Diagnosing Labor Market Search Models: A Multiple-Shock Approach[*]

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Abstract

We construct a multiple-shock version of the Mortensen-Pissarides labor market search model to investigate the basic model's well-known tendency to under predict the volatility of key labor market variables. Data on U.S. job finding and job separation probabilities are used to help estimate the parameters of a three-dimensional shock process comprising labor productivity, job separation, and matching or 'allocative' efficiency. We show that the Mortensen-Pissarides labor market search model requires significantly procyclical and volatile job separations to simultaneously account for high procyclical variations in job finding probabilities as well as relatively small net employment changes. Hence, the model is more fundamentally flawed than its inability to amplify shocks would suggest. This leads us to conclude that the model lacks mechanisms to generate procyclical matching efficiency and labor force reallocation. As for the latter, we conjecture that nontrivial labor force participation and job-to-job transitions are promising avenues of research.

Key words: Labor Market Search; Mismatch; Business Cycles; Unemployment; Job Vacancies.

JEL classification: E24; E32; J40

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1 Introduction

There is now a fairly rich literature using the Mortensen-Pissarides labor market search model to understand business cycle movements in the labor markets[1]. Shimer (2005a) has recently criticized this model, arguing that it requires implausibly large shocks to labor productivity to generate substantial variation in key variables: unemployment, vacancies and the vacancy to unemployment ratio[2]. We explore whether other reasonable sources of exogenous variation, including job separation and job-matching shocks, can satisfactorily resolve this puzzle. In particular, we identify the realizations of a multiple-shock process required for the model to fit the data perfectly. Our results are striking. The perfect-fit experiment strongly indicates that the standard labor market search model is more fundamentally flawed than its inability to amplify shocks would suggest.

Our multiple shock approach allows for exogenous shocks to job separation, labor productivity and matching efficiency that are mutually correlated over the business cycle. In keeping with the most basic Mortensen-Pissarides model, the rate of job separation is exogenous and simply gives the fraction of employed persons that will separate from their jobs, for whatever reason, during a particular period. The shock to the matching function captures the efficiency with which existing labor market institutions pair searching workers with available jobs. We call this the 'allocative efficiency' shock as in Andolfatto (1996). We apply data on monthly separation and job finding probabilities as well as unemployment and job vacancies to estimate the process that governs these shocks. The estimation strategy provides us with empirically plausible variations in labor market transition probabilities of the average U.S. worker.

Although realistic variation in the transition probabilities substantially increase the volatil-

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[1] For a textbook treatment of this class of models see Pissarides (2000). Broadly speaking, we can identify two separate but closely related strands in this literature. The first group, including works by Andolfatto (1996), den Haan, Ramey and Watson (2000) and Merz (1995, 1999), incorporates labor market search into otherwise standard real business cycle environments to improve upon their cyclical implications for labor market variables such as employment. A second group of papers, such as Cole and Rogerson (1999) and Mortensen and Pissarides (1994), focuses on the implications of the standard labor market search model in relation to the empirical evidence on job creation and job destruction provided by Davis and Haltiwanger (1992) and Davis, Haltiwanger and Schuh (1996).

[2] Earlier studies either failed to address the magnitude of the exogenous forcing process (Mortensen and Pissarides (1994), Cole and Rogerson (1999)) or implied counterfactually positive relationship between unemployment and vacancies (Andolfatto (1996), Merz (1995)). Merz (1995) provides two versions of the model, one with constant and one with variable search effort. To be precise, her version with variable search effort gives this counterfactual finding.

ity of key variables in the model, it does so at an enormous descriptive cost. The simulated cyclical behavior of unemployment and vacancies is entirely counterfactual, displaying procyclical unemployment and countercyclical job vacancies. We conduct the 'perfect-fit' experiment to better understand the counterfactual finding. That is, we posit the model as the actual data generating process and subsequently infer the shocks required to match the data perfectly. Our findings are startling. To be consistent with the observed fluctuations in unemployment and job vacancies, the multiple-shock model requires volatile and procyclical job separations to simultaneously account for the pronounced procyclical variation in job finding and the relatively small net employment changes.

These counterfactual findings are due to two reasons. First, the substantial fluctuations observed in market tightness require significantly procyclical and volatile allocative efficiency shocks. Given this, however, observed net employment changes could only be reconciled with significantly procyclical and substantially volatile job separations. This is due to the fact that all of the worker reallocation required to accommodate cyclical variations in employment must involve an unemployment spell in the standard environment. However, it is impossible to reconcile procyclical job separation/destruction with the existing empirical evidence (Blanchard and Diamond (1990), Davis and Haltiwanger (1992), Davis, Haltiwanger and Schuh (1996) and Shimer (2005a)). Therefore, the defects in the labor market search model are even more fundamental than Shimer (2005a) argued. We show that these findings are robust to different calibrations.

We conclude that the basic model lacks sufficiently strong mechanisms to reallocate workers over the course of the business cycle. Our results point to potentially productive extensions of the basic model. If not acceptable, *a priori*, one could search for mechanisms that underly procyclical allocational efficiency and modify the model accordingly. The results also indicate that any such modification be accompanied by a theoretical expansion of the searching-workers pool or the allowance of job-to-job transitions.

This paper is related to various other studies in the literature. Our investigation into the mechanics of the standard labor market search model echoes Shimer's (2005a) diagnostic exploration of Mortensen-Pissarides framework. We argue that the model, even when it has substantial degrees of freedom with multiple shocks and empirically plausible transition probabilities, has counterfactual implications. We emphasize this point further with an experiment that requires our model to be the data generating process and gives us the unique realization of the shocks for perfect fit. Although our objective for this experiment is diagnosis rather than measurement, it is similar to accounting exercises employed in Chari, Keheo and McGrattan (Forthcoming) and Ingram, Kocherlakota and Savin (1994). Our model is identical to Merz (1995) except that we abstract from capital stock. Finally, we discuss our findings and several avenues for future research in conjunction with the literature that tries to resolve the puzzle presented in Shimer (2005a).

The remainder of the paper is organized as follows. Section 2 outlines our version of the Mortensen-Pissarides model. In Section 3, we briefly describe the data and its basic statistical properties. Section 4 discusses our calibration and presents the simulation results. Section 5 analyzes the simulation results and presents our diagnostic procedure. We also interpret our findings in the context of recent literature. Section 6 discusses some robustness issues. We briefly outline our conclusions and set a direction for future research in Section 7.

2 The Model

The economy is inhabited by a continuum of infinitely-lived worker/households distributed uniformly along the unit interval; there is also a continuum of firms. At the beginning of each period, a worker is considered either employed or unemployed. The measure of employed workers is denoted N_t; the measure of unemployed workers is the complement $U_t \equiv 1 - N_t$. The representative household has preferences over state-contingent consumption and employment given by

$$\mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t U\left(C_t, N_t\right), \qquad 0 < \beta < 1, \tag{1}$$

where β is the subjective discount factor. Following Merz (1995), the period utility function is separable in consumption and employment, with

$$U\left(C_t, N_t\right) = \log C_t - \frac{N_t^{1+\frac{1}{\gamma}}}{1+\frac{1}{\gamma}}, \qquad \gamma > 0,$$

where γ defines the wage elasticity of labor supply at a constant marginal utility of wealth (the "Frisch elasticity" of labor supply).

Both workers and firms must undergo a costly search process before jobs are created and output is produced. At the beginning of each period, each unemployed worker searches for a job expending ϕ consumption units in the process. Aggregate period t search costs incurred therefore equal $\phi\left(1-N_t\right)$ consumption units. Firms create job vacancies, but only by expending κ units of output per vacancy per period, generating aggregate "recruiting" costs equal to κV_t. Here, as in the traditional Mortensen-Pissarides framework, all jobs must be posted as vacancies before they can be filled. Once a job is filled, it produces output equal to Z_t generating aggregate output

$$Y_t = Z_t N_t \tag{2}$$

where $Z_t > 0$ is the exogenously determined productivity of labor.

The matching function captures the labor market search frictions. The typical formulation determines the number of job matches formed in a given period, $M\left(V_t, U_t\right)$, as an increasing function M of job vacancies, V_t, and the number of job seekers, U_t, where M exhibits constant returns to scale. With search costs ultimately arising from heterogeneity-induced information problems, we interpret the matching function as a mapping from not only from the quantities of vacancies and searching workers, but also from the degree of mismatch between those two populations. To allow for fluctuations in mismatch, we generalize the matching function to include a multiplicative shock term, χ_t. Hence, the number of matches formed in period t is given by

$$M_t = \chi_t M\left(V_t, U_t\right) = \chi_t V_t^{\alpha}\left(1-N_t\right)^{1-\alpha} \tag{3}$$

where $0 < \alpha < 1$ and χ_t is the period t realization of an unobserved shock process. Increases in χ_t raise the number matches formed given the numbers of searching workers and available positions. From a searching worker's perspective, an increase in χ_t raises the probability of being matched with a vacant position; from the perspective of a single firm, it improves its chances of filling a vacancy. Consequently, fluctuations in χ_t signify improvements or deteriorations in the allocative efficiency of the labor market.

While job matches are being formed, others are dissolved. We assume that the fraction of existing matches dissolved during period t, σ_t, is also determined as the realization of an exogenous stochastic process. The period t change in aggregate employment, i.e. the net employment flow, is defined as the difference between the period gross employment inflow and gross employment outflow:

$$N_{t+1} - N_t = M_t - \sigma_t N_t. \tag{4}$$

Note that each flow is directly impacted by unobserved shocks: the flow into employment by the allocative efficiency term, χ_t, and the outflow by the rate at which workers separate from jobs, σ_t.

The state of the economy in a given period, or (N_t, e_t), consists of the beginning-of-period employment level N_t, and values of the unobserved and exogenous state vector $e_t = (Z_t, \chi_t, \sigma_t)$. We make the standard Markovian assumption which allows agents to form expectations of future-period quantities using knowledge of the current state only. Given the current state, the socially efficient allocation of employment, vacancies, and consumption, $\{N_{t+1}, V_t, C_t\}$, solves the following recursively-defined social planner's problem:

$$\upsilon\left(N_t, e_t\right) = \max_{N_{t+1}, V_t, C_t} \left\{U\left(C_t, N_t\right) + \beta \mathrm{E}_t \upsilon\left(N_{t+1}, e_{t+1}\right)\right\} \tag{5}$$

subject to

$$C_t + \phi\left(1 - N_t\right) + \kappa V_t \leq Z_t N_t. \tag{6}$$

$$N_{t+1} = \left(1 - \sigma_t\right) N_t + \chi_t M\left(V_t, 1 - N_t\right). \tag{7}$$

where $\upsilon\left(N_t, e_t\right)$ is the future discounted social value of employment level N_t and the exogenous state e_t. Equation (6) represents the period t resource constraint prohibiting the sum of current expenditures on consumption, job search, and vacancy creation to exceed current output, and equation (7) describes the trajectory of employment (4) with the matching function (3) determining the current-period flow into employment. Finally, we assume that a VAR(1) process governs the exogenous state e_t:

$$e_{t+1} = A e_t + \varepsilon_{t+1},\ \mathrm{E}(\varepsilon \varepsilon') = \Omega. \tag{8}$$

The autoregressive process for exogenous shocks plays a key role in this paper. We will devise a way to estimate this joint process and try to understand its implications for business cycles in this model.

The corresponding first-order and envelope conditions imply an Euler equation describing an intertemporally efficient vacancy-posting scheme for the economy. Suppressing arguments and letting primes denote one-period-ahead quantities, we write

$$U_C \frac{\kappa}{\chi M_V} = \beta \mathrm{E}_t U_C' \left\{ Z' + \phi + \frac{U_N'}{U_C'} + \frac{\kappa}{\chi' M_V'} \left[(1 - \sigma') - \chi' M_U' \right] \right\} \tag{9}$$

equating the loss in welfare due to vacancy creation with its expected future social benefit. In this expression,

$$\frac{1}{\chi M_V} = \alpha^{-1} \frac{V}{\chi M}$$

gives the average duration of vacancies multiplied by the elasticity of vacancies in matching, $\alpha = \frac{VM_V.}{M}$. The left-hand side of (9), therefore, represents the utility loss associated with a marginal increase in vacancies. The expected gain of the marginal vacancy, given by the right-hand side of (9), derives from many sources. The expression $Z' + \phi + \frac{U_N'}{U_C'}$ gives the one-period-ahead net social benefit of an additional match formed in the current period. The term Z' equals the output flowing from the match; ϕ represents the (constant) search costs foregone by the worker in the match. The final term in the sum, $\frac{U_N'}{U_C'}$, represents the consumption value of the leisure foregone by the newly matched worker. In the basic Mortensen-Pissarides setup this quantity is a constant, whereas we allow it to vary over the business cycle.

The final term in braces represents the net future social benefit arising from the expected persistence of a job match. Given that any single current-period match survives with probability $1 - \sigma'$, future social welfare will increase simply by reducing expected future recruiting costs by the quantity $\frac{\kappa(1-\sigma')}{\chi' M_V'}$. The second term in this sum, $-\chi' M_U'$, represents the future reduction in the future job-finding rate $\frac{\chi M}{U}$ due to the current depletion of the unemployment stock; the expected recruiting cost in future consumption units equals $\frac{\kappa M_U'}{M_V'}$.

As a system, equations (6)–(9) characterize the socially-optimal allocation of employment, vacancies, and consumption given a joint distribution for the exogenous forcing variables or shocks: Z_t, χ_t and σ_t. The traditional Mortensen-Pissarides approach determines these quantities in a market equilibrium with a real wage emerging as the outcome of Nash bargaining between firms and households. The socially optimal allocation characterized above is supported by a similar market allocation mechanism provided that: 1) asset markets are rich enough for households to diversify away employment risk, and 2) the relative bargaining power between households and firms is such that the positive and negative search externalities net out to zero.[3] Although we do not take a position on the precise nature of the allocation mechanism, we maintain that existing market and institutional arrangements direct the realized allocation sufficiently close to the social optimum to establish equations (6)–(9) as a useful instrument of measure.

3 The Data

Before proceeding to shock measurement, we briefly review the salient facts regarding the observed aggregate U.S. labor market measures that bear on our analysis. Given that the model presented in the previous section does not require a labor market participation decision for worker/households, we must choose whether to express our employment and unemployment variables, N_t and $U_t \equiv 1 - N_t$, relative to the labor force or the age 16-and-over population. Although there are valid arguments in favor of both normalizations, we find that the choice little affects our results, and choose the labor force (employment plus unemployment) as our reference population[4]

In the absence of a long time series on actual job vacancies, we follow standard practice and construct vacancies from the Conference Board's help-wanted advertising index. The resulting vacancy series, V_t, is also expressed per member of the labor force[5]. Also, since our model

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[3] Hosios (1990) determines the conditions under which the Pareto-optimum is supported as a decentralized market equilibrium in a static environment; Merz (1995) and Andalfatto (1996) do the same in dynamic general equilibrium settings. The market equilibrium in the current work closely follows those of Merz and Andalfatto.

[4] Specifically we use the unemployment rate (unemployed persons per member of the labor force) constructed as a quarterly average of the seasonally adjusted monthly series from the Current Population Survey (CPS) of the Bureau of Labor Statistics (BLS). The civilian labor force measure is also provided by BLS as part of the CPS. Both series can be downloaded from the CPS home page *http://www.bls.gov/cps.*

[5] We construct a vacancy series by multiplying two seasonally adjusted monthly series – the ratio of help-wanted advertising to unemployed compiled by the Conference Board (downloaded as variable LHELX from the DRI Basic database), and the unemployment rate U (defined above) – and averaging the monthly values to obtain the quarterly series. The commonly reported help-wanted advertising index is a scalar transformation of this series.

abstracts from the capital accumulation decision, we must choose between aggregate output and aggregate consumption – a choice that reflects our desire to preserve a consistent and well-understood labor productivity measure and one that can be more readily compared to those in other studies. Since the aggregate labor input N_t produces all goods and services, including private investment goods and those purchased by government, real GDP provides the appropriate output measure. Therefore, consumption, C, is proxied by real GDP per member of the labor force[6]. We divide this series by the seasonally-adjusted civilian labor force (averaged from monthly to quarterly), appropriately scaled, to express the variable in year 2000 chained dollars per person. Time series data on U, V and C are constructed at the quarterly frequency and run from 1951:1 to 2003:4.

We use real output per person in the non-farm business sector as our productivity measure. This particular series is chosen to ensure comparability with the recent body of literature. It is also a natural way to think about productivity in the standard labor market search model. This series is part of BLS's Major Sector Productivity and Costs program. It is normalized to 100 for 1992.

We also use U.S. labor market transition probabilities for our shock measurement process. These probabilities were constructed by Shimer (2005a), but our discussion follows that of Shimer (2005b). In accord with Shimer (2005b), the relevant labor market states are unemployment (u) and employment (e) with the job finding probability governing the rate at which a worker switches between unemployment and employment and the separation probability determining the rate at which a worker switches between employment and unemployment.

Shimer's (2005b) definitions of job finding and separation probabilities are as follows:

$$f_t = 1 - \frac{U_{t+1} - U^s_{t+1}}{U_t} \tag{10}$$

$$s_t = \frac{U^s_{t+1}}{E_t(1 - f_t/2)} \tag{11}$$

where in a given month t, U_t is the number of unemployed, U^s_t is the number of workers

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[6] Real GDP (billions of chained 2000 dollars, seasonally adjusted annual rate) was downloaded from the Federal Reserve Bank of St. Louis FRED II database at *http://research.stlouisfed.org/fred2/series/GDPC1*

unemployed less than one month, and E_t is the number of workers employed. These definitions correct for time aggregation bias in the job separation probability by allowing for the possibility of short unemployment spells within a given month.

To aggregate monthly transition probabilities, we account for all possible histories of employment states within a quarter. Temporal aggregation of labor market transition probabilities might imply different cyclical features at various frequencies. The idea behind this argument is simple. An unemployed worker at the beginning of a quarter potentially switches between employment and unemployment before being counted as employed at the end of the quarter. Therefore, the quarterly job finding probability of an average worker will not only reflect the cyclical features of monthly job finding probabilities, but also of monthly separation probabilities. Since we are interested in the cyclical properties of labor market variables, it is vital for us to be precise in aggregating Shimer's monthly transition probabilities. We use the following aggregation:

$$\begin{aligned} F_t \; = \; & (1 - f_{(3*(t-1)+1)}) * (1 - f_{(3*(t-1)+2)}) * f_{(3*(t-1)+3)} + \\ & (1 - f_{(3*(t-1)+1)}) * f_{(3*(t-1)+2)} * \left(1 - s_{(3*(t-1)+3)}\right) + \\ & f_{(3*(t-1)+1)} * s_{(3*(t-1)+2)} * f_{(3*(t-1)+3)} + \\ & f_{(3*(t-1)+1)} * (1 - s_{(3*(t-1)+2)}) * \left(1 - s_{(3*(t-1)+3)}\right). \quad \forall t \end{aligned} \tag{12}$$

$$\begin{aligned} S_t \; = \; & (1 - s_{(3*(t-1)+1)}) * (1 - s_{(3*(t-1)+2)}) * s_{(3*(t-1)+3)} + \\ & (1 - s_{(3*(t-1)+1)}) * s_{(3*(t-1)+2)} * \left(1 - f_{(3*(t-1)+3)}\right) + \\ & s_{(3*(t-1)+1)} * f_{(3*(t-1)+2)} * s_{(3*(t-1)+3)} + \\ & s_{(3*(t-1)+1)} * (1 - f_{(3*(t-1)+2)}) * \left(1 - f_{(3*(t-1)+3)}\right). \quad \forall t \end{aligned} \tag{13}$$

Note that in this aggregation four possible histories arise. A simple averaging of monthly probabilities ignores these different experiences.

We summarize the key business cycle features of the data in Table 1. In addition to the labor market variables, we include the properties of the official BLS labor productivity measure for the non-farm business sector[7]. With respect to transition probabilities, we report both averages of f's and s's and F's and S's to facilitate comparison with Shimer (2005a). To describe the business-cycle variation in these quantities, we follow Shimer (2005a) and remove the low-frequency trend in all variables implied by the Hodrick-Prescott filter under a smoothing parameter of 10^5. We apply this procedure to remove movements in the aggregates induced by institutional and technological changes associated with job-matching, so that they are not spuriously assigned to matching function instability arising from cyclical movements in labor market mismatch. Key business cycle features of the U.S. data is summarized in Table 1.

Table 1: U.S. DATA (Quarterly, 1951Q1-2003Q4)

	u	v	v/u	$u \to e$	$u \to e^*$	$e \to u$	$e \to u^*$	z^{**}
Standard Dev.	0.190	0.202	0.381	0.117	0.059	0.075	0.118	0.020
Autocorrelation	0.938	0.947	0.946	0.910	0.916	0.731	0.870	0.889
Cross Correlations								
u		−0.894	−0.971	−0.949	−0.938	0.712	0.889	−0.417
v			0.974	0.898	0.908	−0.689	−0.852	0.369
v/u				0.948	0.948	−0.718	−0.893	0.402
$u \to e$					0.990	−0.578	−0.841	0.406
$u \to e^*$						−0.590	−0.840	0.414
$e \to u$							0.910	−0.518
$e \to u^*$								−0.546

* Quarterly transition probabilities aggregated as in (12) and (13).

** Official BLS labor productivity measure for the non-farm business sector.

From Table 1, we observe that employment, vacancies, and the vacancies-unemployment ratio are all strongly procyclical and persistent; unemployment is strongly countercyclical and

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[7] This series is part of BLS's Major Sector Productivity and Costs program and is downloaded from the Federal Reserve Bank of St. Louis FRED II database at *http://research.stlouisfed.org/fred2/series/GDPC* It is normalized to 100 for 1992.

persistent. These data also affirm the Beveridge curve with a strong contemporaneous correlation between vacancies and unemployment of -0.894. Note that these cyclical properties are all in accord with the qualitative predictions of the standard labor market search model. However, Table 1 also show unemployment and vacancies to be almost 10 times more volatile than labor productivity. Note as well, the extreme volatility of the vacancy-unemployment ratio (market tightness) with a standard deviation of 38 percent around its trend. The extreme volatility observed in unemployment, vacancy and market tightness provides the main motivation of Shimer (2005a) and the subsequent literature on the business cycle implications of standard labor market search models. We will contrast these findings with the implications of a search model in the following section.

Although different ways of aggregation for transition probabilities give strongly correlated series (note that the correlation between u→e and u→e* is 0.99 and $e \rightarrow u \ and \ e \rightarrow u^*$ is 0.91), the relative variation changes significantly. With aggregations according to (12) and(13), quarterly separations become relatively more volatile than quarterly job findings. It is crucial, however, to keep in mind that this reversal of relative variation is a by-product of the aggregation. It does not imply that the fluctuations in unemployment at a higher frequency is dominated by separations.

4 Results

In this section, we explore the cyclical properties of the search model presented in section 2 with two sets of simulation results. First, we subject the model only to a labor productivity shock holding allocational efficiency and job destruction constant. This experiment provides a direct comparison to the standard labor market search model as in Shimer (2005a). The second simulation incorporates the allocational efficiency and job destruction shocks with all three governed by the VAR(1) process. Before presenting our results, we briefly describe calibration of our model.

4.1 Calibration

With a large empirical literature to draw upon and stationary labor market variables at hand, we combine micro-evidence with long-run data averages to calibrate the steady state values of the exogenous shocks and the technology/preference parameters. We begin by setting the steady state values of the labor market variables, N_t, V_t, and U_t, equal to the corresponding data first moments: $N = 0.943$, $V = 0.048$, and $U = .057$. Given these values, we observe that the steady-state version of the equation-of-motion for employment (7),

$$\sigma N = \chi V^{\alpha} U^{1-\alpha}, \tag{14}$$

sharply restricts the steady state values of the shocks, χ_t and σ_t, and matching technology parameter, α. We set α equal to 0.28, which is the value used by Shimer (2005a)[8]. The steady state rate of job separation is chosen to be 6.9 percent of total employment per quarter, or $\sigma = 0.069$, which is the implied quarterly average of job separation probability discussed in the previous section. Under these settings, the steady state employment condition (14) subsequently pins down steady state allocative efficiency level: $\chi = 1.056$. These parameters imply an average vacancy duration, $(M/V)^{-1}$, of 0.85 quarters or about 76 days, which is a bit higher than the value reported by van Ours and Ridder (1992) using data from the Dutch economy. The implied unemployment duration is 0.98 quarters, or about 12.7 weeks, which is consistent with U.S.data.

Without loss of generality, we normalize the steady state of inferred aggregate output to equal one, $ZN = 1$, yielding steady-state labor productivity $Z = 1/N = 1.06$. Under this assumption, the steady state resource constraint becomes

$$C + \phi U + \kappa V = 1.$$

Note that in the absence of search and recruiting costs, i.e. $\phi = \kappa = 0$, labor productivity reduces to the traditional average product of labor definition. Steady state labor productivity equals C^{-1} in that case. (Recall that we must proxy consumption with aggregate output, or

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[8] This is at the low end of the estimates surveyed by Petrongolo and Pissarides (2001).

real GDP.) In the presence of search and recruiting costs, our imputed output measure deviates from measured real GDP somewhat, but we anticipate the magnitude of the difference to be small, with the settings of parameters ϕ and κ largely determining the gap. Unlike the model's other parameters, independent evidence regarding these two parameters is scarce. We follow Andalfatto (1996) in assuming steady state recruiting expenditures to be one percent of output, or $\kappa V = .01$, implying $\kappa = .206$. We assume that steady state search costs for workers are also one percent of aggregate output, $\phi U = .01$, yielding $\phi = .176$. The steady state value of consumption is therefore $C = .98$, or 98 percent of output.

Table 2: Calibrated Parameters

Parameter	Value	Source
β	0.99	4% interest
α	0.28	Shimer (2005a)
γ	1.25	Merz (1995)
ϕ	0.1762	1% of Output
κ	0.2056	1% of Output
χ^{ss}	1.0561	u^{ss} and v^{ss}
z^{ss}	1.0602	Avg. Output = 1
σ^{ss}	0.0609	Shimer (2005b)

Next, we consider the two preference parameters, β and γ, the subjective discount factor and the Frisch elasticity of the labor supply, respectively. We choose $\beta = .99$ to be consistent with a steady-state risk-free real interest rate of 4 percent. We follow Merz's (1995) interpretation of the empirical literature and choose $\gamma = 1.5$ for the Frisch elasticity. Table 2 summarizes our calibration. In section 6, we consider different values of γ and α.

To calibrate the shock process, we first define the data series that comprise the VAR(1) specification. Recall that the job separation probability series is taken from Shimer (2005b) corrected for time aggregation effects. Next, observe that resource constraint along with the data on V, U and C defines the period t productivity shock as follows:

$$Z_t = \frac{C_t + \phi\,(1 - N_t) + \kappa V_t.}{N_t} \tag{15}$$

Although we have reported cyclical properties of U.S. labor market data relative to the official BLS measure of non-farm business sector labor productivity, it is comforting to note that the correlation between this series and the inferred series is nearly perfect – 0.998, specifically.[9]

To measure the allocational efficiency shocks, we make use of job finding probabilities and the matching function. Recall that total flow into employment in a given period is dictated by the matching function, (3). We can rewrite this flow in the following way:

$$\chi_t V_t^{\alpha} (1 - N_t)^{1-\alpha} = \chi_t V_t^{\alpha} (1 - N_t)^{-\alpha} (1 - N_t) = \Pr(u \to e) * (1 - N_t) \tag{16}$$

Hence, total matches equals the job finding probability multiplied by the number of unemployed workers. This decomposition along with data on V, U and $\Pr(u \to e)$ identifies a time series for χ_t.

We depict our series on Z, χ, and σ in Figure 5. With the knowledge that productivity is strongly procyclical, Figure 5 shows allocative efficiency and job separation to be strongly countercyclical. The contemporaneous correlations with Z are -0.55 and -0.69 for χ and σ, respectively. As we have confirmed that the cyclical features of Z are nearly identical to labor productivity as measured by the BLS, we also want to see whether the inferred series of χ and σ imply reasonable fluctuations in unemployment when compared to the BLS measure. Figure 6 compares the actual unemployment rate with the unemployment rate implied by the shocks that are pictured in Figure 5 and inferred using the equation of motion (7). Figure 6 reveals that the shocks we use to estimate the VAR(1) lead to unemployment dynamics that are virtually identical to those implied by the BLS series. Since this paper focuses on fluctuations in unemployment rather than its level, the difference is inconsequential.

Finally, with these data series, we estimate the coefficients of A and Ω using the usual equation-by-equation OLS procedure. Estimates of A and Ω are as follows.

$$A = \begin{bmatrix} 0.9081 & 0.1212 & -0.0360 \\ -0.0214 & 0.3078 & 0.2915 \\ -1.0305 & 0.0438 & 0.7558 \end{bmatrix} \qquad \Omega = \begin{bmatrix} 0.00007 & -0.00003 & -0.00014 \\ -0.00003 & 0.00056 & 0.00047 \\ -0.00014 & 0.00047 & 0.00318 \end{bmatrix} \tag{17}$$

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[9] The correlation between H-P detrended measures is 0.869.

Next we turn to the simulation details of two main experiments.

4.2 Simulating the Benchmark Economy

We start with the analysis of the model presented in Section 2 without stochastic σ and χ. Here we ask whether our benchmark model is consistent with Shimer (2005b), even though we work with a decentralized economy version of the standard labor market search model. Since σ and χ are assumed to be parameters in this benchmark model, we only need to define a stochastic process for Z. Parameters σ and χ are set to their steady state values defined in Table 2. We assume that Z follows a first-order autoregressive process such that the standard deviation and the first-order autocorrelation match the corresponding moments in the data.

Our general solution algorithm is based on Christiano (2002) and relies on the linearized first order condition (9). We posit linear decision rules for log deviations of the endogenous variables V_t, N_{t+1} and C_t around their respective steady states as a function of N_t and $e_t = (Z_t, \chi_t, \sigma_t)$. In the benchmark model, exogenous state only consists of Z_t.

Table 3 presents sample moments from 100 simulations of the model economy where each simulation is 500 periods in length. To facilitate comparison with Table 1, each variable is H-P filtered with smoothing parameter 10^5 and only deviations from trend are reported. We can summarize this table in three broad findings. First, vacancies and market tightness (v/u) are significantly procyclical and unemployment is countercyclical. Second, the Beveridge curve relationship is consistent with the benchmark model as shown by the negative correlation between unemployment and vacancies of -0.846. Finally, variations in the labor market variables are much less than the underlying variation in productivity.

Table 3: Simulations of Benchmark Economy

	u	v	v/u	$u \to e$	$e \to u$	z
Standard Dev.	0.005	0.016	0.020	0.006	0.000	0.020
Autocorrelation	0.900	0.845	0.910	0.910	1.000	0.880
Cross Correlations						
u		−0.846	−0.913	0.085	0.000	−0.890
v			0.989	0.989	0.000	0.996
v/u				1.000	0.000	1.000
$u \to e$					0.000	1.000
$e \to u$						0.000

This last observation regarding the relative standard deviation of vacancies, unemployment and market tightness is the emphasis of Shimer (2005b). He argues that there is virtually no amplification in the standard labor market search model. The third rows of Table 1 and Table 3 confirm this point. Therefore, we conclude that the model with only productivity shocks behaves similarly to the criticized search model, even though we focus on the social planner's problem. In what follows, we refer to this discrepency between the model and the data as the *amplification puzzle*.

4.3 Simulating the Multiple Shock Economy

We now focus on the model presented in section 2 where the exogenous state space consists of (Z_t, χ_t, σ_t). Having introduced two additional shocks to the model, we expect to resolve the amplification puzzle to some extent. In other words, the allocational efficiency and job destruction shocks are expected to improve the model's performance. To show the consequence of introducing the additional shocks to the model, we again report the moments similar to those in Tables 1 and 3. Table 4 presents sample averages of moments from 100 simulations of the model economy where each simulation is 500 periods at length. Once again, we report the percentage deviations from trend.

Table 4: Simulations of Multiple Shock Economy

	u	v	v/u	$u \rightarrow e$	$e \rightarrow u$ (σ)	χ	z
Standard Dev.	0.048	0.386	0.420	0.159	0.109	0.051	0.020
Autocorrelation	0.883	0.732	0.779	0.860	0.840	0.806	0.890
Cross Correlations							
u		−0.690	−0.745	−0.839	−0.800	−0.890	0.270
v			0.990	0.957	0.974	0.670	−0.539
v/u				0.974	0.986	0.716	−0.526
$u \rightarrow e$					0.990	0.855	−0.500
$e \rightarrow u$ (σ)						0.810	−0.530
χ							−0.340

Simulation results of this economy are striking. First, we observe significantly more volatility in all key variables relative to productivity, especially in V and V/U. This is even accompanied by a seemingly correct Beveridge curve relationship, i.e. −0.69 correlation between vacancies and unemployment. However, the model predicts completely counterfactual cyclicality for vacancies, unemployment and market tightness. As cross correlations in Table 4 show, this model implies mildly *procyclical* unemployment and significantly *countercyclical* vacancies and market tightness. Adding two possible channels for possible fluctuations partly resolves the amplification puzzle but it entails seriously counterfactual cyclical features with respect to its key endogenous variables.

5 Discussion

It is no surprise the multiple shock approach produces better results in terms of the volatility of endogenous variables. Beyond that, however, our results provide more questions than answers. When we augment the standard labor market search model with shocks to allocative efficiency and job separation, unemployment and vacancies present counterfactual cyclical properties. Our objective here is to diagnose the reasons behind this erratic behavior to help reconcile the predictions of search models with the cyclical labor market facts. To this end, we present an insightful experiment. In what follows, we compute the innovations of the shock processes that would obtain if the actual data generating process is indeed the multiple shock model[10]. We then analyze the characteristics of the realized shock processes to form economically meaningful conjectures for the counterfactual behavior implied by the simulation in the preceding section. This leads us to productive avenues for future research.

5.1 Required Shocks for Perfect Fit

Given that we apply a linearization-based algorithm to solve for the economy's decision rules, solving for the exogenous shock series that gives the multiple shock model a perfect fit in expectation requires a straightforward inversion of the log-linearized model.

The solution procedure generates log deviations of endogenous variables around the steady-state as a function of N_t and $e_t = (Z_t, \chi_t, \sigma_t)$. Dropping the time subscript to denote steady-state values and using lower-case letters to represent the corresponding log-deviation from steady-state, we define the endogenous variables as follows: $n_t \equiv \ln\left(\frac{N_t}{N}\right)$, $v_t \equiv \ln\left(\frac{V_t}{V}\right)$, and $c_t \equiv \ln\left(\frac{C_t}{C}\right)$. The log-deviations of exogenous variables are similarly defined: $\tilde{z}_t \equiv \ln\left(\frac{Z_t}{Z}\right)$, $\widetilde{\chi}_t \equiv \ln\left(\frac{\chi_t}{\chi}\right)$, and $\widetilde{\sigma}_t \equiv \ln\left(\frac{\sigma_t}{\sigma}\right)$. Similar transformation can be applied to the VAR(1) shock process:

$$\widetilde{e}_{t+1} = A\widetilde{e}_t + \tilde{\varepsilon}_{t+1} \tag{18}$$

where $\widetilde{e}_t = (\tilde{z}_t, \widetilde{\chi}_t, \widetilde{\sigma}_t)'$, A is a 3×3 matrix of constants, and $\tilde{\varepsilon}_t$ is trivariate normal with $\mathrm{E}\tilde{\varepsilon}_t = 0$ and $\mathrm{E}[\varepsilon_t \varepsilon_t'] = \Omega$.

Given values for the parameters comprising the VAR(1) matrix of coefficients A, the decision rules mapping the period t state $(n_t, \widetilde{e}_t)$ into values for the endogenous variables (n_{t+1}, v_t, c_t) are required to be log-linear:

$$\begin{bmatrix} n_{t+1} \\ v_t \\ c_t \end{bmatrix} = \Pi \begin{bmatrix} n_t \\ z_t \\ \widetilde{\chi}_t \\ \widetilde{\sigma}_t \end{bmatrix}, \qquad \Pi = \begin{bmatrix} \pi_{nn} & \pi_{nz} & \pi_{n\widetilde{\chi}} & \pi_{n\widetilde{\sigma}} \\ \pi_{vn} & \pi_{vz} & \pi_{v\widetilde{\chi}} & \pi_{v\widetilde{\sigma}} \\ \pi_{cn} & \pi_{cz} & \pi_{c\widetilde{\chi}} & \pi_{c\widetilde{\sigma}} \end{bmatrix} \tag{19}$$

where the π parameters comprise expressions of technology and preference parameters. Easy

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[10] This exercise is partly in the spirit of Chari, Kehoe and McGrattan (Forthcoming) and Ingram, Kocherlakota and Savin (1994).

manipulation segregates the observed variables from the unobserved exogenous variables:

$$\begin{bmatrix} n_{t+1} - \pi_{nn} n_t \\ v_t - \pi_{vn} n_t \\ c_t - \pi_{cn} n_t \end{bmatrix} = \widehat{\Pi} \begin{bmatrix} z_t \\ \widetilde{\chi}_t \\ \widetilde{\sigma}_t \end{bmatrix}, \qquad \widehat{\Pi} = \begin{bmatrix} \pi_{nz} & \pi_{n\widetilde{\chi}} & \pi_{n\widetilde{\sigma}} \\ \pi_{vz} & \pi_{v\widetilde{\chi}} & \pi_{v\widetilde{\sigma}} \\ \pi_{cz} & \pi_{c\widetilde{\chi}} & \pi_{c\widetilde{\sigma}} \end{bmatrix}. \tag{20}$$

Given data series for employment, vacancies, and consumption, the left-hand side of this expression is a vector of constants in any given period. With values of all model parameters in hand, the matrix $\widehat{\Pi}$ is easily inverted to yield the period t realization of the forcing process: $(\tilde{z}_t, \widetilde{\chi}_t, \widetilde{\sigma}_t)$. Our estimates in (17) and the mapping in (20) yield a unique set of realizations for $(\tilde{z}_t, \widetilde{\chi}_t, \widetilde{\sigma}_t)$. Figure 7 plots the implied time series.

It is clear from the figure that both job destruction and allocative efficiency shocks move in the same direction over the cycle. They are both significantly procyclical and much more volatile than the productivity shock. One interesting observation is that, job destruction shock clearly identifies NBER recession dates. Table 5 presents statistics on the implied shocks[11]. We see that standard deviation of allocative efficiency is required to be almost 8 times that of labor productivity. The relative difference is even larger for the job destruction shock, which displays 30 times the variation observed in labor productivity. All three exogenous shocks are required to be quite persistent.

Table 5: Required Shocks

	z	χ	σ
Standard Deviation	0.016	0.120	0.464
Autocorrelation	0.894	0.928	0.922
Cross Correlations			
z		0.589	0.693
χ			0.859
σ			

There is obviously nothing surprising with the implied series on labor productivity, z. Since we are able to identify χ only indirectly when we use restrictions from the model, we can

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[11] Once again, all variables are log-deviations from their H-P trend, with smoothing parameter, 10^5.

be agnostic about the true nature of the allocative efficiency shock. However, implied job destruction series poses a significant challenge. It is impossible to reconcile a procyclical job destruction shock with the existing evidence (Blanchard and Diamond (1990), Davis and Haltiwanger (1992), Davis, Haltiwanger and Schuh (1996) and Shimer (2005a)). Since we cast this experiment as a diagnostic procedure, we need an answer to the following question: What are the properties of the multiple-shock search model that require it to produce a procyclical and volatile job separation rate to account for U.S. employment fluctuations? A successful answer to this puzzle requires a deeper understanding of the mechanics of the standard labor market search model.

5.2 Diagnosing the Search Model

We begin our analysis by identifying the model mechanisms and the cyclical properties of the observed data that produce the results highlighted above. Motivated by the persistent and procyclical movements of labor productivity Z_t, we first trace out the dynamics generated by the search model in response to a sudden and persistent increase in labor productivity, holding constant allocative efficiency χ_t and the rate of job separation σ_t. In doing so, we make use of the equations (6), (7), and (9).

Consider first, the effects of an innovation to labor productivity. By signaling greater future productivity [as captured by the term Z' in the intertemporal efficiency condition (9)], it encourages an immediate spike in vacancies as firms respond to the higher anticipated productivity benefits of filled positions. Consequently, additional job matches form in the period of impact thereby increasing employment and reducing unemployment. These effects are summarized by an increasing vacancies-unemployment ratio.

The productivity innovation also sets in motion forces that oppose the increasing vacancy-unemployment ratio. To see this, one first notes that the resource constraint (6) translates the anticipated increase in future productivity and employment into higher future consumption through a more rapid output flow.[12] The increases in employment and consumption subsequently reduces the representative worker's marginal willingness to substitute non-market

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[12] The sum of search and vacancy-creation costs, $\phi(1-N_t)+\kappa V_t$, small and the increase in vacancy-creation costs κV_t counteract the reduction in search costs $\phi(1-N_t)$.

activities for consumption, i.e. decreases $\frac{U'_N}{U'_C}$ in equation (9). This offsets, to some extent, an individual firm's vacancy-creation motive and the subsequent increase in employment. Furthermore, the draining of the unemployment pool persists and offsets some of the future benefits of currently high productivity by frustrating future hiring efforts through the term $-\frac{\kappa M'_U}{M'_V}$; this term represents the additional future recruiting costs exacted by the depleted stock of searching workers on the right hand side of (9). Recall that this last quantity (or more precisely, its absolute value) is directly proportional to the vacancy-unemployment ratio–a proxy for the 'tightness' of the labor market. The data, as we have seen, displays extremely large procyclical variation in this ratio, and casts doubt on the model's ability to produce the required cyclical variation in response to realistically sized shocks to labor productivity.[13]

By allowing both matching efficiency and the job separation rate to vary over the business cycle, the preceding diagnostic procedure responds to this tension by, in effect, equating the observed vacancy-unemployment ratio with the socially optimal one in each period. The highly variable and procyclical allocative efficiency shock χ_t implied by this exercise (Table 5 and Figure 7) effectively increases the expected gains of vacancy creation in response to exogenous increases in labor productivity, thus generating additional vacancies while also increasing the rate at which unemployed workers meet up with them. As a result, the flow of workers from unemployment to employment increases, reducing the unemployment pool. The increase in vacancies coupled with falling unemployment, thus gives an additional upward push to the vacancy-unemployment ratio moving the economy along the Beveridge curve in accord with the data. Although the vacancy-employment ratio moves decidedly in the proper direction, it cannot do so with a sizeable increase in net employment, all else constant. However, given that the aggregate employment data reveal relatively small period-to-period changes, a complete picture of the labor market dynamics implied by this model requires a much larger employment outflow to restock the unemployment pool depleted by the enhanced matching efficiency. This element, of course, could only be provided by the required procyclical rate of job separation σ_t (Table 5 and Figure 7). The picture that emerges from this exercise is a more dynamic labor

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[13] This point has already been convincingly demonstrated by Shimer (2005a) using a more conventional Mortensen-Pissarides model with a structurally stable matching function. We are indebted to his work for articulating the opposing forces on the theoretical vacancy-unemployment ratio restraining its response to labor productivity shocks.

market than the standard, single shock search model allows with workers cycling more quickly through labor market states during expansions than in periods of recession.

5.3 A Resolution: Procyclical Mismatch and Reallocation

At this point we could simply accept the results of our experiment with a claim that matching efficiency and job separation are indeed both strongly procyclical. To our knowledge, there is no direct evidence on the efficiency of labor market matching over the business cycle. We could (and do) simply argue that procyclical matching efficiency is a more reasonable outcome than the alternatives. As we have already stated however, sharply procyclical job separation is strongly at odds with existing data. To jointly accept both outcomes would be the equivalent of believing in an extraordinarily unlikely draw from the distribution of shocks. Instead we look for economic meaning in the results to conjecture a plausible solution to the puzzle, and by doing so, propose potentially productive modifications to standard labor market search framework. First, however, we discuss the current state of the literature regarding *amplification puzzle*.

Recent studies have attributed the amplification puzzle to different characteristics of the standard labor market search model with only productivity shock. Shimer (2005a) and Hall (2005) take a particular stand suggesting that the underlying wage determination mechanism is the reason for the lack of amplification in these models. Hall (2004, 2005), Shimer (2004) and Kennan (2006) build on this presumption and introduce wage rigidity either exogenously or through an endogenous mechanism, such as asymmetric information.

As argued extensively in a recent paper by Mortensen and Nagypal (2006), however, wage rigidity *per se* is not the reason for amplification. For instance, assuming no bargaining strength for workers leads to constant wages that are equal to the reservation wage (i.e. the value of leisure). Even in this case, the variability of labor market variables relative to productivity are an order of magnitude smaller (Mortensen and Nagypal (2006), p.15). Therefore, we conclude that our formulation of the problem as a social planner's problem, thereby ignoring wage determination, is not crucial for understanding the amplification puzzle.

Several recent studies also aim to provide a mechanism to amplify the effects of business cycles on unemployment and vacancies (Hagedorn and Manovskii (2006), Krause and Lubik (2006), Nagypal (2006) and Silva and Toledo (2006)). Hagedorn and Manovskii (2006) use an unrealistically high value of leisure to generate amplification, which also implies an excessive unemployment response to a slight increase in the value of leisure (Costain and Reiter (2005) and Hornstein, Krusell and Violante (2005)). Silva and Toledo (2006)'s result depends on a particular constellation of parameter values for separation, hiring and training costs, that is hard to quantify empirically.

In light of our discussion above, we first consider the possibility that the technological efficiency with which labor markets operate to match vacant positions and searching workers varies systematically over the business cycle. In terms of the standard matching model, we question whether the typical constant-returns matching function is structurally stable. In motivating this approach, we take as axiomatic the notion that the matching function owes its existence to concept of mismatch, "an empirical concept that measures the degree of heterogeneity in the labor market across a number of dimensions, usually restricted to skills, industrial sector, and location" (Petrongolo and Pissarides, 2001). In the absence of mismatch, jobs and workers would match instantly. Accordingly, an exogenous reduction in labor market mismatch, given the matching inputs of vacancies and unemployment, increases the number of matches formed, or equivalently, raises the allocative efficiency of labor markets.

To complete the story, procyclical matching efficiency must be paired with a job separation rate that an emerging literature has shown to be only mildly procyclical or nearly acyclical in the aggregate–not pronouncedly procyclical as the multiple-shock model and the U.S. data require.[14] As written, the standard model does not allow for a labor force participation decision or job-to-job transitions. In such a model, all labor force reallocation must be directed through only two labor market states in sequence, employment and unemployment, with no possibility of labor-force dropout and re-engagement or moving between jobs without an interceding unemployment spell. Thus, jobs must be destroyed to release workers into unemployment before being redeployed in a recently created vacancy. The model creates a vicious circle between employment and unemployment.

Introducing a labor force participation decision or the possibility of job-to-job transitions breaks the tight link between job matching and job dissolution in the standard setup by creating an additional pool of workers to draw upon to fill newly-created vacancies. As to whether these

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[14] See Shimer (2005b), for instance.

flows are significant is an empirical question. The distinction between unemployment and not-in-the labor force is relatively vague. However, existing evidence suggests significant job-to-job movements. Nagypal (2004) and Shimer (2005b) argue that job-to-job transitions are crucial for the cyclical worker reallocation. Exploiting dependent interviewing methods introduced in the CPS in 1994, Fallick and Fleischman (2004) find that these flows are large: on average 2.6% of employed workers change employers each month. Moreover, job-to-job transitions turn out to be significantly procyclical. This particular flow cannot be analyzed by standard search models. Thus, on-the-job search seems to be a natural avenue to pursue. Krause and Lubik (2006), Nagypal (2006) and Tasci (2006) are examples of this approach.

Extending the pool of searching workers to include already employed workers work as an incentive for firms to post vacancies. Following the same story provided above, consider what happens in the standard model following a positive productivity shock. As expected, this increases labor demand, hence the number of vacancies posted. Higher number of vacancies imply higher job finding rate and lower unemployment. If all new matches form through the pool of unemployed workers, an increasingly small pool of unemployed workers dampens the incentive for firms to post vacancies. In Nagypal (2006), information frictions generate a bias for firms to hire employed workers, which reduces the counter effect. In Tasci (2006), however, the underlying match heterogeneity in the form of symmetric incomplete information about the quality of the job-worker match implies a measure of workers employed in relatively low quality matches during expansions. These workers have the incentive to accept better quality matches thereby, providing the additional incentive for firms to post vacancies.

6 Robustness

This section explores the robustness of the multiple-shock model experiments. We check whether our results vary with the elasticity of the matching function and the elasticity of labor supply, α and γ respectively. Recall that we have calibrated γ to be 1.25 based on Merz (1995). Since this parameter will determine the response of household labor supply to changes in productivity, it is important to know whether our results are dependent on a particular choice. Similarly we also change the parameter value of the elasticity of matching function and check whether it fundamentally alters our conclusions. We present the relevant tables and figures for this section in the Appendix.

Our alternatives for γ are 1 and 0.5. Simulating the multiple-shock economy with these parameter values changes virtually nothing. The model continues to generate more volatility in labor market variables than the benchmark single-shock economy. Moreover, the counterfactual cyclical implications remain in place with procyclical unemployment and countercyclical job vacancies and labor market tightness. When we repeat our "perfect-fit" experiment, the model continues to require procyclical job separation and allocative efficiency shocks.

Since $a = 0.28$ lies at the lower end of the matching function estimates that Petrongolo and Pissarides (2001) provide, we consider higher alternative values. Increasing the value of α from 0.28 to 0.4 and 0.5, slightly alters our results, generating even less volatility in job vacancies and market tightness. This is expected, given that α also determines the share of the match surplus extracted by workers get in the decentralized manifestation of the model. As the firm's share falls, vacancy-creation becomes less sensitive to the underlying changes in the value of a match. However, the shocks required for a perfect fit conntinue to exhibit procyclical job separation and allocative efficiency. Hence, we conclude that our results remain in place for reasonably different values of Frisch elasticity of labor supply and the elasticity of matching function.

7 Conclusion

We have extended a basic discrete-time version of the Mortensen-Pissarides model of labor market search to include multiple and mutually-correlated sources of exogenous variation. The shock process comprises labor productivity (as in the basic model), job separation, and matching efficiency. The process that governs these shocks is partly estimated using data on job finding and separation probabilities for the U.S. economy. Although the multiple shock process allows the model to generate additional cyclical variation in unemployment, job vacancies, and the vacancies-unemployment ratio (as one would expect), the model does so while producing significant counterfactual implications with mildly procyclical unemployment and significantly countercyclical vacancies and labor market tightness. Having empirically plausible labor market transition probabilities increases variability in endogenous variables, but only at the expense of counterfactual cyclicality in unemployment and job vacancies. Our results, therefore, complement those of Shimer (2005a) and demonstrate that the model is more fundamentally flawed than its inability to amplify shocks would suggest.

We exploit the relative degrees of freedom we have in the model allowed by the multiple-shock structure, to understand the mechanics of the model that generated the empirically implausible implications. We argue that a convincing model of labor markets should simultaneously account for high procyclical variations in job finding probabilities as well as relatively small net employment changes. In the basic model, this is only possible through procyclical job separation. This result leads us to conclude that the Mortensen-Pissarides model of labor market search lacks a mechanism that would give rise to procyclical labor reallocation and procyclical matching efficiency.

We have been silent about the true nature of the allocative efficiency and job separation shocks. Our hope is to stimulate further research into the nature of our findings and to generate even richer theoretical structures which will eventually give us a more thorough picture of the fluctuations in the aggregate labor markets.

Appendix

A Frisch Elasticity of Labor Supply

Table A-1: Multiple Shock Model ($\gamma = 1$)

	u	v	v/u	u→e	$e \to u$ (σ)	χ	z
Standard Dev.	0.048	0.389	0.424	0.160	0.109	0.051	0.020
Autocorrelation	0.883	0.734	0.781	0.861	0.840	0.806	0.890
Cross Correlations							
u		−0.689	−0.746	−0.839	−0.806	−0.886	0.280
v			0.996	0.957	0.974	0.676	−0.548
v/u				0.974	0.986	0.721	−0.535
u→e					0.997	0.858	−0.511
$e \to u$ (σ)						0.822	−0.540
χ							−0.353

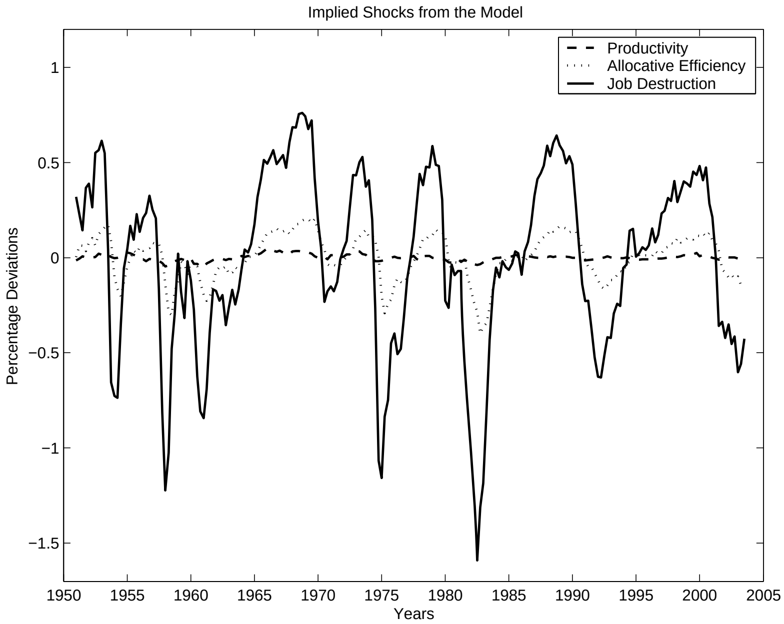

Figure 1:

Table A-2: Multiple Shock Model ($\gamma = 0.5$)

	u	v	v/u	u→e	$e \rightarrow u$ (σ)	χ	z
Standard Dev.	0.047	0.386	0.421	0.158	0.109	0.051	0.020
Autocorrelation	0.882	0.735	0.781	0.861	0.840	0.806	0.890
Cross Correlations							
u		−0.688	−0.745	−0.838	−0.804	−0.881	0.287
v			0.996	0.957	0.974	0.667	−0.550
v/u				0.974	0.986	0.712	−0.538
u→e					0.997	0.852	−0.514
$e \rightarrow u$ (σ)						0.815	−0.543
χ							−0.352

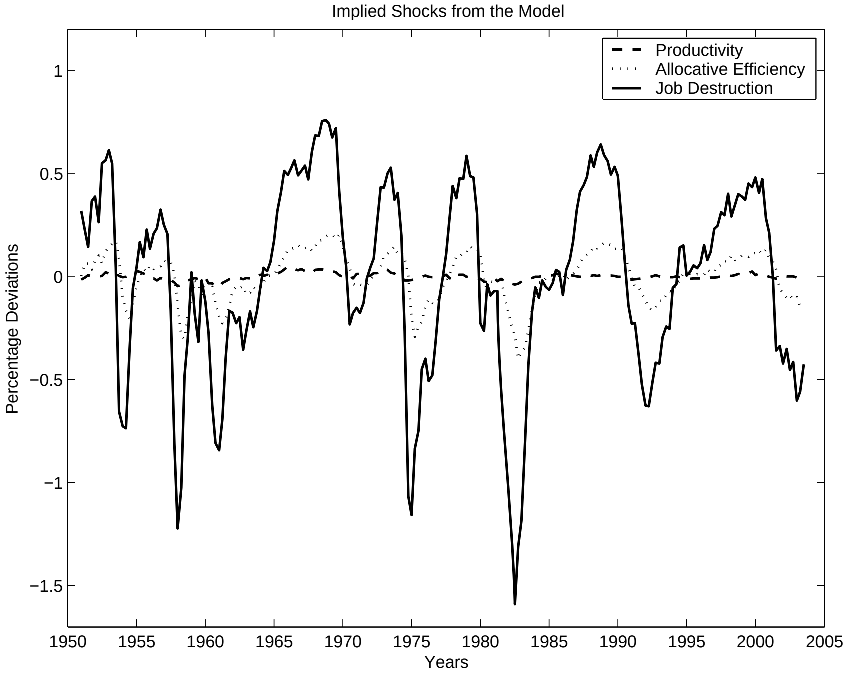

Figure 2:

B Elasticity of Matching Function

Table B-1 : Multiple Shock Model ($\alpha = 0.4$)

	u	v	v/u	u→e	$e\rightarrow u$ (σ)	χ	z
Standard Dev.	0.048	0.262	0.295	0.159	0.109	0.051	0.020
Autocorrelation	0.883	0.707	0.778	0.861	0.840	0.809	0.890
Cross Correlations							
u		−0.650	−0.739	−0.833	−0.795	−0.883	0.24
v			0.992	0.946	0.966	0.649	−0.539
v/u				0.974	0.986	0.719	−0.513
u→e					0.996	0.857	−0.490
$e\rightarrow u$ (σ)						0.819	−0.531
χ							−0.341

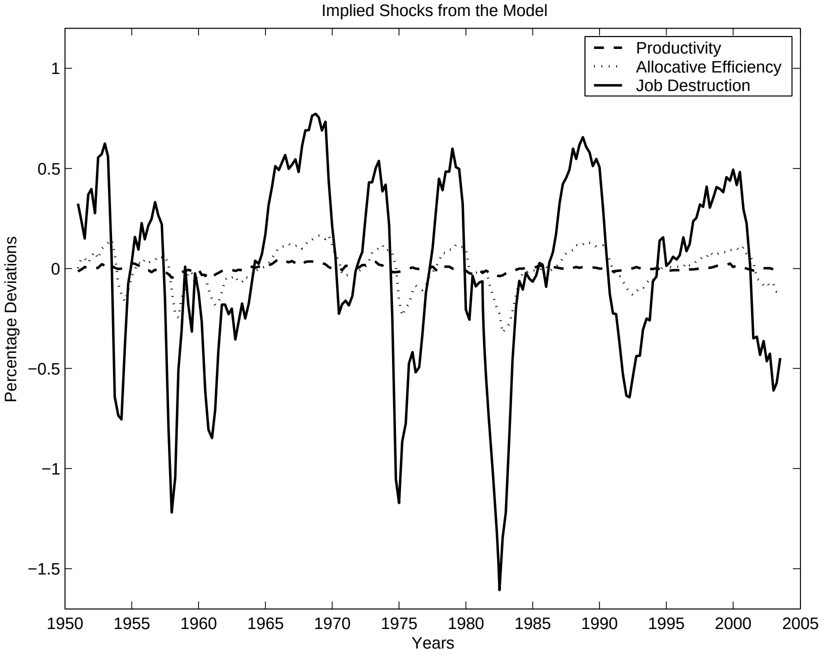

Figure 3:

Table B-2 : Multiple Shock Model ($\alpha = 0.5$)

	u	v	v/u	u→e	$e \to u$ (σ)	χ	z
Standard Dev.	0.048	0.204	0.238	0.160	0.110	0.051	0.020
Autocorrelation	0.880	0.689	0.780	0.861	0.842	0.809	0.890
Cross Correlations							
u		−0.616	−0.734	−0.827	−0.786	−0.878	0.225
v			0.987	0.936	0.959	0.633	−0.543
v/u				0.975	0.986	0.724	−0.514
u→e					0.996	0.859	−0.494
$e \to u$ (σ)						0.820	−0.542
χ							−0.350

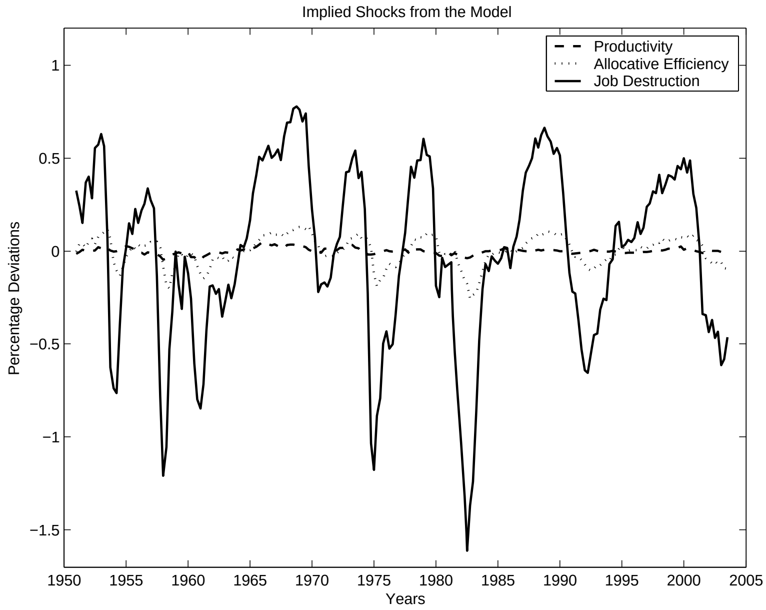

Figure 4:

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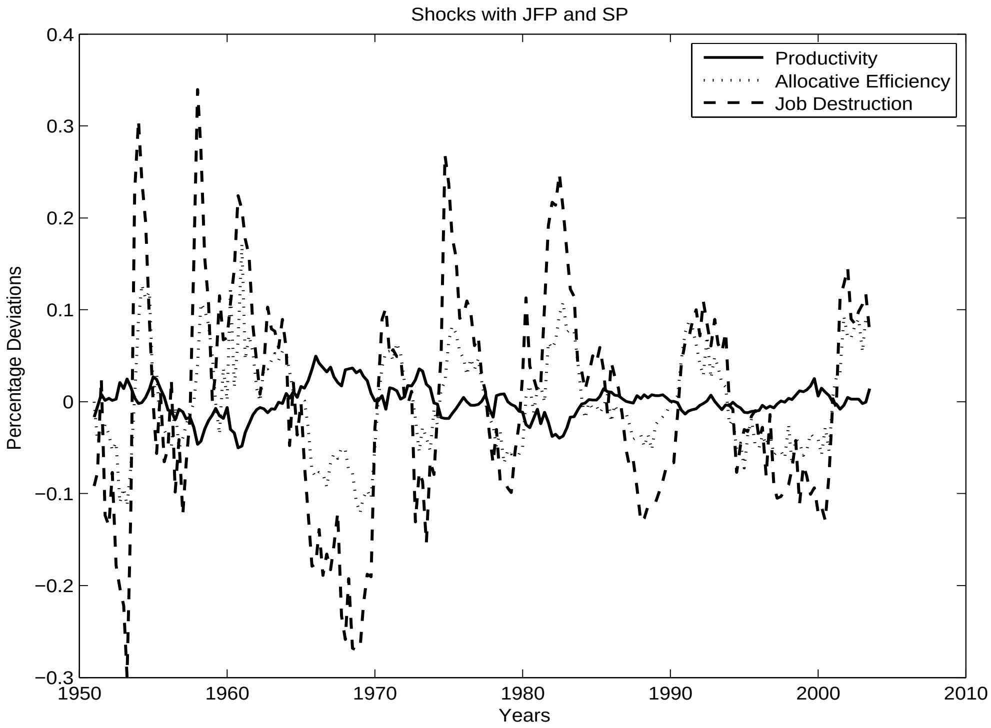

Figure 5: Estimated shocks using job finding (JFP) and separation probabilities (SP).

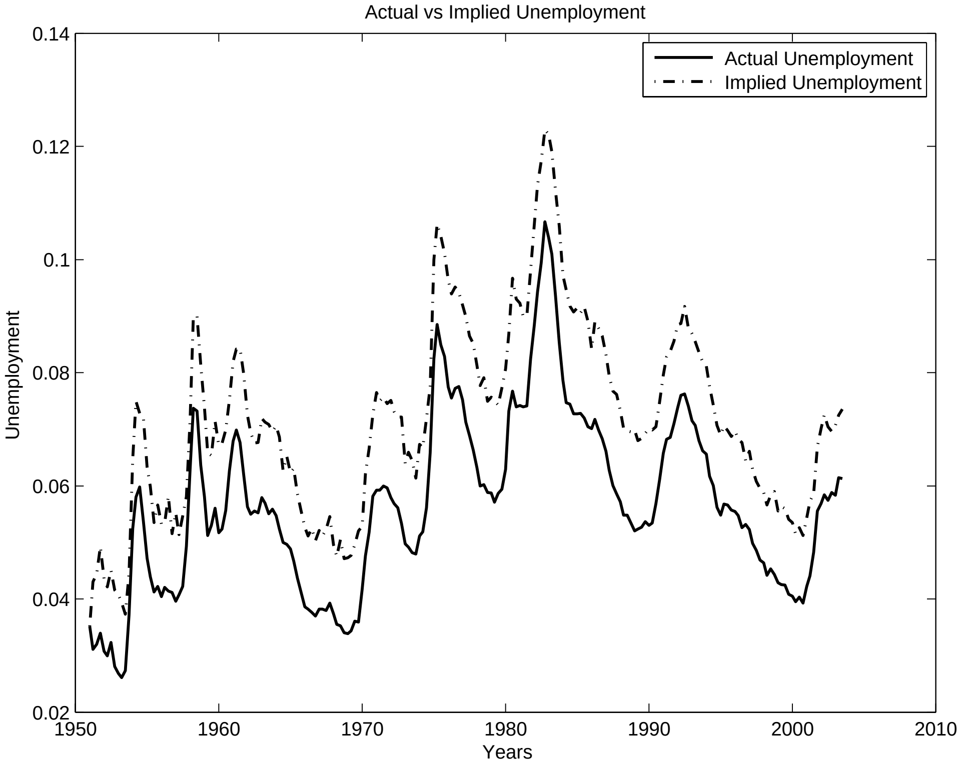

Figure 6: Actual Unemployment versus Implied Unemployment

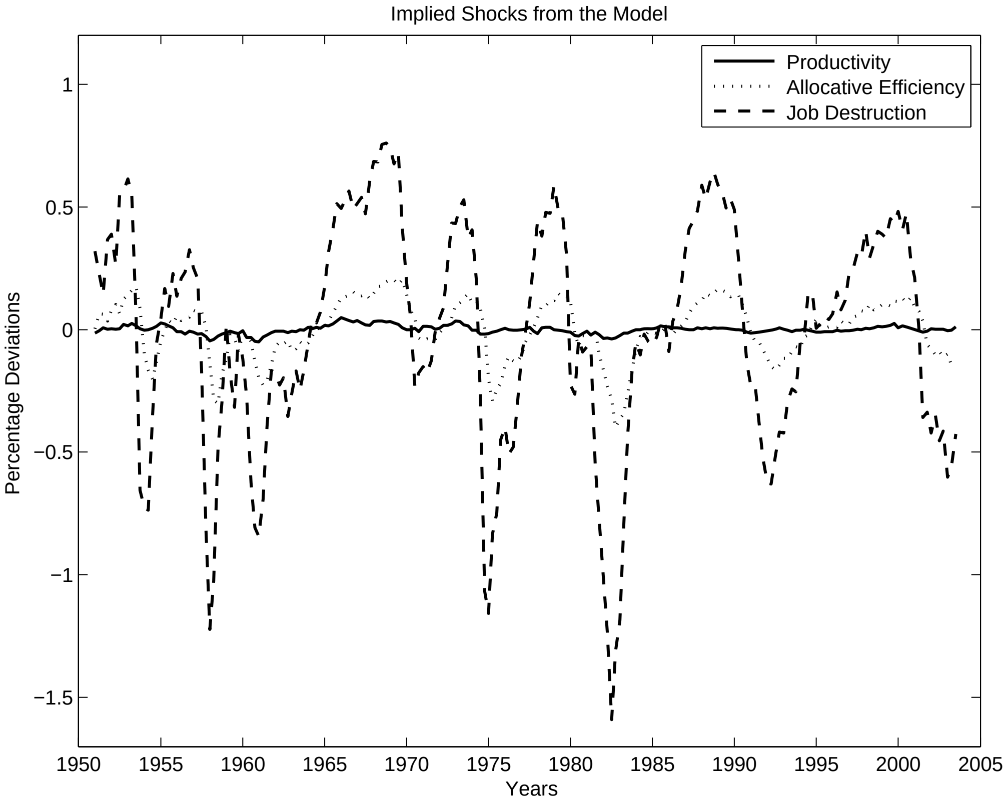

Figure 7: Shocks required for perfect fit.