Trimmed Mean PCE Inflation

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Abstract

Research over the past decade has led to improved measures of core inflation in the Consumer Price Index, or CPI. This paper discusses the application of some of the insights and techniques of that line of research to the Federal Reserve Board of Governors' preferred inflation gauge, the price index for Personal Consumption Expenditures (PCE). The result is a new measure of core PCE inflation—the trimmed mean PCE—and a somewhat different characterization of the economy's recent inflation experience.

Compared to the story told by the usual "excluding food & energy" measure, the trimmed mean PCE tells us that the lows reached in 2003 weren't quite so low and that the highs reached in mid-2004 were really a bit higher. On a 12-month basis, the new measure suggests that core PCE inflation is currently about ½ a percentage point higher than what is being indicated by the "excluding food & energy" inflation rate.

1. Introduction

Speaking of the challenge in interpreting monthly inflation numbers during his tenure on the Federal Reserve Board, former Vice Chairman Alan Blinder has said

> "The name of the game then was distinguishing the signal from the noise, which was often difficult. The key question on my mind was typically: What part of each monthly observation on inflation is durable and what part is fleeting?"[1]

Blinder's conception of a component of monthly inflation that is "durable" as opposed to "fleeting", that represents "signal" rather than "noise", corresponds to what most economists call *core inflation*. Core inflation, understood in this way, represents the underlying trend in inflation, once transitory swings have been smoothed out. Because what is transitory and what is lasting can only be known with the benefit of hindsight, the true core inflation rate for any given month cannot be known with certainty until well after the fact. In real time—as the data arrive and policy decisions need to be made—the best that economists can do is to estimate the core rate of inflation.

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[1] Alan Blinder, "Commentary on 'Measuring short-run inflation for central bankers'", Federal Reserve Bank of St. Louis *Review*, May/June 1997.

Research over the past decade—discussed in more detail below—has led to improved measures of core inflation in the Consumer Price Index, or CPI. To the best of my knowledge, however, the insights and techniques of that line of research have yet to be applied to the Federal Reserve Board of Governors' preferred inflation gauge, the price index for Personal Consumption Expenditures (PCE).[2] The aim of this paper is to rectify that situation. The result is a new measure of core PCE inflation—the trimmed mean PCE—and a somewhat different characterization of the economy's recent inflation experience.

The remainder of the paper is organized as follows. In section 2, I try to provide some heuristic motivation for the use of trimmed means by examining the PCE components which experienced the largest price changes in a typical month. I argue that simply excluding all food and energy items—the conventional method for estimating core inflation—fails to exclude some highly volatile items, while potentially throwing out some useful information. In section 3, I make a more rigorous case for trimming, showing that the distributions of individual component price changes that go into the PCE inflation rate are both fat-tailed and left-skewed. A long tradition in statistics recommends the use of robust measures of location, like the trimmed mean, for distributions having fat tails, while skewness suggests that asymmetric trimming may be optimal. In section 4 I formulate the optimal trimming problem, which entails minimizing a measure of distance between the trimmed mean inflation rate and a proxy for "true" core inflation. Since there is some uncertainty about the correct core proxy, I consider several. I also examine the gains in accuracy that arise from looking at longer price-change horizons. With the optimally trimmed mean PCE inflation rate in hand, I proceed, in section 5, to highlight how the new measure describes the US's recent inflation experience and how that description differs from the one given by the usual "excluding food and energy" core measure. Section 6 offers some conclusions and caveats.

2. Motivation: heuristics

Measures of inflation that exclude food and energy prices are probably the most well-known estimators of core inflation. In fact, the "excluding food & energy" measures (XFE for short) are often spoken of as if they were synonymous with core inflation. Properly speaking, though, they represent just one of many potential core measures. To be sure, because of the high short-run volatility of some food and energy prices, there *is* some rationale for excluding those prices from

[2] Since February, 2000, the Federal Reserve Board's semi-annual monetary policy reports to Congress have described the Board's outlook for inflation in terms of the PCE. Prior to that, the inflation outlook was presented in terms of the CPI. In explaining their preference for the PCE, the Board stated: "The chain-type price index for PCE draws extensively on data from the consumer price index but, while not entirely free of measurement problems, has several advantages relative to the CPI. The PCE chain-type index is constructed from a formula that reflects the changing composition of spending and thereby avoids some of the upward bias associated with the fixed-weight nature of the CPI. In addition, the weights are based on a more comprehensive measure of expenditures. Finally, historical data used in the PCE price index can be revised to account for newly available information and for improvements in measurement techniques, including those that affect source data from the CPI; the result is a more consistent series over time." (Federal Reserve Board of Governors, *Monetary Policy Report to the Congress*, February 17, 2000)

a measure of core inflation, but, as research over the past decade has made clear, much better estimates can be made by taking a more rigorous approach to the problem of which prices to include and which to exclude.[3]

To see some evidence for the claim made in the previous sentence, consider the following data from March, 2005. Of the more than 200 expenditure categories that go into the PCE, Table 1 shows the 10 categories with the biggest increases in price from February 2005 to March 2005.[4] Note that the price changes shown in this table are not annualized—these are one-month percentage changes. By way of comparison, the change in the overall PCE price index from February to March was +0.46%.

Table 1. 10 biggest price increases in March, 2005

Component:	**Change from prior month:**
Gasoline & other motor fuel	8.0%
Purchased fuel oil	5.8%
Airline service	4.2%
Hotels & motels	4.2%
Medical services: Labs	3.2%
Farm fuel	2.5%
Purchased liquid petroleum gas	2.5%
Miscellaneous personal services	2.4%
Watch, clock & jewelry repair	2.4%
Laundry & garment repair	2.4%

Table 2 gives the corresponding list of the 10 components which experienced the largest price decreases in March, 2005:

Table 2. 10 biggest price decreases in March, 2005

Component:	**Change from prior month:**
Eggs	–4.4%
Fresh fruit	–2.6%
Women’s luggage	–1.8%
Men’s luggage	–1.8%
Intrastate toll calls	–1.8%
Photographic equipment	–1.8%
Toys, dolls & games	–1.7%
Household operation: Natural gas	–1.7%
Durable house furnishings: textiles	–1.5%
Lighting supplies	–1.5%

While it’s true that food and energy items show up a number of times on these two lists, there are many other items as well. Moreover, not all food and energy items had price changes as large as

[3] That more-rigorous approach—discussed in greater detail below—was pioneered by Michael Bryan and Stephen Cecchetti (1994). Wynne (1999) is a good survey of these methods.

[4] All data used in this article are from the Bureau of Economic Analysis, via Haver Analytics. See section 3 for more detail.

these. Some food components in particular—such as food consumed away from home—are notoriously stable. For example, the price index for "other purchased meals"—which comprises meals purchased at restaurants and bars—rose by just +0.15% in March. As we'll see below, that sort of small price volatility is typical for food purchased and eaten away from home—making its exclusion from a measure of core inflation questionable, to say the least.

The points to take away from these tables and discussion are two-fold. First, in any given month, excluding *only* food and energy items still leaves many very volatile items in the price index. Second, excluding *all* food and energy items may throw out some useful information.

3. Motivation: statistics

How, then, does one approach the question of which items to exclude or include more rigorously? In a study focusing on the CPI and the PPI, Michael Bryan, Stephen Cecchetti and Rodney Wiggins make a statistical case for the use of trimmed means as a method for estimating core inflation.[5] That case relies on the fact that trimmed means can be more efficient estimators of a distribution's location, as compared to the sample mean, when the distribution is characterized by heavy tails.[6] Intuitively, samples drawn from a heavy-tailed distribution will contain relatively large numbers of extreme values. As a result, the sample means, which are sensitive to outliers, will have a high variance. Trimming some fraction of those extreme observations produces a less volatile estimator, a fact which Bryan, Cecchetti and Wiggins illustrate using Monte Carlo techniques.

This section, therefore, is mainly concerned with establishing that the distributions of price changes which are aggregated each month to yield the PCE inflation rate are, in fact, characterized by very heavy tails. We will also see that those distributions exhibit some degree of left-skewness.

A preliminary step is to make precise how individual prices are aggregated to arrive at the overall PCE price index. We will see that, to a first-order approximation, the PCE inflation rate in any given month can be described as a weighted average of the rates of change of the various component prices, with weights that change from month to month. The monthly data consisting of component price changes and associated weights—what I'll refer to as the underlying *distributions* of price changes—will constitute the basic inputs to the optimal trimming problem described in Section 4.

Distributions of price changes

In any given month, the rate of inflation in a price index like the CPI or the PCE can be thought of as a weighted average of the rates of change in the prices of all the goods and services that make up the index. In contrast to the CPI, which uses expenditure weights which remain fixed for two years, the PCE is a chain-aggregate whose weights vary from month to month. While PCE weights are not precisely expenditure shares, they bear a close relationship to expenditure shares.

[5] Bryan, Cecchetti and Wiggins (1997).

[6] There is an old literature in statistics on the use of trimmed means as robust estimators of location. See, for example, Crow and Siddiqui (1967).

In particular, to a first approximation, the weight a component receives in this month's PCE is an average of (1) its expenditure share last month and (2) what its expenditure share would be if consumers bought this month's quantities at last month's prices.

Let $\{P_{i,t}, Q_{i,t}\}_{i=1}^{N}$ denote a list of prices and real quantities for N component items at date t. The gross rate of growth in the PCE price index, p, from t to $t+1$ obeys the Fisher ideal index formula:

$$\frac{p_{t+1}}{p_t} = \sqrt{\frac{\sum_i Q_{i,t} P_{i,t+1}}{\sum_i Q_{i,t} P_{i,t}} \frac{\sum_i Q_{i,t+1} P_{i,t+1}}{\sum_i Q_{i,t+1} P_{i,t}}} . \tag{1}$$

Define the 1-month inflation rates in the overall index and for each of the N component prices in the usual way by

$$\pi_{t+1} = \frac{p_{t+1}}{p_t} - 1$$

and

$$\pi_{i,t+1} = \frac{P_{i,t+1}}{P_{i,t}} - 1 .$$

Then, (1) implies the approximation

$$\pi_{t+1} \cong \sum_i w_{i,t+1} \pi_{i,t+1}$$

where

$$w_{i,t+1} = \frac{1}{2} \frac{Q_{i,t} P_{i,t}}{\sum_i Q_{i,t} P_{i,t}} + \frac{1}{2} \frac{Q_{i,t+1} P_{i,t}}{\sum_i Q_{i,t+1} P_{i,t}} . \tag{2}$$

That is to say, the PCE inflation rate in any given month is a weighted average of the rates of change of the constituent components, with weights, given by (2), that vary over time. I will often loosely refer to an item's weight as its "share" in the PCE, though one should bear in mind that this is only half true (literally). As (2) shows, a component's weight at date $t+1$ is an average of its expenditure share at t and what it's share would be, hypothetically, if consumers bought $t+1$'s quantities at t's prices.

I refer to a list of component price changes and associated weights, $\{(w_{i,t}, \pi_{i,t}) : i = 1,2,...N\}$, as a distribution of price changes, and, henceforward, I take those distributions to be the primitive objects of study.

Before proceeding to characterize the kurtosis and skewness of the underlying distributions of price changes, a word on the data is in order.

The data

All the data used in this study are publicly available. The source for the detailed underlying price and quantity data—as well as the headline and XFE inflation data used below—is the Bureau of Economic Analysis. The data on the detailed components of the PCE index are as reported in Tables 2.4.4U and 2.4.6U in the "Underlying Detail Tables" section of the BEA's website.[7] The sample period I consider runs from January 1977 to December 2004. Some justification of that choice is in order.

The degree of disaggregation of the items in the "underlying detail" data available from BEA varies depending on the category of commodity and the sample period. Some components have the nature of individual goods (e.g., "eggs"), while others are themselves aggregates (e.g., "fresh fruit"). Data of any degree of disaggregation are available from 1959, and more recent data are available at a higher degree of disaggregation then earlier data. Data at the highest degree of disaggregation (roughly 220 components) are available only since 1990.

As a way of balancing the trade-off between having a longer sample and having more disaggregated data, I chose 1977–2004 as the sample period to use. The year 1977 is notable in that it is the earliest year with prices and quantities for "computers, peripherals & software" and "video equipment and media". The 1977–2004 sample consists of price and quantity data on 186 component items. A complete list of the components is given in Appendix A.

Kurtosis

The degree of kurtosis characterizing the distributions is most important for establishing the usefulness of the trimmed mean to measure core. This section will establish that the distributions of monthly price changes underlying the PCE are, in fact, leptokurtotic.

Focus, for a moment, on the distribution of price changes for a single month. To ease notation, temporarily drop time subscripts as well. Letting $\pi = \sum_{i=1}^{N} w_i \pi_i$ and $\sigma = \sqrt{\sum_{i=1}^{N} w_i (\pi_i - \pi)^2}$, the standard measure of the kurtosis of the distribution is the normalized fourth moment

$$K^1 \equiv \frac{\sum_{i=1}^{N} w_i (\pi_i - \pi)^4}{\sigma^4},$$

which would equal 3 for a standard normal distribution. For the monthly distributions of price changes associated with the PCE, the average kurtosis over the period 1977–2004 is 40.56.

Since K^1 is itself potentially sensitive to extreme values, I consider several "robust" kurtosis measures as well. Generally, these measures—as well as some of the robust skewness measures discussed below—involve the quantiles of the monthly distributions. Appendix B gives explicit

[7] www.bea.doc.gov/bea/dn/nipaweb/nipa_underlying/Index.asp.

formulae for the alternative measures. A detailed discussion of the advantages of these robust measures, along with an economic application, is presented in Kim and White (2004).

Table 3 gives the average values of all the kurtosis measures, along with the values of the statistics under a standard normal distribution.

Table 3. Measures of kurtosis in PCE price distributions, 1977–2004

Kurtosis measure:	Monthly average	Value under normality
Standard	40.56	3
Moors	2.02	1.23
Hogg	4.96	2.59
Crow-Siddiqui	9.67	2.91

According to any of the four measures, the monthly distributions of price changes exhibit excess kurtosis. To put some perspective on these numbers, consider that, for the problem of estimating the location parameter of a single distribution, Hogg recommended trimming 37.5% from each tail of the sample whenever his kurtosis measure (the third in Table 3) exceeded 3.2. (See Prescott, 1978, for a discussion of Hogg's recommendation and an alternative criterion which calls for a 30% trim from each tail when Hogg's kurtosis exceeds 3.)

Skewness

The presence of skewness is inessential to the statistical case for trimming, which is based on the presence of excess kurtosis. A finding of skewness would, however, suggest that we might not wish to constrain our trim to be symmetric. This is intuitive if one considers skewness in conjunction with an extreme form of symmetrically trimmed mean—the median. For many distributions, skewness (as conventionally measured) is associated with a discrepancy between the distribution's mean and median. Suppose that the distributions of price changes underlying the PCE are, on average, left-skewed. It is then likely that the monthly median PCE inflation rate will, on average, exceed the monthly mean PCE inflation rate (*i.e.*, the headline PCE inflation rate). If true core PCE inflation is understood as a measure of the trend of the monthly headline rates, then it is likely that the median will, on average, exceed the true core rate. In other words, the median will be a biased estimate of core PCE inflation.

The conventional measure of skewness is the standardized third moment:

$$S^1 \equiv \frac{\sum_{i=1}^{N} w_i \left(\pi_i - \pi\right)^3}{\sigma^3}$$

As was the case with the conventional measure of kurtosis, the conventional measure of skewness is highly sensitive to outliers. Thus, as I did with kurtosis, I here consider several robust measures of skewness. Formulae for the alternative measures are given in Appendix B, and the reader is again referred to Kim and White (2004) for a discussion of their merits relative to the conventional measure of skewness.

Consideration of robust measures is important in this case, as the three robust measures indicate left-skewness, while the conventional measure indicates a slight right-skewness, on average, in the monthly distributions of price changes. Table 4 presents the average monthly value, over the period 1977–2004, of the various skewness statistics, as well as their average absolute values and averages since 1995. All of these measures would be zero for symmetric distributions.

Table 4. Measures of skewness in PCE price distributions, 1977–2004

Skewness measure:	Monthly average	Average of absolute values	Average since 1995
Standard	+0.36	3.22	+0.47
Bowley	–0.09	0.24	–0.24
Groenveld-Meeden	–0.06	0.20	–0.14
Pearson	–0.04	0.09	–0.07

The numbers given in the first line of the table mirror a finding of Bryan, Cecchetti and Wiggins (1997) in their examination of the distributions of price changes underlying the CPI: when skewness is measured in the standard way, the monthly distributions exhibit considerable absolute skewness, but the skewness changes sign from month to month in such a way that the average amount is quite small. In fact, when skewness is measured in the standard way, 51% of the monthly distributions exhibit negative skewness, while 49% exhibit positive skewness. The average skewness of the distributions is a small positive number.

The story told by the robust measures is slightly different. In particular, average skewness is negative according to all the robust measures. While the average skewness numbers are (in absolute magnitude) smaller than the corresponding average absolute values, the differences are less pronounced than is the case with the standard measure. This may be due to the fact that the incidence of positive and negative skewness is less evenly divided with the robust measures as compared to the standard measure: according to any of the three robust measures, the monthly distributions are negatively skewed 58% of the time, versus 51% of the time for the standard measure.

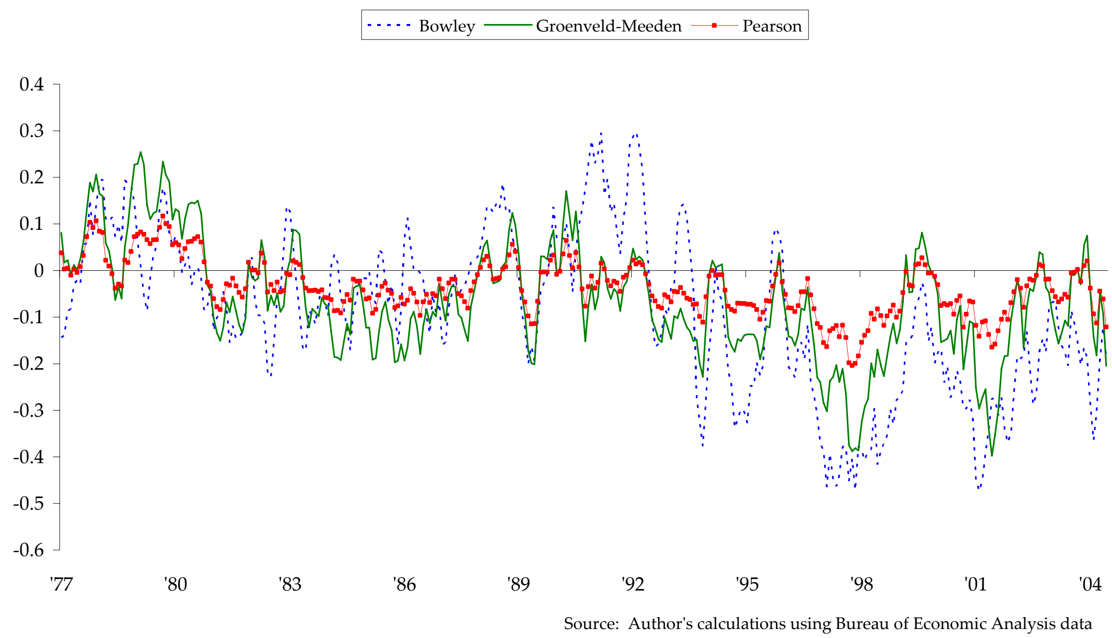
To put these numbers in perspective, note that all three alternatives are bounded between –1 and +1. The Bowley measure, for example, is the difference between the distance from a distribution's 75% quantile to its median and the distance from the median to the 25% quantile, expressed as a fraction of the distribution's inter-quartile range. The first entry in the second line of the table thus has the following interpretation: on average, the 25% quantile of the monthly distributions is further from the median than the 75% quantile by an amount equal to 9% of the inter-quartile range.

Pearson's measure is the difference between a distribution's mean and its median, expressed as a fraction of the distribution's standard deviation, while Groenveld and Meeden's is the same difference, expressed as a fraction of the average absolute deviation between price changes in the distribution and the distribution's median.

Interestingly, the three robust measures indicate that the price change distributions have become more negatively skewed in recent years, particularly since the mid-1990s. The last column of table 3 gives the average monthly skewness values under all four measures in distributions from

1995–2004. Figure 1 plots times series of the three robust measures for the 1977–2004 period; to give some sense of the general trends, the figure presents six-month moving averages of the three measures.

Figure 1: Skewness of monthly price change distributions
6-month moving averages, 1977:07-2004:12



Because the robust measures show more evidence of systematic skewness, particularly in the latter part of the sample, I do not impose a symmetry constraint on the optimal trimming problem. The structure of that problem, and the resulting trim proportions, are described in the next section.

4. The optimal trim

Calculating the trimmed mean PCE inflation rate for a given month involves looking at the price changes for each of the individual components of personal consumption expenditures. The individual price changes are sorted in ascending order, and a certain fraction of the most extreme observations in both tails of the distribution are thrown out, or "trimmed". The trimmed mean inflation rate is then calculated as a weighted average of the remaining components.[8]

[8]As noted in section 3, in the discussion of skewness, the median price change in a distribution can be thought of as an extreme form of trimmed mean. It corresponds to the limiting case where nearly all the price changes in the upper and lower halves of the distribution are trimmed, leaving only the price change of the single component exactly in the middle. Thus, the familiar weighted median CPI, which is produced by the Federal Reserve Bank of Cleveland, is also an extreme type of trimmed mean inflation rate.

More precisely, let $\{(w_{i,t},\pi_{i,t}): i=1,2,...N\}$ denote the distribution of component price changes from a given month, and assume that the N components are ordered such that $\pi_{1,t} \le \pi_{2,t} \le \cdots \le \pi_{N,t}$. For $\alpha \in [0,1]$, let $\hat{i}_t(\alpha) = \min\{I : \sum_{i=1}^{I} w_{i,t} \ge \alpha\}$, and consider a trimming that drops $100\alpha\%$ of the weight from the left tail of each month's distribution and $100\beta\%$ of the weight from the right tail of each month's distribution. This (α,β)-trimmed mean would then be defined by

$$\pi_t^{(\alpha,\beta)} = \frac{1}{1-\alpha-\beta}\sum_{i=\hat{i}_t(\alpha)}^{\hat{i}_t(1-\beta)} w_{i,t}\pi_{i,t}$$

for each date t. The optimal trimming problem is to find the best choice of α and β.

Proxying "true" core inflation

What percentages of components should be trimmed from each tail of the monthly price-change distributions? The amount of data trimmed from each tail of the distribution can be chosen optimally, to minimize the average monthly discrepancy between the trimmed mean inflation rate and a specific proxy for *true* core inflation. In their CPI/PPI study, Bryan *et al.* used a centered 36-month moving average of monthly inflation rates to proxy for true core inflation. That is, the true core inflation rate in any given month was assumed to be the average of that month's inflation rate together with the inflation rates of the prior 18 months and those of the subsequent 18 months. This is a natural choice that treats true core inflation as a smooth underlying trend in actual inflation. I will follow Bryan *et al.* in using this type of trend as one proxy for true core PCE inflation, but I will also consider two alternative core proxies, both informed to some degree by the monetary policy-making process.

The first alternative is a measure of trend PCE inflation that is designed to capture the inflation that the FOMC appears to have responded to by adjusting the Federal Funds rate. This is what might be called an operational definition of core inflation: core inflation is the inflation that policy-makers have typically responded to. As with any trend measure, this measure is constructed by throwing out a certain amount of high frequency movement in monthly PCE inflation. In this particular case, I choose the cut-off frequency between transitory movements in inflation and long-run trend so as to maximize the correlation between the resulting trend series and the Federal Funds rate target set by the FOMC.

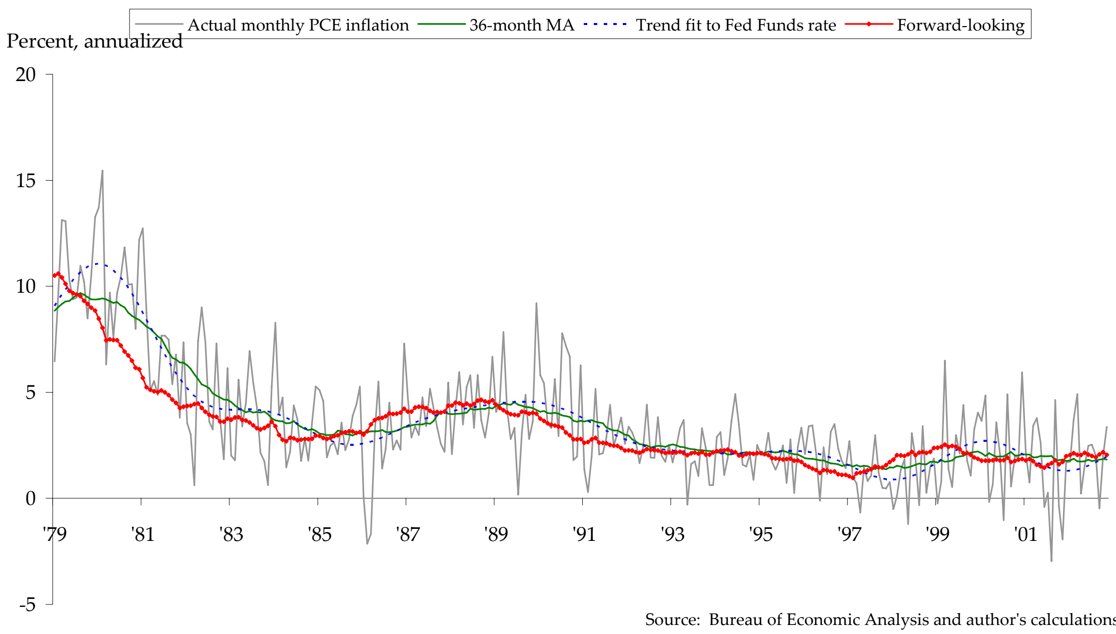
To be precise, this proxy for true core inflation is obtained by applying a band-pass filter to monthly PCE inflation. The correlation-maximizing cut-off described above turns out to be 39 months. Roughly speaking, application of the band-pass filter discards all movements in PCE inflation of duration less than 39 months.[9]

[9] 39 months is in the neighborhood of what is often taken to be the high-frequency cut-off defining business cycles, which many authors, beginning with Baxter and King (1999), have taken to be movements in aggregate variables having a frequency between 3 and 8 years. Our core proxy can thus be thought of as identifying movements in core inflation with movements at or below business cycle frequency. The filter I apply is the default filter suggested by Christiano and Fitzgerald (2003).

The second alternative derives from what policy-makers often describe as their idealized signal for adjusting policy—movements in *future* inflation. This proxy measure is a moving average, like that of Bryan *et al.*, but its value in each month is the average of inflation in that month and the coming 24 months.

Figure 2 shows all three alternative core proxies, together with actual monthly PCE inflation, from 1979 to the end of 2002.[10]

Figure 2: PCE inflation and three proxies for true core PCE inflation



The optimal trimming problem

Given a proxy for true core inflation, the optimal trimming problem chooses the trim points (α, β) to minimize the root mean square distance between the trimmed mean and the core proxy. Letting $\{\overline{\pi}_t\}_{t=1}^{T}$ denote the core proxy series, we solve:

$$\min_{(\alpha,\beta)} \sqrt{T^{-1}\sum_{t=1}^{T}\left(\pi_t^{(\alpha,\beta)} - \overline{\pi}_t\right)^2} \, .$$

[10] Christiano and Fitzgerald recommend that users of their band-pass filter drop two years' worth of observations from the beginning and end of the sample because of the imprecision with which the trend is estimated near the sample end-points. I follow that recommendation here, so that our original 1977–2004 sample period yields proxies for true core inflation over the period 1979–2002.

Following such a procedure, I find that the optimal amount of trimming is somewhat sensitive to the choice of core proxy, though it is always asymmetric. For all three core proxies, the optimal trim discards more data from the upper tails of the distributions than from the lower tails.

The first three rows of Table 5 show the optimal trims from the lower and upper tails, in percent, for the three core concepts described above.[11] The total amount trimmed—the sum of the lower and upper trims—ranges from about 31% to 50%. By comparison, the food and energy items excluded from the conventional XFE measure of core PCE inflation amount to only about 20% of total expenditures.

Table 5. Optimal trimming for various core proxies

Core proxy:	Lower tail trim:	Upper tail trim:
36-month centered moving average	20.6%	25.3%
Trend correlated with Fed Funds rate	14.4%	16.4%
Forward-looking moving average	19.5%	30.5%
Three equally likely	19.4%	25.4%

Without strong prior beliefs about which core proxy is the right one, how does one choose among the three suggested trims? One way to deal with the sensitivity of the recommended trimming to the core proxy used is to calculate the trimming that is optimal in some average sense against all three alternatives. More precisely, we can choose (α, β) to minimize the expected root mean square error, assuming each of the three core proxies is equally likely to be the correct one. That problem is:

$$\min_{(\alpha,\beta)} \sum_{h=1,2,3} \frac{1}{3} \sqrt{T^{-1} \sum_{t=1}^{T} \left(\pi_t^{(\alpha,\beta)} - \overline{\pi}_t^h \right)^2}$$

where $\left\{ \overline{\pi}_t^h \right\}_{t=1}^T$ for $h = 1,2,3$ represent the three proxies for true core inflation.

The results of this calculation are given in the last line of Table 5.[12] In the remainder of the article, unless indicated otherwise, the optimally trimmed mean will employ the trimming given in the last line of Table 5.

Just based on the four sets of optimal trimming proportions in Table 5, one can make a reasonable guess as to how three of the four trimmed mean inflation rates will rank. For example, the left trims for the forward-looking moving average and "Three equally likely" cases are almost identical, while the right trim is much larger for the forward-looking moving average case. We

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[11] As discussed in Section 3, the sample period runs from the beginning of 1977 to the end of 2004. The calculation of the band-pass filtered proxy core measure entails dropping two years' worth of observations from the beginning and end of the sample, so that the root mean square error criterion is minimized using series running from 1979 to 2002.

[12] The numbers are not that different from what one obtains simply by averaging the optimal lower and upper trims given in the first three lines of the table. That simple averaging gives 18.2% and 24.1% as the lower and upper trims.

would thus expect the trimmed mean inflation rate for the forward-looking moving average trimming to be below the trimmed mean inflation rate from the "Three equally likely" trimming. Conversely, the right trims for the "Three equally likely" and 36-month moving average are nearly identical, while the left trim is larger for the 36-month moving average case. We would thus expect the trimmed mean inflation rate for the 36-month moving average trimming to exceed the trimmed mean inflation rate from the "Three equally likely" trimming. Putting these two parts together, we get the following ranking of inflation rates:

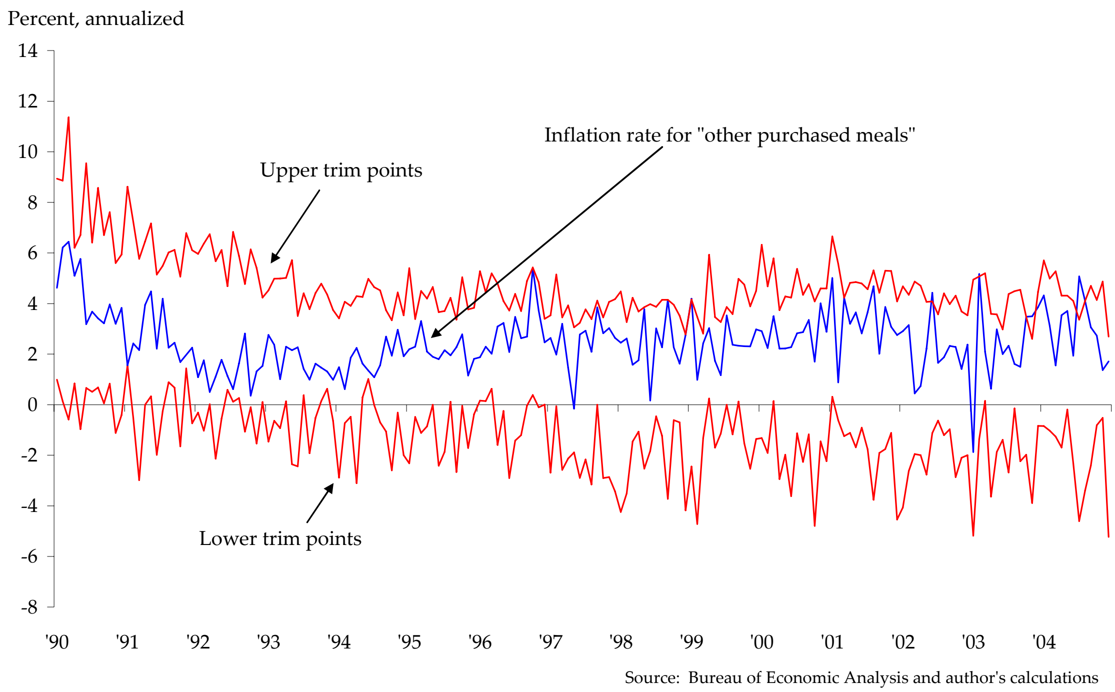
Forward-looking moving average < Three equally likely < 36-month moving average.

As will be shown in Section 5, the data bear out this intuition (see Figure 8). Based simply on the trimming proportions, we cannot say where the "Trend fit to Fed Funds rate" case will fall relative to the other three.

So, which goods get trimmed?

As suggested above, some food components, like food purchased and consumed away from home, are rarely excluded when one approaches the trimming problem rigorously. This is a feature of the inflation data which Bryan and Cecchetti, *op. cit.*, highlighted in their study of the CPI, and it is true of the PCE as well. Figure 3 illustrates this. The figure shows the monthly inflation rate for the PCE component "Other purchased meals" (which comprises meals at restaurants and bars) together with the upper and lower "trim points" for the optimally trimmed mean, from 1990 to the end of 2004.

Figure 3: Food away from home is generally not trimmed



The "trim points" have the following interpretation. In each month, items whose prices rose by more than the upper trim points in the chart are excluded from the optimally trimmed mean that month, as are items whose prices fell by more (or rose by less) than the lower trim points. There are only a handful of months, out of this period of 14 years, in which the purchased meals component was excluded from the optimally trimmed mean.

Food items of this sort are well-represented among the components least often excluded from the optimally trimmed mean. Table 6 gives the top 20 "least-often-excluded" components, for the sample period 1977–2004. Food items actually occupy five of the top ten spots, with "Other purchased meals" coming in first. Out of a sample of 335 months, it's excluded only 13 times. The other dominant category in the least-often-excluded list is housing, which shows up in various forms.

Table 6. The 20 least-often-excluded components, 1977–2004

Component:	**Number of months excluded (out of 335):**
Other purchased meals	13
Owner-occupied stationary homes	17
Casino gambling	34
Tenant-occupied stationary homes	35
Purchased meals: elementary & secondary schools	41
Purchased meals: higher education	41
Food furnished to employees: military	41
Tenant-occupied mobile homes	41
Food furnished to employees: civilian	42
Club and fraternity housing	51
Auto repair	52
Tenant group room and board	53
Tenant group employee lodging	54
Owner-occupied mobile homes	58
Military clothing	76
Domestic service paid in cash	80
Household operation, not elsewhere classified	86
Social welfare including child care	88
Medical care: other professional services	91
Dry-cleaning	92

Table 7 gives a corresponding list of the top 20 "most-often-excluded" items. Food items figure prominently here, too, with "Fresh vegetables" topping the list. Fuels, financial services and electronics items are also prominent on the list.

Table 7. The 20 most-often-excluded components, 1977–2004

Component:	**Number of months excluded (out of 335):**
Fresh vegetables	314
Eggs	312
Computers and peripherals	309
Food produced and eaten on farms	302
Airline services	299

Brokerage charges & investment counseling	297
Software	294
Purchased fuel oil	293
Fresh fruit	292
Poultry	285
Video equipment, excluding TVs	284
Gasoline & other motor fuel	282
Auto insurance net premiums	281
Farm fuel	281
Purchased liquid petroleum gas & other fuel	275
Durable house furnishings: textiles	272
TVs	272
Infants' clothing	271
Commercial bank imputed interest	269
Semi-durable house furnishings	267

How well does the trimmed mean perform?

How much better is the trimmed mean measure of PCE inflation in comparison to the usual XFE measure? Just as Bryan, Cecchetti and their co-authors found regarding the CPI, one can show that the optimally trimmed mean performs much better as an estimator of true core PCE inflation than the usual XFE measure.

In the data running from 1979 through 2002, the gain in accuracy from using the optimally trimmed mean rather than the XFE measure ranges from 0.72 to 0.85 percentage points, at annual rate, depending on the choice of core concept. Assuming the three alternative core proxies are equally likely to be correct, the gain in accuracy is about 0.76 annualized percentage points. That is to say, compared to the usual XFE measure, on average the monthly trimmed mean measure would be expected to come closer to true monthly core inflation by roughly ¾ of a percentage point, when the inflation rates are expressed in annual terms.

The results in the last paragraph compare the performance of 1-month inflation rates, which are quite volatile relative to the slower-moving core series. This is true for both the optimally trimmed mean and the XFE measure, though less so for the former. Cecchetti (1997), looking at the CPI, emphasized the additional noise-reduction that can be had by examining longer-horizon inflation rates. Cecchetti's point is equally valid with regard to the PCE. Looking at 3-, 6-, or 12-month inflation rates improves the accuracy of both the trimmed mean and the XFE measures as gauges of core inflation.

Table 8 shows the value of our measure of fit—the root mean square error, or RMSE—for inflation horizons of 1, 3, 6 and 12 months, for both the optimally trimmed mean and the XFE measure. The RMSEs are calculated for the case where the correct core proxy is uncertain, and the three alternative proxies are equally likely. The optimal trim is calculated under the same assumption, so it matches the last line of Table 5 (19.4% lower tail trim, 25.4% upper tail trim).[13]

Table 8. Root mean square errors for various inflation horizons

	1-month	3-month	6-month	12-month
Optimally trimmed mean	0.99	0.77	0.74	0.79
Excluding food & energy	1.75	1.15	1.00	1.06

For the trimmed mean and the XFE measure, 6-month changes give the smallest RMSE—*i.e.*, the highest accuracy in gauging core inflation. While the longer horizons benefit the XFE measure more than the trimmed mean, the latter is still the more accurate core inflation gauge. For the 3-month inflation horizon, the relative gain in accuracy from using the trimmed mean is almost 0.40 percentage points, and for either the 6-month or 12-month horizons, the gain in accuracy is still more than ¼ of a percentage point, a not-insignificant difference.

The optimally trimmed mean also performs better than the XFE measure in terms of its mean, or average, error, as can be seen in Table 9.[14] Table 9 is analogous in structure to Table 8, but each entry in the table is now an inflation measure's average error, in percentage points:

Table 9. Average errors for various inflation horizons

	1-month	3-month	6-month	12-month
Optimally trimmed mean	0.03	0.05	0.08	0.13
Excluding food & energy	0.21	0.20	0.23	0.28

Figure 4 gives a visual sense of how the trimmed mean performs relative to the XFE measure. The chart shows the annualized 6-month inflation rates in the two measures, together with the average of three alternative proxies to true core inflation. The series are shown for the full sample period used in the calculations of the optimal trim—*i.e.*, the data run from the beginning of 1979 to the end of 2002.

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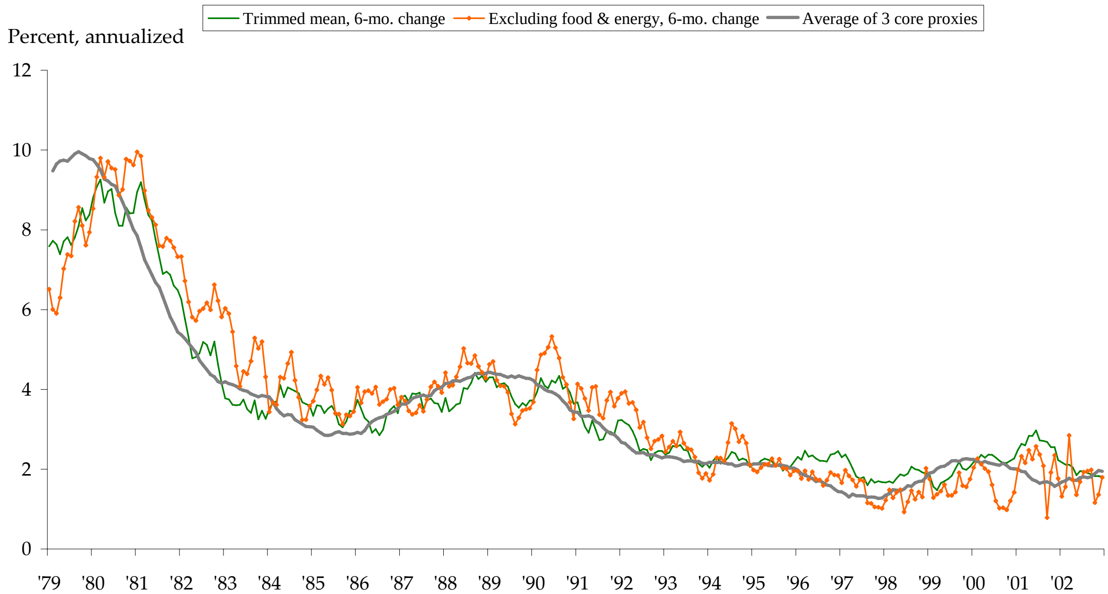
[13] The 3-, 6-, and 12-month inflation rates for the trimmed mean are had by cumulating the optimally trimmed series of 1-month rates to obtain a price index, then taking 3-, 6- and 12-month annualized percentage changes of that price index.

[14] To see the relevance of this point, suppose that true core inflation is zero in two consecutive months. Imagine that one measure (call it A) estimates core inflation as being +¼% in each of the two months, while a second (B) estimates it at +1% in the first month and –1% in the second month. Then B would have a higher RMSE than A—on average, B is 1 percentage point away from the truth, versus ¼ of a percentage point for A—but it would have a smaller mean error than A. B's mean, or average, error is zero (the +1 and –1 cancel out) compared to A's mean error of ¼ of a percentage point. If the trimmed mean and XFE measures followed this pattern—one better in terms of RMSE, the other better in terms of average error—we might be hard pressed to say which was the better measure. Fortunately, Tables 8 and 9 show the trimmed mean is better on both dimensions.

Figure 4: Trimmed mean, Ex Food & Energy, and proxied true core PCE inflation



Source: Bureau of Economic Analysis and author's calculations

One may wonder about the role played here by the asymmetric nature of the optimal trim—after all, analyses like that of Bryan, Cecchetti and Wiggins focus on trims which are constrained to be symmetric. The costs in RMSE from imposing a symmetry constraint over the sample data used here vary, depending on the core concept and the differencing horizon, from negligible ("Trend correlated with Fed Funds rate" core proxy, 3 or 6 month horizon) to 0.22 annualized percentage points ("Forward-looking moving average" core proxy, 12 month horizon). For the benchmark "Three equally likely" case, the costs in RMSE from imposing symmetry are small, ranging from 4 to 7 percentage points, depending on the horizon. The costs in terms of average errors, though, are quite large. The average errors for the best symmetrically trimmed inflation rate in the "Three equally likely" case range from 0.31 annualized percentage points for the 1-month inflation horizon to 0.42 percentage points for the 12-month inflation horizon. They are, in fact, higher at every horizon than the corresponding numbers for the XFE measure shown in Table 9.

5. Recent U.S. inflation

Figures 5–7 show the recent behavior of the trimmed mean PCE inflation rate, together with the more common "excluding food & energy" (XFE) inflation rate. The figures show the 3-, 6- and 12-month inflation rates according to both measures, from the beginning of 2002 through May 2005. Two things to note in particular in the figures are the lower volatility at each horizon of the trimmed mean measure, compared to the XFE measure, and the fact that the trimmed mean inflation rate is generally above the XFE rate over this period. At a 12-month horizon (Figure 7), for example, the trimmed mean inflation rate has run about ½ a percentage point higher than the XFE rate for roughly the past year-and-a-half.

Some other salient points are as follows.

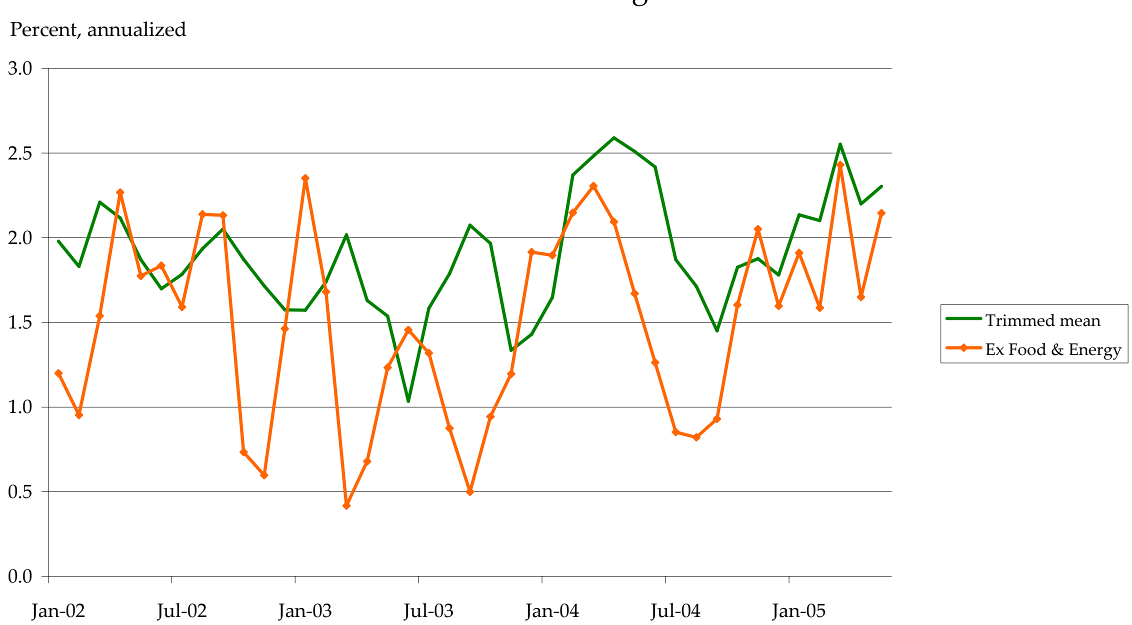
1. While both the trimmed mean and XFE inflation rates decline in 2003, in all three charts, the lows hit by the trimmed mean measure are not nearly as low as those reached by the XFE measure. For example, the 3-month trimmed mean inflation never rate falls below 1% in 2003, while the XFE rate is below 1% in 5 out of the 12 months. The minima of the 6- and 12-month trimmed mean rates are nearer 1.5%.

2. Both the trimmed mean and XFE inflation rates began to climb in early 2004. The highs reached in mid-2004, however, are both higher and more sustained in the trimmed mean measure than in the XFE measure.

3. Inflation decelerated in the second half of 2004, according to either the trimmed mean of the XFE inflation measure. This shows up as a decline in the 3- and 6-month inflation rates and a stabilization in the 12-month rates. The 3- and 6-month trimmed mean rates bottom out around 1.5%, compared with around 1% for the 3- and 6-month XFE troughs. Similarly, the 12-month trimmed mean rate stabilizes at about 2%, or ½ a percentage point higher than the 12-month XFE rate.

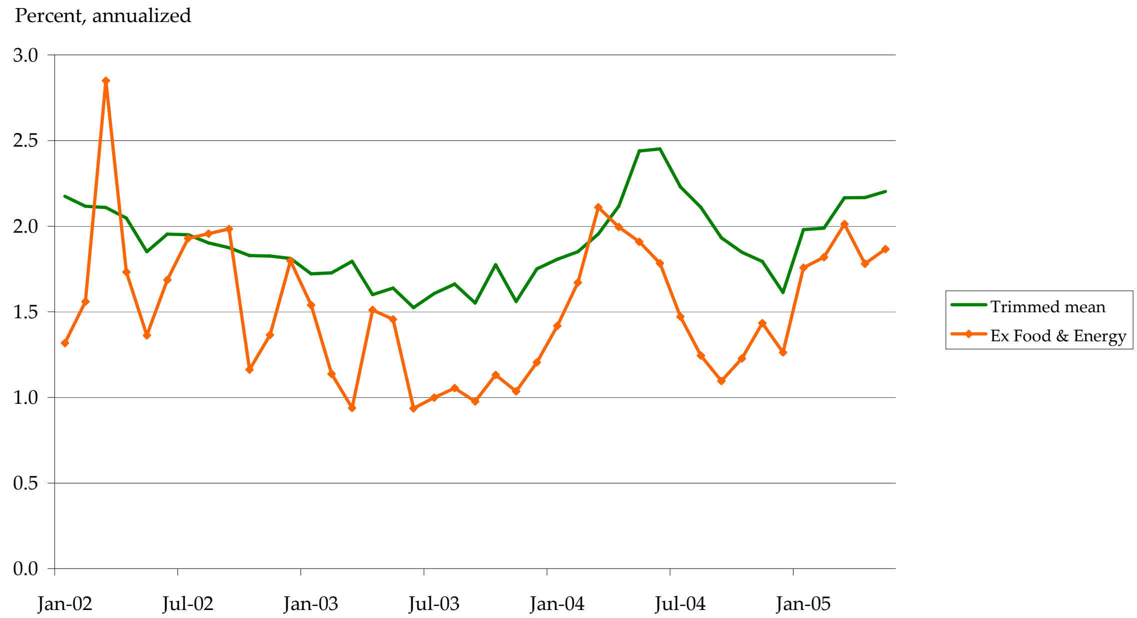
4. While the 12-month inflation rates in both the trimmed mean and XFE measures look stable, the 3- and 6-month rates show some pick-up in inflation since late 2004.

Figure 5: Trimmed mean and Ex Food & Energy PCE inflation; 3-month changes



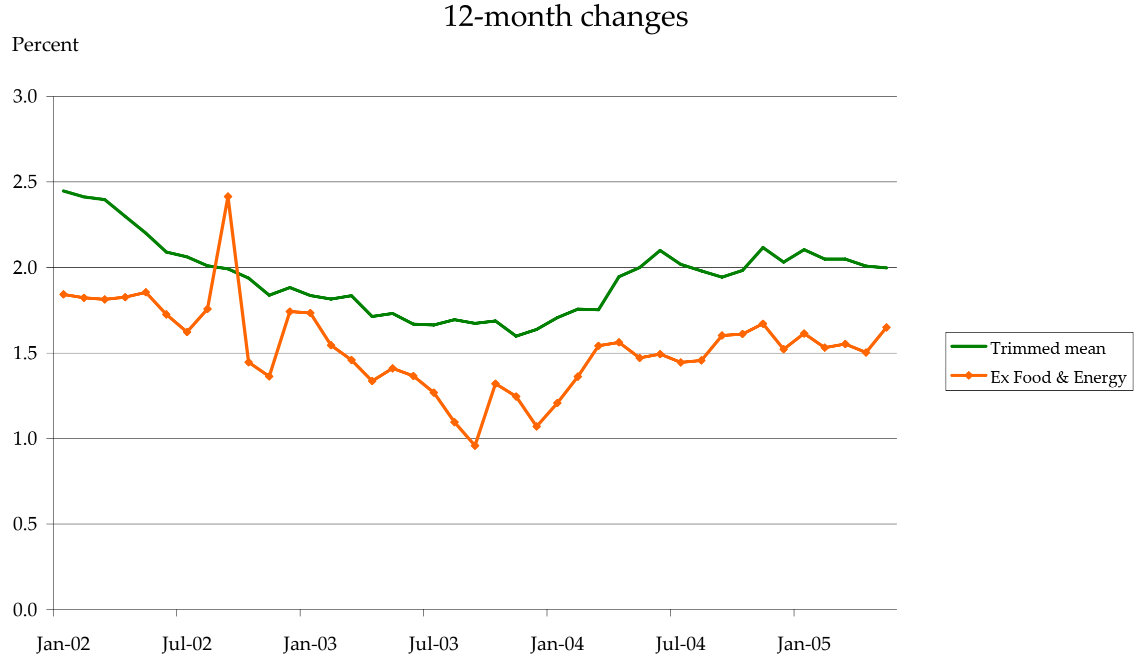
Source: Bureau of Economic Analysis and author's calculations

Figure 6: Trimmed mean and Ex Food & Energy PCE inflation;
6-month changes



Source: Bureau of Economic Analysis and author's calculations

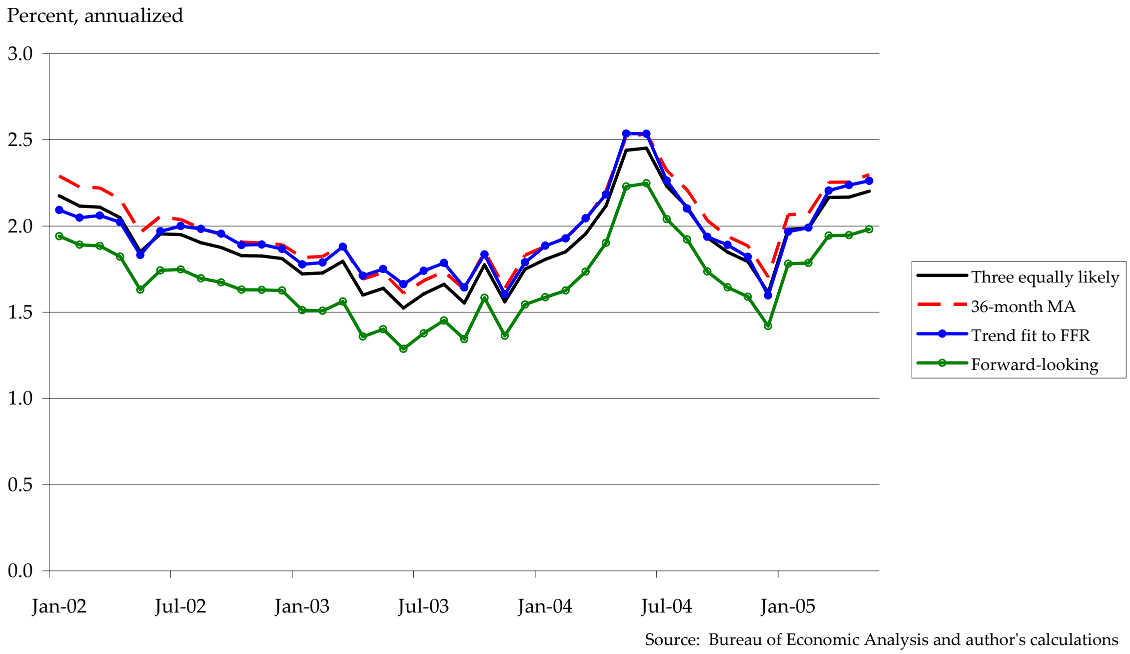
Figure 7: Trimmed mean and Ex Food & Energy PCE inflation;
12-month changes



Source: Bureau of Economic Analysis and author's calculations

The trimmed mean in these figures is the optimal trimmed mean under the assumption that the three core proxies described in Section 4 are all equally likely to be correct. What if one has strong priors about which of the three alternative proxies—the 36-month centered moving average, the band-pass trend fit to the Federal Funds rate or the two-year forward moving average—is correct? Figure 8 plots the recent behavior of the 6-month trimmed mean inflation rate for each of the four trimmings given in Table 5.

Figure 8: Optimally trimmed PCE inflation under alternative proxies for true core inflation; 6-month changes



While the patterns over time in the four series shown are virtually identical, there is difference in levels, particularly for the case of the forward-looking core proxy vis-à-vis the other three. Compared to the benchmark "Three equally likely" case, the optimal trimmed mean inflation rate for the forward-looking core is lower, on average for the period shown, by about 0.21 percentage points. The trimmed mean rate for the 36-month moving average core concept is relatively higher, on average, by about 0.09 percentage points, while that for the trend fit to the Fed Funds rate is higher, on average, by about 0.05 percentage points.[15]

6. Conclusions, caveats and directions for further work

This paper proposes a superior measure of core PCE inflation based on trimmed mean techniques. Trimmed means have previously been applied to the CPI (and comparable price indices in other countries, such as the UK's Retail Price Index), but not, thus far, to the PCE, the preferred inflation gauge of the Federal Reserve Board.

[15] From January 1995 to May 2005, the average discrepancies (relative to the "Three equally likely" case) are: forward-looking, –0.19 percentage points; 36-month moving average, +0.09 percentage points; trend fit to Fed Funds rate, +0.04 percentage points.

As in the work by Bryan, Cecchetti and Wiggins (1997), the statistical case for the application of trimmed means to the PCE is based on the evidence of extreme excess kurtosis in the underlying distributions of price changes that are aggregated to give the PCE inflation rate. I also presented some evidence of systematic skewness in the distributions, with that skewness becoming more pronounced in more recent years. This pattern is apparent only when we move beyond the standard measure of skewness to consider more robust (*i.e.*, outlier-resistant) measures. Thus—and in contrast to Bryan *et al.*—I do not constrain the optimal trim to be symmetric.

The optimal trimming problem, as defined here, chooses the trimming proportions to minimize a measure of distance (the RMSE) between the trimmed mean inflation rate and a proxy for true core inflation. I follow Bryan *et al.* in using a centered 36-month moving average of actual PCE inflation to proxy for true core PCE inflation, but I also consider two alternative proxies. I find that the optimal trims are somewhat sensitive to the choice of core proxy. To deal with this sensitivity, I calculate the optimal trims for the problem of minimizing the expected RMSE, treating each of the three core proxies as equally likely. I use the resulting trimmed mean inflation rate to characterize recent US inflation and find that the story told by the trimmed mean differs somewhat from that told by the more-conventional "excluding food & energy" measure of core inflation.

There are a few caveats to this analysis, which suggest directions for future work.

First, the trimmed mean technique is but one of several recent approaches to the estimation of core inflation. One alternative is to treat core inflation as a type of unobserved common trend or common factor driving the rates of price change of all the individual components underlying a price index. This approach has been explored by Quah and Vahey (1995) and others. Other alternatives—such as weighting component items according to the persistence or variability of their price movements—have also been explored. Wynne (1999) and Mankikar and Paisley (2002) are good surveys of these alternative approaches. Comparing the performance of these alternatives to the trimmed mean is an interesting avenue for further research.

A second caveat pertains specifically to the PCE. Unlike the CPI, the PCE is frequently revised as more comprehensive data become available or as methodological improvements are introduced. These revisions give rise to a discrepancy between past inflation as it looks today (given current vintage data) and as it looked in real time (given the data available at the time). Ideally, the sample used to calculate the optimal trimming proportions would be a real-time data set, since our concern is with how to gauge movements in core inflation from the real-time data we're handed each month. Unfortunately, the real-time data needed for such an exercise has not been compiled.

Finally, it should be noted that the price series for several components of the PCE are based not on market transaction price data, but on imputations of the cost to consumers of non-priced services. For example, many consumers do not pay explicit fees to banks for various sorts of banking services. Rather, consumers pay for the services in the form of lower interest rates earned on deposits. Imputing prices for such services is an inexact science and it is not clear to what extent central banks should pay attention to the behavior of these imputed prices when reckoning the overall pace of inflation. To address concerns about the proper role of imputations in the measurement of inflation, the BEA has recently begun to publish a "market-based PCE" price index that excludes most imputations. While one of the largest imputed components, imputed commercial banking costs, is also one of the most often excluded items from the optimally trimmed mean (see Table 7), I've not considered more generally the extent to which the optimally trimmed mean either excludes or includes non-market-based prices. The construction of a trimmed mean market-based PCE would thus seem to be a natural and fruitful direction for further work.

Appendix A

The following table lists the 186 underlying components whose prices and quantities are used in constructing the monthly distributions of price changes that constitute the basic data for the trimmed mean. To give some sense of the relative importance of the various components, the table also gives each component's expenditure share for the month of December 2004, the last month of the sample.

	Component name	Expenditure share in percent, December 2004[16]
	Durable goods	
1	New domestic autos	0.77
2	New foreign autos	0.51
3	Net purchases of used autos	0.69
4	Trucks, new and net used	2.85
5	Recreational vehicles	0.19
6	Motor vehicle tires & tubes	0.30
7	Motor vehicle accessories & parts	0.40
8	Furniture, mattresses & bedsprings	0.90
9	Major household appliances	0.37
10	Small electric appliances	0.07
11	China, glassware, tableware & utensils	0.41
12	Television receivers	0.20
13	Video equipment & media	0.14
14	Audio equipment	0.34
15	Records, tapes, disks	0.22
16	Musical instruments	0.05
17	Computers and peripherals	0.46
18	Software	0.14
19	Floor coverings	0.23
20	Clocks, lamps & furnishings	0.42
21	Blinds, rods and other durable furnishings	0.07
22	Writing equipment	0.04

[16] The shares given are as a percent of total spending on all items listed in the table, which is slightly less than total PCE, since price data are lacking for three items: net foreign travel, net foreign remittances and household insurance. An entry of 0.00 means the item's share rounded to less than 0.01 percent. Shares may not sum to 100 due to rounding.

23	Tools, hardware and supplies	0.14
24	Outdoor equipment & supplies	0.03
25	Ophthalmic products & orthopedic appliances	0.29
26	Guns	0.03
27	Sporting equipment	0.36
28	Photographic equipment	0.05
29	Bicycles	0.05
30	Motorcycles	0.17
31	Pleasure boats	0.22
32	Pleasure aircraft	0.02
33	Jewelry & watches	0.69
34	Books & maps	0.49
	Non-durable goods	
35	Cereals	0.39
36	Bakery products	0.66
37	Beef & veal	0.36
38	Pork	0.32
39	Other meat	0.25
40	Poultry	0.46
41	Fish & seafood	0.17
42	Eggs	0.08
43	Fresh milk & cream	0.19
44	Processed dairy products	0.45
45	Fresh fruit	0.27
46	Fresh vegetables	0.40
47	Processed fruit & vegetables	0.28
48	Juices & non-alocoholic beverages	0.72
49	Coffee, tea & beverage materials	0.17
50	Fats & oils	0.14
51	Sugar & sweets	0.47
52	Other foods	1.28
53	Pet food	0.34
54	Beer & ale, at home	0.66
55	Wine & brandy, at home	0.21
56	Distilled spirits, oat home	0.18
57	Elementary & secondary school lunch	0.08
58	Higher education school lunch	0.11
59	Other purchased meals	4.74
60	Alcohol in purchased meals	0.58
61	Food furnished to employees: civilian	0.12
62	Food furnished to employees: military	0.01
63	Food produced & consumed on farms	0.01
64	Shoes	0.65
65	Clothing for females	1.79
66	Clothing for infants	0.14
67	Sewing goods for females	0.08
68	Luggage for females	0.04

69	Clothing for males	1.21
70	Sewing goods for males	0.01
71	Luggage for males	0.02
72	Standard clothing issued to military personnel	0.00
73	Gasoline & other motor fuel	2.69
74	Lubricants	0.04
75	Fuel oil	0.15
76	Liquefied petroleum gas & other fuel	0.12
77	Farm fuel	0.00
78	Tobacco products	1.06
79	Soap	0.06
80	Cosmetics & perfumes	0.19
81	Other personal hygiene goods	0.40
82	Semi-durable house furnishings	0.48
83	Cleaning preparations	0.44
84	Lighting supplies	0.12
85	Paper products	0.34
86	Prescription drugs	2.62
87	Non-prescription drugs	0.37
88	Medical supplies	0.05
89	Gynecological products	0.04
90	Toys, dolls & games	0.56
91	Sport supplies, including ammunition	0.17
92	Film & photo supplies	0.04
93	Stationery & school supplies	0.09
94	Greeting cards	0.11
95	Magazines & sheet music	0.24
96	Newspapers	0.23
97	Flowers, seeds & potted plants	0.24
	Services	
98	Owner-occupied mobile homes	0.48
99	Owner-occupied stationary homes	10.41
100	Tenant-occupied mobile homes	0.08
101	Tenant-occupied stationary homes	3.01
102	Tenant landlord durables	0.07
103	Rental value of farm dwellings	0.14
104	Hotels & motels	0.54
105	Clubs & fraternity housing	0.01
106	Higher education housing	0.15
107	Elementary & secondary school housing	0.00
108	Tenant group room & board	0.01
109	Tenant group employee lodging	0.00
110	Electricity	1.48
111	Gas	0.75
112	Water & sewerage maintenance	0.59
113	Refuse collection	0.16
114	Local & cellular telephone	1.24

115	Intrastate toll calls	0.12
116	Interstate toll calls	0.20
117	Domestic service, cash	0.25
118	Domestic service, in-kind	0.02
119	Moving & storage	0.16
120	Rug & furniture cleaning	0.03
121	Electrical repair	0.04
122	Re-upholstery & furniture repair	0.03
123	Postage	0.14
124	Household operation, n.e.c.	0.22
125	Motor vehicle repair	1.66
126	Motor vehicle rental, leasing & other	0.63
127	Bridge, tunnel, ferry & toll roads	0.09
128	Auto insurance	0.61
129	Mass transit systems	0.11
130	Taxicabs	0.05
131	Intercity railway	0.01
132	Intercity buses	0.02
133	Airlines	0.36
134	Other transportation services	0.08
135	Physicians	3.89
136	Dentists	0.93
137	Other professional medical services	2.70
138	Non-profit hospitals	4.35
139	For-profit hospitals	0.89
140	Government hospitals	1.54
141	Nursing homes	1.23
142	Health insurance	1.36
143	Motion picture theaters	0.11
144	Legitimate theater, opera, etc.	0.16
145	Spectator sports	0.17
146	Radio & television repair	0.05
147	Clubs & fraternal organizations	0.28
148	Sightseeing	0.07
149	Private flying	0.01
150	Bowling & billiards	0.04
151	Casino gambling	0.77
152	Other commercial participant amusements	0.24
153	Pari-mutuel net receipts	0.06
154	Other recreation services	2.04
155	Shoe repair	0.01
156	Dry-cleaning	0.08
157	Laundry & garment repair	0.07
158	Beauty shops, health clubs	0.52
159	Barber shops	0.03
160	Watch, clock & jewelry repair	0.02
161	Miscellaneous personal services	0.43

162	Brokerage charges & investment counseling	1.07
163	Bank service charges, trust services & safe deposit box rental	1.11
164	Commercial bank imputed services	1.22
165	Other financial institutions imputed services	1.24
166	Expense of handling life insurance & pension plans	1.13
167	Legal services	1.03
168	Funeral & burial expenses	0.21
169	Labor union expenses	0.14
170	Professional association expenses	0.07
171	Employment agency fees	0.03
172	Money orders	0.03
173	Classified ads	0.01
174	Tax return preparation services	0.11
175	Personal business services, n.e.c.	0.06
176	Private higher education	0.67
177	Public higher education	0.81
178	Elementary & secondary schools	0.36
179	Nursery schools	0.14
180	Commercial & vocational schools	0.46
181	Foundations & non-profit research	0.19
182	Political organizations	0.06
183	Museums & libraries	0.12
184	Foundations to religion & welfare	0.15
185	Social welfare, including child care	1.70
186	Religion	0.69

Appendix B

This appendix provides explicit formulae for the various robust kurtosis and skewness measures used in Section 3. These measures are discussed in more detail in a note by Kim and White (2004), who also perform Monte Carlo simulations illustrating the sensitivity to outliers of the standard kurtosis and skewness measures and the robustness of the alternative measures.

Measures of kurtosis

To describe the three robust kurtosis measures with an economy of notation, fix a distribution of price changes $\{(w_i, \pi_i): i = 1,2,\ldots N\}$ and assume $\pi_1 \le \pi_2 \le \cdots \le \pi_N$. For $\alpha \in [0,1]$ let $\hat{i}(\alpha) = \min\left\{I : \sum_{i=1}^{I} w_i \ge \alpha\right\}$ and $q(\alpha) = \pi_{\hat{i}(\alpha)}$. In words, $q(\alpha)$ is the $100\alpha\%$ quantile of the distribution.

Then, Moors' kurtosis measure is defined as

$$K^2 = \frac{q(7/8) - q(5/8) + q(3/8) - q(1/8)}{q(3/4) - q(1/4)}.$$

Hogg's measure is defined as

$$K^3 = \frac{U(.95) - L(.05)}{U(.50) - L(.50)},$$

where

$$U(\beta) = \frac{1}{\beta} \sum_{\alpha \geq \beta} q(\alpha)$$

and

$$L(\beta) = \frac{1}{\beta} \sum_{\alpha \leq \beta} q(\alpha).$$

Lastly, there is Crow and Siddiqui's measure:

$$K^4 = \frac{q(.975) - q(.025)}{q(.75) - q(.25)}.$$

Measures of skewness

The skewness measure due to Bowley, described verbally in section 3, is defined formally by

$$S^2 = \frac{q(3/4) + q(1/4) - 2q(1/2)}{q(3/4) - q(1/4)}.$$

The measure of Groenveld and Meeden is given by

$$S^3 = \frac{\pi - q(1/2)}{\sum_{i=1}^{N} w_i \left|\pi_i - q(1/2)\right|}$$

where π denotes the mean of the distribution—*i.e.*, $\pi = \sum_{i=1}^{N} w_i \pi_i$. Finally, Pearson's measure is given by

$$S^4 = \frac{\pi - q(1/2)}{\sigma}$$

where $\sigma = \sqrt{\sum_{i=1}^{N} w_i (\pi_i - \pi)^2}$.

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