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The Fiscal Theory of the Price Level

by Charles T. Carlstrom and Timothy S. Fuerst

Charles T. Carlstrom is an economist at the Federal Reserve Bank of Cleveland, and Timothy S. Fuerst is an associate professor of economics at Bowling Green State University and a visiting scholar at the Bank. The authors would like to thank Larry Christiano and Terry Fitzgerald for helpful comments.

Introduction

A traditional function of the central bank is to control the price level. This function is a natural implication of economic theory: The celebrated quantity theory of money can be summarized in Milton Friedman's dictum that "inflation is always and everywhere a *monetary* phenomenon." As reviewed in Robert Lucas' Nobel lecture (1996), there is a wealth of empirical evidence linking price movements to movements in the money stock.

This traditional analysis has been challenged by the *fiscal theory of the price level (FT),* which maintains that the price level is determined by the budgetary policies of the fiscal authority. This attack on the conventional position has come in two parts, *weak-form FT* and *strong-form FT.*

Weak-form FT begins with an obvious link between monetary and fiscal policy. Since seignorage (revenue from money creation) is a possible revenue source, long-run monetary and fiscal policy are jointly determined by fiscal budget constraints. Whether monetary or fiscal policy determines prices involves an assumption about which policymaker will move first, the central bank or the fiscal authority. Weak-form FT assumes that the fiscal authority moves first by committing to a path for primary budget surpluses/deficits, forcing the monetary authority to generate the seignorage needed to maintain solvency. Sargent (1986) describes this as a "game of chicken."

If both the monetary and the fiscal authority refuse to generate the needed seignorage, then the nation's debt-to-GDP ratio will grow at an unsustainable rate. This implies ever-increasing real rates of interest on government debt, as the market demands larger and larger default premiums. This process cannot continue: One of the two players, the fiscal authority or the central bank, must alter its behavior. Weak-form FT assumes the central bank will respond and generate the seignorage needed to avoid default. Using the game-of-chicken analogy, weak-form FT assumes that the monetary authority loses and is forced to "blink."

This version of the fiscal theory predicts that fiscal policy determines future inflation as well. Although this is true, it does so only by determining future money growth. The traditional version of the FT, therefore, is not at odds with the quantity theory, in the sense that prices are still driven by current or future money growth.

Sargent and Wallace's (1981) celebrated example, in which tight money today increases the price level, occurs because future money growth—and hence future inflation—increases. The theory simply posits that the ultimate driver of the money supply is the fiscal authority. In other words, fiscal policy is exogenous, while money supply movements are endogenous.

More recently, a stronger version of the fiscal theory has been posited. Strong-form FT maintains that fiscal policy determines future inflation, but independent of future money growth. Unlike the weak theory, where inflation is still (ultimately) a monetary phenomenon, strong-form FT maintains that fiscal policy affects the price level and the path of inflation independent of monetary policy changes.

This new version of the fiscal theory is possible because, in a wide variety of monetary models, the initial price level is not pinned down; different initial price levels are consistent with different paths for future inflation. In contrast, prices are uniquely determined in the weak form of the FT analyzed by Sargent and Wallace. Strong-form FT assumes that the fiscal budget constraint, and thus fiscal policy, pins down the initial price level. Without this constraint, the initial price level may be indeterminate, even if the money supply is given exogenously—that is, even if the monetary authority moves first by committing to a path for the money stock. This is in sharp contrast with weak-form FT, in which the money supply is endogenous in order to satisfy the government's budget constraint. Strong-form FT assumes that both fiscal and monetary policy are given exogenously and that prices adjust to ensure solvency. In this game of chicken, neither player blinks.

This article begins with a discussion of weak-form FT by reviewing basic budgetary arithmetic and its implications for monetary policy. In particular, the "unpleasant arithmetic" of Sargent and Wallace (1981) is presented. This paper is a natural place to begin, as it provides a powerful demonstration of the restrictions that the government budget may place on monetary policy. Section I analyzes a partial-equilibrium model where real cash balances immediately jump to their steady state—that is, equilibria in which the level of real cash balances remains constant. Section II broadens the analysis to a more fully specified general-equilibrium model, allowing for consideration of equilibria where the level of real balances varies with time and for consideration of strong-form FT. Section III extends the discussion to models in which the central bank targets the interest rate and in which the money supply is endogenous, asking whether this case is an example of weak- or strong-form FT. Section IV presents our conclusions.

I. Weak-Form FT: A Partial-Equilibrium Analysis

This section will present some basic results of the budgetary linkages between monetary and fiscal policy. For illustrative purposes, we assume that the real rate of interest (denoted by r) and the real level of output (normalized to one) are constant. We also assume a form for money demand instead of deducing it from a more completely specified environment. These partial-equilibrium simplifications limit our discussion to steady-state equilibria, where real cash balances immediately jump to their steady-state (constant) level and remain there forever. Since money growth is equal to the inflation rate in such an equilibrium, this partial-equilibrium model can give rise only to weak-form FT. In later sections we extend this analysis to a general-equilibrium model, where this is not necessarily true, so that either weak- or strong-form FT can arise.

Equilibrium is defined by two conditions, fiscal budgetary balance and money-market equilibrium. Money-market equilibrium (real money supply = real money demand) is defined by

(1) $$M_0/P_0 = f(R),$$

where money demand (f) is a function of the nominal interest rate ($R = r + \pi$) and π is the inflation rate. Money demand is a function of inflation only because the real interest rate and output are both assumed to be constant. M_0 is the nominal money stock during the first period of the model, and P_0 is the corresponding nominal price level.

Fiscal budget balance is given by

(2) $$D + S(\pi) = B_0/P_0,$$

where $S(\pi)$ ($S'(\pi) > 0$) denotes the present value of seignorage, and D is the present value of future primary budget surpluses (negative values represent deficits). Annual real seignorage from a constant money growth rate of g (and thus a constant inflation rate of $\pi = g$) is $\pi f(R)$. The present discounted value of seignorage then, is $S = \pi f(\pi)/r$. The accumulated real value of government debt due at time zero, denoted by B_0/P_0, must equal the present value of future primary budget surpluses plus revenues from seignorage.

Total government liabilities are defined as the sum of money (the liability of the central bank) and government bonds (the liability of the Treasury). We assume that the initial level of total government liabilities, $\bar{H} = M_0 + B_0$, is fixed. The ratio $M_0/\bar{H}$ is the fraction of total liabilities that are monetary. This fraction changes via open-market operations by swapping (newly printed) money, M_0, for government debt, B_0, holding $\bar{H}$ fixed. Rewriting equation (2) by substituting out B_0 gives

$$(3) \qquad S(\pi) + \frac{M_0}{P_0} + D = \frac{\bar{H}}{P_0}.$$

Notice that there are two forms of seignorage in this model. One comes from future money growth, $S(\pi)$. The other comes from movements in the current money stock, M_0/P_0. Open-market purchases swap B_0 for M_0 and thus lower the nominal (and real) value of government debt.

Solving for P in equation (1) and substituting into equation (3), we have

$$(4) \quad S(\pi) + D = \frac{(\bar{H} - M_0)}{M_0} f(R).$$

Assuming that S is increasing (that is, we are on the "correct" side of the Laffer curve) and f is decreasing (money demand slopes down), then for a given D and $\bar{H}/M_0$, there is at most one inflation rate (future money growth), π, that satisfies equation (4).

To close the model, we must define monetary and fiscal policy. A policy is defined by choosing two of the following variables: π, D, or $\bar{H}/M_0$. The third variable is determined endogenously to satisfy equation (4).

Weak-form FT assumes *fiscal dominance,* which is defined in the following way: The fiscal authority commits to D, thus forcing the central bank to choose either current (initial) M_0 or future inflation, π, to satisfy equation (4). The central bank can react to a change in fiscal policy by changing either M_0 or π.

If future inflation is held constant, a decrease in D (that is, an increase in the deficit) necessitates increasing the current money stock, M_0 (and hence P_0), lowering the real value of government debt outstanding. If money is held constant, then the monetary authority must react by increasing future inflation. A decrease in D must result in either a one-time increase in money, M_0, and hence P_0 (a one-time jump in inflation), or an increase in future (sustained) inflation, π. We define fiscal dominance as weak-form FT because the price level is still determined by current or future money supply movements. The central bank, however, is driven by the fiscal authority. In terms of the game of chicken, the central bank is forced to blink; that is, the money supply is dictated by fiscal policy and is, therefore, endogenous.

Fiscal dominance is the assumption made by Sargent and Wallace (1981) in their classic paper, "Some Unpleasant Monetarist Arithmetic." They assume that "the path of [primary surpluses, D] is given and does not depend on current or future monetary policies. This assumption is ... about the behavior of the monetary and fiscal authorities and the 'game' that they are playing. Since the monetary authorities affect the extent to which seignorage is exploited as a revenue source, monetary and fiscal policies have to be coordinated. The question is which authority 'moves first' ... who imposes discipline on whom?"[1]

The arithmetic implied by this game of chicken is unpleasant. Tight money today (a low M_0 and more debt, B_0) necessitates loose money (a high π) in the future to pay off the debt. Equivalently, low seignorage today (low M_0) implies high seignorage (high π) tomorrow. An even more unpleasant possibility of weak-form FT is that tight money today could increase *today's* price level. This would occur if money demand were significantly elastic, as higher inflation and, in turn, a higher nominal interest rate lowered real money demand and increased the price level. Solving equation (3) for P_0 yields

$$P_0 = \frac{\bar{H}}{S(\pi) + f(R) + D}.$$

The effect of an increase in future inflation on current prices, $dP_0/d\pi$, depends on the relative magnitude of decreased nominal money (thus lowering prices) versus the decline in real money caused by higher future inflation. Using the fact that $S = \pi f(R)/r$ and $R = r + \pi$, we have

$$(5) \qquad P_0 = \frac{r\bar{H}}{Rf(R) + rD}$$

and

$$\frac{dP_0}{d\pi} = (\eta - 1)P_0 \frac{M_0}{r\bar{H}},$$

where $\eta = -Rf'(R)/f$ is the interest elasticity of money demand.

Notice that $dP_0/d\pi$ has the same sign as $(\eta - 1)$. If money demand is sufficiently elastic (greater than one), then low money supply

1 Sargent (1986), pp. 170–71.

(M_0) today (implying a high level of inflation tomorrow) implies a high price level (P_0) today. The intuition is as follows: Low M_0 tends to lower P_0. But the resulting higher inflation, π, tends to lower real money demand, driving P_0 upward. This second effect overwhelms the first, if and only if the interest rate elasticity of money demand is greater than unity. Empirical estimates of η, however, are uniformly less than one. Thus Sargent and Wallace's unpleasant possibility that tight money leads to higher current prices (a low M_0 leads to a high P_0) is probably only a theoretical curiosity.

The polar opposite of the assumption of fiscal dominance is the assumption of *monetary dominance*. In this case, the central bank commits to π and $\bar{H}/M_0$. The fiscal authority must then choose D to satisfy equation (4)—that is, the fiscal authority is assumed to blink. Since the central bank chooses π and $\bar{H}/M_0$, it also determines P_0. Monetary dominance is the typical assumption in most theoretical monetary models and is not an example of the FT. For example, a standard simplifying assumption in many monetary models is that $B_0 = 0$, implying that D is endogenous and given by $S(\pi) = -D$. In this game of chicken, the monetary authority moves first and the fiscal authority blinks.

II. A General-Equilibrium Model

In this section, we examine the more general case in which the level of real cash balances is not necessarily at the steady state. Here we ask, is it possible for *neither* the monetary nor the fiscal authority to blink? We refer to this case as strong-form FT because movements in inflation do not result from money growth.

To explore this possibility, we consider the simple case where money supply growth is constant. This is an example of monetary dominance in the sense that the central bank moves first. In the previous section, this implied that fiscal policy would be endogenous and dictated by the government budget constraint. Is this still the case? Looking at fixed-money-growth equilibria is useful, since changes in inflation will, by definition, not be driven by money supply changes. To explore these possibilities we require a dynamic (general equilibrium) counterpart to the money-demand equation (1).

The economy consists of infinitely lived households with preferences over consumption and real balances given by

$$\sum_{t=0}^{\infty} \beta^t U(c_t, M_t/P_t),$$

where β is a constant rate of time discount and c_t and M_t/P_t denote consumption and real money balances, respectively. This money-in-the-utility-function framework is quite general and stands as a proxy for the transactions-facilitation role of money.

We assume that preferences are separable and given by

$$U(c_t, m_t) \equiv V(c_t) + \frac{m_t^{1-\varepsilon}}{1-\varepsilon}.$$

The (absolute value of the) interest elasticity of the implied money-demand curve is equal to $\eta = 1/\varepsilon$. This assumption of separability is not as odd as it may seem. In a model with endogenous production, Carlstrom and Fuerst (1999) demonstrate that the model behaves as if utility were separable in consumption and real balances.

The household's intertemporal budget constraint is given by

$$M_{t+1} = M_t + X_t + B_{t-1}(1 + R_{t-1}) - B_t - P_t c_t + P_t y_t,$$

where M_t denotes money balances at the beginning of time t; X_t denotes a monetary transfer from the government (inclusive of lump-sum tax payments); B_{t-1} denotes bondholdings acquired in period $t-1$; R_{t-1} denotes the nominal interest rate from $t-1$ to t; and the endowment is normalized such that $v'(y) = 1$. As preferences are separable, the constant level of income implies that the real rate of interest is constant at $r = (1/\beta) - 1$. Notice that the bond choice at time t, B_t, determines the amount of cash the household has available for the next period's purchases (M_{t+1}).

The Euler equations that define equilibrium are given by

$$(6) \qquad U_c(t)/P_t = (1 + R_t)\beta\, U_c(t+1)/P_{t+1}$$

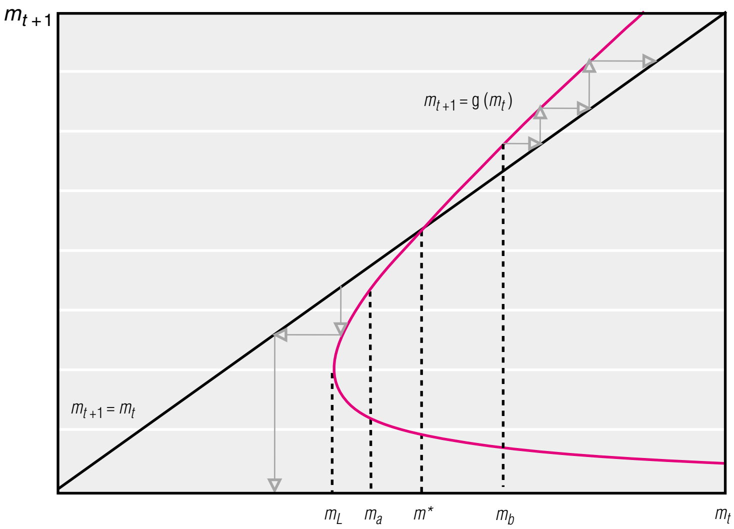
and

$$(7) \qquad \frac{U_c(t)}{P_t} = \beta \left[\frac{U_c(t+1) + U_m(t+1)}{P_{t+1}}\right].$$

Equation (6) arises from optimal bond choice and is the standard Fisherian decomposition of the nominal interest rate into the real interest rate and an inflation premium $[(1 + R) = (1 + \pi)(1 + r)$, or $(R \approx r + \pi)]$. Equation (7) is the choice of next period's money

FIGURE 1

Dynamics of Real Money Balances: $\varepsilon \geq 1$



SOURCE: Authors.

balances (M_{t+1}). Holding on to one dollar today comes at the loss of current consumption (the LHS of equation [7]), but provides for consumption and transaction services next period (the RHS of equation [7]). Combining equations (6) and (7) yields a demand-for-money equation:

$$(8) \qquad U_m(t+1)/U_c(t+1) = R_t.$$

By inverting equation (8) to express m as a function of R, we have the dynamic counterpart of equation (1). Money demand in $t+1$ is a function of the interest rate between t and $t+1$.

To focus on strong-form FT, suppose that the central bank fixes the current money stock at M_0 and the gross money supply growth rate at $G \geq 1$. In fixing the monetary rule in this way, we are assuming monetary dominance, in that money growth is exogenous and will not be deviated from for fiscal reasons. The key question now becomes: Is the path of the price level determined by this exogenous monetary policy? If not, then we have a case of strong-form FT.

Replacing R_t as defined by equation (6) in equation (8) implies that money-market equilibrium is given by

$$(9) \qquad m_{t+1}^{1-\varepsilon} + m_{t+1} = (G/\beta) m_t,$$

where $m_t = M_t/P_t$.[2] Our analysis will examine numerous real balance paths that satisfy equation (9) but are economically meaningful in that real balances remain positive. A steady-state solution is one in which $m_t = m^* \geq 0$ for all t. There is one positive steady state given by

$$(10) \qquad m^* = \left(\frac{\beta}{G-\beta}\right)^{1/\varepsilon}.$$

From equation (9) it is clear that if $\varepsilon < 1$, there is also another steady state in which $m_t = 0$ for all t. This is an equilibrium in which money is not valued. In contrast, if $\varepsilon \geq 1$, then equation (10) describes the only non-negative (and thus permissible) steady state. Therefore, we have two cases.

Case 1, $\varepsilon \geq 1$

This section examines the case where $\varepsilon \geq 1$. Equation (8) implies that the interest elasticity of money demand ($\eta = 1/\varepsilon$) is less than one. In this case, we will show that the general-equilibrium model collapses down to the partial-equilibrium model of the previous section, and thus cannot deliver strong-form FT; equivalently, the assumption of monetary dominance (constant money growth) implies that the fiscal authority must be passive.

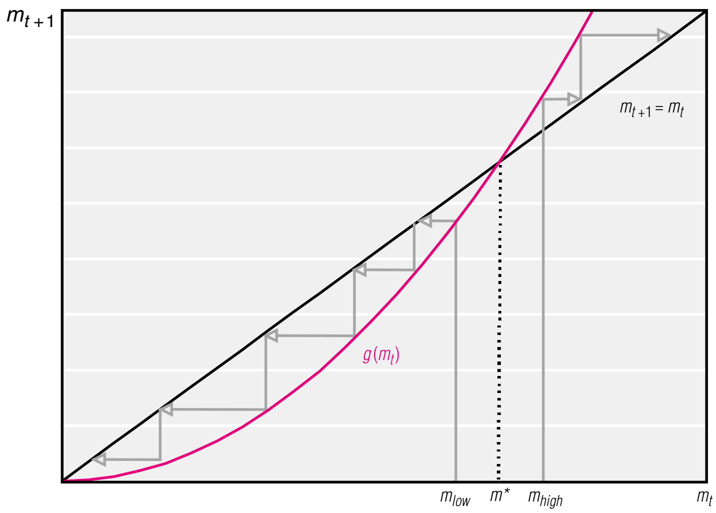
Figure 1 graphs m_{t+1} as a function of m_t [$m_{t+1} = g(m_t)$] to illustrate these dynamics. The arrows indicate how m_t evolves over time. Since there is a unique positive steady state, m^*, paths that begin below m^* (say m_a) imply that real balances become complex-valued in finite time and are thus nonsensical.[3] Real money balances starting to the right (say m_b) explode, eventually violating the transversality condition and thus do not satisfy the necessary conditions for an optimum (see appendix A). Hence, as long as money demand is not too elastic ($\eta = 1/\varepsilon \leq 1$), the current price level is uniquely determined and real balances must jump immediately to the steady state m^*.[4] Thus, since monetary policy was given (fixed G and M_0),

■ 2 $m_{t+1}^{-\varepsilon} = \dfrac{P_{t+1}}{\beta P_t} - 1 = \dfrac{G m_t}{\beta m_{t+1}} - 1.$

■ 3 There are no real solutions to (9) when money balances are to the left of m_L on the graph. The solutions are then all complex which have no economic meaning. This occurs irrespective of whether you take the upper or lower part of the "C" in figure 1. We thank Larry Christiano and Terry Fitzgerald for pointing out an error in the earlier working paper version of this figure.

FIGURE 2

Dynamics of Real Money Balances: $\varepsilon < 1$



SOURCE: Authors.

it is dominant, and fiscal policy must adjust to ensure budgetary solvency (equation [2]). This general-equilibrium example is thus identical to the steady-state example of monetary dominance in the previous section.

Case 2, $\varepsilon < 1$: Strong-Form FT

Suppose instead that $\varepsilon < 1$, so that money-demand elasticity is greater than one ($\eta > 1$). In this case there are two steady-state solutions (equation [10] and $m^* = 0$).

Notice that the only stationary equilibrium with valued money is that in which real balances (and prices) immediately jump to the positive steady state, m^*. If we restrict the analysis to stationary equilibria, because M_0 is given exogenously the fiscal authority must move to maintain fiscal solvency. This once again corresponds exactly to the monetary dominance results in the previous section.

While one can argue that nonstationary equilibria can be ruled out on empirical grounds, there is nothing in the model (if $\eta > 1$) to rule out these nonstationary paths. Figure 2 illustrates the model's dynamics.[5] Unless current real-money balances are given by m^* (that is, M_0/P^*), real balances will either explode or implode over time. To the right of m^*, all paths have real balances exploding as the price level approaches zero (self-fulfilling hyperdeflations). As before, these paths are not equilibria because they violate the household's transversality condition (see appendix A). To the left of m^*, all paths are self-fulfilling hyperinflations: The real value of the money stock goes to zero in the limit. These equilibria cannot be ruled out a priori, since they also converge to a steady state in which money is not valued. Thus, there is an infinite number of equilibria, each indexed by the current price level, P_0. Any initial price level $P_0 > P^*$ is an equilibrium.

Returning to our game-of-chicken analogy, we ask the simple question of whether *anyone* has to blink. As we showed in section I, if the fiscal authority commits to a primary surplus path and the monetary authority commits to a seignorage path (that is, if both agents refuse to blink), then the fiscal solvency condition will be violated. Someone has to move. However, this is not necessarily true in the general equilibrium case, since (with $\varepsilon < 1$) there exist non-steady-state equilibria in which the current price level is free. If both parties refuse to move, then the initial price level will immediately jump to a level satisfying the government's budget constraint.

To see the effect of fiscal policy on the price path, consider the case where there is no money growth ($G = 1$). With no future seignorage revenues, equation (2) gives $P_0 = B_0/D$. Thus, fiscal policy determines the current price level and (from equation [9]) the path of prices. A higher D implies a lower P_0, and vice versa. Despite the exogeneity of monetary policy, fiscal policy maintains a great deal of autonomy, restrained only by the requirement that $m \leq m^*$, so that $D \leq (B_0/M_0) \bullet m^*$. If D exceeded this latter value, there would be no equilibrium if both parties refused to move.

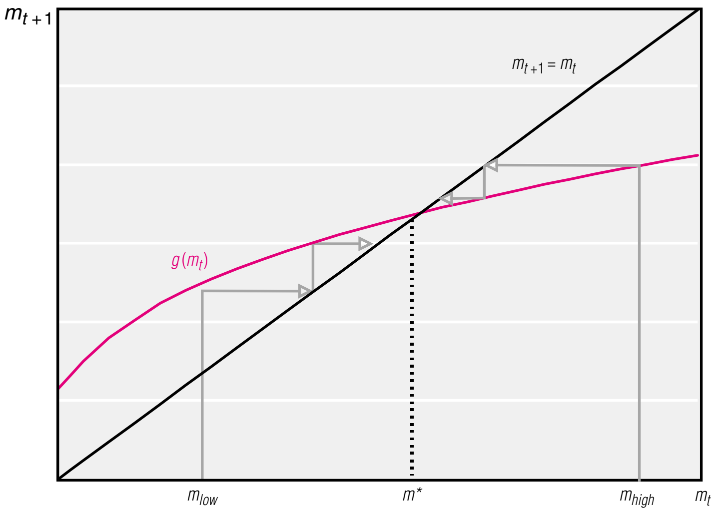
Referring to figure 2, the nonstationary equilibrium paths (where $m < m^*$) have prices rising and inflation increasing. Since the money-demand relationship still holds, the only way for current prices to rise is for the nominal interest rate and inflation to increase (remem-

4 In a private communication, Larry Christiano and Terry Fitzgerald note that if ε is sufficiently large (money demand is sufficiently interest inelastic), then the C-shape in figure 1 is shifted up so that the lower branch cuts the 45-degree line from above (this arises if $\varepsilon > G/(G-\beta)$). In this case, both branches of the g-mapping are relevant so that for a given m_t there is more than one possible m_{t+1}. The strong-form FT will be of no help in eliminating this type of multiplicity. As for empirical relevance, for $\beta = 0.99$, and $G = 1.02$, these pathologies arise only if $\varepsilon > 34$, an interest elasticity less than 0.029!

5 This case is examined in McCallum (1998).

FIGURE 3

Possible Dynamics of Real Money Balances with Nonseparable Preferences



SOURCE: Authors.

ber that money growth is constant). A change in fiscal policy (D) changes current prices by changing the path of future inflation. For example, an increase in the present discounted value of future surpluses (D) lowers current prices and future inflation.

This is a version of strong-form FT; fiscal policy affects the price path even though it has no effect on current or future money growth (nor on real output or the real rate of interest, both of which are assumed to be constant). This strong-form FT occurs because both monetary and fiscal policy are acting in a dominant fashion; in other words, neither party blinks. This is an intriguing possibility—namely, that fiscal policy can influence the price-level path independent of movements in the money stock. But the analysis has two peculiar but interrelated characteristics. First, the model exhibits self-fulfilling hyperinflations. Although this is an interesting theoretical possibility, there is scant empirical evidence for such phenomena.[6] Second, these hyperinflationary paths and the possibility of strong-form FT assume an implausibly high interest elasticity ($\eta = 1/\varepsilon > 1$). We know of no empirical estimates this high.

Nonseparable Preferences: Strong-Form FT

These peculiarities are not robust. For example, following Matsuyama (1990, 1991), suppose we relax the separability assumption on preferences.[7] In this case, it is possible to get the strong-form FT without an implausibly high interest elasticity of money demand *and* without nonstationary (exploding) price paths. The nonseparable counterpart to equation (9) is

$$(11) \quad \frac{G}{\beta} m_t U_c(m_t) = m_{t+1}[U_c(m_{t+1}) + U_m(m_{t+1})].$$

Since consumption is assumed, constant marginal utility is expressed as a function of real cash balances only.

As before, there exists a unique positive steady state. But unlike figure 1, which shows that the economy would immediately jump to this steady state, prices in this example will not necessarily immediately jump to P^*. A sufficient condition for this to occur—that is, for the existence of multiple stationary equilibria—is that the mapping of $m_{t+1} = g(m_t)$ cross the 45-degree line from above, or $0 < g'(m_{ss}) < 1$. Figure 3 shows such a case. The analysis resembles the earlier example where $\varepsilon < 1$ (figure 2), except that *all* initial real balances starting away from the steady state converge to m^* and thus do not have the counterfactual implication that prices will explode over time. Before, real balances beginning to the left of m^* converged to another steady state where money had no value.

Unlike this earlier nonstationary example, there are *no restrictions* on the initial stock of real money: Because these stationary paths converge to the steady state, the transversality condition is never violated. What is the initial level of real balances? On a theoretical level, it is the level of real money chosen at the beginning of time; however, the initial price level is chosen every period. If initial real balances are not determined at the beginning of time, then real balances every period are also undetermined. This leads to what economists call

■ **6** At a theoretical level, these hyperinflationary equilibria could be ruled out by a government promise to guarantee a lower bound on the real value of the currency by backing it with an arbitrarily small (but positive) real asset. Obstfeld and Rogoff (1983) make this point.

■ **7** In addition to considering nonseparability, Matsuyama (1990, 1991) uses a different timing convention. In the model of this article, beginning-of-period cash balances enter into the current-utility functional. In contrast, Matsuyama assumes that end-of-period balances enter into the current-utility functional. See Carlstrom and Fuerst (1999) for a discussion of these issues.

sunspot equilibria, in that purely extraneous information leads to a shift in public beliefs and thus affects the model's equilibrium. The hallmark of sunspot equilibria is the presence of self-fulfilling behavior.

Returning to the details of the nonseparable case, appendix B shows that equation (11) looks like figure 3 [$0 < g'(m_{ss}) < 1$] if and only if

$$(12) \qquad \frac{mU_{cm}}{U_c} < -1,$$

where this ratio is evaluated at the unique positive steady state. This is the ratio of elasticity of the marginal utility of consumption to the level of real balances.

To understand why sunspot equilibria (or self-fulfilling prophecies) are present in this economy under condition (12), let us walk through a simple example. Suppose there is a "sunspot event" at time t (an event independent of market fundamentals) that leads households to increase their holdings of real cash balances by 1 percent (P_t falls by 1 percent). For this sunspot movement to be stationary, the economy must move back toward the steady state: $m_t > m_{t+1} > m_{ss}$. For $m_{t+1} > m_{ss}$, the nominal rate at time t must be below the steady state (recall equation [8]). For real balances to deteriorate between t and $t + 1$ ($m_t > m_{t+1}$), however, the inflation rate must be above the steady state.

But how can the nominal rate be below the steady state, while the inflation rate is above the steady state? If and only if the real rate of interest is sufficiently below the steady state. The logic is as follows: Because the real interest rate (r) is the ratio of the marginal utility of consumption today divided by the marginal utility of consumption tomorrow, r falls by more than the increase in expected inflation; thus, the increase in real money balances significantly decreases the marginal utility of consumption today. This is exactly the restriction in equation (12).

As in the previous section, the only way to escape this indeterminacy is for both the monetary and fiscal authorities to be completely unconcerned with balancing the government's books, in which case the strong-form FT provides the additional restriction needed to uniquely determine equilibrium. By pinning down initial real balances, it essentially eliminates the possibility of sunspot equilibria. Now the only "sunspots" that can change current prices and future inflation are changes in the primary budget surplus, D.[8]

This stationary example, however, makes two unusual assumptions. First, the elasticity in equation (12) is negative—additional cash balances *lower* the marginal utility of consumption. Second, this response (in absolute value) is quite large, *greater than one*. Both of these assumptions are problematic, especially given the results of Carlstrom and Fuerst (1999) that in a production economy with elastic labor, the model economy acts as if $U_{cm} = 0$.[9]

These examples assume that money growth is constant. In this case, it is hard to obtain price-level indeterminacy. What about other monetary rules? For instance, it has long been recognized that interest rate targeting leaves the initial value of nominal money (and prices) free. Section III demonstrates this and asks whether this is still an example of strong-form FT.

III. Endogenous Money: The Case of a Fixed Interest Rate

Most central banks conduct policy by way of directives for the nominal rate of interest. Such a policy implies that the money supply and seignorage are endogenous, opening up some interesting possibilities for the fiscal theory of the price level. By committing to an interest rate peg regardless of fiscal concerns, the central bank is acting in what seems to be a dominant fashion. But since the money supply and seignorage are endogenous, the monetary authority moves last and the fiscal authority maintains a great deal of discretion. On the surface, it is unclear whether monetary policy is acting in a dominant fashion.

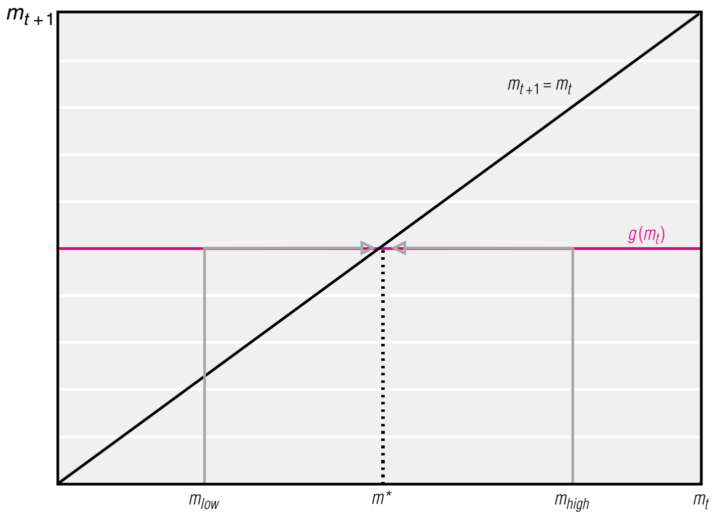
If we return to the game-of-chicken analogy, then in the case of an interest rate target there is another player in the game—the general public. Under such a monetary policy, the central bank agrees to engage in open-market operations to maintain the targeted rate, that is, buy and sell bonds at the request of the public. Thus, the public becomes an important player in the game. So the critical question is, who constrains whom? Does the fiscal authority con-

8 The path of future surpluses (D) may not be completely free. There may be levels of D for which there is no initial price level and no subsequent path of prices that satisfy the fiscal budget contraint. D is not actually a sunspot since it is a market fundamental.

9 There is another example where a constant-money-growth rule may lead to stationary indeterminacy: If the relative coefficient of risk aversion is greater than two (not implausible), then a cash-in-advance (CIA) economy with production implies indeterminacy. (See Carlstrom and Fuerst [1999].) This example suffers because it is extremely sensitive to the CIA assumption and does not arise in a money-in-the-utility-function framework, as assumed above.

FIGURE 4

Dynamics of Real Money Balances with an Interest Rate Peg



SOURCE: Authors.

strain the behavior of the general public? Or does the general public constrain the behavior of the fiscal authority?

This section explores these issues first in the steady-state model of section I, and then in the general-equilibrium model of section II. For simplicity, we will restrict the analysis to a fixed interest rate target (an interest rate peg).

The Steady-State Model

In the steady-state model, the central bank maintains a constant nominal interest rate by picking π, but then allows M_0 to be endogenous. The pegged nominal rate determines the level of real balances in equation (1). Combining equations (1) and (3), we have

$$(13) \qquad S(\pi) + D + f(\bar{R}) = \frac{\bar{H}}{P_0}.$$

Given that the real rate is fixed, a nominal interest rate peg also determines the inflation rate, π. What, then, determines the price level?

One can think of the monetary authority choosing M, the fiscal authority choosing D, and the public choosing real balances and, hence, P. By the definition of an interest rate peg, the central bank moves last since they chose money (endogenously) to ensure that the interest rate remains constant. Given this assumption, there are two cases to consider: the fiscal authority moves first, or the public moves first.

If the fiscal authority moves first, then D is exogenous. Since π and $\bar{H}$ are also given, equation (13) determines P_0 as a function of D: a low D implies a high P_0, and vice versa. Since M_0/P_0 is already determined by equation (3), we can also think of P_0 as a function of M_0. Returning to the game-of-chicken analogy, the general public blinks. Woodford (1994) uses this assumption to eliminate the price-level indeterminacy of operating under an interest rate peg. Notice that the situation resembles weak-form FT since fiscal policy, D, affects prices because it also affects the money supply, M_0.

If the public chooses first, then the fiscal authority must adjust D to satisfy fiscal balance—that is, the fiscal authority blinks. In this case, movements in the public's behavior (different choices for P_0 and M_0) translate directly into price movements. This creates self-fulfilling behavior, or sunspot equilibria: If the public expects a high price level and demands a high level of money balances to satisfy their transactions needs, then the money supply rises and generates the high price level they anticipate. This set of assumptions produces the standard nominal indeterminacy of operating under an interest rate peg: The current money stock (M_0) is free and so is the current price level (P_0).

A General-Equilibrium Model

Now let us consider the effect of an interest rate peg on the general-equilibrium model. With a constant level of consumption, the Fisher equation (6) implies that this corresponds to targeting the inflation rate at some rate π. The counterpart to equation (9) is

$$(14) \qquad m_{t+1}^{-\varepsilon} = \{[(1+\pi)/\beta] - 1\}.$$

Figure 4 graphs m_{t+1} as a function of m_t, a special case of figure 3. Here the initial m is free, but the economy immediately jumps to the steady state given in equation (10).[10]

■ **10** To illustrate how the non-uniqueness of the initial price level leads to sunspot equilibria, note that with uncertainty, equation (13) becomes $E_t(m_{t+1}^{-\varepsilon}) = \{[(1+\pi)/\beta] - 1\}$. A quadratic approximation implies that real money balances will be given by $m_t = m^* + v_t$. There are no restrictions on the shock term v_t which, in principle, can be governed by sunspots.

Let us refer to this initial period as period 0. From equation (13), using the definitions of $\bar{H}$ and $f(R)$, the fiscal solvency constraint is given by

$$(15)\qquad \frac{M_1 - M_0}{P_0} + \frac{S(\pi)}{1+r} + D = \frac{B_0}{P_0}$$

(remember that because the real rate is constant, inflation is also constant).

Since the first bond market does not open until the end of period 0, M_0 and B_0 (since $M_0 + B_0 = \bar{H}$) are given by history. As for M_1, we have that $M_1 = f(\pi)P_1 = f(\pi)(1+\pi)P_0$. There are only two free variables in equation (15), D and M_1.

The situation is symmetric with the steady-state model.[11] The public chooses M_1 in the bond market at the end of period 0. The central bank agrees to exchange money for bonds at the rate desired by the public. We have the same case as before. If the fiscal authority commits to a D path, then the only equilibrium choice for private agents is given by equation (15). If instead the general public moves first, then the fiscal authority must adjust D to satisfy equation (15). In this latter case, once again we have the possibility of sunspot equilibria.

Before closing, we should ask the question: Are these endogenous money cases examples of strong-form or weak-form FT? On one level they appear to satisfy the criteria for strong-form FT in that the fiscal authority is acting in a dominant fashion, as is the central bank since it chooses its goal (for example, an interest rate peg) regardless of fiscal concerns. But at a deeper level, they are really examples of weak-form FT. The monetary authority is not truly dominant because money supply and seignorage are endogenous. That is, if the fiscal authority chooses a different fiscal stance (D), then the monetary authority must change the money supply to ensure the interest rate target is still satisfied. The monetary authority moves last and, in essence, is the one that always blinks, as occurs under weak-form FT. Perhaps more importantly, it is only an example of weak-form FT since the fiscal authority only affects the price level by altering the endogenous supply of money.

IV. Conclusion

This article began with the observation that the implications of weak-form FT on monetary policy are not controversial. If the central bank is passive and the fiscal authority is dominant, then fiscal policy has an enormous influence on the price level. But this traditional form of the FT is also consistent with Friedman's dictum, since fiscal policy affects prices and inflation only through its effect on money.

Recently a much stronger version of this theory has been presented. There are two possibilities in the more recent versions of the fiscal theory of the price level: (1) strong-form FT, in which fiscal policy affects the price level independent of the money supply process, and (2) the case of interest rate targeting, in which the money supply is endogenous.

The strong-form FT, in which *both* the fiscal and monetary authorities move first (neither blinks), relies on large elasticities and thus is little more than an intellectual curiosity. It is difficult to take these examples too seriously.

As for interest rate targeting, our conclusion is more circumspect. This is actually *not* strong-form FT because movements in prices are still governed by movements in money.

This does not imply, however, that the FT has no important implications for monetary policy. There is a long line of research suggesting that interest rate targeting is indeed beneficial. A classic criticism of such a policy, though, is that the endogeneity of the money supply makes the price level unstable. In models with nominal rigidities, this also makes output unstable. FT advocates argue that this is not the case: If the fiscal authority commits to a budgetary path, then the general public must adjust its behavior to ensure equilibrium, and this restriction pins down the price level. If we accept such an argument, then the case for interest rate targeting is greatly strengthened. The government's budget greatly reduces these sunspot equilibria—only changes in D are sunspot equilibria in the sense that they can cause a one-time jump in the price level. But if the more appropriate way to view this game of chicken is to assume that the fiscal authority always moves last, then interest rate targeting remains problematic because it can result in instability.

■ **11** The only difference between the steady-state and the general-equilibrium models is that, in the latter, the timing assumption (the bond market opens at the end of the period) transforms the nominal indeterminacy in the steady-state model into a real indeterminacy (of real balances) in the general-equilibrium model.

Appendix A

This appendix demonstrates that if $G > 1$, then hyperdeflations do not satisfy the household's transversality condition. The transversality condition is given by

$$\text{(A1)} \qquad \lim_{t \to \infty} \beta^t m_t = 0.$$

This requires that real balances grow at a rate less than $1/\beta$. Rewriting equation (9), we have

$$\text{(A2)} \qquad m_{t+1} = \left[\frac{G}{(m_{t+1}^{-\varepsilon} + 1)}\right]\left(\frac{1}{\beta}\right) m_t.$$

Since real balances are exploding along a hyperdeflation, the bracketed term in equation (A2) is growing. Since $G > 1$, this term will eventually exceed one. Therefore, real balances will grow at a rate exceeding $1/\beta$ and will violate the transversality condition.

Appendix B

This appendix provides details of the case with nonseparable preferences (section II). From equation (7), the fundamental equation of the model is given by

$$\text{(B1)} \qquad \frac{G}{\beta} m_t U_c(m_t) = m_{t+1}[U_c(m_{t+1}) + U_m(m_{t+1})].$$

Expressing m_{t+1} as a function of m_t, $m_{t+1} = g(m_t)$, and then totally differentiating equation (B1), yields

$$\text{(B2)} \qquad g'(m_{ss}) = \left[1 - \frac{R/(1+R)}{\eta(1 + mU_{cm}/U_c)}\right]^{-1},$$

where $\eta > 0$ is the interest elasticity of money demand. A necessary and sufficient condition for $0 < g'(m_{ss}) < 1$ (so that we have a mapping as in figure 3), is for $(1 + mU_{cm}/U_c) < 0$. This is just the condition in equation (12).

There are, of course, other possibilities. If $(1 + mU_{cm}/U_c) > 0$, then there are two cases. If η is sufficiently small,

$$\text{(B3)} \qquad \eta < \left(\frac{R}{1+R}\right)\left(\frac{1}{1 + mU_{cm}/U_c}\right),$$

then $g'(m_{ss}) < 0$. This tends to produce oscillatory behavior. Remarkably, Matsuyama (1991) demonstrates that if $g'(m_{ss})$ is sufficiently negative, then there are chaotic dynamics.

In the more likely case that

$$\text{(B4)} \qquad \eta > \left(\frac{R}{1+R}\right)\left(\frac{1}{1 + mU_{cm}/U_c}\right),$$

then $g'(m_{ss}) > 1$, and we are back to a model similar to that in figure 1 or figure 2.

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