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**Designing Stabilization Policy  
in a Monetary Union**

by Russell Cooper and Hubert Kempf



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While the European Monetary Union (EMU) is now a reality, debate among economists nonetheless continues about the design and desirability of monetary unions. Since an essential element of a monetary union is the delegation of monetary power to a single centralized entity, one of the key issues in this debate is whether a monetary union will limit the effectiveness of stabilization policy. If so, monetary union will not necessarily be welfare improving.

In this paper, we study a two-country world economy and consider various designs of monetary union. We argue that the success of monetary union depends on: (i) the commitment ability of the single central bank, (ii) the policy flexibility of the national fiscal authorities and the central monetary authority and (iii) the cross country correlation of shocks. If, for example, the central bank moves before the fiscal authorities, then a monetary union will increase welfare as long as fiscal policy is sufficiently responsive to shocks. However, if the fiscal authorities have a restricted set of tools and/or the monetary authority lacks the ability to commit to its policy, then monetary union may not be desirable.

# Designing Stabilization Policy in a Monetary Union\*

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## Abstract

While the European Monetary Union (EMU) is now a reality, debate among economists nonetheless continues about the design and desirability of monetary unions. Since an the essential element of a monetary union is the delegation of monetary power to a single centralized entity, one of the key issues in this debate is whether a monetary union will limit the effectiveness of stabilization policy. If so, monetary union will not necessarily be welfare improving.

In this paper, we study a two-country world economy and consider various designs of monetary union. We argue that the success of monetary union depends on: (i) the commitment ability of the single central bank, (ii) the policy flexibility of the national fiscal authorities and the central monetary authority and (iii) the cross country correlation of shocks. If, for example, the central bank moves before the fiscal authorities, then a monetary union will increase welfare as long as fiscal policy is sufficiently responsive to shocks. However, if the fiscal authorities have a restricted set of tools and/or the monetary authority lacks the ability to commit to its policy, then monetary union may not be desirable.

# 1 Overview

While the European Monetary Union (EMU) is now a reality, the debate nonetheless continues about the desirability and design of monetary unions. Many countries, such as Argentina, are considering the dollarization of their economy as a possible policy. Further, some economists are advocating the creation of a North American Monetary Union.<sup>1</sup>

The framework for these discussions essentially remains that outlined by Mundell [1961]. The gains to monetary union are associated with the reduction of transactions costs while the costs are associated with reduced effectiveness of stabilization policy once monetary policy is delegated to a single central bank. These issues are paramount in the so-called Delors report (Emerson et al., [1992]) which provided the official arguments in favor of EMU.<sup>2</sup> In his evaluation of the EMU, Feldstein [1997] perfectly exemplifies the dilemma about the evaluation of a monetary union when he writes:

”My own judgement is that, on balance, a European Monetary Union would be an economic liability. The gains from reduced transaction costs would be small and might, when looked at from the global point of view, be negative. At the same time, EMU would increase cyclical instability, raising the cyclical unemployment rate”.

While certainly relevant, we think that this framework misses an important element: the design of monetary union. Clearly there is more to monetary union than the adoption of a single currency. The success of any monetary union, in particular the EMU, will depend in large part on the appropriate design of monetary policy, including its interaction with national fiscal policies. Existing (and defunct) monetary unions have experienced very different macroeconomic performances, particularly with regard to the stabilization of output and inflation. Understanding these outcomes may prove valuable in the consideration of alternative monetary arrangements.

In the debate over monetary union, a number of interrelated questions naturally emerge. First, when is it in the interest of a set of countries or

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<sup>1</sup>On North American Monetary Union, see Buiter [1999], Grubel [1999] and the references therein.

<sup>2</sup>These same points reappear in the analysis of the proposed North American Monetary Union as discussed by Buiter [1999].

regions to share a common currency and to delegate monetary policy to a single central bank? Second, how should the monetary authority operate vis-a-vis the independent fiscal authorities, either national or regional? Third, can this monetary/fiscal system adequately respond to shocks? Under what conditions will stabilization policy be effective in a monetary union?

We address these questions using a variant of the multiple country, overlapping generations model set forth in Cooper-Kempf [1998].<sup>3</sup> The model we explore has two key ingredients: (i) the presence of country specific shocks which generate a basis for stabilization policy and (ii) strategic interaction between the multiple fiscal authorities and the single monetary authority. Thus our approach is quite different from Mundell [1961] which rested on price inflexibility in a static economy without any explicit representation of welfare and without any recognition of the vital importance of the interactions between fiscal and monetary policy.

Our analysis emphasises the institutional framework of monetary union, particularly with regards to the conduct of stabilization policy. We contend that the issues of stabilization policy and design of a monetary union are closely interrelated: the stability of member economies within a monetary union depends on the manner in which the stabilization instruments are allocated. Thus the debate, as framed by Mundell, misses this crucial dimension.

To take just the recent example of the EMU, designers of monetary unions have been cognizant of these issues. For the EMU, the Maastricht treaty contains several clauses specifying in detail the institutional set-up of the EMU: the European system of Central Banks, including the European Central Bank, is strongly independent of any government; its prime objective is price stability; the ECB cannot bail-out any fiscal authority. Further, the Amsterdam Treaty contains a "Growth and Stability Pact" which, in effect, strongly constrains the ability of member countries to run fiscal deficits. A detailed system of punishments has been established in the event a country incurs a deficit which is "excessive" in light of current economic conditions. Clearly the design of a monetary union like EMU matters a lot and cannot be overlooked.

With respect to the key questions noted above, we find that if the mone-

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<sup>3</sup>The point of our earlier exercise was to explore the gains to monetary union arising from two sources: the reduction in transactions costs and the internalization of "beggar thy neighbor" type monetary policies that could arise in a setting with multiple independent monetary authorities.

tary authority has the ability to move prior to the fiscal authorities, then it is able to select a monetary policy that will support the first-best allocation of risk. Essentially, the *strong* central bank uses its commitment to thwart any strategic interactions between the independent fiscal authorities. Given the monetary policy choice, the fiscal authorities, using state contingent tax rates, insure agents against unemployment risk in their countries. In this case, the monetary union creates benefits of reduced transactions costs without jeopardizing stabilization policy: the supposed trade-off does not exist. For these reasons, monetary union is welfare improving.

We explore two alternative institutions in which the monetary union is less successful. In one, we consider the possibility that some policymakers, either the fiscal or monetary authorities, are unable to adequately respond to economic events. Here delegating monetary control to a single central bank is generally costly: unless the country specific aggregate shocks are perfectly correlated, the single central bank will not have enough tools. For the second, we assume that the central monetary authority does not have the ability to credibly commit to its policy. A welfare loss from monetary union may arise from this *weak* central bank that responds to the budget decisions of the independent fiscal authorities.<sup>4</sup> As a consequence, each member country resorts to money creation as a means of financing its fiscal deficit to the sole benefit of its own citizens since the inflation tax is borne by all member countries. In other words, there exists the possibility that a monetary union is so poorly designed that a seignorage game arises between national fiscal authorities, leading to a loss of stabilization and welfare-reducing inflation. In this case, a monetary union can reduce welfare relative to a world economy with multiple currencies.

## 2 First-Best Allocation

This section of the paper presents our model of the world economy and solves for the first-best allocation. This forms a benchmark to which we can compare the allocations obtained by different monetary/fiscal systems.

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<sup>4</sup>As in Sargent-Wallace [1981], a key issue is the determination of the coordination between monetary and fiscal policy: which is chosen first?



## 2.1 Basic Environment

We study an overlapping generations model which builds upon the analysis of Cooper-Kempf [1998] by adding uncertainty and the interaction between fiscal and monetary policy. All agents live for two periods and reside in one of two countries. By assumption, labor is immobile.<sup>5</sup> In each country, a single, nonstorable good is produced. Agents work in youth, producing the country specific good and consume the goods produced in both countries in their old age.

Agents differ within countries due to their employment status: each period a random fraction  $q_t$  of them have an employment opportunity while the remainder are unable to produce.<sup>6</sup> This device allows us to include real shocks to the model and motivates government intervention through the provision of unemployment insurance.<sup>7</sup> We assume that  $q_t$  is a serially uncorrelated shock with  $Eq_t \equiv \bar{q}\epsilon(0, 1)$ .

With regards to their preferences, agents consume the good produced at home and the one produced abroad. As in Cooper-Kempf [1998], we introduce individual specific shocks into preferences to motivate the gains from monetary union. More formally, individuals' in the home country have preferences represented by the following utility function:

$$U^{ih} = U(c_{t+1}^{ih}, c_{t+1}^{if}, n^i) = \theta_{t+1} \ln(c_{t+1}^{ih}) + (1 - \theta_{t+1}) \ln(c_{t+1}^{if}) - g(n^i). \quad (1)$$

Here the superscript  $i$  corresponds to the status of the agent, whether she is employed ( $i = E$ ) or unemployed ( $i = U$ ),  $c_{t+1}^{ih}$  is the level of consumption of the home good when old, and  $c_{t+1}^{if}$  is the level of consumption of the foreign good when old, and  $n^i$  is the supply of labor by agent  $i$ , necessarily equal to zero if  $i = U$ . The function  $g(\cdot)$  represents the disutility of work, and is assumed to be increasing, convex and continuously differentiable. Finally,  $\theta_{t+1}$  is an individual specific taste shock that influences the marginal rate of substitution between home and foreign goods, where  $\theta_{t+1}$  is an i.i.d random

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<sup>5</sup>Thus, relative to the discussion in Mundell [1961], our results are biased against finding gains to monetary union.

<sup>6</sup>Here and throughout the paper, we generally refer directly to home country variables and denote those for the foreign country with \* superscripts. Unless stated otherwise, we assume that  $q_t$  and  $q_t^*$  are nondegenerate random variables.

<sup>7</sup>Clearly this is just one of a multitude of possible shocks. This specification was chosen partly for tractability and partly due to the fiscal obligation that arises from the provision of unemployment insurance, a natural and quite visible form of stabilization policy.

variable with mean  $\bar{\theta}$ . Given the specification of the utility function, it is natural to restrict  $\theta_{t+1} \in [0, 1]$ . This shock is realized in period  $t + 1$ , after the labor supply decision.

The production function is the identity function and there is no capital. As agents are identical at the time of their labor supply decision, they will work the same amount, denoted as  $n_t$ . Hence per capita output of the home good in period  $t$  is simply  $n_t q_t$ .

There is an analogous objective function for foreign agents:

$$U^{*ih} = U(c_{t+1}^{*ih}, c_{t+1}^{*if}, n^{*i}) = \theta_{t+1}^* \ln(c_{t+1}^{*if}) + (1 - \theta_{t+1}^*) \ln(c_{t+1}^{*ih}) - g(n_t^{*i})$$

We assume that  $\theta_{t+1}$  and  $\theta_{t+1}^*$  have the same distribution. Per capita output of the foreign good is then given by  $n_t^* q_t^*$ .

## 2.2 Planner's Problem

The planner's objective is to maximize a weighted average of the welfare of the agents in each generation and in each country. As the countries are completely symmetric, the planner gives equal weight to them.<sup>8</sup> Further, we assume that the planner treats each generation equally. Since there is no physical capital and preferences are intertemporally separable, we look at allocations within a period where the planner decides on the employment levels of generation  $t$  agents and the consumption levels for generation  $t - 1$  agents. In doing so, the planner is assumed to know the current state of the economy, including the realized tastes for each agent and the employment rates in the two countries. Put differently, the planner chooses state contingent consumption allocations for old agents and employment levels for young agents where the aggregate state is denoted by  $s = (q_{-1}, q_{-1}^*, q, q^*)$  and  $\theta$  ( $\theta^*$ ) represents the realization of the taste shock for a home (foreign) agent.<sup>9</sup>

Formally, at the start of any period, the planner selects a contingent consumption profile for home and foreign old agents given by:

$$c(s, \theta) = (c^{Eh}(s, \theta), c^{Ef}(s, \theta), c^{Uh}(s, \theta), c^{Uf}(s, \theta))$$

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<sup>8</sup>Thus the model abstracts from any political considerations that might lead to unequal treatment of countries and/or generations.

<sup>9</sup>Here  $q$  ( $q^*$ ) denotes a current value and  $q_{-1}$  ( $q_{-1}^*$ ) the past value. In principle, the planner could make consumption dependent on past employment status as well. Further,  $\theta$  represents the realized taste shock for an arbitrary home agent and  $\theta^*$  is the realized value for an arbitrary foreign agent.

and

$$c^*(s, \theta^*) = (c^{*Eh}(s, \theta^*), c^{*Ef}(s, \theta^*), c^{*Uh}(s, \theta^*), c^{*Uf}(s, \theta^*))$$

respectively. Further, the planner chooses  $(n(s), n^*(s))$ , the state contingent employment levels of the current generation. Note that these functions are not time dependent as we focus on stationary solutions.

The planner solves

$$\begin{aligned} \max_{c(s, \theta), c^*(s, \theta^*), n(s), n^*(s)} E_{(s, \theta, \theta^*)} \{ & q_{-1} U^{Eh} + (1 - q_{-1}) U^{Uh} + q_{-1}^* U^{*Eh} + (1 - q_{-1}^*) U^{*Uh} \\ & - qg(n(s)) - q^*g(n^*(s)) \} \end{aligned}$$

subject to:

$$E_{\theta, \theta^*} [q_{-1} c^{Eh}(s, \theta) + (1 - q_{-1}) c^{Uh}(s, \theta) + q_{-1}^* c^{*Eh}(s, \theta^*) + (1 - q_{-1}^*) c^{*Uh}(s, \theta^*)] = n(q, q^*)q$$

and

$$E_{\theta, \theta^*} [q_{-1} c^{Ef}(s, \theta) + (1 - q_{-1}) c^{Uf}(s, \theta) + q_{-1}^* c^{*Ef}(s, \theta^*) + (1 - q_{-1}^*) c^{*Uf}(s, \theta^*)] = n^*(q, q^*)q^*$$

for all  $s$ .

The resulting allocation has two important aspects: optimal risk sharing across agents and an efficient level of employment. Formally:

**Proposition 1** *The first-best allocation is characterized by:*

$$\frac{\theta}{c^{Eh}(s, \theta)} = \frac{\theta}{c^{Uh}(s, \theta)} = \frac{1 - \theta^*}{c^{*Eh}(s, \theta^*)} = \frac{1 - \theta^*}{c^{*Uh}(s, \theta^*)}$$

$$\frac{1 - \theta}{c^{Ef}(s, \theta)} = \frac{1 - \theta}{c^{Uf}(s, \theta)} = \frac{\theta^*}{c^{*Ef}(s, \theta^*)} = \frac{\theta^*}{c^{*Uf}(s, \theta^*)}$$

and

$$g'(n)nq = 1 = g'(n^*)n^*q^*.$$

**Proof.** See appendix. ■

From this proposition, the planner allocates the given amount of the two consumption goods across the agents so that the ratio of marginal utilities of consumption between any two agents is state independent. This is the

familiar expression of optimal risk sharing. For our economy this implies that any two agents with the same realization of tastes should consume identical consumption bundles independent of nationality or employment status. As we shall see in the decentralized economies studied below, this condition will be met if, at the end of their first period of life, young agents have the same levels of income regardless of employment status and nationality.

For the employment decision, recall that young agents do not know their tastes. Thus for the planner, the employment levels may only depend on  $(q, q^*)$  as indicated in the proposition.

### 3 A Multi-currency world economy

Given this planner’s solution, we now turn to a discussion of decentralized environments. Our starting point is a multiple currency world economy in which both fiscal and monetary policies are determined by national governments. A key issue is the financing of obligations to the unemployed and risk sharing across countries since this is the essence of ”stabilization policy” in our environment.

This section presents a baseline model of a two-country world economy in which the governments in each of the two countries independently choose their fiscal and monetary policies. In order to capture potential gains to monetary union from a reduction in trading frictions, there is an incomplete markets feature in our model. In each period, goods markets open **before** exchange markets. Thus, old agents are unable to adjust their currency portfolios in response to taste shocks. As we shall see, this leads to a misallocation of resources.<sup>10</sup>

We first solve the individual’s optimization problem. We then characterize optimal government policies and the equilibrium allocations for this institution.

#### 3.1 Individual Optimization

The agent’s decisions over employment and consumption goods are constrained by her income and by cash-in-advance constraints as both goods

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<sup>10</sup>Clearly the assumed timing of markets is key to our results. In our framework, the cost of going to an exchange market for old agents is thus infinite. As we shall see, the implications of this infinite cost can be parameterized by the uncertainty over tastes.

must be purchased in the national currency.<sup>11</sup> We show these constraints in detail for an agent in the home country.

Let  $I_t^i$  represent the income of a home agent of generation  $t$  with employment status  $i = E, U$ . Further, let  $m_t^{ih}$  and  $m_t^{if}$  represent the holdings of home and foreign currency by a home agent of generation  $t$  with employment status  $i = E, U$ . The constraints of a home agent are then given by:

$$m_t^{ih} + e_t m_t^{if} = I_t^i$$

$$p_{t+1} c_{t+1}^{ih} = m_t^{ih}$$

and

$$p_{t+1}^* c_{t+1}^{if} = m_t^{if}$$

for  $i = E, U$ . According to the first constraint, income earned in youth is held either as home currency or foreign currency, where  $e_t$  is the period  $t$  price of foreign currency in terms of home currency. The other constraints relate the holdings of the different currencies to the consumption of final goods and thus reflect the cash-in-advance constraints.

Given income levels, the consumption levels of the individual agents are given by:

$$c_{t+1}^{ih} = \frac{\bar{\theta} I_t^i}{p_{t+1}} \quad \text{and} \quad c_{t+1}^{if} = \frac{(1 - \bar{\theta}) I_t^i}{p_{t+1}^*}$$

for  $i = E, U$ . Note that in contrast to the planner's problem, consumption is not dependent on the realization of the taste shock. It is in this sense that multiple currencies lead to the misallocation of resources and thus a gain from monetary union, a topic we explore in detail below.

Finally, in order to determine the labor supply decisions of agents, we need to be specific about their income levels and thus about government policies. Governments are assumed to levy taxes on labor incomes, where  $\tau_t^F$

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<sup>11</sup>Cooper-Kempf [1998] argue that the cash-in-advance constraints can be generated endogenously by allowing individual governments to choose over legal restrictions regarding the type of currency used in their home markets. Here, for tractability, we impose these constraints directly. Notable contributions by Lucas [1990], Fuerst [1992] and Christiano and Eichenbaum [1992] also use a particular sequencing of exchanges to limit transactions through the introduction of costly market participation.

denotes the period  $t$  tax rate. Thus the income of a representative employed agent is given by:

$$I_t^E = p_t n_t (1 - \tau_t^F). \quad (2)$$

Using this definition of income, the first order condition with respect to employment by a representative generation  $t$  agent is:<sup>12</sup>

$$1 = n_t g'(n_t).$$

So, for Cobb-Douglas utility functions, individual labor supply decisions are independent of the proportional labor income tax. Denote by  $\bar{n}$  this input.

The income of a representative unemployed agent is given by:

$$I_t^U = b_t^F \quad (3)$$

where  $b_t^F$  is the nominal unemployment benefit paid to an unemployed agent. These transfers are financed from the tax revenues collected from the employed agents and the printing of new currency as described further below.

### 3.2 Market Equilibrium given government policies

In each period, a goods market and a money market open in each of the two countries. The market clearing conditions for the good markets, home and abroad respectively, in period  $t + 1$  are:

$$q_{t+1} \bar{n} = q_t c_{t+1}^{Eh} + (1 - q_t) c_{t+1}^{Uh} + q_t^* c_{t+1}^{*Eh} + (1 - q_t^*) c_{t+1}^{*Uh} \quad (4a)$$

$$q_{t+1}^* \bar{n} = q_t^* c_{t+1}^{*Ef} + (1 - q_t^*) c_{t+1}^{*Uf} + q_t c_{t+1}^{Ef} + (1 - q_t) c_{t+1}^{Uf} \quad (4b)$$

The left side of these expressions is the output by employed agents in period  $t + 1$ , using the fact that all employed agents produce  $\bar{n}$  units of output. The right side corresponds to spending on home (foreign) goods by the four types of agents: employed and unemployed in each of the two countries.

For the money markets, the given stock of fiat money must equal the nominal value of output. For the home and foreign countries this implies:

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<sup>12</sup>The derivation of this expression is part of the proof of Proposition 2.

$$M_t = p_t q_t \bar{n} \quad (5a)$$

$$M_t^* = p_t^* q_t^* \bar{n}. \quad (5b)$$

The exchange market clearing condition is simply:

$$q_t^* m_t^{*Eh} + (1 - q_t^*) m_t^{*Uh} = e_t (q_t m_t^{Ef} + (1 - q_t) m_t^{Uf}). \quad (6)$$

Finally, the governments each face a budget constraint. The transfers to the unemployed must be financed by either the collection of tax revenues or the printing of money, denoted  $\Delta_t^F$ . Thus, for the home country,

$$p_t q_t \tau_t^F \bar{n} + \Delta_t^F = (1 - q_t) b_t^F \quad (7a)$$

and for the foreign country:

$$p_t^* q_t^* \tau_t^{*F} \bar{n} + \Delta_t^{*F} = (1 - q_t^*) b_t^{*F}. \quad (7b)$$

These transfers to agents occur prior to exchange market trades. Thus, the transfer of domestic currency to unemployed agents does not imply that they are unable to consume foreign goods.

The allocation given government policies is then characterized by:

**Proposition 2** *In a world economy with multiple currencies, given government transfer policies the (home) consumption allocations are given by:*

$$\begin{aligned} c_{t+1}^{Eh} &= \bar{\theta} \cdot \frac{q_{t+1}}{q_t} \cdot \frac{1 - \tau_t^F}{1 + \sigma_t} \cdot \bar{n}, & c_{t+1}^{Ef} &= (1 - \bar{\theta}) \cdot \frac{q_{t+1}^*}{q_t} \cdot \frac{1 - \tau_t^F}{1 + \sigma_t} \cdot \bar{n}, & (8a) \\ c_{t+1}^{Uh} &= \bar{\theta} \cdot \frac{q_{t+1}}{1 - q_t} \left( 1 - \frac{1 - \tau_t^F}{1 + \sigma_t} \right) \bar{n}, & c_{t+1}^{Uf} &= (1 - \bar{\theta}) \cdot \frac{q_{t+1}^*}{q_t} \left( 1 - \frac{1 - \tau_t^F}{1 + \sigma_t} \right) \bar{n}. & (4) \end{aligned}$$

**Proof.** See appendix. ■

In these expressions, the rate of growth of the money supply is denoted by  $\sigma_t^F$  and is computed directly from the transfers. That is,  $\sigma_t^F M_t = \Delta_t^F \equiv M_t - M_{t-1}$  and  $\sigma_t^{*F} M_t^* = \Delta_t^{*F} \equiv M_t^* - M_{t-1}^*$ . As usual, there are analogous expressions for foreign agents.

### 3.3 Equilibrium Government Policies

The fiscal and monetary authorities within each country are integrated. However, governments act non-cooperatively vis-a-vis one another. Their objective is to maximize the expected lifetime utility of a representative young agent in their own country. They choose tax rates and money creation rates simultaneously each period, after observing the fraction of people currently employed and taking as given the policy choices of the other government.<sup>13</sup>

We find that the equilibrium solution of the non-cooperative game is characterized by:

**Proposition 3** *In a world economy with flexible exchange rates and multiple currencies, the equilibrium policies satisfy:*

$$\xi_t \equiv \frac{1 - \tau_t^F}{1 + \sigma_t^F} = q_t, \quad \xi_t^* \equiv \frac{1 - \tau_t^{*F}}{1 + \sigma_t^{*F}} = q_t^*. \quad (9)$$

**Proof.** See appendix. ■

In this expression,  $\xi_t$  represents the period  $t$  policy choice of the home government. This policy reflects the rate of labor taxation and the rate of money creation. In equilibrium the governments are indifferent with respect to the nature of the taxation: the optimal policy is characterized by combinations of the labor tax and the inflation tax that transfer a given amount to the unemployed. This indeterminacy, reminiscent of the Tinbergen rule, simply reflects the existence of two instruments at the disposal of governments when each government only has a goal: to insure agents against the risk of unemployment.<sup>14</sup>

Using the optimal tax policies, equilibrium consumption allocations are:

$$c_{t+1}^{Eh} = c_{t+1}^{Uh} = \bar{\theta} q_{t+1} \bar{n} \quad \text{and} \quad c_{t+1}^{Ef} = c_{t+1}^{Uf} = (1 - \bar{\theta}) q_{t+1}^* \bar{n} \quad (10a)$$

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<sup>13</sup>Note that the money supplies in each country are the only state variables in the system as there is no physical capital. In addition, policy choices can be contingent on the realized values of the employment shocks in the two countries. The equilibria that we characterize are dependent only on the country specific employment rates and are Markov perfect given this representation of the state space. Note that these restrictions seem natural given that the inherited stocks of fiat money in each country are, in the equilibria we consider, irrelevant for real allocations. Of course, there may exist other equilibria in which inherited money stocks matter.

<sup>14</sup>The fact that there are more instruments than targets in the multi-currency environment is important for understanding conditions under which monetary union will lead to a loss of stabilization.



and

$$c_{t+1}^{*Eh} = c_{t+1}^{*Uh} = (1 - \bar{\theta})q_{t+1}\bar{n} \quad \text{and} \quad c_{t+1}^{*Ef} = c_{t+1}^{*Uf} = \bar{\theta}q_{t+1}^*\bar{n}. \quad (10b)$$

Note that the optimal non-cooperative government policies perfectly insure agents from the current risk of unemployment. Further, the flexible exchange rate system implies that consumptions are equalized across countries as well. In this sense, the multiple currency equilibrium with optimal government policies provides insurance both within and across countries.

However, there are some important differences between this allocation and that obtained by the planner. First, the labor supply of the home (foreign) agents is independent of  $q$  ( $q^*$ ) as the Cobb-Douglas preferences imply that this decision is independent of the tax rate. In the planner's solution, employment varies with  $q$  ( $q^*$ ) in order to stabilize total output. Interestingly, this optimal employment policy would arise if the government had access to lump-sum taxes.

Second, these agents suffer from not being able to adequately tie their demand for currencies to the actual value of the preference parameter  $\theta$ . That is, in the planner's solution, consumption is optimally dependent on the realized taste shocks. In the world economy with multiple currencies, this dependence is not feasible.

Using the expressions for consumption allocations, the expected utility (welfare) generated by this equilibrium is equal to:

$$\begin{aligned} V_F &\equiv E \left[ q_t U \left( c_{t+1}^{Eh}, c_{t+1}^{Ef}, \bar{n} \right) + (1 - q_t) U \left( c_{t+1}^{Uh}, c_{t+1}^{Uf} \right) \right] \\ &= E \left( \theta_{t+1} \ln(\bar{\theta}) + (1 - \theta_{t+1}) \ln(1 - \bar{\theta}) \right) \\ &\quad + \bar{\theta} E \ln(q_{t+1}) + (1 - \bar{\theta}) E \ln(q_{t+1}^*) + \ln(\bar{n}) - \bar{q}g(\bar{n}). \end{aligned} \quad (11)$$

In the remainder of the paper, we use this measure as the benchmark relative to which the various designs of a monetary union will be compared and assessed.

## 4 Strong Central Bank

We now consider a monetary union defined by the presence of a single currency and a **single** monetary authority which represents the interests of

agents in member countries.<sup>15</sup> In this section, we make a key assumption regarding timing: the central bank chooses its policy **before** the fiscal authorities in each period. We term this a **strong** central bank since the timing assumption implies that the monetary authority has commitment power vis-a-vis fiscal authorities. The next section of the paper explores the alternative case in which the central bank moves after the fiscal authorities.<sup>16</sup>

This section first shows the optimization problem of a representative agent in a given country and then explores the conditions for equilibrium given government policies. Then, as in the case of multiple currencies, we explore the equilibria of the game between governments under the assumption of a *strong* monetary union, defined as a monetary union with a strong central bank.

As will become clear, this is an institutional structure that creates gains to monetary union through the presence of a common currency. These gains arise directly from the reduction of trading frictions. Further, the reduction in the number of stabilization tools in each country (due to the creation of a central monetary authority) does **not** limit the extent to which agents are insured against aggregate shocks. Essentially, the fiscal authorities can respond to country specific unemployment shocks and thus efficiently insure the incomes of agents against employment risk. The risk associated with unemployment variations is dissipated in the goods markets.<sup>17</sup> So in contrast to the predictions arising from the work of Mundell, there is no tradeoff between stabilization policy losses and reductions in transactions costs associated with the creation of a monetary union. This section of the paper concludes with some discussion of the robustness of these findings.

## 4.1 Individual Optimization

Given the presence of a common currency, individual's with employment opportunities optimally choose their labor supply. All agents use income

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<sup>15</sup>To maintain symmetry, we assume that this stock of money is initially distributed equally across the two countries.

<sup>16</sup>Interestingly, this parallels the discussion of Sargent and Wallace [1981] who analyze closed economy monetary models with two polar forms of coordination between the fiscal and monetary authorities.

<sup>17</sup>This is clearly a strong result that, as we shall see, depends on the unitary elasticity of demand. Still, the economics of the problem implies that supply shocks are partially offset by price variations which tend to stabilize labor incomes.

received in youth to finance their consumption expenditures when old. In contrast to the world economy with multiple currencies, agents do not have to make portfolio decisions prior to the realization of their taste shocks. It is precisely the relaxation of the cash-in-advance constraint that leads to the gains associated with monetary union.

Formally, the representative young, employed agent of generation  $t$  from the home country solves the following optimization problem:

$$\max_{c_{t+1}^{Eh}, c_{t+1}^{Ef}, n_t} E\{\theta_{t+1} \ln(c_{t+1}^{Eh}) + (1 - \theta_{t+1}) \ln(c_{t+1}^{Ef})\} - g(n_t)$$

subject to

$$p_{t+1}c_{t+1}^{Eh} + p_{t+1}^*c_{t+1}^{Ef} = p_t n_t (1 - \tau_t^S) \equiv I_t^E.$$

Here the tax rate on the income of the home agent is given by  $\tau_t^S$ . In the budget constraint,  $p_t$  ( $p_t^*$ ) represents the period  $t$  price of home (foreign) goods in terms of the common currency.

The first order conditions for the representative, employed home agent are given by:

$$g'(n_t)n_t = 1 \tag{12a}$$

and

$$c_{t+1}^{Eh} = \theta_{t+1} \frac{I_t^E}{p_{t+1}} \quad \text{and} \quad c_{t+1}^{Ef} = (1 - \theta_{t+1}) \frac{I_t^E}{p_{t+1}^*}. \tag{12b}$$

Clearly (5) implies a constant labor supply equal to that obtained in the previous case,  $\bar{n}$ . Thus, with these preferences, the monetary institution has no effect on employment decisions. Since the consumption decision is made after the realization of the taste shock, consumption levels respond to this shock.

The representative unemployed, young agent from the home country decides how to allocate income, denoted by  $I_t^U$ , across the consumption goods in old age after observing the idiosyncratic taste shock. Optimal consumption levels satisfy:

$$c_{t+1}^{Uh} = \theta_{t+1} \frac{I_t^U}{p_{t+1}} \quad \text{and} \quad c_{t+1}^{Uf} = (1 - \theta_{t+1}) \frac{I_t^U}{p_{t+1}^*}. \tag{13}$$

Since these agents are unemployed, their nominal income is equal to the unemployment benefits they receive both from their national fiscal authority, denoted by  $b_t^S$ . There are analogous sets of conditions for employed and unemployed foreign agents.

The fiscal policies are given by the levels of labor taxation and unemployment benefits,  $(\tau_t^S, b_t^S)$  for the home government and  $(\tau_t^{*S}, b_t^{*S})$  for the foreign government. The central monetary authority creates money in period  $t$  which is distributed to the fiscal authorities of the two governments. We let  $\Delta_t^S$  and  $\Delta_t^{*S}$  denote the period  $t$  money transfers to the home and foreign governments.

So, the budget constraint of the home fiscal authority is given by:

$$(1 - q_t)b_t^S = \tau_t^S \bar{n} p_t q_t + \Delta_t^S. \quad (14)$$

The left side of this expression is the level of unemployment benefits paid by the home government. The right side is the sum of their nominal tax revenues and the money created that flows to the home government.

## 4.2 Equilibrium

For this institutional structure, the monetary authority determines the rate of money creation prior to the decisions of the two fiscal authorities. In game theoretical terms, we explore a two-stage game, where the central bank plays first, and then the two national governments play second, non-cooperatively determining their own policy. From (14), the money transfers simply supplement the resources available for the funding of unemployment insurance. Thus, the fiscal authorities, given these transfers, will design UI systems that equate the incomes of employed and unemployed agents, as in the regime of multiple currencies.

Taking these decision rules by the fiscal authorities as given, the monetary authority can use money creation to finance these UI programs and thus redistribute nominal incomes across countries to offset adverse employment shocks. However, in equilibrium, the response of prices to variations in the employment shocks is sufficient for the stabilization of nominal income: active intervention by the central monetary authority is not required. Thus we find:

**Proposition 4** *There exists an equilibrium in the strong monetary union where the rate of money creation is zero, irrespective of the values of the*

shocks, the taxation rates are equal to

$$\begin{aligned}\tau_t^S &= 1 - q_t \\ \tau_t^{*S} &= 1 - q_t^*.\end{aligned}$$

The consumption allocations are given by:

$$c_{t+1}^{Eh} = c_{t+1}^{Uh} = \theta_{t+1} q_{t+1} \bar{n} \quad \text{and} \quad c_{t+1}^{Ef} = c_{t+1}^{Uf} = (1 - \theta_{t+1}) q_{t+1}^* \bar{n} \quad (15a)$$

$$c_{t+1}^{*Eh} = c_{t+1}^{*Uh} = (1 - \theta_{t+1}) q_{t+1} \bar{n} \quad \text{and} \quad c_{t+1}^{*Ef} = c_{t+1}^{*Uf} = \theta_{t+1} q_{t+1}^* \bar{n} \quad (15b)$$

and the employment allocations satisfy  $n = n^* = \bar{n}$  where:

$$g'(\bar{n})\bar{n} = 1.$$

**Proof.** See appendix. ■

Note that the consumption levels for each type of agent depend on the *ex post* realization of the individual specific taste shock  $\theta$ . This dependence of consumption on  $\theta$ , as in the planner's allocation, is surely one of the gains associated with having a common currency.

A very important aspect of the resulting allocation is that agents' nominal income levels are independent of both their individual employment status and against country specific shocks to employment rates ( $q_t, q_t^*$ ) in their youth. The first form of insurance is provided directly by the taxation policy of the fiscal authorities, as in the above proposition. Recall from Proposition 2 that in the economy with independent fiscal and monetary policies, the authorities were indifferent between financing UI benefits with labor income taxes or seignorage. Essentially, the strong monetary union removes one of the two tools so that the fiscal authorities are led to the use of labor income taxes to finance the country specific UI systems.<sup>18</sup>

The second form of insurance is provided by the stabilization of nominal incomes in response to the supply side employment shocks. Given the unitary elasticity of demand, created by the Cobb-Douglas preference structure,

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<sup>18</sup>Thus one might conjecture that an important element in this discussion of stabilization with strong monetary union has to do with the number of instruments relative to targets. In our economy, a single government has two instruments to hit a target. Hence, as demonstrated in Proposition 2, there is an indeterminacy with respect to the nature of intervention. So, monetary union is not destabilizing here because even in the absence of active monetary intervention, the fiscal powers are sufficient. We return to this issue of instruments vs. targets in Section 5.

variations in prices effectively offset the exogenous variations in employment rates leaving the nominal income earned by young employed agents independent of  $q_t$ . If, as we assume, the money supply is equally distributed across the two countries in the initial period, then the young agents in each country will have equal shares of the money supply in all time periods. This guarantees that their nominal incomes are stabilized in the equilibrium with zero money growth.

There are other equilibria under strong monetary union. From Proposition 2, it is apparent that consumption allocations depend jointly on the rate of income taxation and the rate of money creation. The allocation in Proposition 4 picks one such combination of policies. In particular, if the lower support of the country specific employment shocks is sufficiently large, there will exist equilibria with positive money creation along with positive state contingent income taxes. To guarantee that nominal incomes are equal across countries, the nominal transfers to each country will be identical. Importantly, all of these equilibria support the same real allocations of consumption and employment.

In this equilibrium the welfare of the representative agent is:

$$V_S = E \{ \theta_{t+1} \ln(\theta_{t+1}) + (1 - \theta_{t+1}) \ln(1 - \theta_{t+1}) \} \\ + \bar{\theta} E \ln(q_{t+1}) + (1 - \bar{\theta}) E \ln(q_{t+1}^*) + \ln(\bar{n}) - \bar{q}g(\bar{n}). \quad (16)$$

Here the expectation is taken with respect to the individual specific taste shock  $\theta_{t+1}$  and to the employment shocks during the agent's lifetime  $(q_t, q_{t+1}, q_{t+1}^*)$ .

### 4.3 Welfare Comparisons

This expression of welfare allows us to compare the allocation under a strong central bank with that obtained by a world economy with multiple currencies. We find that:

**Proposition 5** *The difference in expected utilities achieved under a strong central bank and a flexible exchange rate world economy with multiple currencies is always positive and equal to:*

$$\Lambda_{SF} \equiv V_S - V_F = E \left\{ \theta_{t+1} \ln \left( \frac{\theta_{t+1}}{\bar{\theta}} \right) + (1 - \theta_{t+1}) \ln \left( \frac{1 - \theta_{t+1}}{1 - \bar{\theta}} \right) \right\} \quad (17) \\ \simeq 1/2 * var(\theta) \left[ \frac{1}{\bar{\theta}} + \frac{1}{(1 - \bar{\theta})} \right].$$

**Proof.** See appendix. ■

Clearly the gain to monetary union comes from the ability of agents' to response to the realization of their taste shocks. Analytically, this gain is reflected in the  $var(\theta)$  term in  $\Lambda_{SF}$ : increases in the variability of the taste shocks will reduce  $V_F$  and thus increase  $\Lambda_{SF}$ .

Relative to the planner's solution, the allocation under a strong central bank succeeds in producing optimal risk sharing: the nominal incomes and thus consumption allocations are equalized given tastes. In effect, the goods markets equalize nominal income across countries and the fiscal authorities equalize incomes across employment status. Thus optimal risk sharing is supported. However, as in the case of multiple currencies, the labor supply decision is not first best. Again this reflects the presence of proportional income taxes or, equivalently, the absence of lump sum taxation.

#### 4.4 Alternative preferences

Under a strong monetary union, inactivity is optimal for the central monetary authority. However, this is a consequence of the fact that Cobb-Douglas preferences have the special property of constant budget shares. In our multi-good setting, this implies that nominal spending on the good produced in a given country is constant. Thus, as the quantity of output varies, the price offsets these movements and nominal income is constant. It is precisely because of this unitary elasticity of demand that nominal incomes are stabilized across countries without the need for transfers from the central bank.<sup>19</sup>

To consider an alternative specification, suppose that preferences are instead given by:

$$\log(\theta c_{t+1}^{Eh} + (1 - \theta)c_{t+1}^{Ef}) - g(n_t).$$

With this preference structure, labor supply is still given by the condition of

$$n_t g'(n_t) = 1$$

but, in contrast to the Cobb-Douglas specification, aggregate budget shares will depend on realizations of  $(q_t, q_t^*)$ . In particular, if agents have identical,

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<sup>19</sup>Note that stabilization of nominal income implies that agents with the same tastes have the same consumption bundle (a condition for efficient allocations) since agents face the same prices.

nonstochastic, symmetric preferences with  $\theta = .5$ , then, in equilibrium,  $p_t = p_t^*$ .<sup>20</sup> This implies that nominal GDP in the home country relative to the nominal GDP of the foreign country is given by:

$$q_t/q_t^*$$

since all agents produce the constant level of output  $\bar{n}$ . So, if shocks are perfectly correlated across countries, then risk sharing remains optimal even with an inactive central bank.

If employment shocks are not perfectly positively correlated, nominal GDP in the home country will not equal nominal GDP in the foreign country. Hence, risk sharing across countries will be imperfect in the absence of central bank intervention. In fact, the central bank should create money and transfer it to the fiscal authority of the government in the country with the low level of nominal GDP. Given these funds, the fiscal authorities will equalize the nominal incomes of employed and unemployed agents. In effect, the optimal central bank rule is to stabilize nominal GDP and not necessarily prices.

In equilibrium, this equalization of nominal incomes will lead to efficient allocations of risk both within and across countries. So, the first best allocation of risk can be supported but, as this example illustrates, this outcome will generally require an active central bank. One might interpret this as a form of fiscal federalism given the ability of the central bank to redistribute real wealth across countries. Of course, the central bank is doing so through money creation rather than direct taxation.

## 5 Weak Monetary Union

The results obtained thus far indicate that concerns over the loss of stabilization policy through the creation of a monetary union may not be justified. However, it is important to realize that these results reflect two strong assumptions about the conduct of monetary policy:

- the central bank was able to choose monetary policy before the national authorities selected fiscal variables,

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<sup>20</sup>Clearly in this example there are no gains to monetary union since  $var(\theta) = 0$ . The point we make about the need for central bank intervention would easily extend to an example with taste shocks as long as the equilibrium required  $p_t = p_t^*$ .



- both the central bank and the fiscal authorities had a rich set of state contingent policy instruments at their disposal.

This section of the paper focuses on stabilization policy under alternative institutional structures which relax these strong assumptions. For these cases, we find that indeed monetary union may entail a loss in stabilization creating a tradeoff between the liquidity gains of joining a monetary union and the welfare losses from relatively ineffective stabilization policy.

In the first case, we retain the assumption that the central bank has commitment power vis-a-vis the fiscal authorities but impose restrictions on the nature of the fiscal interventions. We term this case a "fiscally constrained monetary union". In particular, we assume that the fiscal authorities are unable to set state contingent tax rates thus, in effect, reducing the set of instruments available to the policymakers.

In our second case, we weaken the central bank along two dimensions. First, we restrict the nature of the contingencies associated with monetary transfers so that the distribution of seignorage is the same across countries and independent of employment shocks. Second, we relax the assumption that the central bank has commitment power relative to the fiscal authorities. We term this a "weak central bank" case. We motivate and analyze this alternative extensive form structure where the fiscal authorities move prior to the monetary authority.<sup>21</sup>

## 5.1 A Fiscally Constrained Monetary Union

Up to now, our analysis indicates that the welfare loss from monetary union associated with a reduction in the number of policy instruments may not occur, given that the fiscal authorities retain stabilization tools (income taxes) and that the monetary authority has the ability to create money in order to stabilize nominal GDP. But what if some of these instruments are missing?<sup>22</sup>

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<sup>21</sup>Chari-Kehoe [1997] make a similar point in a two-period, non-stochastic, reduced form model with government debt and money. Their infinite horizon model provides a more formal argument but is essentially a two-period structure.

<sup>22</sup>Or, equivalently what if there are more "shocks" than instruments? In fact, consideration of this question leads one to wonder about the determination of the number of policy instruments. In the absence of a theory of policy instruments, we take the set of tools available to policymakers as exogenous but recognize that this is a rather strong assumption. For example, a country that had previously relied upon seignorage might develop an income tax system upon joining a monetary union.

Suppose, for example, that the fiscal authorities are unable to set tax rates contingent on the realized value of the employment shock: i.e., assume  $\tau = \tau^* = \bar{\tau}$ . In the multiple currency institution, this is not a problem since the condition for efficient risk sharing can be satisfied by the appropriate choice of a money creation rate by the country specific monetary authority.

However, with the creation of a monetary union, the fiscal authorities will lose this stabilization tool. Does a welfare loss result? Is this **fiscally constrained monetary union** still desirable relative to a world economy with multiple currencies? We find that the answers depend critically on the policies available to the monetary authority and, as suggested by Mundell, on the correlation of shocks across countries.

Recall that we have assumed the central monetary authority can make country specific transfers. In this case, it is feasible for the common central bank to make monetary transfers to the individual governments necessary to finance their unemployment insurance schemes. In fact, given the fixed tax rates the monetary authority can print and transfer money to the fiscal authorities to guarantee that the incomes of employed and unemployed agents are equated, *within* a country. However, in the resulting allocation, income levels and thus consumptions of agents *across* countries will not be equated unless the employment shocks are perfectly positively correlated.

Thus, in contrast to the allocations characterized in Proposition 5, risk sharing is imperfect unless  $\text{corr}(q_t, q_t^*) = 1$ . In fact, if shocks are perfectly correlated, the allocation in the fiscally constrained monetary union will be identical to that achieved under strong monetary union. In the language of instruments and targets, the perfect correlation of the shocks implies that one less instrument is needed so that the monetary authority can generate full insurance with its two instruments.

If  $\text{corr}(q_t, q_t^*) \neq 1$ , then there is clearly a loss in stabilization from the creation of a monetary union. In this case, a tradeoff emerges: there are liquidity gains from a monetary union that are increasing in the variability of the taste shocks and stabilization losses that depend on  $\text{corr}(q_t, q_t^*)$ . With the restrictions outlined above imposed on the fiscal authorities ( $\tau = \tau^* = \bar{\tau}$ ), we find:

**Proposition 6** *In a fiscally constrained monetary union, (i) if  $\text{corr}(q_t, q_t^*) = 1$ , the allocation is identical to that obtained under strong monetary union, (ii) if shocks are sufficiently correlated across countries and tastes are sufficiently variable, then the monetary union allocation will dominate the out-*

*come with multiple currencies and (iii) if employment shocks are not perfectly positively correlated and taste shocks are not sufficiently variable, then monetary union will not be welfare improving.*

**Proof.** See the Appendix. ■

Of course, as suggested by Mundell, these considerations lead to predictions about which types of economies will naturally merge to create a monetary union. From these results it is clearly those economies with positively correlated shocks and who trade with one another (so that there are indeed gains from trades and the reduction of transactions costs) which will profit from monetary union.

## 5.2 Constrained Central Bank

Instead of restricting the nature of interventions by the fiscal authorities, suppose that the central bank had limited powers of intervention. In particular, consider a central bank which was forced to equalize monetary transfers to the national governments. While our model contains no rationale for this restriction, to the extent that central banks (such as the ECB) are constrained in this manner, we can evaluate monetary union under these conditions.

As a starting point, consider again Proposition 4. In that setting, the first-best allocation of risk was attained **without** intervention by the central bank. Thus for our baseline model, there is no impact of constraining the central bank.

Now suppose that we consider the implications of a constrained central bank along with the constraints on the fiscal authorities imposed above:  $\tau = \tau^* = \bar{\tau}$ . In this case, a version of Proposition 6 holds. If the correlation of shocks is equal to 1, then the central bank will not have any incentive to differentiate transfers to the countries. By continuity, if this correlation is near 1, then the welfare losses from the additional constraints on the central bank will be minimal and thus monetary union continues to dominate the allocation with multiple currencies. At the other extreme, if the taste shocks are not too variable and employment shocks are not perfectly positively correlated, then monetary union will still not be welfare improving. Clearly, adding more restrictions to monetary union once fiscal constraints are in place will not be welfare improving.

The welfare costs of a constrained central bank will be larger if preferences are not Cobb-Douglas. As we have seen for the specification in which goods

are perfect substitutes, an active monetary policy with unequal transfers to countries is desirable even if the fiscal authorities are unconstrained.

### 5.3 Weak Central Bank

We define a *weak* central bank by two institutional features: (i) the two national fiscal authorities move first but act non-cooperatively and (ii) the central monetary authority moves second and fully finances the total sum of national deficits of the two governments. In other words, such a weak central bank functions under a full bail-out clause in which it is committed to meeting the financial needs of the fiscal authorities of member governments.

There are a number of reasons for being interested in such an institution. First, by looking at this extreme, one can better understand the benefits of imposing a no bail-out clause, as appears to be the case in the European Monetary Union. Second, to the extent that a central bank cannot commit to a no-bail out clause, understanding the outcomes in this extreme case are of interest.<sup>23</sup> Third, there are examples of countries in which independent authorities interacted through a common central bank leading, in some cases, to excessive inflation.<sup>24</sup> Finally, though our model has no interest bearing government debt, the weak monetary union case captures the implications of a monetary authority which monetizes the debt of fiscal authorities.<sup>25</sup>

Given that the three government actors (the two fiscal authorities and the monetary authority) face two constraints, the policy choices must be interrelated. Here, we suppose that the monetary authority must respond to the choices of the independent fiscal authorities. Its strategy is then easily

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<sup>23</sup>In fact, Beetsma and Uhlig [1997] motivate their study of the "Stability Pact" by noting "... there is fear that a high deficit member country or a member country in recession may successfully pressure the ECB into loosening its monetary policy."

<sup>24</sup>There are other studies of the interaction between independent parts of a government that share a common budget constraint. See, for example, Aizenman [1992], Aizenman and Powell [1998], Zarazaga[undated] and Chari and Kehoe [1997] for discussions of this point in the context of macroeconomics.

<sup>25</sup>As noted earlier, this coordination of policy issue arises in Sargent-Wallace [1981]. In the appendix of that paper, they outline a overlapping generations model with government debt, private storage and money as alternative assets. Through restricted participation in asset markets, they characterize an equilibrium in which the real rate of return exceeds the growth of the economy. The consequence of this for the case in which the monetary authority moves after the fiscal authority, in their words, is "Sooner or later in a monetarist economy the result is additional inflation."

defined: it is obligated to monetize deficits.

### 5.3.1 Equilibrium Analysis

To the extent that the interaction between the government entities is the only difference between this structure and the strong central bank case, we can use the conditions from that case to describe the consumption and labor supply decision rules of agents given monetary and fiscal policies. Using these conditions, we can focus on the interaction between the fiscal authorities of the two governments.

As before, each government provides unemployment insurance to agents currently unemployed. These flows are financed by tax revenues from employed agents and from the printing of money. Since the central bank is **required** to finance the deficits of the two governments, the evolution of the money supply is given by:

$$M_{t+1} = M_t + [(1 - q_t)b_t^W - p_t n_t q_t \tau_t^W] + [(1 - q_t^*)b_t^{*W} - p_t^* n_t^* q_t^* \tau_t^{*W}] = M_t + \Delta_t^W + \Delta_t^{*W}. \quad (18)$$

where  $\Delta_t^W$  ( $\Delta_t^{*W}$ ) denote the monetary deficit in the home (foreign) country,  $b_t^W$  the transfer to the unemployed and  $\tau_t^W$  ( $\tau_t^{*W}$ ) the tax rate set in a "weak" environment. Put differently, the change in the money supply equals the sum of the nominal deficits across the two countries.

An important element of this institution is made apparent by these expressions. The weak monetary union introduces an interaction across the fiscal authorities that did not exist in either the multiple currency or strong monetary union cases. Specifically, under a weak monetary union, deficit spending by one country is financed by an inflation tax that is partially paid by the agents of the other country. Thus the "beggar thy neighbor" type seignorage policies that promoted gains to monetary union in Cooper-Kempf [1998] reappear here under a weak central bank. Interestingly, these effects were absent in the other institutional settings. This is a consequence of our timing assumptions: agents were able to go to exchange markets after receiving government transfers. In contrast, Cooper-Kempf [1998] assume that the transfers are received after the exchange markets are closed thus providing a tax base for seignorage.

Taking the response of the weak monetary authority as given, each fiscal authority in period  $t$  chooses the level of taxes and UI benefits to maximize

the expected utility of generation  $t$  agents. In doing so, each government takes the fiscal policy of the other government as given. Further, each government fully perceives the effects of its policies on equilibrium prices.

In general, characterizing additional features of this equilibrium is difficult since the presence of country specific shocks implies that the distribution of the nominal money supply across the two countries is stochastic. Hence in the remainder of this section we make an additional assumption that the economies are symmetric:  $\bar{\theta} = \frac{1}{2}$ .

To characterize the equilibrium, let  $D_t$  represent the level of nominal spending in period  $t+1$  on the home good by generation  $t$  (home and foreign) agents.  $D_t$  is also the nominal income of generation  $t+1$  employment agents. That is, from market clearing:

$$p_{t+1}q_{t+1}\bar{n} = D_t = \bar{\theta}(q_t I_t^E + (1 - q_t)I_t^U) + (1 - \bar{\theta})(q_t^* I_t^{*E} + (1 - q_t^*)I_t^{*U}).$$

Using  $\bar{\theta} = \frac{1}{2}$ , the evolution of  $D_t$  is given by:

$$D_t = \frac{1}{2}[D_{t-1} + D_{t-1}^* + \Delta_t^W + \Delta_t^{*W}]. \quad (19)$$

With this added structure we find:

**Proposition 7** *With  $\bar{\theta} = \frac{1}{2}$ , there is a symmetric equilibrium in which all income taxes are zero and  $\Delta_t^W = 2\frac{(1-q_t)}{(q_t+q_t^*-1)}D_{t-1}$ ,  $\Delta_t^{*W} = 2\frac{(1-q_t^*)}{(q_t+q_t^*-1)}D_{t-1}$  and  $D_t = D_t^*$  for all  $t$ .*

**Proof.** See appendix. ■

According to this proposition, when the central bank is weak, there is no direct taxation by either country to finance transfers to its unemployed people. To the contrary, all transfers are fully monetized.<sup>26</sup> This indeed can be very simply understood. Suppose an adverse shock hits the home economy. The home fiscal authority has two alternative ways to raise compensation to the unemployed: it can either tax its currently employed agents or ask the weak central bank to finance its transfers. Using this inflation tax is a dominant strategy for each of the governments since part of the tax burden is borne by agents outside their country. Nevertheless there is a limit to

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<sup>26</sup>In some countries, such as Argentina, Brazil and Russia, there is vivid evidence of the inflation predicted by this proposition. We discuss this in our conclusions.

the amount raised through monetization since the higher prices lead to a reduction in the utility of young employed agents.

The equilibrium is symmetric: each country runs a deficit in each period which is proportional to the level of nominal spending in that country. These rates of money creation are dependent on the level of unemployment in each of the countries. Since the deficit spending is used to finance a transfer to unemployed agents which is partially paid for by the employed agents, it is natural that the rate of money creation should be an increasing function of the unemployment rate in the home country.

The equilibrium rate of money creation is given by:

$$\sigma_t^W = \frac{2 - q_t - q_t^*}{q_t + q_t^* - 1}. \quad (20)$$

Clearly, money growth is zero if both economies exhibit full employment and is increasing in the rates of unemployment of member countries.

### 5.3.2 Welfare Comparisons

From these propositions, we see that a characteristic of a weak monetary union is that incomes of unemployed and employed agents are not equalized within a country. Hence, as they face the same prices, their consumption allocations will not be the same, for a given realization of the taste shock  $\theta$ . So, the allocation under a weak monetary union will not satisfy the conditions for optimal risk sharing: this institution does not facilitate stabilization through the available policy instruments. Instead, these policies are used strategically in order to induce the central monetary authority to create money. This leads us to study the welfare properties of a weak monetary union, both in relation to a strong monetary union and the allocation obtained with multiple currencies.

**Weak vs. Strong Central Bank** The gain to a strong monetary union relative to a world economy of multiple currencies was driven by the liquidity services associated with a single currency. In the institution of a monetary authority with commitment, there were no insurance losses associated with monetary union: the fiscal authorities provide insurance within a country and the monetary authorities, by maintaining a constant money supply, enabled the price system to insure nominal incomes. In the weak monetary union

institution, the liquidity gains may be offset by the welfare loss associated with the seignorage game between countries. In fact,

**Proposition 8** *The allocation under strong monetary union provides all agents with higher expected utility than the allocation under a weak monetary union.*

**Proof.** See appendix. ■

Clearly the allocation obtained under strong monetary union is feasible with a weak central bank. However, at that configuration of policies, both fiscal authorities would have an incentive to reduce income taxes and rely on money transfers to finance unemployment insurance. This is the point behind Proposition 7. In equilibrium, both governments pursue strategies which lead to inflation and, in equilibrium, lower welfare than the strong monetary union as indicated by this proposition. Thus, there is a prisoner's dilemma structure to this game between the fiscal authorities.

**Weak Central Bank vs. Multiple Currencies** The comparison between a monetary union and an economy with multiple currencies is relevant to determining whether there are gains to creating such a union. Relative to the world economy, monetary union might, in fact, be welfare reducing if the inflation costs are large relative to the liquidity gains.

Given the definitions of incomes for both categories of agents within a country, it is easy to compute the welfare achieved in a weak monetary union. From this, the difference of welfares achieved under weak monetary union and a flexible exchange-rate system is equal to:

$$\Lambda_{WF} \equiv V_W - V_F = E \left\{ \theta_{t+1} \ln \left( \frac{\theta_{t+1}}{\bar{\theta}} \right) + (1 - \theta_{t+1}) \ln \left( \frac{1 - \theta_{t+1}}{1 - \bar{\theta}} \right) \right\} (21) \\ - E \{ q_t \ln (q_t / (q_t + q_t^* - 1)) \}.$$

which, using a Taylor series expansion can be written as

$$\Lambda_{WF} = 2 \cdot var(\theta) - \ln(2\bar{q} - 1) / \bar{q} (22) \\ - \left[ \frac{1(1 - \bar{q})^2 + \bar{q}^2}{2 \bar{q}(2\bar{q} - 1)^2} var(q) + \frac{1(1 - \bar{q})}{2(2\bar{q} - 1)^2} covar(q) \right]$$



The first of these terms represents the gains to monetary union associated with the liquidity benefits of a single currency. The gain to monetary union is zero if there are no taste shocks and is increasing as the variability of the taste shock increases. The second term has two components relating to the realizations of the supply shocks and both are negative. This term captures the loss in welfare from the interaction across countries through seignorage.

**Proposition 9** *If the variability of the taste shock is sufficiently small and both  $\bar{q}$  and  $\bar{q}^*$  are significantly below 1, then economic welfare will be lower under a weak monetary union than in the world economy with multiple currencies. If  $\bar{q}$  and  $\bar{q}^*$  are near 1,  $\text{var}(\theta)$  is small and the variability of taste shocks is large enough, then a weak monetary union dominates a world economy with multiple currencies.*

Comparing this result for a weak monetary union with that for the constrained fiscal structure, Proposition 6, is instructive. For both cases, we see that in order for monetary union to be welfare improving, taste shocks must be sufficiently variable. Otherwise, in the presence of a weak central bank, a country is adversely affected by the inflationary policies of other countries. From Proposition 7, we found that the extent to which countries will inflate via the common central bank depends on their individual countries rates of unemployment. Hence, if unemployment rates are sufficiently low on average and not too volatile, then a monetary union with a weak central bank may indeed be welfare improving as long as taste are sufficiently variable. Interestingly, contrasting Propositions 6 and 9, for the former the costs of monetary union are related to the correlation of shocks while for the latter they depend on the average employment rates.

The main lesson from this section is that a weak monetary authority can create an environment in which fiscal authorities find a cheap source of revenues from the money demands of foreign agents. In equilibrium the resulting welfare loss from excessive inflation may indeed dominate the transactions gains from a single currency. This is more likely if tastes are not too variable ( $\text{var}(\theta) \approx 0$ ) and unemployment rates high so that the rate of money creation by both countries is high.

## 6 Conclusion

The goal of this paper has been to examine the effectiveness of stabilization policy within a monetary union. To do so, we explored the allocations of a stochastic two-country overlapping generations model under alternative monetary/fiscal institutions.

In the extreme case of a strong central monetary authority with commitment power and fiscal authorities with the ability to set state contingent tax rates, a monetary union unambiguously increases welfare. In this institutional design, the delegation of monetary policy to a single central bank does not jeopardize the conduct of stabilization policy. The tradeoff envisioned by Mundell and others does not exist. Further, the gain to monetary union does not rest on labor mobility since in our model, by construction, labor is immobile.

However, there are designs of a monetary union which do not increase welfare precisely because stabilization policy is impaired. In particular, if the set of policy instruments open to fiscal authorities is sufficiently restricted, then monetary union may not increase welfare. Despite having commitment power, the central bank lacks the tools to stabilize in the presence of country specific shocks that are not perfectly correlated.

Further, if the monetary authority is weak and thus influenced by the deficit spending of member countries, then the gains to monetary union may be more than offset by the adverse consequences of inflation. A weak central bank opens the door to strategic interaction by the fiscal authorities and thus to excessive inflation.

These results are normative in that they point to potential gains from monetary union. Will these gains be realized? Cooper-Kempf [1998] argue that though there are gains to a monetary union, the incentives for each country imply that these welfare gains will not be realized without collective action. In particular, Cooper-Kempf [1998] find that a game in which countries, in effect, decide to join a monetary union or not has a prisoners' dilemma structure: the cooperative outcome of monetary union is not a Nash equilibrium of this game. Those results can be extended to the environment studied here. Though Propositions 4, 6 and 10 imply that the allocation under a strong monetary union Pareto dominates that obtained under multiple currencies, the establishment of a monetary union is not a Nash equilibrium. Given a conjectured equilibrium of a monetary union, a country would deviate and impose a requirement that its currency be used in

domestic transactions thus effectively ending the proposed monetary union. From this perspective, the reaping of the gains from monetary union requires collective action.

Some general lessons can be drawn from this analysis. On the issue of commitment vis-a-vis the fiscal authorities, two elements seem important. First, fiscal pacts seem important insofar as they reduce pressures on the monetary authority. While not formally part of our model, clearly restrictions on deficits would limit the "beggar thy neighbor" type inflation that could undermine monetary union. Second, the monetary authority must adopt rules that insulate its decisions from the fiscal pressures exerted by member governments.<sup>27</sup> In this regard, rules that specify growth rates for monetary aggregates seem more desirable relative to rules that respond to fiscal policy through, for example, the stabilization of interest rates.<sup>28</sup>

On the issue of determining which groups of countries might benefit from a monetary union, two factors should be kept in mind. First, one of the gains we have identified pertains to the reduction in consumption misallocations from the adoption of a single currency. Clearly these gains are larger the more the countries trade with one another. Second, unless the central bank is sufficiently strong and fiscal authorities are sufficiently active in the conduct of stabilization policy, the delegation of monetary policy will entail some stabilization losses. If so, our results point to the fact that countries with a high correlation of shocks will suffer less from the centralization of monetary policy.

Even though these findings are rather abstract, we do think that they provide useful insights on actual or possible monetary unions. Let us take the example of a mature and clearly successful monetary union like the US. For the U.S., Rolnick, Smith and Weber [1993] develop an argument linking the clause in the U.S. Constitution which created a central monetary authority with a desire to avoid seignorage games between colonies. On the fiscal side, Poterba [1996] discusses the nature of balanced-budget rules across states. The centralization of monetary authority along with these fiscal constraints seems sufficient to eliminate the seignorage games though there is always the possibility of a link between state budget deficits, transfers from a Federal government and the monetization of any resulting debt.

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<sup>27</sup>For the case of Argentina, Saiegh and Tommasi [1999] discuss the Convertibility Law of 1991 as a means of limiting inflationary pressures.

<sup>28</sup>Clearly this discussion suggests an interesting extension of our analysis would be to evaluate a variety of monetary rules in a version of the model with government debt.

This concern visibly has been absent or neglected by the rulers of South American countries in the 20th century, up to recently. Aizenman [1998] discusses the experiences of Brazil and Argentina from this perspective. Saiegh and Tommasi [1999] discuss the recent experience of Argentina in terms of a weak central bank responding to the actions of independent fiscal entities.<sup>29</sup> Again, our theoretical model makes clear a major root of these hyperinflationary episodes: the laxity of monetary authorities and the uncooperative behavior of fragmented fiscal authorities leading to seignorage games.

A similar fate happened in the former Soviet Union, when it was replaced by the Community of Independent States. In the infancy of the C.I.S., in the early 1990s, local authorities had the ability to finance local expenditures through the creation of rubles by the Russian Central Bank. Essentially, the various republics could, for example, subsidize their enterprises by seignorage revenues. The outcome has been an outburst of money creation by the Russian Central Bank and a total neglect of the building of a proper fiscal system, able to generate funds for public objectives by means of taxation, leading to violent hyperinflation. Inflation has been put under control when the Russian Central Bank stopped bailing out public governments. Again, this is perfectly consistent with our findings.<sup>30</sup>

Turning to the case of the EMU, it is of course far too early to assess its soundness by means of its actual performance. Still, our theoretical reasoning allows us to offer some appreciation of the main institutional features of the EMU.

On the monetary side, the independence of the European System of Central Banks (ESCB) can hardly be blamed as it is clearly a way to reinforce the position of the monetary authority with regard to national political actors and in particular governments. Similarly the no-bail-out provision is clearly the simplest way to forbid the development of seignorage games within the Union which would dramatically impair the capacity of the EMU as a whole to conceive non-inflationary successful stabilization policies. But on the other hand, it also impairs the capacity of the ECB to adopt active measures by means of unequal transfers to specific countries, which can be useful as we have seen. In the whole, even though the commitment power of the ESCB is undoubtedly increased by this provision, it still may not be optimal.

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<sup>29</sup>For the case of Argentina, the fiscal deficits of the individual provinces are actually financed jointly through national taxation and the printing of money.

<sup>30</sup>See, for example, the discussion in V. Koen and M. Marrese [1995] and T. J. Balinos, D. Hoelscher and J. Horder [1997].

On the fiscal side, the discussion turns on the so-called "stability and growth pact", which severely limits the ability of national governments to freely use their fiscal tools and in particular the budget deficit. Again, from our point of view, this provision should be viewed as a mixed blessing, in particular because of the large independence of European monetary policy. We have seen that a successful design allows both a commitment power to the monetary authority and a large set of policy instruments used by the various policymakers. By and large, it can be said that the commitment position of the ESCB is well established. It is then very important that enough instruments are available to accommodate shocks. The no-bail out clause limits the ability of the ECB to accommodate idiosyncratic shocks; the fiscal authorities should then have enough room to respond to these shocks. But clearly the pact limits their ability to use their own instruments. This may therefore well be a too stringent measure, detrimental to the capacity of the EMU as a whole to adequately answer to shocks.

In terms of further analysis, three important extensions come to mind. First, our analysis of stabilization policy looks exclusively at supply shocks. This is relevant since these shocks produce price movements which tend to stabilize nominal incomes. As noted earlier, this feature of markets created an element of stability even with a non-interventionist monetary authority. Clearly, extending the analysis to a setting with demand side shocks and thus no countervailing price movements is of interest.

Second, the issue of stabilization policy in a monetary union could be pursued by analyzing the consequences of fiscal federalism. Such an institutional arrangement adds new fiscal instruments by facilitating interregional transfers. However, the case of strong monetary union in which the central bank has the ability to make state contingent transfers that differ across countries nests any fiscal federalism scheme. Hence, the study of fiscal federalism requires additional restrictions on the conduct of monetary policy.

Third, there is another monetary arrangement that is closely related to monetary union: the adoption of the currency of another country.<sup>31</sup> We term this regime "dollarization" in recognition of the use of the U.S. dollar in a number of Latin America countries, such as Panama and Argentina. This structure is similar to monetary union except that the choice of stabilization through monetary policy remains with one of the countries. Of interest is

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<sup>31</sup>See, for example, the recent analysis of Bencivenga et al. [1999] on dollarization in a setting where capital market integration is central.

understanding how this relationship compares to the environment of multiple currencies and to monetary union.

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# APPENDIX

## A Proof of Proposition 1

The planner solves

$$\max_{c(s,\theta), c^*(s,\theta^*), n(s), n^*(s)} E_s \{ q_{-1} U^{Eh} + (1 - q_{-1}) U^{Uh} + q_{-1}^* U^{*Eh} + (1 - q_{-1}^*) U^{*Uh} - qg(n(s)) - q^*g(n^*(s)) \} \quad (23)$$

subject to:

$$E_{\theta, \theta^*} [q_{-1} c^{Eh}(s, \theta) + (1 - q_{-1}) c^{Uh}(s, \theta) + q_{-1}^* c^{*Eh}(s, \theta^*) + (1 - q_{-1}^*) c^{*Uh}(s, \theta^*)] = n(q, q^*)q$$

and

$$E_{\theta, \theta^*} [q_{-1} c^{Ef}(s, \theta) + (1 - q_{-1}) c^{Uf}(s, \theta) + q_{-1}^* c^{*Ef}(s, \theta^*) + (1 - q_{-1}^*) c^{*Uf}(s, \theta^*)] = n^*(q, q^*)q^*.$$

Denoting  $\lambda$  and  $\lambda^*$  the multipliers associated with the two constraints, the first-order conditions imply:

$$\begin{aligned} \frac{\theta}{c^{Eh}(s, \theta)} &= \frac{\theta}{c^{Uh}(s, \theta)} = \frac{1 - \theta^*}{c^{*Eh}(s, \theta^*)} = \frac{1 - \theta^*}{c^{*Uh}(s, \theta^*)} = \lambda \\ \frac{1 - \theta}{c^{Ef}(s, \theta)} &= \frac{1 - \theta}{c^{Uf}(s, \theta)} = \frac{\theta^*}{c^{*Ef}(s, \theta^*)} = \frac{\theta^*}{c^{*Uf}(s, \theta^*)} = \lambda^* \end{aligned}$$

for all  $s, \theta, \theta^*$ . Further:

$$\lambda = g'(n) \text{ and } \lambda^* = g'(n^*)$$

Using the resource constraint to solve for  $\lambda$  and  $\lambda^*$ , the first-best allocation is characterized by:

$$\frac{\theta}{c^{Eh}(s, \theta)} = \frac{\theta}{c^{Uh}(s, \theta)} = \frac{(1 - \theta^*)}{c^{*Eh}(s, \theta^*)} = \frac{(1 - \theta^*)}{c^{*Uh}(s, \theta^*)}$$

for all  $(\theta, \theta^*)$ , and

$$g'(n)nq = 1 = g'(n^*)n^*q^*$$

as in the proposition.

## B Proof of Proposition 2

### A. The maximization problem of an employed agent

The maximization problem solved by an employed, generation  $t$  Home agent is:

$$\max_{n_t, m_t^{Eh}, m_t^{Ef}} E_\theta \left\{ \theta_{t+1} \ln(c_{t+1}^{Eh}) + (1 - \theta_{t+1}) \ln(c_{t+1}^{Ef}) \right\} - g(n_t) \quad (24)$$

$$\begin{aligned} s.t. \quad p_t(1 - \tau_t)n_t &= m_t^{Eh} + e_t m_t^{Ef} \\ p_{t+1}c_{t+1}^{Eh} &= m_t^{Eh} \\ p_{t+1}^*c_{t+1}^{Ef} &= m_t^{Ef} \end{aligned}$$

The first-order conditions lead to two equalities:

$$g'(n_t) = \frac{p_t(1 - \tau_t)(1 - \bar{\theta})}{e_t p_{t+1}^* c_{t+1}^{Ef}} \quad (25a)$$

$$\frac{\bar{\theta}}{1 - \bar{\theta}} = \frac{p_{t+1}c_{t+1}^{Eh}}{e_t p_{t+1}^* c_{t+1}^{Ef}} \quad (25b)$$

We can rewrite this as:

$$n_t g'(n_t) = \frac{n_t p_t (1 - \tau_t) (1 - \bar{\theta})}{e_t p_{t+1}^* c_{t+1}^{Ef}}$$

which, given the individual budget constraint, can be expressed as:

$$n_t g'(n_t) = \frac{n_t p_t (1 - \tau_t) (1 - \bar{\theta})}{p_t (1 - \tau_t) n_t - m_t^{Eh}} = \frac{\frac{1}{q_t} M_t (1 - \tau_t) (1 - \bar{\theta})}{\frac{1}{q_t} M_t (1 - \tau_t) - m_t^{Eh}}$$

where  $M_t \equiv p_t q_t n_t$  is the total amount of Home money in circulation at period  $t$ . Defining

$$m_t^{Eh} \equiv \phi_t^E \cdot \frac{1}{q_t} M_t (1 - \tau_t)$$

and assuming that  $\phi_t^E = \phi^E$ , it is deduced that:

$$n_t g'(n_t) = \frac{1 - \bar{\theta}}{1 - \phi^E}$$

which implies that  $n_t$  is constant over time and denoted by  $\bar{n}$ .

Remark then that:

$$p_t (1 - \tau_t) n_t = \frac{1}{q_t} M_t (1 - \tau_t) = \frac{m_t^{Eh}}{\phi^E}$$

and we get:

$$\begin{aligned} \frac{\bar{\theta}}{1 - \bar{\theta}} &= \frac{p_{t+1} c_{t+1}^{Eh}}{e_t p_{t+1}^* c_{t+1}^{Ef}} = \frac{m_t^{Eh}}{\frac{1}{q_t} M_t (1 - \tau_t) - m_t^{Eh}} \\ &= \frac{1}{\frac{1}{\phi^E} - 1} = \frac{\phi^E}{1 - \phi^E} \end{aligned}$$

which implies that  $\bar{\theta} = \phi^E$  and  $\bar{n} g'(\bar{n}) = 1$ . It is then easy to obtain the levels of consumption for an employed agent. From above, we can write that:

$$c_{t+1}^{Eh} = \frac{m_t^{Eh}}{p_{t+1}} = \frac{\bar{\theta}}{q_t} \frac{M_t (1 - \tau_t)}{p_{t+1}}$$

Given that  $M_{t+1} \equiv p_{t+1} q_{t+1} n_{t+1}$ , this equality can be rewritten as:

$$c_{t+1}^{Eh} = \frac{m_t^{Eh}}{p_{t+1}} = \frac{\bar{\theta}}{q_t} \frac{M_t}{M_{t+1}} (1 - \tau_t) q_{t+1} \bar{n} = \frac{\bar{\theta} \bar{n}}{q_t} \frac{q_{t+1} (1 - \tau_t)}{(1 + \sigma_t)} \quad (26)$$

### B. The maximization problem of an unemployed agent

The maximization problem solved by an unemployed, generation  $t$ , Home agent is:

$$\max_{m_t^{Uh}, m_t^{Uf}} E_\theta \left\{ \theta_{t+1} \ln(c_{t+1}^{Uh}) + (1 - \theta_{t+1}) \ln(c_{t+1}^{Uf}) \right\} \quad (27)$$

$$\begin{aligned} s.t. \quad I_t^U &= m_t^{Uh} + e_t m_t^{Uf} \\ p_{t+1} c_{t+1}^{Uh} &= m_t^{Uh} \\ p_{t+1}^* c_{t+1}^{Uf} &= m_t^{Uf} \end{aligned}$$

The first-order conditions lead to the following inequality:

$$\frac{\bar{\theta}}{1 - \bar{\theta}} = \frac{p_{t+1} c_{t+1}^{Uh}}{e_t p_{t+1}^* c_{t+1}^{Uf}}$$

Defining  $m_t^{Uh} \equiv \phi_t^U \cdot I_t^U$  and assuming that  $\phi_t^U = \phi^U$ , it follows from the individual budget constraint that:

$$\frac{\bar{\theta}}{1 - \bar{\theta}} = \frac{\phi^U}{1 - \phi^U}$$

which implies that  $\phi^U = \bar{\theta} = \phi^E$ .

Then the following expressions hold:

$$\begin{aligned} q_t e_t m_t^{Ef} &= (1 - \bar{\theta}) (1 - \tau_t) M_t \\ (1 - q_t) e_t m_t^{Uf} &= (1 - \bar{\theta}) (\tau_t + \sigma_t) M_t \end{aligned}$$

implying :

$$e [q_t m_t^{Ef} + (1 - q_t) m_t^{Uf}] = (1 - \bar{\theta}) (1 + \sigma_t) M_t.$$

Using a similar reasoning for the Foreign economy, we get from the clearing condition on the exchange market the following equality;

$$[q_t^* m_t^{*Ef} + (1 - q_t^*) m_t^{*Uf}] = (1 - \bar{\theta}) (1 + \sigma_t^*) M_t^*$$

or equivalently:

$$M_{t+1} = M_{t+1}^* e_t.$$

Given that the equality between money supply and the aggregate nominal product in one country, this immediately implies that:

$$\frac{p_t}{e_t p_{t+1}^*} = \frac{q_t^*}{q_t}. \quad (28)$$

We then easily get the levels of consumption given in the proposition.

## C Proof of Proposition 3

The maximization problem solved by the Home government can be written as follows:

$$\begin{aligned} & \max_{\tau_t^F, \sigma_t^F} E_\theta \left( q_t U \left( c_{t+1}^{Eh}, c_{t+1}^{Ef} \right) + (1 - q_t) \left( U \left( c_{t+1}^{Uh}, c_{t+1}^{Uf} \right) \right) \right) \\ & \text{given} \quad \tau_t^{*F}, \sigma_t^{*F}. \end{aligned}$$

Denoting  $\xi_t \equiv \frac{(1 - \tau_t^F)}{(1 + \sigma_t^F)}$  and given the expressions for the various levels of consumption, this problem can be rewritten as:

$$\begin{aligned} & \max_{\xi_t} E_\theta \left( q_t (\theta_{t+1} \ln \xi_t + (1 - \theta_{t+1}) \ln (1 - \xi_t)) + (1 - q_t) (\theta_{t+1} \ln (1 - \xi_t) + (1 - \theta_{t+1}) \ln (1 - \xi_t)) \right) \\ & \text{given} \quad \tau_t^{*F}, \sigma_t^{*F}. \end{aligned}$$

The first-order condition generates the following equality:

$$\xi_t = q_t.$$

## D Proof of Proposition 4

Any government takes the transfer received by his unemployed agent from the central monetary authority,  $\Delta_t^S$ , as given. The first order conditions for the agents' programs and the resulting consumption levels and supply levels allow us to write the optimization program of the Home government as follows:

$$\begin{aligned} & \max_{\tau_t^S} E \left\{ q_t \ln \left( I_t^E \right) + (1 - q_t) \ln \left( I_t^U \right) - \bar{\theta} \ln \left( p_{t+1} \right) - \left( 1 - \bar{\theta} \right) \ln \left( p_{t+1}^* \right) \right\} \\ & \text{s.t.} \quad I_t^E = p_t \bar{n} \left( 1 - \tau_t^S \right) \\ & \quad \quad I_t^U = p_t \bar{n} \tau_t^S \frac{q_t}{1 - q_t} + \frac{\Delta_t^S}{1 - q_t} \end{aligned} \tag{29}$$

The first-order condition for the home government implies  $I_t^E = I_t^U$ . Using this condition, the home tax rate is given by:

$$\tau_t^S = (1 - q_t) - q_t \frac{\Delta_t^S}{p_t q_t \bar{n}}.$$

Similarly, we obtain

$$\tau_t^{*S} = (1 - q_t^*) - q_t^* \frac{\Delta_t^{*S}}{p_t^* q_t^* \bar{n}}$$

for the foreign government.

Given the decision rules of the fiscal authorities, the central monetary authority solves:

$$\begin{aligned} \max_{\Delta_t^S, \Delta_t^{*S}} E \left\{ q_t \ln(I_t^E) + (1 - q_t) \ln(I_t^U) + q_t^* \ln(I_t^{*E}) + (1 - q_t^*) \ln(I_t^{*U}) \right. \\ \left. - \ln(p_{t+1}) - \ln(p_{t+1}^*) \right\} \end{aligned} \quad (30)$$

$$\begin{aligned} s.t. \quad I_t^E &= p_t \bar{n} (1 - \tau_t^S) \\ I_t^U &= p_t \bar{n} \tau_t^S \frac{q_t}{1 - q_t} + \Delta_t^S \\ I_t^{*E} &= p_t^* \bar{n} (1 - \tau_t^{*S}) \\ I_t^{*U} &= p_t^* \bar{n} \tau_t^{*S} \frac{q_t^*}{1 - q_t^*} + \Delta_t^{*S} \\ \tau_t^S &= (1 - q_t) - q_t \frac{\Delta_t^S}{p_t q_t \bar{n}}, \tau_t^{*S} = (1 - q_t^*) - q_t^* \frac{\Delta_t^{*S}}{p_t^* q_t^* \bar{n}}. \end{aligned}$$

The first-order conditions are:

$$\begin{aligned} \frac{1}{I_t^U} - \frac{dp_{t+1}}{d\Delta_t^S} \frac{1}{p_t} - \frac{dp_{t+1}^*}{d\Delta_t^S} \frac{1}{p_t^*} &= 0, \\ \frac{1}{I_t^{*U}} - \frac{dp_{t+1}^*}{d\Delta_t^{*S}} \frac{1}{p_t^*} - \frac{dp_{t+1}}{d\Delta_t^{*S}} \frac{1}{p_t} &= 0. \end{aligned}$$

From market-clearing conditions, we find:

$$p_{t+1} q_{t+1} \bar{n} = E_{\theta, \theta^*} \left\{ q_t \theta I_t^E + (1 - q_t) \theta I_t^U + q_t^* (1 - \theta^*) I_t^{*E} + (1 - q_t^*) (1 - \theta^*) I_t^{*U} \right\}.$$

Hence:

$$\frac{dp_{t+1}}{d\Delta_t^S} = \frac{\bar{\theta}}{q_{t+1} \bar{n}} \quad \text{and} \quad \frac{dp_{t+1}^*}{d\Delta_t^{*S}} = \frac{1 - \bar{\theta}}{q_{t+1} \bar{n}}$$

and similarly:

$$\frac{dp_{t+1}^*}{d\Delta_t^S} = \frac{1 - \bar{\theta}}{q_{t+1} \bar{n}} \quad \text{and} \quad \frac{dp_{t+1}}{d\Delta_t^{*S}} = \frac{\bar{\theta}}{q_{t+1} \bar{n}}.$$

Then, taking the values for the tax ratios into consideration, the first-order conditions imply that:

$$\frac{1}{p_t \bar{n} q_t + \Delta_t^S} - \frac{\bar{\theta}}{q_{t+1} \bar{n} p_t} - \frac{1 - \bar{\theta}}{q_{t+1}^* \bar{n} p_t^*} = 0, \quad (31a)$$

$$\frac{1}{p_t^* \bar{n} q_t^* + \Delta_t^{*S}} - \frac{\bar{\theta}}{q_{t+1}^* \bar{n} p_t^*} - \frac{1 - \bar{\theta}}{q_{t+1} \bar{n} p_t} = 0. \quad (31b)$$

We assume  $\Delta_t^S = \Delta_t^{*S} = 0$  is a solution and show it is consistent with the first-order conditions. If  $\Delta_t^S = \Delta_t^{*S} = 0$ , we verify from the first order conditions of the individual governments that incomes are equal irrespective of employment status. This equality of incomes requires

$$\tau_t^S = (1 - q_t), \tau_t^{*S} = (1 - q_t^*). \quad (32)$$

Second-order conditions ensure that it is the only solution. The consumption and employment allocations for home agents are then obtained by evaluating (12)-(13) given these policy choices. Similar expressions hold for foreign agents. ■

## E Proof of Proposition 5

**Proof.** This expression for  $\Lambda_{SF}$  comes directly from the difference between (11) and (16). As in Cooper-Kempf [1998], taking the second-order Taylor approximation of this expression, this gain associated to strong monetary union can be viewed as:

$$\Lambda_{SF} = \frac{1}{2} * var(\theta) \left[ \frac{1}{\bar{\theta}} + \frac{1}{(1 - \bar{\theta})} \right].$$

$\Lambda_{SF} > 0$  if  $var(\theta) > 0$  as  $\bar{\theta} \in (0, 1)$ . ■

## F Proof of Proposition 6

We let  $C$ , as a superscript or a subscript, denote variables for the constrained monetary union case. The maximization problem of the monetary authority is similar to the problem explored in the proof of Proposition 4, with the restriction that:  $\tau = \tau^* = \tau^C$ . The first-order conditions then become:

$$\begin{aligned} \frac{1}{p_t \bar{n} \tau^C \frac{q_t}{1-q_t} + \Delta_t^C} - \frac{\bar{\theta}}{q_{t+1} \bar{n} p_{t+1}} - \frac{1-\bar{\theta}}{q_{t+1}^* \bar{n} p_{t+1}^*} &= 0 \\ \frac{1}{p_t^* \bar{n} \tau^C \frac{q_t^*}{1-q_t^*} + \Delta_t^{*C}} - \frac{\bar{\theta}}{q_{t+1}^* \bar{n} p_{t+1}^*} - \frac{1-\bar{\theta}}{q_{t+1} \bar{n} p_{t+1}} &= 0 \end{aligned}$$

(i) Suppose that the monetary transfers ensure perfect income equalization within the monetary union, given the fixed tax rate. Given the definitions of incomes, this implies:

$$I_t^U = I_t^E = \bar{n} (1 - \tau^C) p_t = I_t^{*U} = I_t^{*E} = \bar{n} (1 - \tau^C) p_t^*$$

hence:  $p_t = p_t^*$ . Using the market clearing equalities, the equalization of prices is equivalent to  $q_t = q_t^*$ , or  $\text{corr}(q, q^*) = 1$ . Hence this is a solution to the governments problem iff  $\text{corr}(q, q^*) = 1$ . As we obtain the same allocation as under a strong monetary union, we get that  $V_S = V_C$  iff  $\text{corr}(q, q^*) = 1$ .

(ii) Suppose that  $\text{corr}(q, q^*)$  is near 1 and the variance of taste shocks is positive. From Proposition 5,  $V_S > V_F$  when  $\text{var}(\theta) > 0$ . Since  $V_S = V_C$  when  $\text{corr}(q, q^*) = 1$  by continuity,  $V_C > V_F$  when  $\text{corr}(q, q^*)$  is near 1.

(iii) Suppose that  $\text{var}(\theta) = 0$ . This implies that  $V_S = V_F$  (from Proposition 5). Yet, when  $\text{corr}(q, q^*) \neq 1$ ,  $V_S > V_C$ . Hence when taste shocks are not too volatile and  $\text{corr}(q, q^*) \neq 1$ ,  $V_F > V_C$ .

## G Proof of Proposition 7

Given the expressions for consumption levels and using the fact that  $n_t = \bar{n}$ , the maximization problem of the home government can be rewritten as:

$$\max_{\tau_t^W, \Delta_t^W} \left\{ q_t \ln I_t^E + (1 - q_t) \ln I_t^U - \bar{\theta} \ln p_{t+1} - (1 - \bar{\theta}) \ln p_{t+1}^* + \lambda \tau_t^W \right\} \quad (32)$$

$$\begin{aligned} s.t. \quad I_t^E &= p_t \bar{n} (1 - \tau_t^W) \\ I_t^U &= \frac{1}{1 - q_t} \left[ p_t \bar{n} \tau_t^W q_t + \Delta_t^W \right] \end{aligned}$$



where  $\lambda$  is the multiplier associated with  $\tau_t \geq 0$ . In this optimization problem the home government recognizes the effect of its policies on the equilibrium prices. The market clearing conditions can be written as:

$$1 = \bar{\theta} \frac{I_t}{D_t} + (1 - \bar{\theta}) \frac{I_t^*}{D_t} \quad (33a)$$

$$1 = \bar{\theta} \frac{I_t^*}{D_t^*} + (1 - \bar{\theta}) \frac{I_t}{D_t^*} \quad (33b)$$

where  $D_t$  ( $D_t^*$ ) is  $p_{t+1}q_{t+1}\bar{n}$  ( $p_{t+1}^*q_{t+1}^*\bar{n}$ ),  $I_t = q_t I_t^E + (1 - q_t)I_t^U$  and  $I_t^* = q_t I_t^{*E} + (1 - q_t)I_t^{*U}$ . Using these market clearing conditions, the derivatives of prices with respect to the transfers are:

$$\frac{\frac{\partial p_{t+1}}{\partial \Delta_t^W}}{p_{t+1}} = \frac{1 - q_t}{q_{t+1}} \frac{\bar{\theta}}{\bar{n}} = \frac{\bar{\theta}}{D_t} \quad \frac{\frac{\partial p_{t+1}^*}{\partial \Delta_t^W}}{p_{t+1}^*} = \frac{1 - q_t^*}{q_{t+1}^*} \frac{\bar{\theta}}{\bar{n}} = \frac{(1 - \bar{\theta})}{D_t^*}.$$

The effects of tax rates on prices is zero since spending is independent of the distribution of income within a country.

Using these results the two first-order conditions for the home government are:

$$-\frac{q_t p_t \bar{n}}{I_t^E} + \frac{(1 - q_t) p_t \bar{n} \frac{q_t}{1 - q_t}}{I_t^U} + \lambda = 0 \quad (34a)$$

$$\frac{1}{I_t^U} - \left( \frac{\bar{\theta}^2}{D_t} + \frac{(1 - \bar{\theta})^2}{D_t^*} \right) = 0 \quad (34b)$$

With  $\bar{\theta} = 1/2$ , (33a) and (33b) imply that  $D_t = D_t^*$  for all  $t$ . Further, these conditions imply that

$$D_t + D_t^* = I_t + I_t^* \quad (35)$$

Using (34b) and the analogous condition for the foreign country,

$$I_t^U = I_t^{*U} > D_t$$

since  $\bar{\theta}^2 + (1 - \bar{\theta})^2 < 1$ . Hence, for (35) to hold,

$$I_t^E < D_t \text{ and } I_t^{*E} < D_t^*.$$

This implies that the employed in each country have lower nominal income than the unemployed. Thus in order for (34a) to hold, as well as the analogous condition for the foreign country,  $\lambda > 0$  and  $\lambda^* > 0$ . So, tax rates must be zero in both countries.

As  $\tau_t$  and  $\tau_t^*$  are equal to 0, given the definitions of the monetary transfers, and  $\bar{\theta} = 1/2$ , the FOCs for the home and foreign governments become:

$$\frac{1 - q_t}{\Delta_t^W} - \frac{1}{4} \left[ \frac{1}{q_{t+1} \bar{n} p_{t+1}} + \frac{1}{q_{t+1}^* \bar{n} p_{t+1}^*} \right] = 0 \quad (36a)$$

$$\frac{1 - q_t^*}{\Delta_t^{*W}} - \frac{1}{4} \left[ \frac{1}{q_{t+1} \bar{n} p_{t+1}} + \frac{1}{q_{t+1}^* \bar{n} p_{t+1}^*} \right] = 0 \quad (36b)$$

From the market clearing conditions

$$\begin{aligned} p_{t+1} q_{t+1} \bar{n} &= E_{\theta, \theta^*} \left\{ q_t \theta I_t^E + (1 - q_t) \theta I_t^U + q_t^* (1 - \theta^*) I_t^{*E} + (1 - q_t^*) (1 - \theta^*) I_t^{*U} \right\} \\ p_{t+1}^* q_{t+1}^* \bar{n} &= E_{\theta, \theta^*} \left\{ q_t^* \theta I_t^{*E} + (1 - q_t^*) \theta I_t^{*U} + q_t (1 - \theta) I_t^E + (1 - q_t) (1 - \theta) I_t^U \right\} \end{aligned}$$

we get when  $\bar{\theta} = 1/2$  :

$$p_{t+1} q_{t+1} \bar{n} = p_{t+1}^* q_{t+1}^* \bar{n} = \frac{1}{2} [M_t + \Delta_t^W + \Delta_t^{*W}]$$

and the FOCs can be written:

$$\frac{1 - q_t}{\Delta_t^W} = \frac{1}{2} \frac{1}{q_{t+1} \bar{n} p_{t+1}} \quad (37a)$$

$$\frac{1 - q_t^*}{\Delta_t^{*W}} = \frac{1}{2} \frac{1}{q_{t+1} \bar{n} p_{t+1}}. \quad (37b)$$

This implies:

$$\Delta_t^W = 2(1 - q_t) \cdot \frac{1}{2} [M_t + \Delta_t^W + \Delta_t^{*W}]$$

Finally we get the reaction functions:

$$\Delta_t = \frac{1 - q_t}{q_t} \cdot [M_t + \Delta_t^{*W}] \quad (38a)$$

$$\Delta_t^{*W} = \frac{1 - q_t^*}{q_t^*} \cdot [M_t + \Delta_t^W] \quad (38b)$$

The Nash equilibrium of the game is then given by the following:

$$\Delta_t^W = \frac{1 - q_t}{q_t + q_t^* - 1} \cdot M_t \quad (39a)$$

$$\Delta_t^{*W} = \frac{1 - q_t^*}{q_t + q_t^* - 1} \cdot M_t \quad (39b)$$

This completes the proof as  $2D_{t-1} = M_t$ . ■

## H Proof of Proposition 8

Note that under both designs, the employment and thus output levels are the same. Then, given the welfare function for the representative agent and the strict concavity of the utility function, an allocation of this output generating perfect risk sharing dominates an allocation which does not guarantee perfect risk sharing. ■

## I Proof of Proposition 9

**Proof.** Here we denote the welfare under the weak central bank allocation as  $V_W$ . If  $\text{var}(\theta) = 0$ ,  $V_F = V_S > V_W$ , where the second inequality follows from Proposition 8 as long as  $\bar{q}$  and  $\bar{q}^*$  are below 1 so that there are some states with unemployment in one of the two countries. If the countries never experience unemployment so that  $q_t = q_t^* = 1$  with probability one, then from Proposition 7, there is no inflation. In this case,  $V_W = V_S > V_F$  as long as taste shocks are present. Hence by continuity, if  $q_t$  and  $q_t^*$  near one on average and not too volatile, then by continuity  $V_W > V_F$ . ■