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BANKRUPTCY IN THE LIFE-CYCLE CONSUMPTION MODEL

By Kim J. Kowalewski

Kim J. Kowalewski is an economist at the Federal Reserve Bank of Cleveland. He is currently on leave as a Senior Staff Economist at the Council of Economic Advisers.

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I. Introduction

The standard life-cycle model of consumption assumes that loans are always fully repaid, even when future income is uncertain. Zeldes (1986) shows that these two assumptions imply, in the context of constant relative risk aversion, that current consumption is very sensitive to current income. He concludes that recent tests of the "excess" sensitivity of consumption spending to transitory income are erroneous because the "excess" sensitivity evident in the data is a natural outcome of the life-cycle model and not an indication of liquidity-constrained behavior.

This paper investigates the effects of dropping the assumption that loans are fully repaid with probability one. It solves, using stochastic dynamic programming, a four-period, life-cycle model of individual consumption behavior that includes the possibility of a Chapter 7 or liquidation bankruptcy. One important characteristic of this model is that the partial-equilibrium effects of bankruptcy raise the demand for borrowing and current consumption. This loosens the connection between current consumption and income, thereby providing a rationale for using "excess" sensitivity of consumption to unexpected changes in current income as an indicator of liquidity-constrained behavior.

Bankruptcy creates a discontinuous optimization problem for consumers. A Chapter 7 bankruptcy implies a different set of consumption opportunities than the constraint imposed by the capital market. Bankruptcy entails legal fees, the loss of some assets as payment to creditors, a tarnished credit rating, and possibly the social stigma associated with the failure to repay one's debts. In return, bankruptcy reduces the amount of debts to be repaid, stops wage garnishment and other legal actions of lenders, and allows the bankrupt consumer to keep certain assets as a "fresh start" to life after bankruptcy. In the period of the bankruptcy filing, a consumer optimizes with respect to the bankruptcy constraint, which is different from the constraint he used before he filed for bankruptcy. In the periods following the bankruptcy filing, a consumer may again face different constraints if lenders tighten credit terms to former bankrupts.

Along with bankruptcy, a second feature of the model is an endogenous borrowing rate of interest. The borrowing interest rate is greater than the risk-free lending rate by the explicit default risk created by bankruptcy. The borrowing rate is endogenously set to equate loan demand with supply by the requirement that creditors expect to earn zero profits from lending to the consumer. Thus, the borrowing rate generally rises with the amount borrowed because greater borrowing raises expected loan losses, directly through the amount borrowed and indirectly through the probability of default.

The next section of this paper discusses the assumptions of the model. The third section describes the model's structure. The fourth section discusses the simulation results, and the last section provides summary and concluding remarks.

II. Assumptions of the Theoretical Model

11.1. Definition of Insolvency

One precondition for bankruptcy is financial distress or insolvency, which may be defined as the inability to pay contractual obligations, such as mortgage and installment debts and insurance premiums, in full on a timely basis.[1] Insolvency can arise for a number of reasons. Unplanned income losses, spending needs, and interest-rate increases can place burdensome demands upon a consumer's financial resources. Simple errors by consumers and lenders in evaluating the ability to repay debts also appear to be an important reason for insolvency.[2] For simplicity, only contractual debt payments appear in the model.

Note that insolvency is not defined here as the condition that debts are greater than assets. Insolvency is a flow concept, not a stock concept. Letting y_t be labor income in period t, A_t be the stock of assets owned in period t, and TLP_t be total loan payments due in period t, and excluding all taxes, a consumer is insolvent when discretionary funds (DF_t) are strictly negative:

(1) $DF_t = y_t + A_t - TLP_t < 0.$

11.2 Exogenous Income Uncertainty

An 'obvious necessary condition for insolvency is uncertainty about the future. Without uncertainty, consumers cannot borrow more than they can repay in some states of the world because their future income and creditworthiness are known to creditors. Only uncertainty about future labor income is assumed here because it is the most important source of uncertainty to consumers.[3] Current-period income is known, but all future income is unknown and is assumed to be independently distributed over time. The probability density function of y_t is denoted f_t, and is assumed to be defined over strictly positive y_t. Min_t and max_t are the minimum and maximum values of the income distribution. Although the income probability density function is assumed to be exogenous to the consumer, the probability of insolvency is endogenous because current spending actions affect the ability to weather future income declines and, hence, to avoid bankruptcy.

Although information is imperfect, it is symmetric. Consumers know the credit-supply function, and creditors know the consumer's reputation and income probability density functions. Hence, there are no moral hazard or adverse selection problems of consumers intentionally borrowing more than can be repaid in every state of the world.

11.3 No Alternatives to Bankruptcy

A second precondition for bankruptcy is a lack of viable alternatives to bankruptcy. Insolvency does not always lead to bankruptcy because the consumer may be able to refinance his debt with his current creditor or with a new creditor. If the insolvency is more severe, a consumer may be able to use the services of a consumer-credit advisory service. Many lenders will cooperate with these services in order to limit bankruptcy-related costs and maintain valuable customer relationships. Or, an insolvent consumer may have the option of a wage-earner trusteeship, such as the one administered by the Municipal Court in Cleveland, Ohio, to forestall legal action and arrange a debt repayment plan.[4]

To isolate the response of the optimal consumption path to the possibility of bankruptcy, all of these alternatives will be ignored in this paper. This restriction imposes a specific, though not unreasonable, assumption on lender behavior: all insolvent borrowers are forced into bankruptcy. That is, the consumer files for bankruptcy in period t if and only if $DF_t < 0$. Gale and Hellwig (1985) show that this type of loan contract is incentive-compatible in a one-period model. In the multiperiod model of this paper, this may not be optimal lender behavior. Indeed, borrowers may be given a grace period to make up delinquent payments over time because bankruptcy is costly and hurts customer relations. A more general model would allow the lender greater freedom in managing the loan, but this feature would only obscure the main conclusions of the model.[5]

Voluntary bankruptcy is not allowed in order to keep the model simple.

II.4 Chapter 7 Bankruptcy

Chapter 13 of the Bankruptcy Code is not truly bankruptcy, but a court-sponsored debt repayment plan. The full impact of limited liability is seen in Chapter 7, which is used by the majority of consumers who file for bankruptcy. Chapter 7 bankruptcy in this model takes a very simple form. In return for a discharge of all current debts, the consumer must give creditors all financial assets, plus current-period labor income in excess of that period's minimum labor income. That is, the minimum value of the probability density function for labor income in the period of a bankruptcy filing is the consumer's exempt assets.

11.5 One Bankruptcy per Lifetime

To keep the analysis simple, consumers may file for bankruptcy only once. This is not a severe restriction because only four periods are examined and the bankruptcy law prohibits successive Chapter 7 discharges within six years. This constraint is enforced by assuming that creditors restrict their lending to former consumer bankrupts. The restriction takes the form of requiring consumers to repay all debts with probability one; this is the standard Yaari (1964) life-cycle model analyzed by Zeldes (1986). Without the possibility of default, the cost of borrowing after bankruptcy is the risk-free rate.[6] This reduced borrowing opportunity is the main cost of bankruptcy in the model. A more complete model would include other costs of bankruptcy, such as the loss of nonexempt tangible assets.

11.6 The Characteristics of Assets and Debts

Nonhuman assets are perfectly liquid, predictable, and reversible financial assets that earn the risk-free rate of interest (R-1). Consumers may not own durable goods, but may rent their services. All debt is unsecured and finances spending on nondurable goods and services. There are no collateral requirements, and no bequests.

Given that the consumer files for bankruptcy when current income and assets do not cover current debt payments, the specification of loan maturities is very important. Single-period loans imply a very strict bankruptcy rule, and hence, a strict constraint on borrowing. Moreover, multiperiod loans are the rule rather than the exception for consumer lending. Hence, the longest possible loan maturities are assumed: a loan taken out in the first period is repaid in equal installments over the following three periods; a loan taken out in the second period is repaid over the following two periods; a loan taken out in the third period is repaid in the fourth and last period. Borrowing is not allowed in the last period, and loans cannot be prepaid.

The periodic loan payment (LP_t) for an N-period loan made in period t of size B_t is computed from the present value formula:

$$(2) \quad B_t = LP_t/R_b + LP_t/R_b^2 LP_t/R_b^3 + \ldots + LP_t/R_b^N,$$

where R_b is one plus the borrowing rate of interest.

11.7 Zero-Profit Credit Supply Constraint

The key feature of this model is the possibility of less than full debt repayment in periods before bankruptcy. The implication is that rational creditors must price default risk; before bankruptcy, the supply of credit to consumers cannot be the perfectly elastic function of the risk-free rate of interest found in the Yaari life-cycle model.

A conventional approach is to assume a perfectly competitive, risk-neutral creditor who maximizes expected discounted profits. Revenues are the contractual loan payments, plus any proceeds from a bankruptcy judgement, and costs are the cost of funds. There are no transactions costs, and the supply of funds available to creditors is perfectly elastic at the risk-free rate of interest.

The credit-supply constraint is the first-order condition for maximizing discounted expected profits. It equates the discounted expected cost of funds lent to the consumer with the discounted expected revenues from the loans, with the borrowing rate of interest as the equilibrating mechanism. For simplicity, only one borrowing rate is charged for borrowing in all periods before bankruptcy. That is, creditors make a contingent contract with borrowers that specifies one borrowing interest rate and the amounts to be borrowed in every state of the world each period before bankruptcy.[7] The price of default risk thus is defined as the addition to the risk-free interest rate necessary to equate the discounted expected revenues from lending with the discounted expected cost of funds.[8] In general, the credit-supply curve will be upward sloping because additional borrowing raises expected loan losses. Its slope will depend on the probability density functions of the consumer's labor income and on the demand for credit.

The structure of the credit-supply constraint can be illustrated with a three-period problem and a two-point probability function for labor income.

Let $p_{t,k}$ denote the probability that income in period t is low (k=1) or high (k=2). Assume that bankruptcy is impossible in the first period, the optimal solution implies bankruptcy in the low-income state in periods two and three, borrowing (B_1) occurs in the first period and borrowing ($B_{2,2}$) occurs in the high-income state in period two, and that there are no bequests. The consumer repays B_1 in equal installments (LP_1) in the following two periods and $B_{2,2}$ is fully repaid with interest in the third period. The debt payment $LP_1 = B_1(R_b)^2/(1+R_b)$ from equation (2). Creditors match the maturity of their debts to their assets, repaying first-period borrowing in equal installments of CP_1 in periods two and three and $(R)(B_{2,2})$ in period three. Then the discounted expected cost of funds to the creditor is:

$$CP_1/R + CP_1/R^2 + p_{2,2}(R)(B_{2,2})/R^2,$$

and the discounted expected return from lending is:

$$(p_{2,1})(Y_2\text{-}min_2)/R + (p_{2,2})(LP_1)/R + (p_{2,2})\ (p_{3,1})(Y_3\text{-}min_3)/R^2$$
$$+ (p_{2,2})(p_{3,2})(LP_1 + (R)(B_{2,2})/R^2.$$

If the consumer owned any assets at the time of the bankruptcy filing, some portion of these assets would figure into the loan return.

III. Model Structure

The objective is to maximize the expected present discounted value of utility from consumption over periods one through T, which is four. The consumer begins with an endowment of human and nonhuman wealth, never having filed for bankruptcy, and there is no possibility of bankruptcy in the first period. Current-period income is known when the consumption decision is made. Arrangements are made to borrow in the first three periods, and the cost of borrowing may rise with the amount of borrowing. There is no inflation and no taxes.

The structure of the model can be understood by imagining a solution tree in discrete time.[9] The initial branch of the tree is followed over time unless the consumer cannot meet all of his debt payments. If forced into bankruptcy, the consumer moves onto a new branch of the tree where bankruptcy costs are paid and all debts at the time of the bankruptcy filing are discharged. Once on this new branch, the consumer can never leave it; the consumer faces a strict borrowing constraint that excludes the possibility of additional bankruptcy filings. Clearly, the solutions along these post-bankruptcy branches are independent of those along the initial branch, but not vice versa.

The four-period solution tree is shown in figure 1. The nodes along each branch are labelled (t,j), where t denotes the time period and j denotes the branch number. Branch 0 is the initial branch where bankruptcy is never filed. A branch number greater than 0 refers to the post-bankruptcy branches and indicates the period in which the consumer filed for bankruptcy. For example, the coordinate (3,2) refers to the third period along branch number two and indicates that the consumer filed for bankruptcy in period two. Thus, the time index t is greater than or equal to j along any post-bankruptcy branch. This notation will be used in the formal model specification below.

The model is structured as a two-state, dynamic programming problem with two sets of constraints. The two-state variables are discretionary funds and a bankruptcy indicator variable that denotes the period of a bankruptcy filing. For notational convenience, these two-state variables can be combined into one, $DF_{t,j}$, which denotes discretionary funds at time t along branch j, using the above notation. Consistent with the earlier definition, $DF_{t,j} = y_t + RA_{t-1,j} - TLP_{t,j}$, where $A_{t-1,j}$ is financial assets in period t-1 along branch j and $TLP_{t,j}$ is total loan payments due in period t along branch j. The two sets of constraints are the zero-profit credit supply constraint and the constraints on consumption in the various states. The control variables are new borrowing and new acquisitions of financial assets for each state of the world in each period.

Let $V(DF_{t,j})$ denote the maximum present discounted value of utility from period t to T along branch j. For the post-bankruptcy branches (j > 0) and t < T,

$$(3)\quad V(DF_{t,j}) = \begin{cases} \max\limits_{B,A} U(min_t + B_{t,j} - A_{t,j}) + \delta E V(DF_{t+1,j}), & t = j > 0 \\ \max\limits_{B,A} U(DF_{t,j} + B_{t,j} - A_{t,j}) + \delta E V(DF_{t+1,j}), & t > j > 0 \end{cases}$$

for nonzero j and t = T,

$$V(DF_{T,j}) = \begin{cases} U(min_T) & \text{for } T = j, \\ U(DF_{T,j}) & \text{for } T > j > 0; \end{cases}$$

where δ is the inverse of 1 plus the rate of time preference; $B_{t,j}$ is new borrowing on branch j during period t; U() is the utility of consumption function, defined over nonnegative consumption and twice differentiable with U' > 0, U" < 0; E is the expectation operator over labor income. In the period of the bankruptcy filing (t = j), consumption equals exempt assets plus new borrowing because previous-period net wealth was eliminated by the bankruptcy filing.[10] In the periods following a bankruptcy filing (t > j), consumption equals labor income, plus new borrowing, minus total loan payments due in the period, minus new saving in the financial asset. There is no decision in the last period because there is no bequest motive.

Apart from the shift in the consumption constraint in the period of a bankruptcy filing, (3) is essentially the simple Yaari (1964) model with perfect capital markets; discretionary funds are always strictly greater than zero along these branches. The maximum expected present discounted value of utility, from period j to T, along branch j > 0 will be denoted as $PDVUBr_j$

For the solutions of interest along the initial branch (j = 0) and t < T,

$$(4)\quad V(DF_{t,0}) = \max_{B_{t,0},A_{t,0}} U(DF_{t,0}+B_{t,0}-A_{t,0})+\delta EV(DF_{t+1,0}),$$

subject to the zero-profit, credit-supply constraint, where

$$EV(DF_{t,0}) = \int_{\min_t}^{TLP_{t,0}-RA_{t-1,0}} PDVUBr_t f_t dy_t + \int_{TLP_{t,0}-RA_{t-1,0}}^{\max_t} \{\max_{B_{t,0},A_{t,0}} U(DF_{t,0}+B_{t,0}-A_{t,0}+\delta EV(DF_{t+1,0}) f_t dy_t\}$$

For t = T,

$$V(DF_{T,0}) = U(DF_{T,0}).$$

The transition equation for the state variable $DF_{t,j}$ is

$$DF_{t,j} = \begin{cases} DF_{t-1,j}+Dy_t-DTLP_{t,j}+RDA_{t,j}, & j=0,\ t>j>0 \\ \min_t \qquad , & j=t, \end{cases}$$

where D is the difference operator and $DTLP_{,,j} = LP_{t,j}$.

The major difference between equations (3) and (4) lies in the $EV()$ terms. Because there is no possibility of bankruptcy along the post-bankruptcy branches, the future utility term assumes a simple form in equation (3). However, the possibility of a future bankruptcy filing is a key element of the branch 0 decision problem. The consumer must balance certain consumption today with uncertain consumption tomorrow, where the uncertainty about tomorrow's consumption is complicated by the possibility of bankruptcy. If the consumer never borrows enough to raise the probability of bankruptcy above zero, equation (4) reduces to the simple Yaari life-cycle model.

The probability of bankruptcy in each period, P_t, is defined as follows. Because income is independently distributed over time, the probability, Q_t, reaching node (t,0), for any period t, is simply the product of the marginal probabilities of nonnegative discretionary funds during the first t periods:

$$Q_t = \prod_{k=1}^{t} \Pr(DF_{k,0} \geq 0).$$

The probability of filing for bankruptcy in period t is the product of the probability of not filing for bankruptcy in the first t-1 periods and the probability that discretionary funds are negative period t:

$$P_t = Q_{t-1}[\Pr(DF_{t,0} < 0)],$$

where the bracketed term on the right is defined to be one when t is one. Note that the sum of P_t and Q_t is not one. The difference is the probability of filing for bankruptcy sometime before period t

It is interesting to note that the specification of the dynamic program implicitly uses these conditional densities in the formation of the expectations. This is readily apparent by expanding all of the terms of equation (4) and writing the objective function as the discounted sum of expected utility. The reason comes from the nonlinear shift in the program after bankruptcy. The probability of following a particular path in the tree, that is, obtaining a particular level of utility, depends on previous and current actions. For example, the probability of obtaining the utility value from branch two (bankruptcy in the second period) in period four is the probability of a bankruptcy filing in the second period times the density function of income in the fourth period.

IV. Simulation Results

Because a general, closed-form solution does not exist for this problem, numerical solution of a particular specification is the only feasible solution technique. The utility function of the simulation model is assumed to exhibit constant relative risk aversion:

$$U(C_{i,j}) = (1/(1-A))(C_{i,j})^{1-A}.$$

In accordance with estimation results reported by Zeldes (1986), the value of A is three in all the simulations. The rate of time preference is 20.0 percent, the risk-free rate of interest is five percent, and initial wealth is zero. The probability density function of y_t is assumed to be a three-point, discrete distribution, with

$$y_{t,i} = (MeanY_t)e_i, \quad \text{for } i=1,2,3,$$

where $MeanY_t$ is the mean value of y_t, and e_i is an independent, identically distributed random variable with a mean of one and a probability function:

e_i	probability
0.2	0.02
1.0	0.96
1.8	0.02.

The $MeanY_t$ values are:

Period	MeanY
1	100
2	250
3	400
4	200.

A symmetric distribution for e_i was chosen for simplicity, and the $MeanY_t$ values were chosen to mimic a textbook life-cycle income profile.

Details of the simulation model and its solution are given in Kowalewski (1989).

IV.1 Baseline Simulation

There are four main characteristics of the bankruptcy model. First, relative to the Yaari model, the possibility of bankruptcy shifts consumption from periods late in the life cycle to periods early in the life cycle. As shown in table 1, first-period consumption in the bankruptcy model simulation is about 39 percent greater than first-period consumption in the Yaari model, and the mean value of second-period consumption is over 14 percent greater. Indeed, the time pattern of consumption in the bankruptcy model is similar to that of the certainty equivalent model, also shown in table 1. Consumption is shifted from late to early periods in the life cycle when the rate of time preference is greater than the rate of interest.[11] This stands in sharp contrast to expected consumption in the Yaari model, which is more closely correlated with labor income.

Second, bankruptcy's role as insurance (Arrow 1971) against adverse labor income draws lowers the variance of consumption in every period. The variance of consumption is about 52 percent less in the second period, almost 31 percent less in the third period, and about 10 percent less in the last period.

Third, the present value of expected future utility is greater in the bankruptcy model than in the Yaari model.

Finally, default risk drives a wedge between the borrowing and risk-free rate of interest. The optimal amount of borrowing in the bankruptcy model implies a nonzero probability of bankruptcy in all future periods, with the probabilities falling over time. In the second period, bankruptcy occurs with a probability of 0.02, when the minimum value of labor income results. In the third period, bankruptcy occurs with a probability of 0.0192, when second-period labor income is its mean value and third-period labor income is its minimum value. Bankruptcy occurs in the fourth period with a probability of 0.000008, when second-period labor income is its largest value and both third- and fourth-period labor income are their minimum values. These probabilities create a wedge of 1.725 percentage points between the borrowing and lending rates of interest in the baseline simulation.[12]

IV.2 Changes in the Risk-Free Rate of Interest

Increases in the risk-free rate will raise the borrowing rate of interest directly and indirectly through the default risk premium. This section discusses the impact of changes in the risk-free rate of interest, with all of the other parameters held at their baseline values. Seven experiments were run using odd values of the risk-free rate between 1 and 13 percent. The results are shown in table 2.

The top half of table 2 shows that the borrowing rate and the risk premium increase with the risk-free rate. The relationship between either the risk premium or the borrowing rate of interest and the risk-free rate is linear in this range. A 2 percentage point increase in the risk-free rate raises the risk premium by 0.04 percentage point and the borrowing rate by 2.04 percentage points. The linearity of these relationships is partly due to the assumption of symmetric information. Models with adverse selection, for example Stiglitz and Weiss (1981), would show a nonlinear relationship between the risk-free rate and the borrowing rate of interest. At some sufficiently great interest rate in these models, it is optimal to ration credit by quantity and not by price.

Another reason why the relationships are linear is that the probabilities of bankruptcy in all future periods do not vary across the simulations. The simple, three-point probability function for labor income leaves ample room for borrowing to vary without a change in the probabilities of bankruptcy. If the income probability function were continuous and not uniform, the relationships would not be linear, with successive increases in the risk-free rate implying ever-larger increases in the risk premium and in the borrowing rate of interest.

The bottom half of table 2 compares the elasticities of first-period borrowing and consumption with respect to the risk-free rate in the bankruptcy and Yaari models. The Yaari results assume borrowing and lending rates are equal to the risk-free rate of five percent. The elasticities for both borrowing and consumption are negative and quite small for both models over this range of risk-free interest rates. First-period borrowing in the bankruptcy model is less sensitive to changes in the risk-free rate than it is in the Yaari model.[13] First-period consumption is slightly more elastic in the bankruptcy model, but the differences in the two sets of elasticities is very small. Indeed, the difference between the consumption elasticities is too small to serve reliably as the basis of an econometric test of the two models.

IV.3 Changes in Initial Wealth

The baseline results indicate that the possibility of bankruptcy loosens the relationship between income and consumption found in the Yaari model. As a corollary, the possibility of bankruptcy generally will lower the marginal propensity to consume (MPC) changes in initial wealth. Table 3(a) displays the MPCs of the bankruptcy, Yaari, and certainty-equivalence models. The far left column of the table shows first-period income levels used for the simulations. The MPCs for each model were computed by dividing the successive differences of these income levels into the successive differences of their corresponding first-period consumption levels.

The differences among these MPCs are rather dramatic. Although the values of first- and second-period consumption shown in table 1 are very close in the bankruptcy and certainty-equivalence models, the MPCs of the two models are very dissimilar. The MPCs of the certainty-equivalence model are the lowest of the three models and are constant across income values. The MPCs of the Yaari model are the largest, except when labor income is 175, and they fall monotonically as income rises. The MPCs from the bankruptcy model generally fall between those of the other two models and show an irregular pattern as income rises. They rise from 0.321, when income is 50, to 0.537 when income is 100. They fall for the next two income values, rise when income is 175, and fall when income is 200. When income is 225, the MPC is negative and less than 1, but it increases for income value 250.[14]

The irregular pattern of the MPCs from the bankruptcy model is due to the possibility of bankruptcy, and not to the endogenous borrowing rate of interest. This should be clear from the low interest rate elasticities shown in table 2. Moreover, table 3(b) compares the MPCs from the bankruptcy model when the borrowing rate is endogenous with those from the bankruptcy model when the borrowing rate is exogenously fixed at the risk-free rate of five percent. The MPCs assuming an exogenous borrowing rate have a greater variance, suggesting that the endogenous borrowing rate moderates the consumption response of the model.

Table 4 shows how the borrowing interest rate, the probabilities of bankruptcy, expected borrowing, and expected consumption change as first-period labor income changes. For example, when income increases from 25 to 50, first-period borrowing falls 16.972 units, expected second-period borrowing increases 1.187 units, and so on. The same interpretation holds for the consumption changes. The levels of consumption and borrowing are shown for income equal to 25. Borrowing is either the purchase of new debt or new financial assets; a negative value of borrowing indicates saving in a financial asset.

The time pattern of expected consumption varies greatly as first-period income increases. For income values 50 and 75, increases in income are fairly well-spread across time. The changes are front-loaded because the rate of time preference is greater than the borrowing interest rate. For income values 100 to 200, most of the change in expected consumption occurs in the first two periods. When income is 225, it is close to the expected value of second-period income and it shifts expected consumption away from the first two periods. First-period consumption falls by 27.3 units, producing the negative MPC; second-period expected consumption falls 31.1 units; and third- and fourth-period expected consumption increase by 54.9 and 37.1 units, respectively. This shift in expected consumption implies a shift in expected borrowing, which lowers the probability of bankruptcy in the second period and the borrowing rate of interest by 88 basis points. When income is 250, the change in income again is fairly well-spread across time. Indeed, expected consumption is fairly even across time, as shown in table 3(c).

Table 4 also shows the impact of the discrete nature of the labor income probability function. Between income values 25 and 125, the borrowing rate falls slightly with the amount of first-period borrowing because the probabilities of bankruptcy remain unchanged. When income is 150, the borrowing rate falls a relatively large amount because the drop in the demand for borrowing in the first period lowers the probability of bankruptcy in the second period from 0.02 to zero. The borrowing rate increases slightly for the next two income values before it falls, with the probability of bankruptcy in the third period, at income value 225. For all income values except 175, 200, and 250, the borrowing rate falls with first-period borrowing. This surprising result may be due to the fact that the credit-supply constraint is an inverse cubic equation. This nonlinearity may give the distribution of borrowing across time and states of nature a large impact on the borrowing rate of interest.

IV.4 Changes in the Probability Density Function of Labor Income

Changes in the probability density function of labor income may have two effects. First, a known change in the variance of future income will lead risk-averse consumers to shift the time pattern of consumption. Second, a known change in the probabilities of bankruptcy will change the borrowing rate of interest. In particular, an increase in the probability of a bad income draw will raise the borrowing interest rate and shift consumption from early to later periods in the life cycle.

Table 5 displays the results from symmetrically increasing the tails of the e_i distribution, and compares the first-period borrowing and consumption elasticities in the bankruptcy and Yaari models. The top half of table 5 indicates that the borrowing rate of interest is a positive function of the tail probability. The bottom half of the table indicates that the resulting shifts in first-period borrowing and consumption are extremely small in the bankruptcy model.

IV.5 Changes in the Cost of Bankruptcy

An increase in the cost of bankruptcy in this model will lower the demand for borrowing and hence the borrowing rate of interest. In the aggregate, a greater cost of bankruptcy will lower average bankruptcy filings. The easiest way to change the cost of bankruptcy in this model is to change the amount of labor income that may be kept by the consumer after bankruptcy. Table 6 displays the results of allowing the consumer to keep 25, 50, 75, and 100 percent of his minimum labor income after bankruptcy.

The results indicate that as the cost of bankruptcy increases, the demand for borrowing and the borrowing rate of interest decrease. The magnitudes of the differences should not be taken as reasonable estimates of real-world impacts. The small difference between the results for exempt assets fractions 0.25 and 0.50, and that for 0.74 and 1.00, are due to the small size of exempt assets. The baseline simulation assumes that exempt assets are 100 percent of the minimum value of the labor-income distribution in the period of the bankruptcy filing. These minimum values are already small numbers, and taking fractions of them yields small changes. The relatively large difference between the results for exempt-asset fractions 0.50 and 0.75 again is due to the discrete nature of the labor income density function.

The results in table 6 square with the increase in consumer bankruptcy filings after the Bankruptcy Reform Act of 1978 became effective in October, 1979. This act lowered the cost of bankruptcy to consumers, and came at a time when real consumer income growth was slowing, debt burdens were high, and portfolios were very illiquid.[15] Initially, the sharp increase in bankruptcy filings was due to the insolvent consumers at the margin.[16] Since then, consumer bankruptcy filings have remained at an historically high level because lower bankruptcy costs increased consumer willingness to borrow, which was accommodated by consumer lenders.[17]

V. Summary and Conclusions

This paper develops and analyzes a life-cycle model that incorporates the possibility of insolvency and its resolution with bankruptcy. Insolvency is defined as the inability to repay debts in full out of current income and nonhuman wealth. After-tax labor income is an exogenous random variable, but the probability of insolvency is endogenous to the consumer. The consumer maximizes the present discounted value of expected utility subject to the usual cash flow constraint and a zero-profit credit supply constraint, which equates the demand and supply of credit with the borrowing rate of interest. Loan maturities are generally not one-period, but the number of periods remaining in the life cycle when the loans are made.

Once insolvent, the consumer is immediately forced into a Chapter 7 bankruptcy by creditors. There are two costs of bankruptcy in the model. One is the payment of the delinquent debts with any current-period income greater than its minimum value that period, plus any nonhuman assets. The other, more important cost, is a change in the borrowing constraint: after bankruptcy, all debts must be repaid with probability one. In return for these costs, the bankrupt consumer is discharged from all debts.

The relaxed credit-supply constraint in the bankruptcy model loosens the dependence of current consumption on current income relative to that in the Yaari model. The time path of expected consumption closely follows that of expected labor income in the Yaari model. The time path of consumption in the bankruptcy model is less closely related to that of income, but it is not divorced as in the certainty equivalence model. Indeed, the marginal propensities to consume in the bankruptcy model generally lie between those of the certainty equivalence and Yaari models. An interesting feature of the model is that the marginal propensity to consume may be negative, as increases in current income shift consumption from periods early in the life cycle to later ones.

Increases in the risk-free rate of interest raise the borrowing rate of interest and lower the demand for borrowing. The elasticities of current consumption with respect to the risk-free rate are small and only marginally larger than those of the Yaari life-cycle model. Increases in the variance of future labor income, or equivalently, increases in the probability of bankruptcy, increase the borrowing rate of interest and lower the demand for borrowing. Finally, an increase in the cost of bankruptcy lowers the demand for borrowing and the borrowing interest rate.

The findings of this paper provide support for the strategy of testing for the "excess sensitivity" of current consumption to unexpected changes in current income (see Kowalewski [1985b]). Zeldes (1986) has argued that these tests are invalid because they assume certainty equivalence, which reduces the income sensitivity of consumption. His computer simulations, and those in this paper, show that the excess sensitivity is a characteristic of the Yaari model without certainty equivalence. Adding bankruptcy to the Yaari model without certainty equivalence reduces the income sensitivity of consumption, thereby adding support to the research strategy. Nevertheless, the findings of the "excess sensitivity" papers may be biased because the certainty-equivalence model is not a good approximation to the model with bankruptcy.

The model of this paper can be extended in four important ways. One is the addition of tangible assets. The loss of certain tangible assets in bankruptcy is an important cost of bankruptcy, especially if liquidity constraints tighten after bankruptcy.

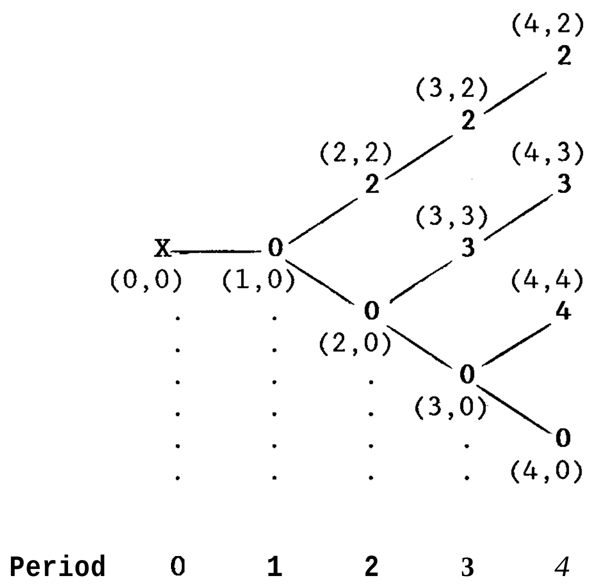
A second extension makes consumption needs a stochastic variable. Accidents and medical problems are an important source of financial problems for consumers who file for bankruptcy. Stochastic consumption needs would lower the demand for borrowing and increase the sensitivity of current consumption to changes in current income.

A third extension is the allowance for asymmetric information. The risk premia found in this paper are very small because they capture the uncertainty only about future income, not about the distribution of income or the integrity of the borrower. The addition of asymmetric information would increase the risk premia and would provide an estimate of the value of information to creditors.

Finally, another extension is the allowance for general equilibrium. This requires, at a minimum, that the cost of funds paid by creditors rise with the amount of borrowing.

FIGURE 1

FOUR-PERIOD SOLUTION TREE



Source: Author

TABLE 1

BASELINE SIMULATION RESULTS
BANKRUPTCY AND YAARI MODELS

Period	Expected Income	Expected Consumption Bankruvtcy	Expected Consumption Yaari	Expected Consumption Certainty Equivalence
1	100	248.761	178.982	249.867
2	250	235.353	205.586	238.990
3	400	214.555	278.386	228.585
4	200	238.656	285.228	218.634

Period	Variance of Expected Consumption Bankruvtcv	Variance of Expected Consumption Yaari	Probability of Bankruvtcy
1	NA	NA	NA
2	325.634	674.586	0.020000
3	1118.576	1614.701	0.019200
4	1612.294	1783.803	0.000008

Present Discounted Value of Expected Future Utility

Bankruptcy	Yaari
-3.03939E-5	-3.846273-5

Interest Rates in the Bankruptcy Model

Borrowing	Risk-Free
6.725%	5.000%

Source: Author

TABLE 2

THE RISK-FREE RATE OF INTEREST SIMULATIONS

The Relationship Between the Risk-free Rate and the Borrowing Rate

Risk-free Rate	Risk Premium	Borrowing Rate
1.000%	1.646%	2.646%
3.000%	1.685%	4.685%
5.000%	1.725%	6.725%
7.000%	1.764%	8.764%
9.000%	1.803%	10.803%
11.000%	1.843%	12.843%
13.000%	1.882%	14.882%

Risk-free Interest Rate Elasticities for First-period Borrowing and Consumption

	BANKRUPTCY MODEL		YAARI MODEL	
Risk-free Rate	Borrowing	Consumption	Borrowing	Consumption
1.000%	NA	NA	NA	NA
3.000%	-0.015	-0.009	-0.017	-0.008
5.000%	-0.040	-0.024	-0.050	-0.023
7.000%	-0.076	-0.045	-0.083	-0.036
9.000%	-0.097	-0.057	-0.114	-0.049
11.000%	-0.119	-0.069	-0.145	-0.062
13.000%	-0.139	-0.080	-0.175	-0.073

Source: Author

TABLE 3

MARGINAL PROPENSITIES TO CONSUME CHANGES IN INITIAL WEALTH

(a)

First-Period Income	Certainty Equivalence	Bankruptcv	Yaari
25	NA	NA	NA
50	0.286	0.321	0.661
75	0.286	0.364	0.630
100	0.286	0.537	0.598
125	0.286	0.506	0.566
150	0.286	0.503	0.536
175	0.286	0.527	0.509
200	0.286	0.413	0.485
225	0.286	-1.092	0.464
250	0.286	0.319	0.446

(b)

First-Period Income	Endogenous Borrowing Rate	Exogenous Borrowing Rate
50	0.321	0.316
75	0.364	0.371
100	0.537	0.518
125	0.506	0.362
150	0.503	0.563
175	0.527	0.495
200	0.413	0.538
225	-1.092	-1.230
250	0.319	0.320

(c)

Expected Consumption

	First-Period Income				
Period	100	150	200	225	250
1	248.761	273.981	297.487	270.181	278.146
2	235.353	264.655	291.416	260.315	267.886
3	214.555	212.978	213.014	267.964	274.132
4	238.656	236.694	237.821	274.963	279.860

Source: Author

TABLE 4

CHANGES IN INITIAL WEALTH
BANKRUPTCY MODEL

Income	Borrowing Interest Rate	Period	Probability of Bankruptcy	Change in Expected Borrowing	Change in Expected Consumption
25	6.748%	1	0.00000	193.196*	218.196*
		2	0.02000	29.124*	207.311*
		3	0.01920	-111.779*	202.083*
		4	0.00001	0.000*	231.230*
50	6.744%	1	0.00000	-16.972	8.028
		2	0.02000	1.187	7.501
		3	0.01920	1.418	6.968
		4	0.00001	0.000	4.061
75	6.737%	1	0.00000	-15.890	9.110
		2	0.02000	2.626	8.539
		3	0.01920	1.037	5.418
		4	0.00001	0.000	3.291
100	6.725%	1	0.00000	-11.573	13.427
		2	0.02000	7.689	12.002
		3	0.01920	0.007	0.087
		4	0.00001	0.000	0.073
125	6.711%	1	0.00000	-12.360	12.640
		2	0.02000	8.212	12.818
		3	0.01920	0.007	0.093
		4	0.00001	0.000	0.079
150	5.891%	1	0.00000	-12.421	12.579
		2	0.00000	12.106	16.484
		3	0.01960	0.189	-1.670
		4	0.00039	0.000	-2.041
175	5.924%	1	0.00000	-11.814	13.186
		2	0.00000	7.731	12.116
		3	0.01960	0.032	0.187
		4	0.00039	0.000	0.122
200	5.964%	1	0.00000	-14.680	10.320
		2	0.00000	9.188	14.646
		3	0.01960	-0.571	-0.151
		4	0.00001	0.000	1.005
225	5.084%	1	0.00000	-52.305	-27.305
		2	0.00000	-50.931	-31.101
		3	0.00040	8.704	54.949
		4	0.00039	0.000	37.143
250	5.105%	1	0.00000	-17.035	7.965
		2	0.00000	1.310	7.571
		3	0.00040	0.619	6.168
		4	0.00039	0.000	4.897

*Levels

Source: Author

TABLE 5

THE EFFECTS OF CHANGES IN THE PROBABILITY DENSITY FUNCTION OF LABOR INCOME

Tail Probability	Variance	Borrowing Rate	Change in Rate
0.02	0.025	6.725%	NA
0.04	0.050	8.500%	1.775%
0.06	0.074	10.326%	1.826%
0.08	0.099	12.201%	1.875%
0.10	0.124	14.125%	1.924%
0.12	0.149	16.095%	1.970%

First-period Borrowing and Consumption Elasticities

Tail	BANKRUPTCY MODEL		YAARI MODEL	
Probability	Borrowing	Consumption	Borrowing	Consumption
0.02	NA	NA	NA	NA
0.04	-0.020	-0.045	-0.169	-0.075
0.06	-0.034	-0.046	-0.273	-0.108
0.08	-0.057	-0.061	-0.366	-0.132
0.10	-0.064	-0.059	-0.456	-0.151
0.12	-0.084	-0.070	-0.548	-0.168

Source: Author

TABLE 6

CHANGES IN THE COST OF BANKRUPTCY

Exempt Asset Percentage	Borrowing	Consumption	Borrowing Interest Rate
25.0%	*128.926*	*228.926*	*5.04862%*
50.0%	*128.978*	*228.978*	*5.04877%*
75.0%	*148.692*	*248.692*	*6.72473%*
100.0%	*148.761*	*248.761*	*6.72481%*

Source: Author

[1]Some consumer lenders argue that insolvency is no longer a precondition for bankruptcy. The Bankruptcy Reform Act of 1978 removed all legal preconditions for bankruptcy, and many lenders feel that the large increase in personal bankruptcy filings after the Act became effective in October, 1979 is evidence that consumers have abused the law by taking advantage of this change and the Act's liberalized exemption limits. Nevertheless, the solution of this abuser problem is trivial and will be ignored.

[2]See for example, Stanley and Girth (1971).

[3]Other sources of uncertainty are possible but complicate the analysis. Stochastic consumption needs raises the possibility of multiple bankruptcy filings per lifetime, which greatly increases the number of solutions required for a problem of given horizon length. Adding stochastic interest rates magnifies the "curse of dimensionality."

[4]See Kowalewski (1982) for a description of this and other alternatives to bankruptcy.

[5]Moreover, a "grace period" variation is a trivial debt restructuring problem unless there is uncertainty about the availability of the grace period.

[6]Due to asymmetric information, it is likely that consumers must pay a greater interest rate after bankruptcy. The use of the risk-free rate is a logical consequence of the assumed borrowing restriction; it is not an crucial assumption because the disutility from the borrowing restriction more than offsets the utility from a lower borrowing rate. A more complete model would allow multiple bankruptcies and greater borrowing interest rates after bankruptcy.

[7]One interest rate for all borrowing before bankruptcy is not restrictive because there is no moral hazard or adverse selection problems. A more realistic assumption would be asymmetric information, which would admit the possibility of credit rationing and time-varying interest rates before bankruptcy. Learning behavior by creditors would be a desirable and complementary feature to add to the model. The assumption of symmetric information is a useful first step that helps to isolate the impact of limited liability.

[8]Using a simple portfolio balance approach, the default premium also may be defined as the extra percentage return necessary to equate the discounted expected loan return with the discounted return from lending the same amount at the risk-free rate of interest. Note that this default premium is not a risk premium as defined by Pratt (1964) because the utility of profits function is linear in profits.

[9]The decision-tree framework of this model is similar to those of Foley and Hellwig (1975) and Watkins (1978). Both take the same view of a consumer following a tree of consumption opportunities, whose branches are determined by discrete, nonlinear changes in the intertemporal budget set. These changes are determined by the employment status of the consumer, which is an

exogenous, stochastic process; insolvency and bankruptcy are ignored.

[10] It may seem odd that the consumer is allowed to borrow in the period of a bankruptcy filing. However, it is logically consistent given the assumption of symmetric information and the constraint that all debts incurred after bankruptcy are fully repaid with probability one. Prohibiting borrowing in the period of a bankruptcy filing would magnify the impact of bankruptcy in the simulation results shown below.

[11] The increase in expected consumption in the last period of the bankruptcy model may be due to the lower probability of bankruptcy in that period.

[12] The size of this wedge may seem small until it is realized that the wedge is only the default premium under symmetric information, and does not include transactions costs or the additional costs created by asymmetric information.

[13] The interest-rate elasticities are somewhat larger if the borrowing rate is used.

[14] Although the MPCs from the bankruptcy and Yaari models are different at these income levels, it is likely that the MPCs from the bankruptcy model approach, and eventually equal, those of the Yaari model as first-period income increases without limit. With sufficiently great first-period income, the consumer will have no need to borrow more than he would have if he was required, as in the Yaari model, to repay all debts with probability one.

[15] See Kowalewski (1982) for a discussion of the financial position of households in the late 1970s and early 1980s.

[16] The initial sharp increase also may be due to consumers anticipating the passage of the act. These consumers may have postponed a bankruptcy filing in order to file under the new act or increased their borrowing before the effective date of the act.

[17] The strength in consumer borrowing since 1982 is discussed in Kowalewski (1986).

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