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FORECASTING THE MONEY SUPPLY
IN TIME SERIES MODELS

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Abstract

In this paper, time series techniques are used to forecast quarterly money supply levels. Results indicate that a bivariate model including an interest rate and M-1 predicts M-1 better than the univariate model using M-1 only and as well as a 5-variable model which adds prices, output, and credit.

The paper also presents evidence on the issue of using seasonally adjusted data in forecasting with time series models. The implications of these results apply to all econometric modeling. Results support the hypothesis that using seasonally adjusted data can lead to spurious correlation in multivariate models.

I. Introduction

The goal of this research is to build a statistical model relating the intermediate targets of monetary policy to inflation and output. The Federal Reserve has used both interest rates and the money supply as intermediate targets in the past 20 years. It has just recently adopted an experimental target range for credit.[1]

This model would be used to monitor the economic relationships that are assumed (predicted) in the construction of the intermediate targets and to develop tests that would suggest when the predicted relationships are rejected by the data. When the assumptions underlying the targets are rejected, the targets should be changed.

This paper reports the results of preliminary work on this project. A 5-variate model is estimated and its forecasts of the money supply are

compared with forecasts from univariate and bivariate models. Estimation procedures developed by Tiao and Box (1981) are used to estimate the simultaneous equation model (SEM) without prior restrictions. Zellner and Palm (1974) argued that time series analysis could be used to test the assumptions underlying econometric models--assumptions about variables being exogenous, about lags in the dynamic structure of the model, and about the correlations between the random elements of economic variables. The problem faced by Zellner and Palm in 1974 was that there were no time series methods available by which one could estimate directly the parameters of an SEM model. The procedures they recommended involved estimating approximations to appropriate transformations of the time series structural model, that is, the final form and the transfer function form. This suggestion by Zellner and Palm led to procedures developed by Granger and Newbold (1977), Wallis (1977), and Chan and Wallis (1978). All of these procedures are computationally burdensome and intuitively inferior to one that can provide direct estimates of the parameters. Because of computational complexity, these procedures were limited to models with 2 or, at most, 3 variables.

Sims (1977, 1980) recommended estimating the vector autoregressive form of the model. The problem with this approach is that it leads to a plethora of parameters in multivariate models. Sims has solved this problem by arbitrarily truncating the order of the autoregression. Others have used the Akaike (1969, 1970) final prediction error in preliminary analysis to specify optional lag lengths for each variable. (See, for example, Hsiao 1982 or Fackler 1982.) This preliminary analysis is in a limited sense the counterpart of the identification stage in the Tiao-Box procedure. A major drawback of this autoregressive approach is that one is constrained to a subset of models that are possible using the more general Tiao-Box procedure.

II. The Vector ARIMA Model

The following is a very brief description of the vector Autoregressive Integrated Moving Average (ARIMA) model. A more detailed description is given in Tiao and Box (1981). In the vector ARIMA model, it is assumed either that each series is stationary or that some suitable difference of the data is stationary. Thus, if z_t is the original k dimensional vector valued time series, then it is assumed that

$$w_{it} = (1 - B)^d (1 - B^S)^{Di} z_{it}$$

is stationary for each component of $\underset{\sim}{z}_t$ for an appropriate choice of d_i and D_i where B is the backshift operator (i.e., $Bz_{it} = z_{it-1}$), S is the seasonal period (e.g., for quarterly data, S = 4), and $d_i(D_i)$ is the number of regular (seasonal) differences necessary to make w_{it} stationary. The model is presented in terms of the stationary series $\underset{\sim}{w}_t$. The general vector ARIMA model is given by

$$\underset{\sim}{\phi}_p(B)\ \underset{\sim}{\phi}_P(B^S)\underset{\sim}{w}_t = \underset{\sim}{\theta}_q\underset{\sim}{\theta}_Q(B^S)\underset{\sim}{a}_t + \underset{\sim}{\theta}_0$$

where

$$\underset{\sim}{\phi}_p(B) = I - \underset{\sim}{\phi}_1 B - \cdots - \underset{\sim}{\phi}_p B^p,$$

$$\underset{\sim}{\phi}_P(B^S) = I - \underset{\sim}{\phi}_1 B^S - \cdots - \underset{\sim}{\phi}_P B^{PS},$$

$$\underset{\sim}{\theta}_q(B) = I - \underset{\sim}{\theta}_1 B - \cdots - \underset{\sim}{\theta}_q B^{QS},$$

$$\underset{\sim}{\theta}_Q(B^S) = I - \underset{\sim}{\theta}_1 B^S - \cdots - \underset{\sim}{\theta}_Q B^{QS},$$

the ϕ_j's, Φ_j's, Θ_j's, θ_j's, and θ_0 are k x k unknown parameter

matrices, and the a_t's are k x 1 vectors of random variables which are

identically and independently distributed as $N(0, \Sigma)$. Thus, it is assumed

that the a_t's at different points in time are independent, but not

necessarily that the elements of a_t are independent at a given point in time.

The Tiao-Box procedure allows one to estimate the structural parameters of

a multivariate simultaneous equation model. The procedure is an interactive

one similar in principle to that used in single equation Box-Jenkins

modeling. The steps involved are: 1) tentatively identify a model by

examining autocorrelations and cross-correlations of the series; 2) estimate

the parameters of this model; and 3) apply diagnostic checks to the

residuals. If the residuals do not pass the diagnostic checks, then the

tentative model is modified and steps two and three are repeated. This

process continues until a satisfactory model is obtained.

III. The Empirical Models

In this section the Tiao-Box procedure is used to estimate the historical

relationships among the intermediate targets and the goals of monetary

policy. The model estimated below includes 3 quantity variables and 2 price

variables from the markets for goods, credit and money. M-1 is used to

measure the money supply (M-1). Credit is measured as funds raised by the

non-financial sector (NFD) including private and government debt. This

measure differs slightly from the actual measure that has been adopted by the

Federal Reserve as an experimental and supplemental target for monetary policy

in 1983. Our variable includes equities issued by nonfinancial corporations

and funds raised in the United States by subsidiaries of foreign

corporations. The quantity of goods is measured as GNP in constant (1972)

dollars (GNP72). The price of output is the implicit GNP deflator (PGNP).

The price of credit is measured as the yield on 3-month Treasury securities (RTB3).

This work is preliminary in many ways. First, we have not checked the sensitivity of our results to alternative measures of the included variables. Certainly, the 3-month Treasury bill note is an arbitrary measure of the yield on credit. Second, we have not checked the sensitivity of our results to the inclusion of other markets. Specifically, much of the work in macroeconomics suggests that the labor market is not in continuous equilibrium and that events in that market are important determinants of fluctuations in both output and inflation. Third, one of the most important tests of any model is how well it does in forecasting out-of-sample. In the last section we compare out-of-sample forecasts for M-1 from alternative time series models, but we do not evaluate forecasts of the other variables nor do we provide a comprehensive comparison of our model's predictions with non-time series procedures. [2]

Using the notation from the introduction, $\underset{\sim}{w}$ is a vector of the 5 economic variables. This vector has an associated random vector, $\underset{\sim}{a}_t$. The model is estimated twice, once using seasonally adjusted datq and once with not-seasonally adjusted data. The w vector includes appropriately differenced logarithms of each variable. The estimates using not-seasonally adjusted data should be considered superior a priori because the seasonal factors are estimated jointly with the other parameters of the model. This is in contrast to using seasonally adjusted data where the seasonal filters applied to the data are different for each variable and the seasonal adjustment procedures do not take account of correlation between series. Wallis (1974) has shown that using data that has been seasonally adjusted with conventional procedures may lead to incorrect inference in dynamic models.

The model estimated using the not-seasonally adjusted data is given in table 1. The model estimated using seasonally adjusted data is given in table 2. When the models are in the general form, they are difficult to interpret because there may be interactions among the autoregressive and moving average operators. Consequently, we express the models in the moving average form as shown in table 3. This leads to the following interpretations.

The Price of Goods. For the not-seasonally adjusted data, the implicit deflator is independent of the rest of the model including contemporaneous correlations. According to these estimates, inflation can be modeled as a univariate ARIMA model with a first-order autoregressive and a first-order moving average term. This model suggests that information from the money supply, credit aggregates, the interest rate and real output will not help predict changes in the price level once we have taken account of information in the history of the price level.

This situation changes dramatically when we examine the same equation from the model estimated with seasonally adjusted data. In this model, inflation responds positively to lagged money supply, negatively to lagged credit, and (although weakly) negatively to lagged interest rates. All of these relationships involve decaying lagged patterns because of the autoregressive terms in the model.

While the positive dependence of inflation on money supply growth will be encouraging to some, we would have more confidence in this result if it was evident in the not-seasonally adjusted model. Part of the model not captured in the parameter matrices is the estimate of the correlations between contemporaneous errors. In neither case is there a significant correlation between the errors from the inflation equation and the other errors. [3]

M-1. The second equation determines the money supply. In table 1 we can

see that the seasonal part of the model required a fourth difference and a fourth-order moving average to represent the seasonal movement in the series.[4] The money supply is determined by a moving average of the error from the M-1 equation and a second-order moving average of the error from the interest rate equation.

The sign of the moving average parameter on the interest rate error is consistent with the money demand literature. The significance of a "scale" variable, usually income or wealth, in almost every model of money demand suggests that there should be significant correlation between M-1 and output. In table 1, the correlation between errors in the money and output equations is not significant. However, there is a strong contemporaneous correlation between the error in the M-1 equation and the error in the credit equation.

Using seasonally adjusted data results in changes that support traditional money demand models. The major differences are a significant positive correlation between the errors from the M-1 and output equations and a 50 percent increase in the estimated interest rate elasticity. There is also a significant effect from credit starting at lag one.

<u>Credit.</u> The third equation determines credit, that is, the amount of funds raised by the nonfinancial sector. In table 3, we see that not-seasonally adjusted credit depends on lagged M-1 growth, on the interest rate lagged 3 quarters and on a first-order moving average error. In all these "quantity" equations, M-1, NFD, and GNP72, the seasonal model involved a fourth-order difference and a fourth-order moving average parameter. The contemporaneous error in the credit equation was significantly correlated with the errors from the M-1 and the real output equations.

The credit equation estimated using seasonally adjusted data differs from the equation in table 1 in that credit does not depend on past M-1 or past

interest rates. Using seasonally adjusted data we find that M-1 depends on past credit but that credit does not depend on past M-1. This is exactly opposite to our findings when we used not-seasonally adjusted data.

The Interest Rate. The fourth equation determines the interest rate, the yield on Treasury bills with 3 months to maturity. In the not-seasonally adjusted model, changes in the interest rate depend only on past errors from the M-1 equation and on past errors from the interest rate equation. There is no significant contemporaneous correlation between the error from the interest rate equation and any of the errors from the other equations.

In the seasonally adjusted model the interest rate depends on past M-1 and credit. In both models the relationship between the interest rate and M-1 is positive indicating a supply relationship. These models suggest that single equation money demand models incorrectly treat the interest rate as exogenous. Again, the error from the interest rate equation is not significantly correlated with contemporaneous errors from any of the other equations.

Real Output. In the not-seasonally adjusted model real output depends on lagged M-1 growth, inflation and interest rates. These estimates clearly reject the hypothesis that real output is independent of anticipated changes in the money supply. There is a weak correlation between contemporaneous errors in M-1 and output, but it is not significant at the 5-percent level.

When seasonally adjusted data is used output depends on past inflation, M-1, credit, and interest rates. This equation is consistent with the hypothesis that accelerating inflation has a significant depressing effect on the trend in output growth. The errors in output are significantly correlated with the errors from the money and credit equations.

Summary of Estimated Models. In every equation, different variables

were significant depending on whether not-seasonally or seasonally adjusted data was used. The contemporaneous correlations between errors were very similar in both models. The strongest contemporaneous correlations were between M-1 and credit and between real output and credit. The contemporaneous correlation between output and money was just barely significant in the seasonally adjusted model and just marginally insignificant in the not-seasonally adjusted model.

One interesting result was that for the seasonally adjusted data, twelve of the twenty off-diagonal terms of the moving average representation were non-zero, while only seven were non-zero for not-seasonally adjusted data. This result supports the (Wallis (1974) claim that the official (Census X-11 variant) seasonal adjustment procedure can induce spurious dynamic correlation between variables.

Using not-seasonally adjusted data results in a forecasting model that is block recursive with two independent leading blocks, the price equation by itself, and the money and interest rate equations. The credit equation depends on the money and interest rate block. The output equation depends on both leading blocks. This result suggests that a bivariate model including just the interest rate and M-1 would predict M-1 as well as the 5-variate model. Both should outperform a univariate model of the money supply process.

Using seasonally adjusted data results in a block recursive forecasting model in which the credit equation forms the leading block, the money and interest equations form the second block, the inflation equation is the third block, and the output equation is the final block. In this case the forecasts of M-1 from the 5-variable model should outperform both the bivariate, including M-1 and the interest rate, and univariate models.

IV. Forecasting the Money Supply in Time Series Models

Three time series models of the money supply were estimated using both seasonally and not-seasonally adjusted data over the period from the first quarter of 1959 to the fourth quarter of 1979, and forecasts were generated over the period from the first quarter of 1980 to the third quarter of 1982. The 3 models are a univariate model of M-1, a bivariate model of M-1 including the yield on 3-month Treasury bills, and the 5-variate model shown in table 1 of section 1.

The results in table 1 show that for not-seasonally adjusted data, the money supply and the interest rate form a leading recursive block in the forecasting model. Therefore, we would expect the bivariate model to do better than the univariate and as well as the 5-variate model. The models for M-1 are displayed in table 4. An interesting feature of these three models is their similarity. The first- and fourth-order moving average terms are almost identical in all three cases. The estimated interest rate elasticity is similar in the multivariate models. In the bivariate model the first-order moving average parameter on the interest rate error is not significantly different from zero, but its exclusion leads to significant serial correlation between errors in the interest rate and M-1.

The results of the forecasting experiment are given in table 5. Panel a. of table 5 shows the results of one-step-ahead forecasts. The results show that the forecasts became slightly better as more variables were added to the model. The differences are small, however, and the Root Mean Square Errors (RMSEs) are disappointingly large. One reason for this may have been the credit controls imposed in the second quarter of 1980 and removed in the third quarter of the same year. We attempted to abstract from the effect of these

controls in two ways.

First, we ran n-step-ahead forecasts, which did not use any actual data after the fourth quarter of 1979. The results were much better and they favored the multivariate models (see panel b.). However, the confidence intervals are so wide on these forecasts that we must ascribe the good performance to coincidence. In panel c. we repeated the n-step-ahead forecasts using the initial values from the first quarter of 1980. The results were much worse, although the multivariate models still outperformed the univariate model.

The second method we used to intervene in the model to correct for credit controls was to replace actual values of M-1 and the interest rate in the second quarter of 1980 and third quarter of 1980 with predicted values. This eliminated errors in those quarters. Panel d. lists the mean error and RMSE for the 8 quarters beginning in the first quarter of 1980. In this case, the mean error was slightly larger than in panel a., but the RMSE was much smaller and more in line with the error normally found in regression models of the money supply.

For the seasonally adjusted data, the models for M-1 are given in table 6. The bivariate and 5-variate models are similar in that the autoregressive terms are close and the first-order moving average terms on the interest rate are roughly the same. The non-significance of the constant in the 5-variate model is due to the addition of the credit term. The univariate model is actually closer to the other two models than it at first appears. This can be seen by transforming this model as follows:

$$(1-.414B-.363b^2)^{-1}\Delta^2 lnM1 = (1-.238B^8)a_{2t}$$

or by dividing the first operator into one of the 1-B factors,

$$(1-.586B+.120B^2-.163B^3+\cdots)\Delta lnM1 = (1-.238B^8)a_{2t}$$

Also, the residuals from both the bivariate and 5-variate models for M-1 had just barely nonsignificant correlations at lag 8. Thus, these models would have a moving average term of lag 8, which would not differ substantially from that of the univariate model if this parameter were included. Thus, the models are quite similiar.

The results of forecasting using the seasonally adjusted models are presented in table 7. The results for the one-step-ahead forecasts agree with the statement that the univariate model should be outperformed by both the bivariate and the 5-variate models and that the 5-variate model should do better than the bivariate model. Also, these RMSEs are smaller than those of the not-seasonally adjusted models. This may be due to the fact that when the data was seasonally adjusted, an attempt was made to adjust for the effects of credit control.[5] We repeated the three additional forecasting experiments from above. The results for the n-step-ahead forecasts from the fourth quarter of 1979 are rather strange in that the univariate model is much better than the other two models. This result is not true when forecasting from the first quarter of 1980 where the 5-variate model is much better. Examining the final result, we see that indeed, even the seasonally adjusted models forecast better past the credit control period.

Overall, these forecasting results from this short period do not distinguish sharply between the three time series models. This may reflect, in part, the particularly volatile period over which the forecasts were run. Besides the credit controls, there was also a change in Federal Reserve operating procedures just before the start of the forecasting period. This change has been associated with higher variance in both interest rates and M-1.

One way to get around this problem would be to "backcast" into the 1950s using the estimated parameters of the model. It may also be instructive to look at different variables. Forecasting output may be more useful in determining the advantage of larger time series models because output depends on more variables in the system than does M-1.

V. Conclusion

In this paper we have used the Tiao-Box procedure to identify and estimate a dynamic simultaneous equation model. The procedure leads to a parsimonious representation of a model including markets for goods, money, and credit. The results from the forecasting experiment were mixed. In 5 of the 8 experiments, the 5-variate model gave better forecasts than the smaller models. In two of the other cases the results were very close. This was a turbulent period for monetary policy. The Federal Reserve adopted a new operating procedure in October 1979. That change in regimes was followed by unpredicted swings in the interest rate and more volatile growth in the money supply. In spite of this, the out-of-sample quarterly prediction error of M-1 was on the order of 1 percent when we intervened for the period of credit controls. This error is of the same magnitude as that which has been found when standard econometric models are used. Overall, there was not much difference between the different models. Perhaps as we gather more information we will be better able to choose between these models.

In the not-seasonally adjusted model, inflation was independen't of all the intermediate targets. This suggests that a different specification of the model will be needed to represent the transmission mechanism going from monetary policy to inflation. Using seasonally adjusted data leads to a model that is more useful for policy evaluation. However, if the dynamic correlations are spurious, caused by an inappropriate seasonal model, then we cannot rely on this model either. One possible approach that we plan to investigate, is to combine inflation and output into nominal GNP and build a model relating nominal GNP to the intermediate targets. In practice, much of the discussion surrounding monetary policy goals is couched in terms of nominal GNP.

Footnotes

1. Fackler and Silver (1982-83) discuss the issues involved in use of credit as an intermediate target for monetary policy. Friedman (1981) and Fackler use vector autoregressive methods with seasonally adjusted data to examine the dynamic relationships among inflation, output, interest rates, M-1 and credit.

2. O'Reilly et al. (1981) reports that univariate ARIMA models did not forecast as well as the DRI large model. The large model forecasts had a root mean square error average 73 percent lower than ARIMA models. They present a multivariate model but do not present comparative statistics for this model. In general, large model forecasts that "do well" do so because of judgmental adjustments to the model forecasts. The vector ARIMA model can be expected to beat non-judgemental forecasts from large econometric models.

3. Throughout this work, we have used a 5-percent critical region to define significance.

4. In preliminary work, we found that if a fourth-order autoregressive term was included in the model, then its estimate was close to 1. Consequently, the data were seasonally differenced.

5. Pierce and Cleveland (1981) discuss the method used by the Federal Reserve to adjust for the effects of credit control.

Table 1 Estimated Model Using Not-Seasonally Adjusted Data: 1959:IQ - 1979:IVQ

$$\phi(B)\underset{\sim}{w}_t = \underset{\sim}{\theta}(B)\underset{\sim}{\Theta}(B^S)\underset{\sim}{a}_t + \underset{\sim}{\theta}_0$$

$$
\begin{bmatrix}
1-.987B & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 \\
0 & -.210B & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 \\
0 & -.278B & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
\nabla \ln PGNP_t \\
\nabla\nabla_4 \ln M1NS_t \\
\nabla\nabla_4 \ln NFDNS_t \\
\nabla \ln RTB3_t \\
\nabla\nabla_4 \ln GNP72NS_t
\end{bmatrix}
=
$$

$$
\begin{bmatrix}
1-.441B & 0 & 0 & 0 & 0 \\
0 & (1+.504B)(1-.558B^4) & 0 & 0 & 0 \\
0 & 0 & (1+.359B)(1-.459B^4) & 0 & -.012B^2 \\
0 & 3.913B(1-.558B^4) & 0 & 1+.617B & 0 \\
-.408B & 0 & 0 & 0 & 1-.823
\end{bmatrix}
\underset{\sim}{a}_t
$$

$$
\underset{\sim}{\theta}_0 = 0, \quad \hat{\rho}\underset{\sim}{a}_t =
\begin{bmatrix}
1 & & & & \\
.146 & 1 & & & \\
.194 & .492 & 1 & & \\
-.023 & .127 & .183 & 1 & \\
-.085 & .203 & .362 & .141 & 1
\end{bmatrix}
$$

a.

Table 2 Estimated Model Using Seasonally Adjusted Data: 1959:IQ   1979:IVQ

$$\phi(B)\underset{\sim}{w}_t = \underset{\sim}{\Theta}(B)\underset{\sim}{a}_t + \underset{\sim}{\theta}_o$$

$$\begin{bmatrix} 1-.851B & -.220B & 0 & 0 & 0 \\ 0 & 1-.433B & -.244B & 0 & 0 \\ 0 & 0 & 1-.850B & 0 & 0 \\ 0 & -4.366B & 0 & 1 & 0 \\ -.354B & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \mathbf{D}\ \ln PGNP_t \\ \mathbf{D}\ \ln M1_t \\ \mathbf{D}\ \ln NFD_t \\ \mathbf{D}\ \ln RTB3_t \\ \mathbf{D}\ \ln GNP72_t \end{bmatrix} = \begin{bmatrix} 1-.470B & 0 & -.254B & 0 & 0 \\ 0 & 1 & 0 & -.018B & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1+.488B & 0 \\ -.347B & 0 & 0 & 0 & 1+.183B \end{bmatrix} a_t + \begin{bmatrix} -.0009 \\ .0018 \\ .0032 \\ -.0358 \\ .0126 \end{bmatrix}$$

$$\hat{\rho}a_t = \begin{bmatrix} 1 & & & & \\ .151 & 1 & & & \\ .006 & .558 & 1 & & \\ -.175 & .033 & .028 & 1 & \\ -.176 & .253 & .316 & .076 & 1 \end{bmatrix}$$

Table 3 Moving Average Representation

$$\underset{\sim}{w}_t = \underset{\sim}{\phi}^{-1}(B^S)\ \underset{\sim}{\phi}^{-1}(B)\ [\underset{\sim}{\theta}(B)\underset{\sim}{\theta}(B^S)a_t + \underset{\sim}{\theta}_0]$$

Not-seasonally adjusted data

$$\underset{\sim}{w}_t = \begin{bmatrix} \frac{1-.441B}{1-.987B} & 0 & 0 & 0 & 0 \\ 0 & (1+.504B)(1-.558B^4) & 0 & .012B^2 & 0 \\ 0 & .210B(1+.504B)(1-.558B^4) & (1+.359B)(1-.459B^4) & -.0025B^3 & 0 \\ 0 & 3.913B(1-.558B^4) & 0 & 1+.617B & 0 \\ -.408B & .278B(1+.504B)(1-.448B^4) & 0 & .003B^3 & 1-.823B^4 \end{bmatrix} a_t$$

Seasonally adjusted data

$$\underset{\sim}{w}_t = \begin{bmatrix} \frac{1-.470B}{1-.851B} & \frac{.220B}{(1-.851B)(1-.433B)} & \frac{.049B^2}{(1-.851B)(1-.433B)(1-.85B)} - \frac{.254B}{1-.851B} & \frac{-.004B^2}{(1-.851B)(1-.433B)} & 0 \\ 0 & \frac{1}{1-.433B} & \frac{.244B}{(1-.433B)(1-.850B)} & \frac{-.018B}{1-.433B} & 0 \\ 0 & 0 & \frac{1}{(1-.850B)} & 0 & 0 \\ 0 & \frac{4.366B}{1-.433B} & \frac{1.065B^2}{(1-.433B)(1-.851B)} & \frac{-.079B^2}{1-.433B} + (1+.488B) & 0 \\ \frac{-.354B(1-.47B)}{1-.851B} - .347B & \frac{-.078B^2}{(1-.851B)(1-.433B)} & \frac{.090B^2}{1-.851B} - \frac{.017B^3}{(1-.851B)(1-.433B)(1-.850B)} & \frac{.0014B^3}{(1-.851B)(1-.433B)} & 1+.183B \end{bmatrix} a_t + \begin{bmatrix} .0111 \\ .0124 \\ .0214 \\ .0181 \\ .0087 \end{bmatrix}$$

Table 4 Time Series Models of M1NS*
(Sample Period: 1959:IQ to 1979:IVQ)

UNIVARIATE:

$$\nabla\nabla_4 \ln M1NS_t = (1 + .473B)(1 - .494B^4)a_{2t}$$

BIVARIATE:

$$\nabla\nabla_4 \ln M1NS_t = (1 + .511B)(1 - .482B^4)a_{2t}$$
$$- (.007B + .016B^2)a_{4t}$$

5-VARIATE:

$$\nabla\nabla_4 \ln M1NS_t = (1 + .504B)(1 - .558B^4)a_{2t}$$
$$- .012B^2\ a_{4t}$$

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* MINS is M-1 not seasonally adjusted

a_2 = Random component of ln MINS

a_4 = Random component from the interest rate equation
not shown in this paper

Table 5 Out-of-Sample Forecasts for MINS
(Billions of dollars)

		Mean Error	RMSE
a.	One-step-ahead forecast 1980: IQ to 1982: IIIQ		
	Univariate	-0.582	7.629
	Bivariate	-0.277	7.526
	5-Variate	-0.380	7.200
b.	n-Step-ahead forecast from 1979: IVQ to 1982: IIIQ		
	Univariate	-4.871	6.994
	Bivariate	-0.381	3.968
	5-Variate	-1.367	4.179
c.	n-Step-ahead forecast from 1980: IQ to 1982: IIIQ		
	Univariate	-9.103	10.611
	Bivariate	-5.939	7.393
	5-Variate	-5.411	6.919
d.	One-step-ahead forecast with intervention from 1979: IVQ to 1982: IIIQ		
	Univariate	-0.967	4.735
	Bivariate	-0.547	4.845
	5-Variate	-0.574	4.934

Table 6   Time Series Models of M-1*

(Sample Period: 1959: IQ to 1979: IVQ)

UNIVARIATE: $^2 lnMl_t = (1 - .414B - .363B^2)(1 - .238B^8)a_{2t}$

BIVARIATE: $(1 - .648B) lnMl_2 = a_{2t} - .0194a_{4t-1} + .00431$

5-VARIATE: $(1 - .433B) lnMl_t = .244 \ NFDL_t + a_{2t}$

$- .0179a_{4t-1} + .0018$

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*a_2 = Random component of $lnMl$

a_4 = Random component of $lnRTB3$

Table 7 Out-of Sample Forecasts for M-1
(Billions of dollars)

	Mean Error	RMSE
a. One-step-ahead forecast 1980:IQ to 1982:IIIQ		
Univariate	-0.422	6.532
Bivariate	0.118	5.644
5-Variate	0.206	5.274
b. n-Step-ahead forecast from 1979:IVQ to 1982:IIIQ		
Univariate	-2.245	4.639
Bivariate	11.536	13.418
5-Variate	8.220	9.920
***c.* n-Step-ahead forecast from 1980: IQ to 1982:IIIQ**		
Univariate	-5.810	7.296
Bivariate	4.573	7.119
5-Variate	1.868	4.774
d. One-step-ahead forecast with intervention from 1979:IVQ to 1982:IIIQ		
Univariate	-0.846	4.541
Bivariate	-0.242	4.880
5-Variate	-0.070	4.160

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