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TESTING FOR SPECULATIVE BUBBLES IN STOCK PRICES

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ABSTRACT

A modified version of West's **(1987)** method for investigating the possibility of speculative bubbles in stock prices is recommended that is computationally simpler and, unlike West's method, tests the "no bubble" hypothesis directly. The proposed method is applied to long-term annual U.S. stock-market data. Contrary to West's findings, no evidence of speculative bubbles in stock prices during **1871-1981** or **1871-1988** is found.

I. INTRODUCTION

It is widely believed that fluctuations in the equilibrium price of an asset reflect changes in corresponding market fundamentals. Accordingly, stock-market booms, depressions and crashes can be explained by changes in stock price fundamentals defined as the expected present value of future dividends. It is possible, however, that self-fulfilling expectations, called speculative bubbles, cause a persistent deviation in stock prices from the path consistent with these fundamentals. Since speculative bubbles are argued to have substantial real effects, it is of interest to investigate their existence (Blanchard and Watson [1982]). This possibility has been empirically investigated by Flood and Garber (1980), Leroy and Porter (1981), Shiller (1981), Grossman and Shiller (1981), Diba and Grossman (1988), Flood, Hodrick and Kaplan (1986), Santoni (1987), and West (1984); evidence in support of, as well as against, the presence of speculative bubbles in stock prices are reported.

The merit of these empirical investigations has been questioned on interpretive grounds by Grossman and Shiller (1981), Blanchard and Watson (1982), Marsh and Merton (1984), Hamilton and Whiteman (1985), and West (1987). These critics emphasize that test procedures have failed to differentiate between the no-bubble hypothesis and the hypothesis that the tested model is incorrectly specified. More specifically, these procedures test the joint hypothesis of "no misspecification and no bubbles," so that a significant test statistic might be incorrectly interpreted as evidence of speculative bubbles when it merely reflects model misspecification. West

(1987) proposed a procedure for separating the "no-misspecification" and the "no-bubble" hypotheses, so that a rejection of the "no-bubble" hypothesis when the "no-misspecification" hypothesis is not rejected would constitute evidence for speculative bubbles. West's procedure combines a direct test of the "no-misspecification" hypothesis, with an indirect and computationally elaborate test of the "no-bubble" hypothesis. Because of the indirect nature of the second test, however, incorrect interpretations can still arise. This is exactly the problem that West's procedure seeks to remedy by testing the two hypotheses separately. Therefore, West's result is itself ambiguous and subject to interpretation criticism.

This paper modifies West's procedure. Our modified procedure tests the "no-bubble" hypothesis directly, thus avoiding ambiguity of interpretation. The modification is based on a method by Plosser, Schwert and White (1982) and by Davidson, Godfrey and Mackinnon (1985). This method is particularly powerful in detecting specification problems in distributed-lag equations of the form considered here (Thursby [1988]). The proposed procedure is then applied to long-term annual data to test for the presence of speculative bubbles in stock prices. Two sample periods with the same starting point were chosen so that one covers the recent market boom and crash and the other does not. Contrary to West's findings, no evidence of speculative bubbles in stock prices during 1871-1981 or 1871-1988 is uncovered.

The plan of the paper is as follows. Section II defines speculative bubbles in the context of the standard efficient market model and explains the proposed test procedure. Section III presents and interprets empirical results. Section IV concludes the paper.

II. THE TEST PROCEDURE

Economists usually believe that the price of an asset must simply reflect market fundamentals. For stock prices, these fundamentals are the expected present value of the future stream of dividends, and they are believed to determine the path of stock prices. It is argued, however, that a deviation from this path is possible even if market participants have rational behavior and expectations (Blanchard and Watson [1982]). Such deviations, if induced by self-fulfilling expectations, are called speculative bubbles (Flood and Garber [1982]). Price bubbles have received considerable empirical attention in recent years. Nonetheless, the issue of bubbles or no bubbles is still considered unsettled given the mixed nature of the results and, more importantly, the criticism that empirical studies on bubbles have received on interpretive grounds. By modifying West's (1987) procedure to test the "no-bubble" and "no-misspecification" hypotheses separately and directly, we hope to avoid this criticism. In the rest of this section, a brief review of the standard efficient-market model and the proposed procedure that utilizes this model are presented.

According to the linear rational expectations model of stock-price determination, the expected real return from holding stocks equals a required real risk adjusted rate of return. Assuming this rate to be constant, a stock price is determined by the arbitrage relationship:

$$P_t = \theta \cdot E((P_{t+1} + d_{t+1}) | \Omega_t) \qquad (1)$$

where $\theta = (1+r)^{-1} < 1$ is the real discount factor, r is the constant required-risk-adjusted rate of return, t denotes time period, P and d are the real stock price and dividend, and $E(\cdot | \Omega_t)$ is the mathematicalexpectation

conditional on the information set Ω available at time t to all market participants.

The forward-looking solution to equation (1) is

$$P_t^* = \sum_1^\infty \theta^i \cdot E(d_{t+i} | \Omega_t), \qquad (2)$$

where P_t^* is the present value of the expected real dividend stream and is referred to as the market fundamental value of a stock in the literature; summations are over i hereafter. It is noted that p_t^* is not the only solution to (1). The general solution is

$$P_t = P_t^* + B_t \qquad (3)$$

where B_t solves the homogeneous expectational difference equation:

$$\theta \cdot E(B_{t+1} | \Omega_t) - B_t = 0. \qquad (4)$$

B_t embodies the notion of a rational speculative bubble and if present, it will cause P_t to deviate from the market fundamental path defined by P_t^*.[1]

To develop a test for the "no bubble" hypothesis, it is necessary to transform (1) and (3) into regression equations.

Equation (1) may be rewritten as

$$P_t = \theta(P_{t+1} + d_{t+1}) + u_t, \qquad (5)$$

where $u_t = \theta[E((P_{t+1} + d_{t+1}) | \Omega_t) - (P_{t+1} + d_{t+1})]$.

When expectations are rational, u_t's are uncorrelated.

Equation (3) may be transformed into a regression equation in the following manner. Rewrite equation (2) as:

$$P_t^* = \sum_1^\infty \theta^i . E(d_{t+i} | H_t) + z_t, \qquad (6)$$

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1. Various specifications for B_t are discussed by Blanchard and Watson (1982).

where

$$z_t = \sum_{1}^{\infty} \theta^i [E(d_{t+i} | \Omega_t) - E(d_{t+i} | H_t)]$$

and H_t is a subset of Ω_t and includes information only on current and past dividends. It is noted that $E(d_{t+i} | H_t)$ is the forecast of future dividends conditional on past dividend history or, more specifically, the autoregressive integrated moving average (ARIMA) forecast of d_{t+i}. Consequently, P_t^* can be expressed as a distributed lag of current and past dividends where the order of this distributed lag is the same as the order of the AR part of the scheme characterizing the dividend process. For example, when the dividend process is identified as:

$$d_t = \phi_o + \phi_1 d_{t-1} + \cdots \phi_q d_{t-q} + \varepsilon t' \qquad (7)$$

we can write equation (6) as:

$$P_t^* = \beta_o + \beta_1 d_t + \cdots \beta_q d_{t-q+1} + V_t, \qquad (8)$$

where $V_t = z_t$ is orthogonal to d_{t-i} for $i \geq 0$.[2] The regression form of equation (3), which is obtained by substituting (8) into (3), is

$$P_t = \beta_\Theta + \beta_1 d_t + \cdots \beta_q d_{t-q+1} + B_t + V_t. \qquad (9)$$

Based on the regression equations (5), (7) and (9), the "no-bubble" hypothesis can be tested according to the following procedure. Equation (5) is rigorously tested for misspecification using a number of tests. If this equation is correctly specified, obviously the arbitrage relationship as stated in equation (1) holds. This, in turn, indicates that equation (9), which is derived from the solution of equation (1), is well-specified provided

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2. When the dividend process is integrated, the distributed lag equations (8) and (9) are written in differenced form. For brevity, we only set forth the undifferenced version of these equations. For a more detailed discussion of the equations derived in this section, see Hansen and Sargent (1981) and West (1987).

that q has been correctly chosen. Therefore, conditional on correct specification of equation (5) and the order of equation (7), testing the "no-bubble" hypothesis is equivalent to testing the specification of equation (9). If bubbles do not exist, $B_t = 0$ and omitting the unobservable bubble term does not affect the consistency of $\hat{\beta}$, the least squares estimate of $\beta=(\beta_0,\beta_1,\ldots\beta_q)$. But if bubbles do exist, omitting the bubble term, B_t, renders $\hat{\beta}$ inconsistent.

Therefore, the "no bubble" hypothesis may be reformulated as:

$$H_0: \text{ plim } \hat{\beta} = \beta \qquad (10)$$

against

$$H_A: \text{ plim } \hat{\beta} \neq \beta$$

where plim denotes probability limit. If properly implemented, this procedure is not subject to interpretation criticism since it verifies the "no-misspecification" hypothesis before testing for the presence of bubbles. Besides, this procedure does not require parametric specification of the bubble term. Therefore, it can detect any bubble that is not orthogonal to the dividend process.[3]

Although West (1987) does not specifically formulate the "no-bubble" hypothesis in terms of the consistency of $\hat{\beta}$, he is, in fact, testing for the consistency of $\hat{\beta}$ indirectly. Based on a method suggested by Hausman (1978), he compares $\hat{\beta}$ with another estimate of β, $\tilde{\beta}$, which is derived from the estimated

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3. If B_t is orthogonal to the dividend process, its omission from equation (9) does not affect the consistency of β and its presence cannot be tested for. The possibility of such a bubble is remote since an overreaction to dividend news is said to be an important factor contributing to the formation of a rational bubble (Shiller [1984]).

coefficients of equations (5) and (7) using a set of constraints from Hansen and Sargent (1981). In the presence of bubbles, there would be a statistically significant difference between $\tilde{\beta}$ and $\hat{\beta}$ due to the inconsistency of $\hat{\beta}$ and since $\tilde{\beta}$ may still be consistent. This indirect test for the consistency of $\hat{\beta}$ has two shortcomings: it is inconsistent, and its indirect computational procedure may cause the likelihood of a type I error to be much larger than the designated significance level (see West [1987], footnote 3 and West [1985], footnote 7 and appendix II).[4] The former could result in a failure to detect bubbles when bubbles are present. The latter could result in a rejection of the "no-bubble" hypothesis when there are no bubbles; this stems from the fact that $\tilde{\beta}$ is a nonlinear function of θ and ϕ's, and its covariance could only be approximated from the variance of θ and the covariance of ϕ's. This could exaggerate the chi-square statistic used by West that utilizes the difference between the covariances of $\hat{\beta}$ and $\tilde{\beta}$ to standardize $\beta - \tilde{\beta}$. Therefore, it is possible that equations (5) and (7) are correctly specified and their parameters are consistently estimated, yet the difference between β and $\tilde{\beta}$ turns out to be statistically significant. Given that the small sample performance of West's application of Hausman method is not known, the results reported in West (1987) are hard to interpret.

To overcome these problems, we recommend a modification of West's procedure that retains the specification tests for equations (5) and (7) but

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4. This test is similar to Hausman (1978) test in spirit but quite different in formulation. The Hausman test compares two estimates of the parameters of the same regression equation rather than different equations. Hauseman's test is consistent in general.

tests the specification of equation (9) directly by applying the differencing test of Plosser, Schwert, and White (1982) (hereafter PSW) to this equation. This test is based on the difference between two different least squares estimates of the parameters of a regression equation, one obtained using the undifferenced data and the other using the differenced data. If the equation is correctly specified, the difference between these two estimates, which are both consistent, would be statistically zero. PSW is a consistent test with good small sample performance and high power for detecting specification problems in equations with distributed lags (Plosser, Schwert and White [1982] and Thursby [1988]). A modified and computationally simple version of this test developed by Davidson, Godfrey and MacKinnon (1985) is used in this study.

The modified procedure for testing for the presence of bubbles is comprised of the following steps. First, the dividend process is identified and estimated using Box and Jenkins (1976) analysis and the Hannan and Quinn (1979) procedure for selecting the order of the autoregressive process (q).[5] Then, diagnostic checks for adequacy of the fit are performed on the residuals by means of the Portmanteau test proposed by Box and Pierce (1970) and modified by Ljung and Box (1978). The stability of the coefficients over the sample period is also examined using a chi square procedure. Secondly, three specification tests are performed on the arbitrage relationship, which is based on the rationality of expectations and a constant discount rate. Rationality is tested by employing Hansen's (1982) specification procedure to

5. Hannan and Quinn (1979) demonstrate that their procedure yields strongly consistent estimates of q and underestimates q less often than other procedures in moderate samples.

equation (5). This procedure uses the fitted values of the dividend process as instruments to estimate the parameters, and then tests instrument-residual orthogonality.[6] The stability of the discount rate (θ) is tested by a chi square procedure that examines the possibility of, different rates for the first and second half of each sample. To see if the residual of this equation are approximately white noise, the standardized first-order residual autocorrelation is tested using the standard normal procedure. Provided that the dividend process and the arbitrage relationship are correctly specified, the "no bubble" hypothesis, as formulated in (10) may be tested by applying PSW's differencing test to equation (9). A significant test statistic would lead to rejecting this hypothesis.

III. THE EMPIRICAL RESULTS

The proposed procedure is applied to U.S. annual stock-market data provided to us by Robert Shiller. The price data is Standard and Poor's composite price index for January, divided by the January Producer Price Index, and scaled so that the 1982 price index equals 100. The dividend series is a four-quarter total dividend per share adjusted to index and made real using the Producer Price Index.[7] Two samples were used, one covering 1871-1981 and the other 1871-1988.

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6. It is noted that the disturbances of equation (5) are heteroskedastic and correlated with the explanatory variable. A consistent estimation of this equation using Hansen's two-step, two-stage instrumental variable method is necessary for testing instrument-residual orthogonality.

7. The dividend data used here is slightly different from those used by Shiller (1981 and 1984) and West (1987). The difference is due to corrections made by Campbell and Shiller as explained in Campbell and Shiller (1987, footnote 21).

The test procedure is carried out in the following order: First, the dividend process is identified, estimated, and checked for misspecification. We let data determine the specification of the process using Box and Jenkins analysis and Hannan and Quinn's Procedure. A second-order AR process adequately characterizes the dividend process; however, since the values of Hannan and Quinn's criterion for selecting the order are very close for q=l and q=2, both specifications are considered. Given the values of q, equation (7) is estimated using the Maximum Likelihood Method, and diagnostic checks are performed on the residuals. Estimates of the residual autocorrelation function up to 24 lags are used to compute the modified Box-Pierce statistic recommended by Ljung and Box (1978).[8] The statistic is insignificant at the five-percent level for AR(1) and AR(2) specifications in both samples, indicating the adequacy of the fits. The chi-square test for stability of the process is also insignificant at the five-percent level, indicating that the no-structural-change hypothesis cannot be rejected. Therefore, the test procedure proceeds using q=l and q=2 as the possible orders for the dividend process.[9]

Secondly, we estimate equation (5) and test its specification using Hansen's method described earlier. The results are reported in table 1. The estimates of the discount rate θ are very similar in all cases. The real return rates implied by these estimates are approximately four and. five percent, the higher rate for 1871-1988 period. The residuals are white noise

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8. Lejung-Box modification improves the performance of Box-Pierce test in moderate sample sizes by using a more accurate measure of the autocorrelation variance.

9. For brievity, the statistics and estimates computed while identifying the order of the dividend process are not reported here.

as indicated by the insignificance of $Z(\rho)$ which is an asymptotically standard normal statistic under the null hypothesis of independent errors; this supports the assertion of expectational rationality. The equation also passes the stability test since the statistics reported in the fourth column, which are distributed as $\chi_{(q)}$, are all insignificant at the five-percent level. Hansen's test statistic for instrument-residual orthogonality, which reflects the consistency of θ as implied by rational expectations, is insignificant at the five-percent level for all cases, indicating that the predicted dividends obtained from equation (7) are orthogonal to the residuals.[10] This statistic is distributed as $\chi_{(q+1)}$. Given the results of the specification tests performed on equations (7) and (5), it is reasonably concluded that the arbitrage relationship and the order of the dividend process are correctly specified.

Finally, we proceed to test the "no bubble" hypothesis by applying the simplified version of the PSW's differencing test proposed by Davidson, Godfrey and MacKinnon (1985) to equation (9). The simplified differencing test is a simple F-test that examines whether the coefficients of the added variables in the augmented version of equation (9) are jointly zero.[11] The coefficient estimates and the test statistic are reported in table 2. In all cases, the modified PSW statistic, which is distributed as $F_{(q,T-2q-1)}$ under the null, is insignificant at the five-percent level, indicating a failure to reject the null hypothesis that β is consistent. Therefore, it is concluded

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10. See Hansen and Sargent (1981) and West (1987) for details of the implications of rational expectations.
11. The added variable corresponding to each original regressor is the sum of one period lag and one period lead values of that regressor, except for the constant term for which no variable is added.

that equation (9) is also correctly specified and the unobservable bubble term B_t does not belong to this equation.

These results support the no-bubble hypothesis for both periods under study. An implication of this finding is that the market boom of 1982-87 and the October crash do not provide evidence for speculative bubbles in stock prices.

IV. CONCLUDING REMARKS

Economists have long conjectured that movements in stock prices can involve speculative bubbles as speculation is often said to be responsible for overpriced markets and their inevitable crashes. Many economists, however, believe that stock-price fluctuations reflect changes in the values of the underlying market fundamentals; bubbles vs. no bubbles is inherently an empirical issue that is yet unsettled.

This study provides evidence that the behavior of stock prices can be explained by market fundamentals, as the employed tests support the view that the standard arbitrage relationship holds and bubbles do not exist. The test procedure advocated here is not subject to the criticism of testing the "no-bubble", "no-misspecification" hypotheses jointly and is capable of detecting a wide class of bubbles. It is also possible to use this test procedure to test for bubbles in any linear rational expectations model. For future research, it may be of interest to apply this test to the U.S. exchange-rate data.

TABLE 1

REGRESSION RESULTS FOR EQUATION (5)*

Sample Period	q	θ	Z(ρ)	Stability Statistic	Hansen's Statistic
1871-1981	1	0.9634	0.695	2.692	3.904
		(0.0156)			
	2	0.9632	0.685	2.863	4.043
		(0.0156)			
1871-1988	1	0.9521	0.673	3.671	1.818
		(0.0149)			
	2	0.9519	0.674	3.698	2.265
		(0.0149)			

*Standard errors are in parentheses. ρ is the estimated first-order residual autocorrelation coefficient.

Source: Authors.

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