1} + \psi_{2t}Z_{2t+1} \le (\Pi_{1t}M^{p}_{1t}-C_{1t}) + (\Pi_{2t}M^{p}_{2t} - \Theta_t C_{2t}) + \Pi_{1t}B^{p}_{1t}(x_t) + \Pi_{2t}B^{p}_{2t}(x_t)$$

$$+ (\psi_{1t}+Y_{1t})Z_{1t} + (\psi_{2t}+\Theta_t Y_{2t})Z_{2t} - (1/2)(\tau_{1t}+\Theta_t\tau_{2t}),\ \text{where } \psi_{it} \equiv Q_{it}/P_{1t}.$$

Agent's Solutions

The agent chooses consumption of both goods, holdings of both currencies, state-contingent claims to both currencies, and titles to both endowments. The future states of the world are uncertain to the consumers, but there is a known, first-order Markov density, $F(x_{t+1}|x_t)$, between the states of the world at times t and t+1. Utility maximization is subject to the wealth constraint and the two CIA constraints. The optimality conditions, listed in appendix A, are identical to those in Hodrick (1989).

The marginal utility of consumption is not necessarily equated to the marginal value of wealth unless the CIA constraint is assumed binding. The choice of money holding will equate the current real value of wealth to the expected marginal utility of money in the next period, which will depend on the marginal values of wealth and money then. The Euler equations for the nonmoney assets differ from those for money, since bonds and stocks provide no return until consumption in the next period has occurred.

Equilibrium

The definition of the equilibrium is identical to Hodrick but for the inclusion of intervention as an additional exogenous process. The equilibrium is defined as the initial stocks of monies and bonds $(M_{i0}, B_{i0},\ i=1,2)$, the stochasic processes for the exogenous variables $(Y_{it}, G_{it}, \tau_{it}, M^G_{it+1}, \zeta_{it+1},$

M_{it+1}, i=1,2, t=0 to ∞), choice variables ($C_{it}, M^P_{it+1}, B^P_{it+1}, Z_{it+1}$, i=1,2, t=0 to ∞), the prices ($\Pi_{it}, \Theta_t, \psi_{it}$, i=1,2, t=0 to ∞), and the pricing functions $n_i(x_{t+1}, x_t)$, i=1,2 such that 1) budget constraints are satisfied, 2) the household's decisions solve the maximization problem, and 3) the following market-clearing conditions are satisfied:

(7a) $Z_{it+1} = \frac{1}{2}$,

(7b) $B_{it+1}(x_{t+1}) = 2B^P_{it+1}(x_{t+1})$, i = 1,2,

(7c) $2C_{it} + G_{it} = Y_{it}$, i = 1,2, and

(7d) $M_{it+1} = M^F_{it+1} + M^G_{it+1} + 2M^P_{it+1}$, i=1,2.

Closed-Form Solutions

In order to show explicitly how intervention can influence the exchange rate and the risk premium in the exchange rate, we assume particular stochastic processes for the exogenous variables. We follow Hodrick regarding the assumed processes, noting the key role played by assumptions about the stochastic independence of exogenous variables. Hodrick examines variation in government's share of output as an independent exogenous variable. Government expenditures influence the amount of output available for consumption.

Below we define the relevant variable as consumption's share of output, which, given the assumptions of the theory, just equals one minus the government's share.[5] Lower-case letters denote logarithms, and ω_{it+1} denotes the logarithm of the gross growth rate of currency i, $\Omega_{it+1}=M_{it+2}/M_{it+1}$. We assume conditional log-normality for outputs and gross money-growth rates.[6] We define the proportion of currency i held by the foreign government by $\zeta_{it} = M^F_{it}/M_{it}$ and assume that the ζ_{it}s and the consumption shares χ_{it} (defined as $[Y_{it}-G_{it}]/Y_{it}$) are conditionally uniform in distribution. Formally, these assumptions are

(8a) $y_{1t+1} = \rho_1 y_{1t} + (1-\rho_1)y_1 + \xi_{1t+1}$,

(8b) $y_{2t+1} = \rho_2 y_{2t} + (1-\rho_1)y_2 + \xi_{2t+1}$,

(8c) $\omega_{1t+1} = \rho_3 \omega_{1t} + (1-\rho_3)\omega_1 + \xi_{3t+1}$,

(8d) $\omega_{2t+1} = \rho_4 \omega_{2t} + (1-\rho_4)\omega_2 + \xi_{4t+1}$,

(8e) $\chi_{1t+1} = \rho_5 \chi_{1t} + (1-\rho_5)\chi_1 + \xi_{5t+1}$,

(8f) $\chi_{2t+1} = \rho_6 \chi_{2t} + (1-\rho_6)\chi_2 + \xi_{6t+1}$, (8g) $\zeta_{1t+1} = \rho_7 \zeta_{1t} + (1-\rho_7)\zeta_1 + \xi_{7t+1}$,

(8h) $\zeta_{2t+1} = \rho_8 \zeta_{2t} + (1-\rho_8)\zeta_2 + \xi_{8t+1}$,

where $0 \le |\rho_i| \le 1$, i=1 to 8, and each ξ_{it+1}, i=1 to 4 is normally distributed with conditional mean equal to zero and conditional variance denoted h_{it}. However, each ξ_{it+1}, i=5 to 8 is distributed uniformly on the interval $[-h_{it}, h_{it}]$ with conditional mean of zero but conditional variance given by $(h_{it})^2/3$. We also assume that the ξ_{it+1}s are independent of each other. The conditional variances are described by the following autoregressive processes:

(9) $E_t(h_{it+1}) = \phi_i h_{it} + (1-\phi_i)h_i$, i=1,2,3,4.

Here the term on the left-hand side is just $E_t[E_{t+1}(\xi_{it+2}{}^2)]$, and the h_is are the unconditional variances. The conditional and unconditional variances of both the foreign money shares and the consumption shares are denoted $(h_{it})^2/3$ and $(h_i)^2/3$, respectively. The state of the economy, x_t, is defined as $\{y_{it}, m_{it+1}, \omega_{it}, \chi_{it}, \zeta_{it+1}, \tau_{it}, h_{jt}$, i=1,2, j=1,8, t=0 to $\infty\}$, and the τ_{it} and

x_t vectors are Markov processes.

As in Hodrick (1989), we assume the following utility function:

(10) $U(C_{1t}, C_{2t}) = [1/(1-\gamma)]C_{1\ t}^{1-\gamma} + [1/(1-\delta)]C_{2\ t}^{1-\delta}$.

Here we have assumed constant relative risk aversion. The magnitude of the parameter of risk aversion (which is also equal to the parameter expressing intertemporal substitution) will influence the response of prices such as the exchange rate to shocks from processes such as intervention.

In addition, we assume that the CIA constraints hold with equality, implying constant unitary velocity of money.' However, Hodrick, Kocherlakota, and Lucas (1989) indicate that relaxing the constraint is not likely to alter velocity greatly. When combined with market clearing, the binding constraints imply the following key relations:

(11) $\Pi_{1t} = Y_{it}/[M_{1t+1}(1-\zeta_{1t+1})]$,

(12) $\Pi_{2t} = \Theta_t Y_{2t}/[M_{2t+1}(1-\zeta_{2t+1})]$.

Here, since endowments must be consumed, changes in end-of-period-t foreign holdings of currency one impact the price of good one in that period by reducing money available for purchases, given the total available, M_{1t+1}. Although set in the goods market before the money is injected, the goods price is influenced by intervention, since the government's purchases indicate the amount of money (net of the amount absorbed by the foreign government) that the government must inject into the asset market.

S_t, the spot market currency one price of currency two, can be expressed as

$$(13)\quad S_t = \frac{\Pi_{2t}}{\Pi_{1t}} = \frac{\Theta_t (Y_{2t}/M_{2t+1}\ [1-\zeta_{2t+1}])}{(Y_{1t}/M_{1t+1}\ [1-\zeta_{1t+1}])}.$$

Use of the optimality conditions yields the general form of S_t:

$$(14)\quad S_t = \frac{Y_{2t}M_{1t+1}(1-\zeta_{1t+1})\beta 2^{\delta}E_t\{E_tY_{2t+1}{}^{1-\delta}\chi_{2t+1}{}^{-\delta}/(Y_{2t}\Omega_{2t+1}[1-\zeta_{2t+2}]/[1-\zeta_{2t+1}])\}}{Y_{1t}M_{2t+1}(1-\zeta_{2t+1})\beta 2^{\delta}E_t\{Y_{1t+1}{}^{1-\gamma}\chi_{1t+1}{}^{-\gamma}/(Y_{1t}\Omega_{1t+1}[1-\zeta_{1t+2}]/[1-\zeta_{1t+1}])\}}.$$

Assuming that money (net of intervention) is independent of the growth rate of money (net of intervention) and the other variables in (14) yields expression (15) for the natural logarithm of the exchange rate

$$(15)\quad s_t = \ln[M_{1t+1}(1-\zeta_{1t+1})] - \ln[M_{2t+1}(1-\zeta_{2t+1})]$$

$$+ \ln E_t[\chi_{2t+1}{}^{-\delta}Y_{2t+1}{}^{1-\delta}] + \ln E_t[\frac{1}{\Omega_{2t+1}[(1-\zeta_{2t+2})/(1-\zeta_{2t+1})]}]$$

$$+ \ln[\frac{1}{E_t(\chi_{1t+1}{}^{-\gamma}Y_{1t+1}{}^{1-\gamma})}] + \ln[\frac{1}{E_t(1/\{\Omega_{1t+1}(1-\zeta_{1t+2})/(1-\zeta_{1t+1})\})}].$$

Expression (15) shows clearly that increases in M_{1t+1} or decreases in ζ_{1t+1} depreciate currency one (S_t is the currency one price of currency two). Either way, the purchasing power of currency one falls. The effect of a higher endowment of good one depends on the parameter γ, which indicates intertemporal substitutability. An increase in the endowment of good one will increase the value of currency one, since cash must be accumulated in advance of purchases. An increase in the expected foreign holdings of currency one in the next period will reduce the amount expected to be available for purchases, increase its future expected value, and thus induce increased demand now, leading to appreciation of currency one.

To arrive at an expression for the logarithm of the exchange rate in terms of observable variables and conditional variances, we utilize the distributions of the exogenous processes and assume that the M_{it+1}s and the ζ_{it+1}s are independent and known at time t.[8] In addition, we replace $\ln(1-\zeta_{it+j})$ by its first-order approximation, $-\zeta_{it+j}$, to yield expression (16).[9] The theoretical values of the coefficients in (16) are given in appendix B.

$$(16)\quad s_t = \alpha_{so} + \alpha_{s1}m_{1t+1} - \alpha_{s2}m_{2t+1} - \alpha_{s3}\Xi_{1t} + \alpha_{s4}\Xi_{2t} - \alpha_{s5}y_{1t} + \alpha_{s6}y_{2t}$$
$$+ \alpha_{s7}\omega_{1t} - \alpha_{s8}\omega_{2t} - \alpha_{s9}h_{1t} + \alpha_{s10}h_{2t} - \alpha_{s11}h_{3t} + \alpha_{12}h_{4t}$$
$$- \alpha_{s13}\zeta_{1t+1} + \alpha_{s14}\zeta_{2t+1} + \alpha_{s15}(h_{7t+1})^2 - \alpha_{s16}(h_{8t+1})^2.$$

Here we define Ξ_{1t} as $\ln\{E_t[\chi_{1t+1}{}^{-\gamma}]\}$ and Ξ_{2t} as $\ln\{E_t[\chi_{2t+1}{}^{-\delta}]\}$.

In expression (16) there are multiple channels through which current monetary conditions influence the exchange rate. An increase in either money supply (M_{it+1}) directly affects s_t and provides information about future money, since the logs of the gross growth rates of money are autocorrelated. An increase in the conditional variance of the endowment for good one, (h_{1t}), will increase the value of currency one to the extent that consumers are risk-averse. An increase in the conditional variance of the growth rate of currency one causes it to appreciate, since the conditional variance influences expectations of future purchasing power. The intervention variables, ζ_{it+1}, do not have the one-for-one influence of the money stock, because they also impact the expected growth rates of money available for purchases. Conditional variance in intervention helps predict variability in the purchasing power of money, since the endowment must be consumed in equilibrium.

Intervention and the Risk Premium in the Exchange Rate

An expression for the risk premium can be developed from the interest-rate parity condition, expressed in equation (17). Arbitrage implies equality between the rates of return on investing currency one in bonds of country one, then converting to currency two and investing in country one bonds, and then selling the proceeds forward.

$$(17)\ \exp(i_{1t}) = (1/S_t)\exp(i_{2t})F_t.$$

F_t is the forward price at time t of delivery and payment in time t+1. A commonly studied expression for the risk premium is $E_t(s_{t+1})-f_t$, which (17) implies is equal to $E_t(s_{t+1}-s_t)-(i_{1t}-i_{2t})$.[10] Expression (18), derived from the optimality conditions, yields the interest rate in country one:

(18) $\exp(-i_{1t}) =$

$$\beta E_t\{\frac{[Y_{1t+1}/\{M_{1t+2}(1-\zeta_{1t+2})\}]\beta 2^{\gamma}E_{t+1}\{\chi_{1t+2}{}^{-\gamma}Y_{1t+2}{}^{1-\gamma}/[Y_{1t+1}(M_{1t+3}/M_{1t+2})([1-\zeta_{1t+3}]/[1-\zeta_{1t+2}])]\}}{[Y_{1t}/\{M_{1t+1}(1-\zeta_{1t+1})\}]\beta 2^{\gamma}E_t\{\chi_{1t+1}{}^{-\gamma}Y_{1t+1}{}^{1-\gamma}/[Y_{1t}(M_{1t+2}/M_{1t+1})([1-\zeta_{1t+2}]/[1-\zeta_{1t+1}])]\}}\}.$$

Assuming independence between total money supplies, intervention variables, and endowment processes and taking logarithms of both sides, we can derive expression (19).

$$\text{(19)}\quad -i_{1t} = \ln\beta + \ln E_t(\frac{M_{1t+1}}{M_{1t+2}}) + \ln E_t[\frac{(1-\zeta_{1t+1})}{(1-\zeta_{1t+2})}] + \ln E_t E_{t+1}(\chi_{1t+2}{}^{-\gamma}Y_{1t+2}{}^{1-\gamma})$$

$$+ \ln E_t E_{t+1}(\frac{1}{[M_{1t+3}]/[M_{1t+2}]}) + \ln E_t E_{t+1}(\frac{1}{[1-\zeta_{1t+3}]/[1-\zeta_{1t+2}]})$$

$$+ \ln E_t[\frac{1}{E_t(\chi_{1t+1}{}^{-\gamma}Y_{1t+1}{}^{1-\gamma})}] + \ln E_t\{\frac{1}{E_t[1/(M_{1t+2}/M_{1t+1})]}\}$$

$$+ \ln E_t\{\frac{1}{E_t[1/\{(1-\zeta_{1t+2})/(1-\zeta_{1t+1})\}]}\}.$$

Utilizing the assumed stochastic processes of the exogenous variables, we arrive at expression (20) for the interest rate in country one. The theoretical values of the coefficients are presented in appendix B.

$$(20)\quad i_{it} = \alpha_{i10} + \alpha_{i11}\Xi_{1t} + \alpha_{i12}\{\ln E_t[\Xi_{1t+1}{}^{-\gamma}]\} + \alpha_{i13}(y_{1t}-y_1) + \alpha_{i14}(\omega_{1t}-\omega_1) + \alpha_{i15}h_{1t} + \alpha_{i16}h_{3t} + \alpha_{i17}(\zeta_{1t+1}-\zeta_1) + \alpha_{i18}(h_{7t})^2.$$

An increase in y_{1t} increases the interest rate in country one if ρ_1 and y are between 0 and 1. The increased demand for money will increase the current purchasing power of money. However, the endowment will return toward its unconditional mean, and the purchasing power of money will fall in the next period. This increase in expected inflation increases i_{1t}. However, the increase in current consumption decreases current marginal utility and leads to intertemporal substitution, which may amplify or reduce this effect. An above-average money growth rate will be followed by another increase in the money supply (although a smaller increase in the growth rate) and thus an increase in expected inflation. An increase in intervention in currency one (increased foreign holding of that currency) increases the purchasing power of the remaining currency one, but will be followed by a decrease in purchasing power as, in the next period, ζ_{1t+1} declines towards its average. Unless swamped by intertemporal substitution, an increase in the conditional variance of good one increases i_{1t}. Risk-averse consumers would desire to hold less of currency one, since good one is more risky. Thus, the purchasing power of currency one rises but is anticipated to fall in the next period. Increased variance of the intervention in currency one increases the interest rate because it implies that the purchasing power of that currency is likely to fall.

Utilizing the analogous expression for i_{2t} and an updated version of s_{t+1}, we derive the expression (21) for the risk premium. The theoretical values of the coefficients are found in appendix B.

$$(21)\quad E_t(s_{t+1}-f_t) = \alpha_{r1}h_{1t} - \alpha_{r2}h_{2t} + \alpha_{r3}h_{3t} - \alpha_{r4}h_{4t}$$

$$- \alpha_{r5}\{E_t\Xi_{1t+1}-\ln E_t(\chi_{1t+2}{}^{-\gamma})\} + \alpha_{r6}\{E_t\Xi_{2t+1}-\ln[E_t(\chi_{2t+2}{}^{-\delta})]\}$$

$$+ \alpha_{r7}(h_{7t+1})^2 - \alpha_{r8}(h_{8t+1})^2.$$

If the conditional variances of both endowments increase by the same amount, the risk premium is unaffected if $\rho_1 = \rho_2$. Analogous statements can be made for the conditional variances of money-growth rates. The extent to which equal changes in conditional variances offset one another depends on the extent to which such changes are expected to be propagated into the future. Increasing the conditional variance of foreign holdings of currency one increases the risk premium, here defined in terms of currency one/currency two. Increasing the other conditional variance has the opposite effect. However, if the conditional variances of both intervention variables increase and are propagated equally into the future, there is no effect on the risk premium. Expression (21) makes clear the need to distinguish between the variation in total money supplies and the components.

VI. Conclusion

In this paper we have modified a model developed by Hodrick (1989) to show how intervention can influence the foreign-exchange risk premium. Unlike previous studies of intervention, we specify the mechanism through which intervention should impact the risk premium in exchange rates. While previous studies of intervention have analyzed sterilized intervention, here we model intervention as changes in foreign governments' holdings of domestic currency. The proportion of currency held by the foreign government as well as the conditional variance of that proportion can influence the level of the exchange rate. The risk premium is shown to be a function of the conditional variance of the intervention variable as well as the conditional variances of the other exogenous variables, including the total money supplies. Future work will test the theory's implications for the intervention/risk premium relationship.

Footnotes

1. See Siebert (1989) for an example of an analysis of the determinants of the risk premium that avoids parameterization of preferences and distributions of the exogenous variables.

2. Of course, the assumptions may be inappropriate for the application at hand. Pagan and Hong (1988) discuss problems with the ARCH formulation as employed by Hodrick (1989). Cumby (1988) cites the assumption of time-separability as a possible explanation of the failure of one particular version of the consumption-based asset pricing model to explain risk premia in forward speculation.

3. Here we do not assume sterilization. Leahy (1989) discusses the significance of earnings on foreign reserves, indicating that such earnings are not large enough to have much of an impact. In any case, the effects of the disposition of such earnings involve issues similar to those raised regarding the impact of portfolio balance effects.

4. See Stockman and Svensson (1987), p. 183 for the solution of a similar model when currencies can be exchanged directly in the "goods" market.

5. Of course, one can argue that these shares are not independent of overall output. However, it may be of interest to follow other empirical work and to examine the relation between variation in consumption and exchange rates (for example, Cumby [1988]).

6. Pagan and Hong (1988) claim that assuming linearity in the conditional mean exaggerates the true volatility in such series. They claim that nonparametric estimation of the conditional mean and conditional variance implies different results. Diebold and Nason (1989) argue that it is unlikely that out-of-sample predictive performance for exchange rates will be improved by taking advantage of nonlinearities in conditional means.

7. See Stockman and Svensson (1987), p. 175 for a discussion of how assumptions about the timing of information alters this result in related models.

8. The assumption that both the M_{it+1}s and the M^F_{it+1}s are known at the start of period t is unnecessary to yield a closed-form solution. A binding CIA constraint implies only that $M_{it+1}(1-\zeta_{it+1})$ is known at the start of the period. However, agents would presumably make use of their knowledge of this net amount in forming their expectations of money variables dated t+2.

9. Although the approximation error involved here may be "small" it may have a large effect on the estimates of conditional variances. Together with footnotes 6 and 8, this highlights the crucial role that must be played by parameterization of the expectational terms in expression (15).

10. Derivation of a similar expression for the risk premium, $E_t(S_{t+1})-F_t$, is discussed in Hodrick (1987), pp. 13-15.

Appendix A

The first-order conditions for the agents' problem flow from the value function

(A1) $V(W_t, \Pi_{1t}M^p_{1t}, \Pi_{2t}M^p_{2t}, x_t) = \max \{ U(C_{1t}, C_{2t})$

$+ \beta \int (W_{t+1}, \Pi_{1t+1}M^p_{1t+1}, \Pi_{2t+1}M^p_{2t+1}, x_{t+1}) F(x_{t+1}|x_t) dx_{t+1} \}$,

where wealth, W_t, is defined as

(A2) $W_t = \Pi_{1t}M^p_{1t} + \Pi_{2t}M^p_{2t} + \Pi_{1t}B^p_{1t}(x_t) + \Pi_{2t}B^p_{2t}(x_t) + (\psi_{1t}+Y_{1t})Z_{1t} + (\psi_{2t}+\Theta_t Y_{2t})Z_{2t}$.

Maximization is with respect to private consumption and choices of money holdings and holdings of bonds and equities. The actual transition probability is assumed to be known. If λ_t is the multiplier for the period-t budget constraint facing the consumer, ν_{1t} is the multiplier for the period-t currency one CIA constraint, and ν_{2t} is the multiplier for the currency two CIA constraint, then the first-order conditions are described by (A3) through (A10):

(A3) $U_{1t} = \lambda_t + \nu_{1t}$,

(A4) $U_{2t} = (\lambda_t + \nu_{2t})\Theta_t$,

(A5) $\lambda_t \Pi_{1t} = \beta E_t[(\lambda_{t+1} + \nu_{1t+1})\Pi_{1t+1}]$, (A6) $\lambda_t \Pi_{2t} = \beta E_t[(\lambda_{t+1} + \nu_{2t+1})\Pi_{2t+1}]$,

(A7) $\lambda_t \psi_{1t} = \beta E_t[\psi_{1t+1} + Y_{1t+1})\lambda_{t+1}]$,

(A8) $\lambda_t \psi_{2t} = \mathrm{BE,}[(\psi_{2t+1} + \Theta_{t+1} Y_{2t+1})\lambda_{t+1}]$,

(A9) $\lambda_t \Pi_{1t} n_1(x_{t+1}, x_t) = \beta \lambda_{t+1} \Pi_{1t+1} F(x_{t+1} | x_t), \ \forall \ x_{t+1},$

(A10) $\lambda_t \Pi_{2t} n_2(x_{t+1}, x_t) = \beta \lambda_{t+1} \Pi_{2t+1} F(x_{t+1} | x_t), \ \forall \ x_{t+1}.$

Appendix B

The theoretical values of the coefficients in expression (16) are

$\alpha_{s0} = (1-\rho_8)\zeta_2 - (1-\rho_7)\zeta_1 + (1-\delta)(1-\rho_2)y_2 - (1-\gamma)(1-\rho_7)y_1 + (1-\rho_3)w_1 - (1-\rho_4)w_2,$

$\alpha_{s1} = \alpha_{s2} = \alpha_{s3} = \alpha_{s4} = 1,$

$\alpha_{s5} = (1-\gamma)\rho_1,$

$\alpha_{s6} = (1-\delta)\rho_2,$

$\alpha_{s7} = \rho_3,$

$\alpha_{s8} = \rho_4,$

$\alpha_{s9} = (1-\gamma)^2/2,$

$\alpha_{s10} = (1-\delta)^2/2,$

$\alpha_{s11} = \alpha_{s12} = 1/29$

$\alpha_{s13} = \rho_7,$

$\alpha_{s14} = \rho_8,$

$\alpha_{s15} = \alpha_{s16} = 1/6.$

The theoretical values of the coefficients for expression (20) are

$\alpha_{i10} = -\ln\beta + w_1 - (1-\gamma)^2(1-\phi_1)h_1/2 - (1-\phi_3)h_3/2 - \rho_1(1-\rho_7)\zeta_1 - (1-\phi_7)(h_7)^3/6,$

$\alpha_{i11} = \alpha_{i12} = 1,$

$\alpha_{i13} = -(1-\gamma)\rho_1(\rho_1-1),$

$\alpha_{i14} = \rho_3^{\,2},$

$\alpha_{i15} = (1-\gamma)^2(1-\phi_1-\rho_1^{\,2})/2,$

$\alpha_{i16} = -(\phi_3+\rho_3{}^2)/2,$

$\alpha_{i17} = \rho_7(1-\rho_7),$

$\alpha_{i18} = (1-\phi_7-\rho_7)/6.$

The theoretical values of the coefficients in expression (21) are

$\alpha_{r1} = (1-\gamma)^2\rho_1{}^2/2,$

$\alpha_{r2} = (1-\gamma)^2\rho_2{}^2/2,$

$\alpha_{r3} = (1+\rho_3{}^2)/2,$

$\alpha_{r4} = (1+\rho_4{}^2)/2,$

$\alpha_{r5} = \alpha_{r6} = 1,\ \alpha_{r7} = \rho_7/6,$

$\rho_{r8} = \rho_8/6.$

References

Backus, D., and Kehoe, P., "On the Denomination of Government Debt: A Critique of the Portfolio Balance Approach," Working Paper No. 359, Federal Reserve Bank of Minneapolis, 1988.

Cumby, Robert E., "Is It Risk? Explaining Deviations from Uncovered Interest Parity," Journal of Monetary Economics, 22 (1988), 279-299.

Danker, Deborah J., Haas, Richard A., Henderson, Dale W., Symansky, Steven A., and Tryon, Ralph W., "Small Empirical Models of Exchange Market Intervention," Staff Study No. 135, Board of Governors of the Federal Reserve System, 1985.

Diebold, Francis X., and Nason, James M., "Nonparametric Exchange Rate Prediction?" Finance and Economics Discusssion Series No. 81, Board of Governors of the Federal Reserve System, 1989.

Domingues, K., "The Informational Role of Official Foreign Exchange Intervention Operations," manuscript, Kennedy School of Government, Harvard University, Cambridge, Mass., 1988.

Dornbusch, Rudiger, "Exchange Rate Risk and the Macroeconomics of Exchange Rate Determination," in The Internationalization of Financial Markets and National Economic Policy, eds. R. Hawkins, R. Levich, and C. Wihlborg, JAI Press, Greenwich, Ct., 1985, 3-27.

Engel, Charles M., and Rodrigues, Anthony P., "Tests of International CAPM with Time-Varying Covariances," National Bureau of Economic Research Working Paper No. 2303, July 1987.

Engle, Robert F., "Autoregressive Conditional Heteroscedasticity with Estimates of the Variance of United Kingdom Inflation," Econometrica, 50 (1982), 987-1007.

Frankel, Jeffrey A., "Recent Estimates of Time-Variation in the Conditional Variance and the Exchange Risk Premium," Journal of International Finance, 7 (1989), 115-125.

Froot, Kenneth A., and Frankel, Jeffrey A., "Forward Discount Bias: Is It an Exchange Risk Premium?" Quarterly Journal of Economics, 104 (1989), 307-325.

Giovannini, Alberto, and Jorion, Phillippe, "The Time Variation of Risk and Return in the Foreign Exchange and Stock Markets," Journal of Finance, 44 (1989), 307-325.

Henderson, Dale W., "Exchange Market Intervention Operations: Their Role in Financial Policy and Their Effects," in Exchange Rate Theory and Practice, ed. John F. Bilson and Richard C. Marston, National Bureau of Economic Research, Unversity of Chicago Press, Chicago, 1984.

_______, and Sampson, Stephanie, "Intervention in Foreign Exchange Markets: A Summary of Ten Staff Studies," Federal Reserve Bulletin, 69 (1983), 830-836.

Hodrick, Robert J., The Empirical Evidence on the Efficiency of Forward and Futures Foreign Exchange Markets, Fundamentals of Pure and Applied Economics, No. 24, Chur, Switzerland: Harwood Academic Publishers, 1987.

_______, "Risk, Uncertainty, and Exchange Rates," Journal of Monetary Economics, (1989, forthcoming).

_______, Kocherlakota, Narayana, and Lucas, Deborah, "The Variability of Velocity in Cash-In-Advance Models," National Bureau of Economic Research Working Paper No. 2891, 1989.

Humpage, Owen, "Intervention and the Dollar's Decline," Economic Review, Federal Reserve Bank of Cleveland, Quarter 2 1988, 2-16.

Johnson, David, "The Currency Denomination of Long-term Debt in the Canadian Corporate Sector: An Empirical Analysis," Journal of International Money and Finance, 7 (1988), 77-90.

Lastrapes, William D., "Exchange Rate Volatility and U.S. Monetary Policy: An ARCH Application," Journal of Money, Credit and Banking, 21 (1989), 66-67.

Leahy, Michael P., "The Profitability of U.S. Intervention," paper presented at the meetings of the Western Economic Association, June 1989.

Loopesko, Bonnie E., "Relationships among Exchange Rates, Intervention and Interest Rates: An Empirical Investigation," Journal of International Money and Finance, 3 (1984), 257-277.

Lucas, Robert E., "Interest Rates and Currency Prices in a Two-Country World," Journal of Monetary Economics, 10 (1982), 335-360.

Lyons, Richard K., "Tests of the Foreign Risk Premium Using the Expected Second Moments Implied by Option Pricing," Journal of International Money and Finance, 7 (1988), 91-108.

Mark, Nelson C., "Time Varying Betas and Risk Premia in the Pricing of Foreign Exchange Contracts," Journal of Financial Economics, 22 (1989), 335-354.

Nelson, D.B., "Conditional Heteroscedasticity and Asset Returns: A New Approach," mimeo, Massachusetts Institute of Technology, 1987.

Obstfeld, Maurice, "The Capitalization of Income Streams and the Effects of Open Market Policies Under Fixed Exchange Rates," Journal of Monetary Economics, 9 (1982), 87-98.

________, "The Effectiveness of Foreign-Exchange Intervention: Recent Experience," National Bureau of Economic Research Working Paper No. 2796, 1989a.

________, "How Integrated Are World Capital Markets? Some New Tests," in Debt, Stabilization and Development: Essays in Memory of Carlos Diaz Alejandro, ed. Ronald Findley, et al., Basil Blackwell, 1989b.

Pagan, A., and Hong, Y., "Non-parametric Estimation and the Risk Premium," Working Paper No. 135, Rochester Center for Economic Research, University of Rochester, 1988.

Pippenger, John E., and Phillips, Llad, "Stabilization of the Canadian Dollar: 1952-1960," Econometrica, 41 (1973), 797-815.

Siebert, Anne, "The Risk Premium in the Foreign Exchange Market," Journal of Money, Credit, and Banking, 21 (1989), 49-65.

Stockman, A., and Svensson, L., "Capital Flows, Investment, and Exchange Rates," Journal of Monetary Economics, 19 (1987), 171-201.

Sweeney, Richard J., "Leaning Against the Wind: The Case of Canadian Exchange Intervention, 1952-1960," Claremont Working Papers, 1981.

Weber, Warren E., "Do Sterilized Interventions Affect Exchange Rates?" Quarterly Review, Federal Reserve Bank of Minneapolis, Summer 1986, 14-23.