VI. Appendix: The Logical Coherence of Fiscal Theory

An important concern regarding the FTPL has to do with its internal logical consistency. When the FTPL uses the intertemporal government budget equation to pin down the price level, is that price level consistent with the one determined by the rest of the economy? In some cases, the answer is no.[72] Do these cases warrant the conclusion that the FTPL is not logically coherent? We think not, as enough interesting examples can be constructed in which the fiscal theory *is* logically coherent. One example is given in the body of this review. The point is also illustrated in several articles of a special issue of *Economic Theory* in 1994. In this appendix, we present another example.

The model we work with is the cash/credit-good model of Lucas and Stokey (1983). We examine a range of parameter values, including the empirically plausible ones, according to estimates reported in Chari, Christiano, and Kehoe (1991). We skip detailed proofs in certain places, though never without providing the intuition for the argument. Readers who wish to see an extensive and rigorous treatment of the properties of the equilibria of this model should consult Woodford (1994). This appendix presents an extended example to illustrate his Propositions 2 and 10 at the level of an advanced undergraduate or first-year graduate economics course.

We first consider the case in which monetary policy is characterized by a constant money-growth rate. We show the model has a unique equilibrium when the non-Ricardian assumption is adopted. We then consider the case in which the monetary authority pegs the interest rate. Like the example in the text, the model has a unique equilibrium when the non-Ricardian assumption is adopted. When that assumption is *not* adopted, the model fails to exhibit a unique equilibrium. In this case, the model reproduces the classic Sargent and Wallace (1975) result: The price level is indeterminate. From a technical standpoint, the non-Ricardian assumption is a device that can eliminate the price-level indeterminacy associated with interest rate pegging that Sargent and Wallace analyze.[73]

The first section below describes the agents of the model and defines equilibrium. The following section addresses the case in which monetary policy is characterized by constant money growth. The final section addresses the case of interest rate pegging.

The Lucas–Stokey Cash/Credit-Good Model

Households

The Household Problem and Constraints

The model abstracts from differences among households by assuming they are all identical. In addition, households are assumed to live infinitely long. This assumption can be interpreted, following Barro (1974), as reflecting that each household actually lives a finite amount of time but cares in a particular way for its offspring.

The preferences of the representative household are given by

$$\sum_{t=0}^{\infty} \beta^t u(c_t),\ u(c)=\log(c),\ 0<\beta<1,$$

$$c=\left[(1-\sigma)c_1^{\upsilon}+\sigma c_2^{\upsilon}\right]^{\frac{1}{\upsilon}},$$

where $0<\sigma<1$ and c_t denotes consumption services. In our analysis, we restrict υ to the range $0<\upsilon<1$. Chari, Christiano, and Kehoe (1991) argue this is the empirically relevant case; based on postwar U.S. data, their point estimates are $\sigma=0.57$ and $\upsilon=0.83$.

Consumption services are generated by the acquisition of two market-produced goods, as indicated. The first, c_{1t}, is called a "cash good" and the other, c_{2t}, is a "credit good." To purchase the cash good, households need to set aside cash in advance.

To make the notion of "in advance" precise, the model adopts a particular timing. Each period is divided into two parts. In the first part (the "morning"), the household participates in an asset market, and in the second part (the "afternoon") the household participates in a goods market. The cash that households need to purchase c_{1t} in the afternoon of a given day must be set aside at the end of asset-market

■ **72** A simple example, in which the traditional quantity theory holds, is presented in the body of this review. In the model of Obstfeld and Rogoff (1983), there is a countable set of equilibria. The Ricardian assumption does not sit comfortably with that model, because the likelihood of the fiscal and monetary authorities choosing a fiscal policy consistent with one of those equilibria seems slim. Buiter (1999) provides another example.

■ **73** This is how Kocherlakota and Phelan (1999) interpret the FTPL.

trading the same morning. They hold these balances idle until the morning of the following day, when actual payment is due. Credit goods work differently. For these goods, the household has no need to accumulate cash in advance. The household simply pays for the goods with cash in the next morning's asset market.

Do not be misled by the labels used to identify these goods. It is not that one can be bought "on credit" in the traditional sense, and the other cannot. Both goods are paid in cash the morning after the purchase. No credit is offered by the seller in either case. The difference is simply that in the case of the cash good, the household must forfeit interest: To buy the cash good, the household must carry idle cash in its pocket throughout the afternoon. From the point of view of the seller, the goods are completely the same. The terms of the transaction are identical—cash only, to be delivered the morning after the sale.

The distinction between cash and credit goods may at first seem artificial. In fact, it is a clever device for capturing the idea that transactions in some goods are more cash-intensive than in others. It will produce a demand for money, one that is a function of the interest rate.

We assume the marginal rate of transformation in production is unity between the two goods. Therefore, the price of the two goods, P_t, is identical in equilibrium. Moreover, in any equilibrium it must be that $R_t \geq 0$ and $P_t > 0$. Market clearing is impossible if either of these two conditions fails to be satisfied.

Let A_t denote the household's financial assets at the end of asset-market trading. In the first period, $t=0$, this is simply a given number, A_0. The household can allocate A_t as follows:

$$\text{(A1)} \quad M_t^d + \frac{B_{t+1}^d}{1+R_t} + T_t \leq A_t, \; t=0, 1, 2, \ldots,$$

where M_t^d denotes money balances; B_{t+1}^d denotes government debt, which costs $B_{t+1}^d/(1+R_t)$ today and pays off B_{t+1}^d in the next period's asset market; and T_t denotes lump-sum taxes. The household does not set M_t^d to zero because it must set aside cash in advance, $P_t\, c_{1t} \leq M_t^d$, if it wishes to consume cash goods. Assets at the beginning of the next period are

$$\text{(A2)} \quad A_{t+1} = M_t^d + P_t\,(y - c_{1t} - c_{2t}) + B_{t+1}^d .$$

Here, M_t^d is the cash balance carried into the previous period's goods market; $P_t y$ denotes the receipts from the sale of y in the previous period's goods market; $P_t\,(c_{1t} + c_{2t})$ represents the bill of goods purchased in the previous period's goods market; and B_{t+1}^d is the receipts from government debt purchased in the previous period's asset market.

It is useful here to follow Woodford's (1994) suggestion to write equations (A1) and (A2) in a slightly different form. "Spending" is defined as

$$S_t = P_t\, c_{1t} + \frac{P_t\, c_{2,t}}{1+R_t} + \left(1 - \frac{1}{1+R_t}\right)(M_t^d - P_t\, c_{1t}).$$

In this measure of spending, excess holdings of money balances, above what are needed for the cash-in-advance constraint, have a positive price if $R_t > 0$. The relative "prices" of c_{1t} and c_{2t} accurately reflect that the former involves sacrificing interest earnings. "Income," I_t, is defined as

$$I_t = \frac{P_t y}{1+R_t} - T_t .$$

Divide both sides of equation (A2) by $(1+R_t)$ and substitute out for $B_{t+1}^d/(1+R_t)$, using equation (A1) to obtain

$$\text{(A3)} \quad A_{t+1} \leq (1+R_t)(A_t + I_t - S_t).$$

The accumulation of household assets obeys the usual simple equation one finds in a non-monetary, single-good model economy.

A lower-bound constraint must be placed on A_t to ensure the household has a bounded consumption set. We impose the assumption that the current value of assets must eventually be non-negative,

$$\text{(A4)} \quad \lim_{T \to \infty} q_T\, A_T \geq 0,$$

where

$$q_T = \frac{1}{(1+R_0)\;(1+R_1)\ldots\;(1+R_{T-1})}, \; q_0 = 1.$$

It is easy to verify that equations (A3) and (A4) are equivalent to the usual single present-value budget constraint for consumption, S_t, and income, I_t.[74]

We suppose that at each date the household chooses $c_{1t+j}, c_{2t+j} \geq 0, M^d_{t+j}\ B^d_{t+1+j} \geq 0$, to maximize its utility, subject to the restrictions just described, and takes A_t, R_{t+j}, P_{t+j}, $j \geq 0$, as given and beyond its control.[75]

Necessary and Sufficient Conditions for Household Optimization

The household first-order conditions are

$$\text{(A5)} \quad \frac{u_{2,t}}{P_t} = \beta \frac{u_{1,t+1}}{P_{t+1}}$$

and

$$\text{(A6)} \quad \frac{u_{1,t}}{u_{2,t}} = 1+R_t.$$

Here, $u_{i,t}$ denotes the partial derivative of utility with respect to c_{it} , $i=1$, 2. To understand why the first of these Euler equations is implied by household optimization, consider the following argument: Suppose the household reduces its purchases of credit goods in the period-t goods market by one dollar and applies that dollar to additional cash-good consumption in period $t+1$. Credit-good consumption today drops by $1/P_t$, which translates into an immediate decrease in utility of $u_{2,t}/P_t$. This reduction of expenditures frees one dollar in the asset market in the next period, which can be applied toward the cash-in-advance constraint for purchasing $1/P_{t+1}$ units of the cash good in next period's goods market. The utility benefit from the standpoint of period t is $\beta u_{1,\ t+1}/P_{t+1}$. If the gain exceeded the cost, the household could not be optimizing, or we would have found a change in its plan that would improve utility.

Similarly, if the gain were less than the cost, the household could raise utility by increasing credit-good consumption in period t and reducing cash-good consumption in period $t+1$. Optimization requires that neither of these strategies raises utility, and this is why the first Euler equation above (A5) is an implication of household optimization.

The second Euler equation (A6) is also implied by household optimization, established by an argument similar to the one in the previous paragraph. The argument exploits the trade-off between cash and credit goods within the same period. The household can increase current-period cash-good consumption by reducing its acquisition of government debt. This reduces its cash receipts in the next period's asset market, thus reducing the cash available for credit-good consumption today. This Euler equation makes considerable sense: When R is high, it implies that u_1 is relatively high, so that c_1 is relatively low. This makes sense because high R raises the cost to households of purchasing c_1.

There is also a condition associated with the cash-in-advance constraint, which we write as

$$\text{(A7)} \quad R_t\left(P_t c_{1t} - M^d_t\right) = 0.$$

As noted above, only the case $R_t \geq 0$ must be considered. Since $P_t c_{1t} - M^d_t$ cannot be negative, equation (A7) is a mathematically concise way of stating that if $R_t > 0$, it follows that $P_t c_{1t} = M^d_t$, while if $R_t = 0$, then all we know is $P_t c_{1t} \geq M^d_t$. From the point of view of the analysis below, the key is that when $R_t > 0$, the cash-in-advance constraint holds as a strict equality. This makes sense: When the interest rate is positive, it is inconsistent with optimization to carry cash in the afternoon that is not absolutely necessary.

In addition to equations (A5)–(A7), the transversality condition is also implied by household optimization:

$$\text{(A8)} \quad \lim_{T\to\infty} q_T A_T = 0.$$

The intuition for this condition is straightforward. To see that the limit cannot be positive, suppose, on the contrary, that it is. In this case, A_t grows faster than the interest rate. It would then be feasible for households to increase spending

74 By recursive substitution, equation (A3) implies

$$q^T A_T \leq A_0 + \sum_{t=0}^{T-1} q_t (I_t - S_t).$$

Driving $T \to \infty$, we obtain

$$\sum_{t=0}^{\infty} q_t S_t \leq A_0 + \sum_{t=0}^{\infty} q_t I_t.$$

This shows that equations (A3) and (A4) imply the standard single-equation budget constraint. To establish the reverse, simply show that if $\{S_t, I_t\}$, $t = 0, 1,...$ satisfy the budget constraint, then they also satisfy (A3) and (A4). That the present value of income is finite will be a feature of equilibrium. Otherwise, demand would be unbounded and no equilibrium could exist.

75 It is easy to verify that in any equilibrium, it must be that $R_t \geq 0$ and $P_t > 0$. Market clearing is impossible if either of these conditions is not satisfied.

in one date without reducing it in another. If this extra spending were financed by a loan, the power of compound interest would cause the resulting debt to spiral upward at a rate equal to the interest rate. However, with total assets rising at an even greater rate, the household's net asset position would remain consistent with equation (A4). The increase in consumption financed in this way raises utility because of nonsatiation, and so we have a contradiction. Thus, optimization implies the above expression cannot be positive, but it also cannot be negative because of the restriction of equation (A4).

For purposes of our analysis, it is convenient to write the transversality condition in a different form. Combining equations (A5) and (A6), we find $u_{1,t}=\beta(1+R_t)u_{1,t+1}P_t/P_{t+1}$. Substituting this into the expression for q_t, we find

$$\text{(A9)}\quad q_t=\left(\frac{P_0}{u_{1,0}}\right)\frac{\beta^t u_{1,t}}{P_t},\ t=0,\ 1,\ 2,\ \ldots.$$

After multiplying both sides of equation (A8) by the positive constant, $u_{1,0}/P_0$, the transversality condition reduces to

$$\text{(A10)}\quad \lim_{T\to\infty}\beta^T u_{1,T}\frac{A_T}{P_T}=0.$$

Equations (A5)–(A10) are not just necessary for optimization, they are also sufficient. This is easily established with a suitably adjusted version of the proof to Stokey, Lucas, and Prescott's theorem 4.15 (1989).

Government

The government purchases no goods, it only participates in the asset market. Its sources of funds in the asset market are new debt issues, tax revenues, and newly created money, $M_t^s-M_{t-1}^s$. It uses these funds to pay its outstanding debt obligations, B_t^s. Equating sources and uses of funds gives us

$$\frac{B_{t+1}^s}{1+R_t}+T_t+M_t^s-M_{t-1}^s=B_t^s.$$

At time t, the government takes M_{t-1}^s and B_t^s as given, $t=0,\ 1,\ldots$. At date 0, $M_{-1}^s+B_0^s=A_0$. Government policy is a sequence of B_{t+1}^s, T_t, and M_t^s that satisfies this flow-budget constraint, which can also be written as

$$\frac{A_{t+1}^s}{1+R_t}+T_t+\frac{R_t}{1+R_t}M_t=A_t^s.$$

Here, A_t^s measures total nominal assets, $A_t^s=B_t^s+M_{t-1}^s$, and $A_0^s=A_0$.

Recursively substituting this expression forward, we find that for each fixed T,

$$\text{(A11)}\quad q_T A_T^s+\sum_{t=0}^{T-1}q_t\left[T_t+\frac{R_t}{1+R_t}M_t\right]=A_0.$$

The presence of $R_tM_t/(1+R_t)$ reflects the interest costs the government saves when it issues money rather than bonds. The government's intertemporal budget equation is represented by the above expression, with $q_T\,A_T^s$ absent and $T-1$ replaced by ∞:

$$\text{(A12)}\quad \sum_{t=0}^{\infty}q_t\left[T_t+\frac{R_t}{1+R_t}M_t\right]=A_0.$$

The only restriction we have placed on government policy is that the flow-budget constraint is satisfied for all possible values of prices, $\{q_t,\ P_t,\ R_t,\ t\geq 0\}$. That is, we require that equation (A11) hold. But no assumption has been made that equation (A12) holds for all possible prices. Government policy is said to be Ricardian if (A12) holds for all possible prices, and it is non-Ricardian if (A12) holds only at equilibrium prices. (We shall see that, at equilibrium prices, [A12] must be satisfied regardless of whether government policy is Ricardian or non-Ricardian. This follows from equation [A8] and the fact that, in equilibrium, $A_t^s=A_t$.)

Equation (A11) converges to equation (A12) if and only if

$$\text{(A13)}\quad q_T A_T^s\to 0.$$

We can equivalently define a Ricardian policy as one that enforces equation (A13) at all possible prices and a non-Ricardian policy as one that does not.

Firms

Firms in this economy are simple. They buy y from households and transform it into cash and credit goods. Given the assumed linearity of the production technology, the resource constraint has the form

$$\text{(A14)}\quad c_{1t}+c_{2t}=y.$$

Equilibrium

A general equilibrium for this economy is a sequence of prices and interest rates, P_t and R_t; a sequence of consumptions, c_{1t}, c_{2t}; and a sequence of money supplies and bonds, M_{t+1} and B_{t+1}, such that households optimize, the government flow-budget constraint is satisfied, and markets clear. Bond-market clearing requires

$$B^s_{t+1} = B^d_{t+1} = 0,$$

and money-market clearing requires

$$M^s_t = M^d_t = M_t,$$

say, for $t \geq 0$. These conditions imply that $A_{t+1} = M^d_t + B^d_{t+1} = A^s_{t+1}$. Goods-market clearing corresponds to the resource constraint, equation (A14).

A feature of equilibrium that will be useful in the analysis is

$$\text{(A15)} \quad 1+R_t = \frac{1-\sigma}{\sigma}\frac{1}{w_t^{1-\upsilon}} \geq 1,\ w_t \equiv \frac{c_{1t}}{c_{2t}},$$

which we obtain from equation (A6) and our parametric form for the utility function. When $R_t > 1$, we can rewrite this to obtain the model's "money-demand" function. The binding cash-in-advance constraint, $c_{1t} = m_t$, and the resource constraint imply $w_t = m_t/(y - m_t)$ where

$$m_t = \frac{m_t}{P_t}$$

Solving equation (A15) for m_t yields

$$\text{(A16)} \quad m_t = \frac{y}{1 + \left[\frac{\sigma}{1-\sigma}\ (1+R_t)\right]^{\frac{1}{1-\upsilon}}}.$$

Constant Money Growth

Here, we consider the set of equilibria associated with a fixed money growth rate policy. We show there is one equilibrium in which inflation is constant and equal to money growth. There is also a continuum of equilibria with explosive inflation.

We suppose the government sets $B^s_{t+1} = 0$ for all $t \geq 0$ by paying off the entire stock of debt in the first period. In addition, it sets $M^s_t = \mu M^s_{t-1}$ for $t = 0, 1, \ldots$, where $\mu \geq 1$. Money growth is accomplished by means of lump-sum tax transfers. In particular,

$$T_0 = B^s_0 - (\mu - 1) M^s_{-1},$$

$$T_t = -(\mu - 1) M^s_{t-1},\ t \geq 1.$$

It is straightforward to verify that, with this specification of policy, there are many price sequences that satisfy equation (A10). Technically, it does not fit into our formal definition of a Ricardian policy, because (A10) is not satisfied for all prices. Under this policy, A_t comprises only the money supply. Thus, equation (A10) would be violated if the price level fell sufficiently rapidly. Still, for practical purposes we will think of this as a Ricardian policy.

It is useful to rewrite the household's dynamic Euler equation by multiplying both sides of equation (A5) by M_t and using $M_{t+1} = \mu M_t$ to obtain

$$\text{(A17)} \quad u_{2,t}\, m_t = \frac{\beta}{\mu} u_{1,t+1} m_{t+1}.$$

A sequence of prices and quantities represents an equilibrium if and only if equations (A7)–(A14) and $P_t > 0$, R_t, c_{1t}, $c_{2t} \geq 0$ are satisfied.

A Characterization Result for Equilibria

We now simplify the equilibrium conditions to obtain a useful set of sufficient conditions for equilibria in which the cash-in-advance constraint binds. In this case, equation (A14) allows us to express equation (A17) as a difference equation in m_t and m_{t+1} alone. Because the cash-in-advance constraint binds, w_t in equation (A15) can be written as

$$w_t = \frac{m_t}{y - m_t}.$$

Using this notation, equation (A17) can be expressed as a difference equation in w_t and w_{t+1},

$$\text{(A18)} \quad a(w_t) = b(w_{t+1}),$$

where

$$\text{(A19)} \quad a(w) = \frac{\sigma w}{(1-\sigma)w^{\upsilon} + \sigma},$$

$$a'(w) = \frac{\sigma}{[(1-\sigma)w^{\upsilon} + \sigma]^2}\left[(1-\sigma)(1-\upsilon)w^{\upsilon} + \sigma\right]$$

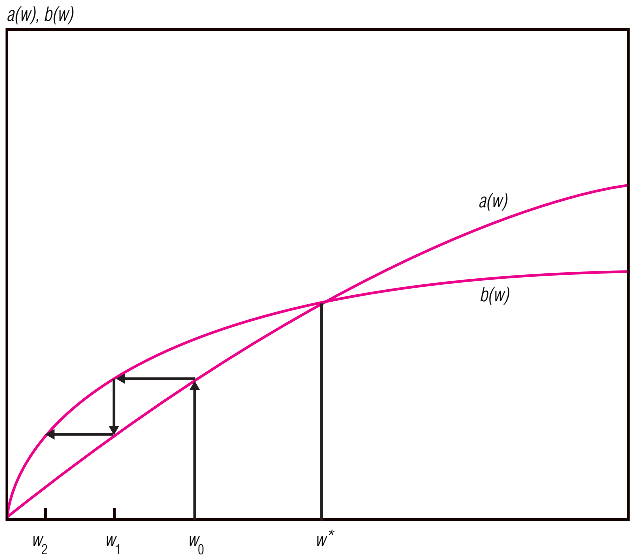
and

$$\text{(A20)} \quad b(w) = \frac{\beta(1-\sigma)}{\mu}\frac{w^{\upsilon}}{(1-\sigma)w^{\upsilon} + \sigma},$$

$$b'(w) = \frac{\beta(1-\sigma)}{\mu}\frac{w^{\upsilon-1}\,\upsilon\sigma}{\left[(1-\sigma)w^{\upsilon} + \sigma\right]^2}.$$

FIGURE A

Equilibrium in the Cash/Credit-Good Model



Here, a' and b' represent the derivatives of a and b, respectively, with respect to w. The transversality condition reduces, in the present notation, to

$$\text{(A21)} \quad \lim_{T\to\infty} \beta^T b(w_T)=0.$$

We are now in a position to state our characterization result.

Proposition A1: Suppose that $w_t \geq 0$, $t=0,1,2,$ ..., satisfies equations (A18), (A21), and (A15). Then, w_t corresponds to an equilibrium.

Proof: Write

$$m_t = y\,\frac{w_t}{1+w_t}\;,\; P_t = \frac{M_t}{m_t}$$

$$R_t = \frac{\sigma}{1-\sigma}\,\frac{1}{w_t^{1-v}}\,,\; c_{2t} = \frac{y}{1+w_t}, c_{1t} = c_{2t}w_t,$$

and verify that all equilibrium conditions are satisfied at these prices and quantities. QED.

Multiple Equilibria with Ricardian Constant Money Growth

We use the characterization result to show there is a continuum of equilibria in this economy when the money growth rate, μ, is constant and greater than unity. From equation (A18), there is exactly one equilibrium with $w_t = w^*$ for all t,

$$\text{(A22)} \quad w^* = \left[\frac{1-\sigma}{\sigma}\,\frac{\beta}{\mu}\right]^{\frac{1}{1-v}}.$$

It is easily verified that this satisfies the conditions of the characterization result. For example, substituting w^* into equation (A15) yields a positive interest rate with $1+R=\mu/\beta$. This is greater than unity by our assumptions on μ and β. Because real balances are constant in this equilibrium, the rate of inflation is equal to μ.

The intuition underlying equation (A22) is straightforward: The relative quantity of cash goods consumed in the equilibrium (that is, w^*) is increasing in $1-\sigma$, which is the relative weight in utility on these goods. It is decreasing in the money growth rate, μ, because increases in μ raise the nominal rate of interest, in turn increasing the cost of the cash good. Finally, consider $v \to 1$. This is easiest to interpret when $\sigma = 1/2$. In this case, the two consumption goods are perfect substitutes. Consequently, if the cash good is more expensive than the credit good, as is the case when $\mu \geq 1$, zero cash goods will be consumed, and $w^* = 0$ as $v \to 1$.

There are other equilibria in which inflation exceeds μ. To show this, we first study the properties of the functions $a(w)$ and $b(w)$.

According to equation (A19), $a(0)=0$ and $a'(0)=1$. Also, $a'(w) > 0$ for all $w \geq 0$. At the same time, equation (A20) indicates that $b(0) = 0$, $b'(w) \to \infty$ as $w \to 0$, and $b'(w) > 0$ for $w > 0$. These observations establish that $a(w)$ and $b(w)$ coincide at $w=0$, with b rising more steeply than a for small values of w.

From the discussion leading up to equation (A22), we know there is a unique value of $w>0$—namely, w^* in equation (A22)—where $a(w) = b(w)$. Since the two functions are continuous for $0<w<w^*$, it follows that $b(w) > a(w)$ for w in this interval, and that a is steeper than b at $w=w^*$. The latter observation can be confirmed by direct differentiation, which yields

$$\frac{a'(w^*)}{b'(w^*)} = \left(\frac{1-\sigma}{\sigma}\right)^{\frac{1}{1-v}}\left(\frac{1-v}{v}\right)\left(\frac{\beta}{\mu}\right)^{\frac{v}{1-v}} + \frac{1}{v} > 1.$$

The strict inequality reflects that the expression immediately after the equality is positive and that $1/v>1$ because $0<v<1$.

Our results on the a and b functions are summarized in figure A. Note how b rises above a and then crosses once. Eventually, the two curves are parallel, since $a'(w)$ and $b'(w)$

both converge to zero as $w \to \infty$. We can use this figure to study the set of equilibria for the model.

Consider an arbitrarily selected $w_0 < w^*$. To determine the value of w_1 implied by equation (A18), draw a vertical line up to $a(w_0)$. Then, identify w_1 such that $b(w_1)$ equals $a(w_0)$. This can be found by following a horizontal line to the left of $a(w_0)$ until it intersects b. The properties of these curves guarantee that such an intersection will occur for a positive value of w. With w_1 in hand, compute w_2 in the same way, and so on.

It should be clear that the sequence of w_t computed in this way converges to 0. Along this path, $b(w_t) \geq 0$ and $b(w_t) \to 0$ as $t \to \infty$. Because b is bounded above along the path, equation (A21) is satisfied. Because w_t declines monotonically, $R > 0$ at w^* and $w_t > 0$ for a given t, equation (A15) implies that $R_t > 0$ for each t. This establishes that the sequence just computed constitutes an equilibrium.[76] The same argument can be applied for each $0 < w_t < w^*$; in each of these equilibria there is a hyperinflation as $w_t \to 0$.[77]

UNIQUE EQUILIBRIUM WITH NON-RICARDIAN, CONSTANT MONEY GROWTH

The previous section showed that with a particular Ricardian policy, constant money growth results in a continuum of equilibria. Here is a particular non-Ricardian policy:

$$T_t = P_t s - \frac{R_t}{1+R_t} M_t ,$$

where s is a positive constant. It is easy to verify that the set of equilibria under this policy is a strict subset of the set of equilibria analyzed in previous section. Thus, we conclude that with constant money growth, a non-Ricardian policy does not lead to an overdetermined price level.

Fixed Interest Rate Policies

This section considers two representations of policy in which the government pegs the nominal rate of interest to a constant value, $R > 0$. In the first representation, fiscal policy is Ricardian and there exists a continuum of equilibria. In the second, policy is non-Ricardian and the equilibrium, if it exists, is unique.

The fixed value of R pins down m (see equation [A16]), c_1, and c_2:

$$c_1 = m, \; c_2 = y - c_1 .$$

As a consequence, the marginal utility of the cash good is constant, so that

$$\frac{P_{t+1}}{P_t} = \beta(1+R)$$

for all t. Consider two specifications of policy,

$$\text{(A23)} \quad T_t = -\frac{R}{1+R} mP_t + \varepsilon A_t$$

and

$$T_t = -\frac{R}{1+R} mP_t + dP_t ,$$

where d is a non-negative constant and $0 < \varepsilon \leq 1$. As we will show, the first policy is Ricardian, while the second is not. These policies may initially appear strange, but the motivation behind them will soon become clear. To determine whether a policy is Ricardian requires us to determine whether equation (A13) holds for all possible prices or only for equilibrium prices.

To investigate this further, it is convenient to write the flow-budget constraint in real terms,

$$\beta a_{t+1} + \tau_t + \frac{R}{1+R} m = a_t .$$

Here, $a_{t+1} = A_{t+1}/P_{t+1}$ and $\tau_t = T_t/P_t$. Substituting the first specification of policy in equation (A23) into the flow-budget constraint gives us

$$\text{(A24)} \quad a_{t+1} = \frac{1-\varepsilon}{\beta} a_t .$$

We seek to understand how $\tilde{a}_t \equiv \beta^t a_t$ evolves as $t \to \infty$. By substituting from equation (A9), we have

$$\text{(A25)} \quad q_T A_T = P_0 \beta^T \frac{A_T}{P_T} = P_0 \tilde{a}_T .$$

Recall that a policy in which $q_T A_T \to 0$ for all

76 Recall, in constructing equation (A18) we assumed the cash-in-advance constraint is binding. This assumption has been verified for $w_0 < w^*$.

77 Our results would not be significantly affected if we allowed labor to be endogenous. Introducing labor as a third argument in the utility function has the effect of adding an extra Euler equation, $-u_3/u_2 = f'(l)$, where $f'(l)$ denotes the marginal product of labor, l, and u_3 denotes the marginal utility of labor. Feasibility restricts l to some subspace, $l \in D$ (for example D might be the unit interval). Also, $y = f(l)$ denotes the production function. Combining the new Euler equation with the resource constraint produces a function, $l = F(w)$, where F has a nice analytical characterization with standard preferences and technology. To find an equilibrium, one would still start by looking for values of w_t that solve the difference equation, $A(w_t) = B(w_{t+1})$. One would then have to verify $F(w) \in D$, in addition to the other conditions listed in the characterization result, to verify that the values of w_t represent an equilibrium.

possible prices corresponds to a Ricardian policy, and one in which this occurs only for equilibrium prices is a non-Ricardian policy. Multiplying both sides of equation (A24) by β^{t+1}, we find

$$\tilde{a}_{t+1} = (1-\varepsilon)\tilde{a}_t = (1-\varepsilon)^t \tilde{a}_0,$$

or, using equation (A25),

$$q_T A_T = (1-\varepsilon)^T A_0 \rightarrow 0.$$

This establishes that the first policy in equation (A23) is Ricardian. The government's policy prevents the debt from exploding too fast, regardless of what happens. As a result, the intertemporal budget equation provides no useful restriction for pinning down prices.

Now consider the second policy in equation (A23). For this policy, total real assets evolve according to

$$a_{t+1} = \frac{1}{\beta}(a_t - d).$$

The policy makes the evolution of total assets exogenous, while letting the private economy determine the breakdown of real assets between money and bonds to be consistent with the interest rate peg. Solve for a_t and then multiply by β^t,

$$\beta^t a_t = a_0 - \frac{d}{1-\beta} + \frac{d}{1-\beta}\beta^t,$$

so that

$$\beta^t a_t \rightarrow \frac{A_0}{P_0} - \frac{d}{1-\beta},$$

where $a_0 = A_0/P_0$. This is a non-Ricardian policy because $\beta^t a_t \rightarrow 0$ for only one value of P_0—the one that satisfies

$$\frac{A_0}{P_0} = \frac{d}{1-\beta}.$$

We can now summarize our results for the interest rate peg. If it is accompanied by a Ricardian policy, the price level is not pinned down by the intertemporal budget equation, nor by the rest of the model. The model pins down only M_t/P_t and P_{t+1}/P_t, but not the numerator and denominator terms. Under the non-Ricardian policy, the intertemporal budget equation supplies the extra equation needed. Once again, the price level is not overdetermined under the non-Ricardian policy.