A General Framework for Design of Experiments: Principles, Techniques, & Applications

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Ernest Seglie
pentagon022009.ppt

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Outline

1. DEX Applications
2. Problem Solving Framework
3. DEX Definitions
4. DEX Principles & Techniques
5. Stat Analysis Principles & Techniques
6. Conclusions

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1. Experiment Design Applications

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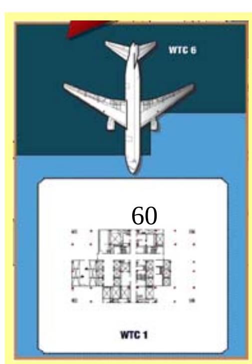
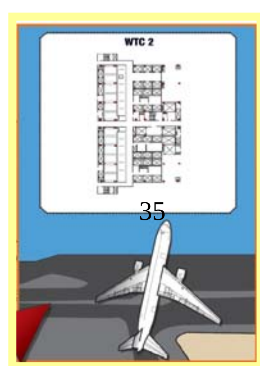
Example 1: WTC Impact Core Damage Assessment

Q. After the plane impact of the WTC South Tower, there was no recorded data as to how many of the interior 47 columns of the building were damaged. A finite-element analysis (FEA) program was written to simulate the impact. The plane was modeled by 1.4 million elements. <u>What factors most affected the performance of this FEA code</u>? What factors could be eliminated as unimportant?





FEMA Report, pp. 2-17, 2-29

Q. How many internal columns were destroyed?
Q. What factors affected the FEA code?

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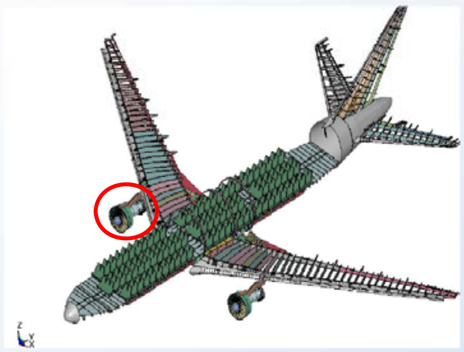
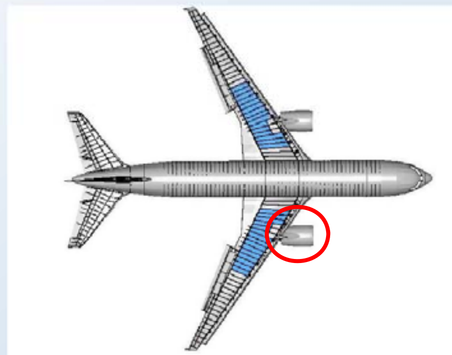
High Fidelity Aircraft Models

Construction: 1.4 Million Elements for Entire Plane
(Labor Intensive)





Applied Res. Assoc.

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Sensitivity Analysis Experiment Design: List of Factors (Component = Engine)

Factors Under Study (k):

1. Flight Speed
2. Flight Impact Location (Vertical)
3. Flight Impact Location (Horizontal)
4. Engine Assignment Set
5. Engine Strength
6. Engine Failure Strain
7. Engine Strain Rate Effects
8. Perimeter Column Strength
9. Perimeter Column Failure Strain
10. Perimeter Column Strain Rate Effects
11. FEA Model Erosion Parameter
12. FEA Contact Parameter
13. FEA Friction Coefficient

Affordable Number of Runs: $n < 50$

DEX = g(k,n)

($k = 13$, $n < 50$)

(Design and data based on research carried out by contractor: Applied Research Associates)

Y = # Core Columns Damaged

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Experiment Design: ?

k = 13, n < 50

Factors →

Runs ↓

	X1	X2	X3	X4	X5	X6	X7	X8	X9	X10	X11	X12	X13	Y
1														
2														
3														
4														
5														
6														
7														
8														
9														
10														
11														
.														
.														
.														
(50)														

?

Experiment Design: 1-FAT

k = 13, n = 1+13 = 14

	X1	X2	X3	X4	X5	X6	X7	X8	X9	X10	X11	X12	X13
Baseline → 1	-	-	-	-	-	-	-	-	-	-	-	-	-
2	+	-	-	-	-	-	-	-	-	-	-	-	-
3	-	+	-	-	-	-	-	-	-	-	-	-	-
4	-	-	+	-	-	-	-	-	-	-	-	-	-
5	-	-	-	+	-	-	-	-	-	-	-	-	-
6	-	-	-	-	+	-	-	-	-	-	-	-	-
7	-	-	-	-	-	+	-	-	-	-	-	-	-
8	-	-	-	-	-	-	+	-	-	-	-	-	-
9	-	-	-	-	-	-	-	+	-	-	-	-	-
10	-	-	-	-	-	-	-	-	+	-	-	-	-
11	-	-	-	-	-	-	-	-	-	+	-	-	-
12	-	-	-	-	-	-	-	-	-	-	+	-	-
13	-	-	-	-	-	-	-	-	-	-	-	+	-
14	-	-	-	-	-	-	-	-	-	-	-	-	+

Example 2: Bullet Casing Forensics

Q. If a casing is collected at a crime scene, is it possible (by comparing the markings on the casing to national image data bases of such casing markings) to identify the type of gun that was used in the crime? Is it possible to identify the individual gun that was used?

Q1. Is a national casing image database feasible?
Q2. Is a casing traceable to an individual gun?

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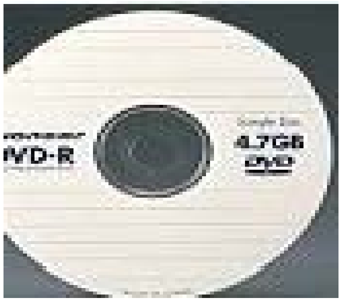
Example 3: Lifetime of CDs

Q. Compact Disks (CDs) do not have an infinite lifetime. The information on all CDs will eventually degrade. How did NIST carry out an accelerated testing program (with accelerants temperature and humidity) to most accurately estimate/predict the failure time at ambient conditions for commercial CDs?



Q. How estimate the lifetime of a CD?

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Example 4: Scatterfield Optical Microscopy

Q. As computer chips get smaller and smaller, their line widths also proportionately decrease. It is important to be able to measure such line widths accurately—it directly affects chip performance. Scatterfield optical spectroscopy is a convenient and relatively inexpensive method for doing such. How can NIST determine the critical parameters that affect the quality of output from this method, and how can best settings be determined for this method so as to make this an optimal metrology tool? <With thanks to Rick Silver, MEL>

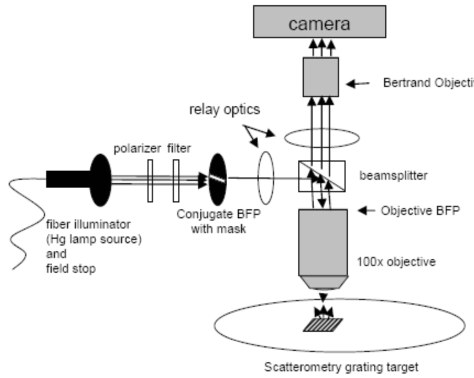

FIG. 1. Schematic of the microscope.

Q. What are optimal settings for Scatterfield Microscope?

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Example 5
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Physics
Plutonium Troubleshooting (SURF)
Am 241/243 Peal Deconv. Alg. Acc.
Cesium 137 Detection
Efficiency of Gamma Ray Emitters
Remote Radiation Detection (SURF)
Sonoluminescent Light Intens.(SURF)
ASP (Adv. Spectrosc. Portal) Monitor.
PRD (Personal Radiation Detectors)
Maritime Radiation Detectors
Soil Leeching Seq. Extraction Prot.

Material Science
MALDI TOF Spectrometry
Nanocantilever Atomic Force Mic.
Dental Polysac Adhesion
Bio Knee Cartilage Regeneration
Ceramic Machining Strength
Comb. Chemistry Tape Peel

Chemistry
Carbon Nanotube Water Pollution
SRM 2396: DNA Base Biomarkers
Gate Dialectrics: SiO2 HRTEM Error
Microarray Sensors for Toxic Gas
DHS: Bio-Agent Detection
Radiocarbon C14 Albuq. CO Pollut.
(Cu-AU) 3D Nanoscale Chem. Imaging
Dual Rotor Turbin Fluid Flow
SO2 Permeation Tube Mass Loss
KC (Key Comparison) Fluid Flow

Elect. & Elect. Eng.
OLES: Bullet Proof Vest Reliability
Eddy Current Probe
IACP/OLES: Safety/Speed Devices Acceptance Samp.
DAC (Digital-to-Analog Converter) Calibration
OLES: Firefighter Infrared Imaging Devices
OLES: Metal Detector Acceptance Sampling

Manufacturing Eng.
Scatterfield Microscopy
Genetic Alg. for Machine Tooling
SMS: Smart Machining System
NIJ/OLES: Forensic Imaging of Gun Casings

Information Tech.
Abilene Network Loss Rate
Motion Imagery Quality Metrics
RAVE Visualization Facility Calibration
Accelerated Testing of Compact Discs
Apache/Linux Web Processing Time
FEA NanoCantilever Sensitivity

Build.& Fire Res.
World Trade Center FEA Core Damage
Cigarette Ignition Propensity
FHWA Highway Concrete Strength (COST)
Tall Building Deflection Safety Codes
HHS CONTAM Home Pollution Dissemination
Solar Sphere Testing of Polymeric Sealants
Optimization of Hot Plate Gap Parameters
Interlab: Thermal Hot Plate Conductivity
Tomographic Flow Detection in Polymer-Bonded Concrete
HUD Lead Paint Test Kit Accuracy
HUD Lead Paint Extraction
Hospital Energy Consumption
Evaluating Strategies for Fire Safety
Paint Peel Strength
Aerosol Spray Flow Rates
Asphalt Roofing Vertical Peel Testing
Remote Detection of Pre-Mold Moisture in Building Mats.
WTC FDS (Fire Dynamics Simulator) Sensitivity
WTC FDS Validation
WTC Impact Sensitivity
WTC FEA Insulation-on-Steel Thermal Propagation
WTC Structural Sensitivity

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Example 6: Bobcat PRD Testing

Q. Personal Radiation Detectors (PRDs) are devices that can be worn by law enforcement and other public safety personnel to alert them to the presence of radioactive material and are fast becoming standard equipment. The primary issue associated with the use of PRDs by law enforcement and public safety personnel is the performance of a PRD in detecting radioactive sources in certain operationally relevant environments. <u>Given a group of PRD models, evaluate their performance over a range of conditions and uses.</u>

Q. Are all PRD models equivalent?

Example 7: International General Aviation Radiation Detection

Q. Handheld and portable radiation detection systems are used by Customs and Border Patrol (CBP) officers to scan general aviation aircraft entering the country from abroad. <u>Compare the performance of the currently depolyed equipment to multiple alternatives.</u>

Q. Are the considered portable radiation systems equivalent?

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2. Problem Solving Framework

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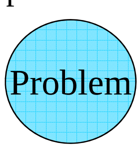
1. What Problem are We Solving?

1. Every problem ➔ a question
2. If we cannot formulate our problem in the form of 1 or 2 discrete questions, then we do not yet have the specificity and/or the consensus to construct a focused experiment design to solve the problem
3. A problem without a question is not a problem--it is a "problem area"
4. Experiment designs attack specific problems--not general problem areas



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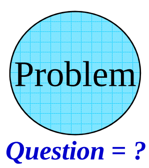

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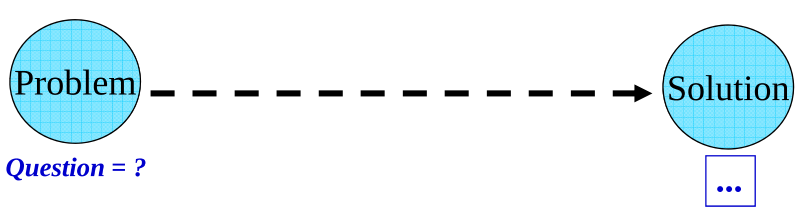
2. What Constitutes a Solution?

1. Much discussion and specificity is needed to define precisely what constitutes a solution to the problem at hand
2. This solution/deliverable is much easier to define if a specific question has been crafted that encapsulates the problem being attacked.



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2. What Constitutes a Solution? (cont.)

3. Examples of solutions/deliverables:

yes/no

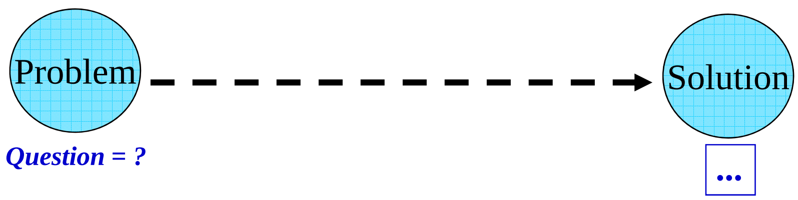
+-

(ranked) list of factors (and interactions?)

fitted function f

k numbers: best settings (x1,x2,x3,...,xk)

go/no-go



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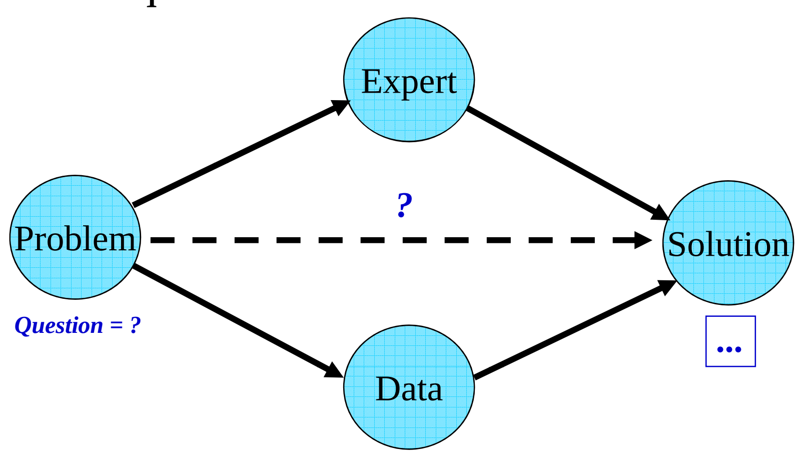
3. How Do We Get to the Solution?

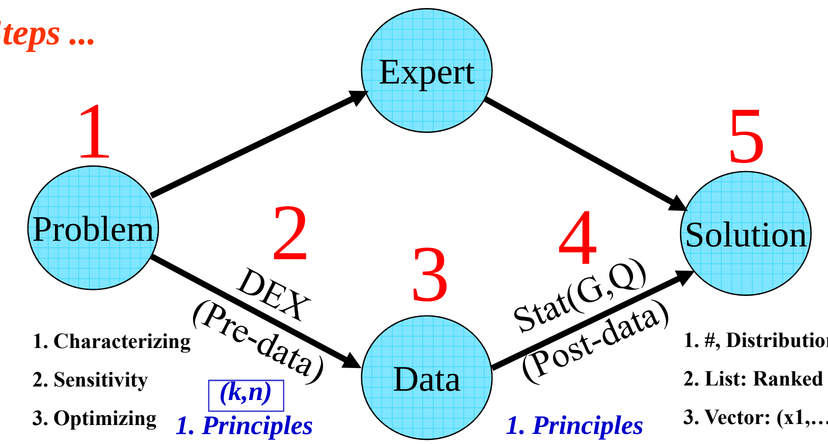
Given a question …



General Problem-Solving Framework/Structure

5 Steps ...

*****



1. Characterizing
2. Sensitivity
3. Optimizing
4. Modeling
5. Comparing
6. Predicting
7. Uncertainty
8. Verifying
9. Validating

(k,n)

1. Principles
2. Techniques

1. Classification
2. Translation
3. Construction
4. Execution
5. Recording

1. Principles
2. Techniques

1. Estimation
2. Testing

1. Graphical
2. Quantitative

1. #, Distribution
2. List: Ranked Factors
3. Vector: (x1,…,xk)
4. f
5. Y/N
6 #
7. SD(#)
8. Y/N, Vector: (x1, …,xk)
9. Y/N, Vector: (x1, …,xk)

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Virtue of Experimental Design

Assures that the data has the capacity to unambiguously answer the Scientific/ Engineering question at hand.

Assures also that the experiment is as rigorous as is statistically and scientifically possible, and is above reproach by the scientific and legal community.

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3. Experiment Design Definitions

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1. Experiment Design

Experimental design is a systematic, rigorous, ***data-based approach*** *to scientific/engineering* ***problem-solving****.*

The goal of experimental design is to generate valid, crisp, unambiguous, and reproducible conclusions about the scientific/engineering process of interest--and to do so in a ***time- and cost-efficient fashion****.*

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2. Scope

1. The scope of an experiment is a set { ...} of factor conditions over which we claim the results of the experiment are true/valid.

2. If we vary no factors (that is, every factor is fixed at a setting), then our scope will be narrow; if we vary many factors our scope will be broad.

3. The scope is dictated by usage--who will use the conclusions of our experiment, and under what conditions will our conclusions be utilized.

4. If a primary factor exists, there will be a tradeoff between primary factor discrimination and scope of conclusions.

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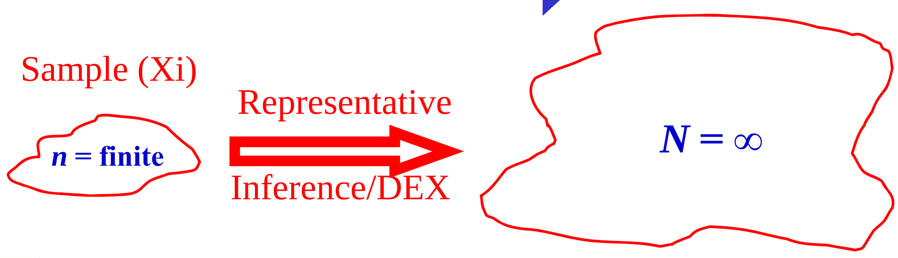
2. Scope (continued)

5. Step 1 is do decide/declare what factors Xi should be included so as to achieve the desired scope--this requires brainstorming and initially yields a superset.
6. Step 2 is to loop through each of these factors Xi and decide what the population is over which we want our conclusions to be valid.

Thus for each factor Xi ... ⇒ Population (Xi)



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3. Robustness

1. The scientific desire to have “general conclusions”-- conclusions in which we do not have to attach qualifiers--leads to the property termed “robustness”

2. To achieve such robustness, expand the scope of your experiment by collecting data over a wide a range of (additional) factors as “reasonable” & affordable.

3. These additional ***robustness*** factors also must be handled with care – i.e. balance, coverage, etc.

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4. Factors & Levels

- ***Factors***
- ***Levels***
- ***Settings***
- ***Parameters***
- ***Variables***
- ***Treatments***

?

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4. Factors, Levels & Response

- *Factors = Parameters = Variables (X_i)*
- *Levels = Settings = Treatments (x_i)*
- *Response (Y)*

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4. Factors, Levels, Response & Runs

$$Y = f(\underbrace{X1, X2, X3, ..., Xk}_{k\ factors})$$

response → Y

nature's f unknown → f

(k,n): k factors, n runs

(k,l,n): k factors, l levels per factor, n runs

A "run": y = f(x1, x2, x3, ..., xk)

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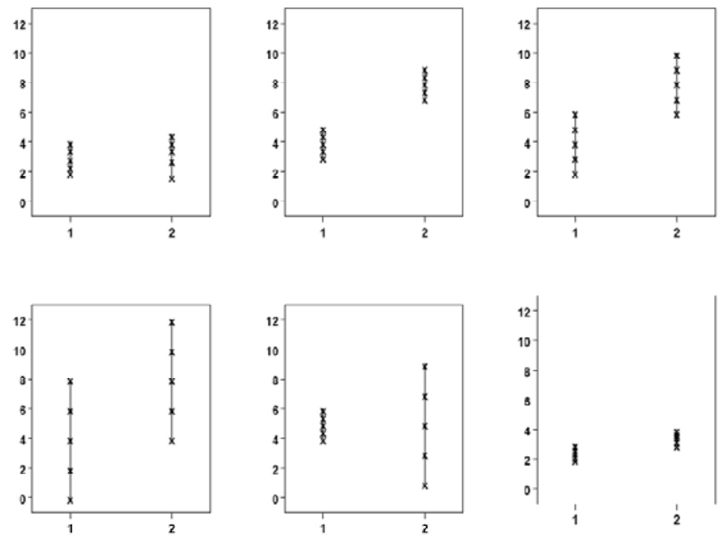
5. Effect

Science: Cause & effect
A factor (ball size) has an effect if ...
A factor (ball size) is significant if ...



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5. Effect (continued)

1. By default, "effect" by definition means shift in *location*
2. Our ability to detect an effect depends on the *intrinsic variability of the data*
3. "Effect" could also mean shift in *variation*
4. We conclude: "a factor has an effect" by computing a minimum statistical significant difference (via statistical hypothesis testing). Of equal importance, is the minimum *engineering* significant difference.

6. Confounding

- Confounding occurs when levels of one factor are directly correlated with levels of another factor

Ball Size X1	Operator X2	Result Y
Small	Ernest	7.24
Small	Ernest	8.15
Small	Ernest	6.98
Small	Ernest	7.40
Large	Gordon	10.23
Large	Gordon	11.05
Large	Gordon	10.78
Large	Gordon	10.11

Ball Size - Operator Confounding:

The observed difference in the results Y cannot be unambiguously attributed to either the ball size or the operator factor

6. Confounding (continued)

- Confounding is a **curse** to any experiment as valid, crisp and unambiguous results cannot be drawn

- Confounding factors may not always be obvious:
  - Environmental factors (Pres, Temp, Humi)
  - Time (radiation sources decay over time)
  - Time (mechanical wear)
  - Time (learning curve)

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6. Confounding (continued)

- Five PRDs, Five operators. Each PRD is operated by a single operator who takes 10 measurements
- Twelve maritime monitors are submitted for a one month test program against a single source.
  - Monitors 1-3 are tested in week 1;
  - Monitors 4-6 are tested in week 2;
  - Monitors 7-9 are tested in week 3;
  - Monitors 10-12 are tested in week 4
- A portal monitor test explores runs at 5 mph and 2 mph. The runs at 5 mph have a source inside a NORM container, The runs at 2 mph have a source behind a NORM container

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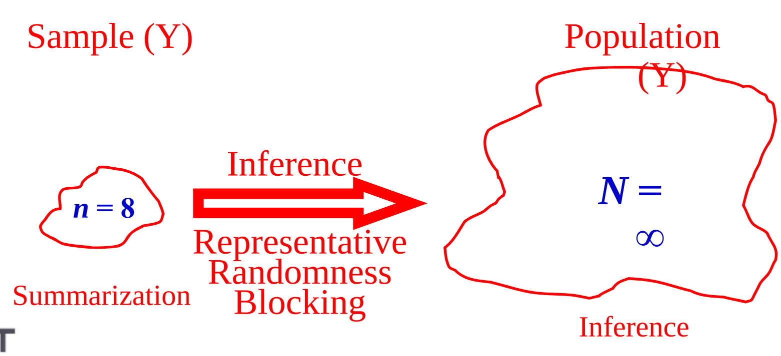
7. Randomization

- A technique for drift protection (insurance) – recall learning effect
- Basis for statistical estimation and inference:
  - Allows one to infer from sample to population
  - Key: Representativeness
  - Fundamental tool: randomization
  - Better tool: blocking (stratification)
  - “Block what you can, randomize what you cannot”



8. Blocking

- A technique applied to robustness factors to assure anti-confounding.
- A robustness factor is a blocking factor if each & every level of the robustness factor has each & every level of the primary factor occurring the same number of times (within-block balance).
- A robustness factor is a blocking factor if each (and every) level of the robustness factor has each (and every) level of the primary factor occurring the same number of times (balance).

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8. Blocking (continued)

(k=3, n=16, l=4)

Design 1

X_2 Tire Brand Distribution	X_3 Car I	II	III	IV
	A	B	C	D
	A	B	C	D
	A	B	C	D
	A	B	C	D

Cell Entry = Additive Type = X_1

Blocking Factor

Design 2

X_2 Tire Brand Distribution	X_3 Car I	II	III	IV
	B	D	A	C
	C	C	B	D
	A	B	D	B
	D	A	C	A

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9. Balance

- A factor is balanced if every level of that factor occurs the same number of times over the n runs.
- Balance is a technique which
  1. minimizes the SD(effect estimates) and
  2. maximizes the Prob{concluding: an effect exists | the reality : an effect exists}
- Balance is intuitively done when have a single factor (k = 1)
- Balance should be done regardless of the number of factors (k >= 1).

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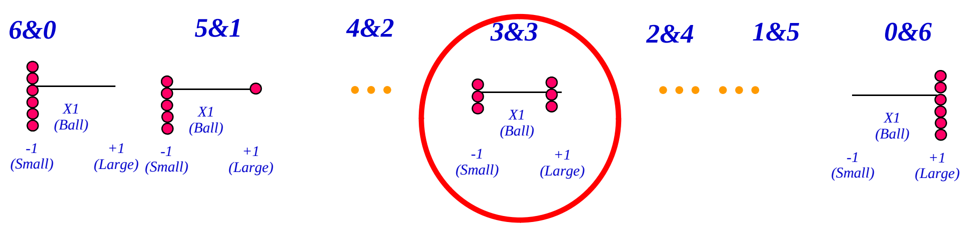
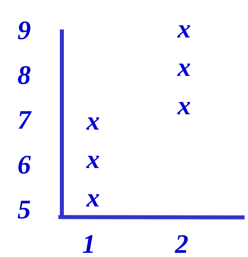
9. Balance (continued)

(k=1, n=6, l=2)



X1	Y
1	5.15
1	6.00
1	6.85
2	7.15
2	8.00
2	8.85



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9. Balance: Min SD(Del)

Del = y2bar - y1bar = 8 - 6 = 2
SD(Del) = ?

SD(Del) = Sqrt(Var(Del))
= Sqrt(Var(y2bar - y1bar))
= Sqrt(Var(y2bar) + Var(y1bar))
*= Sqrt((sigma**2 / n2) + (sigma**2 / n1))*
*= sigma * sqrt(1/n2 + 1/n1)*
*= sigma * sqrt(1/n1 + 1/n2)*

(k=1,n=6)

n1	n2 = 6 - n1	sqrt(1/n1 + 1/n2))
6	0	infinite
5	1	1.20
4	2	0.75
3	3	0.67

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9. Balance: Optimize the t Test

t = Del / SD(Del)
*= (y2bar - y1bar) / [s * sqrt(1 / n1 + 1 / n2)]*

To determine if a statistically significant difference in location exists, the t-test may be employed:

$$t_{stat} = \frac{\bar{y}_2 - \bar{y}_1}{s\sqrt{1/n_1 + 1/n_2}}$$

t_{stat} is compared to t_{crit} where t_{crit} is theoretical value taken from $t_{n1+n2-2}$ distribution

n1	n2	t_{stat}	$t_{crit}=t_{4,\ 0.975}$	Shift?
3	3	$t_{stat} = \frac{\bar{y}_2 - \bar{y}_1}{s\sqrt{1/n_1 + 1/n_2}} = \frac{8-6}{0.85\sqrt{1/3 + 1/3}} =$ 2.88	2.78	Yes
4	2	$t_{stat} = \frac{\bar{y}_2 - \bar{y}_1}{s\sqrt{1/n_1 + 1/n_2}} = \frac{8-6}{0.85\sqrt{1/4 + 1/2}} =$ 2.72	2.78	No
5	1	$t_{stat} = \frac{\bar{y}_2 - \bar{y}_1}{s\sqrt{1/n_1 + 1/n_2}} = \frac{8-6}{0.85\sqrt{1/5 + 1/1}} =$ 2.15	2.78	No

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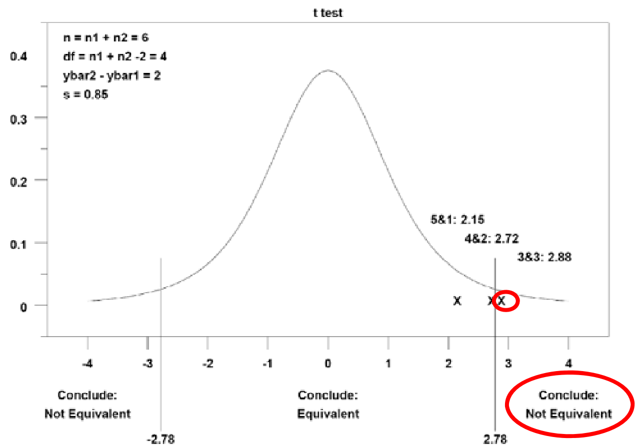
9. Balance: Optimize the t Test



Balance provides optimal statistical discrimination

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10. Orthogonality

- A pair of factors is orthogonal if each of the 2 factors is balanced and if every combination of levels of the 2 factors occurs the ***same*** number of times over the n runs.
- Orthogonality is a DEX technique which
  1. minimizes the SD(effect estimates) and
  2. maximizes the Prob{concluding: an effect exists | the reality : an effect exists}
  3. minimizes the SD(2-term interaction estimates)
  4. maximizes the Prob{concluding: a 2-term interaction effect exists | reality : a 2-term interaction effect exists}

  *5. allows for effective usage of highly efficient fractional factorial designs--especially for sensitivity experiments

  *6. with robustness factors, allows for an optimal “fair sampling” of the robustness factor space--especially for comparative and robustness experiments

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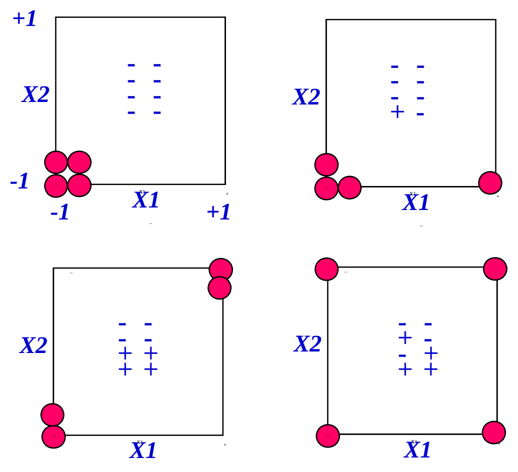
10. Orthogonality (k = 2, n = 4)

(k=2, n=4, l=2)

Design Geometry



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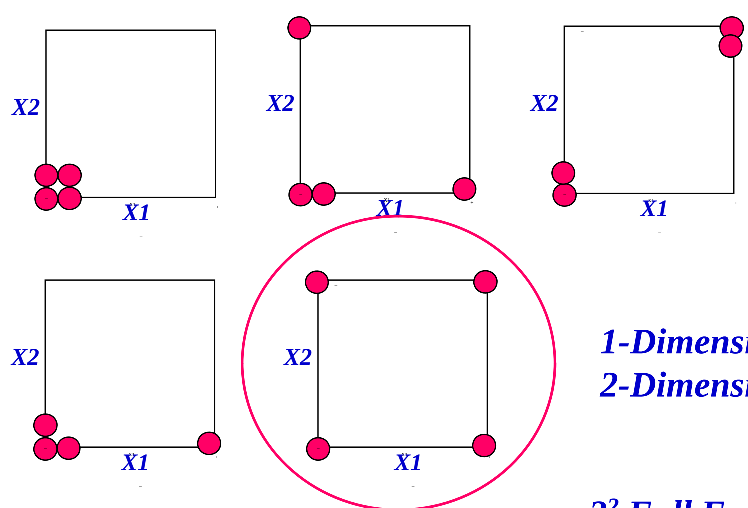
10. Orthogonality (k = 2, n = 4)

(k=2, n=4, l=2)



X1	X2
-1	-1
-1	+1
+1	-1
+1	+1

1-Dimensional Balance
2-Dimensional Balance

2^2 Full Factorial Design

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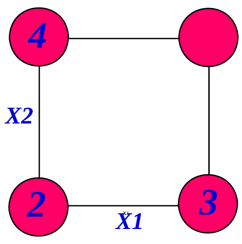
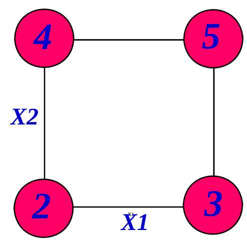
10. Orthogonality (k = 2, n = 3 or 4)

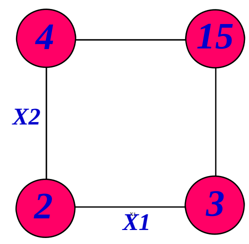
(k=2, n=3)
1FAT



(k=2, n=3)
$\mathbf{2^2}$



(k=2, n=3)
$\mathbf{2^2}$



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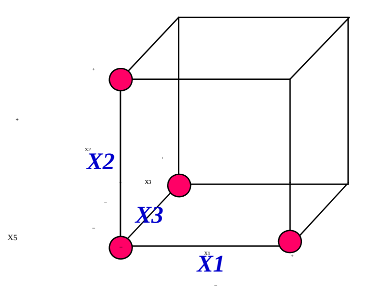
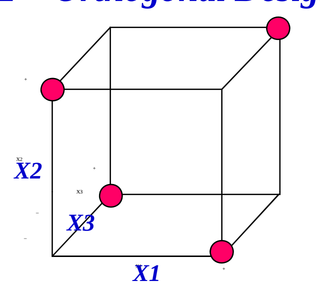
10. Orthogonality (k = 3, n = 4)

(k=3, n=4)
1-FAT Design



X1	X2	X3
-	-	-
+	-	-
-	+	-
-	-	+

(k=3, n=4)
2^{3-1} Orthogonal Design

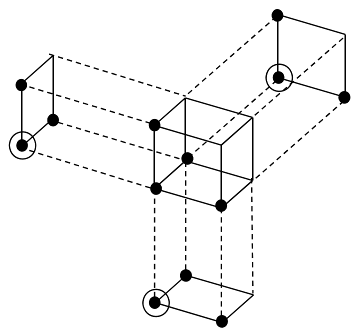


X1	X2	X3
-	-	+
+	-	-
-	+	-
+	+	+

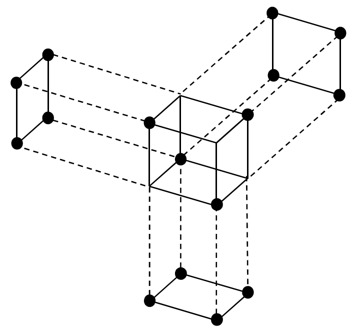
NIST

10. Orthogonality (k = 3, n = 4)

(k=3, n=4)
1-FAT Design



(k=3, n=4)
2^{3-1}Orthogonal Design



For a given number of factors (k = 3)
and a given number or runs (n=4),
not all experiment designs are equally good

NIST

10. Orthogonality (k = 5, n = 6 or 8)

1-FAT Design

X1	X2	X3	X4	X5
-	-	-	-	-
+	-	-	-	-
-	+	-	-	-
-	-	+	-	-
-	-	-	+	-
-	-	-	-	+

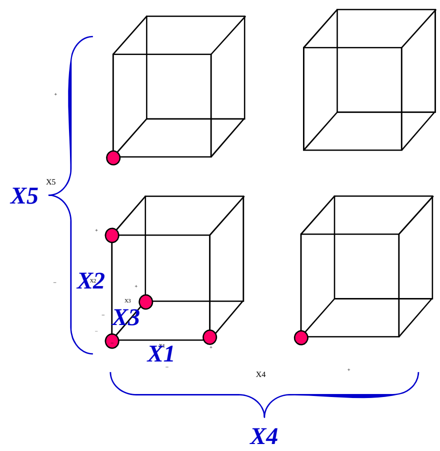
2^{5-2}Orthogonal Design

X1	X2	X3	X4	X5
-	-	-	+	+
+	-	-	-	-
-	+	-	-	+
+	+	-	+	-
-	-	+	+	-
+	-	+	-	+
-	+	+	-	-
+	+	+	+	+

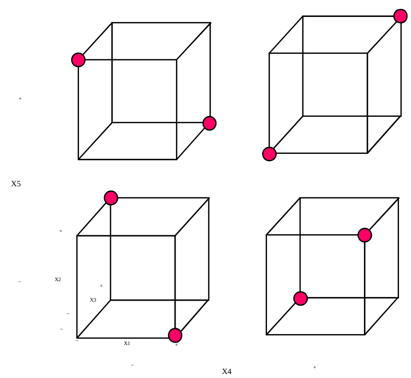
NIST

10. Orthogonality (k = 5, n = 6 or 8)

1-FAT Design



2^{5-2} Orthogonal Design



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10. Orthogonality (k = 7, n = 8)

1-FAT Designs

X1	X2	X3	X4	X5	X6	X7
-	-	-	-	-	-	-
+	-	-	-	-	-	-
-	+	-	-	-	-	-
-	-	+	-	-	-	-
-	-	-	+	-	-	-
-	-	-	-	+	-	-
-	-	-	-	-	+	-
-	-	-	-	-	-	+

2^{7-4} Orthogonal Design

X1	X2	X3	X4	X5	X6	X7
-	-	-	+	+	+	-
+	-	-	-	-	+	+
-	+	-	-	+	-	+
+	+	-	+	-	-	-
-	-	+	+	-	-	+
+	-	+	-	+	-	-
-	+	+	-	-	+	-
+	+	+	+	+	+	+

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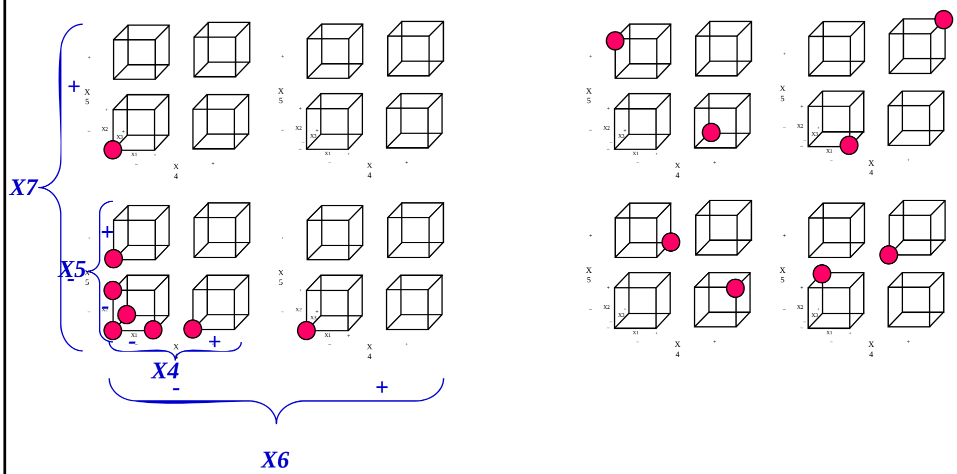
10. Orthogonality (k = 7, n = 8)

1-FAT Design | **2^{7-4} Orthogonal Design**



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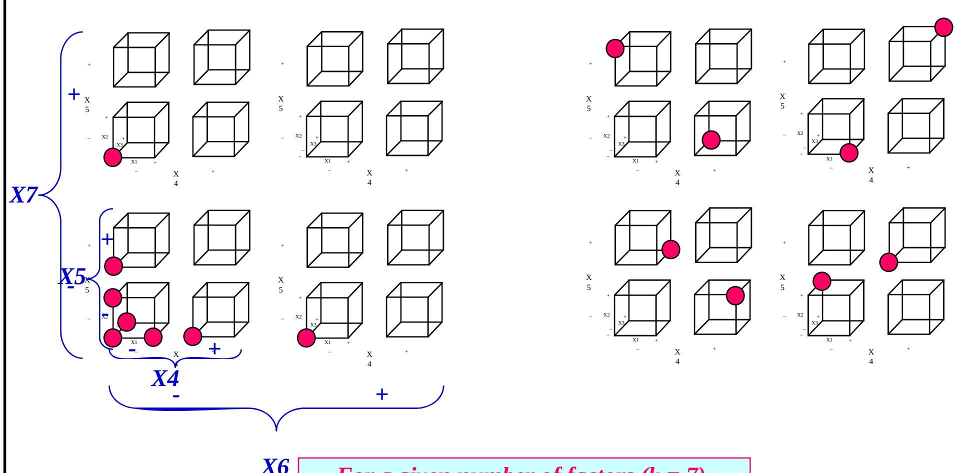
10. Orthogonality (k = 7, n = 8)

1-FAT Design | **2^{7-4} Orthogonal Design**



For a given number of factors (k = 7)
and a given number or runs (n = 8),
not all experiment designs are equally good

10. Orthogonality (k = 13, n = 16)

World Trade Center

Index	X1	X2	X3	X4	X5	X6	X7	X8	X9	X10	X11	X12	X13
1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1
2	+1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1
3	-1	+1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1
4	-1	-1	+1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1
5	-1	-1	-1	+1	-1	-1	-1	-1	-1	-1	-1	-1	-1
6	-1	-1	-1	-1	+1	-1	-1	-1	-1	-1	-1	-1	-1
7	-1	-1	-1	-1	-1	+1	-1	-1	-1	-1	-1	-1	-1
8	-1	-1	-1	-1	-1	-1	+1	-1	-1	-1	-1	-1	-1
9	-1	-1	-1	-1	-1	-1	-1	+1	-1	-1	-1	-1	-1
10	-1	-1	-1	-1	-1	-1	-1	-1	+1	-1	-1	-1	-1
11	-1	-1	-1	-1	-1	-1	-1	-1	-1	+1	-1	-1	-1
12	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	+1	-1	-1
13	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	+1	-1
14	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	+1

Index	X1	X2	X3	X4	X5	X6	X7	X8	X9	X10	X11	X12	X13
1	-1	-1	-1	-1	-1	-1	-1	-1	+1	+1	+1	+1	+1
2	+1	-1	-1	-1	+1	-1	+1	+1	-1	-1	-1	-1	+1
3	-1	+1	-1	-1	+1	+1	-1	+1	-1	-1	+1	+1	-1
4	+1	+1	-1	-1	-1	+1	+1	-1	+1	+1	-1	-1	-1
5	-1	-1	+1	-1	+1	+1	+1	-1	-1	+1	-1	+1	-1
6	+1	-1	+1	-1	-1	+1	-1	+1	+1	-1	+1	-1	-1
7	-1	+1	+1	-1	-1	-1	+1	+1	+1	-1	-1	+1	+1
8	+1	+1	+1	-1	+1	-1	-1	-1	-1	+1	+1	-1	+1
9	-1	-1	-1	+1	-1	+1	+1	+1	-1	+1	+1	-1	+1
10	+1	-1	-1	+1	+1	+1	-1	-1	+1	-1	-1	+1	+1
11	-1	+1	-1	+1	+1	-1	+1	-1	+1	-1	+1	-1	-1
12	+1	+1	-1	+1	-1	-1	-1	+1	-1	+1	-1	+1	-1
13	-1	-1	+1	+1	+1	-1	-1	+1	+1	+1	-1	-1	-1
14	+1	-1	+1	+1	-1	-1	+1	-1	-1	-1	+1	+1	-1
15	-1	+1	+1	+1	-1	+1	-1	-1	-1	-1	-1	-1	+1
16	+1	+1	+1	+1	+1	+1	+1	+1	+1	+1	+1	+1	+1

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10. Orthogonality (k = 13, n = 16)

(k = 13, n = 16)

Orthogonal (n = 16)

All 13:

8 8

\- X_i +

All $\binom{13}{2}$: X_j

\+ 4 4

\- 4 4

\- X_i +

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4. Experiment Design Principles & Techniques

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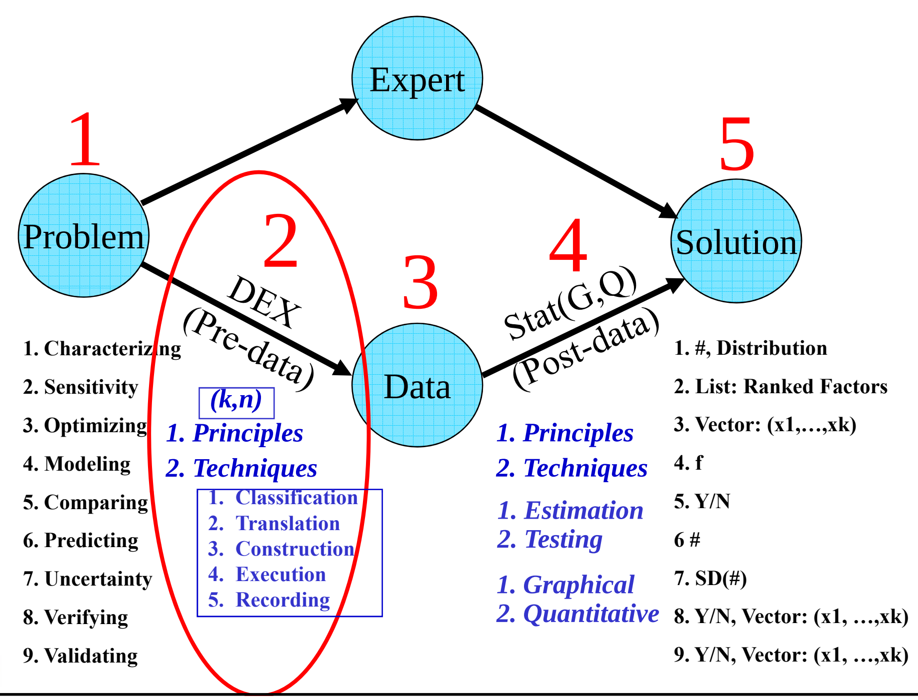
General Problem-Solving Framework/Structure



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(k,n)

Every design has a k and an n.
k = number of factors being varied
n = number or runs

k dictates the scope
n dictates the affordability

necessary: n >= 1 + k
better : n >= 1 + k + C(k,2)

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DEX Principles & Techniques

Principles	Techniques
Construct Efficient Designs	Elicit Dominant Project Goal(s)
Construct Effective Designs	Elicit Project Scope & Constraints
Infer about Population	Randomize
Avoid Biased Factors	Randomize, Block, Balance Orthogonal
Maximize Test Sensitivity	Balance
Estimate All Model Parameters	n(distinct) >= k+1
Allow for Expanded Model	Record Additional (e.g., Ambient) Variables
Estimate Sigma Model-Free	Replicate
Estimate Main Effects & Int.	Full Factorial or High Res. Fractional Factorial Designs
Save $	Pilot Study, Fract. Fact. Designs
Make Conclusions Robust	Design in Many Robustness Factors
Assess Drift	Controls, Replicate across Time
Avoid Bias from Drift	Drift-Reducing Designs
Avoid Confounding	Full Factorial or High Res. Fractional Factorial Designs
Minimize SD(Estimates)	Sample Where Variation Is
Realistically Sample the Process	Design in Vicissitudes (Multiple Sets)
Reduce Effect Uncertainty	Youden Pairs for Homogeneity
Assess Repeatability	Replication
Assess Reproducibility	Multiple Sets (Across Days)

dexprinciplesandtechniques.ppt dexprintech.dp

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Framework Step 2: DEX: 5 Steps Unto Itself

2.1 Classification

2.2 Translation

2.3 Construction

2.4 Execution

2.5 Recording

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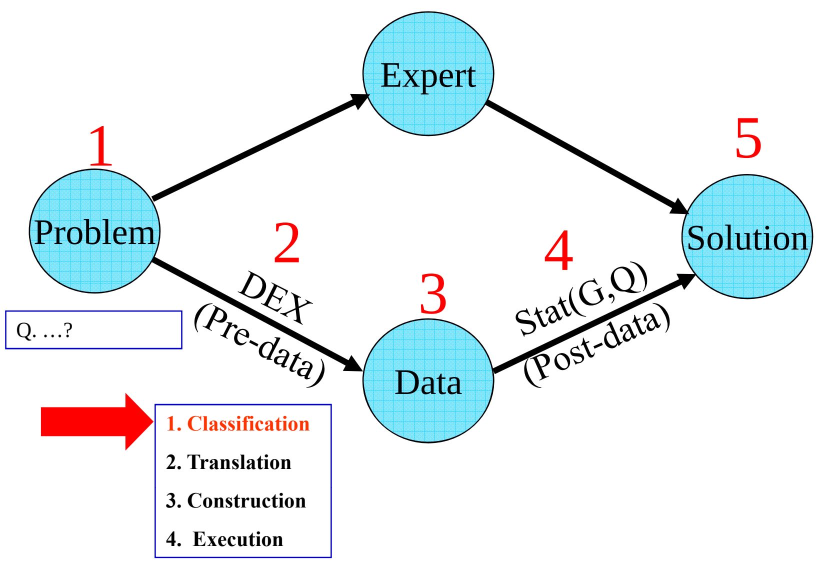
2.1: Problem Classification



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Problem Classification

Is this Factor Significant?

(➔ 1. Comparative Robust Inference)

Most Important Factors?

2. (Screening/Sensitivity)

$$Y = f(X1, X2, X3, \ldots, Xk)$$

Good Approximating Function?

3. (Regression)

Best Settings of the k Factors?

4. (Optimization)

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Problem Classification

Comparative	Screening/Sensitivity
Focus: 1 primary factor *Q1. Does that factor have an effect (Y/N)?* *Q2. If yes, then best setting for that that factor = ? (vector)* *Constraint: Want conclusions to be robust over all other factors* *Designs: CRD, RBD, LSqD,TPD* *BHH, Ch. 4*	*Focus: all factors* *Q1. Most important factors (ranked list)* *Q2. Best settings (vector)* *Q3. Good model (function)* *Designs: 2^kD, 2^{k-p}D,TD* *BHH, Ch. 5-6*
Regression	**Optimization**
Focus: all factors *Q1. Good model (function)* *Continuous factors* *Designs: BBD,XOD* *BHH, Ch. 10-11*	*Focus: all factors* *Q1. Best settings (vector)* *Continuous factors* *Designs: RSD, CD, BBD* *BHH, Ch. 12*

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Problem Classification

Critical: The choice of design is dictated by the problem classification

Comparative/Robust: ***CRD, RBD, LSD, TPD***

Screening/Sensitivity: ***2^kD, 2^{k-p}D, TD***

Regression: ***BBD, XOD***

Optimization: ***RSD, CD, BBD***

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Problem Classification

Comparative

Focus: 1 primary factor
Q1. Does that factor have an effect (Y/N)?
Q2. If yes, then best setting for that factor = ? (vector)
Constraint: Want conclusions to be robust over all other factors
Designs: CRD, RBD, LSqD, TPD
BHH, Ch. 4

Screening/Sensitivity

Focus: all factors
Q1. Most important factors (ranked list)
Q2. Best settings (vector)
Q3. Good model (function)
Designs: 2^kD, 2^{k-p}D, TD
BHH, Ch. 5-6

Regression

Focus: all factors
Q1. Good model (function)
Continuous factors
Designs: BBD, XOD
BHH, Ch. 10-11

Optimization

Focus: all factors
Q1. Best settings (vector)
Continuous factors
Designs: RSD, CD, BBD
BHH, Ch. 12

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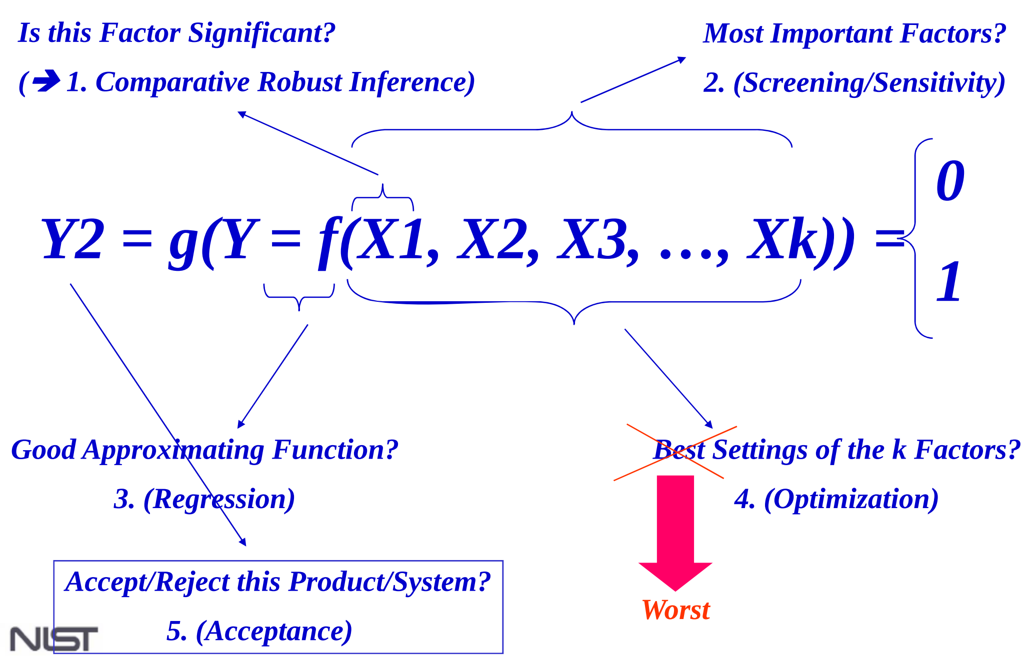
Problem Classification (Revisited)



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Problem Classification (Revisited)

*

Comparative

Focus: 1 primary factor

Q1. Does that factor have an effect (Y/N)?

Q2. If yes, then best setting for that that factor = ? (vector)

Constraint: Want conclusions to be robust over all other factors

Designs: CRD, RBD, LSqD,TPD

Screening/Sensitivity

Focus: all factors

Q1. Most important factors (ranked list)

Q2. Best settings (vector)

Q3. Good model (function)

Designs: 2^kD, 2^{k-p}D, TD

Regression

Focus: all factors

Q1. Good model (function)

Continuous factors

Designs: BBD,XOD

Optimization

Focus: all factors

Q1. Best settings (vector)

Continuous factors

Designs: RSD, CD, BBD

Acceptance

Focus: all population points => all t-tuples of settings

Q1. Accept the product/system as safe?

Q2. Points ➔ failure

Q3. t-tuples of settings ➔ failure

Q4. Factors affecting safety?

Designs: 2^{k-p}D,CD

Many real-world problems should be done in 2 stages:

1. *exploratory (= sensitivity analysis)*
2. *ultimate objective*

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Problem Classification

Critical: The choice of design is dictated by the problem classification

Comparative/Robust: ***CRD, RBD, LSD, TPD***

Screening/Sensitivity: ***2^kD, 2^{k-p}D, TD***

Regression: ***BBD, XOD***

Optimization: ***RSD, CD, BBD***

Acceptance: ***2^kD, 2^{k-p}D, CD***

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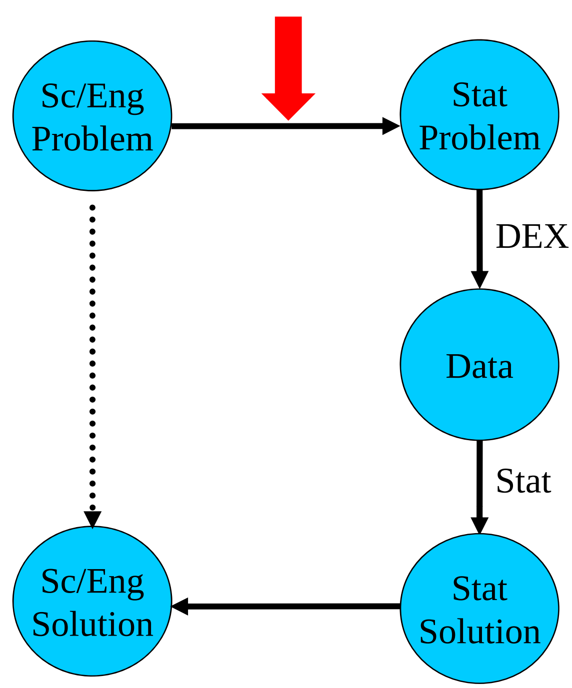
2.2 Translation: Sc/Eng ➔ Stat



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2.2 Translation: Minimal Info for an Experiment

Specificity is the key ...

For one's own project/problem:

1. Title = ______________________________

____

____

2. Problem/Question = ______________________________

____

____

3. Number k of Factors to Vary = ______

4. Sample Size n = ______

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2.2 Translation: DEX Worksheet

Date:

Experiment Design Worksheet:

1. Project/Problem Title:

2. Researcher:

3. Project Background & Importance:

4. General Project Question:

5. Specific Project Question (This Experiment Only):

6. (Generic) Stat Goal(s):

7. Scope of Conclusions:

DEX Essentials: **Generic Stat Model: $Y = f(X_1,X_2,...,X_k) + e$**

8. Response Variable Y :
9. Current Typical Value for Y :
10. Project Target Value for Y :
11. Project Min. Eng. Significant D for Y :
12. Project Min. Eng. Residual SD for :

13. Run Time & Cost per Observation :
14. Total Available Experiment Time & Budget:
15. Constraint: Max Affordable Number of Runs — n <=
16. Number of Factors to Vary/Investigate — k =

17. Factors & Factor Levels:

Factor	C/D	Range	#Levels	Levels
X_1				
X_2				
X_3				
X_4				
X_5				
X_6				
X_7				

18. General DEX Category (Pred&Unc, Comp, Scr/Sens., Regr., Optim., Robust/V&V):
19. Specific DEX:

dexworksheet.doc

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2.3 Construction

1. Construction is (relatively) easy
2. Having multiple designs "on the table" is useful
3. 2-level designs (especially 2-level fractional factorial designs are very powerful for doing sensitivity problems.
4. Many tabulated designs (e.g, Box, Hunter & Hunter, p. 410/272)

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2.3 Construction (How to start)

Start (conceptually) with the full factorial design with all levels of all k factors

If affordable (n), then design may be done.

If not affordable (n), then ...

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2.3 Construction (4 ways to reduce n)

*

1. Reduce the number of factors (k)
2. Reduce the number of levels (l)
3. Reduce the number of replications (r)
4. (Orthogonal fractional factorial designs)

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2.3 Construction (orthogonal fractional factorial designs)

Benchmark Problem

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Benchmark "FEA" Problem for Comparing/Evaluating Designs

5 Factor Model ("Truth")

$Y = f(X_1, X_2, X_3, X_4, X_5) = 65.5 +$ C

$0.5 \{ -1.375 X_1 + 19.5 X_2 - 0.625 X_3 + 10.75 X_4 - 6.25 X_5 +$ ME

$1.375 X_1X_2 + 0.75 X_1X_3 - 0.875 X_1X_4 + 0.125 X_1X_5 +$

$0.875 X_2X_3 + 13.25 X_2X_4 + 2 X_2X_5 +$ 2TI

$1.125 X_3X_4 + 0.975 X_3X_5 - 11 X_4X_5 +$

$1.5 X_1X_2X_3 + 1.375 X_1X_2X_4 - 1.875 X_1X_2X_5 -$

$0.75 X_1X_3X_4 - 2.5 X_1X_3X_5 + 0.625 X_1X_4X_5 +$

$1.125 X_2X_3X_4 + 0.125 X_2X_3X_5 - 0.245 X2X_4X_5 +$ 3TI

$0.125 X_3X_4X_5 +$

$0.0 X_1X_2X_3X_4 + 1.5 X_1X_2X_3X_5 + 0.625 X_1X_3X_4X_5 +$

$1 X_1X_3X_4X_5 - 0.625 X_2X_3X_4X_5 -$ 4TI

$0.5 X_1X_2X_3X_4X_5 \}$ 5TI

Box, Hunter, & Hunter, p. 377

Q. Most important factors = ?

Benchmark "FEA" Problem for Comparing/Evaluating Designs(4)

(k = 5, n= 32/16/8/6)

Experiment Design

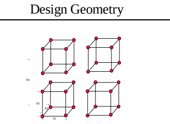
Problem: Determine Most Important Factors in a k = 5 Factor Experiment

Design Name: 2^5 *Full Factorial Design*, *n = 32*

Design Tableau:

X1	X2	X3	X4	X5	Y	X1	X2	X3	X4	X5	Y
-	-	-	-	-	61	-	-	-	-	+	56
+	-	-	-	-	53	+	-	-	-	+	63
-	+	-	-	-	63	-	+	-	-	+	70
+	+	-	-	-	61	+	+	-	-	+	65
-	-	+	-	-	53	-	-	+	-	+	59
+	-	+	-	-	56	+	-	+	-	+	55
-	+	+	-	-	54	-	+	+	-	+	67
+	+	+	-	-	61	+	+	+	-	+	65
-	-	-	+	-	69	-	-	-	+	+	44
+	-	-	+	-	61	+	-	-	+	+	45
-	+	-	+	-	94	-	+	-	+	+	78
+	+	-	+	-	93	+	+	-	+	+	77
-	-	+	+	-	66	-	-	+	+	+	49
+	-	+	+	-	60	+	-	+	+	+	42
-	+	+	+	-	95	-	+	+	+	+	81
+	+	+	+	-	98	+	+	+	+	+	82

Design Geometry:



Effect Estimators:

$\hat{\beta}_2 = 19.50$
$\hat{\beta}_{24} = 13.25$
$\hat{\beta}_{45} = -11.00$
$\hat{\beta}_4 = 10.75$
$\hat{\beta}_5 = -6.25$
$\hat{\beta}_1 = -1.38$
$\hat{\beta}_3 = -0.63$

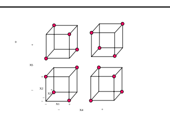
$\hat{\beta}_2 = 19.5$

Design Name: 2^{5-1} *Fractional Factorial Design*, *n = 16*

Design Tableau:

X1	X2	X3	X4	X5	Y
-	-	-	-	+	56
+	-	-	-	-	53
-	+	-	-	-	63
+	+	-	-	+	65
-	-	+	-	-	53
+	-	+	-	+	55
-	+	+	-	+	67
+	+	+	-	-	61
-	-	-	+	-	69
+	-	-	+	+	45
-	+	-	+	+	78
+	+	-	+	-	93
-	-	+	+	+	49
+	-	+	+	-	60
-	+	+	+	-	95
+	+	+	+	+	82

Design Geometry:



Effect Estimators:

$\hat{\beta}_2 = 20.50$
$\hat{\beta}_4 = 12.25$
$\hat{\beta}_{24} = 10.75$
$\hat{\beta}_{45} = -9.50$
$\hat{\beta}_5 = -6.25$
$\hat{\beta}_1 = -2.00$
$\hat{\beta}_3 = 0.00$

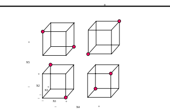
$\hat{\beta}_2 = 20.5$

Design Name: 2^{5-2} *Fractional Factorial Design*, *n = 8*

Design Tableau:

X1	X2	X3	X4	X5	Y
-	-	-	+	+	44
+	-	-	-	-	53
-	+	-	-	+	70
+	+	-	+	-	93
-	-	+	+	-	66
+	-	+	-	+	55
-	+	+	-	-	54
+	+	+	+	+	82

Design Geometry:



Effect Estimators:

$\hat{\beta}_2, \hat{\beta}_{14} = 20.25$
$\hat{\beta}_4, \hat{\beta}_{12} = 13.25$
$\hat{\beta}_{23}, \hat{\beta}_{45} = -12.75$
$\hat{\beta}_1, \hat{\beta}_{24}, \hat{\beta}_{35} = 12.25$
$\hat{\beta}_{25}, \hat{\beta}_{34} = 6.25$
$\hat{\beta}_5, \hat{\beta}_{13} = -3.75$
$\hat{\beta}_3, \hat{\beta}_{15} = -0.75$

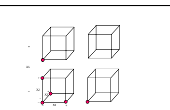
$\hat{\beta}_2 = 20.25$

Design Name: *1-Factor-at-a-Time Design*, *n = 6*

Design Tableau:

X1	X2	X3	X4	X5	Y
-	-	-	-	-	61
+	-	-	-	-	53
-	+	-	-	-	63
-	-	+	-	-	53
-	-	-	+	-	69
-	-	-	-	+	56

Design Geometry:



Effect Estimators:

$\hat{\beta}_1 = -8.00$
$\hat{\beta}_3 = -8.00$
$\hat{\beta}_4 = 8.00$
$\hat{\beta}_5 = -5.00$
$\hat{\beta}_2 = 2.00$

$\hat{\beta}_2 = 2.0$

Conclusions: 1-Factor-at-a-Time Designs are Poor. Orthogonal Designs are Excellent.

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2.4 Execution

Randomization

Blocking

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2.5 Recording

The best of designs can be negated by poor recording / database practices

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5. Statistical Analysis Principles & Techniques

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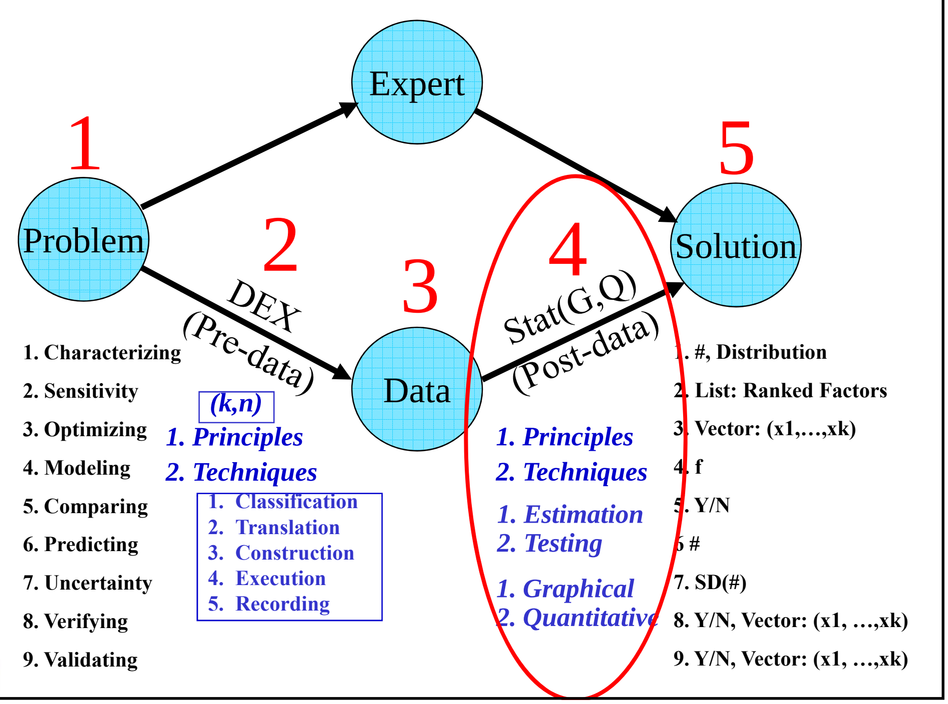
General Problem-Solving Framework/Structure



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Stat Analysis Principles & Techniques

Principles	Techniques
Client Knows/Understands the Conclusions	Graphics
Conclusions Not Approach-Dependent	Data-Based Analysis Simple Statistics Multiple Approaches Graphics + Quantitative
General Conclusions (Robust) "True in General"	Subsetting Robustness Plot
Analysis => Comparison => Juxtaposition	Multiplotting Robustness Plot
Conclusion Validity <= Assumption Validity	Test Underlying Assumptions
Insight Maximization (Know the Data)	EDA
Conclusions Validity <= Model Validity	Minimize Modeling Data-Based Graphics Let Data Speak for Self
Questions More Important than Methodology	Every Plot Should Have Lead Question Question-Driven Graphics
Conclusions More Important than Methodology	Every Plot Should Have Trailing Conclusions Conclusions-Driven Graphics
Analysis Graphics => Presentation Graphics	Every Conclusion Should Have a Best Presentation Graphic

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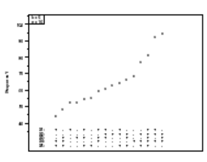
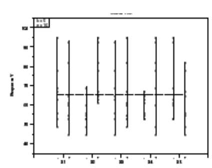
10 Step Graphical Analysis of 2-Level Designs (Dataplot)

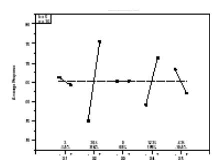
1. Ordered Data Plot



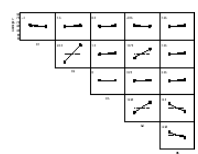
2. Scatter Plots



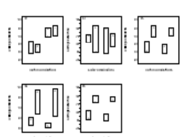
3. Main Effects Plot



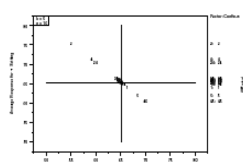
4. Interaction Effects Matrix



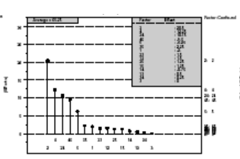
5. Block Plots



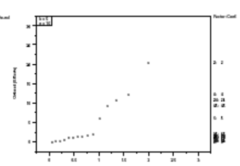
6. Youden Plot



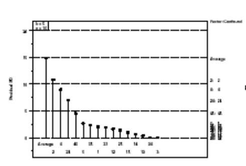
7. Pareto Plot



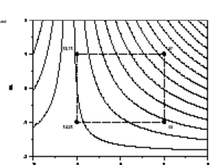
8. Half-Normal Plot



9. Cumulative ResSD Plot



10. Contour Plot



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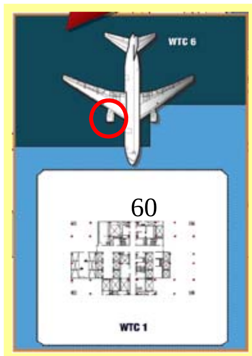
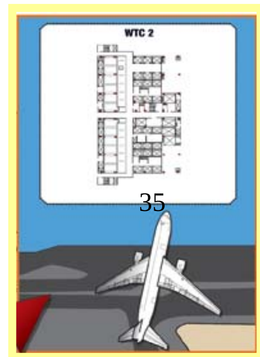
WTC Impact Core Damage Assessment

Q. After the plane impact of the WTC South Tower, there was no recorded data as to how many of the interior 47 columns of the building were damaged. A finite-element analysis (FEA) program was written to simulate the impact. The plane was modeled by 1.4 million elements. What factors most affected the performance of this FEA code? What factors could be eliminated as unimportant?





FEMA Report, pp. 2-17, 2-29

Q. What factors affect quality of FEA code predictions?

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Factors ...

2. Sensitivity Analysis Experiment Design: List of Factors (Component = Engine)

Factors Under Study (k):

1. Flight Speed
2. Flight Impact Location (Vertical)
3. Flight Impact Location (Horizontal)
4. Engine Assignment Set
5. Engine Strength
6. Engine Failure Strain
7. Engine Strain Rate Effects
8. Perimeter Column Strength
9. Perimeter Column Failure Strain
10. Perimeter Column Strain Rate Effects
11. FEA Model Erosion Parameter
12. FEA Contact Parameter
13. FEA Friction Coefficient

Affordable Number of Runs: $n < 50$

$DEX = g(k,n)$

$(k = 13, n < 50)$

(Design and data based on research carried out by contractor: Applied Research Associates)

Y = # Core Columns Damaged

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Design …

Experiment Design: 1-FAT

k = 13, n = 1+13 = 14

Baseline → (run 1)

	X1	X2	X3	X4	X5	X6	X7	X8	X9	X10	X11	X12	X13	Y
1	-	-	-	-	-	-	-	-	-	-	-	-	-	
2	+	-	-	-	-	-	-	-	-	-	-	-	-	
3	-	+	-	-	-	-	-	-	-	-	-	-	-	
4	-	-	+	-	-	-	-	-	-	-	-	-	-	
5	-	-	-	+	-	-	-	-	-	-	-	-	-	
6	-	-	-	-	+	-	-	-	-	-	-	-	-	
7	-	-	-	-	-	+	-	-	-	-	-	-	-	
8	-	-	-	-	-	-	+	-	-	-	-	-	-	
9	-	-	-	-	-	-	-	+	-	-	-	-	-	
10	-	-	-	-	-	-	-	-	+	-	-	-	-	
11	-	-	-	-	-	-	-	-	-	+	-	-	-	
12	-	-	-	-	-	-	-	-	-	-	+	-	-	
13	-	-	-	-	-	-	-	-	-	-	-	+	-	
14	-	-	-	-	-	-	-	-	-	-	-	-	+	

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Design …

Experiment Design: 2^{13-9} Orthogonal Fractional Factorial wcp

($k = 13$, $n = 17^{+}$)

Run	I												C
	X1	X2	X3	X4	X5	X6	X7	X8	X9	X10	X11	X12	X13
1	1	1	-1	1	1	-1	-1	1	1	-1	-1	-1	1
2	1	1	-1	-1	-1	1	1	-1	-1	1	1	-1	1
3	1	-1	-1	1	-1	1	-1	1	-1	1	-1	1	-1
4	1	-1	-1	-1	1	-1	1	-1	1	-1	1	1	-1
5	-1	1	1	1	1	1	-1	-1	-1	-1	1	-1	-1
6	-1	1	1	-1	-1	-1	1	1	1	1	-1	-1	-1
7	-1	-1	1	1	-1	-1	-1	-1	1	1	1	1	1
8	-1	-1	1	-1	1	1	1	1	-1	-1	-1	1	1
9	0	0	0	0	0	0	0	0	0	0	0	0	0
10	1	1	1	1	-1	1	1	-1	1	-1	-1	1	-1
11	1	1	1	-1	1	-1	-1	1	-1	1	1	1	-1
12	1	-1	1	1	1	-1	1	-1	-1	1	-1	-1	1
13	1	-1	1	-1	-1	1	-1	1	1	-1	1	-1	1
14	-1	1	-1	1	-1	-1	1	1	-1	-1	1	1	1
15	-1	1	-1	-1	1	1	-1	-1	1	1	-1	1	1
16	-1	-1	-1	1	1	1	1	1	1	1	1	-1	-1
17	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1

"Figure" 1.4 Data from 2^{13-9} (with center point) orthogonal experiment design for engine/core-column impact study

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Data ...

(k = 13, n = 17^+)

Run	I X1	X2	X3	X4	X5	X6	X7	X8	X9	X10	X11	X12	C X13	Core Damage Y
1	1	1	-1	1	1	-1	-1	1	1	-1	-1	-1	1	0.313
2	1	1	-1	-1	-1	1	1	-1	-1	1	1	-1	1	0.154
3	1	-1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	0.162
4	1	-1	-1	-1	1	-1	1	-1	1	-1	1	1	-1	0.449
5	-1	1	1	1	1	1	-1	-1	-1	-1	1	-1	-1	0.295
6	-1	1	1	-1	-1	-1	1	1	1	1	-1	-1	-1	0.015
7	-1	-1	1	1	-1	-1	-1	-1	1	1	1	1	1	0.019
8	-1	-1	1	-1	1	1	1	1	-1	-1	-1	1	1	0.424
9	0	0	0	0	0	0	0	0	0	0	0	0	0	0.28
10	1	1	1	1	-1	1	1	-1	1	-1	-1	1	-1	0.08
11	1	1	1	-1	1	-1	-1	1	-1	1	1	1	-1	0.035
12	1	-1	1	1	1	-1	1	-1	-1	1	-1	-1	1	0.043
13	1	-1	1	-1	-1	1	-1	1	1	-1	1	-1	1	0.095
14	-1	1	-1	1	-1	-1	1	1	-1	-1	1	1	1	0.288
15	-1	1	-1	-1	1	1	-1	-1	1	1	-1	1	1	0.067
16	-1	-1	-1	1	1	1	1	1	1	1	1	-1	-1	0.063
17	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	0.301

"Figure" 1.4 Data from 2^{13-9} (with center point) orthogonal experiment design for engine/core-column impact study

$\bar{Y}$ = .175

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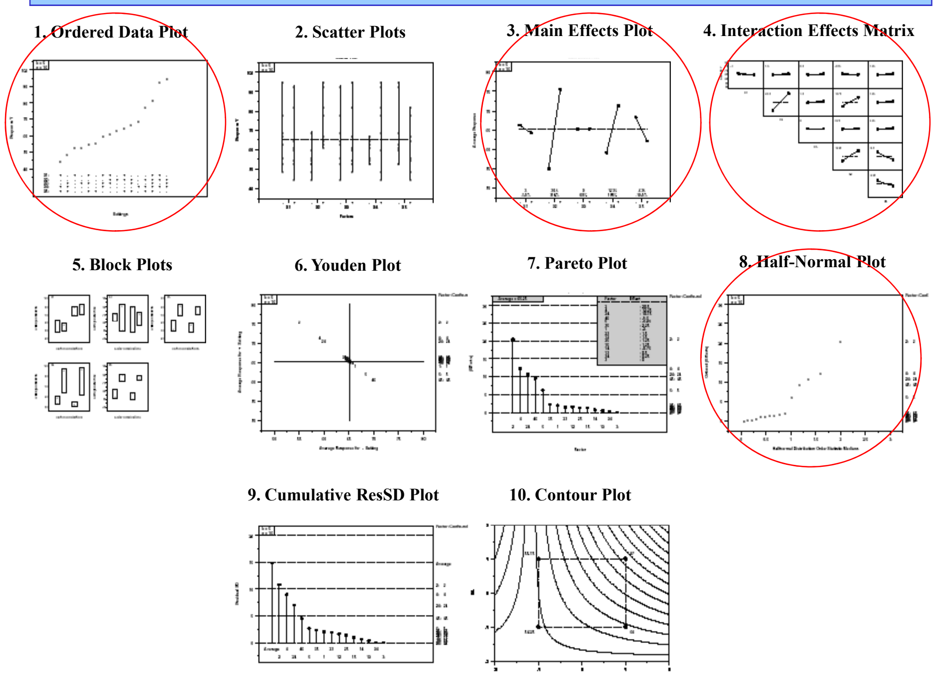
Analysis ...

10 Step Graphical Analysis of 2-Level Designs (Dataplot)



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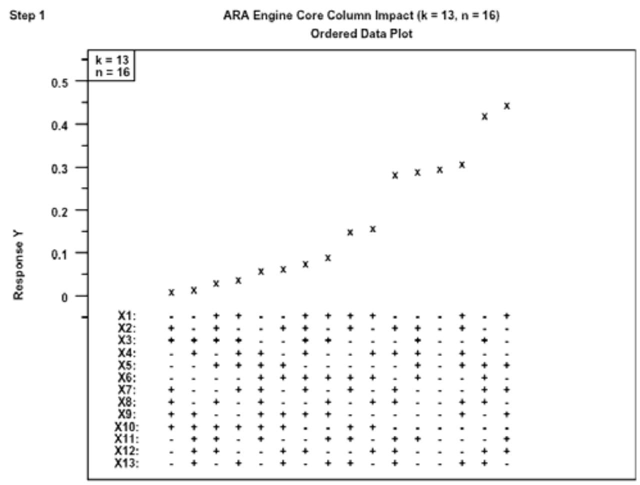
DEXPLOT.DP

Analysis ...

5. Data Analysis (Graphical): Most Important Factor, “Best” Setting

Ordered Data Plot



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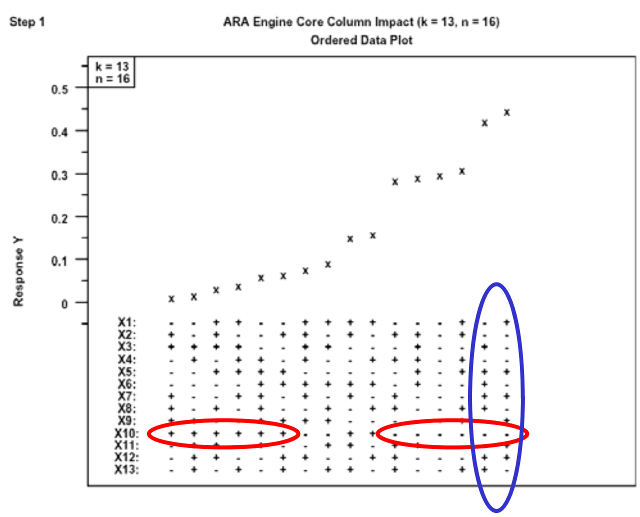
Analysis ...

5. Data Analysis: Most Important Factor, Best Setting

Ordered Data Plot



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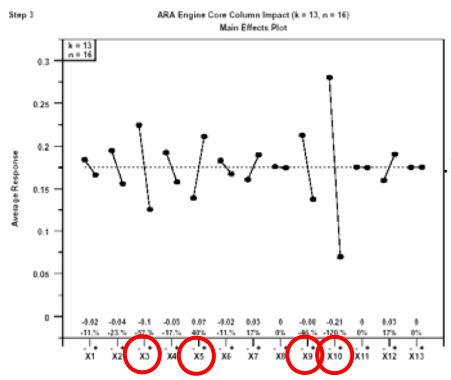
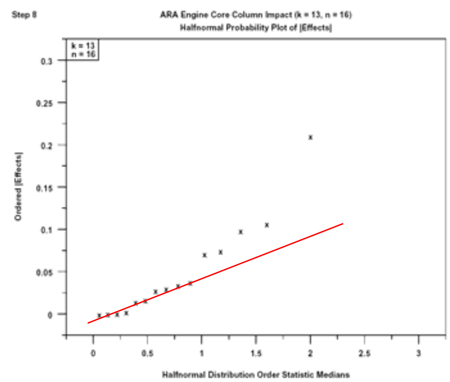
5. Data Analysis: Estimation of Factor Effects

Main Effects Plot

Halfnormal Probability Plot of |Effects|



.175



Least Squares Estimates

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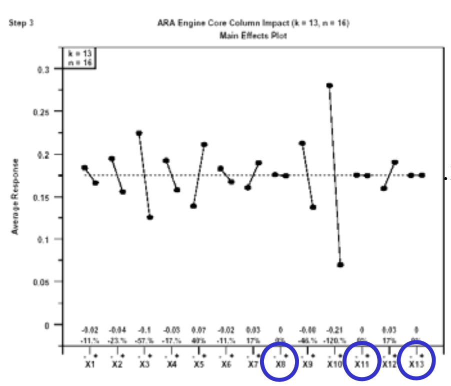
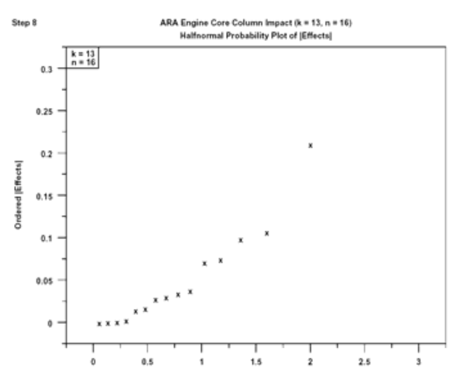
5. Data Analysis: Estimation of Factor Effects

Main Effects Plot

Halfnormal Probability Plot of |Effects|



.175



Least Squares Estimates

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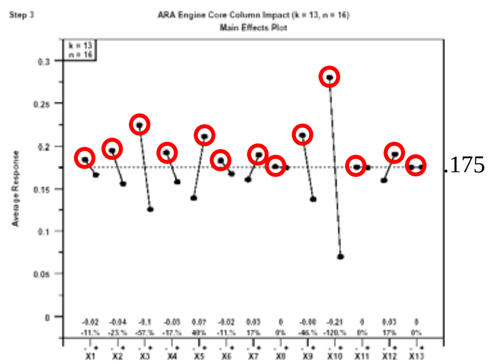
5. Data Analysis: "Best" Settings

Main Effects Plot



- - - - + - + . - - . + .

5. Data Analysis: Confounding

Interaction Effects Matrix

X10
X1*X9
X2*X4
X3*X6
X7*X12
X8*X13

1 + 13 + 78

Analysis ...

6. Conclusions

X10	-.21	(-120%)	(Perimeter Column Strain Rate Effects)
X3	-.10	(-57%)	(Impact Location: Horizontal)
X9	-.08	(-46%)	(Perimeter Column Failure Strain)
X5	+.07	(+40%)	(Engine Strength)
X2	-.04	(-23%)	(Impact Location: Vertical)

with least important factors being

X13	.00	(0%)	(FEA Friction Coefficient)
X11	.00	(0%)	(FEA Erosion Parameter)
X8	.00	(0%)	(Perimeter Column Strength)

Additional ARA Runs: LHC => f

FEA for *plane*: 1.4 million elements

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6. Conclusions

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Conclusions

1. Approach: A structured problem-solving approach exists, with generic and relevant questions, issues, & methodologies
2. Design: Design is more important than analysis
3. (k,n): Every design has a (k,n) (specificity)
4. Problem Categories: Scientific problems often generically fall into 4 categories--these categories have corresponding designs
5. Designs & Conclusions: Designs makes a difference in terms of the quality of estimates and validity of conclusions
6. Orthogonal: 1FAT designs are poor; orthogonal designs are excellent
7. Fractional: If the number of runs n is an issue then orthogonal fractional factorial designs are excellent
8. 2^{k-p}: 2-level orthogonal fractional factorial designs are remarkably insightful and extremely n-efficient

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References

1. Experiment Design Textbook:
   Box, Hunter, & Hunter: Statistics for Experimenters, Wiley, (esp. p. 410 for edition 1 and p. 272 for edition 2)
2. Experiment Design Methodology:
   NIST/SEMATEC e-Handbook of Statistics, Chapter 5: Improve
   http://www.itl.nist.gov/div898/handbook/
   http://www.itl.nist.gov/div898/handbook/pri/section5/pri59.htm
3. 10-Step Graphical Analysis of 2^{k-p} Designs:
   Dataplot macro: DEXPLOT.DP
   http://www.itl.nist.gov/div898/software/dataplot.html

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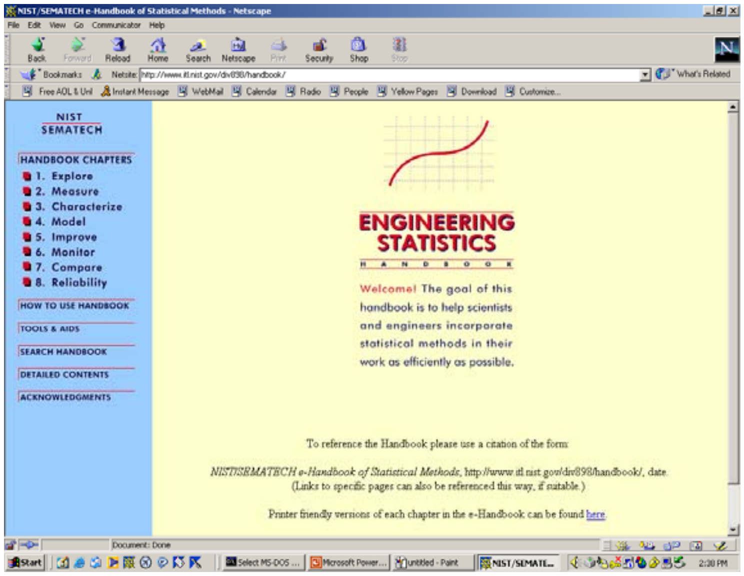
NIST e-Handbook of Engineering Statistics



http://www.itl.nist.gov/div898/handbook/

(3000 pages; 3 million page views / month)

5. Improve
5. Advanced Topics
9. An EDA Approach to DEX

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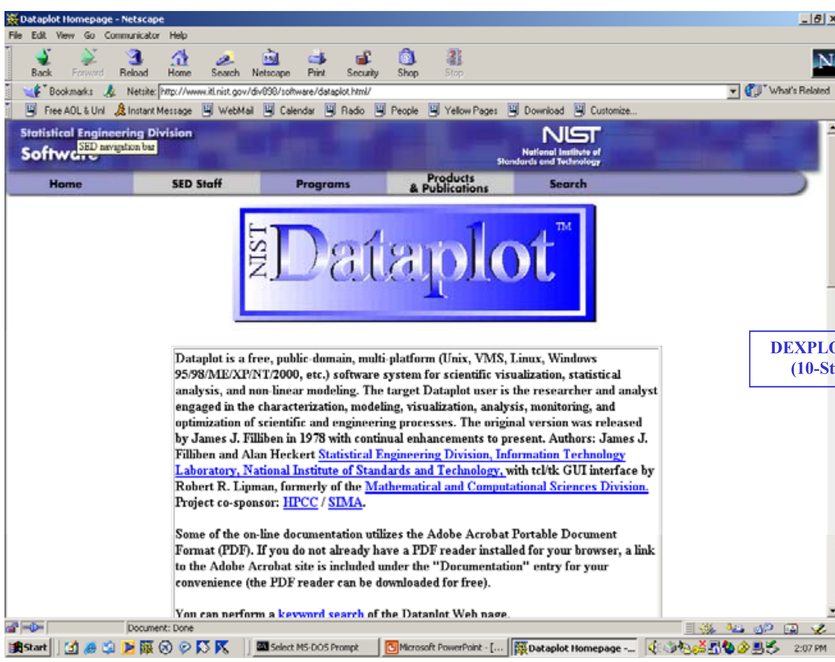
NIST Dataplot (Filliben/Heckert)



DEXPLOT.DP (10-Step)

http://www.itl.nist.gov/div898/software/dataplot.html/

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