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FORECASTING GNP USING MONTHLY M1

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Abstract

In this paper, we present an application of multivariate time series forecasting in which the data consist of a mixture of quarterly and monthly series. In particular, we use monthly series of M1 to forecast quarterly values of the nominal gross national product (GNP). Results from estimating models over the period 1959:IQ through 1979:IVQ indicate that models involving only movements in monthly M1 series provide approximately the same explanatory power as one using quarterly M1. When these models are used to forecast GNP over the time period 1980:IQ through 1984:IIIQ, the results are mixed. For one-quarter-ahead change, four-quarter-ahead change, and one-year change forecasts, the Root Mean Square Error (RMSE) for all the models (including a univariate model of GNP) have approximately the same RMSE (for a given forecast horizon) for the entire period. However, when we examine the period 1983:IIIQ through 1984:IIIQ, the models using M1 provide better forecasts than the univariate model, in terms of RMSE, for four-quarter and one-year change forecasts. Also, the models using monthly M1 data, perform at least approximately equal to the model using quarterly M1 data, **and** in some cases substantially better. All of the multivariate models used in this study indicate that the growth in GNP was smaller than expected relative to changes in M1 over the entire period. GNP growth had a larger variance from 1980:IVQ

to 1983:IIQ than was expected based on all models used in this study.

Comparisons of forecast errors among different studies is often difficult because of the different time periods involved and because of the different amount of data available when the forecasts are actually made. However, comparisons of the forecasts errors for these models to results from other studies using St. Louis type equations indicate that the models presented in this study appear to perform slightly better than the St. Louis models for one-quarter forecasts in terms of RMSE. Also, results for one-year change forecasts are apparently better than the median of five early-quarter forecasts by the ASA/NBER survey, Chase, Data Resources, Inc. (DRI), Wharton, and BEA.

I. Introduction

Sometimes data are available at different periodicities for the series involved in a multivariate forecasting effort. It is desirable to use this information optimally in developing forecasts. For example, if part of the data is available monthly and the rest quarterly, then there is a possibility of developing earlier forecasts by using the monthly data rather than quarterly summary data for those series. Also, it might be possible to develop better forecasts using the individual monthly series rather than a quarterly-aggregated series.

In this study, we are interested in the possible use of the monthly money supply (M1) series to forecast quarterly nominal GNP. We have chosen to examine the relationship between M1 and GNP because the instruments of monetary control affect the money supply and then, it is hoped, the ultimate target GNP. During most of the period in which the Federal Reserve has established explicit target ranges for the monetary aggregates, M1 has been regarded as the primary measure. While there are some questions concerning the recent stability of the relationship between M1 and GNP, Batten and Thornton (1983), as a result of a comparison of M1 and M2, indicate that as of 1983 there was no conclusive evidence that this relationship had deteriorated enough to justify using M2 in place of M1. Judd and Motley (1984) agreed with this conclusion.

As we will demonstrate in this paper, the relationship between M1 and GNP appears to have restabilized between 1983:IIQ and 1984:IIIQ. This result supports the study by Judd and Motley (1984) that states that the change in velocity during the early 1980s was caused by the sharp decline in nominal interest rates that occurred at that time. By 1983:IIQ, Judd and Motley point out, the interest rates would no longer have this impact, and thus, velocity and any other relationship between M1 and GNP, should have returned to normal.

Some of the questions addressed in this analysis are: 1) can we develop forecasts of GNP using only the first monthly M1 series (or first and second month), which are as good as, or better than, those using the quarterly M1 series and 2) can we develop forecasts of GNP using the three individual monthly M1 series, which are better than those developed using the quarterly M1 series. To investigate this question, we use autoregressive moving average (ARMA) and multivariate ARMA time series methods to develop models relating 1) GNP and its past history, 2) GNP and monthly M1 series, and 3) GNP and quarterly M1.

We also are interested in determining whether the forecasts derived from time series methods are as accurate as forecasts developed using other techniques. This comparison of our results to other results is complicated by the fact that often other studies are done over different time periods and have different amounts of data available when the forecasts are actually produced.

In this paper, we compare our results to the results of two papers using St. Louis type equations. The results should be interpreted carefully, because these earlier studies were carried out over a slightly different time period than our study. Also, the data available at the time of these studies may have been revised since then. We also compare our results to a study by McNees and Ries (1983) that used the median forecast of a group of five forecasts--ASA/NBER survey, Chase, DRI, Wharton, and BEA. While the data from the McNess and Ries study can be used to calculate statistics for the same period as part of our study, the results must be interpreted carefully, because the amount of information available when the forecasts used in that study were produced is most likely different from the information used in our study.

II. Multivariate ARMA Time Series Models

The following is a very brief description of multivariate ARMA time series models; Tiao and Box (1981) provide a more detailed description. The general multivariate ARMA model of order (p,q) is given by:

(1) $\underline{\phi}_p(B)\underline{Z}_t = \underline{\theta}_q(B)\underline{a}_t + \underline{\theta}_0,$

where

$$(2) \qquad \underline{\phi}_p(B) = \underline{I} - \underline{\phi}_1 B - \ldots - \underline{\phi}_p B^p,$$

$$\underline{\theta}_q(B) = \underline{I} - \underline{\theta}_1 B - \ldots - \underline{\theta}_q B^q,$$

where

B = backshift operator (i.e., $B^s z_{i,t} = z_{i,t-s}$),

$\underline{I}$ = k x k identity matrix,

$\underline{z}$ = vector of k variables in the model,

$\underline{\phi}_j$'s and $\underline{\theta}_j$'s = k x k matrixes of unknown parameters,

$\underline{\theta}_0$ = k x 1 vector of unknown parameters, and

$\underline{a}$ = k x 1 vector of random errors that are identically and independently distributed as $N(0,\Sigma)$.

Thus, it is assumed that the $a_{j,t}$'s at different points in time are independent, but not necessarily that the elements of $\underline{a}_t$ are independent at a given point in time.

The n-period-ahead forecasts from these models at time t ($\underline{z}_t(n)$) are given by:

$$(3) \qquad z_t(n) = \underline{\phi}_1[\underline{z}_{t+n-1}] + \ldots + \underline{\phi}_p[\underline{z}_{t+n-p}] + [\underline{a}_{t+n}] - \underline{\theta}_1[\underline{a}_{t+n-1}] - \ldots - \underline{\theta}_q[\underline{a}_{t+n-q}],$$

where for any value of t,n,m, $[\underline{x}_{t+n-m}]$ implies the conditional expected values of the random variables $\underline{x}_{t+n-m}$ at time t. If n-m is less than or equal to zero, then the conditional expected values are the actual values of the random variables and the error terms. If n-m is greater than zero, then the expected values are the best forecasts available for these random variables and error terms at time t. Because the error terms are uncorrelated with present and past information, the best forecasts of the error terms for n-m greater than zero are their conditional means, which are zero. The forecasts can be generated iteratively with the one-period-ahead forecasts that depend only on known values of the variables and error terms. The longer-length forecasts, in turn, depend on the shorter-length forecasts.

III. Models For Forecasting GNP

The variables in the models developed in this paper are the money supply M1 and GNP in current dollars, both seasonally adjusted. The money supply is represented by four series - M1 which is the quarterly money supply and M1A, M1B, and M1C which are monthly series. M1A is the first month of the quarter, M1B is the second month of the quarter, and M1C is the third month of the quarter. Thus, models involving M1A and/or M1B would be models involving information that would be available either two months or one month earlier than the quarterly data. Models involving M1C will be used to test whether there are more efficient ways of using the information within a quarter than just combining the information into one quarterly number.

The univariate model used in this paper was estimated using Box-Jenkins modeling (Box and Jenkins 1976). The multivariate models were estimated using the Tiao-Box procedure to estimate the parameters of a multivariate simultaneous equation model; The procedure is an interactive one similar in principle to that used in single Box-Jenkins modeling. See Tiao and Box

(1981). The steps involved are: (1) tentatively identify a model by examining autocorrelations and cross-correlations of the series, (2) estimate the parameters of this model, and (3) apply diagnostic checks to the residuals. These diagnostic checks include checks of correlations in the residuals, normality of residuals, etc. If the residuals do not pass the diagnostic checks, then the tentative model is modified and steps 2 and 3 are repeated. This process continues until a satisfactory model is obtained.

The models resulting from applying these techniques to the change in the logarithm of the GNP, quarterly M1, and monthly M1 series from 1959:IQ through 1979:IVQ are in the appendix. In this analysis, the change in a monthly series is defined as the difference between the current value and the corresponding value in the previous quarter. Table 1 gives the sample standard deviations for the GNP equation from the within sample estimation of these models. From table 1, we see that the change in any of the monthly M1 series has approximately as much information concerning the behavior of the change in GNP as the change in the quarterly M1 series during the estimation period.

These models were then used to forecast from 1980:IQ through 1984:IIIQ. The forecasting period is broken into two periods because of one-time events in the early 1980s (such as the imposition of credit controls in 1980 and the Depository Institutions Deregulation and Monetary Control Act of 1980 and the shift in monetary policy and high interest rates during the 1980s), indicating that 1980:IQ through 1983:IIQ might not be representative of the estimation period. Forecasts were developed for three situations: 1) one-quarter-ahead, 2) four-quarter-ahead (a forecast of the change in GNP four quarters ahead of the current quarter), and 3) one-year-change (that is, the change over the next four quarters combined). All of these forecasts were generated using only current or past information. The results are presented in tables 2, 3, and 4.

From table 2, we see that, in terms of RMSE, there is essentially no difference in the performance of all the models used in this study for one-quarter forecasts. For the latter period, the univariate model does have a smaller RMSE than all but one of the multivariate models. Also, we see that there is a substantial difference between the RMSEs from 1980:IQ through 1983:IIQ and those from 1983:IIIQ through 1984:IIIQ. The RMSEs in the latter period are, at most, 20 percent larger than the corresponding within-sample standard deviations. In the former period, the RMSEs are up to 80 percent larger than the standard deviations. The RMSEs for these models can be compared with other results for forecasting GNP. For example, Batten and Thornton (1983) used a version of the St. Louis equation involving a monetary measure (either M1 or M2) and high-employment government expenditures. These models were estimated for 1962:IIQ through 1979:IVQ and then used to forecast for 1980:IQ through 1983:IQ. The resulting RMSEs (when expressed in units corresponding to those used in this study) were 0.0173 for the model using M1, and 0.0150 for the model using M2. Both of these models used contemporaneous values of the monetary variable and the high-employment government expenditures variables. Also, Hafer (1984) used a variant of the St, Louis model using M1 or a debt measure (total domestic nonfinancial debt) and high-employment federal expenditures, relative price of energy, and a strike variable. These models were estimated for 1960:IQ through 1981:IVQ and then used to forecast 1982:IQ through 1983:IVQ. The resulting RMSEs for these two models were 0.0148 and 0.0155. Again, these models used contemporaneous values of the independent variables. Although it is difficult to compare the results of the current study with these earlier studies because of different time periods, the results of this study do compare favorably with previous results. The largest RMSE of any of the models in this study for one-quarter forecast is 0.0139. Also, the models presented in the contemporaneous study did not use current values of M1.

From table 3, we see that again all of the models provide roughly equal forecasts for the entire time period for four-quarter-ahead forecasts. However, all of the models involving M1 have slightly smaller RMSEs than the univariate model. For the latter period, all of the models using the M1 series have RMSEs that are moderately smaller than the univariate model's RMSE. The model with only M1A does slightly worse than the other models. This result indicates that once we know the M1 value for the second month of the quarter, we can forecast the four-quarter-ahead change in the log of GNP just as well as if we knew and used the quarterly M1 value. There is a slight indication that for this latter period, we can obtain a better forecast when we have an entire quarter's information on M1 by using the individual monthly data series instead of the quarterly series. However, this difference is very small, and given the small sample (five quarters), the result could be due to random effects.

When we examine the one year change forecasts (table 4), we see that again there is no substantial differences among the models in the entire time period. However, the univariate model does have a smaller RMSE then most of the models. This does not continue in the latter period. In fact, the univariate model has the largest RMSE in this latter period. In contrast to the four-quarter-ahead forecasts, the forecast using only M1A has a much smaller RMSE then any of the other models. Also, all the models using monthly

M1 data, except for the four-variate model, have smaller RMSEs than the quarterly model in this latter period.

As a comparison to these forecasts, McNees and Ries (1983) presented the errors made in the median of early-quarter forecasts by the ASA/NBER survey, Chase, DRI, Wharton, and BEA. These forecasts had a RMSE of 0.0476 and a mean error of 0.0213 from 1980:IVQ through 1983:IIQ. The largest RMSE over this time period for the models presented in this study was 0.0428. The largest mean error was -0.0146. Thus, the forecasts given by these models compare favorably with the median forecasts as reported in McNees and Ries. This conclusion must be made in the knowledge that the forecasters used in the McNess and Ries study would have had a different set of information than used in the models developed in this study. In particular, these forecasters would have based their forecasts on data that has since been revised. The forecasts developed in our study used the latest data available.

To examine the results of the one-period-ahead forecasts further, we examine three statistics that test whether the estimated models provide an adequate representation for the post-sample periods. If the model remains constant over time, then the following statistics have the indicated approximate distributions:

(4) $$\frac{1}{\sigma_i^2 T} \sum_{t=1}^{T} a_{it}^2 \sim \chi_T^2,$$

(5) $$\frac{1}{\sigma_i \sqrt{T}} \sum_{t}^{T} a_{it} \sim N(0,1), \text{ and}$$

(6) $$\frac{1}{\sigma_i^2 T} \sum_{t=1}^{T} (a_{it} - a_i)^2 \sim \chi_T^2,$$

where σ_i is the estimated within-sample standard deviation for the ith model, T is the number of observations in the post-sample period being tested, and $\bar{a}_i$ is the mean forecast error in the post-sample period.

Equation (4) is the sum of the square of the forecast errors standardized by the appropriate within-sample variance. If either the mean or the variance of the change in the log of GNP has changed, then this statistic will be affected. This statistic thus tests for changes in both the variance and the mean of the series. This statistic can also be used to test whether the RMSE is statistically larger than the within-sample standard deviation, because it is the mean square error. Equation 5 is the sum of the forecast errors standardized by the within-sample standard deviation from the appropriate model. If the mean of the change in the log of GNP has shifted relative to the estimated models, then this statistic will be affected. Equation 6 is the sum of the square of the deviation of the individual forecast errors from their mean, standardized by the appropriate within-sample variance estimate. This statistic will be affected if the variance of the change in the log of GNP changes in the post-sample period relative to the models. The results of applying these tests to each of the models estimated in this paper are in tables 5 through 7.

From table 5, we see that for the entire post-sample period and the 1980:IVQ to 1983:IIQ period, all the tests are significant at the 5 percent level at least. This implies that either the mean or the variance (or both) of the GNP series has changed relative to all of the models being used in this study. For the period 1983:IIIQ to 1984:IIIQ, none of the models has significant results. Examining table 6, we see that the mean forecast error for the univariate model is not significantly different from zero for any of the periods being studied here. However, the rest of the models have a significant negative mean forecast error for the entire post-sample period and for the earlier subperiod. Also, in the second subperiod, the mean errors for all the multivariate models are negative, although not significant. This means that on average all of the multivariate models are overforecasting the change in GNP for the entire post-sample period. Thus, the models are indicating that GNP has not grown as rapidly as expected relative to growth in M1.

Table 7 indicates that all of the models have significantly larger out-of-sample variances relative to in-sample variances. Thus, the growth of GNP in this period has been more variable than expected.

IV. Summary

The results of this paper are mixed -- that is, if we examine all the 1980s, the conclusions are different from those obtained if we examine only 1983:IIIQ through 1984:IIIQ. In the entire period, the univariate model of GNP forecasts as well as, if not better than, any of the multivariate models, despite the fact that multivariate models provided better-fitting models during the estimating period. We believe that this is due to the one-time events that occurred during the early 1980s. Events of this sort would naturally affect relationships among variables more than they would affect the relationship of one variable to its own past.

The evidence from 1983:IIIQ through 1984:IIIQ appears to indicate that these disturbances have worked their way through the economy, and that the models estimated through 1979:IVQ are once again applicable for forecasting. The results for this period seem to indicate that indeed, if we wish to forecast nominal GNP for more than one-quarter ahead, it is worthwhile to consider adding a measure of M1 to the forecasting model. Because of the small number of observations (five) in this period, this conclusion is weak, and further study is necessary when more data become available.

The results in this latter period do appear to indicate, that by using monthly M1 data, we can forecast quarterly GNP as well as, or better than by using quarterly M1 data. The forecasts from the first two monthly M1 series would be available before the quarterly M1 series, providing us earlier forecasts that are at least as accurate. For the one-year-change forecasts, the forecasts using monthly M1 data are actually substantially better than those from the quarterly model. This conclusion must be further tested as more data become available because of the small sample size in this latter period.

The results in this study also indicate that the growth in M1 during this time was slower than would have been expected, relative to models involving the growth of M1. This seems to have leveled off in the second subperiod studied, but the difference is still slightly negative, although not significantly so. Also, the variance of the growth in GNP was significantly larger from 1980:IVQ to 1983:IIQ, relative to the in-sample variance of all the models used in this study.

Table 1 Within-Sample Standard Deviations of GNP

Model	Sample standard deviation
Univariate	.0095
Bivariate with quarterly $M1_{t-1}$	.0081
Bivariate with $M1A_{t-1}$	.0082
Bivariate with $M1B_{t-1}$	.0082
Bivariate with $M1C_{t-1}$	.0080
Bivariate with $M1A_{t-1}$ and $M1B_{t-1}$	.0082
Four-variate with $M1A_{t-1}$, $M1B_{t-1}$, and $M1C_{t-1}$	.0079

Table 2 One-Quarter Forecasts

	Time period					
	1980:IQ-1984:IIIQ		1980:IQ-1983:IIQ		1983:IIIQ-1984:IIIQ	
Model	Mean error	RMSE	Mean error	RMSE	Mean error	RMSE
Univariate	.0004	.0122	-.0004	.0136	.0024	.0071
Bivariate with $M1_{t-1}$	-.0051	.0125	-.0056	.0136	-.0037	.0089
Bivariate with $M1A_{t-1}$	-.0041	.0116	-.0047	.0129	-.0025	.0069
Bivariate with $M1B_{t-1}$	-.0048	.0125	-.0055	.0135	-.0028	.0092
Bivariate with $M1C_{t-1}$	-.0046	.0121	-.0055	.0128	-.0023	.0098
Trivariate with $M1A_{t-1}$ and $M1B_{t-1}$	-.0047	.0135	-.0049	.0148	-.0043	,0083
Four-variate with $M1A_{t-1}$, $M1B_{t-1}$, and $M1C_{t-1}$	-.0055	.0129	-.0060	.0139	-.0043	.0095

NOTE: RMSE is the root mean square error of the forecast.

Table 3 Four-Quarter-Ahead Forecasts

	Time period					
	1980:IVQ-1984:IIIQ		1980:IVQ-1983:IIQ		1983:IIIQ-1984:IIIQ	
Model	Mean error	RMSE	Mean error	RMSE	Mean error	RMSE
Univariate	.0012	.0130	-.0004	.0147	.0048	.0082
Bivariate with $M1_{t-1}$	-.0012	.0126	-.0016	.0146	-.0001	.0062
Bivariate with $M1A_{t-1}$	.0004	.0129	-.0010	.0147	.0035	.0075
Bivariate with $M1B_{t-1}$	-.0005	.0123	-.0014	.0142	.0013	.0063
Bivariate with $M1C_{t-1}$	-.0000	.0126	-.0012	.0146	.0026	.0066
Trivariate with $M1A_{t-1}$ and $M1B_{t-1}$	-.0013	.0125	-.0018	.0145	-.0001	.0062
Four-variate with $M1A_{t-1}$, $M1B_{t-1}$, and $M1C_{t-1}$	-.0017	.0127	-.0019	.0149	-.0011	.0057

Table 4 One-Year-Change Forecasts

Model	1980:IVQ–1984:IIIQ Mean error	1980:IVQ–1984:IIIQ RMSE	1980:IVQ–1983:IIQ Mean error	1980:IVQ–1983:IIQ RMSE	1983:IIIQ–1984:IIIQ Mean error	1983:IIIQ–1984:IIIQ RMSE
Univariate	.0034	.0338	-.0051	.0375	.0221	.0235
Bivariate with $M1_{t-1}$	-.0140	.0368	-.0137	.0428	-.0145	.0177
Bivariate with $M1A_{t-1}$	-.0075	.0350	-.0112	.0418	.0007	.0086
Bivariate with $M1B_{t-1}$	-.0118	.0355	-.0127	.0419	-.0097	.0135
Bivariate with $M1C_{t-1}$	-.0100	.0329	-.0112	.0389	-.0074	.0114
Trivariate with $M1A_{t-1}$ and $M1B_{t-1}$	-.0138	.0362	-.0144	.0425	-.0126	.0144
Four-variate with $M1A_{t-1}$, $M1B_{t-1}$, and $M1C_{t-1}$	-.0159	.0360	-.0146	.0411	-.0187	.0207

Table 5 Tests For RMSE Changes

Model	1980: IVQ–1984: IIIQ	1980: IVQ–1983: IIQ	1983: IIIQ–1984: IIIQ
Univariate	31.33[a]	28.69[a]	2.79
Bivariate with $M1_{t-1}$	45.25[b]	39.47[b]	6.04
Bivariate with $M1A_{t-1}$	38.02[b]	34.65[b]	3.54
Bivariate with $M1B_{t-1}$	44.15[b]	37.95[b]	6.29
Bivariate with $M1C_{t-1}$	43.47[b]	35.84[b]	7.50
Trivariate with $M1A_{t-1}$ and $M1B_{t-1}$	51.50[b]	45.61[b]	5.12
Four-variate with $M1A_{t-1}$, $M1B_{t-1}$ and $M1C_{t-1}$	50.66[b]	43.34[b]	7.23

a. Significant at 0.05 level.
b. Significant at 0.01 level.

Table 6 Tests For Mean Changes

Model	1980: IVQ-1984: IIIQ	1980: IVQ-1983: IIQ	1983: IIIQ-1984: IIIQ
Univariate	0.18	-0.16	0.56
Bivariate with $M1_{t-1}$	-2.74[b]	2.58[b]	-1.02
Bivariate with $M1A_{t-1}$	-2.18[a]	-2.14[a]	-.68
Bivariate with $M1B_{t-1}$	-2.55[a]	-2.51[a]	-.76
Bivariate with $M1C_{t-1}$	-2.51[a]	-2.57[b]	-1.44
Trivariate with $M1A_{t-1}$ and $M1B_{t-1}$	-2.50[a]	-2.24[a]	-1.17
Four-variate with $M1A_{t-1}$, $M1B_{t-1}$ and $M1C_{t-1}$	-3.03[b]	-3.31[b]	-1.22

a. Significant at 0.05 level.
b. Significant at 0.01 level.

Table 7 Tests For Variance Changes

Model	1980: IVQ–1984: IIIQ	1980: IVQ–1983: IIQ	1983: IIIQ–1984: IIIQ
Univariate	31.30[a]	28.67[a]	2.47
Bivariate with $M1_{t-1}$	37.72[b]	32.78[b]	4.99
Bivariate with $M1A_{t-1}$	33.27[a]	31.15[b]	3.08
Bivariate with $M1B_{t-1}$	37.64[b]	31.65[b]	5.71
Bivariate with $M1C_{t-1}$	37.17[b]	29.24[b]	5.43
Trivariate with $M1A_{t-1}$ and $M1B_{t-1}$	45.25[b]	40.59[b]	3.75
Four-variate with $M1A_{t-1}$, $M1B_{t-1}$ and $M1C_{t-1}$	41.48[b]	41.85[b]	5.79

a. Significant at 0.05 level.
b. Significant at 0.01 level.

Appendix

Univariate model

$$(1-.3098B)\nabla \ln(GNP_t) = .0137 + a_t$$

$$\sigma_a^2 = .0009053$$

Bivariate model with quarterly GNP and M1

$$\nabla \ln(GNP_t) = .315\nabla \ln(M1_{t-1}) + .561\nabla \ln(M1_{t-2}) + a_{1t} +.0095$$

$$\nabla \ln(M1_t) = .627\nabla \ln(M1_{t-1}) + a_{2t} + .0046$$

$$\hat{\Sigma} = \begin{bmatrix} .000065 & .000016 \\ .000016 & .000029 \end{bmatrix}$$

Bivariate model with quarterly GNP and first month of quarter M1 (M1A)

$$\nabla \ln(GNP_t) = .429\nabla \ln(M1A_{t-1}) + .318\nabla \ln(M1A_{t-2}) + a_{1t} +.0110$$

$$\nabla \ln(M1A_t) = .366\nabla \ln(M1A_{t-1}) + a_{2t} +.0078$$

$$\hat{\Sigma} = \begin{bmatrix} .000068 & .000020 \\ .000020 & .000056 \end{bmatrix}$$

Bivariate model with quarterly GNP and second month of quarter M1 (M1B)

$$\nabla \ln(GNP_t) = .334\nabla \ln(M1B_{t-1}) + .475\nabla \ln(M1B_{t-2}) + a_{1t} +.0103$$

$$\nabla \ln(M1B_t) = .556\nabla \ln(M1B_{t-1}) + a_{2t} +.0055$$

$$\hat{\Sigma} = \begin{bmatrix} .000068 & .000020 \\ .000020 & .000036 \end{bmatrix}$$

Bivariate model with quarterly GNP and third month of quarter M1 (M1C)

$$\nabla \ln(GNP_t) = .334\nabla \ln(M1C_{t-1}) + .482\nabla \ln(M1C_{t-2}) + a_{1t} +.0102$$

$$\nabla \ln(M1C_t) = .420\nabla \ln(M1C_{t-1}) + a_{2t} +.0073$$

$$\hat{\Sigma} = \begin{bmatrix} .000068 & .000020 \\ .000020 & .000036 \end{bmatrix}$$

Appendix continued

Trivariate model with quarterly GNP and first and second month of quarter M1

$$\nabla \ln(GNP_t) = .435\nabla \ln(M1A_{t-1}) + .367\nabla \ln(M1B_{t-3}) + a_{1t} + .0105$$

$$\nabla \ln(M1A_t) = -.864\nabla \ln(M1A_{t-1}) - .394\nabla \ln(M1A_{t-2}) + 1.390\nabla \ln(M1B_{t-1}) + .551\nabla \ln(M1B_{t-2}) + a_{2t} + .0041$$

$$\nabla \ln(M1B_t) = .520\nabla \ln(M1B_{t-1}) + a_{3t} + .0062$$

$$\hat{\Sigma} = \begin{bmatrix} .000068 & & \\ .000012 & .000024 & \\ .000018 & .000023 & .000032 \end{bmatrix}$$

Four-variable model with quarterly GNP and first, second, and third month of quarter M1

$$\nabla \ln(GNP_t) = .258\nabla \ln(M1C_{t-1}) + .264\nabla \ln(M1A_{t-2}) + .418\nabla \ln(M1C_{t-2}) + a_{1t} + .0105$$

$$\nabla \ln(M1A_t) = -.484\nabla \ln(M1A_{t-1}) + 1.069\nabla \ln(M1C_{t-1}) + .315\nabla \ln(M1C_{t-1}) + a_{2t} + .0012$$

$$\nabla \ln(M1B_t) = -.432\nabla \ln(M1B_{t-1}) + .899\nabla \ln(M1C_{t-1}) + .271\nabla \ln(M1C_{t-2}) + a_{3t} + .0032$$

$$\nabla \ln(M1C_t) = .322\nabla \ln(M1C_{t-1}) + .226\nabla \ln(M1C_{t-2}) + a_{4t} + .0057$$

$$\hat{\Sigma} = \begin{bmatrix} .000062 & & & \\ .000008 & .000015 & & \\ .000014 & .000014 & .000025 & \\ .000014 & .000014 & 000029 & .000043 \end{bmatrix}$$

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