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# Getting the Most Out of a Mandatory Subordinated Debt Requirement

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Recent advances in asset pricing—the reduced-form approach to pricing risky debt and derivatives—are used to quantitatively evaluate several proposals for mandatory bank issue of subordinated debt. We find that credit spreads on both fixed and floating rate subordinated debt provide relatively clean signals of bank risk and are not unduly influenced by non-risk factors. Fixed rate debt with a put is unacceptable, but making the putable debt floating resolves most problems. Our approach also helps to clarify several different notions of "bank risk".

**JEL Classification:** G28, G12, G18
**Key Words:** subordinated debt, banks, asset pricing

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# 1 Introduction

For the better part of two decades economists have debated the merits of regulations that would require a minimum level of subordinated debt in a bank's capital structure. Proponents view a subordinated debt requirement as a reform that can resolve agency problems created or exacerbated by federal safety net guarantees. Typically, proposals for a sub-debt requirement cite increased market discipline and reduced taxpayer exposure to loss as the primary benefits. Moreover, proposals such as Wall (1989) and Calomiris (1997) attempt to resolve regulator principal-agency problems through features in their proposed sub-debt structures that force earlier official recognition of a institution's insolvency. Policymakers, too, have begun to seriously consider subordinated debt proposals. Both the U.S. and European Shadow Financial Regulatory Committees have their own proposals, and the Federal Reserve System convened a Study Group on Subordinated Notes and Debentures (1999). A joint report by the Fed and Treasury mandated by the Financial Services Modernization Act of 1999 (Gramm-Leach-Bliley) concluded that "existing evidence supports efforts to use subordinated debt as a way to encourage market discipline" and also held out the possibility that, pending further research, "the Secretary or the Board may recommend such a policy to Congress." (Board of Governors of the Federal Reserch System and the Secretary of the U.S. Department of the Treasury (2000)). Furthermore, section 121 of the GLB Act in fact requires that large holding companies controlling a financial subsidiary must have at least one issue of rated debt outstanding (albeit not necessarily subordinated).

Proponents of subordinated debt have suggested that it will increase market discipline in two ways. First, the interest rates on debt issues will provide information to supervisors, in some proposals triggering prompt corrective action, in others merely providing supplemental information. This is referred to as indirect market discipline, as it relies on the proper response by the supervisors. Second, subordinated debt provides direct market discipline, as banks taking riskier strategies face higher debt prices, and thus find funding more difficult. They also face direct pressure from existing bond holders.

The literature on subordinated debt has addressed these issues separately. Theoretical studies by Winton (1995), Hart and Moore (1995), Bolton and Scharfstein (1996), and Dewatripont and Tirole (1994) have explored the direct incentive effects, while empirical studies by Avery, Belton, and Goldberg (1988), Flannery and Sorescu (1996), and Evanoff and Wall (2001) have looked at how bank risk affects subordinated debt yields. Berger, Davies, and Flannery (2000) study how market and supervisory assessments of risks differ, while Maclachlan (2001) and others have questioned the ability of regulators to credibly apply indirect discipline. Levonian (2001) assesses the relative merits of subordinated debt against other types of stricter capital requirements. Thomson and Osterberg (1991) show how even in the absence of information and incentive effects, subordinated debt reduces taxpayer exposure to loss.

There has been virtually no work, however, on striking the proper balance between direct and indirect market discipline, nor on the possible trade-offs between the two, despite the very different weights placed on the two in different proposals. Short-term debt would require the bank to constantly re-enter the market and prove its worth, but it is unclear as to whether the debt would be as sensitive to credit changes as longer-term debt. Making the debt putable would remove discretion from regulators, but might interfere with the spread's signaling properties. At this point we have no way to quantitatively assess these trade-offs, or consequently to assess the different proposals.

Of course, properly assessing the signal-to-noise properties will illuminate not only the trade-offs between different designs, but also the pitfalls of inferring risk given any particular proposal. Thus, to what extent does the standard approach of looking at the spread between the yield on sub debt and comparable Treasury bonds go wrong by ignoring maturity? Would regulators incorrectly view one bank as riskier merely because it had longer-term bonds? The problem exists not just across banks but across time as well. To what extent would changes in the risk-free term structure show up as changes in the credit spread? If we had extensive experience with a rich market of subordinated debt and associated derivatives, we might answer these questions with further empirical work. When it becomes a question of security design, however, we need another route to assess the quantitative effects involved.

In this paper we evaluate the information content of subordinated debt under various proposals, using recent advances in asset pricing theory. Because we have a model, we can hold fixed the underlying risk of the bank and examine how the price (or yield) of subordinated debt varies with maturity, with the underlying risk-free term structure, and with the addition of embedded options such as puts. This approach stops short of a full evaluation, as it does not quantitatively assess the incentive effects. For example, while we quantify the information lost by adding putabiliy, we cannot quantify the benefit from reducing forebearance. We find:

- Credit spreads vary by maturity. In spite of this observation, in ranking banks by their risk, the maturity of the subordinated debt does not matter much.
- Subordinated debt of very short maturity (under two years) is sensitive to credit shocks, but the impact of credit shocks among longer-maturity bonds is less apparent.
- If two banks have the same short-term credit spread, their inherent risk may be quite different, since other factors of the spread process, such as their mean reversion factors, could be substantially different. We identify several examples of this.
- Theoretically, many factors besides bank risk can affect credit spreads, and identifying a simple mapping from spreads to risk is nontrivial. We find, however, that many of these extraneous effects are not quantitatively significant. For example, changes in the shape of the risk-free term structure, or in interest rate volatilities do not have a major impact on credit spreads.
- When a put is added to a fixed-rate bond, the effects of maturity and volatility become significant. Changes in the put premium dominate the credit spread, so that observed spreads reflect interest rate movements rather than bank risk.
- Adding a put to floating-rate bonds, however, removes most of the effect of the risk free term structure. Credit spreads once again primarily reflect bank risk.

From these findings we conclude that fixed-rate putable debt should not be adopted. Although any conclusions for standard fixed-rate debt or putable floating-rate debt are more dependent on the range of our data and our model specification, our evidence strongly suggests that little, if any, information is lost by adding a put to floating rate debt. Furthermore, the credit spreads on fixed and floating putable debt reflect primarily credit risk, not changes in the underlying risk free interest rate.

The paper proceeds as follows. Section 2 briefly describes the different subordinated debt proposals. Section 3 presents a two-factor model for pricing riskless bonds and a three-factor model for pricing subordinated debt and related derivatives. Section 4 describes the estimation procedures, and using swap and swaption data, calibrates the two-factor model for riskless bonds. In addition, we use subordinated bond data to estimate the term structure of credit spreads for five large banks. The resulting estimates for these models are used as benchmark parameters in the evaluation of the proposals. Section 5 explores the alternative proposals and section 6 compares them.

## 2 The Different Proposals

A variety of subordinated debt plans have been proposed over the years (see the Study Group on Subordinated Notes and Debentures (1999) for a summary of many of them), and new plans are proposed regularly. It would be impractical to examine all of the proposals, and indeed the differences between them often do not directly matter for the pricing questions we take up in this paper. Whether the amount of sub debt issued is tied to total assets or deposits, how much counts as regulatory capital, or whether banks or bank holding companies issue the debt, won't directly affect the relationships we explore. Such differences can have an important indirect effect by changing the risk of the bank, and thus of the sub debt, but this will be subsumed in the risk factor.

We concentrate on four different proposals which between them span most of the sub debt plans. First, we look at the standard, noncallable, fixed-rate debt with semiannual coupons. Because a key question is the impact of maturity on prices, we look at debt maturities of one through nine years. Secondly, we look at floating rate debt of the same maturities. The debt would be floating, paying the 3-month LIBOR rate semi-annually.

Next, as in Evanoff (1993), we add a put feature to the fixed-rate debt. Lastly, we consider floating-rate debt putable at par. Since a put on fixed-coupon debt will sometimes be exercised solely because of shifts in the term structure, Wall (1989) argues that putable floating-rate debt may give a cleaner signal of credit events.

These different assets will be priced based on model parameters calculated for five different banks which had enough outstanding issues of subordinated debt to allow calibration: Chase, JP Morgan, Wachovia, Bank One, and Bankers Trust.[1]

## 3 Pricing Credit Derivatives

The methods we use for pricing the proposed subordinated debt instruments consist of three parts. First, for modeling the underlying risk-free term structure, we use a standard two-factor Heath Jarrow Morton model, which matches the initial term structure and allows the future dynamics of the term structure to evolve with different levels, slopes, and curvatures.

Second, to model risky debt, we use a "reduced form" approach, where defaults occur at surprise stopping times.[2] In this framework, the default process of risky debt is modeled directly

[1]For a discussion and some evidence on the question of whether the bank or the bank holding company should issue the sub debt, see Jagtiani, Kaufman, and Lemieux (2001).

[2]This contrasts with more fundamental models based on Merton (1974) where default is endogenized, but

rather than through the asset process for the firm, and assumptions are made regarding the recovery rate in default. Combining the default process and recovery rate with assumptions on the riskless term structure process leads to models for risky debt and their derivative products.

Third, the methodology used to price the options embedded in some proposals requires pricing American options present in putable debt. Since we have a three-factor model, we resort to simulation methods of Longstaff and Schwartz (2001) to price the embedded options.

The model that we use for pricing credit sensitive claims incorporates significant information on the term structure of interest rates, where much information is available, and is less demanding on the term structure of credit for an individual bank. In particular, the riskless term structure can readily be observed, and volatility information, which is embedded in the prices of a wide array of liquid derivative prices such as caps, floors, and swaptions, can easily be extracted. In contrast the credit information for a particular bank typically comes in the form of a few prices of traded bonds and default swap quotations. Given this sparse information, the model for the spread is less demanding.

Specifically, for the term structure of interest rates we adopt a two-factor Heath, Jarrow, and Morton (1992) model where forward rates are initialized to the observed values, and volatilities are humped functions of their maturities, consistent with empirical evidence (Amin and Morton (1994)). In contrast, credit spreads are not initialized to given values. Rather, they are generated by an additional factor. The dynamics of the short credit spread are specified as a mean reverting process, correlated with interest rates. Given the dynamics of the riskless term structure and the short credit spread, a three-factor model for the risky forward rate is developed. Using market prices of risky debt, the parameters of the short credit spread can then be calibrated so that the model produces prices of risky debt that are close to market prices. The resulting model we obtain for the term structure of credit spreads is flexible enough to permit upward, downward, and humped shapes.

## 3.1 Pricing Riskless Bonds

Partition the time interval into increments of width $\Delta t$ years and label the time periods by consecutive integers. Let $f(t,T)$ be the forward rate at period $t$, for the time period $[T, T+1]$. Hence, expressed in years, the actual time is $[T\Delta t, (T+1)\Delta t]$. The forward rates, under the risk neutral equivalent martingale measure, are updated as follows:

$$f(t+1,T) = f(t,T) + \sum_{n=1}^{2}[\mu_{f_n}(t,T)\Delta t + \sigma_{fn}(t,T)\sqrt{\Delta t}Z_{t+1}^{(n)}] \quad (1)$$

where $\{Z_j^{(n)}, n = 1,2\}$ are independent standard normal random variables and the volatility structures are given by:

$$\sigma_{f1}(t,T) \quad = \quad a_1^{T-t}b_1 \quad (2)$$

which have been less successful at pricing risky debt. For examples of this approach see Kim, Ramaswamy, and Sundaresan (1993), Longstaff and Schwartz (1995), and Nielsen, Sao-Requejo, and Santa-Clara (1993). Structural models are attractive on theoretical grounds, as they link the valuation of financial claims to economic fundamentals. They have proved to be hard to implement, however, because of the difficulty in valuing the firm's assets, characterizing and measuring the firm's volatility, and because of the complexity of the capital structure of the firm. Moreover, as shown by Eom, Helwege, and zhi Huang (2000), these models tend to generate spreads that are too low for high-quality borrowers.

$$\sigma_{f2}(t,T) = a_1^{T-t} b_2 + a_2^{T-t} c_2, \tag{3}$$

where $a_j = e^{-\kappa_j \Delta t}$ for $j = 1, 2$.

These volatility structures imply that the volatility function for forward rates can be a humped function of the maturity, which is consistent with empirical evidence. The drift terms in equation (1) are completely determined by the volatility structures and are given by the discrete version of the Heath, Jarrow, and Morton (1992) restriction, which is:

$$\mu_{fn}(t,T) = \frac{\Delta t}{2}\sigma_{fn}^2(t,T) + \sigma_{fn}(t,T) \sum_{j=t+1}^{T-1} \sigma_{fn}(t,j)\Delta t, \qquad n = 1, 2 \tag{4}$$

**Proposition 1**
*(i) If the dynamics of the forward rates are given by equation (1), with the volatility structures given by equations (2) and (3), and the initial forward rate curve $\{f(0,T)|T = 0, 1, 2, \cdots\}$, given, then the dynamics of the term structure can be represented by a process Markovian in two state variables, $r(t)$ and $u(t)$. The dynamics of these state variables are:*

$$\begin{aligned} r(t) &= f(0,t) + a_1(r(t-1) - f(0,t-1)) + b(t) + c_1 u(t) + d_1\sqrt{\Delta t}\epsilon_t & (5) \\ u(t) &= a_2 u(t-1) + a_2\sqrt{\Delta t}\eta_t, & (6) \end{aligned}$$

*where $b(t)$ is a time varying deterministic function defined in the appendix and*

$$\begin{aligned} d_1 &= \sqrt{(b_2 a_1 + c_2 a_2)^2 + (b_1 a_1)^2} \\ c_1 &= (a_2 - a_1)c_2, \end{aligned}$$

*In addition, $u(0) = 0$ and the two random variables, $\eta_t$ and $\epsilon_t$, are standard normal random variables with correlation $\rho_1 = \frac{b_2 a_1 + c_2 a_2}{d_1}$.*
*(ii) The price at time $t$ of a riskfree zero coupon bond that pays \$1 at time $t+n$, $P(t,t+n)$, is:*

$$P(t,t+n) = e^{-A_n(t) - B_n r(t)\Delta t - C_n u(t)\Delta t} \tag{7}$$

*where $A_1(t) = 0$, $B_1 = 1$, $C_1 = 0$, and*

$$\begin{aligned} B_n &= 1 + B_{n-1}a_1 \\ C_n &= c_1 B_{n-1} + a_2 C_{n-1} \\ A_n(t) &= A_{n-1}(t+1) + B_{n-1}[f(0,t+1) + b(t+1) - a_1 f(0,t) - a_1(f(0,t) + b(t+1)]\Delta t \\ &\quad -\frac{1}{2}[(B_{n-1}d_1 + C_{n-1}\rho_1 a_2)^2\Delta t^3 + (C_{n-1}a_2)^2(1-\rho_1^2)\Delta t^3]. \end{aligned}$$

**Proof:** See Appendix.

In this representation, the dynamics of interest rates follow a mean reverting process, where the long run average of the short rate is itself stochastic, following its own mean reverting process. Similar processes have been considered by Jegadeesh and Pennacchi (1996) and by Hull and White (1994). The advantage of this process over a simpler one-factor extended Vasiceck model is that forward rates are not perfectly correlated, and the volatility structure of forward rates can be humped.

### 3.2 Pricing Risky Bonds

Duffie and Singleton (1999) price risky bonds under the assumption that if a default occurs, recovery is proportional to the predefault market value of the debt. Under this assumption they show that the price of a defaultable bond can be obtained as the martingale expectation of the promised face value and coupons, where all payoffs are discounted by a specific discount rate that embodies the riskless time value, the loss arrival rates, and the fractional recovery.

Specifically, assume the time to default is generated by a Cox process.[3] The intensity of the process, under the risk neutral measure $Q$, is $\lambda(t)$. Hence, the chance of default over some small time interval, $\Delta t$ is $1-e^{-\lambda(t)\Delta t} \simeq \lambda(t)\Delta t$. Let $\tau$ be the random variable representing the period in which default takes place. If default occurs, the firm recovers a fraction, $\phi(t)$, of the value that the bond would have had, if there had been no default. Let $G(t,T)$ represent the period $t$ value of a risky bond that promises to pay \$1 at period $T$. Then the date $t$ price of a risky bond that matures in $n$ periods, assuming the bond has not defaulted, is:

$$\begin{aligned} G(t,t+n) &= P(t,t+1)E_t^Q[G(t+1,t+n)] \\ &= e^{-r(t)\Delta t}[(1-\lambda(t)\Delta t) \times E_t^Q[G(t+1,t+n)|\tau > t+1] + \\ &\quad \lambda(t)\Delta t \times E_t^Q[G(t+1,t+n)|\tau = t+1] \\ &= e^{-r(t)\Delta t}[(1-\lambda(t)\Delta t) + \lambda(t)\Delta t\phi(t)]E_t^Q[G(t+1,t+n)|\tau > t+1] \end{aligned}$$

Notice that $[(1-\lambda(t)\Delta t)+\lambda(t)\Delta t\phi(t)] \simeq e^{-\lambda(t)\Delta t(1-\phi(t))}$. Denote $s(t) = \lambda(t)(1-\phi(t))$ , then the risky bond can be written as:

$$G(t,t+n) = e^{-(r(t)+s(t))\Delta t}E_t^Q[G(t+1,t+n)|\tau > t+1].$$

Indeed, Duffie and Singleton's valuation formula for risky bonds is identical to the formula for risk-free bonds, with the exception that the riskless rate, $r(t)$, is replaced by an adjusted short rate given by $R(t) = r(t) + s(t)$, where the spread, $s(t)$, reflects the local default rate and the fractional loss rate given a default. The advantage of this approach is that once the higher rate is used as a discount rate, valuation can proceed as if the claim never defaults. This result makes it possible to transfer all the standard term structure models for default-free bonds to risky bonds, merely by parameterizing $R(t)$ instead of $r(t)$.

Using the reduced form modeling approach with fractional recovery implies that to price a risky bond, we need to specify the process for interest rates and for the short credit spread, under the risk neutral measure. In our approach, the interest rate process, under the risk neutral measure, is given by our two-factor discrete time model, given in equations (5) and (6), where the initial yield curve is given, and the volatility structure for forward rates is humped. The dynamics of the short credit spread process that we adopt is given by:

$$s(t+1) = \alpha_0 + \alpha_1 s(t) + \alpha_2(\zeta_{t+1} - \alpha_3)^2,$$

where $\zeta_t$ is a standard normal variable with $Cor(\zeta_t, \epsilon_t) = \rho_2$ and $Cor(\zeta_t, \eta_t) = \rho_3$ . The full model for establishing risky forward rates is then:

$$r(t+1) = f(0,t+1) + a_1[r(t) - f(0,t)] + b(t) + c_1 u(t) + d_1\sqrt{\Delta t}\epsilon_{t+1} \quad (8)$$

[3] Roughly speaking, a Cox process is a Possion process with a stochastic intensity parameter. The form used in finance differs slightly from the standard form. See Lando (1998).

$$u(t+1) = a_2 u(t) + d_2\sqrt{\Delta t}\eta_{t+1} \quad (9)$$
$$s(t+1) = \alpha_0 + \alpha_1 s(t) + \alpha_2(\zeta_{t+1} - \alpha_3)^2. \quad (10)$$

In this representation, the short credit spread is mean reverting, persistent, and if the constraints $\alpha_0 \geq 0$, $\alpha_1 \geq 0$, and $\alpha_2 \geq 0$ are imposed, then the spread does not become negative. It may be thought of as an autoregressive process with noncentral $\chi^2$ shocks. In addition, the short spread can have arbitrary correlation with the two state variables characterizing interest rates. The correlation between the short spread and interest rate , $\rho_{rs}$, say, is given by:

$$\rho_{rs} = \frac{-\sqrt{2}\alpha_3\rho_2}{\sqrt{1+2\alpha_3^2}}.$$

Hence, $\alpha_3$ influences the correlation between interest rates and spreads. Further, $\alpha_3$ controls the skewness of the short spread; if its value is positive, then negative innovations on $\zeta_{t+1}$ have larger influences on the short spread than equivalent positive innovations. In this sense, $\alpha_3$ controls the skewness of the spread distribution.

**Proposition 2**

*If the interest rate and the instantaneous credit spread, under the risk neutral measure, are given by (8), (9) and (10), then, risky zero coupon bond prices at date t can be expressed as:*

$$G(t,t+n) = P(t,t+n)e^{-\overline{D}_n s(t)\Delta t - \overline{E}_n}, \quad (11)$$

*where*

$$\overline{D}_n = 1 + \overline{D}_{n-1}\alpha_1, \ \overline{D}_1 = 1$$
$$\overline{E}_n = \overline{E}_{n-1} + \overline{D}_{n-1}(\alpha_0 + \alpha_2\alpha_3^3)\Delta t + \frac{1}{2}ln(1 + 2\overline{D}_{n-1}\alpha_2\Delta t)$$
$$-\frac{(\overline{B}_{n-1}d_1\sqrt{\Delta t}\rho_2 + \overline{C}_{n-1}d_2\sqrt{\Delta t}\rho_3 - 2\overline{D}_{n-1}\alpha_2\alpha_3)^2(\Delta t)^2}{2(1+2\overline{D}_{n-1}\alpha_2\Delta t)} + \frac{1}{2}(\overline{B}_{n-1}d_1 + \overline{C}_{n-1}d_2\rho_1)^2(\Delta t)^3$$
$$+\frac{(C_{n-1}d_2)^2(1-\rho_1^2)\Delta t^3}{2} - \frac{\Delta t^3}{2}(\overline{B}_{n-1}d_1\sqrt{1-\rho_2^2} + \overline{C}_{n-1}d_2\frac{\rho_1 - \rho_2\rho_3}{\sqrt{1-\rho_2^2}})^2$$
$$-\frac{\Delta t^3}{2}(\overline{C}_{n-1}d_2\frac{\sqrt{1-\rho_1^2-\rho_2^2-\rho_3^2+2\rho_1\rho_2\rho_3}}{\sqrt{1-\rho_2^2}})^2), \ \overline{E}_1 = 0$$
$$\overline{B}_n = 1 + a_1\overline{B}_{n-1}, \ \overline{B}_1 = 1$$
$$\overline{C}_n = c_1\overline{B}_{n-1} + a_2\overline{C}_{n-1}, \ \overline{C}_1 = 0.$$

**Proof** : See appendix.

Knowing the zero coupon bond prices, we can obtain the term structure of credit spreads, which at date $t$ is defined as the difference between the yields of defaultable and default-free bonds, and is given by the expression:

$$s(t,t+n) = -ln(G(t,t+n))/(n) - (-ln(P(t,t+T))/(n))$$
$$= \frac{1}{n}ln\frac{P(t,t+n)}{G(t,t+n)}$$
$$= \frac{1}{n}(\overline{D_n}s(t)\Delta t + \overline{E_n}).$$

It is clear from the above formula that while the credit spread, $s(t, t+n)$, is not explicitly dependent on the level of interest rates, it is influenced by the correlation effects with the interest rate process. While the level and shape of the term structure of credit spreads is influenced by all the parameters, the way in which shocks to the short credit spread, $s(t)$, are transmitted along the term structure of credit spreads, is solely determined by the maturity and the parameter $\alpha_1$.

By taking particular diffusion limits of the riskless dynamics, our model can be made to converge to continuous time diffusions that include the two-factor Hull and White (1993) model. Similarly, by taking diffusion limits of the credit spread process, particular diffusion limits can be obtained.[4] For our purposes, however, the discrete time models will suffice.

A special case of the above model occurs when the innovation for the credit event is taken to be the same innovation that affects interest rates. Now the credit spread follows the dynamics:

$$s(t+1) = \alpha_0 + \alpha_1 s(t) + \alpha_2(\epsilon_{t+1} - \alpha_3)^2. \tag{12}$$

Equation (12) is a special case of equation (10) where $\rho_2 = 1$ and $\rho_3 = \rho_1$. Risky discount bond prices for this model differ from the more general model in that the correlation between interest rates and spread innovations are controlled through $\alpha_3$, and the correlation between the innovations of the long-run average of the short rate process and the spread innovation is not required. The correlation between interest rates and credit spreads is controlled by the level of $\alpha_3$. That is,

$$\rho_{rs} = -\frac{2\alpha_3}{\sqrt{2 + 4\alpha_3^2}}. \tag{13}$$

When $\alpha_3$ is positive(negative), the correlation is negative (positive).

Notice that if we were to consider two different firms, this model does not imply that the credit spreads are perfectly correlated. To see this, denote the credit spread of a second firm by:

$$\tilde{s}(t+1) = \tilde{\alpha_0} + \tilde{\alpha_1}\tilde{s}(t) + \tilde{\alpha_2}(\epsilon_{t+1} - \tilde{\alpha_3})^2. \tag{14}$$

The correlation between $s(t)$ and $\tilde{s}(t)$ is:

$$\rho_{s\tilde{s}} = -\frac{2(1 + 2\alpha_3\tilde{\alpha_3})}{\sqrt{2 + 4\alpha_3^2}\sqrt{2 + 4\tilde{\alpha_3}^2}}. \tag{15}$$

. The correlation is 1 only when $\alpha_3$ equals $\tilde{\alpha_3}$.

Our three-factor model for pricing risky debt has the following properties: credit spreads are nonnegative and arbitrarily correlated with interest rates, they exhibit skewness, and analytical solutions are avaiable for all maturities.

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[4]For example, if we take

$$\begin{aligned} \overline{\alpha}_0 \Delta t &= \alpha_0 + \alpha_2(1 + \alpha_3^2) \\ \overline{\alpha}_1 \Delta t &= 1 - \alpha_1 \\ \overline{\alpha}_2 \sqrt{\Delta t} &= \alpha_2 \\ \overline{\alpha}_3 &= \alpha_3, \end{aligned}$$

the short-term spread process converges to a Gaussian mean reverting process.

### 3.3 Pricing American Options

Since some of the subordinated debt proposals involve the issuance of derivative contracts, such as putable bonds, we need pricing mechanisms for such contracts. Longstaff and Schwartz (2001) provide a simple method for valuing American options using simulation, and we use their methods. The key to their approach is to use least squares to estimate the conditional expected payoff to the optionholder from continuation. Since the expectation can be represented as a linear function of the elements of basis functions, it can be approximated using the first $M$ basis functions. Once the basis functions have been specified, they show how regression methods can be used to accurately approximate the conditional expectation. Interestingly, Longstaff and Schwartz show that many applications require only a few basis functions, and that the exact choice of basis functions is not that material.[5]

In our application, we have three state variables, from which the entire term structures of riskless and risky debt can be reconstructed. We used functions of these state variables as independent variables for the regressions. For the case of put options on coupon bonds, we computed the price of the underlying risky and riskless bond and added them to the list of state variables. Simple polynomial functions of these state variables were used for the regressions. After some experimentation we ended up using the following variables, $\{r, r^2, r^3, s, s^2, s^3, u, u^2, u^3\}$. The addition of other variables, including cross products of the state variables, did not provide additional accuracy.

## 4 Estimating the Parameters for the Credit Spread Model

Next we obtain parameter values by calibrating the model to the available data. We do this in two steps, first obtaining the term structure parameters, and then obtaining the credit spread parameters. One result of this exercise is a realization that we must be precise about several things often treated rather cavalierly. Chief among these is the definition of the credit spread. To obtain a consistently defined spread across several types of contracts, we restrict ourselves to the par spread: the difference between par yields, calculated as the coupon payment that makes the value of a newly issued bond equal to par.

### 4.1 Calibrating the Interest Rate Parameters

To implement the model described above, we first estimate the parameters of the two-factor interest rate process using term structure data.

For constructing the yield curve, we use futures and swap data on LIBOR. This means our work is not exactly comparable with work that uses the spread over U.S. Treasury yields. The LIBOR market gives us a richer and deeper set of traded derivatives that make estimating the volatility structure easier. Swap rates also have some claim to being a reasonable benchmark, as the market is active and quotes are readily available.[6] For the short end of the curve (up to one-year maturity), we use the five nearest futures contracts on any given date. These futures

[5]For example, using Hermite, Legendre,and Chebyshev polynomials or even simple powers of the state variable lead to accurate results.

[6]For more on pricing swaps and their relation to Treasuries, see Grinblatt (2001),Collin-Dufresne and Solnik (2001), and Duffie and Singleton (1997). For a discussion of the extent to which swaps have already replaced Treasuries as a benchmark, see Mengle and Smithson (2001)

rates are interpolated, and then convexity corrected to obtain the forward rates for three, six, nine, and twelve-month maturities. The rest of the yield curve out to five years is estimated using the forward rates bootstrapped at six month intervals from market swap rates. The futures and swap data are obtained from DataStream.

The data for this study consists of USD swaption prices. Specifically, from Datastream, the swaptions data set comprises volatilities of swaptions of maturities six months, one, two, three, four, and five years, with the underlying swap maturities of one, two, three, four, and five years each (in all, there are 30 swaption contracts). As per market convention, a swaption is considered at-the-money when the strike rate equals the swap rate for an equal maturity swap.

Like Amin and Morton (1994), Driessen, Klaassen, and Melenberg (2000), Longstaff, Santa-Clara, and Schwartz (2001), and Moraleda and Pelsser (2000), we estimate model parameters from cross sectional options data. This means that at any date we fit models to the prices of swaptions for different maturities and underlying swap expirations. Our objective function is to minimize the sum of squared percentage errors between theoretical and actual prices using a nonlinear least squares procedure.

Using data on September 30, 1999, the following estimates were obtained for our two factor volatility structure:

$$\kappa_1 = 0.044978,\ \kappa_2 = 3.407608,\ b_1 = 0.00014383,\ b_2 = -0.012441,\ c_2 = 0.018797$$

Figure 1 shows the volatility hump for the forward rate volatilities, and table 1 shows the percentage errors in the 30 estimated swaption prices. Pricing errors of about 3 percent or less are deemed reasonable, as this typically corresponds to the bid-ask spread in this market. Actual market prices of swaptions are quoted in terms of implied Black volatilities and a 3 percent pricing error approximates a one half Black vol. As the table makes clear, the predicted prices generally fall well within the bid–ask spread.

Figure 1 Here

Table 1 Here

These parameter estimates are used as benchmark values for the analysis that follows.

## 4.2 Calibrating the Credit Spread Parameters

To estimate the remaining parameters of the model requires credit spread information. To obtain an indication of the parameter values, we estimated the credit spread of five different banks. The data were taken from Bloomberg.[7] The terms and prices of these issues on September 30 1999, are shown in Table 2 along with the percentage errors of the fitted prices, which were obtained by minimizing the sum of squared percentage errors.

Table 2 Here

The values for the credit spread parameters are reported in Table 3.

[7] For an analysis of the accuracy and comparability of different sources for bank subdebt pricing, see Hancock and Kwast (2001).

Table 3 Here

These estimated values are used as parameters for an illustrative case, for which we can then price a variety of hypothetical debt contracts being considered under various proposals.

Notice that our model fits the data very well. On a par bond (price of \$100) the mean absolute error from the model is \$0.8. This compares with an average bid–ask spread of high–grade corporate bonds of about \$0.2 obtained by Chakravarty and Sarkar (1999).

Table 2 also shows the actual and fitted yields-to-maturity for all the bonds. As can be seen from the table, the differences are relatively small. The errors are symmetric, and the mean absolute percentage difference in prices is 0.87% or 87 cents on a \$100.00 bond (median, 0.62%). For yields, the mean absolute difference is 2 basis points. On a per bank basis, the root mean squared error is 60.66. This compares, for example, with Duffee's per firm value of 7.99.

# 5 Evaluating the Proposals

In this section we use the calibrated model to generate theoretical prices of a variety of subordinated debt contracts that have been proposed. How prices and yields change with bank risk and nonrisk factors depends on contract design. A good proposal has yields that are sensitive to risk factors and insensitive to nonrisk factors.

## 5.1 Spreads for Straight Subordinated Debt

The first proposal we consider is fixed-coupon subordinated debt. As an initial base case, figure 2 shows the credit spread curves for zero-coupon risky bonds of increasing maturities for our five banks.

Figure 2 Here

The credit spreads are both significant and differ noticeably across banks: the difference between Wachovia and Bankers Trust is over 50 basis points. Although the credit ratings of the banks in this sample show little variation (all are Moody's A1 or A2), it is nice to note that the bank with the highest spread is rated A2 and that with the lowest is rated A1. The amount of information in these rating differences is unclear; in fact, while Moody's rates Bank One above J.P. Morgan (A1 vs. A2), Standard and Poor's ranks Morgan several steps higher (AA- vs. A).

The spread curve also decreases with maturity for all but one bank, but only slightly. Even in that case however, the five basis point spread of Wachovia is far less than the term spread seen in the LIBOR market on the same date.

For comparative static results, looking at the simpler case of zero coupon bonds is instructive. Zeroes provide some clear results for how changes in the risk process $s(t)$ affect yield spreads. As the underlying process parameters $\alpha_1, \alpha_2$, and $\alpha_3$ change, the risk spread, and how that spread spread depends on maturity, also changes. Figures 3a-3c plot spread against maturity for values that encompass the range of parameter values we find in the data.

Figure 3a plots the effect of maturity on spread for different levels of $\alpha_1$, the mean reversion parameter. This has a noticeable, but expected, effect. For high levels of $\alpha_1$, the yield spread becomes upward sloping. Conversely, low levels of $\alpha_1$ imply fast mean reversion—so the bank is expected to be less risky in future, and the spread declines with maturity. With a high $\alpha_1$, mean revision is slow, and today's high probability of loss compounds over time, increasing the spread with maturity.

Figure 3b, with results for different values of $\alpha_2$, shows a similar picture. A higher $\alpha_2$ increases both the mean and variance of the process, and for high enough values this leads to an increasing yield spread.

Figure 3c shows the effects of different values of $\alpha_3$. Here the qualitative effects are less noticeable: in all cases the spread has a "u" shape. The most important impact of $\alpha_3$ is on the correlation of the spread with the risk-free interest rate, which we discuss in more detail below.

A revealing chart plots the sensitivity of the yield spread to changes in the instantaneous spread $s(0)$. (This is just the derivative of 12). Figure 4 shows this as a function of maturity. Roughly, this chart shows how much of an increase in the instantaneous failure rate $s(0)$, shows up in the subdebt spread. The result depends heavily on maturity: less than 10 percent of the increase shows up in a 10 year bond. Differences in $\alpha_1$ matter as well, particularly for shorter maturities—the effect is generally washed out for longer. Thus, maturity has a big impact on a bond's ability to signal a particular type of risk change, though, as we discuss in section 6, this is not always the only, or the most relevant, risk to consider.

Zeroes make up a very small proportion of the subordinated debt market (Sironi (2001) finds only 1.6 percent of European bank Subdebt issues are zeroes). Most are coupon bonds of varying maturities. Figure 5 shows the *par credit spreads* for the risky bonds. They also have a slight downward slope. Unlike the zero coupon spread curves, these par spreads do depend on the term structure. Since the term structure at the time was upward sloping, the par credit spreads are all slightly higher, though with no noticeable change in slope.

Figure 6 shows how the shape of the riskless yield curve affects par credit spreads. It shows the par credit spread for a hypothetical five year par coupon subordinated debt issue, based on the parameter estimates for Chase. The risk free yields make their presence felt: the long end of the credit spread slopes up or down depending on the slope of the initial term structure. A careful look at the scale, however, reveals that the difference is on the order of two basis points. The credit spread is insensitive to the risk-free term structure, by these calculations.

The effect of a shock to the state variable, $s(t)$, on the par spread depends on the maturity of the bond, on $\alpha_1$, on the size of the shock to the short credit spread, and on the shape of the riskless yield curve. The size of the shock is magnified by $\alpha_2$ and its distance from $\alpha_3$, and its correlation with the riskless interest rate also depends on $\alpha_3$. In terms of the parameters observed, however, changes in the state variable $s(t)$ do not have a large impact on the credit spread. Figure 7 shows the changes in credit spread as $s(0)$ varies, given the parameters for the five banks in our sample, assuming a maturity of five years. The effects are mostly on the order of less than one basis point.

Any conclusion about the proper design of subdebt cannot be made without a comparison to other proposals, but these results do bear on the question of subdebt maturity. The answer, however, depends on the purpose of the debt. If regulators desire only a rough ranking of banks by risk, perhaps to confirm or deny examination ratings, maturity does not matter. It is unlikely that the regulator would misclassify banks by ignoring maturity. If one desires a more active monitoring that detects changes in a bank's risk, then maturity does matter and shorter is better. The subdebt spread becomes highly sensitive only for maturities shorter than two years. Furthermore, these signals are robust to noise in the sense of varying little with the shape of the risk-free yield curve.

## 5.2 Spreads for Floating Rate Debt

Floating rate subordinated debt pays out a fixed spread above a floating benchmark, which we take as LIBOR. The certainty equivalents of the LIBOR cash flows can be determined, and the spread can then be determined such that the bond is issued at par. The change in this par spread over time is the signal that we now investigate. Duffie and Liu (2001) study the term structure of yield spreads between floating rate and otherwise identical fixed rate bonds. The credit spreads of the two need not be the same. For example, if the term structure slopes upward, investors expect that floating rate coupons will increase over time. Default risk, however, also increases over time. As the later and higher coupons are also the most likely to be lost to default, investors who hold the floating rate bond must be compensated by a floating point spread that is slightly larger.

For our five banks, the difference between par spreads on fixed and floating bonds is small for all maturities, averaging one-third of a basis point. This confirms the Duffie and Liu result that the magnitude of the differences is small. Since credit spreads based on floating rate bonds are so similar to those based on fixed rates, the signaling mechanisms of the two are similar and the implications in the previous section follow through.

## 5.3 Spreads for Putable Fixed Coupon Bonds

We now consider the proposal that calls for banks issuing American style putable fixed coupon bonds, where the coupon is set so the bond is initially priced at par. If the performance of the put is guaranteed regardless of the condition of the bank, then the putable bond provides a money-back guarantee. This assumption is unreasonable since the conditions under which the put might be optimally exercised include states of nature where the credit condition of the bank has deteriorated, and in these states of nature, the firm may not be able to deliver on its obligations. In light of this, we assume that if the bond defaults before the option is exercised, the option is made worthless. That is, the put option is an American put option which is knocked out if the bank defaults.

Notice that if the put option was a European option, its value would not depend on the recovery rate at all. Payouts at the expiry date would only be obtained if no default occurred over the entire period. In contrast, for the American knockout, the decision to put the bond or not is based on the current market price of the defaultable bond, and that value is influenced by the recovery value of the bond.

To price this contract, we simulate the hazard rate and the short interest rate dynamics, and based on the hazard rate establish whether a default occurs in a particular period. If a default occurs, then the option is worthless, and a payout of $0 is assigned to the path thereafter. If no default occurs, we compute the risky bond price based on the state variables and the intrinsic value of the put option. We then use these values in the Longstaff/Schwartz simulation model.[8]

Of course, the putable bond may be exercised because interest rates have increased. Hence, the put feature does not provide a claim that precisely targets credit risk, and as a result the option adjusted spread for this instrument will clearly be affected by the riskless yield curve. In this regard, the correlation effects between the credit spread and interest rates become important.

[8]Actually, when discounting here, it is correct to discount at the riskless rate, not the riskless rate adjusted for the spread. This is the case because the defaults are explicitly accounted for in the assignment of payouts to the paths.

Because the putable bond may be exercised because of changes in the risk free rate, maturity becomes a larger factor in the spread, as a longer maturity gives more time for interest rates to move enough to make the option exercisable. Figure 8 shows the credit spread for the different banks in our sample. Maturity differences are often greater than 10 basis points, and in one case, greater than 20 basis points. It now becomes possible to misclassify banks if maturity is ignored.

The serious concern about putable coupon bonds is that they will be exercised when the risk-free rate falls, not only when the credit spread rises. The initial term structure becomes important because it influences the direction rates will move and thus the probability that the put will be exercised. In our case, however, the actual effect is quite small: the change is a few basis points at most. The problem shows up much more when the effect of the change in interest rate volatility is plotted, as in figure 9. Particularly for the longer maturities, the effect of increasing interest rate volatility is easily more than 50 basis points. The added volatility makes the put more valuable, and thus the observed credit spread decreases. While the volatility has no direct effect on the pure credit spread, to the extent that credit shocks are correlated with interest rate shocks, the higher volatility makes the bank debt riskier. The put not only adds noise both to the exercise of the option and to the value of the spread, it also directly masks the risk signal: the spread decreases as the bank gets riskier.

### 5.4 Spreads for Putable Floating Rate Bonds

The final proposal that we consider argues for the creation of putable floating rate bonds. As in the previous case, we assume that the put contracts are made worthless if they are not exercised prior to default. The advantage of using a floating rate bond is that at reset dates, the discount from par is fully attributed to credit risk. If credit risk has remained the same, then the price of the bond at reset dates fully reflects credit risk, not interest rate risk, and the decision by bondholders to exercise the put will be based on credit events alone. Figure 10 plots the credit spreads for putable floating rate bonds.

Attaching the put option to a floating-rate bond removes the influence of the initial term structure and of maturity on the credit spread. There is also no longer much of an effect from changes in interest rate volatility. Making the bond floating accomplishes the objective of removing the influence of riskfree interest rates on the put.

The possibility still remains, though, that the put may drive the credit spread in perverse directions if the credit spread process changes. The effects, however, are quite negligible. Figures 11a, 11b, and 11c isolate put's effect on the spread from changes in $\alpha_1$, $\alpha_2$, and $\alpha_3$. These never even reach one-half of a basis point. In part, this is because for our sample, the put was out of the money, and not worth much. Had it been more valuable, the effects would have been larger—but our calibration does establish that for a representative time, the put has little effect on the spread. One further consequence is that the sensitivity of the spread to changes in the state variable (instantaneous spread) is little affected by the put, as demonstrated by figure 12.

## 6 Comparing the Proposals

In order for subordinated debt to increase market discipline, it must provide the correct signals to supervisors. An important criterion for evaluating different proposals must be information content—how hard will it be to extract the correct information about bank risk from subordinated debt? One straightforward signal is the credit spread—the spread between yields on subordinated debt and comparable risk-free rates (in our paper, LIBOR).

Looking at the spread is the standard approach, but not the only one. Some studies have used option valuation techniques to back out a risk measure from subordinated debt prices (Gorton and Santomero (1990), Schellhorn and Spellman (1996)), and often emphasize the nonlinear relation between risk and credit spread, as higher risk may correspond to either larger or smaller spreads (Bliss (2000) also stresses this). Though our approach is in some ways complementary to this branch of the literature in that we work directly with an asset pricing model, we concentrate on spreads rather than on implied volatilities for several reasons. First, the negative relation between risk and credit spread, though theoretically possible, has not been observed in subordinated debt prices. Furthermore, such models, based on the fundamental approach to pricing risky debt, are notoriously unreliable at pricing credit spreads, and this calls into question their mapping between risk and prices. Separate from the academic aspects of the problem, it is our view that proposals using spreads are more likely to be adopted by regulators, either because the proposals specifically mandate spreads (as in Calomiris), or because the regulators would be reluctant to commit to a specific model for backing out implied risk, perhaps because it would be less verifiable than a publicly observable credit spread.

From the perspective of information contained in the credit spread, there is reason to be optimistic about standard, fixed coupon subordinated debt. Credit spreads definitely vary between banks—in some cases by over 50 basis points in our small sample. Furthermore, some of the more obvious sources of noise do not seem important. Neither changes in maturity or in the risk-free term structure affect credit spreads substantially. Nor do shifts in the volatility of interest rates matter much for the credit spread. As far as one can generalize from our limited sample, while supervisors should be aware of the slope of the yield curve and the bond's maturity, those factors are unlikely to affect the risk ranking among banks. The credit spread reflects risk, not today's yield curve.

The downside is that credit spreads are not very sensitive to changes in the instantaneous credit spread. We argue below that this is not a disadvantage, but it does point out that sub debt is unlikely to be a reliable indicator of high frequency variation in bank quality.

Floating rate subordinated debt behaves almost identically to fixed coupon debt. To the extent that some banks seem to prefer floating rate subdebt (Sironi (2001) finds 25 percent of bank sub debt in Europe is floating), the regulatory burden might be decreased if banks are allowed a choice between fixed and floating debt. Most proposals specify fixed coupon debt, but there seems to be no good reason for this.

Making debt putable has been a controversial suggestion. A particular concern is the possibility that fixed coupon debt is put for reasons other than credit events. We don't directly address this problem, but our results suggest that this is a serious problem. Maturity has more of an impact, and it becomes harder to rank banks. Even less encouraging for proponents, interest rate volatility matters greatly, even when the put is out of the money. Thus, a fixed coupon bond with a put attached fails in providing both direct and indirect discipline: adding the put means the market will discipline the bank at inappropriate times, and also makes it harder to extract information from the credit spread. The first of these concerns has been expressed for years, but we confirm the quantitative significance of the problem.

Making the putable debt floating avoids many problems of fixed rate debt, and maturity and term structure disappear as influences on the credit spread. While a potential disadvantage of putable floating debt could lie in its response to the variability of the credit spread, we find this is not the case. Adding the put to floating debt does not distort the credit spread signal. While our conclusion in part depends on our sample, it at a minimum establishes a base for a range of parameters.

### 6.1 Broader Lessons: Processes, Pricing, and Proposals

One important contribution an asset pricing approach brings to the study of mandatory subordinated debt proposals is so simple and obvious that it is likely to be overlooked. It focuses on the importance of the stochastic process driving the combined default/recovery probability. The spread on a risky bond depends on several factors. Today's short spread, or instantaneous default probability, measures only one aspect of bank risk. Longer term spreads depend on other things as well. The mean reversion parameter determines how fast a bank will recover from today's problems. The variance of the process determines the chance of future bad shocks. The correlation with interest rates determines a bank's sensitivity to interest rates, including whether increases or decreases hurt the bank more. It is important to keep these parameters conceptually separate. A bank with little risk of failing in the very near future (low $s(0)$) may be very vulnerable to higher interest rates in the future.

A possible downside to subordinated debt became apparent in our simulations. The credit spread was also relatively insensitive to several variables that should have mattered more, such as $s(0)$, the instantaneous credit spread (compounding default probability and recovery rate). The lesson is that the probability of failure between now and the end of the day is not the only, nor even the dominant, factor in pricing a risky bond. Rather, the entire process matters, since it is the expected future evolution of $s(0)$ that governs the default probability over the next ten years. A case in point is Wachovia—despite a relatively high instantaneous default probability, it has the lowest credit spread, primarily because the shock reverts quickly (low $\alpha_1$) to a low mean (low $\alpha_0$). Bankers Trust has a high spread because it reverts quickly to a high mean (high $\alpha_1$ with high $\alpha_0$).

This has important implications for interpreting credit spreads. The bonds, having payoffs stretching over several years, provide information about default probabilities over several years—they provide average, rather than point-in-time data. For most purposes that is preferable—we want to know if the bank is going to be around next year, and are less interested in the precise odds that it won't be around tomorrow.

The probability of failure over time—and the consequent credit spread—depends on the stochastic process for $s(t)$, not merely its current value. That means the mean reversion parameter $\alpha_1$ matters, as it tells, roughly, how long the bad shock will persist. A variable that turns out to be quantitatively important is $\alpha_3$, which has a large influence on the correlation between credit shocks and interest rate shocks. This parameter may be affected by banks' strategies—how hedged they are, and the extent to which they diversify and invest in cyclically volatile industries.

Of course there are reasons for being skeptical of results so far: they are for a limited number of banks calibrated at one particular date. The model attributes all of the spread to credit risk, ignoring liquidity premiums or tax consequences. Yet despite these drawbacks, which we hope to address in future work, we feel our calibration provides reasonable numbers that help assess the relative merits of mandatory subordinated debt proposals. Beyond the specific results, though, we have set up a framework that can capture, or at least raise the issue, of the trade-offs inherent in designing these instruments. Any attempt to balance the competing objectives needs to assess, and preferably assess quantitatively, the effects of the contract provisions. Only by matching security design with the desired objectives will a useful subordinated debt program emerge.

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# Appendix

**Proof of Proposition 1**

Substituting equations (2) and (3) into equation (1), the forward rate can be written as

$$\begin{aligned} f(t,T) &= f(t-1,T) + \sum_{n=1}^{2} \mu_{fn}(t-1,T)\Delta t + \sum_{n=1}^{2} \sigma_{fn}(t-1,T)\sqrt{\Delta t} Z_t^{(n)} \\ &= f(0,T) + \sum_{n=1}^{2}\sum_{j=0}^{t-1} \mu_{fn}(j,T)\Delta t + \sum_{n=1}^{2}\sum_{j=0}^{t-1} \sigma_{fn}(j,T)\sqrt{\Delta t} Z_{j+1}^{(n)}, \end{aligned}$$

where $\mu_{fn}(t,T), n = 1, 2$ is the drift term for each factor, given in equation (4).

Substituting the volatility functions in equations (2) and (3) into the drift term, and defining $a_1 = e^{-\kappa_1 \Delta t}, a_2 = e^{-\kappa_2 \Delta t}$, we have, upon simplification:

$$\begin{aligned} \mu_{f1}(t,T) &= \frac{b_1^2 a_1^{T-t}\Delta t}{2(a_1-1)}(a_1^{T-t} + a_1^{T-t+1} - 2a_1) \qquad (16) \\ \mu_{f2}(t,T) &= \frac{b_2^2 a_1^{T-t+1}\Delta t}{1-a_1} - \frac{1+a_1}{2(1-a_1)} b_2^2 a_1^{2(T-t)}\Delta t \\ &+ \frac{c_2^2 a_2^{T-t+1}\Delta t}{1-a_2} - \frac{1+a_2}{2(1-a_2)} c_2^2 a_2^{2(T-t)}\Delta t \\ &+ \frac{c_2 b_2((1-a_2)a_2^{T-t}a_1 + (1-a_1)a_1^{T-t}a_2 + (a_1 a_2 - 1)(a_1 a_2)^{T-t})\Delta t}{(1-a_1)(1-a_2)} \qquad (17) \end{aligned}$$

From the above calculation we see that the drift term $\mu_f(t,T)$ of the forward rate is a deterministic function of $t$ and $T$. Let $h(t,T)$ represent this function. That is:

$$h(t,T) = \sum_{n=1}^{2}\sum_{j=0}^{t} \mu_{fn}(j,T)\Delta t. \qquad (18)$$

Then:

$$\begin{aligned} f(t,T) &= f(0,T) + \sum_{n=1}^{2}\sum_{j=0}^{t-1} \mu_{fn}(j,T)\Delta t + \sum_{n=1}^{2}\sum_{j=0}^{t-1} \sigma_{fn}(j,T)\sqrt{\Delta t} Z_{j+1}^{(n)} \\ &= f(0,T) + h(t-1,T) + b_1 a_1^{T-t}\Phi_1(t) + b_2 a_1^{T-t}\Phi_2(t) + c_2 a_2^{T-t}\Phi_3(t), \end{aligned}$$

where,

$$\Phi_1(t) = \sum_{j=1}^{t-1} a_1^{t-j}\sqrt{\Delta t} Z_{j+1}^{(1)}$$

$$\Phi_2(t) = \sum_{j=1}^{t-1} a_1^{t-j}\sqrt{\Delta t} Z_{j+1}^{(2)}$$

$$\Phi_3(t) = \sum_{j=1}^{t-1} a_2^{t-j}\sqrt{\Delta t} Z_{j+1}^{(2)}.$$

For $T = t$, we have

$$r(t) = f(0,t) + h(t-1,t) + b_1\Phi_1(t) + b_2\Phi_2(t) + c_2\Phi_3(t). \qquad (19)$$

Further, we also have

$$r(t+1) = f(0,t+1) + h(t,t+1) + b_1\Phi_1(t+1) + b_2\Phi_2(t+1) + c_2\Phi_3(t+1). \qquad (20)$$

Notice

$$\begin{aligned}
\Phi_1(t+1) &= a_1\Phi_1(t) + a_1\sqrt{\Delta t}Z^{(1)}_{t+1} \\
\Phi_2(t+1) &= a_1\Phi_2(t) + a_1\sqrt{\Delta t}Z^{(2)}_{t+1} \\
\Phi_3(t+1) &= a_2\Phi_3(t) + a_2\sqrt{\Delta t}Z^{(2)}_{t+1}.
\end{aligned}$$

Substituting these terms into the equation (20) we obtain

$$\begin{aligned}
r(t+1) &= f(0,t+1) + h(t,t+1) + a_1(r(t) - f(0,t) - h(t-1,t)) \\
&\quad +(a_2 - a_1)c_2\Phi_3(t) + b_1a_1\sqrt{\Delta t}Z^{(1)}_{t+1} + b_2a_1\sqrt{\Delta t}Z^{(2)}_{t+1} + c_2a_2\sqrt{\Delta t}Z^{(2)}_{t+1}. \qquad (21)
\end{aligned}$$

Let $\ell(t) = h(t-1,t)$. The exact expression for $\ell(t)$ can be obtained by substituting in equations (16) and (17) into equation (18) for $T = t$ to obtain:

$$\begin{aligned}
\ell(t) &= \sum_{j=0}^{t-1}[\mu_{f1}(j,t) + \mu_{f2}(j,t)]\Delta t \\
&= \frac{b_1^2a_1^2(a_1^{2t} - 2a_1^t + 1)\Delta t^2}{2(a_1-1)^2} + \frac{1}{2}\left[\frac{b_2a_1(a_1^t - 1)\Delta t}{(a_1-1)} + \frac{c_2a_2(a_2^t - 1)\Delta t}{(a_2-1)}\right]^2 \qquad (22)
\end{aligned}$$

The dynamics of the variables $r(t)$ and $u(t)$ then follow by transforming the state variable $\phi_3(t)$, to $u(t)$ using $\phi_3(t) = u(t)$.

These two state variables also determine the term structure. Reconsider the forward rate equation, it can be written as:

$$\begin{aligned}
f(t,T) &= f(0,T) + h(t-1,T) + a_1^{T-t}(b_1\Phi_1(t) + b_2\Phi_2(t)) + c_2a_2^{T-t}\Phi_3(t) \\
&= f(0,T) + h(t-1,T) + a_1^{T-t}(r(t) - f(0,t) - h(t-1,t)) + c_2(a_2^{T-t} - a_1^{T-t})ku(t).
\end{aligned}$$

This completes the proof of the first part of Proposition 1.

Now consider the second part of the Proposition. The riskless discount bond price at time $t$ can be obtained by induction. For $n = 1$ the bond price equation leads to $P(t,t+1) = e^{-r(t)\Delta t}$ which is the correct result. Now assume that:

$$P(t,t+n) = e^{-A_n(t) - B_nr(t)\Delta t - C_nu(t)\Delta t}.$$

Then,

$$\begin{aligned}
P(t,t+n+1) &= e^{-r(t)\Delta t}E[P(t+1,t+n+1)] \\
&= e^{-r(t)\Delta t}E[e^{-A_n(t+1) - B_nr(t+1)\Delta t - C_nu(t+1)\Delta t}].
\end{aligned}$$

After computing the expectation, we obtain the result.

**Proof of Proposition 2**

The results can be obtained by induction. When $n=1$, $G(t,t+1)=e^{-r(t)\Delta t-s(t)\Delta t}$, which is correct. Assume $G(t,t+n)=e^{-\bar{A}_n(t)-\bar{B}_n r(t)\Delta t-\bar{C}_n u(t)\Delta t-\bar{D}_n s(t)\Delta t}$. Then,

$$\begin{aligned}
G(t,t+n+1) &= e^{-r(t)\Delta t-s(t)\Delta t}E[G(t+1,t+n+1)] \\
&= e^{-r(t)\Delta t-s(t)\Delta t}E[e^{-\overline{A}_n(t+1)-\overline{B}_n r(t+1)\Delta t-\overline{C}_n u(t+1)\Delta t-\overline{D}_n s(t+1)\Delta t}] \\
&= e^{-(1+\overline{B}_n a_1)r(t)\Delta t-(1+\overline{D}_n\alpha_1)s(t)\Delta t-(\overline{B}_n c_1+\overline{C}_n a_2)u(t)\Delta t} \\
&\quad \cdot e^{-\overline{A}_n(t+1)-\overline{B}_n(f(0,t+1)+l(t+1)-a_1(f(0,t)+l(t)))\Delta t-\overline{D}_n(\alpha_0+\alpha_2\alpha_3^2)\Delta t} \\
&\quad \cdot E[e^{-\overline{B}_n d_1(\Delta t)^{3/2}\epsilon_{t+1}-\overline{C}_n d_2(\Delta t)^{3/2}\eta_{t+1}-\overline{D}_n\alpha_2\Delta t(\zeta_{t+1}^2-2\alpha_3\zeta_{t+1})}]
\end{aligned}$$

Let

$$\begin{aligned}
\eta_{t+1} &= \rho_3\zeta_{t+1}+\sqrt{1-\rho_3^2}\,\overline{\eta}_{t+1},\ \ E[\zeta_{t+1}\overline{\eta}_{t+1}]=0 \\
\epsilon_{t+1} &= \rho_2\zeta_{t+1}+\sqrt{1-\rho_2^2}\,\overline{\epsilon}_{t+1},\ \ E[\zeta_{t+1}\overline{\epsilon}_{t+1}]=0.
\end{aligned}$$

Since $E[\eta_{t+1}\epsilon_{t+1}]=\rho_1$, $E[\overline{\eta}_{t+1}\overline{\epsilon}_{t+1}]=\frac{\rho_1-\rho_3\rho_2}{\sqrt{(1-\rho_3^2)(1-\rho_2^2)}}$. Denote $\rho_4=E[\overline{\eta}_{t+1}\overline{\epsilon}_{t+1}]$. We can write $\overline{\eta}_{t+1}=\rho_4\overline{\epsilon}_{t+1}+\sqrt{1-\rho_4^2}\,\overline{\overline{\eta}}_{t+1}$, $E[\overline{\zeta}_{t+1}\overline{\overline{\eta}}_{t+1}]=0$. Since $E[\zeta_{t+1}\overline{\eta}_{t+1}]=0$, and $E[\zeta_{t+1}\overline{\epsilon}_{t+1}]=0$ we have $E[\zeta_{t+1}\overline{\overline{\eta}}_{t+1}]=0$. Now we can calculate the expectation using three uncorrelated variables, $\zeta_{t+1},\ \overline{\epsilon}_{t+1},\ and\ \overline{\overline{\eta}}_{t+1}$. Specifically,

$$\begin{aligned}
& E[e^{-\overline{B}_n d_1\epsilon_{t+1}(\Delta t)^{\frac{3}{2}}-\overline{C}_n d_2\eta_{t+1}(\Delta t)^{\frac{3}{2}}-\overline{D}_n\alpha_2\Delta t(\zeta_{t+1}^2-2\alpha_3\zeta_{t+1})}] \\
=\ & E[e^{-\overline{B}_n d_1(\Delta t)^{\frac{3}{2}}(\rho_2\zeta_{t+1}+\sqrt{1-\rho_2^2}\overline{\epsilon}_{t+1})-\overline{C}_n d_2(\Delta t)^{\frac{3}{2}}(\rho_3\zeta_{t+1}+\sqrt{1-\rho_3^2}\overline{\eta}_{t+1})-\overline{D}_n\alpha_2\Delta t(\zeta_{t+1}^2-2\alpha_3\zeta_{t+1})}] \\
=\ & E[e^{-(\overline{B}_n d_1\sqrt{1-\rho_2^2}+\overline{C}_n d_2\sqrt{1-\rho_3^2}\rho_4)\overline{\epsilon}_{t+1}(\Delta t)^{\frac{3}{2}}-\overline{C}_n d_2\sqrt{1-\rho_3^2}\sqrt{1-\rho_4^2}\overline{\overline{\eta}}_{t+1}(\Delta t)^{\frac{3}{2}}} \\
& \cdot e^{-\overline{D}_n\alpha_2\zeta_{t+1}^2\Delta t+(2\alpha_3\alpha_2\overline{D}_n-(\overline{B}_n d_1\sqrt{\Delta t}\rho_2+\overline{C}_n d_2\sqrt{\Delta t}\rho_3))\zeta_{t+1}\Delta t}] \\
=\ & e^{\frac{\Delta t^3}{2}(\overline{B}_n d_1\sqrt{1-\rho_2^2}+\overline{C}_n d_2\sqrt{1-\rho_3^2}\rho_4)^2+\frac{\Delta t^3}{2}(\overline{C}_n d_2\sqrt{1-\rho_3^2}\sqrt{1-\rho_4^2})^2} \\
& \cdot e^{-\frac{1}{2}ln(1+2\alpha_2\overline{D}_n\Delta t)+\frac{(\overline{B}_n d_1\sqrt{\Delta t}\rho_2+\overline{C}_n d_2\sqrt{\Delta t}\rho_3-2\alpha_2\alpha_3\overline{D}_n)^2\Delta t^2}{2(1+2\alpha_2\overline{D}_n\Delta t)}}.
\end{aligned}$$

Eventually, we have

$$G(t,t+n+1)=e^{-\overline{A}_{n+1}(t)-\overline{B}_{n+1}r(t)\Delta t-\overline{C}_{n+1}u(t)\Delta t-\overline{D}_{n+1}s(t)\Delta t}.$$

Comparing $P(t,t+n)$ and $G(t,t+n)$, we see that $B_n=\overline{B}_n$ and $C_n=\overline{C}_n$. Hence we can write $G(t,t+n)=P(t,t+n)e^{-\overline{D}_n s(t)\Delta t-\overline{E}_n(t)}$.

Actually,

$$\overline{E}_n(t)=\overline{A}_n(t)-A_n(t)=\overline{E}_n.$$

The time-varying parts cancel out in $\overline{E}_n(t)$. Therefore,

$$G(t,t+n)=P(t,t+n)e^{-\overline{D}_n s(t)\Delta t-\overline{E}_n}$$

.

Table 1: Percentage Errors in the Estimated Prices of Swaptions*

| Contract | Market Vol. | Theoretical Vol. | Difference | Market Price | Theoretical Price | % Error |
|---|---|---|---|---|---|---|
| 6mx1 | 15.5 | 15.6414 | 0.1414 | 0.002486 | 0.002508 | 0.9111 |
| 6mx2 | 16.1 | 16.5372 | 0.4372 | 0.005185 | 0.005326 | 2.7125 |
| 6mx3 | 16.1 | 16.4354 | 0.3354 | 0.007705 | 0.007865 | 2.0807 |
| 6mx4 | 16.1 | 16.1456 | 0.0456 | 0.010104 | 0.010132 | 0.2829 |
| 6mx5 | 16.1 | 15.8067 | -0.2933 | 0.012377 | 0.012152 | -1.8196 |
| 1yx1 | 17.1 | 16.6335 | -0.4665 | 0.003937 | 0.00383 | -2.7218 |
| 1yx2 | 16.8 | 16.9068 | 0.1068 | 0.007665 | 0.007714 | 0.6339 |
| 1yx3 | 16.4 | 16.6756 | 0.2756 | 0.011018 | 0.011203 | 1.6767 |
| 1yx4 | 16 | 16.3184 | 0.3184 | 0.014036 | 0.014314 | 1.9858 |
| 1yx5 | 15.8 | 15.9555 | 0.1555 | 0.016919 | 0.017086 | 0.9821 |
| 2yx1 | 17.7 | 16.9607 | -0.7393 | 0.00565 | 0.005415 | -4.1566 |
| 2yx2 | 16.6 | 16.9269 | 0.3269 | 0.010355 | 0.010558 | 1.9602 |
| 2yx3 | 16.2 | 16.5905 | 0.3905 | 0.014806 | 0.015161 | 2.3995 |
| 2yx4 | 15.9 | 16.2216 | 0.3216 | 0.018881 | 0.019262 | 2.014 |
| 2yx5 | 15.6 | 15.7694 | 0.1694 | 0.022647 | 0.022892 | 1.0817 |
| 3yx1 | 17.6 | 17.0755 | -0.5245 | 0.006553 | 0.006359 | -2.9582 |
| 3yx2 | 16.5 | 16.8938 | 0.3938 | 0.012001 | 0.012286 | 2.37 |
| 3yx3 | 16.1 | 16.5515 | 0.4515 | 0.017099 | 0.017575 | 2.7852 |
| 3yx4 | 15.6 | 16.0721 | 0.4721 | 0.021613 | 0.022263 | 3.0069 |
| 3yx5 | 15.2 | 15.6692 | 0.4692 | 0.025636 | 0.026422 | 3.0683 |
| 4yx1 | 17.3 | 16.2614 | -1.0386 | 0.007078 | 0.006657 | -5.9486 |
| 4yx2 | 16.2 | 16.0363 | -0.1637 | 0.012884 | 0.012755 | -1.0019 |
| 4yx3 | 15.7 | 15.5608 | -0.1392 | 0.018342 | 0.01818 | -0.8792 |
| 4yx4 | 15.2 | 15.1738 | -0.0262 | 0.023038 | 0.022999 | -0.1713 |
| 4yx5 | 14.7 | 14.8149 | 0.1149 | 0.027062 | 0.027272 | 0.7757 |
| 5yx1 | 17 | 15.7716 | -1.2284 | 0.007323 | 0.0068 | -7.1486 |
| 5yx2 | 15.9 | 15.3364 | -0.5636 | 0.013473 | 0.013 | -3.5094 |
| 5yx3 | 15.3 | 14.9874 | -0.3126 | 0.018902 | 0.018519 | -2.0237 |
| 5yx4 | 14.7 | 14.6451 | -0.0549 | 0.023502 | 0.023415 | -0.3699 |
| 5yx5 | 14.3 | 14.279 | -0.021 | 0.027786 | 0.027745 | -0.1457 |

* Table 1 shows the percentage errors in swaption prices for September 30, 1999. The optimization procedure is described in the text.

Table 2: Comparisons of Fitted and Actual Bond Prices and Yields *

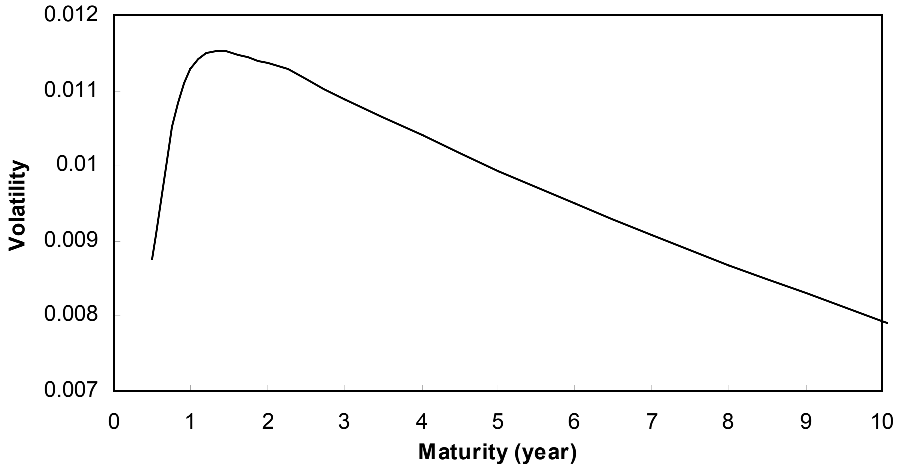
| Bank | Coupon | Maturity | IssueDate | Price | Pred Price | % Error | Yield | Pred Yield | Diff(bp) |
|---|---|---|---|---|---|---|---|---|---|
| Chase(BHC) | 6 | 2/15/09 | 8/15/99 | 92.16 | 90.94 | -1.32 | 8.98 | 9.36 | 38.74 |
| | 6.375 | 2/15/08 | 8/15/98 | 95.51 | 94.36 | -1.20 | 8.62 | 9.02 | 39.56 |
| | 6.375 | 4/1/08 | 10/1/98 | 95.51 | 96.63 | 1.17 | 7.30 | 6.98 | -32.02 |
| | 7.125 | 2/1/07 | 8/1/97 | 99.98 | 99.86 | -0.12 | 8.00 | 8.05 | 4.44 |
| | 7.125 | 6/15/09 | 12/15/97 | 99.98 | 99.64 | -0.34 | 7.84 | 7.93 | 9.07 |
| | 7.25 | 6/1/07 | 12/1/97 | 101.1 | 101.65 | .55 | 7.72 | 7.55 | -17.67 |
| | 7.5 | 2/1/03 | 8/1/93 | 102.03 | 102.62 | 0.58 | 7.94 | 7.40 | -53.97 |
| | 8.625 | 5/1/02 | 11/1/92 | 104.36 | 107.53 | 3.03 | 3.60 | 0.90 | -269.50 |
| J.P. Morgan | 6 | 1/15/09 | 7/15/99 | 91.65 | 91.03 | -0.67 | 8.92 | 9.12 | 19.16 |
| | 6.25 | 12/15/05 | 6/15/96 | 95.92 | 96.37 | 0.47 | 8.45 | 8.26 | -18.93 |
| | 6.25 | 1/15/09 | 7/15/94 | 93.26 | 92.77 | -0.53 | 8.72 | 8.87 | 15.10 |
| | 6.7 | 11/1/07 | 5/1/98 | 97.21 | 98.09 | 0.91 | 7.71 | 7.44 | -26.88 |
| | 6.875 | 1/15/07 | 7/15/97 | 98.09 | 98.32 | 0.23 | 8.28 | 8.20 | -8.45 |
| | 7.25 | 1/15/02 | 7/15/92 | 101.14 | 102.13 | 0.98 | 8.66 | 7.19 | -146.40 |
| | 7.625 | 9/15/04 | 3/15/95 | 102.67 | 101.85 | -0.80 | 7.11 | 7.62 | 50.76 |
| Wachovia | 5.625 | 12/15/08 | 6/15/99 | 90.11 | 90.52 | 0.46 | 8.77 | 8.64 | -12.78 |
| | 6.15 | 3/15/09 | 9/15/99 | 93.52 | 92.41 | -1.18 | 8.94 | 9.30 | 35.50 |
| | 6.25 | 8/4/08 | 2/4/99 | 94.42 | 94.30 | -0.13 | 8.26 | 8.30 | 3.87 |
| | 6.375 | 4/15/03 | 10/15/93 | 98.98 | 101.33 | 2.38 | 8.05 | 6.57 | -147.85 |
| | 6.375 | 2/1/09 | 8/1/94 | 95.06 | 94.84 | -0.23 | 8.42 | 8.49 | 6.67 |
| Bank One | 6 | 2/17/09 | 8/17/99 | 91.38 | 90.17 | -1.32 | 9.24 | 9.63 | 38.86 |
| | 6.125 | 2/15/06 | 8/15/96 | 94.44 | 94.33 | -0.12 | 9.57 | 9.63 | 5.26 |
| | 6.375 | 1/30/09 | 7/30/94 | 93.73 | 93.10 | -0.68 | 8.82 | 9.01 | 19.59 |
| | 7.25 | 8/1/02 | 2/1/93 | 101.59 | 101.58 | -0.01 | 6.27 | 6.27 | 0.69 |
| | 7.6 | 5/1/07 | 11/1/97 | 101.8 | 103.78 | 1.94 | 7.68 | 7.07 | -61.19 |
| | 7.625 | 1/15/03 | 7/15/93 | 101.88 | 103.01 | 1.11 | 7.91 | 6.91 | -99.19 |
| | 8.74 | 9/15/03 | 3/15/92 | 105.82 | 105.61 | -0.20 | 8.80 | 8.98 | 17.56 |
| Bankers Trust | 6.7 | 10/1/07 | 4/1/98 | 95.88 | 96.42 | 0.56 | 7.93 | 7.76 | -16.36 |
| | 7.125 | 7/31/02 | 1/31/93 | 101.1 | 100.38 | -0.71 | 6.67 | 7.52 | 84.35 |
| | 7.125 | 3/15/06 | 9/15/96 | 98.56 | 97.03 | -1.55 | 9.24 | 9.96 | 71.68 |
| | 7.5 | 1/15/02 | 7/15/92 | 101.52 | 101.90 | 0.37 | 8.45 | 7.89 | -56.03 |
| | 8.125 | 4/1/02 | 10/1/95 | 103.04 | 105.74 | 2.62 | 4.02 | 1.85 | -216.85 |

* Table 2 shows the actual and fitted prices for all subordinated debt bond issues as on September 30 1999.

Table 3: Parameter Estimates for the Credit Spread Models for Each Bank

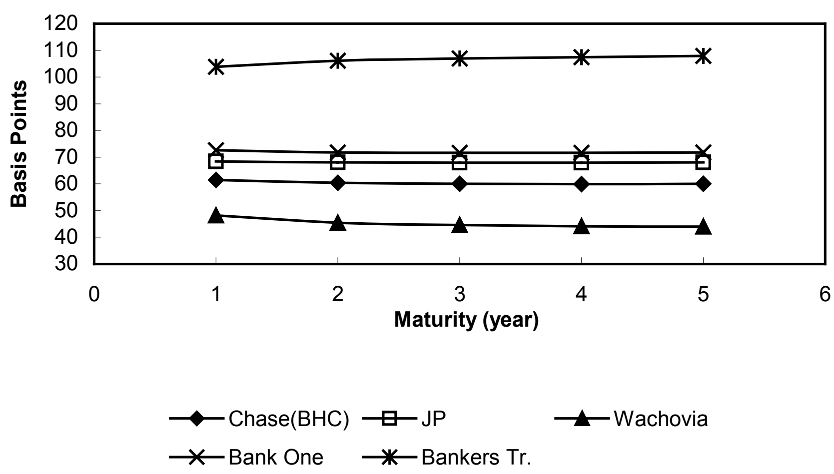
| | Chase(BHC) | JP | Wachovia | Bank One | BankersTr |
|---|---|---|---|---|---|
| $s_0$ | 0.00704 | 0.007054 | 0.007156 | 0.007497 | 0.00745 |
| $\alpha_0$ | 0.001917 | 0.000486 | 0.000722 | 0.001088 | 0.006468 |
| $\alpha_1$ | 0.289229 | 0.251739 | 0.313746 | 0.536976 | 0.113232 |
| $\alpha_2$ | 0.001221 | 0.004444 | 0.002075 | 0.001884 | 0.002842 |
| $\alpha_3$ | 0.885446 | 0.031403 | 0.153819 | 0.348979 | 0.137815 |
| $\rho_{r,s}$ | -0.781406 | -0.044367 | -0.212562 | -0.442567 | -0.1913 |
| $SSE$ | 0.001454 | 0.000343 | 0.000733 | 0.000727 | 0.001022 |

**Figure 1: Volatilities of Forward Rate**



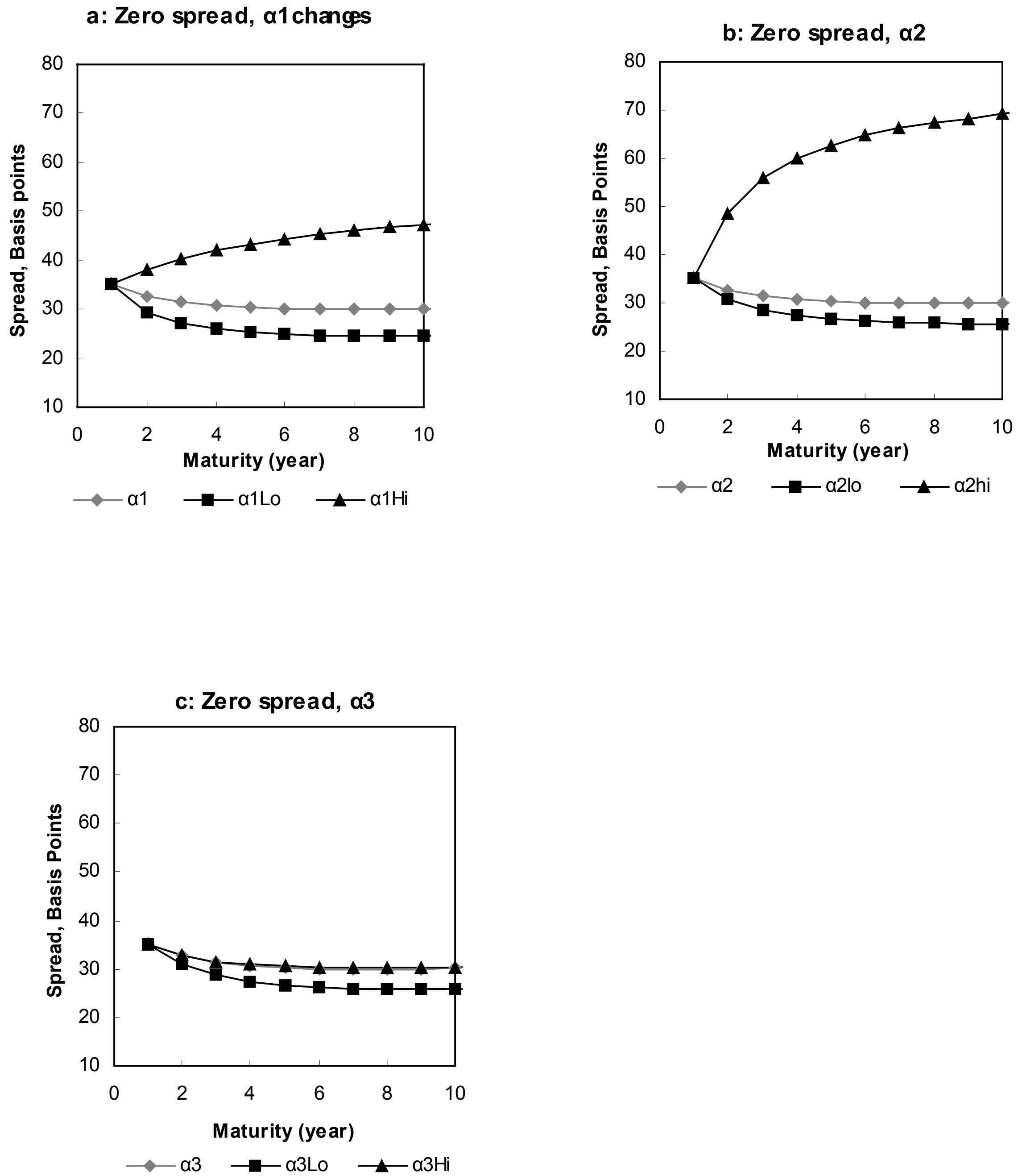

The volatility structure of forward rates was estimated using data on the LIBOR term structure and concurrent swaption prices as discussed in the text. The estimated parameters are $\kappa_1$=0.044978; $\kappa_2$=3.407608; $b_1$=0.00014383; $b_2$=-0.012441; $c_2$=0.018797.

**Figure 2: Zero Credit Spreads**

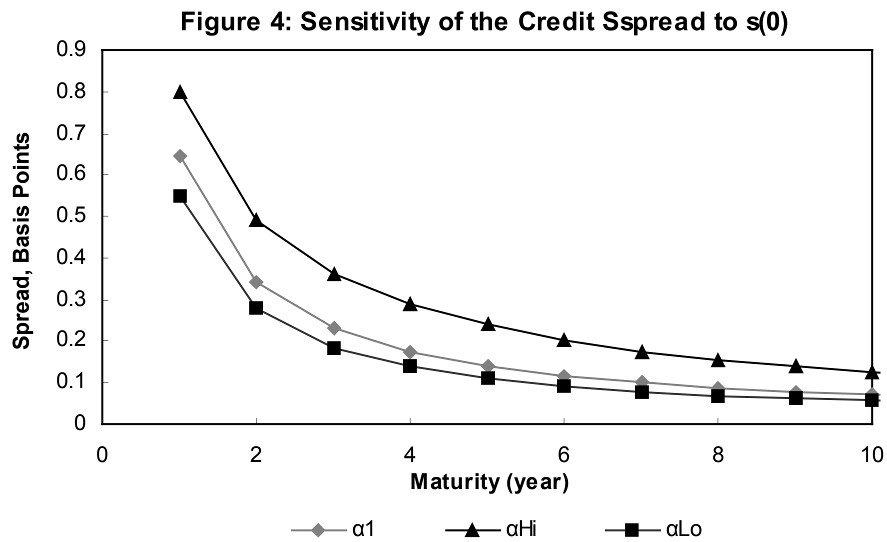



The credit spreads for the five banks at maturities from 1 to 5 years are based on the model described in the text and the parameters in table 3.

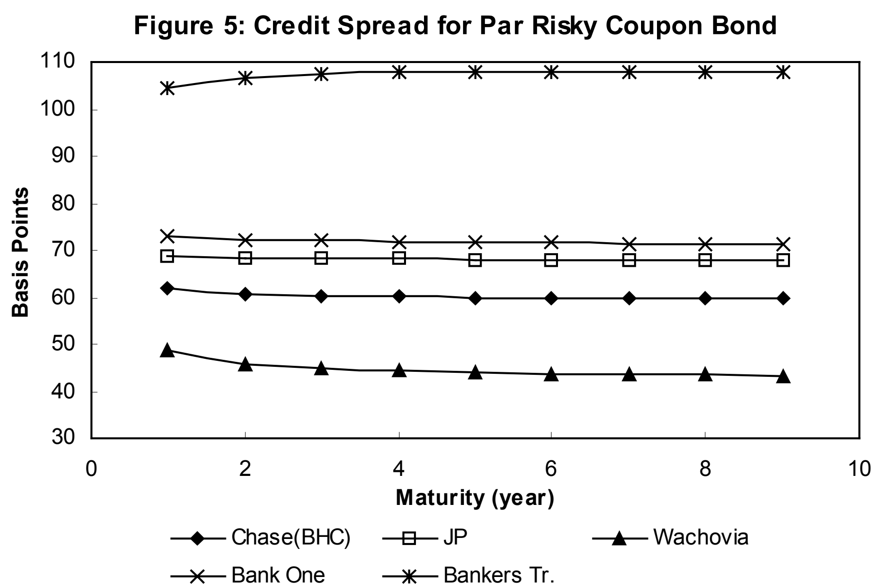
**Figure 3. Changes in Zero Credit Spreads as Parameters Vary**




Panels a, b, and c show the term structure of credit spreads for different values of the parameters. $\alpha1=0.289220$, $\alpha1Lo=0.1$, $\alpha1Hi=0.6$. $\alpha2=0.001221$, $\alpha2Lo=0.0008$, $\alpha2Hi=0.005$, $\alpha3=0.885446$, $\alpha3Lo=0.003$, $\alpha3Hi=0.9$. The base values are the parameters for Chase Manhattan.

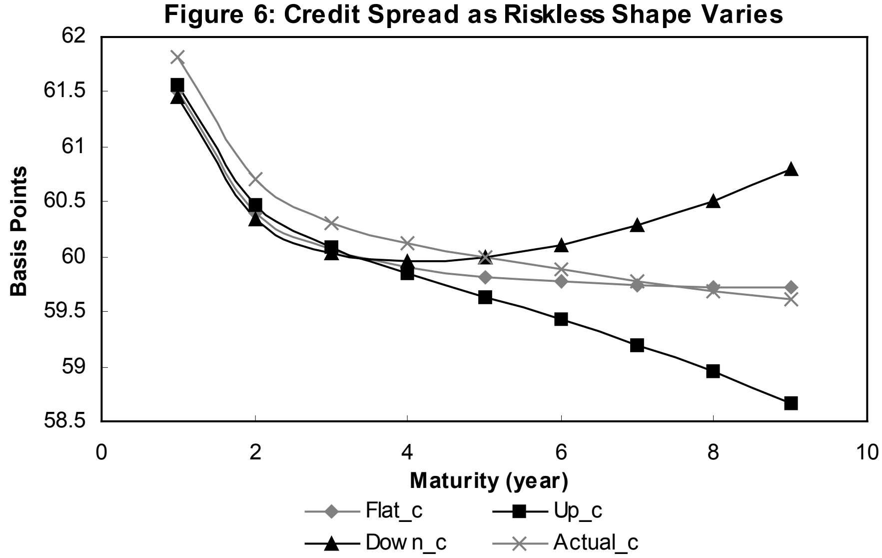



S(0) change in the instantaneous spread. The change in credit spread given a unit change in s(0), for maturities ranging from 6 months to 10 years. The same values are used for α1 as in figure 3.



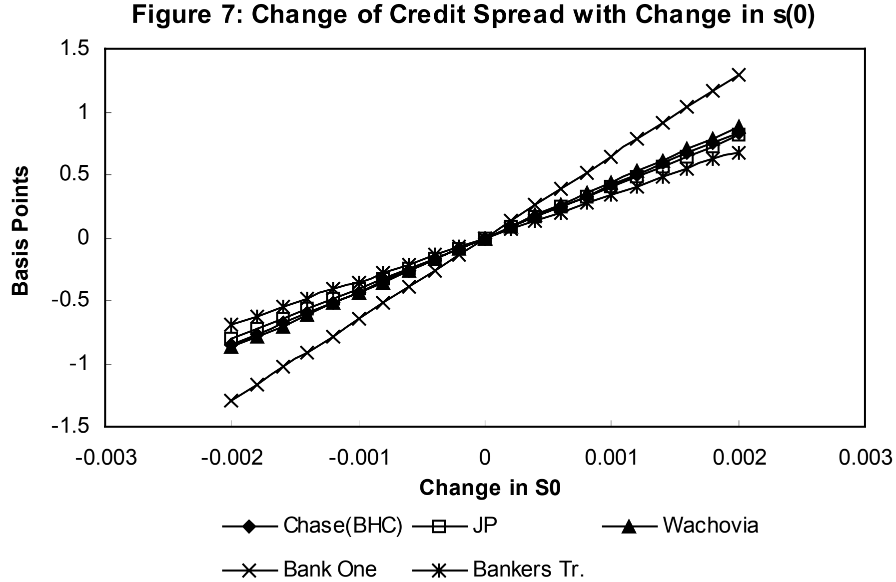

The credit spread for coupon bonds at par of maturities from 1 to 9 years, based on the parameters in table 3.

**Figure 6: Credit Spread as Riskless Shape Varies**

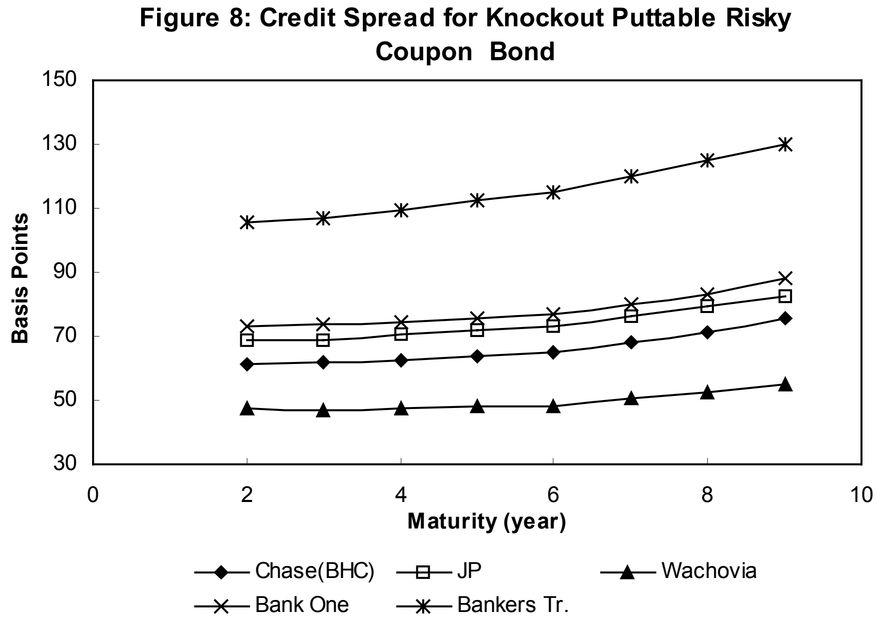



The credit spread is plotted against maturity for the Chase Manhattan parameters of table 3, varying the shape of the underlying risk-free term structure. Actual denotes the actual term structure for September 30, 1999. Flat denotes a constant 5% term structure. The downward sloping yield curve is given by $(5 - 0.4k)\%$ for maturity $k$, and the upward sloping by $(5 + 0.4k)\%$.

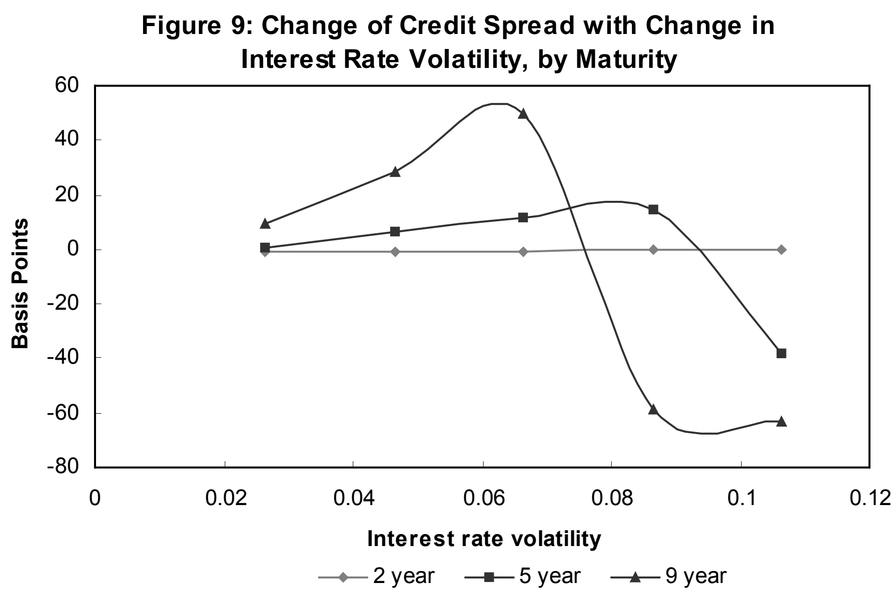
**Figure 7: Change of Credit Spread with Change in s(0)**




The change in the credit spread for a coupon bond at par, with a maturity of 5 years, calculated for different banks using the parameters given in table 3.

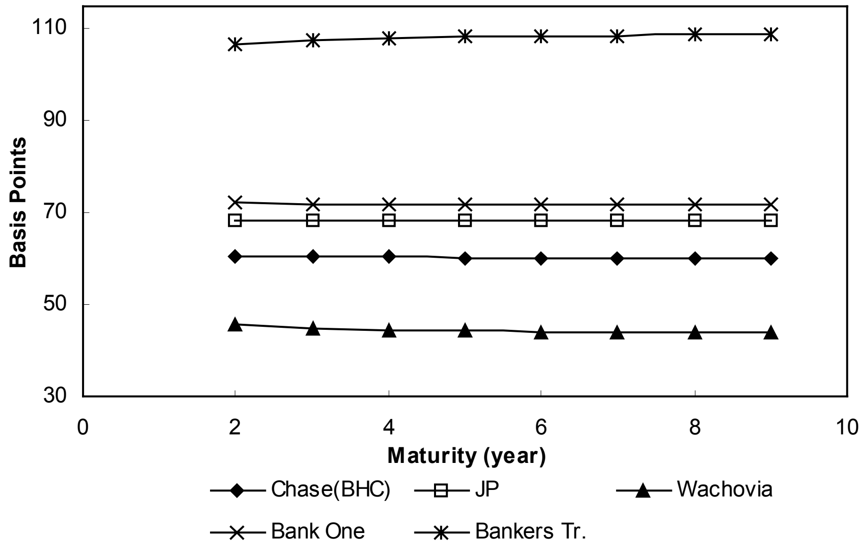



The coupon is set so that the bond is initially priced at par, given the American put, which is knocked out if the bank defaults before the bond matures. The bond is puttable at par. The option is priced using the Longstaff/Schwartz simulation method described in the text.



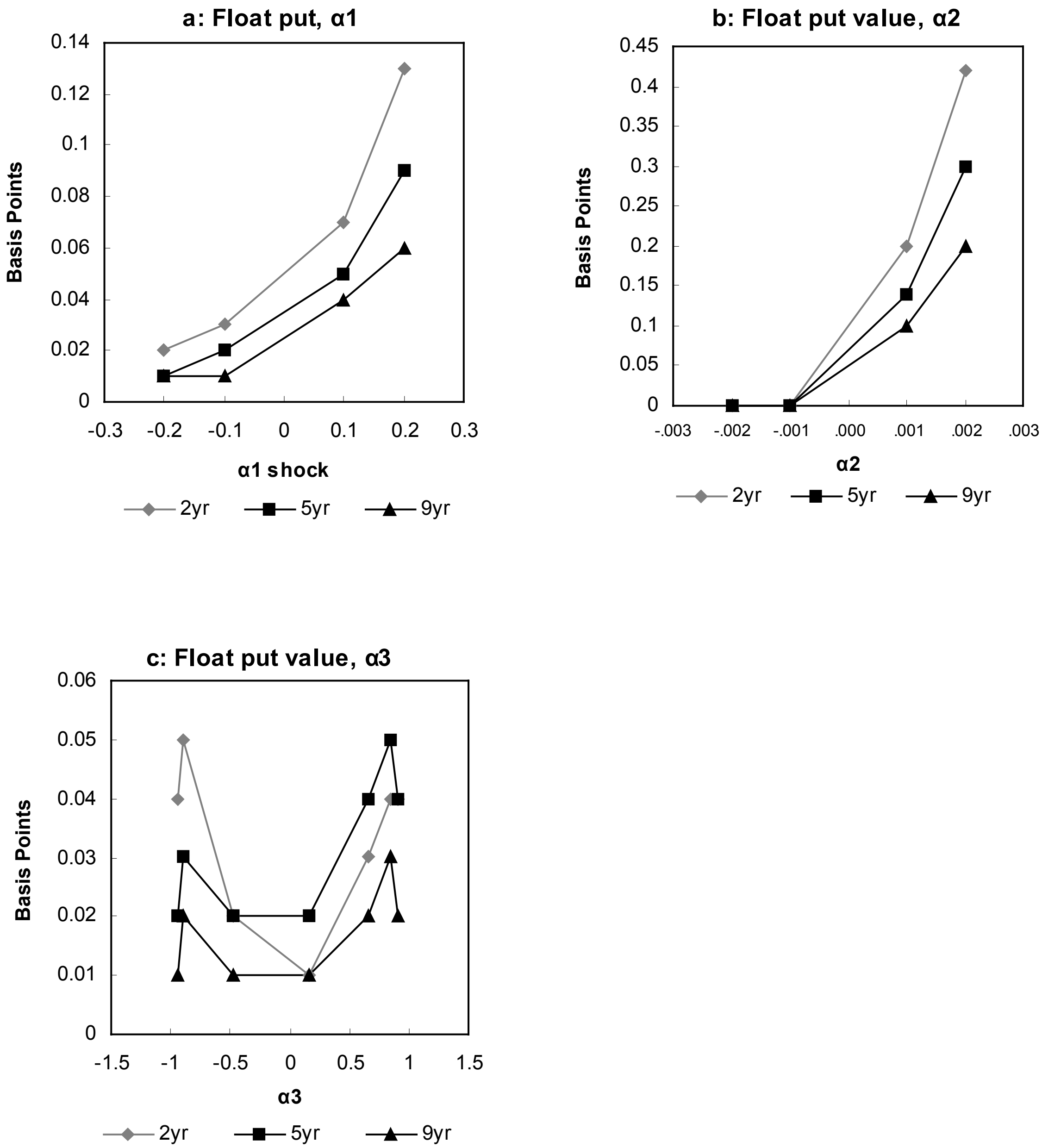

Using Chase Manhattan parameters from table 3, the volatility of the risk-free interest rate varies from 0.026358 to 0.106358.

**Figure 10: Credit Spread for Par Floating Rate Risky Bond with Knockout Put Option**



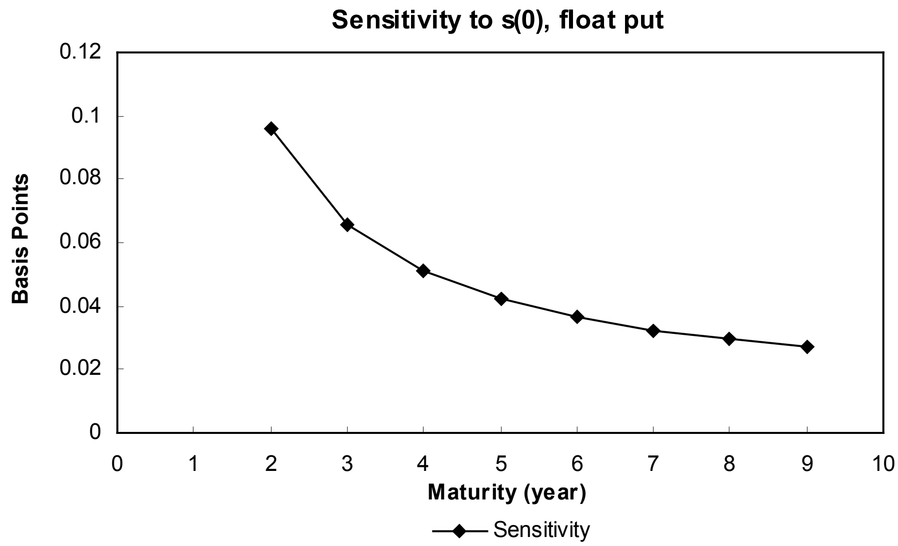

The initial coupon is set so that the bond is initially priced at par, given the American put, which is knocked out if the bank defaults before the bond matures. The bond is puttable at par. The option is priced using the Longstaff/Schwartz simulation method described in the text.

**Figure 11. Changes in Credit Spread for Puttable Floating Rate Bond with Changes in Credit Process Parameters**




Panels a, b, and c show the change in the credit spread for floating rate bonds with a knockout put for maturities 2, 5, and 9 years for various values of the parameters α1, α2, and α3 of the credit spread process. The other parameters as for Chase Manhattan in table 3.

**Figure 12. Sensitivity of Credit Spread for Puttable Floating Rate Bonds to Changes in the Instantaneous Credit Spread s(0)**




The change in credit spread given a unit change in s(0), for maturities ranging from 2 years to 9 years. The same values are used for $\alpha 1$ as in figure 3.

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