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A TIME VARYING MULTIVARIATE AUTOREGRESSIVE MODELING OF ECONOMETRIC TIME SERIES*

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ABSTRACT: A time varying multivariate autoregressive modeling of econometric time series is shown. Deviations from trend data are modeled. Kozin's orthogonal Legendre polynomial time varying representation, a Householder transfomation method of least squares modeling and the use of Akaike's AIC for subset selection are the key ideas in this method. Frequency domain relative power contribution computations yield an interpretation of the changing with time econometric relationships in the analysis of the U.S. hog, corn and farm wage series.

KEY WORDS: Multiple time series, AIC, nonstationary time series, Legendre polynomial, relative power contribution, Householder transformation.

1. INTRODUCTION

Economic, environmental and engineering data are often collected at equally spaced time intervals. In many problems such time series may be available from several related variables. It is of interest to model and analyze such series jointly to understand the dynamic relationships among them and to enhance the accuracy of forecasts.

The series of concern here are of relatively short duration, 100 not 1000 observations, and each series can exhibit slowly varying as well as relatively rapidly varying components. Our emphasis is on the modeling of the relatively rapid fluctuations. In the context of economic data, this is a concern for modeling the dynamic rather than the relatively static interrelationships. To enable that modeling, each time series is individually detrended and the remaining simultaneous multivariate time series are modeled without the constraining assumptions of stationarity. It may be anticipated that the relationships between logically related economic components may change in time. We do allow for that contingency and model both time invariant and time changing relationships.

The theory and practice of the modeling of multivariate time series has been developed for example in Quenouille (1957), Whittle (1963), Akaike (1968, 1971), Hannan (1970), Zellner and Palm (1974), Brillinger (1975), Box and Tiao (1977), Dunsmuir and Hannan (1976), Hsiao (1979) and Tiao and Box (1982) with quite divergent points of view. Our approach has been strongly influenced by the aforementioned work of Akaike and the successful application of that work to the control of rotary cement kilns, ship's yaw motion and supercritical thermal power plants (Otomo, Nakagawa and Akaike (1972), Ohtsu, Kitagawa and Horigome (1979) and Nakamura and Akaike (1981)). Consequently, we adopt the autoregressive (AR) model as the basic statistical model. The Householder transformation algorithm is exploited in the adaption of the Kozin's (1979) orthogonal polynomial time varying coefficient model for multivariate time series. Also, we use Akaike's AIC for subset selection to ameliorate the curse of the overparameterization that is implicit in multivariate time series modeling and to choose the best of the time invariant or time-varying models fitted to the data.

The analysis is in Section 2. Section 3 is a worked example of the US Hog series. That series was modeled earlier by Quenouille (1957) and Box and Tiao (1977). Comments are in Section 4.

2. THE ANALYSIS

2.1 DETRENDING

Assume that each of D simultaneous time series is in the form

$$y_d(n) = t_{d(n)} + x_{d(n)} + \varepsilon(n): \quad d=1,\dots,D; \; n=1,\dots,N. \tag{2.1}$$

In (2.1) $y_d(n)$, $n=1,\dots,N$ is the dth component series, $t_d(n)$ is a relatively slowly varying trend component, $x_d(n)$ is a relatively rapidly varying component and $\varepsilon_d(n)$ is a zero-mean constant variance uncorrelated observation error. It is assumed that the observation errors in each time series are orthogonal to each other. Then, let

$$z_d(n) = y_d(n) - t_d(n|N); \quad d=1,\dots,D; \; n=1,\dots,N . \tag{2.2}$$

In (2.2) $z_d(n)$ denotes the detrended dth component series with $d=1,\dots,D$ and $t_d(n|N)$ is the smoothed trend component. (The estimation of the smoothed trend may be done by our smoothness priors-recursive computation procedure, Kitagawa and Gersch (1982).)

2.2 MULTIVARIATE TIME VARYING MODEL

Denote the D component vector of deviations from the trends by $z(n) = (z_1(n), z_2(n), \dots, z_D(n))'$. The notion A' denotes the transpose of A. The data $z(n)$, $n=1,\dots,N$ is modeled in the form

$$z(n) = A_0(n)z(n) + A_1(n)z(n-1) + \ldots + A_L(n)z(n-L) + \xi(n) \,, \qquad (2.3)$$

with $E(\xi(n)\xi(n')')$ a diagonal matrix and

$$A_0(n) = \begin{bmatrix} 0 & & & & \\ a_0(2,1,n) & 0 & & & \\ \cdot & \cdot & \cdot & & \\ \cdot & & \cdot & \cdot & \\ \cdot & & & \cdot & \cdot \\ a_0(D,1,n) & & & a_0(D,D-1,n) & 0 \end{bmatrix} . \qquad (2.4)$$

The model in (2.3) and (2.4) is an instantaneous responses model. An equivalent non-instantaneous response time varying AR coefficient model computed from (2.3) and (2.4) is

$$z(n) = \sum_{\ell=1}^{L} [I-A_0(n)]^{-1} A_\ell(n) z(n-\ell) + [I-A_0(n)]^{-1}\xi(n) \,. \qquad (2.5)$$

In component form, the model in (2.3) is

$$z_d(n) = \sum_{j=1}^{D} \sum_{\ell=0}^{\ell(d,j)} a_\ell(d,j,n) z_j(n-\ell) + \xi_d(n) \quad \begin{matrix} d=1,\ldots,D \\ n=1,\ldots,N \end{matrix} \qquad (2.6)$$

with $a_0(i,j,n) = 0$, for $i \leq j$, $j=1,\ldots,D$.

The components of the multivariate time varying AR model in (2.6) are computed using a Householder transformation algorithm and an AIC subset selection procedure-version of the Kozin (1977) orthogonal polynomial time varying AR model method. Many details on multivariate stationary time series AR modeling are in Kitagawa and Akaike (1982). Householder transformation programs are in TIMSAC 78 (Akaike, Kitagawa, Arahata and Tada, 1978). A brief description follows.

2.2.1 Householder Transformation-Scalar AR Modeling

Consider the stationary scalar AR model

$$x(n) = a_1x(n-1) = \dots + a_Lx(n-L) + \varepsilon(n) \quad . \tag{2.7}$$

In (2.7) $\varepsilon(n)$ is a zero mean uncorrelated sequence with unknown variance σ^2. The data $x(n)$, $n=1,\dots,N$ is observed. Define the matrix Z and the vectors y and by

$$Z = \begin{bmatrix} x(L) & x(L-1) & \dots & x(1) \\ x(L+1) & x(L) & \dots & x(2) \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ x(N-1) & x(N-2) & \dots & x(N-L) \end{bmatrix}, \quad Y = \begin{bmatrix} x(L+1) \\ x(L+2) \\ \cdot \\ \cdot \\ \cdot \\ x(N) \end{bmatrix}, \quad a = \begin{bmatrix} a_1 \\ \cdot \\ \cdot \\ \cdot \\ a_L \end{bmatrix} . \tag{2.8}$$

The Householder transformation solves $\min_a || Za - y ||^2$ for the unknown AR coefficient vector a and the variance σ^2, where $||.||^2$ denotes the Euclidean norm. Write $X = [Z|y]$ as the $(N-L)x(L+1)$ matrix with the vector y appended to the right of the matrix Z. The Householder transformation, realized with orthogonal matrix P, reduces the matrix X to an upper triangular form.

$$X \dashrightarrow PX = S = \begin{bmatrix} S_{11} & \dots & S_{11} & S_{1L+1} \\ & \cdot & \cdot & \cdot \\ & & \cdot & \cdot \\ & & \cdot & \cdot \\ & & S_{LL} & S_{LL+1} \\ & & & S_{L+1L+1} \end{bmatrix} . \tag{2.9}$$

For the structure of P and computation of PX, see Golub (1965). From the orthogonality of P, $||Za-y||^2=||PZa-Py||^2$. This can be rewritten as

$$||PZa-Py||^2 = \left|\left| \begin{bmatrix} S_{11} & \dots & S_{1L} \\ & \cdot & \cdot \\ & & \cdot \\ & & \cdot \\ & & S_{LL} \end{bmatrix} \begin{bmatrix} a_1 \\ \cdot \\ \cdot \\ \cdot \\ a_L \end{bmatrix} - \begin{bmatrix} S_{1L+1} \\ \cdot \\ \cdot \\ \cdot \\ S_{LL+1} \end{bmatrix} \right|\right|^2 + S^2_{L+1L+1} \quad . \tag{2.10}$$

The minimum of this quantity is S^2_{L+1L+1}. The least squares estimate of the coefficient vector a solves

$$\begin{bmatrix} S_{11} & \cdots & S_{1L} \\ & \ddots & \vdots \\ 0 & & S_{LL} \end{bmatrix} \begin{bmatrix} a_1 \\ \vdots \\ a_L \end{bmatrix} = \begin{bmatrix} S_{1L+1} \\ \vdots \\ S_{LL+1} \end{bmatrix} . \tag{2.11}$$

Given the triangular matrix X, AR models of order $k \leq L$ can be obtained by solving

$$\begin{bmatrix} S_{11} & \cdots & S_{1k} \\ & \ddots & \vdots \\ 0 & & S_{kk} \end{bmatrix} \begin{bmatrix} a_1 \\ \vdots \\ a_k \end{bmatrix} = \begin{bmatrix} S_{1L+1} \\ \vdots \\ S_{kL+1} \end{bmatrix} . \tag{2.12}$$

An estimate of the innovations variance for the kth order model is obtained from

$$\sigma^2(k) = \frac{1}{N-L} \sum_{i=K+1}^{L+1} S^2_{iL+1} . \tag{2.13}$$

The quantity N-L in (2.13) signifies that in Householder transformation least squares method, the first L data points are used for initial conditioning. Subset autoregression models of the AR model of order $K \leq L$ are simply obtained. Let $\{i(1),\ldots,i(k)\}$ be a subset of $\{1,2,\ldots,L\}$. The corresponding subset AR model is

$$x(n) = \sum_{j=1}^{k} a_j x(n-i(j)) + \varepsilon(n) . \tag{2.14}$$

The least squares estimates of the model coefficients are obtained by first transforming the matrix X to S' with $S'_{p,i(j)} = 0$ ($p=j+1,\ldots,L+1$; $j=1,..,k$) and then solving the linear equation

$$\sum_{j=\ell}^{k} S'_{\ell,i(j)} a_j = S'_{\ell,L+1} \quad (\ell=1,\ldots,k). \tag{2.15}$$

The estimate of the innovation variance is given by

$$\sigma^2(i(1),\ldots,i(k)) = \frac{1}{N-L} \sum_{i=k+1}^{L+1} S'^2_{i,r+1} \; . \tag{2.16}$$

The value of Akaike's AIC statistics for a particular model is defined by (Akaike, 1973, 1974),

$$\mathrm{AIC}(k) = (N-L) \log \sigma^2(k) + 2(k+1) \; . \tag{2.18}$$

The AIC is used to select the best of alternative parametric models. The model which attains the minimum AIC is selected as the AIC best model. Note that the AIC's of the $k \leq L$ order AR models and of individual subset AR models can be computed directly from the S matrix without solving the linear equations for the particular models.

2.2.2 Kozin's Model, A Householder Transformation Realization.

Consider the time varying AR coefficient model

$$x(n) = a_1(n)x(n-1) + \ldots a_L(n)x(n-L) + \varepsilon(n) \; , \tag{2.19}$$

with

$$a_i(n) = \sum_{j=1}^{J} a_{ij} f_j(n), \; i=1,\ldots,L \tag{2.20}$$

$$f_j(n) = \sum_{s=0}^{j} (-1)^s \binom{j+s}{s} \binom{j}{s} \frac{n^{(s)}}{N^{(s)}} \; , \; n = 0,1,\ldots,N \; .$$

In (2.20) $n^{(s)} = n!/(n-s)!$, $N^{(s)} = N!/(N-s)!$ and N is the number of data points.

The functions $f_j(n)$ are the orthogonal family of discrete Legendre polynomials (Milne (1949)). They are orthogonal on the interval [0,N]. This method for characterizing the time varying AR coefficient model was used by Kozin (1977).

In a notation that is more convenient for Householder transformation computation, rewrite (2.19) in the form

$$x(L+K) = \sum_{\ell=1}^{L} a_\ell(L+k)x(L+k-\ell) + \varepsilon(L+k) \tag{2.21}$$

$$= \sum_{\ell=1}^{L} a(\ell,j)f(j,L+k)x(L+k-\ell) + \varepsilon(L+k)\ , \ n=L+k,\ k=1,\dots,N.$$

In (2.21) the subscript notation a_{ij} and $f_j(n)$ in (2.20) are changed to a(i,j) and f(j,n). Then, re-index n in f(k,n) by f(j,n-L) and expand to the more transparent representation for Householder transformation computation,

$$\begin{aligned} x(n) = {} & a(1,1)f(1,n)x(n-1) + \dots + a(1,j)f(J,n)x(n-1) \\ & + a(2,1)f(1,n)x(n-2) + \dots + a(2,J)f(J,n)x(n-2) \\ & \vdots \\ & + a(L,1)f(1,n)x(n-L) + \dots + a(L,J)f(J,n)x(n-L) + \varepsilon(n)\ . \end{aligned} \tag{2.22}$$

Now, define the LJ and N vectors a and y: a = (a(1,1) ... a(1,J),a(2.1) ... a(2,J) ...a(L,1) ... a(L,J))', y=(x(L+1),...,x(L+N)' and the NxJL matrix Z

$$Z = \begin{bmatrix} f(1,1)x(L) & \dots & f(J,1)x(L) & f(1,1)x(L-1) & \dots & f(J,1)x(1) \\ f(1,2)x(L+1) & \dots & f(J,2)x(L+1) & f(1,2)x(L) & \dots & f(J,2)x(2) \\ \vdots & & \vdots & \vdots & & \vdots \\ f(1,N)x(L+N-1) & \dots & f(J,N)x(L+N-1) & f(1,N)x(L+N-2) & \dots & f(J,N)x(N) \end{bmatrix} . \tag{2.23}$$

As before, append the vector y to the matrix Z, use the Householder transformation and solve for the least squares estimates of the components a(i,j), i=1, ...,L j=1,...,J of the a vector. In the new notation, the time varying coefficients $a_i(n)$ in the representation (2.19) are

$$a_i(n) = \sum_{j=1}^{J} a(i,j)f(j,n) \quad . \tag{2.24}$$

Given the maximum AR model order L and the maximum polynomial order J, time varying AR models of order $k \leq L$ with polynomial orders $J' \leq J$ and subsets of those models can be readily computed by the methods in section (2.2.1). The AIC best of those models is then also readily selected.

2.2.3 Time Varying Multivariate AR Models

The extension of the use of the Householder transformation to the computation of multivariate time varying AR coefficient models is straight forward. Each coefficient in the multivariate AR representation is computed independently of the other coefficients. The role of the AIC is critical in subset selection to reduce the potential JxD^2xL time varying AR coefficients in the model (2.5) to a more reasonable number. To fit the model described in (2.3),(2.6), set up the (N-L)xJxDx(L+1) matrix X,

$$X = \begin{bmatrix} (f(L)'z(L))' & (f(L)z'(L-1))' & \ldots & (f(L)'z(1))' & z(L+1)' \\ (f(L+1)'z(L+1))' & (f(L+1)'z(L))' & \ldots & (f(L+1)'z(2))' & z(L+2)' \\ \vdots & & & & \\ (f(N-1)'z(N-1))' & (f(N-1)'z(N-2))' & \ldots & (f(N-1)'z(N-L))' & z(N)' \end{bmatrix} . \tag{2.25}$$

The JD column vectors f(n)'z(s) in (2.25) signify the lexicographic ordering of the product of the jth polynomial at time s, j=1,...,J, for each of the components $z_d(s)$. d=1,...,D at time s. The matrix X is reduced to an upper triangle matrix S. The model (2.6) is obtained by applying a subset regression algorithm to the submatrix of S,

$$\begin{bmatrix} S_{i,1} & \cdots & S_{JLD+Ji,1} \\ & \ddots & \vdots \\ & & S_{JLD+Ji,JLD+Ji} \end{bmatrix}. \tag{2.26}$$

More details and a worked example of the time invariant version of the multivariate model fitting computation are in the program MULMAR in TIMSAC-78 (Akaike, Kitagawa, Arahata and Tada, 1978).

3. AN EXAMPLE

3.1. U.S. HOG, CORN AND WAGE SERIES.

We modeled the 5 variate U.S. hog, corn and labor wage series that was studied earlier by Quenouille (1957) and Box and Tiao (1979). The data are annual. There are 82 observations in each time series in the interval 1867 to 1948. Quenouille logarithmically transformed and scaled the data to produce numbers of comparable magnitude in each series. The original data with superimposed trend and deviations from the trend are shown in Figure 1. The trends were computed by the smoothness priors-recursive computational method (Kitagawa and Gersch, 1982). Visual examination of the deviations data suggests that the hog supply is negatively correlated with the hog price and corn price, and leading the corn price. It is positively correlated with and leading corn supply with increasing time more positively correlated and with increasing lead with farm wage rate. Hog price is positively correlated and with increasing time with increasing lead with corn price. Hog price and farm wages appear to be positively correlated to slow movements early in the time series and to more rapid changes later in the series. These observations were made by examining plots of the deviations data pairwise, sliding them in time in relation to each other, inverting them etc.

3.2. TIME INVARIANT AND TIME VARYING MODELS.

A time invariant analysis was conducted first. A lag L=5 and Lag L=2 were fitted to the data n=6,...,82. The corresponding AIC's and number of parameters fitted were AIC(5,5)=2812.2, AIC(5,2)=2814.7; params(5,5)=62, params(5,2)=47. The AIC of the Lag L=2 model fitted to the data n=3,...,82 was AIC(2,2)=2936 with params(2,2)=31. The notation, AIC(n-1, Lag L), signifies that the lag L model was fitted to the data n, n+1,...,N. L data points are required for initial conditions in fitting a lag L model by the Householder transformation method that implies that $(n-1) \leq L$. We fitted lag models L and L' with L'<L

and computed AIC(L,L) and AIC(L,L') in order that the alternative contender AIC best models be computed on the same data span, (L+1,L+2,...,N). params(,) denotes the number of parameters fitted in the model.

To check the stability of fitted models, compare the L=2 models fitted to the n=6,...,8 data and n=3,...,82 data by multiplying AIC(5,2) by the ratio 80/77 to obtain an approximate data length corrected AIC of 2924.4. The computational results suggest that the lag L=2 model be tentatively adopted as a satisfactory time invariant model.

INSERT TABLE 1 HERE

Table 1 is a listing of the fitted instantaneous model parameters, the innovations matrix, the innovations correlation matrix and the equivalent AR model. The model coefficients are of no particular interest. Note, however that in the instantaneous and AR models there are 31 and 39 parameters respectively, (out of 60 possible parameters). The large negative correlation between the corn price and corn supply series suggests that one of those variables might be eliminated. We did some additional computing and used the AIC to make the decision. The AIC computed for the model with deleted variables is the sum of the AIC's computed for the model purged of the deleted variables and the AIC of the model fitted with the deleted variables. This computation reflects the assumption that the two sets of variables are statistically independent. This total AIC must be compared with the AIC computed with all the variables present. The AIC preferred model is the one with the smallest AIC value.

Carrying out this procedure with the corn price and corn supply time series deleted (one of those series was deleted at a time) indicated that the full model be retained. Very likely the high correlation between those series and their relationship to the remaining series is influenced by other unobserved time series. The weather is such a candidate time series.

INSERT TABLE 2 HERE

Table 2 shows the results of the fitting of the time varying model, parametric in the Legendre polynomial order for the L=2 model. IPOL=0 corresponds to the time invariant model. The AIC preferred model is taken to be the L=2, IPOL=1 model. We do not accept the L=2,IPOL=3 model with the slightly smaller AIC value because of the very large number of parameters included in that model and because the 4th and 5th regressands are fitted with 40 and 50 variables respectively. This is an excessive parameterization when only 80 data points are available for the modeling.

The graphical results of frequency domain computations are revealing. Figure 2A is the power spectral density shown on a logarithmic scale of the deviation from the trend series. The computations are performed on the series individually using an AR model spectral density computation method. These graphs are compatible with our visual appraisal of the individual series. The high and low frequency bumpiness in the appearance of first three series are in agreement with the bumpiness of the power spectral densities for those series. The 4th series, corn price does appear to be quite random and indeed the AR modeled spectral density reveals a white noise series. The farm wage series is dominated by relatively slow behavior. Correspondingly the spectral density of that series is dominated by the energy at low frequencies. The frequency scale is linear over the interval $0 \leq f \leq 1/2$ years.

Frequency domain relative power contributions, RPC, were also done. An AR modeled spectral density power matrix was computed both for the stationary and nonstationary time series model cases. Only the diagonal terms of the residual covariance matrix were used in the computation. In that way, the total power spectral density at frequency f in each time series can be decomposed into the RPC's from each of the D time series. This idea was used in an exploratory control problem setting to determine which of the variables fitted in a multivariate AR model might be most useful for control purposes, Akaike 1968. In Oritani, 1979, the RPC was used in an economic time series analysis problem.

In the context of this paper the RPC is decomposition of the total power of the time series $z_d(n)$ at frequency f, $0 \leq f \leq 1/2$ to the contribution from each of the individual time series $\{z_d(n)\ d=1,\ldots,D\}$. Figure 2B shows the RPC contributions for the stationary model. The successive contributions of the time series $z_d(n)$ $d=1,\ldots,D$ are plotted so that the contribution of the dth time series at frequency f is the difference between the dth and the (d-1)st time series. The RPC are plotted for each variable. Thus for $\{z_2(n)\}$ the hog price series, the principal contributions are the corn price series at the low frequency and the price of hog-series at the high frequencies. The RPC contributions to the price of corn and the supply are dominated respectively by the price of corn and the supply of corn. The farm wage series has relatively little power at the high frequencies. The most significant contributions to the farm wage series at low frequencies are the corn price and corn supply series.

Perhaps the most interesting use of the RPC is in exposing the changing relationships betwen the different time series with time. In figures 2C,D,E the RPC's of the 5 deviations from trend time series are shown for times n=12,42, and 72. The general impression from those graphs is that the interrelationships between those series change with time. The farm wage series have little relative influence on the other series. At n=12, the hog supply series is most influenced by regression upon itself. This influence diminishes with time. At n=72, the corn supply series becomes the most influential series on hog supply. At n=12, the hog price series is most influenced at low frequencies by corn price. It is dominated at frequency $\simeq$ 1/6 years by hog supply. The hog supply influence diminishes with time. The low frequency influence of corn price on hog price increases with time and the hog price series is increasingly sensitive to itself at high frequency. Corn price is dominated by a peak influence of hog supply at frequency $\simeq$ 1/7 years. Otherwise, corn price is most sensitive to previous corn price with that influence diminishing with increasing time, particularly at higher frequencies. At n=72, the corn supply series is a high frequency influence on the corn price series. The corn supply series is primarily influenced by itself. At n=12 hog supply has a peak influence at frequency $\simeq$ 1/7 years, at n=72 corn price has a peak influence at frequency $\simeq$ 1/7 years. The farm wage series is primarily a low frequency series. At n=12 it is dominated by itself. At n=42 and n=72 the corn price and corn supply series are the dominant influences on farm wage rate.

4. COMMENTS

A phenomenological approach to the understanding of the dynamic relationships between simultaneously related short span time series was taken. The simultaneous deviations from the relatively slowly changing trends of the series are modeled. The relationships between the trend components of economic time series may be interpreted as "static" relationships. The key model fitting ideas in our approach are the modeling of the deviations from trend data, the employment of the AR model, the use of the Householder transformation for least squares computations for time invariant and time varying models, the use of Akaike's AIC to select subset AR models and an engineering-like frequency domain view with the RPC to visually expose the sensitivity of one time series to the influence of other time series.

The AIC yields several checks on the stability of the computations with respect to model order, eliminated variables, polynomial order in the time varying AR model and finally on choosing between time invariant and time varying AR models. The availability of such verification is essential with the inevitably large number of coefficients present in the fitting of multivariate models.

Our analysis does seem reasonable. The interpretation of our computational analysis is compatible with our interpretation of a visual examination of the data and of the computed results. Also, the plots of the RPC's can be related to the appearances of time series pairwise.

As suggested in Tiao and Box (1972) the modeling of multivariate time series is still a challenging problem. We have achieved a new and highly desired modeling of time changing relationships between econometric variables. Only the conventional orthogonal polynomial and subset selection tools of regression analysis have been used. If we assume that the system which generated the data were infinitely complex and reasonably regular and that our model were only an approximation to that system, strong consistency of the parameter estimates and minimum expected one-step-ahead prediction error properties could be proved. At this point, those analyses are not as compelling as is examination of the phenomenological properties of the methods. It does appear that several simultaneous short duration time series can be reasonably modeled by our method.

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LEGENDS

Figure 1 US HOG CORN AND WAGE SERIES

A Original data and superimposed trend.

B Deviations from the trend for each of the Hog supply, hog price, corn price, corn supply and farm wage rate series.

Figure 2 RELATIVE POWER CONTRIBUTIONS

A Power spectral density

B RPC, stationary model

C RPC, nonstationary model n=12

D RPC, nonstationary model n=42

E RPC, nonstationary model n=72

TABLES

TABLE 1. TIME INVARIANT INSTANTANEOUS AND AR MODELS

TABLE 2. TIME VARYING AR COEFFICIENT MODEL, L=2

TABLE 1: TIME INVARIANT INSTANTANEOUS AND AR MODELS

INSTANTANEOUS RESPONSE

1.000	.0	.0	.0	.0
.341	1.000	.0	.0	.0
.0	.0	1.000	.0	.0
-.433	.0	.442	1.000	.0
.0	-.161	-.137	-.222	1.000

COEFFICIENTS

.528	.263	.110	.399	.0
-.844	.347	.486	.559	.489
.0	-.212	.624	.934	.0
.0	.0	.229	.0	.0
.291	.0	.194	.288	.693

.0	.0	-.284	-.110	-.082
.0	-.302	.0	.0	.0
.0	-.430	.0	.363	.0
.0	.0	.0	.0	.0
.0	.0	-.098	.0	-.119

AR COEFFICIENTS

.528	.263	.110	.399	.0
-1.024	.257	.449	.422	.489
.0	-.212	.624	.934	.0
.229	.207	.001	-.240	.0
.178	.058	.351	.431	.771

.0	.0	-.284	-.110	-.082
.0	-.302	.097	.037	.028
.0	-.430	.0	.363	.0
.0	.190	-.123	-.208	-.035
.0	-.065	-.110	.010	-.122

INNOVATIONS COVARIANCE MATRIX

431.8	-147.4	0.0	186.9	17.9
-147.4	2110.7	0.0	-63.8	325.3
0.0	0.0	6106.7	-2701.4	237.3
186.9	-63.8	-2701.4	3027.0	292.1
17.9	325.3	237.3	292.1	574.7

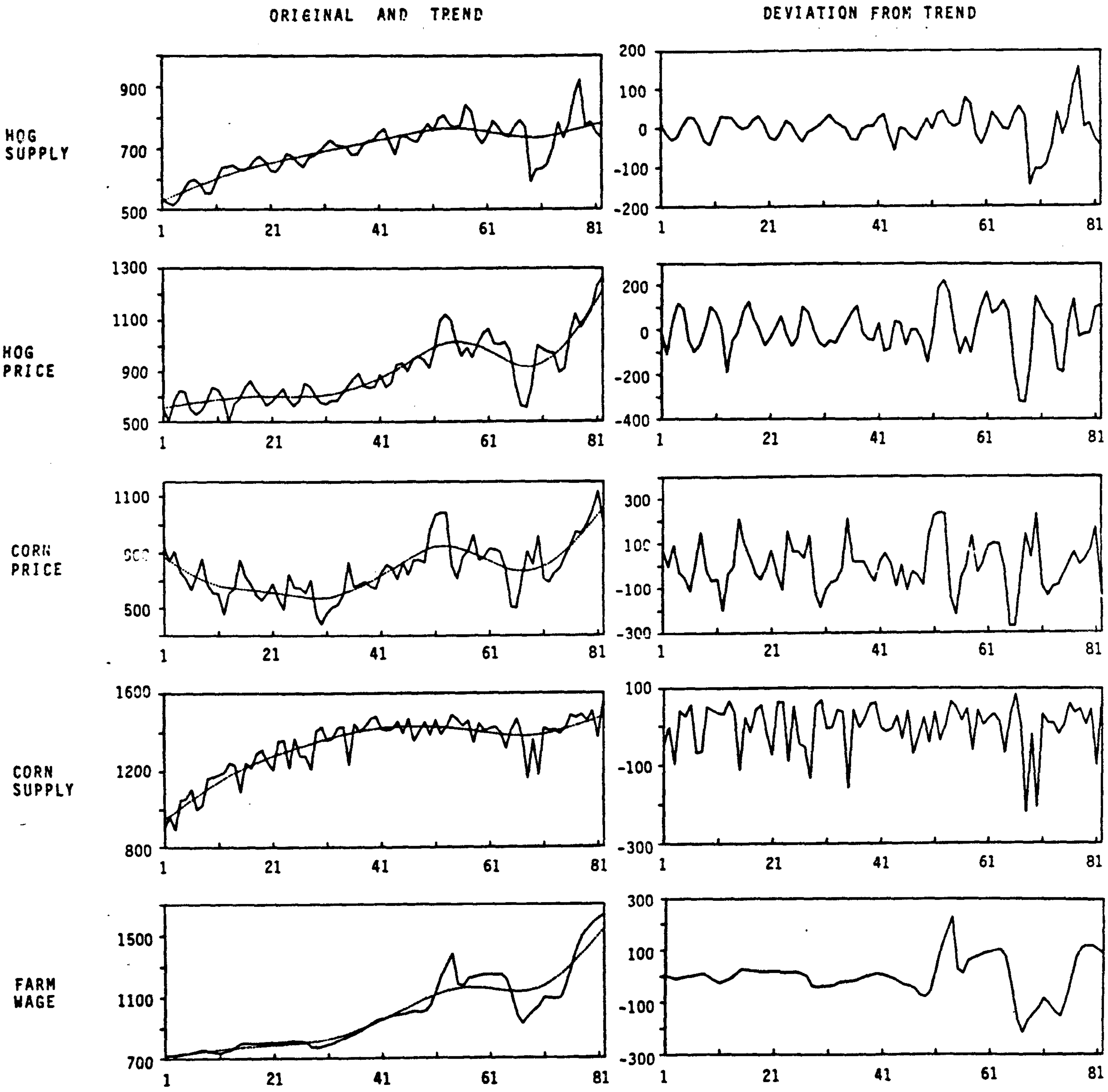
INNOVATIONS CORRELATION MATRIX

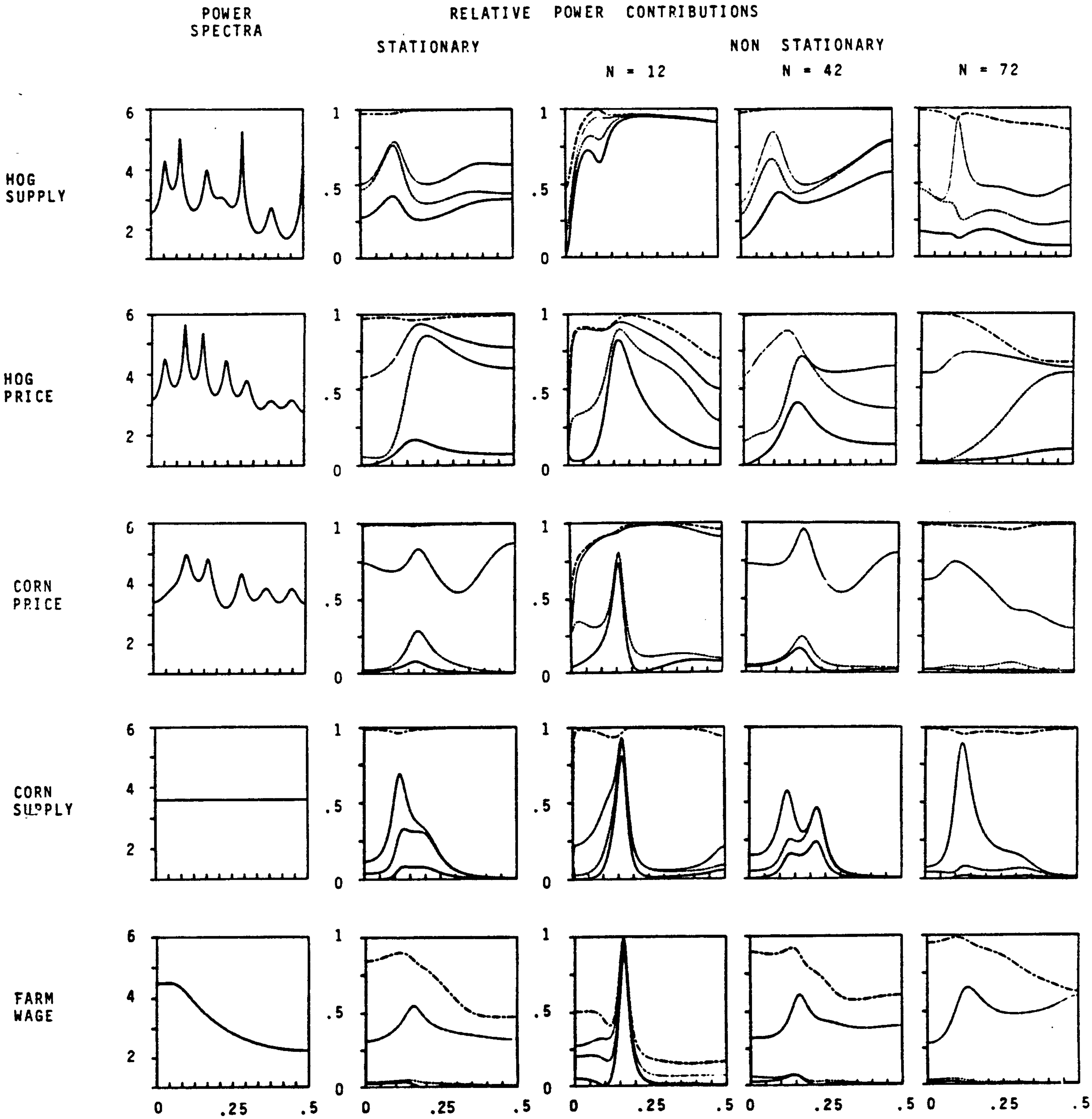
1.000	-.154	0.0	.164	.036
-.154	1.000	0.0	-.025	.295
0.0	0.0	1.000	-.628	.127
.164	-.025	-.628	1.000	.221
.036	.295	.127	.221	1.000

TABLE 2: TIME VARYING AR COEFFICIENT MODEL, L=2

IPOL-ORDER	NUMBER OF PARAMETERS	AIC
0	31 (7,7,5,3,9)	2936.84
1	66 (14,16,10,6,20)	2880.52
2	111 (18,27,15,24,27)	2888.01
3	188 (28,36,32,40,52)	2819.07

Figure 1


ORIGINAL AND TREND
DEVIATION FROM TREND
HOG SUPPLY
HOG PRICE
CORN PRICE
CORN SUPPLY
FARM WAGE
1
21
41
61
81
900
700
500
200
100
0
-100
-200
1300
1100
900
500
-400
1100
500
300
-300
1600
1200
800
1500
700


POWER
SPECTRA
RELATIVE POWER CONTRIBUTIONS
STATIONARY
NON STATIONARY
N = 12
N = 42
N = 72
HOG SUPPLY
HOG PRICE
CORN PRICE
CORN SUPPLY
FARM WAGE
6
4
2
1
.5
0
.25
.5