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This paper presents a dynamic model of a bank's optimal choices of imposing a binding liquidity-coverage-ratio (LCR) constraint. Our baseline balance-sheet dynamics starts with portfolio separation and no LCR constraint. Under a scenario in which regulators prohibit banks from applying securities to fulf ll the LCR constraint, portfolio separation continues to hold, but deposit holdings depend on the extent to which the LCR constraint is binding. When banks are allowed to apply securities toward satisfying the constraint, portfolio separation can break down and lead to ambiguous effects on optimal dynamic loan and deposit paths. Our results indicate that under special cases in which portfolio separation holds, the LCR constraint affects bank-sheet dynamics in ways not previously recognized. As regulators move forward in implementing Basel III style LCR, it is imperative to understand the effects of the LCR constraint on bank balance-sheet dynamics.

Keywords: Liquidity-coverage-ratio (LCR), binding constraint, portfolio separation.

JEL Code: G21.

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1. Introduction

A key component of the Basel III standards currently slated for phased adoption by the end of 2018 is a liquidity-coverage-ratio (LCR) constraint. This regulatory constraint, which is tentatively scheduled for implementation during 2015, will require banks to meet a short-term requirement to hold a "stock of high-quality liquidity assets" sufficient to at least cover "total net cash outflows over the next 30 calendar days (Bank for International Settlements, 2010), with a long-term liquidity requirement to be in place by 2018. Under the short-term requirement currently outlined under Basel III, the ratio of qualifying assets must be at least 100 percent of the 30-day net cash outflow. Although a basic framework has been established for determining how an LCR constraint will be applied, specific details regarding computation of the numerator and denominator of the ratio have yet to be determined. Indeed, recent press reports [see, for instance McGrane (2012, Borak (2012), and Enrich (2012)] suggest the possibility that several regulatory adjustments in definitions of the statutory LCR requirement might take place by the 2015 implementation date.

Under the terms of the basic framework, however, it is clear that the numerator of the liquidity-coverage ratio will consist of cash reserves and an allowable portion of banks' security portfolio—essentially a specified set of low-risk and unencumbered securities judged to be readily marketable without resort to "fire sales" during periods of financial stress. Although the Basel agreement indicates that the denominator of the liquidity-coverage ratio is intended to be based on likely forward-looking liquidity requirements, a reading of the requirements (BIS, 2010) indicates that in fact the determination of a 30-day liquidity requirement will be based retroactively on outflows observed in prior periods. Thus, the denominator of a bank's current LCR requirement realistically will be based primarily on deposit outflows observed during a preceding interval.

The dependence of the contemporaneous period's LCR requirement on deposit outflows during the preceding period suggests a dynamic aspect of the Basel III LCR regulation that has not yet received attention from researchers. The objective of this paper is to explore fundamental implications of the intertemporal link that application of an LCR constraint will create for a bank's optimal dynamic paths of deposits and loans. Toward this end, the analysis that follows builds on the work on dynamic models of bank behavior developed initially by Goodfriend (1983), Cosimano (1987, 1988), Cosimano and Van Huyck (1989), and extended since by Elyasiani et al. (1995), Hülsewig et al. (2006) and Kopecky and VanHoose (Forthcoming). Within a model of the behavior of a perfectly competitive bank facing costs of intertemporal adjustments of its deposits and loans, we consider the effects on a bank's optimal balance-sheet adjustments of the introduction of an LCR constraint in which both reserves and a portion of securities appear in the numerator of a liquidity coverage ratio and in which the denominator of the constraint depends on deposit outflows observed in the prior period.

As discussed by Baltensperger (1980), models of any bank's deposit outflows indicate that the key determinant of their magnitudes during a given interval is the stock of deposits issued by the bank that period. This relationship suggests that the bank's observed liquidity-coverage ratio depends on the stock of deposits at that bank during the prior period, which in turn implies that an LCR constraint introduces a source of intertemporal balance-sheet dynamics presently absent from a bank's decision-making process. To highlight how this occurs, we begin the paper by introducing a simple dynamic model in which portfolio separation holds, as in Sealey (1985), so that each bank's optimal deposit and loan paths are independent. Introduction of an LCR constraint in which a portion of banks' securities portfolios can be applied toward meeting the requirement, we find, generally tends to cause portfolio separation to break down, resulting in interdependence of dynamic deposit and loan decisions that previously had been separable.

Indeed, the resulting general solutions for the deposit and loan paths with both reserves and securities being admissible to satisfy an LCR constraint are quite complicated and not readily amenable to tractable analysis regarding effects of alternations in the LCR requirement or the allowed portion of a bank's securities portfolio. In the context of special cases of the model that maintain portfolio separation and, hence, independence of the bank's selection of optimal intertemporal deposit and loan paths, however, we are able to obtain more concrete results regarding of regulatory variations in the nature of an LCR constraint. For instance, when only reserves may be applied to satisfy an LCR requirement, portfolio separation continues to apply. Under either an exogenous increase in average net deposit outflows each period or a regulatory tightening of the liquidity constraint causes a bank's optimal deposit path to exhibit less intertemporal variability but causes the bank's current deposit choice to respond more strongly to changes in the spread between the security rate and deposit rate generated by external market shocks. This result suggests that imposing the most basic, cash-only-based LCR constraint has mixed effects on a bank's deposit stability.

In the case in which regulators allow banks as well to apply a portion of securities to satisfying the LCR requirement, we can obtain solutions readily amenable to analysis for a special case in which banks' equity positions are assumed fixed over the horizon relevant to the model and in which banks are also bound by a Basel-style risk-based capital requirement linking loans to capital. In the more general of these special cases, policy variations in the portion of a bank's security portfolio that can be applied toward satisfying the LCR constraint are theoretically ambiguous. Even for a special case in which the marginal resource cost of adjusting reserves within each period approaches a constant value, the effect of varying the share of allowable securities is ambiguous. A bank's optimal deposit path again becomes more persistent while becoming more sensitive to variations in the security rate generated by market shocks. Indeed, in this special case, as the share of securities that banks may direct toward satisfying the LCR constraint increases, banks essentially become more nearly bifurcated institutions. Each bank directs its share of deposits to lending levels constrained by a risk-based capital ratio and otherwise behaves essentially like a money market mutual fund, varying issuance of deposits that are more elastic to changes in the security rate, with customer preferences ultimately determining the scale of its securities portfolio.

We present and analyze our basic dynamic banking model without and with an LCR constraint in the following section. Section 3 provides the analysis that yields the policy implications that we are able to identify as applicable to the above-referenced special cases of the model. Section 4 summarizes our essential conclusions, discusses qualifications, and offers suggestions for future research.

2. The Banking Model without and with a Required Liquidity Coverage Ratio

Our analysis of the effects of a required LCR constraint is conducted within context of the following framework. An individual bank *i*'s profits at time *t+j* are given by

$$\pi^i_{t+j} = r^s_{t+j} S^i_{t+j} + r^l_{t+j} L^i_{t+j} + r^R R^i_{t+j} - r^d_{t+j} D^i_{t+j} - \frac{\phi_1}{2}\left(L^i_{t+j}\right)^2 - \frac{\phi_2}{2}\left(L^i_{t+j} - L^i_{t+j\text{-}1}\right)^2$$
$$- \frac{\theta_1}{2}\left(D^i_{t+j}\right)^2 - \frac{\theta_2}{2}\left(D^i_{t+j} - D^i_{t+j\text{-}1}\right)^2 - \frac{\Gamma}{2}\left(R^i_{t+j}\right)^2, \tag{1}$$

where
S^i_{t+j} ≡ securities, L^i_{t+j} ≡ loans, R^i_{t+j} ≡ reserves, D^i_{t+j} ≡ deposits, r^k_{t+j} ≡ market interest rate on asset *k* = *S*, *L*, *D*, and r^R ≡ administered interest rate received on excess reserves. All parameters are nonnegative constants.

The profit function in (1) indicates that the bank incurs both quadratic intraperiod resource costs and quadratic intertemporal adjustment costs for deposit and loans. The bank incurs no quadratic intertemporal resource costs for reserves. It faces no explicit resource expense in adjusting its securities. As shown by Elyasiani et al. (1995), who build on the insights of Sealey (1985), in this type of banking model the consequence is that when the bank maximizes its discounted stream of profits subject solely to the balance-sheet constraint in each period,

$$L^i_{t+j} + S^i_{t+j} + R^i_{t+j} = D^i_{t+j}, \tag{2}$$

portfolio separation holds; the bank's choices of deposits , loans, and reserves are chosen independently from one another, with S^i_{t+j} adjusting as necessary to satisfy the balance-sheet constraint. Specifically, as outlined in the Appendix, if the bank maximizes its expected discounted present value, $E_{t+j}\left[\pi^i_{t+j} + \beta\pi^i_{t+j+1} + \ldots\right]$, where β is the discount factor, subject the balance-sheet constraints (2), the solutions for the optimal paths of deposits, loans, and reserves are given by

$$D^i_{t+j} = \eta_1 D^i_{t+j\text{-}1} + \left(\frac{\eta_1}{\theta_2}\right)\left(r^s_{t+j} - r^d_{t+j}\right) + \left(\frac{\eta_1}{\theta_2}\right)\sum_{n=0}^{\infty}\left(\frac{1}{\eta_2}\right)^n E_{t+j}\left(r^s_{t+j+n} - r^d_{t+j+n}\right), \tag{3a}$$

$$L^i_{t+j} = \eta_3 L^i_{t+j\text{-}1} + \left(\frac{\eta_3}{\phi_3}\right)\left(r^l_{t+j} - r^s_{t+j}\right) + \left(\frac{\eta_3}{\phi_3}\right)\sum_{n=0}^{\infty}\left(\frac{1}{\eta_4}\right)^n E_{t+j}\left(r^l_{t+j+n} - r^s_{t+j+n}\right), \tag{3b}$$

and

$$R^i_{t+j} = \left(\frac{1}{\Gamma}\right) r^R, \tag{3c}$$

with the solution for securities implied by (2), where the η_m (m = 1, 2, 3, 4) are positive-valued characteristic roots, with $0 \le \eta_1, \eta_3 \le 1$ and with η_2 and η_4 denoting unstable roots with values exceeding $\frac{1}{\beta} > 1$. T he values of the characteristic roots are given by $0 < \eta_1 = \left(\frac{1}{2\beta\theta_2}\right)\left(\left[\theta_1 + (1+\beta)\theta_2\right] - \sqrt{\left[\theta_1 + (1+\beta)\theta_2\right]^2 - 4\beta\theta_2^2}\right) < 1,$

$\eta_2 = \left(\frac{1}{2\beta\theta_2}\right)\left(\left[\theta_1 + (1+\beta)\theta_2\right] + \sqrt{\left[\theta_1 + (1+\beta)\theta_2\right]^2 - 4\beta\theta_2^2}\right) > 1,$

$0 < \eta_3 = \left(\frac{1}{2\beta\phi_2}\right)\left(\left[\phi_1 + (1+\beta)\phi_2\right] - \sqrt{\left[\phi_1 + (1+\beta)\phi_2\right]^2 - 4\beta\phi_2^2}\right) < 1,$, and

$\eta_4 = \left(\frac{1}{2\beta\phi_2}\right)\left(\left[\phi_1 + (1+\beta)\phi_2\right] + \sqrt{\left[\phi_1 + (1+\beta)\phi_2\right]^2 - 4\beta\phi_2^2}\right) > 1.$

Equation (3a) indicates that, as in Cosimano and Van Huyck (1989) and Elyasiani et al. (1995), under portfolio separation the bank's contemporaneous deposit choice depends positively on lagged deposits, the spread between the current spread on the security rate and the deposit rate, and expected future spreads between these two interest rates. Equation (3b) implies that as in Cosimano (1987, 1988), Elyasiani et al. (1995), and Hülsewig et al. (2006), the current lending choice by the bank is positively related to lagged loans, the contemporaneous spread between the loan rate and security rate, and expected future spreads between this pair of interest rates. Equation (3c) shows that in this simple setup, the bank desires an amount of reserves that solely depends, positively, and on the central bank's interest rate on reserves. Thus, if that rate remains unchanged, the bank's desired reserve holdings remain fixed over time.

Of course, this banking model is not intended to be fully realistic. For instance, if a bank also faced intertemporal reserve adjustment costs, equation (3c) would involve intertemporal dynamics analogous to those in (3a) and (3b). In addition, the bank realistically might have other motives affecting its reserve holdings. For instance, it might have to meet a reserve requirement constraint, and if the bank were based in the United States it might adjust its reserves in conjunction with sweeps between two different types of deposit accounts [see, for example, Dutkowsky and VanHoose (2011)]. Furthermore, a traditional motive for holding reserves in excess of any required reserves is as a buffer against unanticipated customer demands, such as deposit withdrawals [see, for instance, Baltensperger's (1980) survey of such approaches]. The bank's reserve choice is intentionally simplified in this model to allow for a stark and tractable comparison with a banking environment in which a regulatory LCR constraint is binding.

In such a regulatory setting, we assume that the bank faces an additional constraint, given by

$$\alpha D^{i}_{t+j\text{-}1} \leq \gamma S^{i}_{t+j} + R^{i}_{t+j} \,. \tag{4}$$

which when a binding constraint holds as an equality. According to this constraint, that the contemporaneous sum of banks' holdings of an allowed portion ($0 \leq \gamma \leq 1$) of securities and of reserves must be sufficient to cover the observed deposit outflow in the previous period, where α was the observed fraction of deposits that flowed from the bank during that prior interval. As a simplification, α is assumed to be a constant, exogenous fraction in every period—which would be consistent with the assumption of a uniform distribution governing deposit outflows. Hence, a rise in α suggests greater liquidity demands on the bank by its depositors. Note that if the LCR constraint were expressed as a fraction of the previous period's outflow from deposits, then a change the value of α also could reflect a variation in the regulatory constraint.

When an LCR constraint is binding, the bank maximizes its expected discounted present value subject to both sets of constraints (2) and (4). Note, however, that under the assumption that the both sets of constraints are binding, the two constraints can be combined by substituting out reserves to obtain

$$S^i_{t+j} = \frac{D^i_{t+j} - \alpha D^i_{t+j-1} - L^i_{t+j}}{(1-\gamma)} . \tag{5}$$

Hence, using (5) to substitute for S^i_{t+j} throughout the expression for the expected discounted stream of profits $E_{t+j}\left[\pi^i_{t+j} + \beta\pi^i_{t+j+1} + \ldots\right]$ and to substitute for S^i_{t+j} in (4) –as an equality—and maximizing with respect to deposits and loans, [with reserves then ultimately determined by (4) and securities again determined by (2)] yields expected-discounted-value solutions under both the balance-sheet and LCR constraints.

To help in understanding the nature of the solutions that ultimately must emerge in the presence of the LCR constraint, note that (5) implies that there are terms involving both D^i_{t+j} in the expression for $\beta\pi^i_{t+j+1}$. Thus, the fact that the regulatory liquidity constraint relates contemporaneous reserves and securities (for γ > 0) to lagged deposits period after period means that the presence of this constraint also implies that the bank's contemporaneous deposit choice must impinge on expected future loans, reserves, and securities. Furthermore, in the presence of intertemporal loan adjustment costs, the contemporaneous deposit decision of the bank also must influence its current lending decision. These facts explain the fact that the Euler equations for deposits and loans generally are interdependent:

$$\begin{aligned}
&\left[\left(1-\gamma\right)^{2}\theta_{2}+\alpha\gamma\Gamma\left(1-\gamma\right)+\alpha\gamma\Gamma\right]D^{i}_{t+j-1}\\
&\quad-\left\{\left(1-\gamma\right)^{2}\left[\theta_{1}+\theta_{2}\left(1+\beta\right)\right]+\alpha^{2}\beta\Gamma\left(1-\gamma\right)+\alpha^{2}\beta\Gamma+\Gamma\gamma^{2}\right\}D^{i}_{t+j}\\
&\quad+\left[\beta\theta_{2}\left(1-\gamma\right)^{2}+\alpha\beta\gamma\Gamma\right]E_{t+j}D^{i}_{t+j+1}+\Gamma\gamma^{2}L^{i}_{t+j}-\alpha\beta\gamma\Gamma E_{t+j}L^{i}_{t+j+1}\\
&=-\left(1-\gamma\right)r^{s}_{t+j}+\gamma\left(1+\alpha\beta\right)\left(1-\gamma\right)r^{R}+\left(1-\gamma\right)^{2}r^{d}_{t+j}+\alpha\beta\left(1-\gamma\right)E_{t+j}r^{s}_{t+j+1},
\end{aligned} \tag{6a}$$

and

$$\begin{aligned}
&\left(1-\gamma\right)^{2}\phi_{2}L^{i}_{t+j-1}-\left\{\left(1-\gamma\right)^{2}\left[\phi_{1}+\phi_{2}\left(1+\beta\right)\right]-\gamma^{2}\Gamma\right\}L^{i}_{t+j}+\left(1-\gamma\right)^{2}\beta\phi_{2}E_{t+j}L^{i}_{t+j+1}\\
&\qquad+\gamma^{2}\Gamma\left[\alpha D^{i}_{t+j-1}+D^{i}_{t+j}\right]=\left(1-\gamma\right)r^{s}_{t+j}-\left(1-\gamma\right)^{2}r^{l}_{t+j}+\gamma\left(1-\gamma\right)r^{R}
\end{aligned} \tag{6b}$$

Thus, portfolio separation no longer necessarily holds in the setting in which securities can partially be applied and reserves fully applied to satisfy the binding regulatory LCR constraint,. Instead, portfolio interdependence generally prevails.

3. Evaluating the Dynamic Implications of the Basel Liquidity Constraint

What does the model imply about the effects of imposing a binding LCR constraint on the bank? Not surprisingly, the answer to this question hinges on parameters governing both the regulatory constraint and the bank's costs.

3.1 Exclusion of Securities from Satisfying the Liquidity Constraint

There is one special case in which imposing the LCR constraint fails to break down portfolio separation at the bank. Examination of equations (6) indicates that in the special case γ = 0, in which regulators do not permit banks to apply any securities to satisfy the liquidity constraint but instead must require banks to rely solely on reserves to do so, deposit and loan choices remain independent. To understand why this is so, note that in this case with γ = 0, the fact that (4) is a binding constraint implies that the bank's reserves are given by

$$R^i_{t+j} = \alpha D^i_{t+j-1} . \tag{7}$$

Consequently, with reserves predetermined each period based on the prior interval's deposit outflow, there is no source of deposit-loan interdependence added by imposing the LCR requirement. Portfolio separation applies as before, as do the solutions for deposits and loans in (3a) and (3b). The reserve solution in (7) replaces (3c), and securities adjust to satisfy the balance-sheet constraint.

Thus, the following are the resulting independent solutions for a bank's deposits and loans at any given time *t* for this special case of decision-making under the liquidity constraint:

$$D^i_t = \lambda_1 D_{t-1} + \left(\frac{\lambda_1}{\theta_2}\right)\left(r^s_t - r^d_t\right) + \left(\frac{\lambda_1}{\theta_2}\right)\sum_{i=0}^{\infty}\left(\frac{1}{\lambda_2}\right)^i E_t\left(r^s_{t+i} - r^d_{t+i}\right) , \tag{8a}$$

and

$$L^i_t = \lambda_3 L_{t-1} + \left(\frac{\lambda_3}{\phi_2}\right)\left(r^l_t - r^s_t\right) + \left(\frac{\lambda_3}{\phi_2}\right)\sum_{i=0}^{\infty}\left(\frac{1}{\lambda_4}\right)^i E_t\left(r^l_{t+i} - r^s_{t+i}\right) , \tag{8b}$$

where

$$0 < \lambda_1 = \left(\frac{1}{2\beta\theta_2}\right)\left(\left[\theta_1 + (1+\beta)\theta_2 + 2\alpha^2\beta\Gamma\right] - \sqrt{\left[\theta_1 + (1+\beta)\theta_2 + 2\alpha^2\beta\Gamma\right]^2 - 4\beta\theta_2^2}\right) < 1,$$

$$\lambda_2 = \left(\frac{1}{2\beta\theta_2}\right)\left(\left[\theta_1 + (1+\beta)\theta_2 + 2\alpha^2\beta\Gamma\right] + \sqrt{\left[\theta_1 + (1+\beta)\theta_2 + 2\alpha^2\beta\Gamma\right]^2 - 4\beta\theta_2^2}\right) > 1,$$

$$0 < \lambda_3 = \eta_3 = \left(\frac{1}{2\beta\phi_2}\right)\left(\left[\phi_1 + (1+\beta)\phi_2\right] - \sqrt{\left[\phi_1 + (1+\beta)\phi_2\right]^2 - 4\beta\phi_2^2}\right) < 1,$$

and

$$\lambda_4 = \eta_4 = \left(\frac{1}{2\beta\phi_2}\right)\left(\left[\phi_1 + (1+\beta)\phi_2\right] + \sqrt{\left[\phi_1 + (1+\beta)\phi_2\right]^2 - 4\beta\phi_2^2}\right) > 1.$$

Note immediately that with a current period's reserves related to the prior period's deposits through the LCR requirement with γ = 0, the bank's optimal deposit path implied by (8a) depends on the bindingness of the liquidity constraint determined by the magnitude of α, but the bank's optimal path for loans implied by (8b) does not. The reason for this is can be seen by substitution of (7) into the balance-sheet constraint, yielding the constraint that the bank's balance-sheet choices must satisfy at any given time *t*:

$$L_t^i + S_t^i + \alpha D_{t-1}^i = D_t^i. \quad (9)$$

Under this overall constraint, the bank's choices for deposits over time clearly are governed by the magnitude of α, but the bank can optimally vary loans and securities simultaneously. With the securities not eligible for inclusion to satisfy the LCR requirement, these assets can be varied as necessary to satisfy the bank's balance-sheet constraint. This fact leaves the bank free to dynamically adjust loans optimally—and independently from deposit adjustments and from the liquidity constraint and hence without regard to the magnitude of α. Indeed, the solution for the dynamic loan path in (8b) is identical to the solution given by (3b) for the unconstrained case.

From the solution for λ_1 in (8a), it can be shown that

$$\frac{\partial \lambda_1}{\partial \alpha} = \left(\frac{\Gamma}{\theta_2}\right)\left(1-\left(\frac{1}{2}\right)\frac{\left[\theta_2+(1+\beta)\theta_2+2\alpha^2\beta\Gamma\right]}{\sqrt{\left[\theta_2+(1+\beta)\theta_2+2\alpha^2\beta\Gamma\right]^2-\beta\theta_2^2}}\right)>0$$

holds unambiguously since the discount factor β is less than unity. In addition, the solution for λ_2 yields $\frac{\partial \lambda_2}{\partial \alpha} > \frac{\partial \lambda_1}{\partial \alpha} > 0$. Taken together, these facts suggest that when only reserves are eligible to satisfy a regulatory LCR requirement, greater bindingness of that constraint boosts the magnitudes of the coefficients on lagged deposits and on the current spread between the security and deposit rates in (8a) while reducing the sizes of the coefficients on expected future rate spreads.

Thus, either an exogenous increase in average net deposit outflows or a regulatory tightening of the liquidity constraint induces the bank to opt for lessened adjustments of contemporaneous deposits to changes in lagged deposits. In the face of a more binding LCR requirement, therefore, the bank's optimal deposit path exhibits greater persistence and hence less intertemporal variability, *ceteris paribus*.

At the same time, however, the bank's contemporaneous deposit level responds more strongly to changes in the current spread generated by external market shocks. In the knowledge that it will have to maintain a more persistent deposit level over time to support reserves to satisfy the regulatory liquidity constraint, the bank responds to a more binding constraint by boosting its current deposits by a larger among in response to shock-induced changes in the current rate spread.

Nevertheless, at time *t* the bank also recognizes that the greater persistence of its deposit base is necessary to satisfy the LCR requirement will give it less margin for adjusting its deposits in light of changes in future rate spreads. Consequently, its contemporaneous deposit level becomes less sensitive to anticipated changes in these future spreads.

The first and third of these reactions of the bank's deposit path are consistent with a tightening of a purely reserve-based LCR requirement with an aim to generate greater intertemporal stability of bank deposits. The second reaction, however, is not. Greater bindingness of the LCR constraint increases the reaction of bank deposits to contemporaneous shocks to the spread between an open-market rate and the bank deposit rate.

3.2 Inclusion of Securities for Satisfying the Liquidity Constraint

Currently, the Basel III plan for implementation of the LCR constraint will allow banks to utilize designated high-quality securities to satisfy the regulatory requirement alongside reserves. In the context of our model, this corresponds to a regulatory environment in which $0 < \gamma \leq 1$ in (4) and (5). In this circumstance, portfolio separation unambiguously breaks down.

Why is this so? The answer is that a bank's securities holdings now must be adjusted each period in part to satisfy the LCR constraint; the bank can no longer adjust securities as required to satisfy solely the balance-sheet constraint. Furthermore, contemporaneous securities holdings must be adjusted in response to lagged deposits, which implies as well that the bank anticipates that it will have to adjust its future securities holdings in response to its current deposit choice. At the same time, generally the bank's reserve holdings must reflect adherence to the LCR constraint as well. Taken together, these facts imply that the bank's contemporaneous loans must be adjusted in accordance with the liquidity-constraint-influenced levels of deposits, securities, and reserves. Thus, all four balance-sheet items ultimately are contemporaneously interdependent, which in turn yields the intertemporal interdependence of deposits and loans suggested by equations (6).

In this case,

$$\begin{aligned}
D_t &= (\upsilon_1 + \upsilon_3) D_{t-1} + \upsilon_1 \upsilon_3 D_{t-2} \\
&+ \sum_{i=0}^{\infty} \sum_{k=0}^{\infty} \left(\frac{1}{\upsilon_2}\right)^{i+1} \left(\frac{1}{\upsilon_4}\right)^{k+1} E_t \left\{ \left(\frac{(1-\gamma)\left[-\beta(1-\gamma)^2 \phi_2 + \alpha\beta^2\gamma\Gamma\left(\theta_2(1-\gamma)^2 + \alpha\gamma\Gamma\right) + \alpha\beta\left\{(1-\gamma)^2\left[\phi_1 + (1+\beta)\phi_2\right] - \Gamma\gamma^2\right\}\right]}{\beta^2\left[\theta_2(1-\gamma)^2 + \alpha\gamma\Gamma\right](1-\gamma)^2\phi_2} \right) r^s_{t+j} \right. \\
&\quad + \left(\frac{(1-\gamma)}{\beta\left[\theta_2(1-\gamma)^2 + \alpha\gamma\Gamma\right]} \right) r^d_{t+j} - \left(\frac{\alpha\beta\gamma\Gamma}{\beta^2\left[\theta_2(1-\gamma)^2 + \alpha\gamma\Gamma\right](1-\gamma)\phi_2} \right) r^l_{t+j} \\
&\quad + \left(\frac{(1-\gamma)\left[\alpha\beta(1-\gamma)\phi_2 + \left\{(1-\gamma)^2\left[\phi_1 + (1+\beta)\phi_2\right] - \Gamma\gamma^2\right\} - \Gamma\gamma^2\right]}{\beta^2\left[\theta_2(1-\gamma)^2 + \alpha\gamma\Gamma\right](1-\gamma)^2\phi_2} \right) r^s_{t+j-1} \\
&\quad + \left(\frac{\alpha\beta(1-\gamma)}{\beta\left[\theta_2(1-\gamma)^2 + \alpha\gamma\Gamma\right]} \right) r^s_{t+j+1} - \left(\frac{(1-\gamma)^2\left\{(1-\gamma)^2\left[\phi_1 + (1+\beta)\phi_2\right] - \Gamma\gamma^2\right\}}{\beta\left[\theta_2(1-\gamma)^2 + \alpha\gamma\Gamma\right]} \right) r^d_{t+j-1} \\
&\quad \left. - \left(\frac{(1-\gamma)^4\phi_2}{\beta\left[\theta_2(1-\gamma)^2 + \alpha\gamma\Gamma\right]} \right) \left[r^s_{t+j-2} - (1-\gamma) r^d_{t+j-2}\right] + \left(\frac{(1-\gamma)^2\Gamma\gamma^2}{\beta^2\left[\theta_2(1-\gamma)^2 + \alpha\gamma\Gamma\right](1-\gamma)^2\phi_2} \right) r^l_{t+j-2} \right\},
\end{aligned} \tag{10}$$

and

$$
\begin{aligned}
L_t &= (\upsilon_1+\upsilon_3)L_{t-1}+\upsilon_1\upsilon_3 L_{t-2} \\
&+\sum_{i=0}^{\infty}\sum_{k=0}^{\infty}\left(\frac{1}{\upsilon_2}\right)^{i+1}\left(\frac{1}{\upsilon_4}\right)^{k+1} E_t\left\{\left(\frac{(1-\gamma)\beta\left\{\left[\theta_2(1-\gamma)^2+\alpha\gamma\Gamma\right]-\alpha^2\Gamma\gamma^2\right\}}{\beta^2\left[\theta_2(1-\gamma)^2+\alpha\gamma\Gamma\right](1-\gamma)^2\phi_2}\right)r^s_{t+j}\right. \\
&\qquad +\left(\frac{\left\{(1-\gamma)^2\left[\theta_1+(1+\beta)\theta_2\right]+\alpha^2\beta\Gamma(2-\gamma)+\gamma^2\Gamma\right\}+(1-\gamma)\alpha\gamma^2\Gamma-(1-\gamma)\alpha\beta\gamma\Gamma}{\beta^2\left[\theta_2(1-\gamma)^2+\alpha\gamma\Gamma\right](1-\gamma)^2\phi_2}\right)r^s_{t+j-1} \\
&\qquad -\left(\frac{(1-\gamma)}{\beta^2\left[\theta_2(1-\gamma)^2+\alpha\gamma\Gamma\right](1-\gamma)\phi_2}\right)r^l_{t+j} \\
&\qquad +\left(\frac{\alpha\beta(1-\gamma)\gamma\Gamma}{\beta\left[\theta_2(1-\gamma)^2+\alpha\gamma\Gamma\right]}\right)r^s_{t+j+1}-\left(\frac{(1-\gamma)^2\alpha\Gamma\gamma^2}{\beta^2\left[\theta_2(1-\gamma)^2+\alpha\gamma\Gamma\right](1-\gamma)^2\phi_2}\right)r^d_{t+j-1} \\
&\qquad -\left(\frac{(1-\gamma)\gamma\Gamma}{\beta^2\left[\theta_2(1-\gamma)^2+\alpha\gamma\Gamma\right](1-\gamma)^2\phi_2}\right)\left[r^d_{t+j-2}-(1-\gamma)r^d_{t+j-2}\right] \\
&\qquad \left.+\left(\frac{\Gamma\gamma^2\left\{(1-\gamma)^2\theta_2+\alpha\beta\Gamma(2-\gamma)\right\}}{\beta\phi_2}\right)\left[r^l_{t+j-2}-(1-\gamma)r^d_{t+j-2}\right]\right\}.
\end{aligned}
\tag{11}
$$

Note that equations (10) and (11) suggest, as in Elyasiani et al. (1995), that when portfolio interdependence arises—in this model, from application of an regulatory LCR constraint that is at least partially satisfied by securities holdings—the bank's optimal deposit and loan paths depend on forward expectations of all relevant interest rates as well as multiple lags of those rates. Furthermore, contemporaneous levels of deposits and loans depend on both one- and two-period lags of previous levels. As discussed by Elyasiani et al. (p. 960), the two-period lags appear because, with portfolio interdependence, increasing, loans (deposits) results in both a direct loan (deposit) adjustment cost and an *indirect* deposit (loans) adjustment cost that impinges on the cost of adjusting loans (deposits) and consequently lengthens the loan (deposit) lag structure.

Of course, (10) and (11) are only semi-reduced-form solutions in terms of the characteristic roots υ_1, υ_2, υ_3, and υ_4, which, as discussed in the Appendix, must satisfy a four-equation non-linear system involving the underlying structural parameters of the model. Very lengthy expressions for these characteristic roots obtained via application of Mathematica are available from the authors upon request. The resulting solutions for the coefficients on lagged levels and lagged, current, and expected future interest rates are exceedingly complex and not readily amenable to further analysis. Thus, in an effort to examine further the implications of an LCR constraint that may be at least partially satisfied by securities holdings, in the subsections that follow we consider special cases of the more general framework.

3.2 Inclusion of Securities for Satisfying the Liquidity Constraint Coupled with a Binding Risk-Based Capital Requirement and Fixed Equity Capital

Under the Basel III plan currently scheduled for phased-in implementation to be completed by the end of 2018, the LCR constraint is to be employed alongside risk-based capital regulation. What if risk-based capital requirements are binding and a bank cannot adjust its equity capital position during the time horizon relevant for our dynamic model?

To answer this question, suppose that the bank's lending is constrained each period by a risk-based capital requirement, $L^{i}_{t+j} = \frac{\overline{E}}{\psi}$, where $\overline{E}$ is equity capital that is assumed invariant over the model's applicable time frame and that generates fixed costs consequently irrelevant to the bank's marginal balance-sheet choices, and where ψ is the required capital ratio specified for loans, with securities assumed to be riskless and consequently not included among risk-weighted assets encompassed by capital regulation. Essentially, in this case the bank's functions are subdivided by regulation, with the binding capital requirement limiting its lending and the LCR constraint influencing its optimal deposit path and hence its choices of securities and reserves each period in light of the cost parameters it faces. Because loans are bound by the capital constraint to this constant value each period, the bank's optimal path for deposits becomes

$$D_t^i = \kappa_1 D_{t-1} + \left(\frac{\kappa_1}{\beta\theta_2(1-\gamma)^2 + \alpha\beta\gamma\Gamma}\right)\left(r_t^s - r_t^d\right) + \left(\frac{\kappa_1}{\beta\theta_2(1-\gamma)^2 + \alpha\beta\gamma\Gamma}\right)\sum_{i=0}^{\infty}\left(\frac{1}{\kappa_2}\right)^i E_t\left(r_{t+i}^s - r_{t+i}^d\right) \tag{12}$$

where

$$0 < \kappa_1 = \left(\frac{1}{2\beta\left[\theta_2(1-\gamma)^2 + \alpha\beta\Gamma\right]^2}\right)\left(\begin{array}{c}\left\{\left[\theta_1 + (1+\beta)\theta_2\right](1-\gamma)^2 + 2\alpha^2\beta\Gamma + \gamma\Gamma\left(\gamma - \alpha^2\beta\right)\right\} \\ -\sqrt{\left\{\left[\theta_1 + (1+\beta)\theta_2\right](1-\gamma)^2 + 2\alpha^2\beta\Gamma + \gamma\Gamma\left(\gamma - \alpha^2\beta\right)\right\}^2 - 4\beta\theta_2^2(1-\gamma)^4}\end{array}\right) < 1,$$

and

$$\kappa_2 = \left(\frac{1}{2\beta\left[\theta_2(1-\gamma)^2 + \alpha\beta\Gamma\right]^2}\right)\left(\begin{array}{c}\left\{\left[\theta_1 + (1+\beta)\theta_2\right](1-\gamma)^2 + 2\alpha^2\beta\Gamma + \gamma\Gamma\left(\gamma - \alpha^2\beta\right)\right\} \\ +\sqrt{\left\{\left[\theta_1 + (1+\beta)\theta_2\right](1-\gamma)^2 + 2\alpha^2\beta\Gamma + \gamma\Gamma\left(\gamma - \alpha^2\beta\right)\right\}^2 - 4\beta\theta_2^2(1-\gamma)^4}\end{array}\right) > 1.$$

Examination of these coefficients indicates that, when bank equity is fixed and banks are bound by risk-based capital requirement the constrains lending, a policy variation in γ, the share of securities permissible for satisfying the LCR constraint, has analytically ambiguous effects on the coefficients of (12).

Note that in the limiting special case in which $\Gamma \to 0$, so that the marginal resource cost associated with managing reserves approaches a constant value, equation (9) reduces to

$$D_t^i = \kappa_1 D_{t-1} + \left(\frac{\overline{\kappa}_1}{\beta\theta_2(1-\gamma)^2}\right)\left(r_t^s - r_t^d\right) + \left(\frac{\overline{\kappa}_1}{\beta\theta_2(1-\gamma)}\right)\sum_{i=0}^{\infty}\left(\frac{1}{\overline{\kappa}_2}\right)^i E_t\left[r_{t+i}^s - (1-\gamma)r_{t+i}^d\right] \tag{12'}$$

where

$$0 < \overline{\kappa}_1 = \left(\frac{1}{2\beta\theta_2(1-\gamma)^2}\right)\left(\left[\theta_1 + (1+\beta)\theta_2\right] - \sqrt{\left[\theta_1 + (1+\beta)\theta_2\right]^2 - 4\beta\theta_2^2}\right) < 1,$$

and

$$0 < \bar{\kappa}_2 = \left(\frac{1}{2\beta\theta_2(1-\gamma)^2} \right) \left(\left[\theta_1 + (1+\beta)\theta_2 \right] + \sqrt{\left[\theta_1 + (1+\beta)\theta_2 \right]^2 - 4\beta\theta_2^2} \right) > 1.$$

Equation (12′) indicates that for this special case, the coefficients governing the optimal deposit path are all increasing in γ. With $\Gamma \to 0$, portfolio separation holds for both securities and reserves. Thus, when the bank is permitted to apply a larger share of securities to satisfy the LCR constraint, it is optimal for the bank to issue more deposits when current or expected future security-deposit rate spreads increase. The bank utilizes funds from the extra deposits it issues both to help satisfy the LCR constraint and also earn more interest income. Given that portfolio separation applies to reserves as well as securities, the bank is free to adjust reserves as required to satisfy the balance sheet constraint. Thus, the bank exhibits a reduced degree of adjustment of contemporaneous deposits in relation to deposits in the previous period.

Note, however, that in this special case of the binding-capital-regulation/fixed-equity version of the model, the optimal deposit path tends to be become unbounded when policymakers raise the value of γ toward unity. This is so because with $\Gamma \to 0$, there are no further constraints—from banks' perspectives—on overall scale. Increasing γ toward unity gives banks an incentive to keep expand deposits without bound and apply securities toward meeting the LCR constraint in place of reserves. This fact suggests that if Γ is empirically "small," coupling a tougher, more fully binding system of capital requirements with application of an LCR constraint that banks could largely meet using interest-bearing securities would lead to bifurcated banks using deposits to fund constrained loans and otherwise operating like mutual funds by expanding their deposits and security holdings to the limits of the market as determined by depositors' preferences.

Otherwise, even with equity capital fixed and bank lending capital-constrained over time to simplify the dynamics of deposit adjustment, changes in the share of securities eligible to satisfy the LCR constraint lead to theoretically ambiguous effects on the optimal deposit path. Naturally, allowing for endogenous equity would make lending once more dynamically endogenous as well, leading to results analogous to (10) and (11) in their complexity and associated theoretical uncertainties regarding the effects of tightening the LCR constraint and altering the share of securities that banks can use to satisfy it.

4. Conclusion

Analysis of our dynamic model of the application of a contemporaneous LCR constraint based on deposit-related outflows from prior period suggests that when regulators require securities to be excluded from computations of banks' liquidity coverage ratio, a toughening of the LCR requirement has mixed effects on the stability of a bank's deposit base. On one hand, a bank's optimal desired deposit path becomes more intertemporally persistent, but on the other hand, the bank's desired deposit level becomes more responsive to variations in the spread between security and deposit rates generated by external market shocks. In our model, if regulators allow banks to apply a portion of their securities portfolios to satisfying the LCR constraint, the general outcome is a breakdown in portfolio separation. The consequence is a potentially complex interdependence between a bank's optimal deposit and loan paths that can make the implications of policy variations in LCR requirements analytically intractable—or at least very complicated. Evaluation of a special case in which banks are constrained to fixed equity during the horizon of the dynamic model and in which a Basel-style risk-based capital requirement constrains lending suggests that increasing the allowed share of securities again leads under most circumstances to mixed effects on the stability of a bank's deposits. The optimal deposit path tends to exhibit greater persistence when more securities can be utilized to satisfy the LCR requirement, but deposits respond more elastically to variations in the market security rate.

Irrespective of the above specific implications of our study, the fundamental implication of our analysis is that subjecting banks to an LCR requirement has heretofore unexplored implications for bank balance-sheet dynamics. As a consequence, imposing an LCR constraint involving varying parameters applied to computations of the numerator and denominator of the requirement may generate bank responses that are not necessarily fully consistent with a policy objective of greater dynamic stability of bank deposits and loans. In several cases, we find that the results could be greater intertemporal persistence of banks' deposits but at the same time increased responsiveness of deposits to interest rate variations potentially arising from external market shocks.

It is important to keep in mind that we have obtained these conclusions in the context of a model in which portfolio separation is assumed to be applicable in advance of imposing an LCR constraint. As discussed by VanHoose (2010, pp. 38-40), there is relatively little empirical evidence regarding whether portfolio separation currently holds for banking firms. The general- and special-case results we have obtained hinge on the commonly presumed presence of portfolio separation in today's banking system—a condition that our analysis suggests likely will not generally hold true under an LCR requirement in which securities can partially satisfy. If we had begun our analysis by following Elyasiani et al. (1995) and presuming portfolio interdependence at the outset, we would have been confronted with a host of analytical complexities at the outset that would only expand with the introduction of another source of intertermporal balance-sheet interdependence operating through an LCR constraint.

Our paper offers two key implications for future work seeking to understand the impacts of a Basel-III-style liquidity-coverage-ratio requirement. First, analysis of the effects of an LCR constraint are incomplete without some explicit recognition of the dynamic effects of the constraint that operate via the computation of the LCR-requirement's denominator using past experience on cash outflows that depend directly on a bank's lagged deposits. Thus, future theoretical and empirical work examining an LCR constraints impacts on bank behavior must allow for a time dimension. Second, the paucity of empirical work to date regarding the relevance of the portfolio-separation condition to banking firms and the dynamic interdependence of banks' asset and liability decisions cannot remain the prevailing norm if researchers truly wish to understand the impacts of Basel III. The increasingly pervasive array of regulatory constraints adopted and proposed under the Basel Accords will alter the intertemporal margins across which banks must operate, so there is a pressing need for a much better empirical understanding of the real-world dynamics of banks' balance sheets.

APPENDIX

After substituting the liquidity-coverage-ratio constraint and the balance sheet constraint, $E_{t+j}\left[\pi^i_{t+j} + \beta\pi^i_{t+j+1} + \ldots\right]$ can be written in the following form:

$$
\begin{aligned}
\pi^i_{t-j} + \beta E_{t+j}\pi^i_{t+j+1} + \ldots = {} & r^s_{t+j}\left[\frac{D^i_{t+j} - \alpha D^i_{t+j-1} - L^i_{t+j}}{(1-\gamma)}\right] + r^l_{t+j}L^i_{t+j} \\
& + r^R\left[\alpha D^i_{t+j\text{-}1} - \left(\frac{\gamma D^i_{t+j} - \alpha D^i_{t+j-1} - \gamma L^i_{t+j}}{(1-\gamma)}\right)\right] - r^d_{t+j}D^i_{t+j} \\
& - \frac{\phi_1}{2}\left(L^i_{t+j}\right)^2 - \frac{\phi_2}{2}\left(L^i_{t+j} - L^i_{t+j\text{-}1}\right)^2 - \frac{\theta_1}{2}\left(D^i_{t+j}\right)^2 - \frac{\theta_2}{2}\left(D^i_{t+j} - D^i_{t+j\text{-}1}\right)^2 \\
& - \frac{\Gamma}{2}\left[\alpha D^i_{t+j\text{-}1} - \left(\frac{\gamma D^i_{t+j} - \alpha D^i_{t+j-1} - \gamma L^i_{t+j}}{(1-\gamma)}\right)\right]^2 \\
& + E_{t+j}\Bigg\{\beta r^s_{t+j+1}\left[\frac{D^i_{t+j+1} - \alpha D^i_{t+j} - L^i_{t+j+1}}{(1-\gamma)}\right] + \beta r^l_{t+j+1}L^i_{t+j+1} \\
& + \beta r^R\left[\alpha D^i_{t+j} - \left(\frac{\gamma D^i_{t+j+1} - \alpha D^i_{t+j} - \gamma L^i_{t+j+1}}{(1-\gamma)}\right)\right] \\
& - \beta r^d_{t+j+1}D^i_{t+j+1} - \frac{\beta\phi_1}{2}\left(L^i_{t+j+1}\right)^2 - \frac{\beta\phi_2}{2}\left(L^i_{t+j+1} - L^i_{t+j}\right)^2 \\
& - \frac{\beta\theta_1}{2}\left(D^i_{t+j+1}\right)^2 - \frac{\beta\theta_2}{2}\left(D^i_{t-j+1} - D^i_{t-j}\right)^2 \\
& - \frac{\beta\Gamma}{2}\left[\alpha D^i_{t+j} - \left(\frac{\gamma D^i_{t+j+1} - \alpha D^i_{t+j} - \gamma L^i_{t+j+1}}{(1-\gamma)}\right)\right]^2 + \ldots\Bigg\}
\end{aligned}
\tag{A1}
$$

The Euler equation for deposits is determined by

$$
\begin{aligned}
\frac{\partial\left(\pi^i_{t+j}+\beta E_{t+j}\pi^i_{t+j+1}+\ldots\right)}{\partial D^i_{t+j}} &= \frac{1}{\left(1-\gamma\right)} r^s_{t+j} - r^d_{t+j} \\
&-\theta_1 D^i_{t+j} - \theta_2\left(D^i_{t+j} - D^i_{t+j-1}\right) \\
&+\Gamma\frac{\gamma}{\left(1-\gamma\right)}\left[\alpha D^i_{t+j-1} - \left(\frac{\gamma D^i_{t+j} - \alpha D^i_{t+j-1} - \gamma L^i_{t+j}}{\left(1-\gamma\right)}\right)\right] \\
&-\left[\frac{\alpha\beta}{\left(1-\gamma\right)}\right] E_{t+j} r^s_{t+j+1} + \beta\theta_2\left(E_{t+j} D^i_{t+j+1} - D^i_{t+j}\right) \\
&-\beta\Gamma\frac{\alpha}{\left(1-\gamma\right)}\left[\alpha D^i_{t+j} - \left(\frac{\gamma E_{t+j} D^i_{t+j+1} - \alpha D^i_{t+j} - \gamma L^i_{t+j+1}}{\left(1-\gamma\right)}\right)\right] = 0.
\end{aligned}
\tag{A2}
$$

Multiplying through by $\left(1-\gamma\right)^2$ and rearranging terms yields (A3) as the general Euler equation for deposits, special cases of which are considered in the text depending on assumptions regarding different parameters:

$$
\begin{aligned}
&\left[\left(1-\gamma\right)^2\theta_2 + \alpha\gamma\Gamma\left(1-\gamma\right) + \alpha\gamma\Gamma\right] D^i_{t+j-1} - \left\{\left(1-\gamma\right)^2\left[\theta_1 + \theta_2\left(1+\beta\right)\right] + \alpha^2\beta\Gamma\left(1-\gamma\right) + \alpha^2\beta\Gamma + \Gamma\gamma^2\right\} D^i_{t+j} \\
&\quad + \left[\beta\theta_2\left(1-\gamma\right)^2 + \alpha\beta\gamma\Gamma\right] E_{t-j} D^i_{t+j+1} + \Gamma\gamma^2 L^i_{t+j} - \alpha\beta\gamma\Gamma E_{t+j} L^i_{t+j+1} \\
&= -\left(1-\gamma\right) r^s_{t-j} + \left(1-\gamma\right)^2 r^d_{t+j} + \alpha\beta\left(1-\gamma\right) E_{t+j} r^s_{t+j+1}.
\end{aligned}
\tag{A3}
$$

The Euler equation for loans, which yields (6b) in the text, is given by

$$\frac{\partial\left(\pi^{i}_{t+j}+\beta E_{t+j}\pi^{i}_{t+j+1}+\ldots\right)}{\partial L^{i}_{t+j}}=-\frac{1}{(1-\gamma)}r^{s}_{t+j}+r^{l}_{t+j}-\phi_1 L^{i}_{t+j}-\phi_2\left(L^{i}_{t+j}-L^{i}_{t+j\text{-}1}\right)$$
$$-\frac{\gamma\Gamma}{(1-\gamma)}\left[\alpha D^{i}_{t+j\text{-}1}-\left(\frac{\gamma D^{i}_{t+j}-\alpha D^{i}_{t+j-1}-\gamma L^{i}_{t+j}}{(1-\gamma)}\right)\right]$$
$$+\beta\phi_2\left(E_{t+j}L^{i}_{t+j+1}-L^{i}_{t+j}\right)=0. \tag{A4}$$

For the model's most general case (again, with special cases discussed in the text emerging under assumptions about values of specific parameters), we have a two-equation dynamic system given by

$$\left(\begin{array}{l}\left[(1-\gamma)^2\theta_2+\alpha\gamma\Gamma(1-\gamma)+\alpha\gamma\Gamma\right]H^2-\left\{(1-\gamma)^2\left[\theta_1+\theta_2(1+\beta)\right]+\alpha^2\beta\Gamma(1-\gamma)+\alpha^2\beta\Gamma+\Gamma\gamma^2\right\}H\\ \quad+\left[\beta\theta_2(1-\gamma)^2+\alpha\beta\gamma\Gamma\right]\end{array}\right)E_{t+j}D^{i}_{t+j+1}$$
$$+\Gamma\gamma\left(\gamma H-\alpha\beta\right)E_{t+j}L^{i}_{t+j+1} \tag{A5a}$$
$$=-(1-\gamma)r^{s}_{t+j}+(1-\gamma)^2 r^{d}_{t+j}+\alpha\beta(1-\gamma)E_{t+j}r^{s}_{t+j+1},$$

and

$$\left((1-\gamma)^2\phi_2H^2-\left\{(1-\gamma)^2\left[\phi_1+\phi_2(1+\beta)\right]-\gamma^2\Gamma\right\}H+(1-\gamma)^2\beta\phi_2\right)E_{t+j}L^{i}_{t+j+1} \tag{A5b}$$
$$+\gamma^2\Gamma\left[\alpha H^2+H\right]E_{t+j}D^{i}_{t+j+1}=(1-\gamma)r^{s}_{t+j}-(1-\gamma)^2 r^{l}_{t+j},$$

The two-equation system in (A5) can be rewritten in terms of the lag operator *H* as the following:

$$\left(1-\Lambda_1 H+\Lambda_2 H^2\right) E_{t+j} D_{t+j+1}+\left(-\Omega_3+\Omega_4 H\right) E_{t+j} L_{t+j+1}=\Upsilon_{t+j} \tag{A6a}$$

and

$$\left(1-\Omega_1 H+\Omega_2 H^2\right) E_{t+j} L_{t+j+1}+\left(\Lambda_3 H+\Lambda_4 H^2\right) E_{t+j} D_{t+j+1}=\Psi_{t+j} \tag{A6b}$$

where

$$\Lambda_1 \equiv \frac{\omega_1}{\omega_3}, \Lambda_2 \equiv \frac{\omega_2}{\omega_3}, \Lambda_3 \equiv \frac{\alpha\gamma^2\Gamma}{\xi_3}, \Lambda_4 \equiv \frac{\gamma\Gamma}{\xi_3}, \Omega_1 \equiv \frac{\xi_1}{\xi_3}, \Omega_2 \equiv \frac{\xi_2}{\xi_3}, \Omega_3 \equiv \frac{\alpha\beta\gamma\Gamma}{\omega_3}, \Omega_4 \equiv \frac{\gamma^2\Gamma}{\omega_3}$$

$$\omega_1 \equiv (1-\gamma)^2\left[\theta_1+\theta_2(1+\beta)\right]+\alpha^2\beta\Gamma(1-\gamma)+\alpha^2\beta\Gamma+\Gamma\gamma^2,$$

$$\omega_2 \equiv (1-\gamma)^2\theta_2+\alpha\gamma\Gamma(1-\gamma)+\alpha\gamma\Gamma,$$

$$\omega_3 \equiv \beta\theta_2(1-\gamma)^2+\alpha\beta\gamma\Gamma,$$

$$\xi_1 \equiv (1-\gamma)^2\left[\phi_1+\phi_2(1+\beta)\right]-\gamma^2\Gamma,$$

$$\xi_2 \equiv (1-\gamma)^2\phi_2,$$

$$\xi_3 \equiv (1-\gamma)^2\beta\phi_2,$$

$$\Upsilon_{t+j} \equiv -(1-\gamma)r^s_{t+j}+(1-\gamma)^2 r^d_{t+j}+\alpha\beta(1-\gamma)E_{t+j}r^s_{t+j+1}, \text{ and}$$

$$\Psi_{t+j} \equiv (1-\gamma)r^s_{t+j}-(1-\gamma)^2 r^l_{t+j}.$$

As discussed in the text, equations (A6) imply that if γ = 0—in which case no portion of the bank's securities is eligible to count toward meeting the liquidity-coverage-ratio constraint— $\Lambda_3=\Lambda_4=\Omega_3=\Omega_4=0$, and portfolio separation will hold. Thus, it is through intertemporal security adjustments via the liquidity-ratio requirement that portfolio interdependence arises in the model. In the general case in which γ may exceed unity in the liquidity-coverage-ratio constraint, the solutions to the above two-equation system must satisfy the following relationships (using Cramer's rule):

$$\left[\left(1-\Lambda_1 H+\Lambda_2 H^2\right)\left(1-\Omega_1 H+\Omega_2 H^2\right)-\left(\Lambda_3 H+\Lambda_4 H^2\right)\left(-\Omega_3+\Omega_4 H\right)\right]E_{t+j}D_{t+j+1} \tag{A7a}$$
$$=\left(1-\Omega_1 H+\Omega_2 H^2\right)\Upsilon_{t+j}-\left(-\Omega_3+\Omega_4 H\right)\Psi_{t+j}$$

and

$$\left[\left(1-\Lambda_1 H+\Lambda_2 H^2\right)\left(1-\Omega_1 H+\Omega_2 H^2\right)-\left(\Lambda_3 H+\Lambda_4 H^2\right)\left(-\Omega_3+\Omega_4 H\right)\right]E_{t+j}L_{t+j+1} \tag{A7b}$$
$$=\left(1-\Lambda_1 H+\Lambda_2 H^2\right)\Psi_{t+j}-\left(\Lambda_3 H+\Lambda_4 H^2\right)\Upsilon_{t+j}.$$

It follows from equations (A7) that in the general case, there are four characteristic roots—υ_1, υ_2, υ_3, and υ_4—that must satisfy

$$\left(1-\upsilon_1 H\right)\left(1-\upsilon_2 H\right)\left(1-\upsilon_3 H\right)\left(1-\upsilon_4 H\right)= \tag{A8}$$
$$\left(1-\Lambda_1 H+\Lambda_2 H^2\right)\left(1-\Omega_1 H+\Omega_2 H^2\right)-\left(\Lambda_3 H+\Lambda_4 H^2\right)\left(-\Omega_3+\Omega_4 H\right)$$

Expanding both sides of (A8) implies that the characteristic roots must satisfy

$$\upsilon_1+\upsilon_2+\upsilon_3+\upsilon_4=\Lambda_1+\Omega_1+\Lambda_3\Omega_3; \tag{A9a}$$
$$\upsilon_1\upsilon_2+\upsilon_1\upsilon_3+\upsilon_1\upsilon_4+\upsilon_2\upsilon_3+\upsilon_2\upsilon_4+\upsilon_3\upsilon_4=\Lambda_2+\Omega_2+\Lambda_1\Omega_1+\Lambda_4\Omega_3-\Lambda_3\Omega_4; \tag{A9b}$$
$$\upsilon_1\upsilon_2\upsilon_3+\upsilon_1\upsilon_2\upsilon_4+\upsilon_1\upsilon_3\upsilon_4+\upsilon_2\upsilon_3\upsilon_4=\Lambda_1\Omega_2-\Lambda_2\Omega_1-\Lambda_4\Omega_4;\text{ and} \tag{A9c}$$
$$\upsilon_1\upsilon_2\upsilon_3\upsilon_4=\Lambda_2\Omega_2. \tag{A9d}$$

The nonlinear system of restrictions in (A9) must be solved for the general case in which all parameters are non-zero.

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