III. Fiscal Theory in General Equilibrium

Here we address issues that could not be addressed in the one-period example. The first issue will likely concern any reader who has made it this far. Part II showed how an equation that is not usually used in the context of price determination—the government's intertemporal budget equation—can pin down the price level. But don't we already have a theory of the price level? If we adopt the non-Ricardian assumption on policy, won't the price level be *overdetermined?* It might be, depending on how we flesh out the rest of the economy. If the price level were overdetermined, there would be no equilibrium, except in the fortuitous case in which the government happens to pick just the right value for s. If this were the situation for all reasonable ways of modeling the rest of the economy, the FTPL would be in trouble: It would not be a logically coherent macroeconomic model. But this is not the case. In the following sections, we flesh out the economy in what appears to be a reasonable way, and we find the price level is uniquely determined. (This issue is addressed more rigorously in the appendix.)

We then turn to an issue of greater concern. We present evidence suggesting that to use the FTPL, one must take the non-Ricardian assumption *very* seriously. Seemingly minute departures from that assumption—in the direction of allowing for some sensitivity in the surplus to the real debt—collapses the FTPL's ability to pin down the price level.

The final section revisits the central bank's ability to control average inflation under the FTPL. It shows that conventional views about how to control average inflation could actually spark an explosive hyperinflation under the FTPL. So, although the central bank can feasibly control average inflation, its method of doing so must be designed with care.

Is the Price Level Overdetermined in the FTPL?

We begin this section by providing a general discussion of the issues involved in determining the price level. We then turn to a specific example in which the price level is uniquely determined by the FTPL.

General Discussion

It is easy to find examples of the FTPL in which the price level is overdetermined. Recall the equation of exchange, discussed previously. For convenience, we reproduce it here: $MV = PY$. In traditional, old-fashioned monetarism, V is assumed to be fixed by technology, Y is determined exogenously, and monetary policy takes the form of a choice of M. In this model, P is obviously determined by the equation of exchange. If the rest of the economy were characterized by these assumptions, then a logically coherent FTPL would be impossible.[32]

Each of these assumptions, however, has been rejected on empirical grounds. First, V exhibits substantial fluctuations in the data; the assumption that V is fixed is replaced in modern models by the assumption that V is an increasing function of the nominal rate of interest. Additionally, expected inflation plays an important role in determining R. With these two features, a logically coherent FTPL is possible. These changes cause expected future values of P to enter the equation of exchange through V. This creates the possibility that there are many P processes that can satisfy the equation, leaving room for the non-Ricardian assumption to pin down one of them. This possibility is illustrated through an example in the appendix.[33]

Second, the assumption that Y is exogenous has been questioned. There is general agreement that at least short-run movements in Y are influenced by movements in V, P, and M. When models are constructed to capture this, expected future values of P enter into the determination of Y. As in the example of the

■ **32** Finite-horizon models in which the simple quantity equation holds in the last period, but in which velocity is an increasing function of the nominal interest rate in the previous periods also have the property that the price level is overdetermined (see Buiter [1999]). The reasoning is similar to that used in our text. In those models, the equilibrium price level must satisfy a first-order difference equation. All equilibrium prices are then pinned down by the fact that the price level is pinned down in the last period. The likelihood that this price level coincides with the one produced by the intertemporal budget equation of the government seems small. The most likely outcome is that the price level will be overdetermined.

■ **33** Brock (1975), Obstfeld and Rogoff (1983), and Matsuyama (1991) present similar examples. An example taken from Woodford (1994) can be found in the appendix.

previous paragraph, there can be multiple P processes that satisfy the equation of exchange. Again, this leaves room for the non-Ricardian assumption to pin down one of them.[34]

Third, there is a nearly universal consensus that exogenous M poorly characterizes monetary policy. For example, Taylor (1993) has argued that, in practice, monetary policy is best thought of as a rule for setting the rate of interest. In this case, M becomes an endogenous variable. We can see in the equation of exchange that if R is the exogenous policy variable (as opposed to M), then V is pinned down. But now there are two endogenous variables, M and P, in this equation. Generally, under these circumstances, P and M are not pinned down. There is, in a sense, a missing equation. Again, there is room for the FTPL to fit in.

An Example

Next, we present a simple, multiperiod model economy in which the price level is uniquely determined in the FTPL. There is no last period, and time is indexed by $t = 0, 1, 2, ...$ Suppose that output, Y, is the same for each date, t. Money demand depends on the rate of interest,

$$(3.1) \quad \frac{M_t}{P_t} = AR_t^{-\alpha},\ \alpha > 0.$$

The parameter A captures other factors (like income) that affect money demand but are assumed to be constant here. M_t is the money stock at the beginning of period t; P_t is the price level during period t; and R_t is the nominal rate of interest on government bonds held from the beginning of period t to the beginning of period $t+1$. The Fisher equation holds

$$(3.2) \quad 1+r = (1+R_t)\frac{P_t}{P_{t+1}}.$$

The expression on the right $(1+r)$ is the real rate of interest on bonds paying a nominal rate of return, R_t, and $r > 0$ is the rate at which households discount future utility. This pins down the real rate of interest.

A reasonable specification of monetary policy is that the central bank targets the nominal rate of interest. For purposes of exposition, we adopt an extreme version of this specification, in which the central bank pegs the rate of interest to a constant, $R > 0$. The central bank accomplishes this by supplying whatever amount of money the private economy demands at this rate of interest.

The interest rate peg pins down seignorage:

$$s_t^m \equiv \frac{M_t - M_{t-1}}{P_t} = \frac{M_t}{P_t} - \frac{P_{t-1}}{P_t}\frac{M_{t-1}}{P_{t-1}}.$$

Imposing the money-demand and Fisher equations [(3.1) and (3.2)] and the policy rule $R_t=R$, we find[35]

$$(3.3) \quad s^m = AR^{-\alpha}\ \frac{R-r}{1+R},\ t = 0,1,2, ...$$

Consistent with the FTPL, we assume the primary budget surplus, s_t^f, is non-Ricardian. We adopt the simplest such policy, one in which s_t^f is simply a constant, s^f. Thus, net government revenues from all sources, excluding interest payments, are given by

$$(3.4) \quad s_t = s = s^f + s^m > 0.$$

To complete the description of the government, we present the period-t budget constraint. We assume government debt is composed of one-period discount bonds; that is, the amount of borrowing in period t is $B_{t+1}/(1+R)$, and the amount paid in period $t+1$ is B_{t+1}. The period-t government budget constraint is

$$(3.5) \quad \frac{B_{t+1}}{1+R} + P_t s = B_t,\ t = 0, 1, 2, ...$$

The terms on the left of the equality represent the government's sources of funds, and the terms on the right denote the uses of funds to pay off the debt.[36] It is convenient to rewrite this expression in real terms, that is, in terms of $b_t \equiv B_t/P_t$. Dividing equation (3.5) by P_t, taking the Fisher equation (3.2) into account, and rearranging the terms, we obtain

$$(3.6) \quad b_{t+1} = (1+r)(b_t - s).$$

34 A cash-in-advance model displayed in Christiano, Eichenbaum, and Evans' (1998) illustrates this. Because this is a cash-in-advance model, velocity is fixed and the factors discussed in the previous paragraph are ruled out. Chistiano, Eichenbaum, and Evans show that for different specifications of the monetary policy rule for selecting M_t, the model has a continuum of equilibria. If the non-Ricardian assumption were adopted in this model, the equilibrium would be pinned down. For other examples like this, see Carlstrom and Fuerst (1998).

35 For simplicity, we assume the interest rate peg was in place in period –1, too. A more rigorous treatment which does not make this assumption can be found in the appendix.

36 An alternative representation, which has some theoretical advantages, expresses the government's budget equation in terms of its total nominal liabilities, $B_t + M_{t-1}$. We work with this alternative representation in the appendix.

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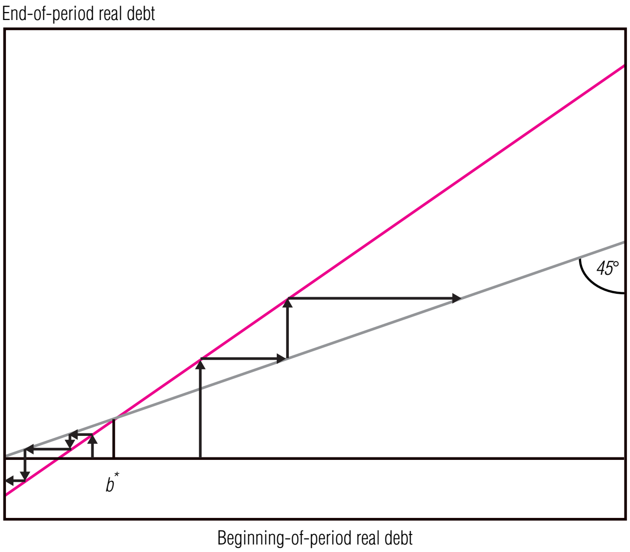
FIGURE 1

Debt Evolution Under Non-Ricardian Fiscal Policy



Finally, we develop the multiperiod analog of $B'=0$ in part II. Recall the logic we used there: First, $B'>0$ is not optimal, since households could increase utility by raising consumption and financing it with a reduction of B'. A negative value of B' is also not optimal, since we have removed it from the feasible set by assumption. We continue to assume that holdings of government bonds must be non-negative; that is, households only lend to the government, they do not borrow from it.

The analog of $B'=0$ in this setting is

$$(3.7) \quad \lim_{T\to\infty} \frac{B_T}{(1+R)^T} = 0.$$

We establish that household optimization implies this condition by the same reasoning used to establish $B'=0$. The limit cannot be positive, for otherwise households could increase utility by reducing their holdings of government debt. To see this, suppose the limit *is* positive. Eventually, government debt would grow at the rate of interest, that is,

$B_t = B_{t^*}(1+R)^{t-t^*}$, $t \geq t^*$ for some t^*.

At this point, the government is engaged in what is called a Ponzi scheme with households. The principal and interest on debt coming due are financed entirely and forever with newly issued debt. Under these circumstances, households can do better by saying no to the Ponzi game, consuming the principal and interest on debt coming due in one period and then never holding any more government debt. The household is better off, because the action allows a one-time increase in consumption without the need to reduce consumption at any other date. An optimizing household would not pass up an opportunity like this; therefore, household optimization implies the limit cannot be positive. But the limit cannot be negative either, because $B_t<0$ is not allowed. Equation (3.7) is called the *transversality condition*. It is convenient for us to express this condition in real terms, after substituting out for the nominal rate of interest from the Fisher equation (3.2). Using that equation, we find[37]

$$(1+R)^t = (1+r)^t \frac{P_t}{P_0}, \; t=1, 2, \ldots,$$

so that $B_T/(1+R)^T = P_0 b_T/(1+r)^T$. The transversality condition can then be written as

$$(3.8) \quad \lim_{T\to\infty} \frac{b_T}{(1+r)^T} = 0, \; b_T = \frac{B_T}{P_T}.$$

We have now stated the entire model. The household's part is given by equations (3.1), (3.2), and (3.8) and by the condition $B_t \geq 0$. The government is summarized by its policy, equation (3.4), and by its flow-budget constraint, equation (3.6). Does this economy uniquely determine the price level? To see that it does, first note that the money-demand equation and the government's policy of pegging the interest rate have the effect of pinning down real balances, but not M or P separately. Double M and P, and those equations remain satisfied. The same is true of the Fisher equation: Double P at all dates, and it continues to hold, too. So, the level of the money stock and the price level are not pinned down. It turns out that the non-Ricardian specification of government policy, together with the household's transversality condition, is sufficient to pin down the price level uniquely.

To see that the price level is uniquely determined, consider figure 1, which illustrates the government budget equation, $b' = (1+r)(b-s)$. The vertical axis measures b' and the horizontal axis measures b. (The 45-degree line is included in the figure for convenience.) The intercept for the budget equation is negative, and it cuts the 45-degree line from below. Its slope is steeper than 45 degrees because we assume $r>0$.

37 For example, for $t=2$,

$$(1+R)^2 = \left[(1+R)\frac{P_1}{P_2}\right]\left[(1+R)\frac{P_0}{P_1}\right]\frac{P_2}{P_0} = (1+r)^2\frac{P_2}{P_0}.$$

Figure 1 shows what happens to b over time for any initial value of b.[38] Denote the value of b where the budget equation intersects the 45-degree line by b^*,

$$(3.9)\quad b^* = \frac{1+r}{r}\, s = \sum_{t=0}^{\infty} \frac{s}{(1+r)^t}\,.$$

As the last equality indicates, b^* is the present value of future surpluses.

The value of b in period 0, b_0, is now an endogenous variable. Although the nominal debt, B_0, is predetermined at date 0, the price level is not. Consider three possibilities: Suppose $0 \leq b_0 < b^*$. Figure 1 indicates that b quickly spirals into the negative zone, violating the non-negativity constraint on the household's holdings of debt.[39] Next consider $b_0 > b^*$. In this case, figure 1 indicates the debt diverges to plus infinity. To see how the debt's growth rate evolves, divide equation (3.6) by b_t:

$$\frac{b_{t+1}}{b_t} = (1+r)\left(1 - \frac{s}{b_t}\right).$$

As b_t grows, s becomes relatively small, and the growth rate of b_t eventually converges to $1+r$. At this point, the debt becomes so large that s is, by comparison, insignificant. The government is now running a Ponzi scheme. For the reasons we have given above, it is not in the household's interest to participate in this scheme (technically, the household's transversality condition, equation [3.8], is violated). Since households will not hold this debt, we conclude that all $b_0 > b^*$ do not correspond to equilibria.

This leaves only $b_0 = b^*$ to consider. Since the level of real debt is fixed in this case, the transversality condition is now trivially satisfied. Thus, only $P_0 = B_0 / b^*$ is consistent with equilibrium. We conclude this version of the FTPL is an internally consistent theory of the price level.

Is the FTPL Fragile?

The assumptions underlying economists' theories are, at best, only approximations. We don't think of them as being *exactly* true. Therefore, we trust theories more if their central implications do not change when we alter the assumptions a little. But, if key implications evaporate with small changes—particularly changes that are arguably in the direction of greater empirical plausibility—then there is reason for concern. In this case, we say a theory is *fragile*.

Here, we describe one concern about the fragility of the FTPL, based on Canzoneri, Cumby, and Diba (1998). We show that small, plausible perturbations of non-Ricardian policy collapse the FTPL's ability to pin down the price level.[40] Consider the following alternative to the canonical non-Ricardian policy of setting s to a constant. Suppose $s = \varepsilon b$, where $0 < \varepsilon \leq 1$. With this policy, $b' = (1+r)(1-\varepsilon)b$, or

$$\frac{b_t}{(1+r)^t} = (1-\varepsilon)^t \frac{B_0}{P_0},$$

so that the transversality condition is satisfied for all $P_0 > 0$. Clearly, this is a Ricardian policy; the FTPL does not pin down the price level. Now, this policy may appear to be a significant perturbation of the policy $s_t = s$. Perhaps so, but it has close cousins in which the perturbation appears to be much smaller.

Consider the following alternative to the canonical non-Ricardian policy:

$$(3.10)\quad s_t = \begin{cases} -\dfrac{\xi}{1+r} + \dfrac{1+r-\gamma}{1+r}\, b_t & b_t > \bar{b}\,, \\[2ex] s & b_t \leq \bar{b} \end{cases}$$

where

$$0 \leq \gamma < 1,$$

$$\frac{1+r}{r}\, s < \bar{b} < \frac{\xi}{1-\gamma}\,.$$

In this case, as long as real debt remains below some upper bound, $\bar{b}$, then the policy is the constant-surplus policy that we have been analyzing. But, as soon as b_t exceeds $\bar{b}$, fiscal policy adjusts to bring the debt back in line.

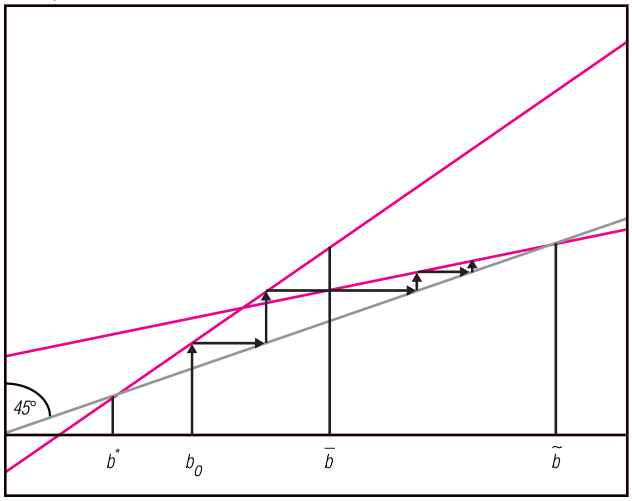
■ **38** To see this, specify an initial value for b on the horizontal axis. Proceed vertically to the budget line, move horizontally to the 45-degree line, move vertically to the budget line, and so on.

■ **39** Implicitly, we have ruled out the possibility that a negative b implies a negative P. This cannot be, since P_0 is positive for $0 \leq b_0 < b^*$. The Fisher equation (3.2) then pins down the price path for P_t and cannot produce a negative P_t if $P_0 > 0$.

■ **40** Woodford (1998a) argues there may be a local sense in which the FTPL's ability to pin down the price level survives the sort of perturbations we consider here. He also discusses versions of the model with learning which may survive these perturbations.

FIGURE 2

Debt Evolution Under Perturbed Fiscal Policy



Although the algebraic representation of this policy may seem forbidding, it is easy to analyze it with the help of figure 2. For $b_t > \bar{b}$, real debt evolves according to $b_{t+1} = \gamma b_t + \alpha$.

If real debt followed this equation forever, it would eventually converge to $\tilde{b} = \xi/(1-\gamma)$. This equation, as well as equation (3.6), is graphed in figure 2.

Figure 2 simulates the evolution of real debt under the fiscal policy in equation (3.10). The simulation is initiated with the indicated value of b_0. Real debt initially follows the steep line with slope $1+r>1$ until it passes $\bar{b}$, at which point it follows the flatter line with slope $\gamma<1$. All paths with $b_0 \geq b^*$ are consistent with the transversality condition because they converge to a finite value, either b^* or $\tilde{b}$. With the given change in policy, the FTPL cannot pin down the price level.

This perturbation of the non-Ricardian policy seems realistic. At low levels of debt, fiscal policy is exogenous, as it is in the canonical non-Ricardian policy. If the debt gets out of line, then fiscal policy adjusts to bring it under control. This rings true in light of the U.S. experience in the 1980s and 1990s and the provisions of the Maastricht Treaty, which limit the real debts of European Union member countries.

The FTPL with Stochastic Fiscal Policy

Thus far, we have illustrated non-Ricardian fiscal policy with $s_t = s$, a constant. But the essence of non-Ricardian fiscal policy is simply that s_t is *not* calibrated to satisfy the intertemporal budget equation for all prices; it is compatible with a much larger class of specifications for s_t than $s_t = s$. Here, we study non-Ricardian policies in which surpluses, s_t, are random. We use this specification to make three points.

First, Barro's (1979) famous policy of absorbing fiscal shocks by raising taxes in the future can be represented as non-Ricardian fiscal policy.[41] This is an important example, partly because it clarifies the definition of non-Ricardian policy as it is used in the FTPL. Clarification is necessary because one might mistakenly attach other meanings to the term "non-Ricardian," based on economists' everyday usage of the term "Ricardian."

Second, unless policy takes the form advocated by Barro, fiscal shocks cause the inflation rate to fluctuate randomly about its average. The average value of inflation is determined by the value of the monetary authority's interest rate peg.

Third, we describe an important result from Woodford (1996, 1998a). He shows that under the FTPL, instability in fiscal policy *must* affect the price level, no matter how committed the monetary authority is to price stability.[42] We call this striking result "Woodford's *really* unpleasant arithmetic," to contrast it with Sargent and Wallace's famous title. Woodford's arithmetic is even tougher than that of Sargent and Wallace, who argue that if the central bank is weak, then the fiscal authority can push it into producing price instability. However, Sargent and Wallace's pessimistic ("unpleasant") conclusion is balanced by their optimism that, if the central bank just hangs tough, the problem of price stability will be solved. From the perspective of the FTPL, Woodford argues that no matter how tough the central bank is, it still cannot stabilize the price level.[43]

■ **41** It seems obvious that the Barro policy can also be represented as Ricardian, so we do not discuss it here.

■ **42** Implicitly, we have in mind non-Ricardian policies other than those advocated by Barro (1979).

■ **43** Recall, however, our second point, that the central bank *can* control the average inflation rate.

Random Fiscal Policy

Suppose the surplus obeys the first-order autoregressive representation,

$$(3.11)\quad s_{t+1} = (1-\rho)s + \rho s_t + \varepsilon_{t+1}.$$

In this equation, ε_{t+1} is an independently and identically distributed white-noise process independent of $s_{t-j}, j \geq 0$. A positive realization of ε_t induces a change in the date-t government surplus and in the expected value of future government surpluses. Let ψ_j denote this effect at date $t+j$ for $j \geq 0$:

$$(3.12)\quad \psi_j \varepsilon_t = E_t s_{t+j} - E_{t-1} s_{t+j},\ j \geq 0,\ \psi_0 \equiv 1.$$

E_t denotes the expectation operator, conditional on information available at date t ($E_t s_t = s_t$). When the surplus has the time-series representation, equation (3.11), then $\psi_j = \rho^j$. Of course, equation (3.12) applies more generally, even when s_t does not have the time-series representation given in equation (3.11). The present value of ε_t's impact on current and expected future surpluses can be defined as

$$\psi\left(\frac{1}{1+r}\right)\varepsilon_t \equiv \varepsilon_t + \frac{\psi_1}{1+r}\varepsilon_t + \frac{\psi_2}{(1+r)^2}\varepsilon_t + \ldots.$$

(Note here that $\psi(\cdot)$ is a function.) In the case of equation (3.11), this is

$$\psi\left(\frac{1}{1+r}\right) = \frac{1+r}{1+r-\rho}.$$

When $\rho = 0$, so that s_t is independently and identically distributed, then the present-value term is just unity. In this case, the effect of an innovation in the surplus is limited to the current surplus only. As ρ increases above zero, then the present-value term increases to take into account the future effects of innovation. Negative values of ρ cause the present-value terms to fall as innovations in the current surplus generate expected reductions in the future surplus.

It is interesting to compare the fiscal policy considered in equation (3.11) with that advocated by Barro (1979). He argues that a negative shock to government finances (due, for instance, to war) should be met by a large increase in debt, coupled with a constant increase in the labor tax rate that is sufficient to pay off the interest and principal on that debt over time. In particular, he advocates fiscal policies of the form

$$\psi\left(\frac{1}{1+r}\right) = 0.$$

For example,[44]

$$\psi_0 = 1,\ \psi_1 = -(1+r), \text{ or,}$$

$$\psi_0 = 1,\ \psi_i = -\left(\frac{1+r}{2}\right)^i, i \geq 1;$$

that is,

$$s_t = s + \varepsilon_t - (1+r)\varepsilon_{t-1}, \text{ or,}$$

$$s_t = s + \varepsilon_t - \sum_{i=1}^{\infty}\left(\frac{1+r}{2}\right)^i \varepsilon_{t-i}.$$

These examples head off misunderstandings about the definition of "non-Ricardian" policy. In everyday discussion, the word "Ricardian" is used in a variety of senses. For instance, economists may refer to a policy as Ricardian when a current tax cut is financed by increases in future taxes that are large enough in present value to match the current cut.[45] It is clear from the preceding discussion that this type of policy can be part of a non-Ricardian regime.

Under the fiscal policy just discussed, the price level is insulated from fiscal shocks. Shocks to the real primary surplus are financed by appropriate movements in the opposite direction later. In the next section, we will show that when $\psi \neq 0$, surplus shocks are at least partially financed by movements in the price level.

■ **44** See, for example, Woodford (1998a), footnote 18.

■ **45** Hayashi (1989) is one example.

Inflation with Random Fiscal Policy

We continue to assume that policy pegs $R_t = R$, so that the seignorage component of s_t is the constant value given in equation (3.3). As a result, the random nature of s_t in equation (3.11) reflects randomness in fiscal policy. The Fisher equation still holds, although it must be adjusted to take into account uncertainty,

$$1+r=(1+R)E_t\frac{P_t}{P_{t+1}},$$

where E_t is the conditional expectation, given information available at time t. This expression shows the central bank controls the expected rate of deflation through its choice of R. This translates into control over the average rate of deflation by the fact $E[E_t(P_t/P_{t+1})] = E(P_t/P_{t+1})$. Imposing the suitably adjusted version of the household's transversality condition, equation (3.8), on the government's flow-budget equation, the intertemporal budget equation becomes[46]

$$(3.13)\quad \frac{B_t}{P_t}=E_t\sum_{j=0}^{\infty}\frac{s_{t+j}}{(1+r)^j}=s\left(\frac{1+r}{r}\right)\left(\frac{1-\rho}{1+r-\rho}\right)+\left(\frac{1+r}{1+r-\rho}\right)s_t .$$

We now have a completely specified theory of the price level and inflation. One way to understand it is to use the model to simulate a sequence of prices for a given realization of primary surpluses. Suppose we have a time series, $s_0, s_1, ..., s_T$, from equation (3.11) and an initial level of nominal debt, B_0.[47] P_0 is computed by evaluating equation (3.13) at $t=0$. B_1 is then computed from the government's flow-budget equation, $B_{t+1}=(1+R)(B_t-P_t s_t)$, for $t=0$. A sequence, $P_0, P_1, ..., P_T$, is obtained by performing these calculations in sequence for $t=0,1,2, ..., T$.

The interest rate peg guarantees that the expected rate of inflation (actually, deflation) is constant in these simulations. As a result, the rate of inflation itself will be approximately uncorrelated over time, an artifact of the constant interest rate peg. If the interest rate rule were instead dependent on past interest rates and/or past inflation, then persistence would presumably appear in the model's inflation process.[48]

One can gain further insight into equation (3.13) by subtracting $E_{t-1}B_t/P_t$:

$$(3.14)\quad \frac{B_t}{P_t}-E_{t-1}\left(\frac{B_t}{P_t}\right)=\left(\frac{1+r}{1+r-\rho}\right)(s_t-\mathrm{E}_{t-1}s_t)$$

$$=\psi\left(\frac{1}{1+r}\right)\varepsilon_t .$$

This says that a date-t shock in the primary surplus induces a contemporaneous change in the real value of the debt equal to the present value of the shock.[49] Since B_t is predetermined at time t, the change is brought about entirely by a change in the price level.[50]

■ **46** To see how this is derived, consider first the expression to the right of the first equality in equation (3.13). Note from the Fisher equation (3.2):

$$\frac{1}{P_{t+j}(1+R_{t+j})}=E_{t+j}\left[\frac{1}{(1+r)P_{t+j+1}}\right],\text{ all } t,j\geq 0.$$

Then, the government's flow-budget constraint can be written

$$\frac{B_{t+j}}{P_{t+j}}=s_{t+j}+\frac{B_{t+j+1}}{P_{t+j}(1+R_{t+j})}=s_{t+j}+\frac{1}{1+r}=E_{t+j}\frac{B_{t+j+1}}{P_{t+j+1}},$$

or, after applying the law of iterated mathematical expectations,

$$E_t\frac{B_{t+j}}{P_{t+j}}=E_t s_{t+j}+\frac{1}{1+r}E_t\frac{B_{t+j+1}}{P_{t+j+1}}\ (^{**}).$$

Substitute this, for $j=1$, into the period-t flow-budget constraint of the government:

$$\frac{B_t}{P_t}=s_t+\frac{1}{1+r}\frac{B_{t+1}}{P_t(1+R)},$$

$$=s_t+\frac{1}{1+r}+E_t\frac{B_{t+1}}{P_{t+1}}.$$

Applying (**) repeatedly to this expression, for $j=1, 2, ...$ results in the expression to the right of the first equality in equation (3.13), if we apply the transversality condition, $\lim_{T\to\infty} E_0\, b_T/(1+r)^T=0$.

To obtain the expression to the right of the second equality in (3.13), first solve equation (3.11) to find

$$E_t\frac{S_{t+j}}{(1+r)^j}=s\left[\left(\frac{1}{1+r}\right)^j-\left(\frac{\rho}{1+r}\right)^j\right]+\left(\frac{\rho}{1+r}\right)^j s_t,$$

for $j=0, 1, 2, ...$ Then substitute this into equation (3.13) and apply the geometric sum formula.

■ **47** It should be obvious how this procedure could be adapted to accommodate any other time-series representation for s_t.

■ **48** Loyo (1999) emphasizes this in his discussion of the persistent rise in inflation observed in Brazil in the 1980s.

■ **49** The analysis of price determination under the FTPL is similar to the analysis of consumption in the permanent-income hypothesis. See Christiano (1987).

■ **50** Divide both sides of equation (3.14) by B_t and take into account that $E_{t-1}B_t = B_t$ to set

$$\frac{1}{P_t}-E_{t-1}\left(\frac{1}{P_t}\right)=\frac{1}{B_t}\,\Psi\left(\frac{1}{1+r}\right)\varepsilon_t .$$

Fiscal policies like equation (3.14) underscore the fact that movements in the price level are an alternative to Barro's way of financing shocks to the primary surplus. A jump in the price level acts as a capital levy on holders of government bonds, which helps to finance government spending just as surely as the sort of taxes included in the primary surplus. We describe an environment with this type of efficient fiscal policy in part IV.

*Woodford's **Really** Unpleasant Arithmetic*

Woodford's argument—that instability in fiscal policy must affect the price level—is a simple proof by contradiction. Suppose the monetary authority could perfectly stabilize inflation and the price level. This implies $P_{t+1} = P_t$, so that the nominal rate of interest is fixed and equal to the real rate. This, in turn, implies that seignorage, s^m, is zero and, as a result, $s_t = s_t^f$. Now, suppose fiscal policy is stochastic, with $\psi\,[1/(1+r)] \neq 0$. According to equation (3.14), P_t responds to innovations in s_t. But this contradicts our assumption that P_t is constant. It follows that with shocks to fiscal policy, it may not be feasible for the monetary authority to insulate the price level from those shocks.

Bear in mind that the monetary authority *can* control the expected rate of inflation in the FTPL. For Woodford's *really* unpleasant arithmetic to be truly unpleasant, shocks to the realized price level must have socially inefficient consequences. This is not the case in many economic environments, where only the expected inflation rate matters (see, for example, Chari, Christiano, and Kehoe [1991]). Shocks to the realized price level are costly in environments with nominal rigidities and in environments with heterogeneous agents.[51]

The FTPL and the Control of Average Inflation

The previous section described how the monetary authority can control the average rate of inflation by pegging the nominal interest rate to an appropriate value.[52] In policy discussions about inflation, it is sometimes suggested that inflation can be controlled more effectively with an interest rate rule that responds aggressively to inflation. In this section, we show how such a monetary policy could, in fact, lead to disaster if fiscal policy were non-Ricardian.

Suppose the monetary authority adjusts the interest rate according to the rule

$$1+R_t = \alpha_0 + \alpha_1 \pi_t, \;\; \pi_t = P_t / P_{t-1}\,.$$

The monetary authority implements this rule by adjusting the money supply so that money demand is satisfied at the targeted rate of interest. An "aggressive" interest rate rule is one in which α_1 is large. For example, Taylor (1993) has argued that α_1 should be around 1.5. This means that if inflation rises 1 percentage point, then the central bank raises the nominal interest rate by 1.5 percentage points. According to conventional wisdom, an aggressive interest rate rule such as this is a good way to keep inflation under control. As we shall see, this is not necessarily true if policy is non-Ricardian.

We suppose the rest of the economy corresponds to the example in "Is the Price Level Overdetermined in the FTPL?" (page 11). As in that model economy, we assume there is no uncertainty, since it is not essential to the analysis here. Combining the interest rate rule with the Fisher equation (3.2), we obtain the following expression, which must hold in equilibrium:

$$\pi_{t+1} = \frac{\alpha_0}{1+r} + \frac{\alpha_1}{1+r}\,\pi_t\,.$$

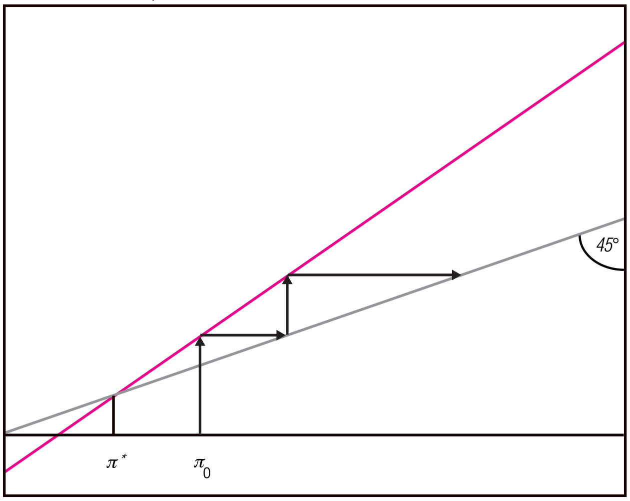
Consider an aggressive interest rate rule with $\alpha_1/(1+r) > 1$. The relationship between π_{t+1} and π_t is illustrated in figure 3. There is a particular inflation rate, π^*, such that if $\pi_t = \pi^*$, then $\pi_{t+1} = \pi^*$. However, if the initial inflation rate is greater than π^*, then π_t grows without

51 See Woodford (1996) for an environment with endogenous production and sticky prices. With sticky prices, shocks to the aggregate price level distort the allocation of resources across the production of different goods. They also distort aggregate output.

52 See "Inflation with Random Fiscal Policy" on page 17.

FIGURE 3

Evolution of Inflation under Aggressive Interest Rate Rule and Non-Ricardian Policy



bound. This possibility is shown in figure 3, in which inflation starts at π_0 in period 0 and then explodes.

As in our previous model economy, the initial price level is determined by fiscal policy according to the intertemporal budget equation (3.9). Technically, s is no longer constant because variations in inflation cause seignorage to vary over time, too. However, we assume that seignorage revenues are small enough to ignore, so that s comprises only s^f. We continue to assume that s^f is constant.[53] The price level in period 0 is determined by $P_0 = B_0/b^*$, where B_0 is the initial nominal debt and b^* is defined in equation (3.9).

With P_0 determined by the intertemporal budget equation and P_{-1} determined by history, π_0 is uniquely pinned down. However, there is no way to rule out the possibility that this value of π_0 lies to the right of π^*, in which case inflation explodes.[54]

One way to gain insight into the mechanics of this exploding inflation is to focus on the government's budget constraint, equation (3.5). From that equation, we see that a higher nominal interest rate leads to a more rapid increase in the nominal debt, B_{t+1}. Assuming the outlook for the fiscal primary surplus does not change, the real value of the debt remains constant. With the nominal debt growing more quickly and its real value constant, inflation must rise.

The central bank's monetary policy responds to the rise in inflation by driving the interest rate up even further, leading to an additional increase in inflation. This circular, self-reinforcing process produces the explosion in inflation.

The possibility just outlined, whereby an aggressive interest rate rule leads to exploding inflation, may seem peculiar. Loyo (1999) refers to it as a "tight money paradox." According to the model, if the central bank, instead of being aggressive, adopts a more accommodating stance by choosing a value of α_1 substantially less than unity, then exploding inflation cannot occur. In the previous section, with $\alpha_1 = 0$, inflation fluctuated around a constant value. It is easy to confirm, using the logic of figure 3, that the same is true for $0 < \frac{\alpha_1}{1+r} < 1$. Relative to a simple monetarist perspective, it is certainly a paradox that adopting an aggressive stance against inflation by increasing α_1 could convert stable inflation into an exploding inflation.[55] However, we have just seen that it can occur in a coherent economic model. Moreover, Loyo argues the model captures the driving forces behind Brazil's inflation take-off in the early 1980s. Although we are skeptical that tough monetary policy caused Brazil's high inflation, the hypothesis certainly does seem intriguing.

Woodford (1998b, pp. 399–400) uses the exploding-inflation scenario to understand the nature of fiscal policy in the United States over the past two decades. He observes that econometric estimates of the Federal Reserve's policy rule in the 1980s and 1990s place α_1 substantially above unity (see Clarida, Gali, and Gertler [1998]), and there is no evidence of instability in U.S. inflation. He concludes that policy in the United States during this time must not have been non-Ricardian.

■ **53** Here we are assuming the economy is in the "cashless limit" discussed by Woodford (1998 a,b,c) and defined in footnote 30 of the present paper.

■ **54** In this situation, both fiscal policy and monetary policy are active in the sense defined by Leeper (1991). Our analysis is consistent with Leeper's, which concludes that for almost all values of fiscal policy (s^f), there is no stationary equilibrium inflation rate.

■ **55** Tight money paradoxes also exist in environments with Ricardian fiscal policy. For example, Sargent and Wallace (1981) showed tight monetary policy may lead to an immediate rise in inflation in such an environment. See Kocherlakota and Phelan (1999) for a discussion of a tight money paradox in the FTPL.