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Investment and Trade Patterns in a Sticky-Price, Open-Economy Model*

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Abstract

This paper develops a tractable two-country DSGE model with sticky prices à la Calvo (1983) and local-currency pricing. We analyze the capital investment decision in the presence of adjustment costs of two types, the capital adjustment cost (CAC) specification and the investment adjustment cost (IAC) specification. We compare the investment and trade patterns with adjustment costs against those of a model without adjustment costs and with (quasi-) flexible prices. We show that having adjustment costs results into more volatile consumption and net exports, and less volatile investment. We document three important facts on U.S. trade: a) the S-shaped cross-correlation function between real GDP and the real net exports share, b) the J-curve between terms of trade and net exports, and c) the weak and S-shaped cross-correlation between real GDP and terms of trade. We find that adding adjustment costs tends to reduce the model's ability to match these stylized facts. Nominal rigidities cannot account for these features either.

JEL codes: F31, F37, F41

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1 Introduction

Adjustment costs on capital accumulation often feature in modern macroeconomic models of the business cycle. For instance, the Q theory of investment as developed by Lucas and Prescott (1971) and Abel (1983) among others formalizes the idea that capital investment becomes more attractive whenever the value of a unit of additional capital is higher relative to its acquisition cost. International business cycle models have also adopted the Q theory of investment based on convex adjustment costs. However, while there is a broad agreement on the importance of investment for trade, there is less clarity on the role that adjustment costs play in these models.

In the standard international real business cycle model (IRBC) of Backus, Kehoe and Kydland (BKK) (1995, p. 340), the connection between investment and trade is rather straightforward: "resources are shifted to the more productive location (...). This tendency to 'make hay where the sun shines' means that with uncorrelated productivity shocks, consumption will be positively correlated across countries, while investment, employment, and output will be negatively correlated. With productivity shocks that are positively correlated, as they are in our model, all of these correlations rise, but with the benchmark parameter values none change sign."

Heathcote and Perri (2002) elaborate further on this point, explaining that a domestic productivity shock causes domestic investment to increase by much more than the increase in foreign consumption, so the domestic country draws more resources from abroad and the domestic trade deficit widens at the same time as domestic output is raising. Hence, the IRBC model implies that the trade balance is countercyclical as in the data. Engel and Wang (2007) in a richer model with adjustment costs and durable goods also find that the IRBC framework can deliver the countercyclical trade balance observed in the data.

Raffo (2008, p. 21), however, notes that the IRBC model accounts for this empirical pattern "due to the strong terms of trade effect generated by the change in relative scarcity of goods across countries". This prediction is clearly counterfactual, since the empirical correlation between output and terms of trade (ToT) is close to zero for most countries. Furthermore, output and specially consumption volatility in the BKK (1995) and Heathcote and Perri (2002) models tends to be significantly lower than in the data. In turn, as this paper shows, models that do match the real U.S. GDP volatility tend to overshoot and generate too much investment volatility, even though consumption remains too smooth.

Therefore, the role of the Q theory extension for trade in this class of models requires further consideration. While capital accumulation provides a powerful mechanism to smooth consumption intertemporally, capital adjustment costs are likely to induce smoother investment patterns and a more volatile consumption series. In other words, the Q extension has links to a long tradition on investment theory, but it also has implications for the model's ability to generate empirically-consistent consumption and investment paths.

Another strand of the international macro literature has emphasized the role of deviations of the law of one price (LOOP) as a distortion that leads to a misallocation of expenditures across countries and, therefore, to sizable effects on trade. The most common international new neoclassical synthesis (INNS) model is built around the assumptions of monopolistic competition among firms, price stickiness à la Calvo (1983) and local-currency pricing. An influential paper in this strand of the literature is Chari, Kehoe and McGrattan (CKM) (2002), which incorporates a form of adjustment costs with the explicit purpose of calibrating the volatility of consumption. But the paper focuses on the behavior of the real exchange rate rather than on trade dynamics.

We think that the CKM (2002) paper - by its own right a Q theory extension of the INNS model - raises the issue of how adjustment costs together with deviations of the LOOP affects the ability of the model to replicate the trade patterns in the data. For instance, the particular cost function that CKM (2002) used is not necessarily the only one being proposed. Christiano, Eichenbaum and Evans (CEE) (2005) have popularized an alternative adjustment cost specification linked to investment growth rates instead of the capital-to-output ratio, recently used by Justiniano and Primiceri (2008) among others.[1] To our knowledge, however, the trade predictions of the Q-INNS model with complete international asset markets have not been consistently evaluated against: a) different specifications of the adjustment cost function (including the case without adjustment costs), and b) an approximation of the flexible price environment conventionally assumed in the Q-IRBC literature.

In this paper, we develop a two country DSGE model with the distinctive features of the Q-INNS model with sticky prices and local-currency pricing to help us understand the role of adjustment costs and pricing in trade. We also examine whether there is any interaction between deviations of the LOOP and adjustment costs that can affect the dynamics of net exports. In other words, this paper aims to provide a broad assessment of whether the Q theory extension of the INNS model can simultaneously be reconciled with the empirical evidence on investment and trade.

`[Insert Table 1 about here]`

We focus our analysis on several important features of the international business cycle data summarized in Table 1. First, investment is around three times more volatile than real GDP, while consumption and the net exports share are significantly less volatile (see, e.g., BKK, 1995). All series tend to be quite persistent. Second, the trade balance is countercyclical. This feature is quite robust across countries, as corroborated by the empirical evidence provided by Engel and Wang (2007). They find that among 25 OECD countries, the mean correlation between real GDP and the real net export share is -0.24 and the median is -0.25.

Third, as noted by Ghironi and Melitz (2007) and Engel and Wang (2007), the cross-correlation between real GDP and the real net exports share is S-shaped. Fourth, there is evidence of the J-curve in the cross-correlation between ToT and net exports; a relationship extensively discussed in BKK (1994). Finally, there are weak cross-correlations between real GDP and ToT. This feature is quite robust across countries, as confirmed by the empirical evidence provided by Raffo (2008). He finds that for 14 OECD countries plus the EU-15, the mean correlation between real GDP and ToT is 0.08 and the median is 0.11. We also document that the cross-correlation between real GDP and ToT is S-shaped.

2 The Baseline Open Economy

Here, we specify the structure of our baseline, two-country stochastic general equilibrium model. The model itself is fully described in Martínez-García and Søndergaard (2008a, 2008b).

[1] CEE (2005) and Justiniano and Primiceri (2008) are closed economy models. For an application in an open economy model, see e.g. Martínez-García and Søndergaard (2008b).

2.1 The Households' Problem

Each country is populated by a continuum of infinitely lived (and identical) households in the interval $[0, 1]$. The domestic households' maximize a utility function which is additively separable in consumption, C_t, and labor, L_t, i.e.

$$\sum\nolimits_{\tau=0}^{\infty} \beta^{\tau} \mathbb{E}_t \left[\frac{1}{1-\sigma^{-1}} \left(C_{t+\tau}\right)^{1-\sigma^{-1}} - \frac{1}{1+\varphi} \left(L_{t+\tau}\right)^{1+\varphi} \right], \tag{1}$$

where $0 < \beta < 1$ is the subjective intertemporal discount factor. The elasticity of intertemporal substitution and the inverse of the Frisch elasticity of labor supply satisfy that $\sigma > 0$ $(\sigma \neq 1)$ and $\varphi > 0$, respectively. The households' maximization problem is subject to the sequential budget constraints,

$$P_t \left(C_t + X_t\right) + \int Q\left(s^{t+1} \mid s^t\right) B\left(s^{t+1}\right) ds_{t+1} \leq B_t + W_t L_t + Z_t K_t, \tag{2}$$

and the law of motion for capital,

$$K_{t+1} \leq (1-\delta) K_t + \Phi\left(X_t, X_{t-1}, K_t\right) X_t. \tag{3}$$

where X_t is domestic real investment, and K_t stands for domestic real capital. Moreover, W_t is the domestic nominal wage, Z_t defines the nominal rental rate on capital, and P_t is the domestic consumption price index (CPI).

We denote s_{t+1} the event that occurs at time $t+1$ and $s^{t+1} = (s^t, s_{t+1})$ the history of events up to that point. We assume complete international asset markets. Therefore, $B_t \equiv B(s_t)$ is the nominal payoff after the event s^t is realized on a claim over the portfolio held at the end of period $t-1$. This portfolio includes a proportional share on the nominal profits generated by the domestic firms as well as a complete set of contingent claims, traded internationally. $Q_{t,t+1} \equiv Q\left(s^{t+1} \mid s^t\right)$ is the price for such a one period contingent claim.[2] The foreign households maximize their lifetime utility subject to an analogous sequence of budget constraints and the law of motion for capital.

We assume that there is no trade in either domestic or foreign shares, imposing *de facto* a strict home bias in portfolios. Sole ownership of the local firms rests in the hands of the local households. Embedded in the specification of the budget constraint lies also the assumption that both labor and capital markets are both homogenous and perfectly competitive within a country, but segregated across countries. In other words, the factors of production are immobile across borders.

Adjustment Costs. We investigate the no adjustment costs (NAC) specification as a starting point, i.e.

$$\Phi\left(X_t, X_{t-1}, K_t\right) = 1. \tag{4}$$

We also explore two special cases of adjustment cost functions that have become popular in the literature. On one hand, the capital adjustment cost (CAC) case favored by CKM (2002); on the other hand, the investment adjustment cost (IAC) case preferred by CEE (2005).[3]

[2] It can be shown that the price of the claim is also equivalent to the domestic stochastic discount factor (SDF) for one-period ahead nominal payoffs, $M_{t,t+1} \equiv M\left(s^{t+1} \mid s^t\right)$, divided by its conditional probability, $\mu\left(s^{t+1} \mid s^t\right)$.

[3] One of the perceived advantages of the IAC specification is that it contributes to generate a hump-shaped response of investment to monetary shocks. In part, that explains its popularity in the literature. This issue is orthogonal to our discussion

The capital adjustment cost (CAC) specification implies that the function $\Phi(\cdot)$ in (3) takes the following form,

$$\Phi\left(\frac{X_t}{K_t}\right) = 1 - \frac{1}{2}\chi \frac{\left(\frac{X_t}{K_t} - \delta\right)^2}{\frac{X_t}{K_t}}, \tag{5}$$

where $\frac{X_t}{K_t}$ is the investment-to-capital ratio, and δ is the depreciation rate. Among the relevant properties of this adjustment cost function, we note that,

$$\begin{aligned}
\Phi'\left(\frac{X_t}{K_t}\right) &= -\frac{1}{2}\chi \left[\frac{2\frac{X_t}{K_t}\left(\frac{X_t}{K_t} - \delta\right) - \left(\frac{X_t}{K_t} - \delta\right)^2}{\left(\frac{X_t}{K_t}\right)^2}\right], \\
\Phi''\left(\frac{X_t}{K_t}\right) &= -\frac{1}{2}\chi \left[\frac{2\left(\frac{X_t}{K_t}\right)^3 - 2\left(2\frac{X_t}{K_t}\left(\frac{X_t}{K_t} - \delta\right) - \left(\frac{X_t}{K_t} - \delta\right)^2\right)\frac{X_t}{K_t}}{\left(\frac{X_t}{K_t}\right)^4}\right].
\end{aligned}$$

In steady state, the adjustment costs dissipate and investment equals the replacement of depreciated capital. This implies that $\Phi(\delta) = 1$, $\Phi'(\delta) = 0$, and $\Phi''(\delta) = -\frac{\chi}{\delta}$. The investment adjustment cost function (IAC) specification implies the following functional form,

$$\Phi\left(\frac{X_t}{X_{t-1}}\right) = 1 - \frac{1}{2}\kappa \frac{\left(\frac{X_t}{X_{t-1}} - 1\right)^2}{\frac{X_t}{X_{t-1}}}. \tag{6}$$

Among the relevant properties of the IAC function, we note that,

$$\begin{aligned}
\Phi'\left(\frac{X_t}{X_{t-1}}\right) &= -\frac{1}{2}\kappa \left[\frac{2\frac{X_t}{X_{t-1}}\left(\frac{X_t}{X_{t-1}} - 1\right) - \left(\frac{X_t}{X_{t-1}} - 1\right)^2}{\left(\frac{X_t}{X_{t-1}}\right)^2}\right], \\
\Phi''\left(\frac{X_t}{X_{t-1}}\right) &= -\frac{1}{2}\kappa \left[\frac{2\left(\frac{X_t}{X_{t-1}}\right)^3 - 2\left(2\frac{X_t}{X_{t-1}}\left(\frac{X_t}{X_{t-1}} - 1\right) - \left(\frac{X_t}{X_{t-1}} - 1\right)^2\right)\left(\frac{X_t}{X_{t-1}}\right)}{\left(\frac{X_t}{X_{t-1}}\right)^4}\right].
\end{aligned}$$

In steady state, the adjustment costs also dissipate in this case, and net investment growth is zero. This also implies that $\Phi(1) = 1$, $\Phi'(1) = 0$, and $\Phi''(1) = -\kappa$. The same adjustment cost formula applies to the foreign households' problem.

The law of motion for capital in steady state is the same independently of whether we add adjustment costs to the model or not. Hence, it can be said that the adjustment cost functions in (5) and (6) alter the dynamics of the model in the short-run, but they do not distort its long-run properties.

Aggregation Rules and the Price Indexes. The home and foreign consumption bundles of the domestic household, C_t^H and C_t^F, as well as the domestic investment bundles, X_t^H and X_t^F, are aggregated by means

(since we only investigate the role of real shocks). We add it for the sake of completeness.

of a CES index as,

$$C_t^H = \left[\int_0^1 C_t(h)^{\frac{\theta-1}{\theta}} dh\right]^{\frac{\theta}{\theta-1}}, \ C_t^F = \left[\int_0^1 C_t(f)^{\frac{\theta-1}{\theta}} df\right]^{\frac{\theta}{\theta-1}}, \tag{7}$$

$$X_t^H = \left[\int_0^1 X_t(h)^{\frac{\theta-1}{\theta}} dh\right]^{\frac{\theta}{\theta-1}}, \ X_t^F = \left[\int_0^1 X_t(f)^{\frac{\theta-1}{\theta}} df\right]^{\frac{\theta}{\theta-1}}, \tag{8}$$

while domestic aggregate consumption and investment, C_t and X_t, are defined with another CES index as,

$$C_t = \left[\phi_H^{\frac{1}{\eta}} \left(C_t^H\right)^{\frac{\eta-1}{\eta}} + \phi_F^{\frac{1}{\eta}} \left(C_t^F\right)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}, \tag{9}$$

$$X_t = \left[\phi_H^{\frac{1}{\eta}} \left(X_t^H\right)^{\frac{\eta-1}{\eta}} + \phi_F^{\frac{1}{\eta}} \left(X_t^F\right)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}. \tag{10}$$

The elasticity of substitution across varieties produced within the same country is $\theta > 1$, and the elasticity of intratemporal substitution between the home and foreign bundles of varieties is $\eta > 0$. The share of the home goods in the domestic aggregators is ϕ_H, while the share of foreign goods is $\phi_F = 1 - \phi_H$. We define the aggregators for the foreign household consumption and investment similarly. However, the model introduces symmetric home bias in consumption and investment, by requiring the share of the home goods in the foreign aggregator to be $\phi_H^* = \phi_F$ and the share of foreign goods in the foreign aggregator to be $\phi_F^* = \phi_H$ (see, e.g., Warnock, 2003).

After aggregation, investment goods can only be used for local production and capital, K_t and K_t^*, is immobile at that point. One justification for this assumption is provided by the compositional differences across countries in the investment bundles. Technological differences across countries prevent the use of foreign capital goods, K_t^*, because it does not have the right mix of input varieties for the domestic country. The same can be said for domestic capital in the foreign country. In other words, implicit barriers preventing technological diffusion explain the immobility of capital. However, while capital itself is effectively nontraded, all local and foreign varieties used to bundle up either consumption or investment can still be traded internationally.

The model presented in Martínez-García and Søndergaard (2008a, 2008b) does not impose an investment irreversibility constraint. Therefore, a unit of capital can always be disassembled and its input varieties traded. As a result, investment resources can still be shifted across countries, in spite of the immobility of capital. Trade patterns will reflect those movements. The symmetry of the aggregators implies that the corresponding price indexes for the investment and consumption bundles are identical. Hence, with identical aggregation rules and no irreversibility constraints, the relative price of investment in units of consumption is one, as reflected in the budget constraint in equation (2). In our framework, the value of capital is entirely determined by the rate of transformation of aggregate investment into capital goods which, in turn, depends on the adjustment cost function in (3) and the marginal Q (as we shall discuss shortly).

Under standard results on functional separability, the indexes which correspond to our specification of aggregators for the CPIs are,

$$P_t = \left[\phi_H \left(P_t^H\right)^{1-\eta} + \phi_F \left(P_t^F\right)^{1-\eta}\right]^{\frac{1}{1-\eta}}, \tag{11}$$

$$P_t^* = \left[\phi_H^* \left(P_t^{H*}\right)^{1-\eta} + \phi_F^* \left(P_t^{F*}\right)^{1-\eta}\right]^{\frac{1}{1-\eta}}, \tag{12}$$

and the price sub-indexes are,

$$P_t^H = \left[\int_0^1 P_t(h)^{1-\theta} dh\right]^{\frac{1}{1-\theta}},\ P_t^F = \left[\int_0^1 P_t(f)^{1-\theta} df\right]^{\frac{1}{1-\theta}}, \tag{13}$$

$$P_t^{H*} = \left[\int_0^1 P_t^*(h)^{1-\theta} dh\right]^{\frac{1}{1-\theta}},\ P_t^{F*} = \left[\int_0^1 P_t^*(f)^{1-\theta} df\right]^{\frac{1}{1-\theta}}, \tag{14}$$

where P_t^H and P_t^F are the price sub-indexes for the home- and foreign-produced bundle of goods in units of the home currency. Similarly for P_t^{H*} and P_t^{F*}. We define the real exchange rate, RS_t, as,

$$RS_t \equiv \frac{S_t P_t^*}{P_t}, \tag{15}$$

where S_t denotes the nominal exchange rate.

The Optimality Conditions. Given the structure described in $(7)-(8)$, the solution to the sub-utility maximization problem implies that the home and foreign households' demands for each variety are given by,

$$C_t(h) = \left(\frac{P_t(h)}{P_t^H}\right)^{-\theta} C_t^H,\ X_t(h) = \left(\frac{P_t(h)}{P_t^H}\right)^{-\theta} X_t^H,\ \forall h \in [0,1], \tag{16}$$

$$C_t(f) = \left(\frac{P_t(f)}{P_t^F}\right)^{-\theta} C_t^F,\ X_t(f) = \left(\frac{P_t(f)}{P_t^F}\right)^{-\theta} X_t^F,\ \forall f \in [0,1], \tag{17}$$

while the demands for the bundles of home and foreign goods are simply equal to,

$$C_t^H = \phi_H \left(\frac{P_t^H}{P_t}\right)^{-\eta} C_t,\ X_t^H = \phi_H \left(\frac{P_t^H}{P_t}\right)^{-\eta} X_t, \tag{18}$$

$$C_t^F = \phi_F \left(\frac{P_t^F}{P_t}\right)^{-\eta} C_t,\ X_t^F = \phi_F \left(\frac{P_t^F}{P_t}\right)^{-\eta} X_t. \tag{19}$$

These equations, combined with the analogous counterparts for the foreign country, characterize the demand functions in the model. The real exports and imports of domestic goods in the model can be inferred from equations $(16)-(19)$ and their foreign counterparts as follows,

$$EXP_t \equiv \int_0^1 \left[C_t^*(h) + X_t^*(h)\right] dh = \left[\int_0^1 \left(\frac{P_t^*(h)}{P_t^{H*}}\right)^{-\theta} dh\right] \phi_H^* \left(\frac{P_t^{H*}}{P_t^*}\right)^{-\eta} \left[C_t^* + X_t^*\right], \tag{20}$$

$$IMP_t \equiv \int_0^1 \left[C_t(f) + X_t(f)\right] df = \left[\int_0^1 \left(\frac{P_t(f)}{P_t^F}\right)^{-\theta} df\right] \phi_F \left(\frac{P_t^F}{P_t}\right)^{-\eta} \left[C_t + X_t\right], \tag{21}$$

where $\phi_H^* = \phi_F$ under the symmetric home bias assumption. Real exports and imports in (20) – (21) are defined from the point of view of the domestic country, but they are naturally the counterpart of real imports and exports in the foreign country. The market clearing conditions at the variety level allows us to express the domestic and foreign aggregate output, Y_t and Y_t^*, from the demand-side as follows,

$$\begin{aligned} Y_t &= \int_0^1 \left[C_t(h) + X_t(h) + C_t^*(h) + X_t^*(h)\right] dh \\ &= \left[\int_0^1 \left(\tfrac{P_t(h)}{P_t^H}\right)^{-\theta} dh\right] \phi_H \left(\tfrac{P_t^H}{P_t}\right)^{-\eta} (C_t + X_t) + \left[\int_0^1 \left(\tfrac{P_t^*(h)}{P_t^{H*}}\right)^{-\theta} dh\right] \phi_H^* \left(\tfrac{P_t^{H*}}{P_t^*}\right)^{-\eta} (C_t^* + X_t^*), \end{aligned} \tag{22}$$

$$\begin{aligned} Y_t^* &= \int_0^1 \left[C_t(f) + X_t(f) + C_t^*(f) + X_t^*(f)\right] df \\ &= \left[\int_0^1 \left(\tfrac{P_t(f)}{P_t^F}\right)^{-\theta} df\right] \phi_F \left(\tfrac{P_t^F}{P_t}\right)^{-\eta} (C_t + X_t) + \left[\int_0^1 \left(\tfrac{P_t^*(f)}{P_t^{F*}}\right)^{-\theta} df\right] \phi_F^* \left(\tfrac{P_t^{F*}}{P_t^*}\right)^{-\eta} (C_t^* + X_t^*), \end{aligned} \tag{23}$$

where $\phi_F^* = \phi_H$ under the symmetric home bias assumption. Equations (22) – (23) tie the aggregate output in both countries to consumption as well as to relative prices.

Under complete international asset markets, the intertemporal first-order conditions result in the following (well-known) equilibrium condition,

$$RS_t = \nu \left(\frac{C_t^*}{C_t}\right)^{-\sigma^{-1}}, \tag{24}$$

derived by backward induction, where $\nu \equiv \frac{S_0 P_0^*}{P_0} \left(\frac{C_0^*}{C_0}\right)^{\sigma^{-1}}$ is a constant that depends on the initial conditions. Equation (24) is often referred as the *international risk-sharing condition*. The intertemporal first-order conditions also pin down the price of any given Arrow-Debreu security. Let I_t be the (gross) one-period riskless nominal interest rate in terms of the domestic currency, and I_t^* be the corresponding rate in terms of the foreign currency. Under complete asset markets, we can price a one-period nominal bonds using the price of the contingent claims as follows,

$$\frac{1}{I_t} = \beta \mathbb{E}_t \left[\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma^{-1}} \frac{P_t}{P_{t+1}}\right], \tag{25}$$

$$\frac{1}{I_t^*} = \beta \mathbb{E}_t \left[\left(\frac{C_{t+1}^*}{C_t^*}\right)^{-\sigma^{-1}} \frac{P_t^*}{P_{t+1}^*}\right]. \tag{26}$$

The equilibrium conditions of the households' problem also include a pair of labor supply functions (the intratemporal first-order conditions) which can be expressed as,

$$\frac{W_t}{P_t} = (C_t)^{\sigma^{-1}} (L_t)^{\varphi}, \tag{27}$$

$$\frac{W_t^*}{P_t^*} = (C_t^*)^{\sigma^{-1}} (L_t^*)^{\varphi}, \tag{28}$$

plus the appropriate no-Ponzi games, transversality conditions, the budget constraints and the law of motions for capital in both countries.

Finally, the equilibrium conditions are completed with a pair of equations that account for the capital-investment decisions of households. The capital-investment equations, however, depend on the choice of the adjustment cost function $\Phi(\cdot)$. Let us define the marginal Q, Q_t and Q_t^*, as the shadow value of a unit of capital.[4] Then, it is possible to write a pair of equilibrium conditions for a generic adjustment cost function in the domestic country as,

$$Q_t = \beta \mathbb{E}_t \left\{ \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma^{-1}} \left[\frac{Z_{t+1}}{P_{t+1}} + Q_{t+1} \left((1-\delta) + \frac{\partial \Phi \left(X_{t+1}, X_t, K_{t+1} \right)}{\partial K_{t+1}} X_{t+1} \right) \right] \right\},$$

$$Q_t \left[\Phi \left(X_t, X_{t-1}, K_t \right) + \frac{\partial \Phi \left(X_t, X_{t-1}, K_t \right)}{\partial X_t} X_t \right] = 1 - \beta \mathbb{E}_t \left\{ \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma^{-1}} \left[Q_{t+1} \frac{\partial \Phi \left(X_{t+1}, X_t, K_{t+1} \right)}{\partial X_t} X_{t+1} \right] \right\}.$$

A similar set of derivations allows us to write the following system of equations for the foreign country,

$$Q_t^* = \beta \mathbb{E}_t \left\{ \left(\frac{C_{t+1}^*}{C_t^*} \right)^{-\sigma^{-1}} \left[\frac{Z_{t+1}^*}{P_{t+1}^*} + Q_{t+1}^* \left((1-\delta) + \frac{\partial \Phi \left(X_{t+1}^*, X_t^*, K_{t+1}^* \right)}{\partial K_{t+1}^*} X_{t+1}^* \right) \right] \right\},$$

$$Q_t^* \left[\Phi \left(X_t^*, X_{t-1}^*, K_t^* \right) + \frac{\partial \Phi \left(X_t^*, X_{t-1}^*, K_t^* \right)}{\partial X_t^*} X_t^* \right] = 1 - \beta \mathbb{E}_t \left\{ \left(\frac{C_{t+1}^*}{C_t^*} \right)^{-\sigma^{-1}} \left[Q_{t+1}^* \frac{\partial \Phi \left(X_{t+1}^*, X_t^*, K_{t+1}^* \right)}{\partial X_t^*} X_{t+1}^* \right] \right\}.$$

In the no adjustment costs (NAC) case, the domestic capital-investment decision is summarized as,

$$1 = \beta \mathbb{E}_t \left\{ \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma^{-1}} \left[\frac{Z_{t+1}}{P_{t+1}} + (1-\delta) \right] \right\}, \tag{29}$$

$$Q_t = 1, \tag{30}$$

where the marginal Q is known to be equal to one. Similarly for the foreign country,

$$1 = \beta \mathbb{E}_t \left\{ \left(\frac{C_{t+1}^*}{C_t^*} \right)^{-\sigma^{-1}} \left[\frac{Z_{t+1}^*}{P_{t+1}^*} + (1-\delta) \right] \right\}, \tag{31}$$

$$Q_t^* = 1. \tag{32}$$

Under capital adjustment costs (CAC), the pair of conditions added to account for the capital-investment decisions of households are,

$$Q_t = \beta \mathbb{E}_t \left\{ \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma^{-1}} \left[\frac{Z_{t+1}}{P_{t+1}} + Q_{t+1} \left((1-\delta) - \Phi' \left(\frac{X_{t+1}}{K_{t+1}} \right) \left(\frac{X_{t+1}}{K_{t+1}} \right)^2 \right) \right] \right\}, \tag{33}$$

$$Q_t = \left[\Phi \left(\frac{X_t}{K_t} \right) + \Phi' \left(\frac{X_t}{K_t} \right) \frac{X_t}{K_t} \right]^{-1}. \tag{34}$$

[4] The marginal Q or Tobin's Q is equivalent to the Lagrange multiplier on the law of motion relative to the Lagrange multiplier on the budget constraint expressed in real terms.

A similar set of derivations allows us to write the following system of equations for the foreign country,

$$
\begin{aligned}
Q_t^* &= \beta \mathbb{E}_t \left\{ \left(\frac{C_{t+1}^*}{C_t^*} \right)^{-\sigma^{-1}} \left[\frac{Z_{t+1}^*}{P_{t+1}^*} + Q_{t+1}^* \left((1-\delta) - \Phi' \left(\frac{X_{t+1}^*}{K_{t+1}^*} \right) \left(\frac{X_{t+1}^*}{K_{t+1}^*} \right)^2 \right) \right] \right\}, && (35) \\
Q_t^* &= \left[\Phi \left(\frac{X_t^*}{K_t^*} \right) + \Phi' \left(\frac{X_t^*}{K_t^*} \right) \frac{X_t^*}{K_t^*} \right]^{-1}. && (36)
\end{aligned}
$$

Under investment adjustment costs (IAC), the pair of conditions added to account for the capital-investment decisions of households are,

$$
\begin{aligned}
Q_t &= \beta \mathbb{E}_t \left\{ \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma^{-1}} \left[\frac{Z_{t+1}}{P_{t+1}} + Q_{t+1} (1-\delta) \right] \right\}, && (37) \\
Q_t &= \left[\Phi \left(\frac{X_t}{X_{t-1}} \right) + \Phi' \left(\frac{X_t}{X_{t-1}} \right) \frac{X_t}{X_{t-1}} \right]^{-1} \left[1 + \beta \mathbb{E}_t \left\{ \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma^{-1}} \left[Q_{t+1} \Phi' \left(\frac{X_{t+1}}{X_t} \right) \left(\frac{X_{t+1}}{X_t} \right)^2 \right] \right\} \right] && (38)
\end{aligned}
$$

A similar set of derivations allows us to write the following system of equations for the foreign country,

$$
\begin{aligned}
Q_t^* &= \beta \mathbb{E}_t \left\{ \left(\frac{C_{t+1}^*}{C_t^*} \right)^{-\sigma^{-1}} \left[\frac{Z_{t+1}^*}{P_{t+1}^*} + Q_{t+1}^* (1-\delta) \right] \right\}, && (39) \\
Q_t^* &= \left[\Phi \left(\frac{X_t^*}{X_{t-1}^*} \right) + \Phi' \left(\frac{X_t^*}{X_{t-1}^*} \right) \frac{X_t^*}{X_{t-1}^*} \right]^{-1} \left[1 + \beta \mathbb{E}_t \left\{ \left(\frac{C_{t+1}^*}{C_t^*} \right)^{-\sigma^{-1}} \left[Q_{t+1}^* \Phi' \left(\frac{X_{t+1}^*}{X_t^*} \right) \left(\frac{X_{t+1}^*}{X_t^*} \right)^2 \right] \right\} \right] && (40)
\end{aligned}
$$

These equations are fundamental in our model because they characterize the investment dynamics as well as the evolution of the marginal Q. The connection between investment decisions and marginal Q is not novel, neither is new the use of convex adjustment costs to develop a Q theory of investment. However, we highlight in this paper the role that these interactions play in an open economy environment, and emphasize the impact of different adjustment costs in the transmission mechanism for real shocks.

2.2 The Firms' Problem

There is a continuum of firms in each country located in the interval $[0, 1]$. Each firm supplies the home and foreign market, and sets prices under local currency pricing (henceforth, LCP). Firms engage in third-degree price discrimination across markets (re-selling is infeasible) and, furthermore, enjoy monopolistic power in their own variety. Relocation is not allowed. Frictions in the goods market are modelled with nominal price stickiness à la Calvo (1983). At time t any domestic or foreign firm is forced to maintain its previous period prices in the domestic and foreign markets with probability $\alpha \in (0, 1)$. Alternatively, the firm receives a signal to optimally reset each price with probability $(1 - \alpha)$.

We assume that firms employ a homogeneous of degree one, Cobb-Douglas technology, i.e.

$$
\begin{aligned}
Y_t(h) &= A_t (K_t(h))^{1-\psi} (L_t(h))^{\psi}, \ \forall h \in [0, 1], && (41) \\
Y_t^*(f) &= A_t^* (K_t^*(f))^{1-\psi} (L_t^*(f))^{\psi}, \ \forall f \in [0, 1], && (42)
\end{aligned}
$$

where A_t is the domestic productivity shock and A_t^* is the foreign productivity shock. The productivity shocks in logs, $a_t \equiv \ln(A_t)$ and $a_t^* \equiv \ln(A_t^*)$ follow two $AR(1)$ processes of the form,

$$a_t = (1-\rho_a)\,\overline{a} + \rho_a a_{t-1} + \varepsilon_t^a, \quad |\rho_a| < 1, \tag{43}$$

$$a_t^* = (1-\rho_a)\,\overline{a}^* + \rho_a a_{t-1}^* + \varepsilon_t^{a*}, \quad |\rho_a| < 1, \tag{44}$$

where ε_t^a and ε_t^{a*} are zero mean, correlated (i.e. $corr(\varepsilon_t^a, \varepsilon_t^{a*}) \neq 0$), and normally-distributed innovations with a common standard deviation (i.e. $\sigma(\varepsilon_t^a) = \sigma(\varepsilon_t^{a*})$). The unconditional means of the productivity shocks are denoted generically $\overline{a}$ and $\overline{a}^*$, and normalized to one.

The labor share in the production function is represented by the parameter $\psi \in (0,1]$. By market clearing and the immobility of capital across borders, it follows that the aggregate capital supplied by the households in each country corresponds to the aggregate demand by the local firms, i.e.

$$K_t = \int_0^1 K_t(h)\,dh, \; K_t^* = \int_0^1 K_t(f)\,df.$$

Similarly, we get by market clearing and the immobility of labor across countries that,

$$L_t = \int_0^1 L_t(h)\,dh, \; L_t^* = \int_0^1 L_t(f)\,df.$$

Solving the cost-minimization problem of each individual firm yields an efficiency condition linking the capital-to-labor ratios to factor price ratios as follows,

$$\frac{K_t}{L_t} = \frac{K_t(h)}{L_t(h)} = \frac{1-\psi}{\psi}\frac{W_t}{Z_t}, \; \forall h \in [0,1], \tag{45}$$

$$\frac{K_t^*}{L_t^*} = \frac{K_t^*(f)}{L_t^*(f)} = \frac{1-\psi}{\psi}\frac{W_t^*}{Z_t^*}, \; \forall f \in [0,1], \tag{46}$$

as well as a characterization for the nominal marginal costs,

$$MC_t = \frac{1}{A_t}\frac{1}{\psi^\psi (1-\psi)^{1-\psi}} (W_t)^\psi (Z_t)^{1-\psi}, \tag{47}$$

$$MC_t^* = \frac{1}{A_t^*}\frac{1}{\psi^\psi (1-\psi)^{1-\psi}} (W_t^*)^\psi (Z_t^*)^{1-\psi}. \tag{48}$$

The factors (capital and labor) are homogenous within a country and immobile across borders, and the local factor markets are perfectly competitive.

Factor prices equalize in each country (but not necessarily across countries), i.e. $W_t(h) = W_t$ and $Z_t(h) = Z_t$ for all $h \in [0,1]$ as well as $W_t^*(f) = W_t^*$ and $Z_t^*(f) = Z_t^*$ for all $f \in [0,1]$. Since the production function is homogeneous of degree one (constant returns-to-scale), this implies that all local firms choose the same capital-to-labor ratio as implied by equations (45) – (46). The evolution of relative factor prices, $\frac{W_t}{Z_t}$ and $\frac{W_t^*}{Z_t^*}$, determines the firm's incentives to become more or less capital-intensive in each period.

A re-optimizing domestic firm h chooses a domestic and a foreign price, $\widetilde{P}_t(h)$ and $\widetilde{P}_t^*(h)$, to maximize the expected discounted value of its net profits,

$$\sum_{\tau=0}^{\infty} \mathbb{E}_t \left\{ \alpha^{\tau} M_{t,t+\tau} \left[\left(\widetilde{C}_{t,t+\tau}(h) + \widetilde{X}_{t,t+\tau}(h) \right) \left(\widetilde{P}_t(h) - MC_{t+\tau} \right) + \left(\widetilde{C}^*_{t,t+\tau}(h) + \widetilde{X}^*_{t,t+\tau}(h) \right) \left(S_{t+\tau} \widetilde{P}^*_t(h) - MC_{t+\tau} \right) \right] \right\}, \tag{49}$$

where $M_{t,t+\tau} \equiv \beta^{\tau} \left(\frac{C_{t+\tau}}{C_t} \right)^{-\sigma^{-1}} \frac{P_t}{P_{t+\tau}}$ is the domestic stochastic discount factor (SDF) for τ-periods ahead nominal payoffs, subject to a pair of demand constraints in each goods market,

$$\widetilde{C}_{t,t+\tau}(h) + \widetilde{X}_{t,t+\tau}(h) = \left(\frac{\widetilde{P}_t(h)}{P^H_{t+\tau}} \right)^{-\theta} \left(C^H_{t+\tau} + X^H_{t+\tau} \right), \ \forall h \in [0,1], \tag{50}$$

$$\widetilde{C}^*_{t,t+\tau}(h) + \widetilde{X}^*_{t,t+\tau}(h) = \left(\frac{\widetilde{P}^*_t(h)}{P^{H*}_{t+\tau}} \right)^{-\theta} \left(C^{H*}_{t+\tau} + X^{H*}_{t+\tau} \right), \ \forall h \in [0,1], \tag{51}$$

where $\widetilde{C}_{t,t+\tau}(h)$ and $\widetilde{C}^*_{t,t+\tau}(h)$ indicate the consumption demand for any variety h at home and abroad respectively, given that prices $\widetilde{P}_t(h)$ and $\widetilde{P}^*_t(h)$ remain unchanged between time t and $t+\tau$. Similarly, $\widetilde{X}_{t,t+\tau}(h)$ and $\widetilde{X}^*_{t,t+\tau}(h)$ indicate the households' investment demand.[5] We characterize the objective of the re-optimizing foreign firm f as,

$$\sum_{\tau=0}^{\infty} \mathbb{E}_t \left\{ \alpha^{\tau} M^*_{t,t+\tau} \left[\left(\widetilde{C}_{t,t+\tau}(f) + \widetilde{X}_{t,t+\tau}(f) \right) \left(\frac{\widetilde{P}_t(f)}{S_{t+\tau}} - MC^*_{t+\tau} \right) + \left(\widetilde{C}^*_{t,t+\tau}(f) + \widetilde{X}^*_{t,t+\tau}(f) \right) \left(\widetilde{P}^*_t(f) - MC^*_{t+\tau} \right) \right] \right\}, \tag{52}$$

where $M^*_{t,t+\tau} \equiv \beta \left(\frac{C^*_{t+\tau}}{C^*_t} \right)^{-\sigma^{-1}} \frac{P^*_t}{P^*_{t+\tau}}$ is the foreign SDF for τ-periods ahead nominal payoffs. The demand constraints of the foreign firm are,

$$\widetilde{C}_{t,t+\tau}(f) + \widetilde{X}_{t,t+\tau}(f) = \left(\frac{\widetilde{P}_t(f)}{P^F_{t+\tau}} \right)^{-\theta} \left(C^F_{t+\tau} + X^F_{t+\tau} \right), \ \forall f \in [0,1], \tag{53}$$

$$\widetilde{C}^*_{t,t+\tau}(f) + \widetilde{X}^*_{t,t+\tau}(f) = \left(\frac{\widetilde{P}^*_t(f)}{P^{F*}_{t+\tau}} \right)^{-\theta} \left(C^{F*}_{t+\tau} + X^{F*}_{t+\tau} \right), \ \forall f \in [0,1], \tag{54}$$

given that prices $\widetilde{P}_t(f)$ and $\widetilde{P}^*_t(f)$ remain unchanged between time t and $t+\tau$.

As usual, the firm's problem is solved under the implicit assumption that domestic and foreign markets ought to be supplied with as much of the variety as it is demanded at the prevailing prices. In other words, rationing is not an option and so it is possible that firms with sticky prices will incur losses in some periods that would have to be 'covered' by their shareholders, the local households.

[5] We derive the demand for variety h in the home and foreign markets by combining the first-order conditions in (16) – (17) and the analogous counterparts for the foreign country.

The Optimality Conditions (Optimal Pricing Policy). The necessary and sufficient first-order conditions for the domestic firm producing variety h give us the following pair of price-setting formulas,

$$\sum_{\tau=0}^{\infty} \alpha^{\tau} \mathbb{E}_t \left[M_{t,t+\tau} \left(\widetilde{C}_{t,t+\tau}(h) + \widetilde{X}_{t,t+\tau}(h) \right) \left(\widetilde{P}_t(h) - \frac{\theta}{\theta - 1} MC_{t+\tau} \right) \right] = 0, \tag{55}$$

$$\sum_{\tau=0}^{\infty} \alpha^{\tau} \mathbb{E}_t \left[M_{t,t+\tau} \left(\widetilde{C}^*_{t,t+\tau}(h) + \widetilde{X}^*_{t,t+\tau}(h) \right) \left(S_{t+\tau} \widetilde{P}^*_t(h) - \frac{\theta}{\theta - 1} MC_{t+\tau} \right) \right] = 0. \tag{56}$$

Similarly, the first-order conditions for the foreign firm producing variety f give us the following price-setting formulas,

$$\mathbb{E}_t \left[\sum_{\tau=0}^{\infty} \alpha^{\tau} M^*_{t,t+\tau} \left(\widetilde{C}_{t,t+\tau}(f) + \widetilde{X}_{t,t+\tau}(f) \right) \left(\frac{\widetilde{P}_t(f)}{S_{t+\tau}} - \frac{\theta}{\theta - 1} MC^*_{t+\tau} \right) \right] = 0, \tag{57}$$

$$\mathbb{E}_t \left[\sum_{\tau=0}^{\infty} \alpha^{\tau} M^*_{t,t+\tau} \left(\widetilde{C}^*_{t,t+\tau}(f) + \widetilde{X}^*_{t,t+\tau}(f) \right) \left(\widetilde{P}^*_t(f) - \frac{\theta}{\theta - 1} MC^*_{t+\tau} \right) \right] = 0. \tag{58}$$

Using the Calvo randomization assumption and the inherent symmetry of all the firms, the price sub-indexes on domestic varieties, P^H_t and P^{H*}_t, become,

$$P^H_t = \left[\alpha \left(P^H_{t-1} \right)^{1-\theta} + (1-\alpha) \left(\widetilde{P}_t(h) \right)^{1-\theta} \right]^{\frac{1}{1-\theta}}, \tag{59}$$

$$P^{H*}_t = \left[\alpha \left(P^{H*}_{t-1} \right)^{1-\theta} + (1-\alpha) \left(\widetilde{P}^*_t(h) \right)^{1-\theta} \right]^{\frac{1}{1-\theta}}, \tag{60}$$

while the price sub-indexes on foreign varieties, P^F_t and P^{F*}_t, are computed as,

$$P^F_t = \left[\alpha \left(P^F_{t-1} \right)^{1-\theta} + (1-\alpha) \left(\widetilde{P}_t(f) \right)^{1-\theta} \right]^{\frac{1}{1-\theta}}, \tag{61}$$

$$P^{F*}_t = \left[\alpha \left(P^{F*}_{t-1} \right)^{1-\theta} + (1-\alpha) \left(\widetilde{P}^*_t(f) \right)^{1-\theta} \right]^{\frac{1}{1-\theta}}. \tag{62}$$

Equations (59) – (62) are a convenient way to reformulate (13) – (14). These expressions are fundamental to aggregate the pricing decisions of firms and characterize their effect on the dynamics of inflation.

The production functions in (41) – (42) can be re-written as,

$$Y_t(h) = A_t \left(\frac{K_t}{L_t} \right)^{1-\psi} L_t(h), \ \forall h \in [0,1],$$

$$Y^*_t(f) = A^*_t \left(\frac{K^*_t}{L^*_t} \right)^{1-\psi} L^*_t(f), \ \forall f \in [0,1],$$

since capital-to-labor ratios are equated across all firms.[6] If we define aggregate output in each country as

[6] Since pricing decisions are not synchronized, the labor and capital used in each firm and the output produced are likely to differ.

$Y_t \equiv \int_0^1 Y_t(h)\,dh$ and $Y_t^* \equiv \int_0^1 Y_t(f)\,df$, we get by market clearing in the labor market that,

$$\begin{aligned} Y_t &= A_t (K_t)^{1-\psi} (L_t)^{\psi}, &(63)\\ Y_t^* &= A_t^* (K_t^*)^{1-\psi} (L_t^*)^{\psi}. &(64)\end{aligned}$$

Combining equations (63) – (64) with the efficiency conditions in (45) – (46) and the labor supply equations from the households' problem in (27) – (28), we can express the real rental rate of capital in terms of productivity shocks, consumption, output and capital,

$$\begin{aligned} \frac{Z_t}{P_t} &= \frac{1-\psi}{\psi}\frac{W_t}{P_t}\frac{L_t}{K_t} = \frac{1-\psi}{\psi}(A_t)^{-\frac{1+\varphi}{\psi}}(C_t)^{\sigma^{-1}}(Y_t)^{\frac{1+\varphi}{\psi}}(K_t)^{-\left(\frac{1+(1-\psi)\varphi}{\psi}\right)}, &(65)\\ \frac{Z_t^*}{P_t^*} &= \frac{1-\psi}{\psi}\frac{W_t^*}{P_t^*}\frac{L_t^*}{K_t^*} = \frac{1-\psi}{\psi}(A_t^*)^{-\frac{1+\varphi}{\psi}}(C_t^*)^{\sigma^{-1}}(Y_t^*)^{\frac{1+\varphi}{\psi}}(K_t^*)^{-\left(\frac{1+(1-\psi)\varphi}{\psi}\right)}. &(66)\end{aligned}$$

Manipulating this pair of conditions a little bit more allows us to re-write the real wages in terms of real returns on capital as well as productivity shocks, consumption, output and capital, i.e.

$$\begin{aligned} \frac{W_t}{P_t} &= \left(\frac{1-\psi}{\psi}\right)^{-\frac{(1-\psi)\varphi}{1+(1-\psi)\varphi}}(A_t)^{-\frac{\varphi}{1+(1-\psi)\varphi}}(C_t)^{\frac{\sigma^{-1}}{1+(1-\psi)\varphi}}(Y_t)^{\frac{\varphi}{1+(1-\psi)\varphi}}\left(\frac{Z_t}{P_t}\right)^{\frac{(1-\psi)\varphi}{1+(1-\psi)\varphi}}, &(67)\\ \frac{W_t^*}{P_t^*} &= \left(\frac{1-\psi}{\psi}\right)^{-\frac{(1-\psi)}{1+(1-\psi)\varphi}\varphi}(A_t^*)^{-\frac{\varphi}{1+(1-\psi)\varphi}}(C_t^*)^{\frac{\sigma^{-1}}{1+(1-\psi)\varphi}}(Y_t^*)^{\frac{\varphi}{1+(1-\psi)\varphi}}\left(\frac{Z_t^*}{P_t^*}\right)^{\frac{(1-\psi)\varphi}{1+(1-\psi)\varphi}}. &(68)\end{aligned}$$

These two equations suffice for the purpose of replacing real wages out of the marginal cost equations in (47) – (48).

2.3 The Monetary Policy Rules

We assume a cashless limit economy as in Woodford (2003). Monetary policy has an impact on inflation by regulating short-term nominal interest rates, and it has real effects because it interacts with nominal rigidities. Since the Taylor (1993) rule has become the trademark of modern monetary policy, we assume that the monetary authorities set short-term nominal interest rates accordingly, i.e.

$$\begin{aligned} i_t &= \rho_i i_{t-1} + (1-\rho_i)\left[\bar{i} + \psi_\pi(\pi_t - \bar{\pi}) + \psi_y(y_t - \bar{y})\right], &(69)\\ i_t^* &= \rho_i i_{t-1}^* + (1-\rho_i)\left[\bar{i}^* + \psi_\pi(\pi_t^* - \bar{\pi}^*) + \psi_y(y_t^* - \bar{y}^*)\right]. &(70)\end{aligned}$$

We define all the variables in (69) and (70) in logs. Hence, $i_t \equiv \ln(I_t)$ and $i_t^* \equiv \ln(I_t^*)$ are the monetary policy instruments of both countries, $\pi_t \equiv \ln(P_t) - \ln(P_{t-1})$ and $\pi_t^* \equiv \ln(P_t^*) - \ln\left(P_{t-1}^*\right)$ are the (gross) CPI inflation rates in logs, while $y_t \equiv \ln(Y_t)$ and $y_t^* \equiv \ln(Y_t^*)$ denote output in logs. The variables identified with an upper bar on top and no time-subscript are the corresponding steady states.

These symmetric policy rules target deviations of output and inflation from their long-run trends, and ignore the possibility of discretionary monetary policy shocks. The weights assigned to deviations of output and inflation are $\psi_y > 0$ and $\psi_\pi > 0$, respectively. In keeping with much of the literature, we augment the rule proposed by Taylor (1993) with an interest rate smoothing term regulated by the inertia parameter $0 < \rho_i < 1$. Nonetheless, the rule studied by Taylor (1993) can be seen as a special case of equations (69) and (70) where $\rho_i = 0$, $\psi_y = 0.5$ and $\psi_\pi = 1.5$.[7]

3 Investment, Trade and ToT

We posit the existence of a deterministic, zero-inflation steady state (with zero net exports). We log-linearize the equilibrium conditions around this steady state and report them in the Appendix. We refer the interested reader to Martínez-García and Søndergaard (2008a) for details on the derivation of the steady state and the system of log-linearized equilibrium conditions. Here, we put the emphasis on the log-linear equations that characterize the terms of trade and the net exports share over GDP in the model. As a notational convention, from now any variable identified with lower-case letters and a caret on top will represent a transformation (expressed in log deviations relative to steady state) of the corresponding variable in upper-case letters.

International Relative Prices. Domestic terms of trade, ToT_t, represent the value of the imported good (quoted in the domestic market) relative to the value of the domestic good exported to the foreign market, but expressed in units of the domestic currency. Similarly for the foreign terms of trade, ToT_t^*. This conventional definition of the terms of trade measures the 'foreign market' cost of replacing one unit of imports with one unit of exports of the locally-produced good, and can be formally expressed as follows,

$$ToT_t \equiv \frac{P_t^F}{S_t P_t^{H*}} = D_t \frac{P_t^F}{P_t^H}, \tag{71}$$

$$ToT_t^* \equiv \frac{S_t P_t^{H*}}{P_t^F} = D_t^* \frac{P_t^{H*}}{P_t^{F*}} = \frac{1}{ToT_t}, \tag{72}$$

where D_t and D_t^* capture the deviations of the law of one price (LOOP) across the border for the domestic and foreign bundle of goods respectively, i.e.

$$D_t \equiv \frac{P_t^H}{S_t P_t^{H*}},\ D_t^* \equiv \frac{S_t P_t^{F*}}{P_t^F},$$

We also define a pair of international relative prices, T_t and T_t^*, as,

$$T_t \equiv \frac{P_t^F}{P_t^H}, \tag{73}$$

$$T_t^* \equiv \frac{P_t^{H*}}{P_t^{F*}} = \frac{1}{D_t D_t^* T_t}, \tag{74}$$

The relative price T_t represents the value of the imported good (quoted in the domestic market) relative to the value of the domestic good sold in the domestic market. Similarly for the foreign relative price, T_t^*. The ratios T_t and T_t^* are the 'local market' cost of replacing one unit of imports with one unit of the locally-produced and locally-supplied good.

Terms of trade, ToT_t and ToT_t^*, and the international relative prices, T_t and T_t^*, ought to be identical for each country pair if the LOOP condition holds across countries, i.e. if $D_t = D_t^* = 1$. The joint

[7] Taylor's (1993, p. 202) rule for the U.S. "rises if inflation increases above a target of 2 percent or if real GDP rises above trend GDP (which equals 2.2 percent per year). If both the inflation rate and real GDP are on target, then the federal funds rate would equal 4 percent, or 2 percent in real terms."

assumption of nominal rigidities and local-currency pricing (LCP) implies that the LOOP fails. Therefore, while the distinction between ToT and other international relative prices is unnecessary in a standard Q-IRBC model (with flexible prices), it becomes relevant for our understanding of the patterns of trade in a Q-INNS environment.

After log-linearizing the definitions in (71) – (72) and (73) – (74), we get that,

$$\begin{aligned}
\widehat{tot}_t &= \widehat{d}_t + \widehat{t}_t, \\
\widehat{t}_t &= \widehat{p}_t^F - \widehat{p}_t^H, \\
\widehat{tot}_t^* &= -\widehat{tot}_t = \widehat{d}_t^* + \widehat{t}_t^*, \\
\widehat{t}_t^* &= -\left(\widehat{p}_t^{F*} - \widehat{p}_t^{H*}\right) = \widehat{p}_t^{H*} - \widehat{p}_t^{F*},
\end{aligned}$$

where $\widehat{d}_t \equiv \left(\widehat{p}_t^H - \widehat{s}_t - \widehat{p}_t^{H*}\right)$ and $\widehat{d}_t^* \equiv \left(\widehat{s}_t + \widehat{p}_t^{F*} - \widehat{p}_t^F\right)$ are the deviations of the LOOP. With this log-linear equalities, we define the model-consistent measure of world terms of trade as $\widehat{t}_t^W \equiv \widehat{p}_t^{F,W*} - \widehat{p}_t^{W*}$, where $\widehat{p}_t^{F,W*} \equiv \phi_F \widehat{p}_t^F + \phi_H \widehat{p}_t^{F*}$ and $\widehat{p}_t^{W*} \equiv \phi_F \widehat{p}_t + \phi_H \widehat{p}_t^*$. After some algebra, we find that world terms of trade is approximately proportional to the difference between the international relative prices, $\widehat{t}_t$ and $\widehat{t}_t^*$, i.e.

$$\widehat{t}_t^W \approx (1 - \phi_F)\, \phi_F \left[\widehat{t}_t - \widehat{t}_t^*\right], \tag{75}$$

where $\phi_H = 1 - \phi_F$. This approximation is based on the log-linearization of the consumption-price indexes in (11) and (12), which implies that $\widehat{p}_t \approx \phi_H \widehat{p}_t^H + \phi_F \widehat{p}_t^F$ and $\widehat{p}_t^* \approx \phi_F \widehat{p}_t^{H*} + \phi_H \widehat{p}_t^{F*}$. *Ceteris paribus*, an increase in the world terms of trade, $\widehat{t}_t^W$, shifts world consumption and investment spending away from the foreign goods and into the domestic goods, pushing upwards the net exports share for the domestic country.

Using the definition of the international relative prices $\widehat{t}_t$ and $\widehat{t}_t^*$ we can alternatively re-write the world terms of trade as,[8]

$$\widehat{t}_t^W \approx 2\,(1 - \phi_F)\, \phi_F \widehat{tot}_t - \widehat{d}_t^W, \tag{76}$$

where we define a model-consistent measure of world deviations of the LOOP as $\widehat{d}_t^W \equiv (1 - \phi_F)\, \phi_F \left[\widehat{d}_t - \widehat{d}_t^*\right]$. These calculations show that movements in the world terms of trade can be thought as coming from fluctuations in the world measure of deviations of the LOOP or from fluctuations in a conventional measure of domestic ToT. In a standard Q-IRBC model with flexible prices, $\widehat{d}_t = \widehat{d}_t^* = \widehat{d}_t^W = 0$ and ToT is simply proportional to world terms of trade.

It is important to distinguish between $\widehat{t}_t^W$ and $\widehat{tot}_t$ at least on two accounts. First, because if we compare the full-blown Q-INNS model with deviations of the LOOP to the data, we must recognize that the relevant international relative price for expenditure-switching effects, $\widehat{t}_t^W$, does not exactly correspond to the data available on ToT. Second, because it shows that the expenditure-switching implied by changes in international relative prices is influenced by the the same distortions that introduce deviations of the LOOP in the model.

Another important international relative price is the real exchange rate defined in (15), i.e. $RS_t \equiv \frac{S_t P_t^*}{P_t}$.

[8] The number 2 in equation (76) simply reflects our assumption that the world economy is formed by two countries of mass one each.

Using the log-linearization of the consumption-price indexes in (11) and (12), it can be shown that,

$$\begin{aligned}\widehat{rs}_t &\approx \frac{1}{\phi_F}\widehat{t}_t^W - \widehat{tot}_t \\ &\approx (1-2\phi_F)\,\widehat{tot}_t - \frac{1}{\phi_F}\widehat{d}_t^W,\end{aligned} \tag{77}$$

where $(\phi_H - \phi_F) = (1-2\phi_F)$. This expression is also quite illuminating in itself. It neatly shows that real exchange rate fluctuations arise from two channels: on one hand, compositional differences in the basket of goods across countries due to home bias in preferences; on the other hand, deviations from the LOOP. In a flexible price model the real exchange rate is purely proportional to terms of trade, and that severely restricts the ability of Q-IRBC models (if they rely on this channel alone) to match the empirical features of the real exchange rate and ToT.

In our benchmark model, independently of whether we allow for deviations of the LOOP or not, equation (77) implies that we can express world terms of trade as being proportional to the real exchange rate itself plus the ToT, i.e.

$$\widehat{t}_t^W \approx \phi_F\left(\widehat{tot}_t + \widehat{rs}_t\right). \tag{78}$$

In other words, the deviations of the LOOP can be fully subsumed into a linear combination between a conventional measure of ToT and the real exchange rate, both of which are observable in the data (unlike $\widehat{t}_t^W$). Equation (78) suggests that in models with deviations of the LOOP this is really a crucial variable to pin down the expenditure switching effects due to international relative price movements. While the exploration of the dynamics of the real exchange rate goes beyond the scope of this paper, we refer the interested reader to Martínez-García and Søndergaard (2008b) for a deeper exploration of the issue in the Q-INNS model.

Net Exports Share over GDP. In a two-country model, suffices to determine the net exports share of the domestic country. The share of the foreign country is simply the additive inverse of the domestic share. A simple log-linearization of real exports and imports in equations (20) and (21) allows us to obtain the following pair of equations,

$$\begin{aligned}\widehat{exp}_t &\approx -\eta\left(\widehat{p}_t^{H*} - \widehat{p}_t^*\right) + (1-\gamma_x)\,\widehat{c}_t^* + \gamma_x\widehat{x}_t^*, \\ \widehat{imp}_t &\approx -\eta\left(\widehat{p}_t^F - \widehat{p}_t\right) + (1-\gamma_x)\,\widehat{c}_t + \gamma_x\widehat{x}_t,\end{aligned}$$

where the relative price distortion at the variety level embedded in the price sub-indexes (13) – (14) turns out to be only of second-order importance. In other words, the first-order effects of relative price dispersion at the variety level are negligible in our log-linearization of real exports and imports.

The net export share over GDP in deviations from its steady state is defined as,

$$\begin{aligned}\widehat{tb}_t &\equiv \phi_F\left(\widehat{exp}_t - \widehat{imp}_t\right) \\ &\approx -\eta\left(\phi_F\left[\left(\widehat{p}_t^{H*} - \widehat{p}_t^*\right) - \left(\widehat{p}_t^F - \widehat{p}_t\right)\right]\right) - (1-\gamma_x)\,\phi_F\left(\widehat{c}_t - \widehat{c}_t^*\right) - \gamma_x\phi_F\left(\widehat{x}_t - \widehat{x}_t^*\right).\end{aligned} \tag{79}$$

In steady state, ϕ_F is the domestic imports share over domestic GDP in real terms and ϕ_H^* is the foreign imports share (where foreign imports are equal to domestic exports) over foreign GDP in real terms. Given the symmetric home bias assumption, i.e. $\phi_H^* = \phi_F$, and the fact that the steady state is symmetric, i.e. $\overline{Y} = \overline{Y}^*$, the weighted difference between real exports and imports in (79) can be interpreted as the net exports share over GDP.[9]

We define two measures of world price sub-indexes as $\widehat{p}_t^{H,W} \equiv \phi_H \widehat{p}_t^H + \phi_F \widehat{p}_t^{H*}$ and $\widehat{p}_t^{F,W*} \equiv \phi_F \widehat{p}_t^F + \phi_H \widehat{p}_t^{F*}$, and two measures of the relative price sub-indexes as $\widehat{p}_t^{H,R} \equiv \widehat{p}_t^H - \widehat{p}_t^{H*}$ and $\widehat{p}_t^{F,R} \equiv \widehat{p}_t^F - \widehat{p}_t^{F*}$. We already used $\widehat{p}_t^{F,W*}$ and $\widehat{p}_t^{W*}$ to define a model-consistent measure of world terms of trade. Here, we use these definitions coupled with the log-linearization of the consumption-price indexes in (11) and (12) in order to express the relative prices embedded in equation (79) in the following terms, i.e.

$$\begin{aligned}
\widehat{p}_t^{H*} - \widehat{p}_t^* &= \widehat{p}_t^{H,W} - \widehat{p}_t^W - \phi_H \left(\widehat{p}_t^{H,R} - \widehat{p}_t^R\right), \\
\widehat{p}_t^F - \widehat{p}_t &= \widehat{p}_t^{F,W*} - \widehat{p}_t^{W*} + \phi_H \left(\widehat{p}_t^{F,R} - \widehat{p}_t^R\right),
\end{aligned}$$

where the relative CPI is $\widehat{p}_t^R \equiv \widehat{p}_t - \widehat{p}_t^*$.

The log-linearization of the CPIs in both countries can be re-written as,

$$\begin{aligned}
\phi_H \left[\widehat{p}_t^H - \widehat{p}_t\right] + \phi_F \left[\widehat{p}_t^F - \widehat{p}_t\right] &\approx 0, \\
\phi_F \left[\widehat{p}_t^{H*} - \widehat{p}_t^*\right] + \phi_H \left[\widehat{p}_t^{F*} - \widehat{p}_t^*\right] &\approx 0.
\end{aligned}$$

Based on these relationships, we can infer that,

$$\phi_F \left[\left(\widehat{p}_t^{H,W} - \widehat{p}_t^W\right) - \phi_H \left(\widehat{p}_t^{H,R} - \widehat{p}_t^R\right)\right] + \phi_H \left[\left(\widehat{p}_t^{F,W*} - \widehat{p}_t^{W*}\right) - \phi_F \left(\widehat{p}_t^{F,R} - \widehat{p}_t^R\right)\right] \approx 0. \tag{80}$$

Using the approximation derived in (80) and the definition of the world terms of trade, $\widehat{t}_t^W \equiv \widehat{p}_t^{F,W*} - \widehat{p}_t^{W*}$, we can write after some algebra the relevant relative prices as follows,

$$\begin{aligned}
\phi_F \left(\widehat{p}_t^{H*} - \widehat{p}_t^*\right) &\approx -\phi_H \left[\widehat{t}_t^W - \phi_F \left(\widehat{p}_t^{F,R} - \widehat{p}_t^R\right)\right], \\
\phi_F \left(\widehat{p}_t^F - \widehat{p}_t\right) &\approx \phi_F \left[\widehat{t}_t^W + \phi_H \left(\widehat{p}_t^{F,R} - \widehat{p}_t^R\right)\right],
\end{aligned}$$

which naturally implies that,

$$\begin{aligned}
\phi_F \left[\left(\widehat{p}_t^{H*} - \widehat{p}_t^*\right) - \left(\widehat{p}_t^F - \widehat{p}_t\right)\right] &\approx -\phi_H \left[\widehat{t}_t^W - \phi_F \left(\widehat{p}_t^{F,R} - \widehat{p}_t^R\right)\right] - \phi_F \left[\widehat{t}_t^W + \phi_H \left(\widehat{p}_t^{F,R} - \widehat{p}_t^R\right)\right] \\
&= -\left(\phi_H + \phi_F\right) \widehat{t}_t^W = -\widehat{t}_t^W.
\end{aligned}$$

Hence, replacing this expression into equation (79) we infer that the net exports share can be calculated as,

$$\widehat{tb}_t \approx \eta \widehat{t}_t^W - \left(1 - \gamma_x\right) \phi_F \left(\widehat{c}_t - \widehat{c}_t^*\right) - \gamma_x \phi_F \left(\widehat{x}_t - \widehat{x}_t^*\right). \tag{81}$$

This equations gives a precise meaning to our early claim that the world terms of trade, $\widehat{t}_t^W$, is the model-consistent measure of international relative prices that explains the expenditure-switching across countries.

[9] A simple look at equations (76) and (77) on one hand and equation (79) on the other hand, suggests that there is a trade-off between quantities (net exports) and international relative prices which crucially depends on the parameterization of the steady state imports share ϕ_F.

The differences in consumption and investment across countries reflect also the income effects on the demand, and their contribution to shifting resources across countries.

In other words, our measure of the net exports share is equivalent to the difference between the log of real exports and imports (in deviations relative to their respective steady states), scaled by the parameter ϕ_F. Adjustment in trade comes directly through movements in the world terms of trade, $\widehat{t}_t^W$, or from relative adjustments in consumption and investment across countries. This is the central equation in our analysis of the trade patterns. A well-known fact in IRBC and INNS models alike is that investment movements tend to be much larger than movements in consumption (see, e.g., BKK, 1995, Heathcote and Perri, 2002, CKM, 2002, and Raffo, 2008).

Here, we revisit the old question of what role does investment play in trade, but we do so with a two-sided strategy. On one hand, we look at the role of adjustment costs in the formation of capital investment. We recognize that adjustment costs have a role to play in modulating the volatility of investment and consumption, and therefore can alter the trade dynamics. On the other hand, we recognize that Q-INNS models with deviations of the LOOP could lead to distortions in the allocation of expenditures across countries. We consider this additional channel to evaluate and quantify the impact of those distortions on net trade flows.

Our previous discussion on the characterization of an appropriate international relative price measure allows us to re-write equation (81) as,

$$\widehat{tb}_t \approx 2\eta (1-\phi_F) \phi_F \widehat{tot}_t - \eta \widehat{d}_t^W - (1-\gamma_x) \phi_F (\widehat{c}_t - \widehat{c}_t^*) - \gamma_x \phi_F (\widehat{x}_t - \widehat{x}_t^*),$$

which illustrates mechanically the way in which the world relative price distortion, $\widehat{d}_t^W$, operates on the trade balance. In the flexible price case, obviously, all that is needed is a conventional measure of domestic ToT to account for the expenditure-switching effects. In turn, equation (78) allows us to express net exports as a function of observable international relative prices for a more generic model (presumably with deviations of the LOOP), i.e.

$$\widehat{tb}_t \approx \eta \phi_F \left(\widehat{tot}_t + \widehat{rs}_t \right) - (1-\gamma_x) \phi_F (\widehat{c}_t - \widehat{c}_t^*) - \gamma_x \phi_F (\widehat{x}_t - \widehat{x}_t^*).$$

This alternative expression has an added value. It indicates that in a broad class of Q-INNS models the international relative price effects on expenditure-switching cannot be fully accounted with conventional ToT or the real exchange rate alone, as it occurs in most Q-IRBC models with flexible prices. Instead, both variables are needed to capture those relative price effects.

The analytic derivation of the net exports share in equation (81) and the conventional ToT implicit in equation (78) does not constitute a model in itself. All the other variables on the right- and left-hand side are endogenous, and their dynamics are determined by the full-blown model reviewed in this paper (see the summary of the log-linearization in the Appendix)[10]. The fact that these relationships, (81) and (78), hold up to a first-order approximation gives us a way to apportion the blame whenever the model does not deliver empirically-consistent predictions, and a way to mechanically identify how the different mechanisms

[10] In fact, notice that the log-linear system of equations described in the Appendix can be simulated excluding equations (81) and (78). Then, the endogenous variables $\widehat{tb}_t$ and $\widehat{tot}_t$ can be fully characterized as functions of the endogenous variables in that 'core' model.

for the propagation of shocks operate. Here, we exploit these relationship to focus our attention on the role of investment in trade, and how it is influenced by the presence of adjustment costs and/or large fractions of firms ‘unable’ to update their prices in every period.

4 Quantitative Findings

4.1 Model Calibration

Our benchmark is Q-INNS model, and the choice of parameter values is summarized in Table 2. For comparison purposes, we follow quite closely the parameterization of a similar Q-INNS model in CKM (2002). We refer the interested reader to their paper for a complete discussion of the calibration. Here, we only comment on those parameters that we calibrate differently.

[Insert Table 2 about here]

The Calvo price stickiness parameter, α, is assumed to be 0.75. This implies that the average price duration in our model is 4 quarters. This is comparable with CKM (2002), where a quarter of the firms re-set prices every period and those prices remain fixed for a total of 4 periods. We also study the implications of the model under (quasi-) flexible prices. We do not simulate an exact solution for a comparable Q-IRBC model. Instead, we approximate the flexible price scenario by bringing the Calvo parameter, α, down to 0.00001 in our benchmark Q-INNS model. This implies that 99.999% of the firms are able to re-optimize, and only a negligible fraction of them is subject to maintain its previous period prices in each period.[11]

The inverse of the Frisch elasticity of labor supply, φ, is set to 3 instead of 5 as in CKM (2002). This is compatible with the available micro evidence (see, e.g., Browning, Hansen and Heckman, 1999, and Blundell and MaCurdy, 1999), but not consistent with a balanced growth path. This choice is meant to reduce the sensitivity of CPI inflation to movements in consumption and investment (see, e.g., Martínez-García and Søndergaard, 2008b). The parameterization of the monetary policy rule is slightly different than in CKM (2002). The interest rate inertia parameter, ρ_i, equals 0.85, while the weight on the inflation target, ψ_π, equals 2, and the weight on the output target, ψ_y, is 0.5. Our Taylor rule targets current inflation, instead of expected inflation as in CKM (2002). The rule also includes interest rate smoothing and gives more weight to inflation than the one proposed by Taylor (1993).

In CKM (2002), the volatility and cross-correlation of the real shocks are fixed for all the experiments to approximate the properties of the Solow residuals in the data. Then, they select in each of their experiments the volatility of the monetary innovations to match the volatility of U.S. real GDP, and the correlation of domestic and foreign monetary innovations to match the observed cross-correlation. They do so because their aim is to investigate whether a combination of real and monetary shocks that accounts for the movements in real GDP is also consistent with the movements of other macroeconomic variables.

In our model, business cycle fluctuations are exclusively driven by real shocks. We could calibrate our real shocks to approximate the stochastic properties of the Solow residuals. Instead, we adapt the calibration

[11] The experiment, here, does not imply that $\widehat{d}_t^W$ is equal to zero in the (quasi-) flexible price case. In fact, it would not be. Therefore, we should not think about this experiment as if it were equivalent to a standard Q-IRBC model, even though the patterns it displays along the dimensions of interest are similar to those found in the IRBC literature. Instead, the (quasi-) flexible price case merely reflects the limiting behavior of the Q-INNS model whenever the share of firms affected by the nominal rigidities becomes marginal (close to zero).

strategy of CKM (2002) by setting the parameters of the stochastic processes to approximate the features of U.S. real GDP in the data. Our experiments, therefore, evaluate to what extent consumption, investment, trade and conventional ToT can be replicated with a model that accounts for some key empirical moments on U.S. real GDP.

We assume two $AR(1)$ exogenous processes for the real shocks, and set their persistence, ρ_a, at 0.9 by default. The structure of the stochastic processes and the calibration of the real shock persistence are, therefore, similar to CKM (2002). In experiments with an approximate flexible price scenario, the persistence of output that can be attained lies below the empirical persistence of U.S. real GDP. Alternative choices for ρ_a at best provide marginal improvements in output persistence. Sticky price models with adjustment costs generate a better match with the data. As a sensitivity check, we report only a different calibration of ρ_a at 0.75 for the model with sticky prices and no adjustment costs. In this case, the alternative parameterization produces a significant enough improvement on the fit of output persistence (see Table 3).

Unlike for persistence, we can match the volatility and cross-correlation of real GDP precisely in each one of our experiments. We set the standard deviation of the real innovations to get the exact output volatility in the U.S. data (i.e., 1.54%). In addition, we calibrate the cross-country correlation of the innovations to replicate the observed cross-correlation of U.S. and Euro-zone GDP (i.e., 0.44).

Finally, CKM (2002) choose the adjustment cost parameter to match the empirical ratio of the standard deviation of consumption relative to the standard deviation of output in the data, while Raffo (2008) uses it to reproduce the volatility of investment relative to output. We select either the capital adjustment cost parameter, χ, or the investment adjustment cost parameter, κ, depending on the model specification, to ensure that investment volatility is as volatile as in the data (i.e., 3.38 times as volatile as U.S. real GDP). This is consistent with the aim of adopting a Q theory extension that delivers the best possible fit for investment.

4.2 Model Exploration

From equation (81) we know that the net export share must be linked to investment, consumption and world terms of trade. From equation (77) we also know that a complex interaction exists between world terms of trade, domestic ToT and world deviations of the LOOP. Based on the calibration described before and the log-linearization of the equilibrium conditions reported in the Appendix, we are able to simulate the model and gain further insight on the nature of those relationships. We are also able to assess the performance of the benchmark model relative to the observable data. The contemporaneous business cycle moments are summarized in Table 3.

[Insert Table 3 about here]

Our model results are broadly consistent with the existing literature. We find that in none of our experiments does the volatility of consumption get above 55% of the volatility of U.S. real consumption. Similar patterns can be found in BKK (1995), Heathcote and Perri (2002) and Raffo (2008). CKM (2002) do match the consumption volatility, but do so by driving the adjustment cost parameter up at the expense of making investment significantly smoother than in the data. Although consumption is slightly more volatile under investment adjustment costs (IAC) than capital adjustment costs (CAC), the improvement we find is insufficient to close the gap.

The trade off between investment and consumption volatility becomes particularly stark when we compare the IAC and CAC specifications against the no adjustment costs (NAC) case. Without adjustment costs the full power of capital accumulation as a mechanism for intertemporal smoothing comes to light. The consumption volatility produced by any variant of the model with sticky or flexible prices is less than 20% of the empirical volatility, while investment volatility is at least 67% higher. Overall, consumption volatility appears little affected by the choice of (quasi-) flexible prices or sticky prices.

The model also has difficulties matching the volatility net exports. In the (quasi-) flexible price experiments, adding adjustment costs to the model makes difficult the intertemporal smoothing of consumption difficult, and this leads to a higher reliance on trade for risk-sharing and smoothing consumption. We call this the intratemporal smoothing channel. This results in higher volatilities for the net exports share. In the sticky price experiments, the same patterns emerge. However, there is no clear evidence that the relative price distortion leads to systematively higher volatility of net exports. In any event, the predicted volatility for net exports is no more than 53% of the data. This is one dimension in which the INNS model performs better with the Q theory extension. The effect of sticky prices is not so significant, whenever we compare it against the competing (quasi-) flexible price scenario.

When we look at the findings on persistence, the picture that we get is also somewhat familiar. In the (quasi-) flexible price case, output persistence falls below the empirical numbers for U.S. real GDP. We observe something similar for consumption, investment and net exports. This is, nonetheless, consistent with the results in BKK (1995). We find that adding adjustment costs of the CAC type or using no adjustment costs (NAC) at all, does not substantially alter the persistence generated by the model persistence (except for net exports). However, the results are more mixed when we experiment with adjustment costs of the IAC type. The IAC specification leads the model to produce higher persistence on GDP and investment. Unfortunately, it also generates counterfactually low first-order autocorrelations for consumption and net exports.

In the sticky price case with some form of adjustment costs, the model delivers persistence values for all variables that are in line with the data. The differences between the CAC and IAC specifications are only marginal for this particular moment. The no adjustment cost case (NAC), however, cannot replicate the same patterns even when we look at a different calibration of the persistence of the real shock (i.e., $\rho_a = 0.75$) to enhance the model's ability to fit the output persistence. Surprisingly, the model with sticky prices and no adjustment costs also produces a counterfactual, negative first-order autocorrelation. This is another dimension in which the Q-INNS model performs better (or not worst) than a competing with (quasi-) flexible prices.

We match the cross-correlation of U.S. real GDP with our calibration of the cross-correlation of real shock innovations. Whether the model relies on sticky prices or not, the cross-country correlations of consumption and investment are very stable. It should be pointed out that all models generate very high cross-correlations of consumption, around twice as much as in the data. This finding is consistent with BKK (1995) and Heathcote and Perri (2002).[12]

Most notably, we also find that the only models without adjustment costs can account (qualitatively at least) for the fact that the correlation of investment across countries is lower than the cross-country

[12] In a complete asset markets model, this strong consumption cross-correlation has implications for the behavior of the real exchange rate through the perfect international risk-sharing condition in (24). Since that goes beyond the scope of this paper, we refer the interested reader to Martínez-García and Søndergaard (2008b) for additional insight.

correlation of output. Once again, whether prices are flexible or sticky seems to make little difference. BKK (1995) and Heathcote and Perri (2002) show that this stylized fact is not easy to match with a standard calibration of the IRBC model (without adjustment costs). This is, therefore, the first piece of evidence that comes out against the implementation of the Q theory extension by means of either the CAC or the IAC adjustment cost specifications.

On the Contemporaneous Correlations of ToT and Net Exports. The last three correlations reported in Table 3 are, however, the litmus test for each one of the experiments that we consider in this paper. The only models that can account qualitatively for the empirical evidence of countercyclical net exports are models without adjustment costs (NAC). BKK (1995) and Heathcote and Perri (2002) attest to the same pattern in standard IRBC models without adjustment costs. Our benchmark shows that it can deliver countercyclical trade patterns, but the effects are weaker than in those two papers. Adding adjustment costs of either type pushes the correlation up and changes its sign. However, Engel and Wang (2007) and Raffo (2008) using different models in the Q-IRBC tradition can deliver countercyclical trade patterns. Our model with sticky prices does not differ significantly from the patterns uncovered under flexible prices.

The contemporaneous correlation between output and the net exports share is quite sensitive to the calibration of the model. Even minor differences in the structure of the economy or the calibration of the model could explain why Engel and Wang (2007) and Raffo (2008) can account for this feature, while our model does not. For example, Raffo (2008, p. 21) notes that: "Higher substitution between intermediates translates into lower response of the terms of trade. (...), net exports are already procyclical. In the limiting case of perfect substitute intermediates, this economy resembles a one-good economy and net exports are systematically procyclical." The elasticity of intratemporal substitution, η, plays an analogous role in our model (see also BKK, 1995, Figure 11.4). Equation (81) suggests that, indeed, the calibration of this parameter is crucial to determine the sensitivity of trade to ToT. Therefore, this can affect whether trade becomes procyclical or countercyclical.

We leave the exploration of the role of this and other structural parameters in the calibration for future research. Suffice to say here that there is evidence in our results that including adjustment costs in order to reduce the volatility of investment and increase the volatility of consumption (and net exports) may push the contemporaneous correlation between GDP and net exports higher. The effect may be even strong enough to turn net exports into a procyclical variable. This suggests that the Q theory extension has to be undertaken with great care.

Consistent with the findings of Raffo (2008), the model produces high and positive contemporaneous correlations between output and ToT. This is true for all variants of the model. However, we find that the model with sticky prices tends to generate lower correlations between output and ToT (closer to the data). And adding adjustment costs helps further. Based on this contemporaneous correlations alone, the Q-INNS model appears to be a better fit. However, as we shall see shortly, the interpretation becomes more complex when we look at the shape of the cross-correlations function.

The model with flexible prices and no adjustment costs generates a contemporaneous correlation of 0.26 between ToT and net exports, which is still far apart from the value of -0.03 observed in the data. Adding adjustment costs in a (quasi-) flexible price environment makes matters much worst. In turn, adding adjustment costs and sticky prices helps reduce the correlation. Even though no model does better than the flexible price model without adjustment costs, the Q-INNS model with adjustment costs of the IAC type also does well. Once again, the interpretation is less straightforward once we look at the cross-correlations function.

BKK (1994, p. 94) point out also that "the contemporaneous correlation between net exports and the terms of trade is weaker, moving from -0.41 in the benchmark case to -0.05" with a higher elasticity of intratemporal substitution between foreign and domestic goods. We already quoted an argument reminiscent of this made by Raffo (2008) when discussing the countercyclical nature of trade. Recalling our previous discussion we could say that there are other parameters that should and do matter, as equation (81) indicates. In any event, we would still argue that the importance of adjustment costs cannot be discounted.

On the Cross-Correlation Function. Figures 1 and 2 plot the cross-correlations between real GDP and the real net exports share. Not surprisingly the data reveals the same type of S-shaped pattern that Engel and Wang (2007) emphasize in their paper. We already noted that we are able to find countercyclical trade patterns only in experiments without adjustment costs. When we compare the cross-correlations in the data against those generated by the different versions of the model that we explore here, we find that the mismatch runs deeper. The only model that can qualitatively approximate the S-shaped pattern found in the data is the one with (quasi-) flexible prices and no adjustment costs. The graph indicates that allowing for price stickiness to play a dominant role moves us away from the empirical evidence.

`[Insert Figures 1 and 2 about here]`

Most notably, we see from Figure 2 that adding adjustment costs of either type to the model alters the shape of the cross-correlations in a fundamental way. The predicted cross-correlations are shaped like a tent, with its peak around the contemporaneous correlation between GDP and net exports. While adding sticky prices to the mix matters, clearly the dominant effect comes from the adjustment costs. Engel and Wang (2007) have a model that also matches qualitatively these facts, and they do so instead with adjustment costs. Our models are not immediately comparable, but their paper suggests that there is still room to reconcile the Q theory extension with this empirical pattern.

Our reading of the results is that the (quasi-) flexible price case scenario without adjustment costs brings us back a flavor of the BKK (1995) results where investment resources are being shifted across countries in search of higher productivity and higher returns. When we add adjustment costs, we also cap the size of these effects. In our calibrations, we set the adjustment cost parameter to make sure that investment is not too volatile. The side-effect is that the trade balance becomes procylical and the cross-correlations peak contemporaneously.

Figures 3 and 4 plot the cross-correlations between real GDP and the ToT. Raffo (2008) argues that the BKK framework delivers a contemporaneous correlation between GDP and ToT that is simply counterfactual (too high). We document a very low contemporaneous correlation between both variables, but also show that the empirical cross-correlation are S-shaped. The results of the model are, however, disappointing. On one hand, we confirm that the contemporaneous correlations are way off mark. On the other hand, we note that all the model variants display a tent shape which is inconsistent with the data. Combining price stickiness with adjustment costs (particularly of the CAC type) allows us to qualitatively fit the cross-correlations of real GDP with current and lagged terms of trade, but the correlations with ToT leads of $3-4$ periods are more than twice as high as in the data. These features are a challenge in the IRBC literature (see, e.g., Raffo, 2008), and they are not better off with the benchmark INNS model.

`[Insert Figures 3 and 4 about here]`

The J-curve has been extensively discussed in the IRBC literature, specially since BKK (1994) showed that their framework was powerful enough to replicate this pattern. We still find evidence of a J-curve effect in the data, as reported in Figures 5 and 6, although the strength of the correlation diminishes beyond a 4-period lead (one year ahead). Our quantitative findings are consistent with the intuition of BKK (1994), since the best qualitative fit for the cross-correlations between ToT and the net exports share comes from the (quasi-) flexible price scenario without adjustment costs. Adding adjustment costs and/or sticky prices does not only alter the shape of the cross-correlation function, it also shifts its peak from leads to either contemporaneous or lagged correlations.

`[Insert Figures 5 and 6 about here]`

We see a common message emerge from Figures 1 through 6. Our experiment with (quasi-) flexible prices and no adjustment costs tends to approximate well the good and the bad features of the IRBC model. It qualitatively tracks the J-curve effect and the S-shaped pattern of the cross-correlation between GDP and net exports. It also produces an excessively high correlation between output and ToT, and cannot track the S-shaped pattern of the cross-correlation between these two variables at different leads and lags. Whenever we try to pull the model closer to our Q-INNS benchmark by making price stickiness or adjustment costs a more relevant feature of the dynamics, we end up with a worse model prediction along some of these dimensions.

`[Insert Figures 7, 8, 9 and 10 about here]`

Figures 7 through 10 confirm the familiar story that emerges in Table 3. We find that the judicious combination of adjustment costs and sticky prices used in our calibration of the benchmark Q-INNS model often improves the fit of the data. In these figures we see that such a calibrated model tracks rather well the tent shape form of the cross-correlation between output on one hand and consumption and investment on the other hand. The fact that the model matches the cross-correlations of output with investment better than with consumption is not completely unexpected. After all, we calibrate the adjustment cost function to match other moments of the investment data. However, it is reassuring to see that in turn this calibration is not distorting the features of investment along other important dimensions.

It must be noted that in most of our experiments, and Figures 7 through 10 are a good example of that, the performance of the IAC and CAC specifications is not all that different. If the model were to include other shocks, particularly monetary shocks, the differences are significative, well-known and noticeable. For instance, the impulse response function for investment becomes hump-shape only with the IAC specification (see, e.g., CEE, 2005). For real shocks, however, there seems to be little gain in choosing one type over the other (at least based on our results here).

When we compare the evidence from Figures 1 through 6 with the evidence from Figures 7 through 10, the message becomes clearer. The Q-INNS framework is capable of closely approximating a number of relevant features for consumption and specially investment. The Q theory extension is instrumental to deliver a significant improvement on the fit of investment data. However, one of the big challenges for the model from the quantity-side is on the patterns of trade. More research needs to be done to fully understand the complex interactions between deviations of the LOOP and fluctuations of the marginal Q in this environment. We raise the challenge here, and leave it for future research.

5 Concluding Remarks

The findings in this paper suggest that a Q theory extension of the standard INNS model has important, although conflicting implications for our ability to replicate observed trade patterns. On the one hand, adding adjustment costs makes investment costlier and, therefore, results in a smoother investment series and a more volatile consumption series. At the same time, the net exports share becomes more volatile. And while the model does not perfectly match the properties (volatility, persistence and cross-country correlations) of consumption, investment and net exports, adding adjustment costs appears to lead us in the right direction overall.

On the other hand, we see that the model with adjustment costs cannot replicate well-known features of the trade data such as the J-curve (see, e.g., BKK, 1994), the S-shaped cross-correlation of GDP and net exports (see, e.g., Engel and Wang, 2007), and the weak and S-shaped cross-correlation between GDP and ToT (see, e.g., Raffo, 2008).

Furthermore, our analysis suggests that a full-blown INNS model with sticky prices and LCP does not do any better than an alternative variant with (quasi-) flexible prices. In fact, the (quasi-) flexible price experiment without adjustment costs delivers results similar to those documented in the standard IRBC literature and tracks qualitatively the S-shaped cross-correlation of GDP and net exports and the J-curve.

An open question is what role monetary policy plays. In the standard INNS model, with or without the Q theory extension, the size and effect of the relative price distortion resulting from nominal rigidities (price stickiness and LCP) depends on the path of inflation and, in turn, on the choice of monetary policy. We have taken as given a version of the Taylor rule with interest rate inertia and selected a very specific calibration. The predictions of the model for trade are conditional on that calibration of the Taylor rule, and are likely to be different for alternative rules or parameterizations. We leave a close examination of the interplay between the role of monetary policy and trade dynamics for future research.

We interpret the findings of the paper mainly as a cautionary tale and not as a final word on the issue. To sum up: We need to be mindful of the fact that adjustment costs together with nominal rigidities can have unintended consequences for the trade dynamics in the standard INNS model. Therefore, we have to think deeply about how to reconcile the Q-INNS model with the empirical evidence on trade.

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Appendix: Log-Linearized Equilibrium Conditions

The 'Core' Model:

$$\widehat{c}_t \approx \mathbb{E}_t\left[\widehat{c}_{t+1}\right] - \sigma\left(\widehat{i}_t - \mathbb{E}_t\left[\widehat{\pi}_{t+1}\right]\right),$$

$$\widehat{c}^*_t \approx \mathbb{E}_t\left[\widehat{c}^*_{t+1}\right] - \sigma\left(\widehat{i}^*_t - \mathbb{E}_t\left[\widehat{\pi}^*_{t+1}\right]\right),$$

$$\widehat{c}_t - \widehat{c}^*_t \approx \sigma \widehat{rs}_t,$$

$$\begin{cases} \widehat{q}_t \approx 0, \text{ if NAC}, \\ \widehat{q}_t \approx \chi\delta\left(\widehat{x}_t - \widehat{k}_t\right), \text{ if CAC}, \\ \widehat{q}_t \approx -\kappa\widehat{x}_{t-1} + (1+\beta)\kappa\widehat{x}_t - \beta\kappa\mathbb{E}_t\left[\widehat{x}_{t+1}\right], \text{ if IAC}, \end{cases}$$

$$\begin{cases} \widehat{q}_t \approx (1-\delta)\beta\mathbb{E}_t\left[\widehat{q}_{t+1}\right] + \left[(1-(1-\delta)\beta)\mathbb{E}_t\left(\widehat{r}^{z}_{t+1}\right) - \left(\widehat{i}_t - \mathbb{E}_t\left(\widehat{\pi}_{t+1}\right)\right)\right], \text{ if NAC or IAC}, \\ \widehat{q}_t \approx \beta\mathbb{E}_t\left[\widehat{q}_{t+1}\right] + \left[(1-(1-\delta)\beta)\mathbb{E}_t\left(\widehat{r}^{z}_{t+1}\right) - \left(\widehat{i}_t - \mathbb{E}_t\left(\widehat{\pi}_{t+1}\right)\right)\right], \text{ if CAC}, \end{cases}$$

$$\begin{cases} \widehat{q}^*_t \approx 0, \text{ if NAC}, \\ \widehat{q}^*_t \approx \chi\delta\left(\widehat{x}^*_t - \widehat{k}^*_t\right), \text{ if CAC}, \\ \widehat{q}^*_t \approx -\kappa\widehat{x}^*_{t-1} + (1+\beta)\kappa\widehat{x}^*_t - \beta\kappa\mathbb{E}_t\left[\widehat{x}^*_{t+1}\right], \text{ if IAC}, \end{cases}$$

$$\begin{cases} \widehat{q}^*_t \approx (1-\delta)\beta\mathbb{E}_t\left[\widehat{q}^*_{t+1}\right] + \left[(1-(1-\delta)\beta)\mathbb{E}_t\left(\widehat{r}^{z*}_{t+1}\right) - \left(\widehat{i}^*_t - \mathbb{E}_t\left(\widehat{\pi}^*_{t+1}\right)\right)\right], \text{ if NAC or IAC}, \\ \widehat{q}^*_t \approx \beta\mathbb{E}_t\left[\widehat{q}^*_{t+1}\right] + \left[(1-(1-\delta)\beta)\mathbb{E}_t\left(\widehat{r}^{z*}_{t+1}\right) - \left(\widehat{i}^*_t - \mathbb{E}_t\left(\widehat{\pi}^*_{t+1}\right)\right)\right], \text{ if CAC}, \end{cases}$$

$$\widehat{k}_{t+1} \approx (1-\delta)\widehat{k}_t + \delta\widehat{x}_t,$$

$$\widehat{k}^*_{t+1} \approx (1-\delta)\widehat{k}^*_t + \delta\widehat{x}^*_t,$$

$$\widehat{\pi}_t \approx \beta\mathbb{E}_t\left(\widehat{\pi}_{t+1}\right) + \\ + \left(\tfrac{(1-\alpha)(1-\alpha\beta)}{\alpha}\right)\left[\begin{array}{c} \left(\sigma^{-1} + (1-\gamma_x)\varphi\left(\frac{\varphi\psi^2+(1-\psi)(1+\varphi)^2}{\varphi\psi+(1-\psi)\psi\varphi^2}\right)\right)\left[\phi_H\widehat{c}^W_t + \phi_F\widehat{c}^{W*}_t\right] + \\ +\gamma_x\varphi\left(\frac{\varphi\psi^2+(1-\psi)(1+\varphi)^2}{\varphi\psi+(1-\psi)\psi\varphi^2}\right)\left[\phi_H\widehat{x}^W_t + \phi_F\widehat{x}^{W*}_t\right] + \\ +2\phi_H\phi_F\widehat{rs}_t + (\phi_H - \phi_F)\eta\varphi\left(\frac{\varphi\psi^2+(1-\psi)(1+\varphi)^2}{\varphi\psi+(1-\psi)\psi\varphi^2}\right)\widehat{t}^W_t - \\ -\left(\frac{(1-\psi)(1+\varphi)}{\psi}\right)\widehat{k}^W_t - \left(\frac{1+\varphi}{\psi}\right)\left[\phi_H\widehat{a}_t + \phi_F\widehat{a}^*_t\right] \end{array}\right],$$

$$\widehat{\pi}^*_t \approx \beta\mathbb{E}_t\left(\widehat{\pi}^*_{t+1}\right) + \\ + \left(\tfrac{(1-\alpha)(1-\alpha\beta)}{\alpha}\right)\left[\begin{array}{c} \left(\sigma^{-1} + (1-\gamma_x)\varphi\left(\frac{\varphi\psi^2+(1-\psi)(1+\varphi)^2}{\varphi\psi+(1-\psi)\psi\varphi^2}\right)\right)\left[\phi_F\widehat{c}^W_t + \phi_H\widehat{c}^{W*}_t\right] + \\ +\gamma_x\varphi\left(\frac{\varphi\psi^2+(1-\psi)(1+\varphi)^2}{\varphi\psi+(1-\psi)\psi\varphi^2}\right)\left[\phi_F\widehat{x}^W_t + \phi_H\widehat{x}^{W*}_t\right] - \\ -2\phi_F\phi_H\widehat{rs}_t - (\phi_H - \phi_F)\eta\varphi\left(\frac{\varphi\psi^2+(1-\psi)(1+\varphi)^2}{\varphi\psi+(1-\psi)\psi\varphi^2}\right)\widehat{t}^W_t - \\ -\left(\frac{(1-\psi)(1+\varphi)}{\psi}\right)\widehat{k}^{W*}_t - \left(\frac{1+\varphi}{\psi}\right)\left[\phi_F\widehat{a}_t + \phi_H\widehat{a}^*_t\right] \end{array}\right],$$

$$\widehat{\pi}^R_t \approx \beta\mathbb{E}_t\left(\widehat{\pi}^R_{t+1}\right) + \left[\left(\tfrac{(1-\alpha)(1-\alpha\beta)}{\alpha}\right)\widehat{rs}_t + \left(\tfrac{\phi_H-\phi_F}{\phi_H\phi_F}\right)\left(\beta\mathbb{E}_t\left(\widehat{t}^W_{t+1}\right) - \left(\tfrac{1+\beta\alpha^2}{\alpha}\right)\widehat{t}^W_t + \widehat{t}^W_{t-1}\right)\right], \ \widehat{\pi}^R_t \equiv \widehat{\pi}_t - \widehat{\pi}^*_t,$$

$$\widehat{r}^{z}_t \approx \tfrac{1}{\sigma}\widehat{c}_t + \tfrac{1+\varphi}{\psi}\widehat{y}_t - \left(\tfrac{1+(1-\psi)\varphi}{\psi}\right)\widehat{k}_t - \tfrac{1+\varphi}{\psi}\widehat{a}_t,$$

$$\widehat{r}^{z*}_t \approx \tfrac{1}{\sigma}\widehat{c}^*_t + \tfrac{1+\varphi}{\psi}\widehat{y}^*_t - \left(\tfrac{1+(1-\psi)\varphi}{\psi}\right)\widehat{k}^*_t - \tfrac{1+\varphi}{\psi}\widehat{a}^*_t,$$

$$\widehat{y}_t \approx \eta\widehat{t}^W_t + (1-\gamma_x)\widehat{c}^W_t + \gamma_x\widehat{x}^W_t, \ \widehat{c}^W_t \equiv \phi_H\widehat{c}_t + \phi_F\widehat{c}^*_t, \ \widehat{x}^W_t \equiv \phi_H\widehat{x}_t + \phi_F\widehat{x}^*_t, \ \widehat{k}^W_t \equiv \phi_H\widehat{k}_t + \phi_F\widehat{k}^*_t,$$

$$\widehat{y}^*_t \approx -\eta\widehat{t}^W_t + (1-\gamma_x)\widehat{c}^{W*}_t + \gamma_x\widehat{x}^{W*}_t, \ \widehat{c}^{W*}_t \equiv \phi_F\widehat{c}_t + \phi_H\widehat{c}^*_t, \ \widehat{x}^{W*}_t \equiv \phi_F\widehat{x}_t + \phi_H\widehat{x}^*_t, \ \widehat{k}^{W*}_t \equiv \phi_F\widehat{k}_t + \phi_H\widehat{k}^*_t,$$

$$\widehat{i}_t \approx \rho_i\widehat{i}_{t-1} + (1-\rho_i)\left[\psi_y\widehat{y}_t + \psi_\pi\widehat{\pi}_t\right],$$

$$\widehat{i}^*_t \approx \rho_i\widehat{i}^*_{t-1} + (1-\rho_i)\left[\psi_y\widehat{y}^*_t + \psi_\pi\widehat{\pi}^*_t\right],$$

Net Exports Share and Terms of Trade:

$$\widehat{tb}_t \equiv \eta\widehat{t}^W_t - (1-\gamma_x)\phi_F\left(\widehat{c}_t - \widehat{c}^*_t\right) - \gamma_x\phi_F\left(\widehat{x}_t - \widehat{x}^*_t\right),$$

$$\widehat{tot}_t \cong \tfrac{1}{\phi_F}\widehat{t}^W_t - \widehat{rs}_t.$$

Tables and Figures

Table 1: Stylized Facts in the U.S. Data.

Table 1. Stylized Facts in the U.S. Data

			Cross-correlation of real GDP with								
Variable	Std. Dev.	Autocorr.	x_{t-4}	x_{t-3}	x_{t-2}	x_{t-1}	x_t	x_{t+1}	x_{t+2}	x_{t+3}	x_{t+4}
GDP	1.54	0.87	0.31	0.51	0.70	0.87	1.00	0.87	0.70	0.51	0.31
Investment	5.21	0.91	0.29	0.47	0.66	0.84	0.94	0.88	0.75	0.58	0.37
Consumption	1.24	0.87	0.51	0.66	0.79	0.87	0.85	0.69	0.51	0.33	0.16
Net Exports	0.38	0.83	−0.46	−0.49	−0.51	−0.52	−0.48	−0.38	−0.22	−0.06	0.11
ToT	1.72	0.69	−0.14	−0.11	−0.05	−0.01	0.07	0.16	0.18	0.17	0.20
			Cross-correlation of ToT with								
Net Exports			−0.15	−0.16	−0.18	−0.14	−0.03	0.14	0.25	0.31	0.35

We collect U.S. quarterly data spanning the post-Bretton Woods period from 1973q1 through 2006q4 (for a total of 136 observations per series). The U.S. dataset includes real output (rgdp), real private consumption (rcons), real private fixed investment (rinv), real exports (rx), the export price index (px), real imports (rm), the import price index (pm), and population size (n). The U.S. import price index and the U.S. export price index cover only the sub-sample between 1983q3 and 2006q4 (for a total of 94 observations). All data is seasonally adjusted.

Real output (rgdp), real private consumption (rcons) and real private fixed investment (rinv): Data at quarterly frequency, transformed to millions of U.S. Dollars, at constant prices, and seasonally adjusted. Source: Bureau of Economic Analysis.

Real exports (rx) and real imports (rm). Data at quarterly frequency, transformed to millions of U.S. Dollars, and seasonally adjusted. Source: Bureau of Economic Analysis.

Import price index (pm) and export price index (px). Data at quarterly frequency, indexed (2000=100), but not seasonally adjusted. Source: Bureau of Labor Statistics. (We compute a conventional measure of terms of trade, tot = pm/px, based on the data for the import and the export price indexes. We seasonally-adjust the resulting series with the multiplicative method X12.)

Working-age Population between 16 and 64 years of age (n): Data at quarterly frequency, expressed in thousands, and seasonally adjusted. Source: Bureau of Labor Statistics. For U.S. working-age population, we take the difference between civilian non-institutional population 16 and over and civilian non-institutional population 65 and over. We also seasonally-adjust the resulting series with the multiplicative method X12.

The real output (rgdp), real private consumption (rcons), real private fixed investment (rinv), real exports (rx), and real imports (rm) are expressed in per capita terms dividing each one of these series by the population size (n). We compute the terms of trade ratio, tot, and the real net export share over GDP, ((rx - rm)/rgdp)*100, based on the data for real imports, real exports, import and export price indexes and real GDP. We express all variables in logs and multiply them by 100, except the real net export share (which is computed in percentages). Finally, all series are Hodrick-Prescott (H-P) filtered to eliminate their underlying trend. We use the H-P smoothing parameter at 1600 for our quarterly dataset.

Table 2: Parameters Used in the Benchmark Calibration.

Table 2. Parameters Used in the Benchmark Calibration			
		Benchmark	CKM (2002)
Structural Parameters:			
Discount Factor	β	0.99	0.99
Elasticity of Intratemporal Substitution	η	1.5	1.5
Elasticity of Substitution across Varieties	θ	10	10
Elasticity of Intertemporal Substitution	σ	1/5	1/5
(Inverse) Elasticity of Labor Supply	φ	3	5
Domestic Home Bias Parameter	ϕ_H	0.94	0.94
Foreign Home Bias Parameter	ϕ_F	0.06	0.06
Calvo Price Stickiness Parameter	α	0.75	$N = 4$
Depreciation Rate	δ	0.021	0.021
Capital/Investment Adjustment Cost	χ, κ	varies	varies
Labor Share	ψ	2/3	2/3
Parameters on the Taylor Rule:			
Interest Rate Inertia	ρ_i	0.85	0.79
Weight on Inflation Target	ψ_π	2	2.15
Weight on Output Target	ψ_y	0.5	0.93/4
Exogenous Shock Parameters:			
Real Shock Persistence	ρ_a	0.9	0.95
Real Shock Correlation	$corr\left(\varepsilon_t^a, \varepsilon_t^{a*}\right)$	varies	0.25
Monetary Shock Correlation	$corr\left(\varepsilon_t^m, \varepsilon_t^{m*}\right)$	-	varies
Real Shock Volatility	$\sigma\left(\varepsilon_t^a\right), \sigma\left(\varepsilon_t^{a*}\right)$	varies	0.007
Monetary Shock Volatility	$\sigma\left(\varepsilon_t^m\right), \sigma\left(\varepsilon_t^{m*}\right)$	-	varies
Composite Parameters:			
Steady State Investment Share	$\gamma_x \equiv \frac{(1-\psi)\delta}{\left(\frac{\theta}{\theta-1}\right)\left(\beta^{-1}-(1-\delta)\right)}$	0.203	(0.203)

This table summarizes our benchmark parameterization. Additional results on the sensitivity of certain parameters can be obtained directly from the authors upon request. The proper comparison is with CKM's (2002) specification with a Taylor rule. The composite parameters are inferred based on the parametric choices described for our benchmark and for the model of CKM (2002).

For the most part, we follow the calibration strategy of CKM (2002). In CKM (2002), prices are fixed for 4 periods. In our model, a Calvo parameter of 0.75 implies an average contract duration of 4 periods. CKM's (2002) Taylor rule targets expected inflation and current output, while in our model it targets current inflation and current output. CKM (2002) also allow for discretionary monetary policy shocks, while we only consider real shocks. For more details on the parametric choices, specially for the adjustment cost parameter and the volatility and correlation of innovations, read also the calibration section.

Table 3: Baseline Model: Selected Business Cycle Moments.

Table 3. Selected Business Cycle Moments of the Baseline Model

		Sticky Prices ($\alpha = 0.75$)				Flexible Prices ($\alpha = 0.00001$)		
		IAC	CAC	NAC	NAC	IAC	CAC	NAC
Variable	U.S. Data			(ρ_a= 0.9)	(ρ_a= 0.75)			
Std. dev.								
GDP*	1.54	1.54	1.54	1.54	1.54	1.54	1.54	1.54
Investment**	5.21	5.21	5.21	7.08	7.09	5.21	5.21	6.62
Consumption	1.24	0.60	0.53	0.22	0.15	0.68	0.51	0.24
Net Exports	0.38	0.17	0.14	0.10	0.07	0.20	0.13	0.04
Autocorrelation								
GDP	0.87	0.91	0.89	0.54	0.71	0.77	0.69	0.70
Investment	0.91	0.94	0.88	0.40	0.67	0.89	0.69	0.69
Consumption	0.87	0.82	0.83	0.75	0.76	0.48	0.70	0.76
Net Exports	0.83	0.84	0.84	−0.12	−0.03	0.45	0.71	0.94
Cross-correlation								
GDP*	0.44	0.44	0.44	0.44	0.44	0.44	0.44	0.44
Investment	0.33	0.57	0.55	0.37	0.40	0.54	0.56	0.41
Consumption	0.33	0.65	0.63	0.69	0.66	0.68	0.62	0.62
Correlation								
GDP, Net Exp.	−0.47	0.49	0.49	−0.18	−0.11	0.41	0.52	−0.06
GDP, ToT	0.07	0.31	0.21	0.37	0.44	0.47	0.53	0.49
ToT, Net Exp.	−0.03	0.27	0.52	0.42	0.35	0.97	1.00	0.26
Calibration								
$\sigma\left(\varepsilon_t^a\right) = \sigma\left(\varepsilon_t^{a*}\right) =$		0.0207	0.0189	0.0127	0.01785	0.0143	0.0134	0.0115
$corr\left(\varepsilon_t^a, \varepsilon_t^{a*}\right) =$		0.4625	0.4475	0.4875	0.44	0.4775	0.465	0.457
$\chi, \kappa =$		3.35	11.15	−	−	2.12	13.25	−

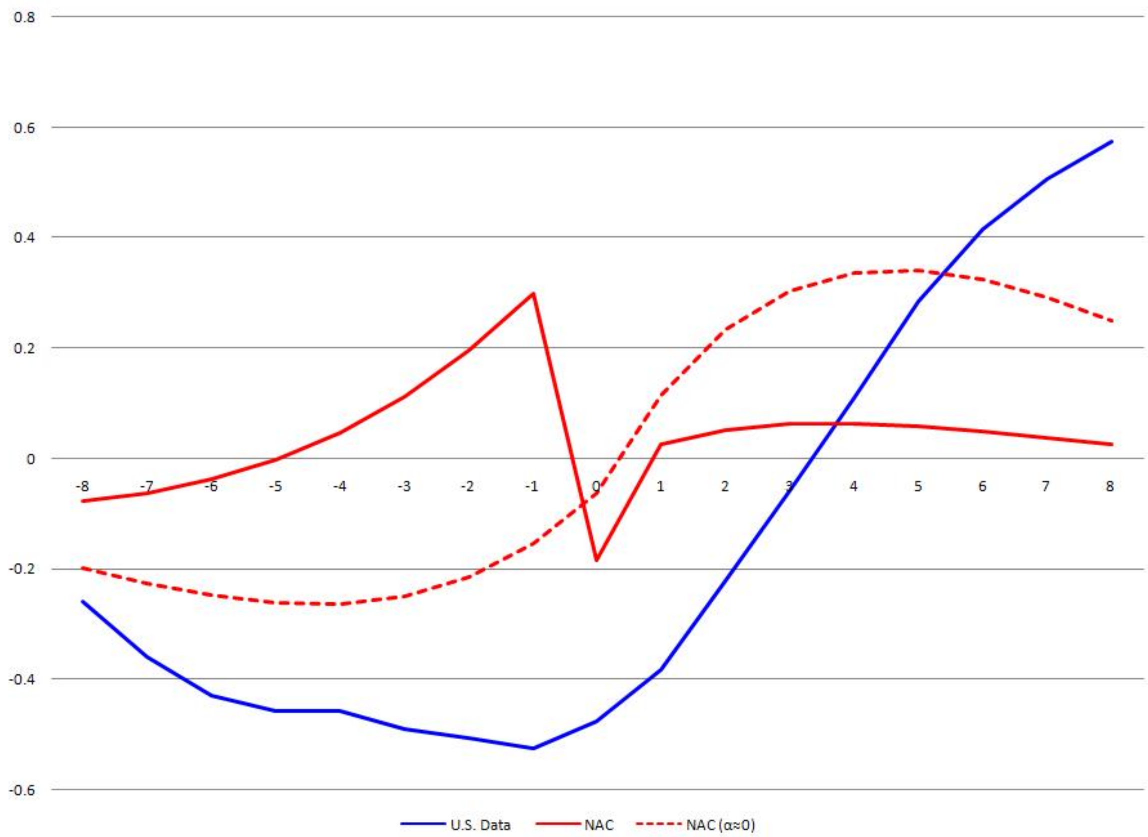
This table reports the selected theoretical moments for each series given our benchmark parameterization. All statistics are computed after each simulated series is H-P filtered (smoothing parameter=1600). NAC denotes the no adjustment cost case, CAC denotes the capital adjustment cost case, and IAC denotes the investment adjustment cost case. We use Matlab 7.4.0 and Dynare v3.065 for the stochastic simulation.

(*) We calibrate the volatility and cross-correlation of the real shock innovations to match the observed volatility and cross-country correlation of GDP.

(**) We calibrate the adjustment cost parameter, whenever available, to match the observed volatility of U.S. investment.

Data Sources: The Bureau of Economic Analysis and the Bureau of Labor Statistics. For more details, see the description of the data in the footnotes to Table 1. Sample period: 1973q1-2006q4 (except for ToT, which covers only 1983q3-2006q4).

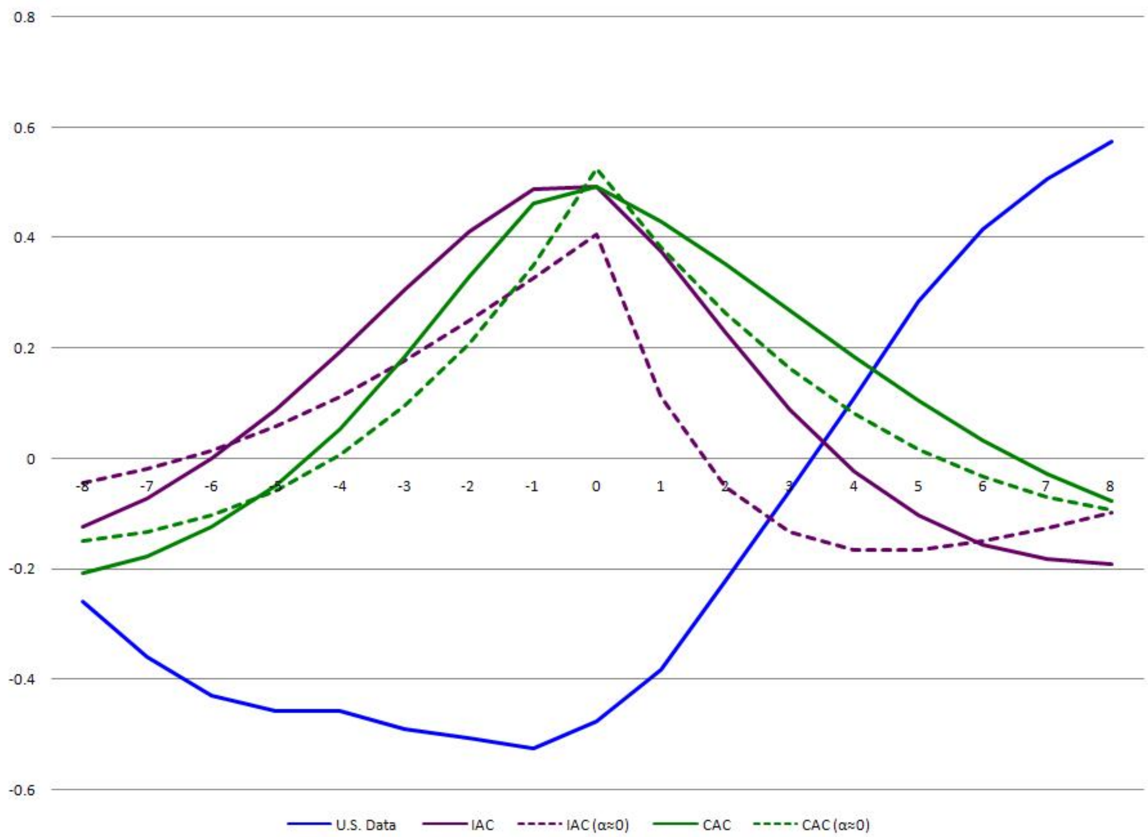
Figure 1: Cross-Correlations of GDP with Net Exports (without Adjustment Costs).



This graph plots the cross-correlation of output at t and net exports at t+s given our parameterization. All cross-correlations are computed after each simulated series is H-P filtered (smoothing parameter=1600). NAC denotes the no adjustment cost case, while $\alpha \approx 0$ denotes the experiment with (quasi-) flexible prices. We use Matlab 7.4.0 and Dynare v3.065 for the stochastic simulation.

Data Sources: The Bureau of Economic Analysis and the Bureau of Labor Statistics. For more details, see the description of the data in the footnotes to Table 1. Sample period: 1973q1-2006q4.

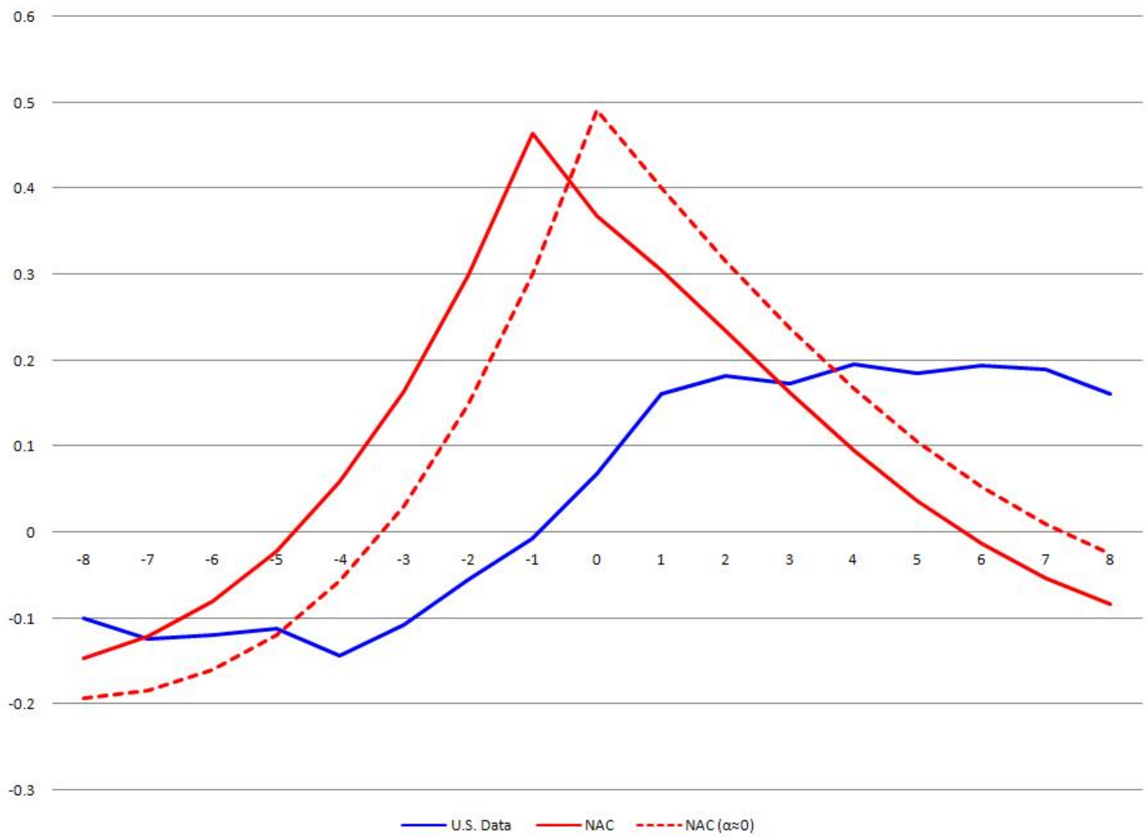
Figure 2: Cross-Correlations of GDP with Net Exports (with Adjustment Costs).



This graph plots the cross-correlation of output at t and net exports at t+s given our parameterization. All cross-correlations are computed after each simulated series is H-P filtered (smoothing parameter=1600). CAC denotes the capital adjustment cost case, IAC denotes the investment adjustment cost case, while $\alpha \approx 0$ denotes the experiment with (quasi-) flexible prices. We use Matlab 7.4.0 and Dynare v3.065 for the stochastic simulation.

Data Sources: The Bureau of Economic Analysis and the Bureau of Labor Statistics. For more details, see the description of the data in the footnotes to Table 1. Sample period: 1973q1-2006q4.

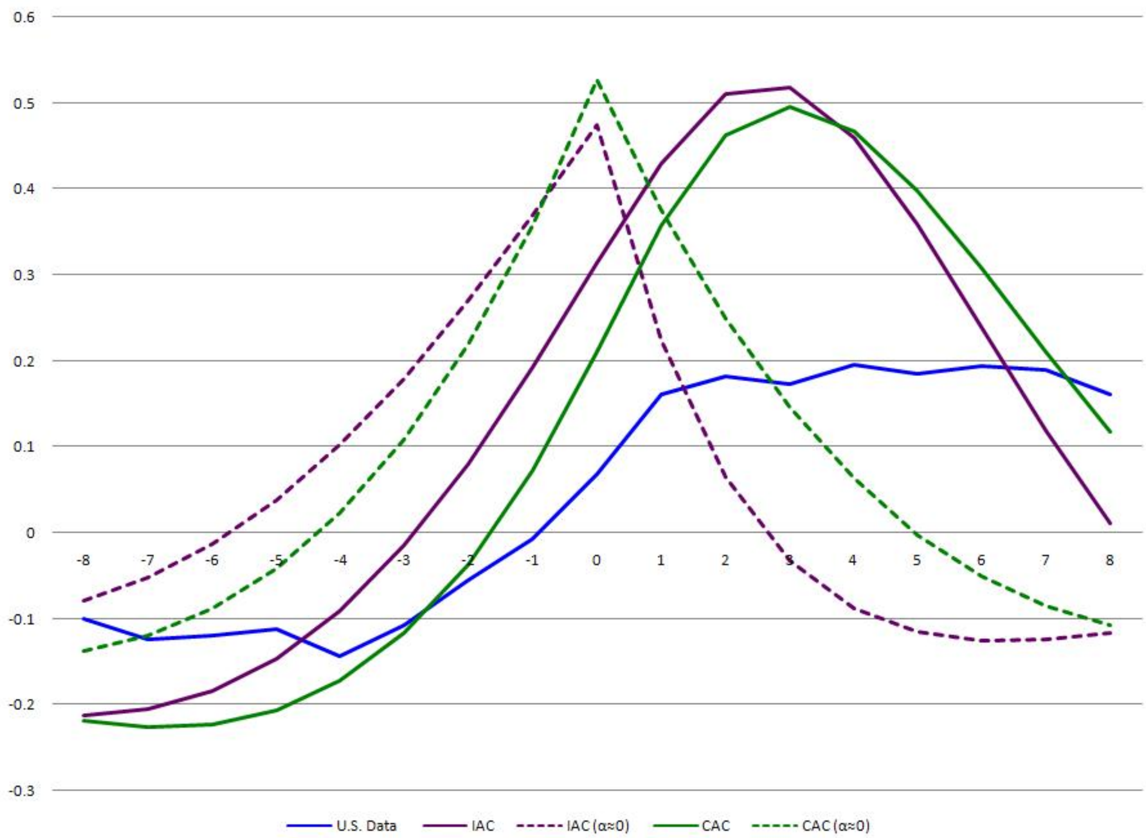
Figure 3: Cross-Correlations of GDP with ToT (without Adjustment Costs).



This graph plots the cross-correlation of output at t and terms of trade at t+s given our parameterization. We look at conventional terms of trade, ToT, rather than world terms of trade, Tw. ToT and Tw are proportional to each other under flexible prices, but they differ under sticky prices and local-currency pricing due to the failure of the law of one price. All cross-correlations are computed after each simulated series is H-P filtered (smoothing parameter=1600). NAC denotes the no adjustment cost case, while $\alpha \approx 0$ denotes the experiment with (quasi-) flexible prices. We use Matlab 7.4.0 and Dynare v3.065 for the stochastic simulation.

Data Sources: The Bureau of Economic Analysis and the Bureau of Labor Statistics. For more details, see the description of the data in the footnotes to Table 1. Sample period: 1983q3-2006q4.

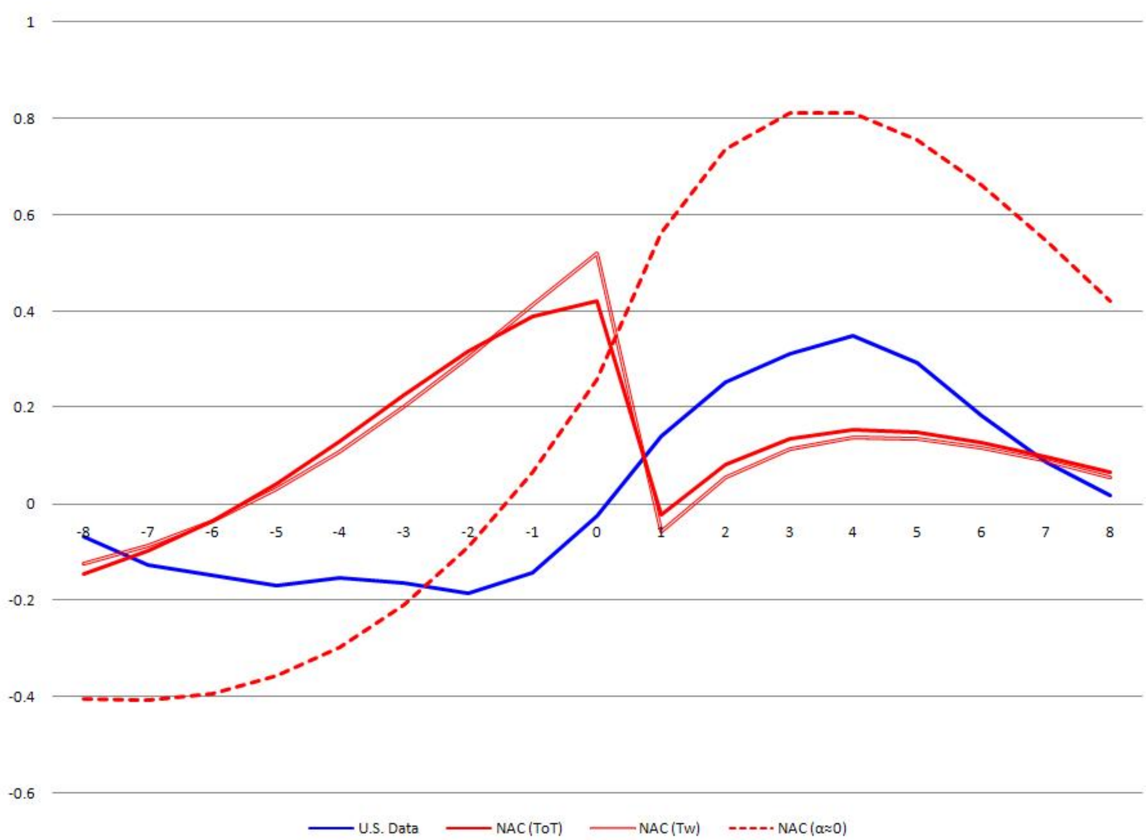
Figure 4: Cross-Correlations of GDP with ToT (with Adjustment Costs).



This graph plots the cross-correlation of output at t and terms of trade at t+s given our parameterization. We look at conventional terms of trade, ToT, rather than world terms of trade, Tw. ToT and Tw are proportional to each other under flexible prices, but they differ under sticky prices and local-currency pricing due to the failure of the law of one price. All cross-correlations are computed after each simulated series is H-P filtered (smoothing parameter=1600). CAC denotes the capital adjustment cost case, IAC denotes the investment adjustment cost case, while $\alpha \approx 0$ denotes the experiment with (quasi-) flexible prices. We use Matlab 7.4.0 and Dynare v3.065 for the stochastic simulation.

Data Sources: The Bureau of Economic Analysis and the Bureau of Labor Statistics. For more details, see the description of the data in the footnotes to Table 1. Sample period: 1983q3-2006q4.

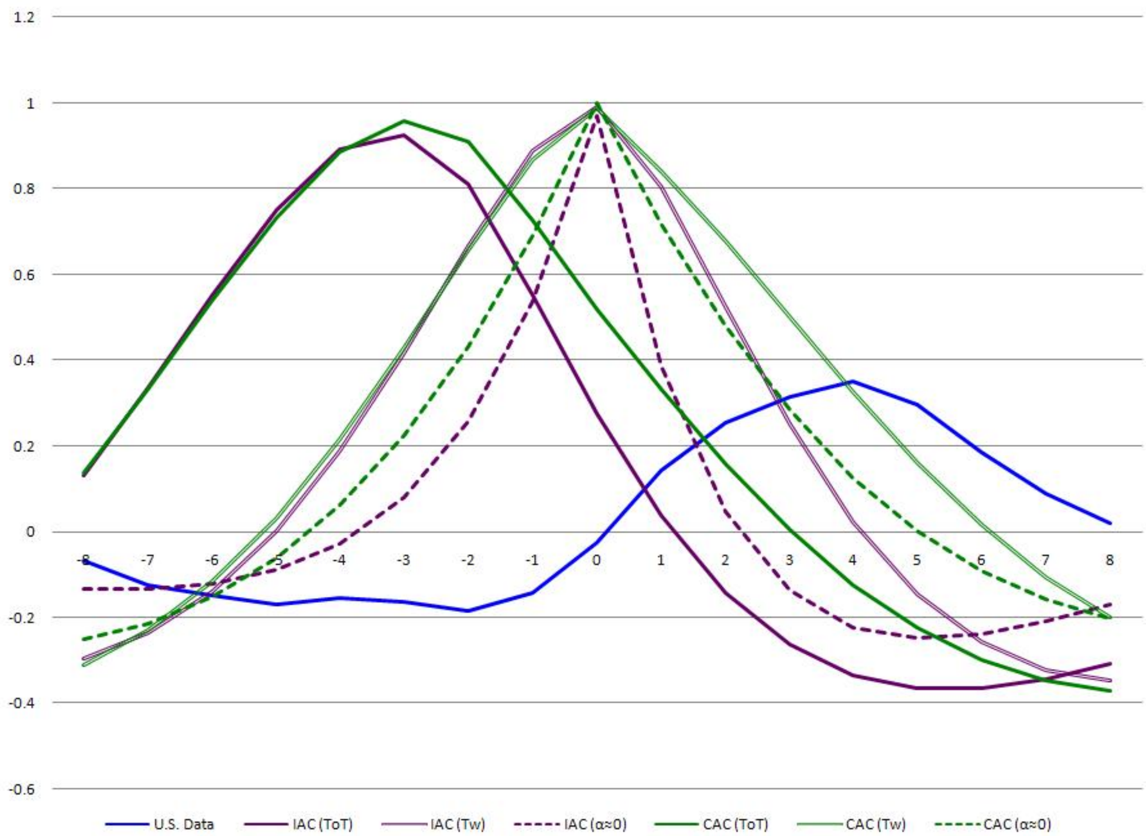
Figure 5: Cross-Correlations of ToT with Net Exports - J-Curve (without Adjustment Costs).



This graph plots the cross-correlation of terms of trade at t and net exports at t+s given our parameterization. We distinguish between conventional terms of trade, ToT, and the world terms of trade, Tw, which captures the relative price effects in the share of net exports. ToT and Tw are proportional to each other under flexible prices, but they differ under sticky prices and local-currency pricing due to the failure of the law of one price. All cross-correlations are computed after each simulated series is H-P filtered (smoothing parameter=1600). NAC denotes the no adjustment cost case, while $\alpha \approx 0$ denotes the experiment with (quasi-) flexible prices. We use Matlab 7.4.0 and Dynare v3.065 for the stochastic simulation.

Data Sources: The Bureau of Economic Analysis and the Bureau of Labor Statistics. For more details, see the description of the data in the footnotes to Table 1. Sample period: 1983q3-2006q4.

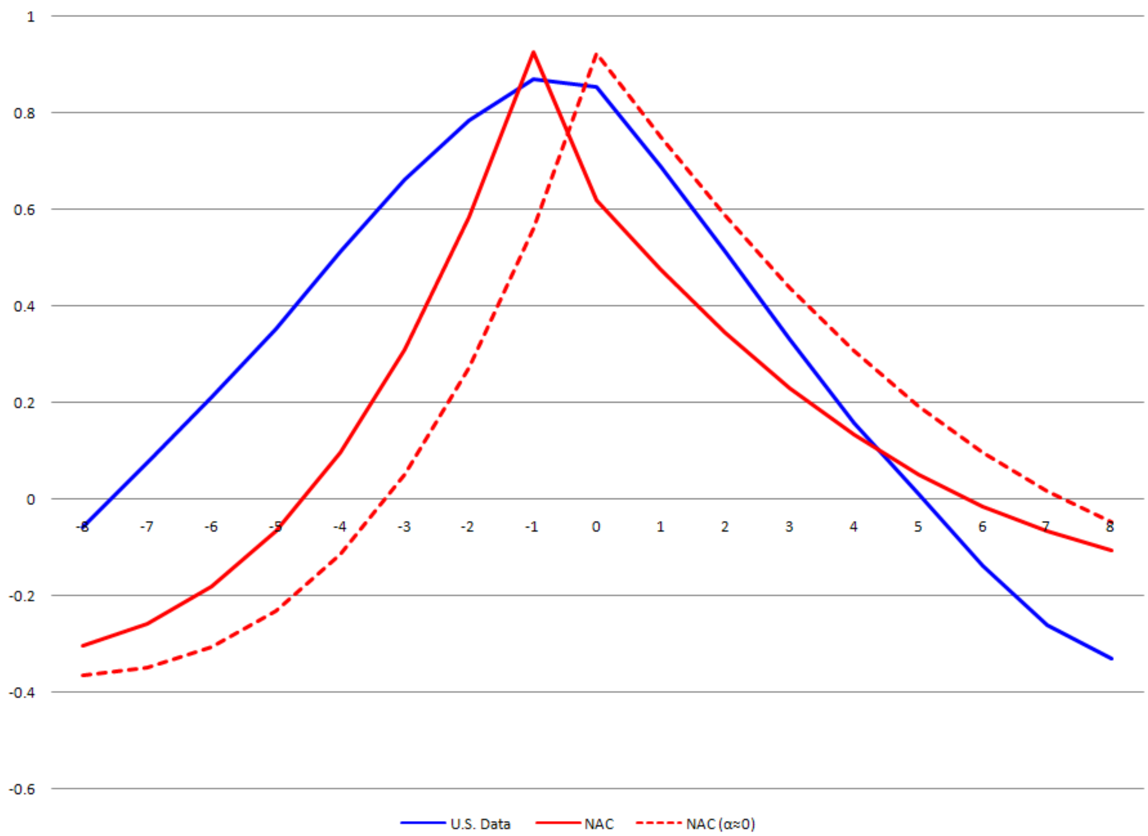
Figure 6: Cross-Correlations of ToT with Net Exports - J-Curve (with Adjustment Costs).



This graph plots the cross-correlation of terms of trade at t and net exports at t+s given our parameterization. We distinguish between conventional terms of trade, ToT, and the world terms of trade, Tw, which captures the relative price effects in the share of net exports. ToT and Tw are proportional to each other under flexible prices, but they differ under sticky prices and local-currency pricing due to the failure of the law of one price. All cross-correlations are computed after each simulated series is H-P filtered (smoothing parameter=1600). CAC denotes the capital adjustment cost case, IAC denotes the investment adjustment cost case, while $\alpha \approx 0$ denotes the experiment with (quasi-) flexible prices. We use Matlab 7.4.0 and Dynare v3.065 for the stochastic simulation.

Data Sources: The Bureau of Economic Analysis and the Bureau of Labor Statistics. For more details, see the description of the data in the footnotes to Table 1. Sample period: 1983q3-2006q4.

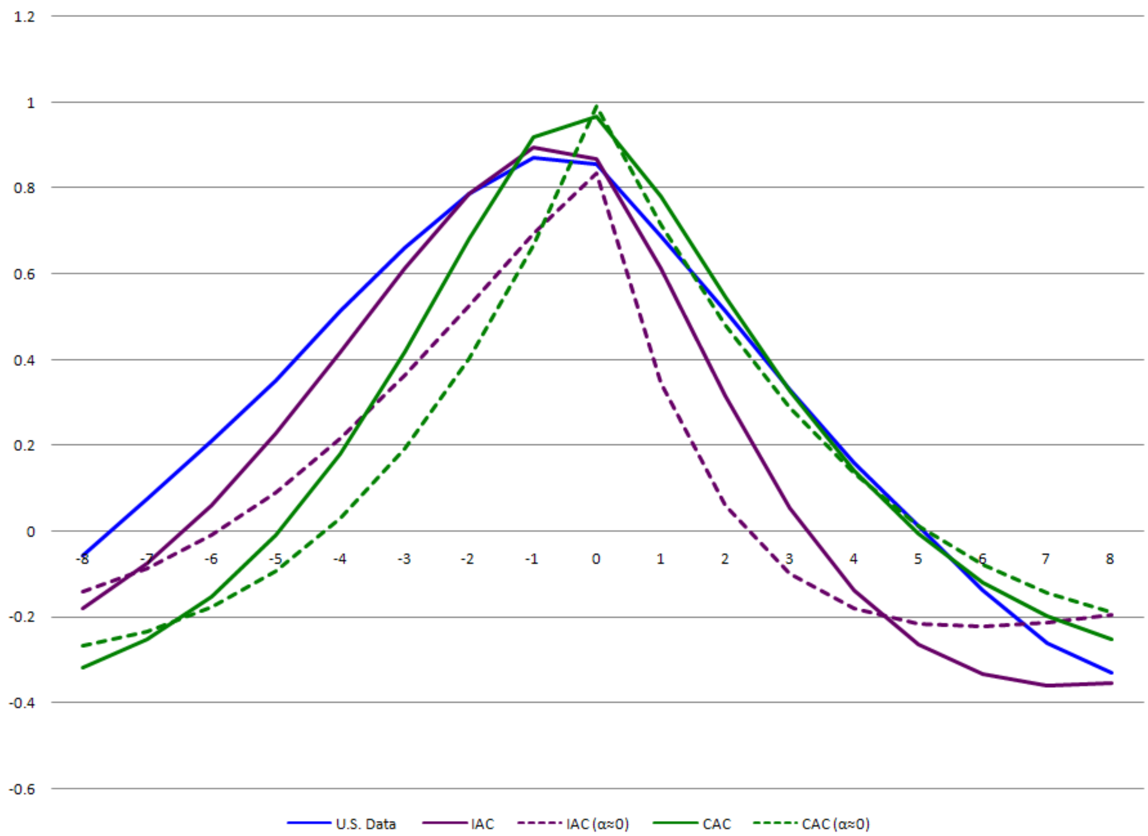
Figure 7: Cross-Correlations of GDP with Consumption (without Adjustment Costs).



This graph plots the cross-correlation of output at t and consumption at t+s given our parameterization. All cross-correlations are computed after each simulated series is H-P filtered (smoothing parameter=1600). NAC denotes the no adjustment cost case, while $\alpha \approx 0$ denotes the experiment with (quasi-) flexible prices. We use Matlab 7.4.0 and Dynare v3.065 for the stochastic simulation.

Data Sources: The Bureau of Economic Analysis and the Bureau of Labor Statistics. For more details, see the description of the data in the footnotes to Table 1. Sample period: 1973q1-2006q4.

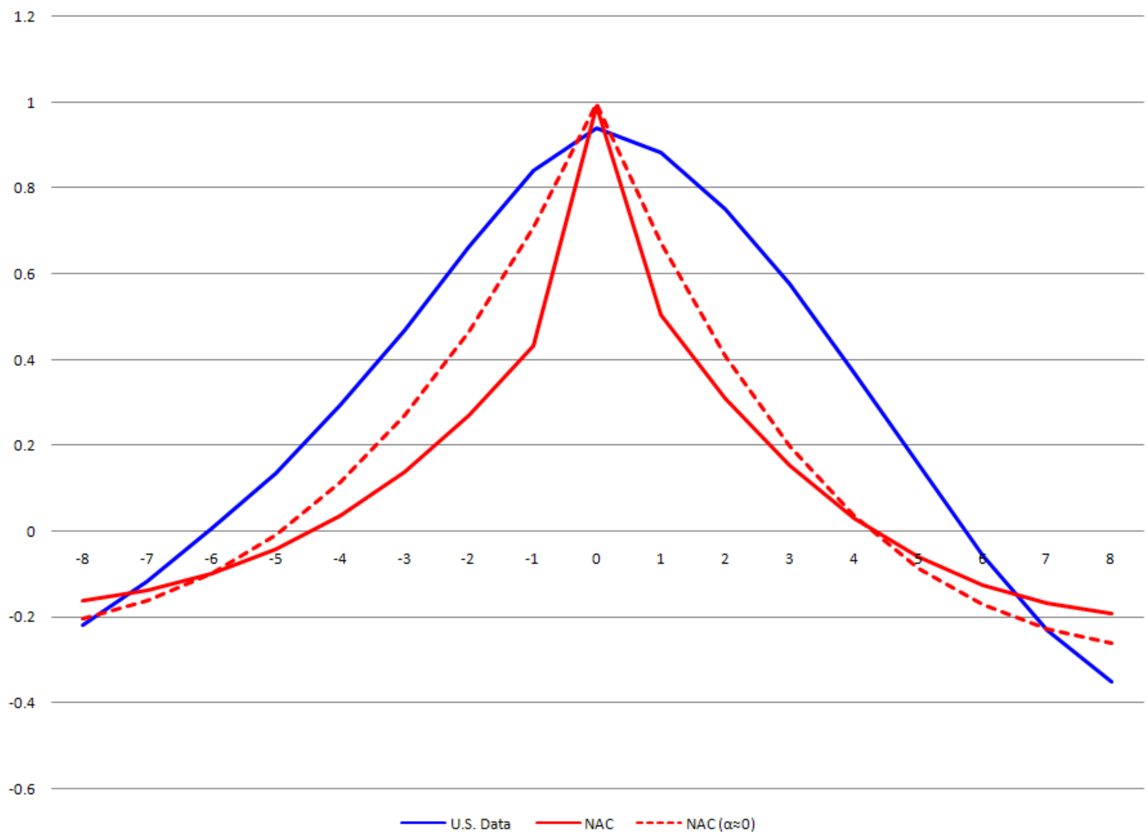
Figure 8: Cross-Correlations of GDP with Consumption (with Adjustment Costs).



This graph plots the cross-correlation of output at t and consumption at t+s given our parameterization. All cross-correlations are computed after each simulated series is H-P filtered (smoothing parameter=1600). CAC denotes the capital adjustment cost case, IAC denotes the investment adjustment cost case, while $\alpha \approx 0$ denotes the experiment with (quasi-) flexible prices. We use Matlab 7.4.0 and Dynare v3.065 for the stochastic simulation.

Data Sources: The Bureau of Economic Analysis and the Bureau of Labor Statistics. For more details, see the description of the data in the footnotes to Table 1. Sample period: 1973q1-2006q4.

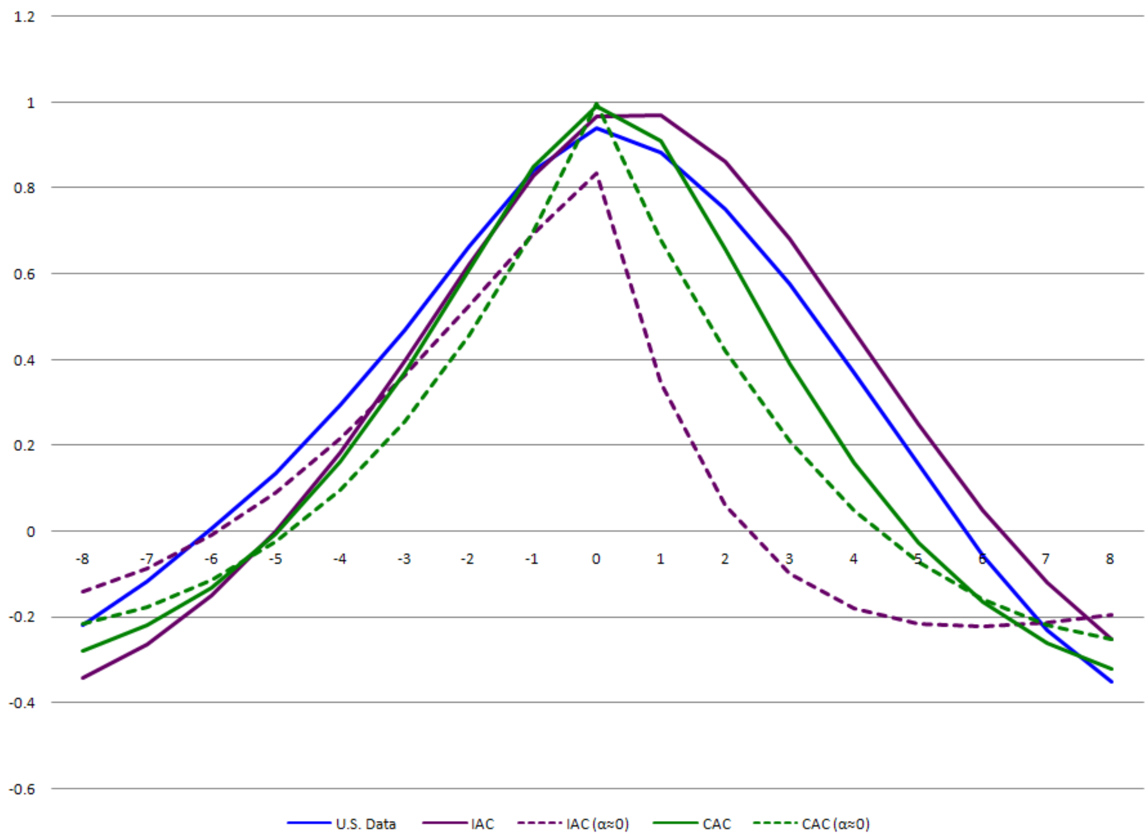
Figure 9: Cross-Correlations of GDP with Investment (without Adjustment Costs).



This graph plots the cross-correlation of output at t and investment at t+s given our parameterization. All cross-correlations are computed after each simulated series is H-P filtered (smoothing parameter=1600). NAC denotes the no adjustment cost case, while $\alpha \approx 0$ denotes the experiment with (quasi-) flexible prices. We use Matlab 7.4.0 and Dynare v3.065 for the stochastic simulation.

Data Sources: The Bureau of Economic Analysis and the Bureau of Labor Statistics. For more details, see the description of the data in the footnotes to Table 1. Sample period: 1973q1-2006q4.

Figure 10: Cross-Correlations of GDP with Investment (with Adjustment Costs).



This graph plots the cross-correlation of output at t and investment at t+s given our parameterization. All cross-correlations are computed after each simulated series is H-P filtered (smoothing parameter=1600). CAC denotes the capital adjustment cost case, IAC denotes the investment adjustment cost case, while $\alpha \approx 0$ denotes the experiment with (quasi-) flexible prices. We use Matlab 7.4.0 and Dynare v3.065 for the stochastic simulation.

Data Sources: The Bureau of Economic Analysis and the Bureau of Labor Statistics. For more details, see the description of the data in the footnotes to Table 1. Sample period: 1973q1-2006q4.