IV. Fiscal Theory and the Optimal Degree of Price Instability

The FTPL literature has drawn attention to the possibility that some price instability may be desirable when unavoidable shocks to the government budget constraint occur (Sims [1999], Woodford [1998a]). When there is nominal government debt, unanticipated shocks to the price level act as capital levies on bondholders. The idea is that it is efficient to absorb unanticipated shocks with capital levies rather than by changing distortionary taxes.

We illustrate these observations in the simplest possible model.[56] Relative to the one-period model of part II, this model incorporates two essential complications. First, we must take into account the distortionary effects on the bond-accumulation decision that may arise from price-level instability. For this reason, we adopt a two-period model. The bond-accumulation decision is taken in the first period, and the government-spending shock and price-level uncertainty occur in the second period. Second, the model must capture the notion that taxes are distortionary. Accordingly, we assume the labor supply is endogenous and taxes are raised using a proportional tax on labor income.

The model is an example of the FTPL because government policy—the choice of labor tax rates—is non-Ricardian. We illustrate how FTPL advocates study the optimal degree of price stability by examining the "best" equilibrium of such a model (see, for example, Sims [1999] and Woodford [1998a]). The literature on optimal fiscal and monetary policy (see, for example, Lucas and Stokey [1983]) calls this equilibrium the *Ramsey equilibrium*.

First, we describe the model. To simplify the analysis, the model does not include money; as a consequence, the model again illustrates price determination in an economy with no government-provided money. Next, we characterize the best (that is, the Ramsey) equilibrium in this economy. We then present a numerical example to illustrate the role of price instability in bringing about efficient resource allocation in the model. We assess the results in a summary section.

The Model

The economy comprises firms, households, and a government. Households and firms interact in competitive markets. The government must finance an exogenously given level of expenditures by levying a distortionary tax rate on labor and possibly by issuing debt. There is no uncertainty in the first period. However, there is uncertainty in the second period's level of government spending. Spending could be high or low, with probability 1/2 each, with the uncertainty being resolved at the beginning of the second period. Consistent with the non-Ricardian assumption, the government commits to its policies before the first period. Trade occurs by barter, and there is no money in the model.[57]

Firms have access to a linear production technology,

$$y = n,\ \ y^h = n^h,\ \ y^l = n^l,$$

where y and n denote output and labor, respectively, in the first period, and y^i, n^i denote output and labor in the second period, $i = h, l$. The superscript h or l indicates the second period when government spending is high or low, respectively. The linearity in the production function guarantees the real wage is always unity in equilibrium; henceforth, we simply impose this result and do not refer to firms any more.

Preferences of households over consumption and labor during the two periods take the form

$$(4.1)\quad U(x) = c - \frac{1}{2}n^2 + \frac{1}{2}\beta\left\{\left[c^h - \frac{1}{2}(n^h)^2\right] + \left[c^l - \frac{1}{2}(n^l)^2\right]\right\},\ 0 \le \beta \le 1,$$

where c denotes consumption in the first period and β is the discount rate, with $\beta = (1+r)^{-1}$. Similarly, c^i denotes consumption in the second period, conditional on the realization of government spending, $i = h, l$. β is the discount rate of the household, and the fraction "1/2"

56 The example illustrates the results on the desirability of tax smoothing and volatile prices reported in Chari, Christiano, and Kehoe (1991).

57 We could imagine there is "inside money" and trade is accomplished through an efficient exchange of IOUs.

preceding β corresponds to the probability of the h or l state of the world. Finally,

$$(4.2)\quad x = (c, c^h, c^l, n, n^h, n^l).$$

The linear-quadratic structure of preferences is chosen to ensure a simple analysis. The household's period-1 budget constraint is

$$(4.3)\quad \frac{B'}{1+R} + Pc \leq B + P(1-\tau)n,$$

where P is the period-1 price level, B is the nominal bonds the household inherits from the past, and R is the nominal rate of interest. Also, τ denotes the tax rate on labor and B' denotes bonds acquired from the government. The household's budget constraint in the second period, conditional on the realization of uncertainty, is

$$(4.4)\quad P^h c^h \leq B' + P^h(1-\tau^h)n^h,$$

$$P^l c^l \leq B' + P^l(1-\tau^l)n^l.$$

Again, superscripts indicate the realization of the exogenous government-spending shock. There is no government-supplied money in this economy.

The household maximizes utility by its choice of non-negative values for B', c, c^h, c^l, n, n^h, and n^l. It must respect the budget constraints just specified, and it takes prices and the interest rate as given and beyond its control. The Euler equations associated with the household's optimal choice of labor and bonds are

$$(4.5)\quad n = 1-\tau,\ n^h = 1-\tau^h,\ n^l = 1-\tau^l,$$

$$\frac{1}{(1+R)P} = \frac{1}{2}\beta\left(\frac{1}{P^h} + \frac{1}{P^l}\right).$$

The last of these equations tell us that the expected gross real rate of return on bonds must be $1/\beta$. That this is true, independent of the intertemporal pattern of consumption, reflects our assumption that utility is linear in consumption.

The government's budget constraints in the first and second periods are given by

$$(4.6)\quad \frac{B'}{1+R} + P\tau n \geq B + Pg$$

$$P^h \tau^h n^h \geq B' + P^h g^h$$

$$P^l \tau^l n^l \geq B' + P^l g^l.$$

Here, g denotes government consumption in the first period, and g^i denotes period-2 government consumption, $i = h, l$. In the equations of our model, R appears everywhere as $(1+R)/P^h$, $(1+R)/P^l$, or $B'/(1+R)$.[58] Thus, we cannot pin down R, P^h, P^l, and B' separately. For this reason, we adopt the normalization $R = 0$ from here on. Government policy is a vector of three numbers, π, where

$$\pi = (\tau, \tau^h, \tau^l).$$

This is a non-Ricardian policy because there is no set of values for π that will satisfy the government's intertemporal budget equation (see below) for all prices.

Combining the government and household budget equations, we obtain the resource constraints:

$$(4.7)\quad c + g \leq n,\ c^h + g^h \leq n^h,\ c^l + g^l \leq n^l.$$

There are 10 variables to be determined in equilibrium: P, P^h, P^l, B', c, c^h, c^l, n, n^h, and n^l. They are determined by the three household budget constraints ([4.3] and [4.4]), evaluated with a strict equality; the four household Euler equations ([4.5]); and the three resource constraints ([4.7]). These 10 equations, together with the requirement $P, P^h, P^l > 0$, characterize the equilibrium (if one exists!) associated with a given government policy. The mapping from π to these variables is single valued. We denote the function relating the last six variables to π by $x(\pi)$, where x is defined in equation (4.2).

58 The statement is obviously true in the case of the household Euler equation in (4.5). To see that it is also true of equations (4.3),(4.4), and (4.6), replace B' with $\widetilde{B} = B'/(1+R)$ and divide the period-2 budget constraints by $1+R$.

The Ramsey Equilibrium

The Ramsey equilibrium is associated with the policies, π, that solve the problem

$$\max_{\pi} U[x(\pi)],$$

subject to the requirement that prices be strictly positive, $B' \geq 0$, and the elements in x be non-negative.[59] The Ramsey equilibrium is easy to compute in this model economy.

After substituting out for the endogenous variables in terms of π in equations (4.7) and (4.5), the utility function is represented by

$$(4.8) \quad U[x(\pi)] = -\tau^2 - \frac{1}{2}\beta\left[(\tau^b)^2 + (\tau^l)^2\right] + \kappa,$$

where κ is a constant.[60] To complete the statement of the Ramsey problem, we need a simple representation of the restrictions placed on π by the positive-price requirement. Before we do this, we must confront a technical issue.

It is well known in the literature on Ramsey equilibria that it is efficient to renege on the initial nominal debt, B, by selecting policies that produce an infinite first-period price level. Allowing this would plunge us into exotic mathematical issues, distracting us from the central focus of the example: the desirability of letting prices in the second period react to the realization of government spending in that period. With this in mind, we simply fix the period-1 price level at P=1. Since the nominal debt, B, is given from the past, it follows that we have fixed the initial real debt. It is important to emphasize, however, that we do not fix the second-period price levels.[61]

The restriction on π implied by P=1 is easy to determine by expressing the government's first-period intertemporal budget equation in terms of π. Combine the household's intertemporal Euler equation (4.5) with the government's budget constraints (4.6),

$$(4.9) \quad B \leq \tau(1-\tau) - g + \frac{1}{2}\beta\left[\tau^b(1-\tau^b) - g^b + \tau^l(1-\tau^l) - g^l\right],$$

where we have imposed P=1. The restrictions on second-period prices come from the intertemporal government budget equations that obtain

$$(4.10) \quad \tau^b(1-\tau^b) - g^b \geq 0, \; \tau^l(1-\tau^l) - g^l \geq 0$$

in those periods. The Ramsey problem, modified to incorporate the restriction P=1 is set up in Lagrangian form:

$$\max_{\tau,\,\tau^b,\tau^l} \; -\tau^2 - \frac{1}{2}\beta\left[(\tau^b)^2 + (\tau^l)^2\right]$$

$$+ \lambda\left\{\tau(1-\tau) - g + \frac{1}{2}\beta\left[\tau^b(1-\tau^b) - g^b + \tau^l(1-\tau^l) - g^l\right] - B\right\}$$

$$+ \mu^b\left[\tau^b(1-\tau^b) - g^b\right] + \mu^l\left[\tau^l(1-\tau^l) - g^l\right],$$

where λ, μ^b, and $\mu^l \geq 0$ are Lagrange multipliers.[62] The necessary and sufficient conditions associated with the maximum are the inequality constraints on the multipliers, λ, μ^b, and $\mu^l \geq 0$; the inequality constraints in equations (4.9) and (4.10); the "complementary slackness" conditions,

$$(4.11) \quad 0 = \lambda\left\{\tau(1-\tau) - g + \frac{1}{2}\beta\left[\tau^b(1-\tau^b) - g^b + \tau^l(1-\tau^l) - g^l\right] - B\right\},$$

$$0 = \mu^b\left[\tau^b(1-\tau^b) - g^b\right],$$

$$0 = \mu^l\left[\tau^l(1-\tau^l) - g^l\right];$$

59 See Bizer and Judd (1989), Chari, Christiano, and Kehoe (1990), Judd (1989), and Lucas and Stokey (1983), among others.

60 Here, $\kappa = 2\left[\frac{1}{2} - g + \frac{1}{2}\beta\left(1 - g^h - g^l\right)\right]$.

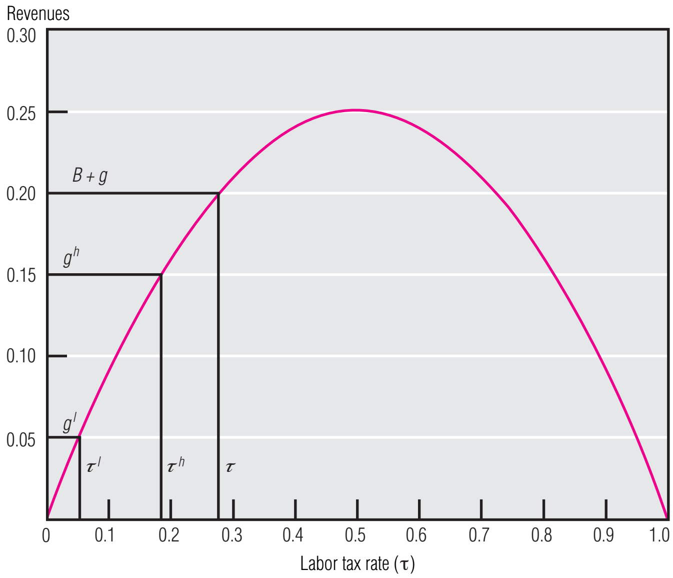
To see how we obtain this expression, note that $c-(1/2)n^2$ is $y-g-(1/2)\,n^2$ after using the resource constraint, $y=c+g$. Imposing $n=1-\tau$, then, yields that $c-(1/2)\,n^2$ is $(1/2)(1-\tau^2)-g$.

61 Chari, Christiano, and Kehoe (1991) confront the same problem, which they deal with by setting the initial debt to zero. In our context, that creates a problem because it leaves us with no ability to pin down P.

62 Our model differs from Sims' (1999) model in two respects. First, ours has only two periods, while Sims' has an infinite horizon. (It is trivial to extend our model to the infinite horizon.) Second, we model agents at the level of preferences and technology, while Sims adopts a reduced-form representation analogous to the one in Barro (1979). Our reduced-form utility function coincides with Sims', but our budget constraint does not. Sims models taxes as lump-sum in the budget constraint, whereas we take into account the distortionary effects of taxation. For example, Sims would have τ in the budget constraint, rather than $\tau(1-\tau)$, as we do. The conclusions of the analysis are not sensitive to these differences.

FIGURE 4

Revenues from Labor Tax



and the three first-order conditions corresponding to τ, τ^h, τ^l. After rearranging, these are

$$(4.12)\quad \lambda = \frac{2\tau}{1-2\tau},$$

$$\mu^h = \beta\left[\frac{\tau^h}{1-2\tau^h} - \frac{\tau}{1-2\tau}\right],$$

$$\mu^l = \beta\left[\frac{\tau^l}{1-2\tau^l} - \frac{\tau}{1-2\tau}\right].$$

We solve the (constrained) Ramsey problem by finding multipliers, λ, μ^h, μ^l, and policies, τ, τ^h,τ^l, that satisfy these conditions.

Once the Ramsey policies have been identified, n, n^h, and n^l are obtained from equation (4.5) and c, c^h, and c^l from equation (4.7). Then, B', P^h, and P^l are obtained by solving equation (4.6). Several qualitative features of the solution are immediately apparent. First, the weak inequality in equation (4.9) is satisfied as a strict equality.[63] This is not surprising—otherwise, taxes would be higher than necessary and, given the form of preferences, this would be counterproductive. Also, because the period-0 intertemporal budget equation is satisfied as a strict equality, it would have been optimal to inflate away the initial debt by setting $P=\infty$, had we not imposed the requirement the government pay off B with $P=1$.[64] Second, ignoring the requirements of the non-negativity constraints on prices in the second period, the optimal outcome is $\tau=\tau^h=\tau^l$. To see this, note that the first-order conditions in this case are equation (4.12) with $\mu^h \equiv \mu^l \equiv 0$. Inspecting the second two equations in (4.6), it is obvious that $P^h > P^l$ as long as $g^h > g^l$. Third, in practice, the constant tax rate policy is not necessarily feasible, since it may conflict with the positive-price requirement. In this case, however, the price fluctuations across states of nature are even more extreme.

Suppose, for example, the constant tax rate policy is inconsistent with the first of the two inequalities in equation (4.10). Then, $\mu^h > 0$, $\tau^h > \tau$, and, by equation (4.11), $\tau^h(1-\tau^h)-g^h=0$. The latter implies the government inflates away the debt completely in state h, with $P^h=\infty$. To ensure that households still have an incentive to accumulate debt in the first period, equation (4.5) indicates P^l must satisfy $P^l = (\beta/2)P(1+R)=\beta/2$ in this case. That is, the real rate of return on debt into state l must be high.

A Numerical Example

This section studies a numerical example to illustrate the properties of P^h and P^l in the Ramsey equilibrium. A natural benchmark to consider is the no-debt equilibrium: τ is selected so that $B'=0$, and τ^h and τ^l are selected so the constraints in equation (4.10) are satisfied as exact equalities. With this as a benchmark, we evaluate the Ramsey equilibrium in which $B'>0$ and consider P^h and P^l.

To see how taxes are determined in the benchmark equilibrium, consider figure 4, which graphs $\tau n=\tau(1-\tau)$ for $\tau \in (0,1)$. We have a single-peaked Laffer curve in our model economy. The horizontal lines indicate the

63 Here is a proof by contradiction. Suppose the weak inequality in equation (4.9) were a strict inequality. Then, by the first expression in equation (4.11) and in equation (4.12), we have $\lambda=0$ and $\tau=0$. The strict inequality in equation (4.9) implies that at least one of the weak inequalities in (4.10) is strict. That implies, by (4.11), the associated multiplier is zero. Equation (4.12) implies the associated tax rate is zero, but this contradicts the non-negativity of the primary surplus in that period.

64 In that case, the constraint would have been equation (4.11) without the term $-B$.

TABLE 1

Two Equilibria

Variable	Benchmark, no-debt equilibrium (B'=0)	Ramsey equilibrium
P	1	1
P^h	—	64.67
P^l	—	0.49
n	0.72	0.82
n^h	0.82	0.82
n^l	0.95	0.82
c	0.52	0.62
c^h	0.67	0.67
c^l	0.90	0.77
τ	0.28	0.19
τ^h	0.18	0.19
τ^l	0.05	0.19
$g+B$	0.20	0.20
g^h	0.15	0.15
g^l	0.05	0.05
B'	0	0.05
Utility	–0.0941	–0.0674

revenue requirements in the first and second periods. We assume the first-period revenue requirement, $B+g$, is 0.20. The second-period revenue requirement is $g^h = 0.15$ when government spending is high and $g^l = 0.05$ when government spending is low. The benchmark equilibrium requires that τ, τ^h, and τ^l be set as indicated on the horizontal axis. In particular, $\tau = 0.28$, $\tau^h = 0.18$, and $\tau^l = 0.05$, after rounding. The value of equation (4.8) in this equilibrium is –0.0941, ignoring κ and setting $\beta = 0.97$. Tax rates are very uneven over time and over states of nature.

Now consider the Ramsey tax rates. We proceed under the conjecture (subsequently verified) that they are optimally chosen to be a constant, τ^*, across dates and states. We use the fact, established in the previous section, that the constraint, equation (4.9), is binding. Two constant tax rates solve equation (4.9) evaluated as a strict equality. Given preferences, equation (4.8), we go with the lower one, $\tau^* = 0.19$, after rounding. To verify this solves the Ramsey problem, we must confirm that equation (4.10) is satisfied. Indeed it is, with $\tau^*(1-\tau^*) - g^h = 0.0008$ and $\tau^*(1-\tau^*) - g^l = 0.10$.

Solving the first expression in equation (4.6), we find that $B' = 0.05$. In addition, we find from the second two expressions in equation (4.6) that $P^h = 64.67$ and $P^l = 0.49$. Essentially, the government reneges on the debt in period h and pays an attractive 100 percent rate of return in state l. Finally, the utility of this equilibrium is –0.0674. These results, plus the consumption and labor allocations, are summarized in table 1. By issuing debt, it is possible to stabilize employment and consumption across dates. By issuing the debt in nominal terms and allowing the price level to fluctuate, it is possible to make the payoff on that debt state-contingent in real terms.

Summary

We have described a model in which an efficient fiscal program issues nominal debt and then allows the price level to fluctuate. Although we demonstrated this finding in an economy with no government-provided money, this feature of our model plays no fundamental role in the result. The same result was obtained by Chari, Christiano, and Kehoe (1991) and by Woodford (1998a) using models with money.

In our model, the equilibrium is equivalent to one in which the government issues debt whose payoff is denominated in real terms in the first period, and where the payoff is explicitly contingent on the realization of government spending in the second period.[65] From this perspective, the natural question is, why not use the state-contingent-debt strategy, rather than going to the trouble of issuing nominal debt and allowing the state contingency to arise because of fluctuations in the price level?

65 Lucas and Stokey (1983) emphasize the desirability of this type of debt.

To address this question, we must invoke considerations that are not included in the model. One advantage of the nominal-debt strategy is that it is likely to have lower costs of administration and information acquisition, because the appropriate response of the real payoff on the debt to shocks is achieved automatically as a by-product of price fluctuations generated in the market-clearing process (Sims [1999] and Woodford [1998a]).

This may provide an overly optimistic view of the nominal-debt strategy. For example, if there are sticky prices, then fluctuations in the price level could distort resource allocations. In addition, price volatility may interfere with private contracts by inducing reallocations of wealth among private agents. Presumably, a version of the Ramsey problem that incorporates those costs would still exhibit price fluctuations, though they would likely be smaller.[66] Designing a fiscal system that properly balances benefits and costs would presumably be very difficult, reducing the cost advantages of the nominal-debt strategy we allude to above.

There is another reason to question the advantages of both the nominal and real state-contingent-debt strategies. Unless the government has substantial ability to commit to its policies, either strategy could backfire, a possibility that can be seen in the example. It is efficient in the first period for the government to inflate away the debt. But when time moves forward one period, the second period becomes the first period. When that time arrives, it is again in the government's interest to inflate away the debt! Households that understand this in the first period may well choose not to accumulate debt in the first place.[67]

Now, this case was excluded in our analysis because of the assumption that policy is non-Ricardian: The policy is just a sequence of numbers (tax rates) through time, and the possibility of adjusting them ex post is ruled out. Is this a realistic assumption? Does it assume that governments have more commitment power than they actually have? The literature on Ramsey policy has generally concluded the answer is yes, and has moved on to equilibrium concepts that do not presume as much commitment power.[68]

In principle, one can make the case that the degree of commitment needed for the policy to work is not implausibly large. This might be so if the required price fluctuations occurred automatically, in a way that legislatures have difficulty interfering with. For example, Judd (1989) suggests that price movements in the U.S. economy correspond roughly to the requirements of an efficient fiscal program. He notes that good shocks to the government budget constraint, such as technology shocks, tend to produce a negative shock to the price level, generating transfers to holders of government bonds. Similarly, bad shocks, like a jump in government spending due to war or natural disaster, tend to drive the price level up, taxing government bond holders.

Our point is *not* that the degree of commitment required for the volatile price strategy is necessarily too great. Our point is only that commitment is a fundamental assumption of the volatile price strategy. In the absence of commitment, the strategy is likely to backfire.

■ **66** It would be interesting to investigate this question in quantitative models. There is a possibility that the efficient degree of volatility in prices would be reduced to zero if price volatility introduced distortions. Chari, Christiano, and Kehoe (1991) argue that, in principle, there are many ways to achieve state contingency in fiscal policy. If there were costs to using the price level for this, then the efficient thing to do would be to use another way. Only if there were costs associated with all ways of achieving state contingency in fiscal policy would some volatility in prices be desirable.

■ **67** Sims (1999) considers a proposal that Mexico adopt the U.S. dollar as its national currency. He criticizes the proposal on the grounds that, with the Mexican national debt denominated in a foreign currency, the Mexican government loses the fiscal benefits of the policy described in the text. That is, it would not be able to periodically renege on and subsidize holders of its debt through fluctuations in the Mexican price level. Our point here is that giving up this option may not be very costly to Mexico, if the Mexican government lacks credibility. Indeed, giving up the option may be a good thing. In the absence of credibility, attempts to use the option may lead to the disastrous situation in which everyone refuses to buy Mexican government debt.

■ **68** Research on optimal policy that presumes a lack of commitment includes Chari and Kehoe (1990) and Stokey (1991). In contrast, Lucas (1990) argues forcefully in favor of implementing Ramsey-optimal capital taxation, a theme continued in Atkeson, Chari, and Kehoe (1999).