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Vehicle Currency*

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Abstract

While in principle, international payments could be carried out using any currency or set of currencies, in practice, the US dollar is predominant in international trade and financial flows. The dollar acts as a 'vehicle currency' in the sense that agents in non-dollar economies will generally engage in currency trade indirectly using the US dollar rather than using direct bilateral trade among their own currencies. Indirect trade is desirable when there are transactions costs of exchange. This paper constructs a dynamic general equilibrium model of a vehicle currency. We explore the nature of the efficiency gains arising from a vehicle currency, and show how this depends on the total number of currencies in existence, the size of the vehicle currency economy, and the monetary policy followed by the vehicle currency's government. We find that there can be very large welfare gains to a vehicle currency in a system of many independent currencies. But these gains are asymmetry weighted towards the residents of the vehicle currency country. The survival of a vehicle currency places natural limits on the monetary policy of the vehicle country.

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1. Introduction

A universal feature of international monetary systems is the predominance of one currency in facilitating international trade and financial flows. Since the middle of the 20th century, the US dollar has played the role of an international currency. But before the first world war, the British pound was the most accepted international currency, and before that, in the seventeenth and eighteenth centuries, it was the Dutch guilder. In frictionless models of international trade there is no reason for exchange between countries to take place in any particular currency. In practice, however, the presence of transactions costs of trading leads agents to make and receive payments in a currency which has a high trade volume, and is widely acceptable to all countries. A very large proportion of international exchange in currencies has the US dollar on one side of the transaction (BIS, 2008). In this sense the dollar acts as a 'vehicle currency'. It is cheaper for payments between agents in small countries with thinly traded currencies to be made indirectly using US dollars than to use direct bilateral trade in their own currency markets.

While the efficiency benefits of a vehicle currency in avoiding transactions costs of trade are clear, they also introduce an asymmetry into the international monetary system by giving a central role to one currency. This may give the residents of the country issuing that currency an advantage, either in the ease with which payments may be made, or through the direct gains from issuing a currency which is in demand by residents of other currencies. In addition, by their very nature, vehicle currencies are likely to become locked in a way which gives the issuer of the currency a natural monopoly. On the other hand, the historical record shows that the international system does abandon international currencies and adopts alternative currencies. Is it likely, for instance, that the vehicle currency role of the dollar will be given up in favor of the euro in the future? The option of using alternative currencies as vehicles may place a constraint on the policy actions of monetary and fiscal authorities of vehicle currency countries.

The economics literature has long recognized the benefits of a vehicle currency as a solution to a problem of transactions costs (e.g. Krugman, 1980, Black, 1991). But this literature has almost wholly been either simply descriptive, or based on partial equilibrium models in which relative prices or trades are exogenous. There are few general equilibrium models analyzing the way in which a vehicle currency facilitates international exchange (see below for references). In the absence of such a framework, it is not possible to assess the efficiency gains to a vehicle currency, nor to address the nature of the asymmetry inherent in such a system, or the limits on economic policies that are necessary to maintain the role of a vehicle currency.

This paper develops a dynamic general equilibrium model of a vehicle currency. In our model, a vehicle currency arises as an equilibrium precisely in the manner described in the narrative descriptions; by eliminating costly bilateral exchange in small currency markets, a vehicle currency can reduce the transactions cost of exchange. But the advantage of a fully specified general equilibrium model is that we can be precise about the trading mechanism underlying a vehicle currency equilibrium, the effect of a vehicle currency on equilibrium exchange rates, and the nature and magnitude of gains to a vehicle currency.

In addition, we use the model to analyze the specific gains to the issuer of such a currency. Finally, we can explore how a vehicle currency arises, and the constraints on monetary policy necessary for a vehicle currency to survive.

We build a multi-country monetary exchange economy model. The money of a particular country is required to finance purchases in that country, through a cash-in-advance constraint. But the way in which agents acquire foreign currencies may differ. We model foreign exchange trade as a costly process that takes place through 'trading post' technologies. Trading posts have been modelled by Shapley and Shubik (1977), Starr (2000) and Howitt (2005). They represent locations where agents can go in order to buy or sell one currency for another; that is, they facilitate bilateral trade in currencies. But trading posts are costly to set up. In a purely symmetric world, there would be one trading post for all possible bilateral pair of currencies. Trading possibilities would be the same for the holders of any currency, so that currencies and countries would be treated equally. But in a world with a large number of currencies, this environment would involve significant real resources used up in setting up trading posts.

An alternative equilibrium is where one country operates as a 'vehicle currency'. This offers significant efficiencies, since less resources are used up in trading. At the same time however, it confers significant benefits on the vehicle currency issuer. The main object of the paper is to explore this trade off.

Our model has $N > 3$ countries, labeled $1, 2, ..., N$. In a Symmetric Trading Equilibrium, there are $N(N-1)/2$ bilateral foreign exchange trading posts, and agents from any country can use their currency directly to buy the currency of any other country. In a Vehicle Currency Equilibrium, country 1 acts as an intermediary. There are only $N-1$ trading posts, with currency 1 being on one side of all currency trades. Agents from any country $i > 1$ who wish to purchase currency $j \notin \{i, 1\}$ must first purchase currency 1 and then use currency 1 to purchase currency j.

The gains to a Vehicle Currency Equilibrium come from being able to facilitate all possible trades while reducing the number of trading posts by $(N/2-1)(N-1)$. For large N, these gains may be substantial. The gains are reflected in smaller bid-ask spreads in currency markets. But the gains are unevenly distributed. Residents of the issuing country have the same opportunity set as in a Symmetric Trading Equilibrium, since they can directly buy the currency of any other country. But residents of the peripheral countries (i.e. all countries $i > 1$) must visit two trading posts in order to complete an exchange with another peripheral country. This imposes additional costs of trade. We find that a Vehicle Currency Equilibrium always benefits residents of country 1. But residents of peripheral countries may lose or gain.

The model points to three key features in the assessment of the gains to a vehicle currency. The first is the number of currencies. The more independent countries and currencies, the greater are the transactions cost gains to using a vehicle currency in exchange. With only a small number of currencies, a vehicle currency will not offer much welfare gain for peripheral countries, because the costs of indirect exchange will offset the gains to reduced transactions costs for peripheral countries. The second key feature is the size of countries. Larger countries have a natural advantage as providers of the vehicle currency because they engage in more international trade than smaller countries, leading to larger volume in foreign exchange markets. Finally, the monetary policy followed by the authority of the vehicle currency is a crucial determinant of the size and distribution of the gains to a vehicle currency. A higher rate of inflation in the vehicle country shifts the transactions gains away from the rest of the world, and towards vehicle currency residents. But if the vehicle country is large, the use of a vehicle currency may still offer substantial benefits, even with quite a high rate of inflation. There is a natural trade-off between size and inflation.

We use the model to explore the degree to which a vehicle currency is sustainable. Because the model combines fixed costs and 'network externalities', there are many Nash equilibria of the conventional type that are robust to deviation by individual agents. In order to explore the robustness of a vehicle currency equilibrium we investigate the incentives for deviation by aggregate groups of agents. We show that the robustness of a vehicle currency depends in very intuitive way on the three features just described. There is a three-way trade-off between monetary policy, country size, and the number of currencies that are required to prevent peripheral countries from deviation from vehicle currency equilibrium. We show that the introduction of a single currency area among peripheral countries (such as the euro) tends to significantly tighten the constraints imposed on a vehicle currency in order to maintain robustness of the vehicle currency equilibrium. This is because a single currency area simultaneously reduces the number of existing currencies, reducing the transactions costs gains to a vehicle currency, and increases the economic size of the area issuing a peripheral currency. Both these effects tend to work together to make a vehicle currency less robust.

There is a relatively small literature on the nature of an international currency. Krugman (1980) defines a vehicle currency in the same way that is used here, within a partial equilibrium setting, and explores alternative trading patterns. Rey (2001) examines how increasing returns to scale technologies in financial markets may give rise to an international currency. Hartmann (1998) looks at a model of a vehicle currency in financial markets and endogenizes a bid-ask spread. A different literature on search and money has explored the use of international currencies in an environment where agents can choose the currency they will hold to make purchases (e.g. Matsuyama et al. 1993, Wright and Trejos 2001). This differs from ours principally in that we assume the existence of a cash-in-advance constraint for all goods purchases, but look specifically at the nature of trade between currencies.[1]

The paper is organized as follows. Section 2 develops the basic model. Section 3 analyzes the equilibrium where all bilateral trading posts exist. Section 4)analyzes the equilibrium with a vehicle currency and explores the comparison with the symmetric equilibrium. Section 5 explores the robustness of the Vehicle Currency Equilibrium. Some conclusions then follow.

[1] Head and Shi (2003) construct a search-based model of two countries in which goods trade for money, and monies also trade for one another. Goldberg and Tille (2005) use the term 'vehicle currency' to refer to a sitution where a firm may set a price for sale to a foreign customer in the currency of a third currency. This is quite different from the sense in which we use the term.

2. The Model

2.1. Technology and Preferences

Time is discrete, indexed by $t = 0, 1, ...$ There are $N \geq 3$ countries, indexed by $i = 1, 2, ..., N$. The world population is normalized to unity. Country i has population n_i, so that $\Sigma_{i=1}^{N} n_i = 1$. We call n_i the size of country i. The world economy has a continuum of goods of measure one. Country i is endowed with measure n_i of these types of goods, with each resident being endowed with one unit of a particular type of good. Thus, the endowment per capita is the same across countries (i.e., 1).[2] All goods are perishable at the end of a period.

Within a country, all households are alike. Let c_{ij} represent the consumption by a country i resident of each of the n_j goods produced by country j. Because all goods endowed to a country are symmetric, a country i household's total utility in a period from consuming country j goods is $n_j u(c_{ij})$. Such a household has the following intertemporal utility function:

$$U^i = \sum_{t=0}^{\infty} \beta^t \left[\sum_{j=1}^{N} n_j u(c_{ijt}) \right],$$

where $\beta \in (0, 1)$ is the discount factor. Throughout the analysis, we will assume that $u(c) = \ln(c)$.

Until the end of section 5, we assume that each country has its own currency. Residents of a country receive lump-sum transfers only from their own country's monetary authority. Let m_{ij} be the stock of currency j held by a country i household, *normalized by the total stock of currency j*. If country i residents hold all their own currency, then symmetry within a country implies that $m_{ii} = 1/n_i$. The gross rate of growth of currency i is defined as γ_i. Proceeds from money growth are transferred to domestic households. Let τ_i denote the transfer to each country i household, normalized by money stock. This implies that

$$\tau_i = (\gamma_i - 1) / (\gamma_i n_i).$$

2.2. Monetary Exchange at Trading Posts

Purchases of country i's goods must use only currency i. This represents a cash in advance constraint at the national level. Therefore, in order to consume country j's good, a household in country i must obtain currency j. The purpose of imposing this constraint is to focus on the exchange between currencies, rather than between currencies and goods.[3]

Currency trade is organized in bilateral trading posts. At a trading post, one currency is exchanged for another. There can be many agents on each side of a trading post. We order the two currencies at a post in ascending order and refer to a trading post with

[2] This modelling of country size and endowments allows us to vary the size of a country without affecting the endowment per capita (or "productivity") of that country.

[3] One way to view this assumption is as a result of a legal restriction on settlement with domestic currency within a domestic market.

currencies k and j as post kj, where $k < j$. There cannot be instantaneous arbitrage between trading posts or shorting on a currency.[4]

Operating a trading post involves a fixed cost. In order to operate trading post kj, the manager of a trading post must incur a fixed cost ϕ in both goods k and j. There is also a cash-in-advance constraint on trading posts - the fixed cost in each country's good needs to be paid in that country's money. Examples of this fixed cost include the wage cost of workers who operate the post and the amortized amount of the initial cost of setting up the post. For simplicity, we abstract from the flexible cost that depends positively on the trading volume at the post.

Trading posts are a contestable market (see Tirole, 1988, p308). That is, anyone can set up a trading post and offer prices for the exchange between two currencies, but only one successful manager will run a trading post with zero net profit. The manager of each trading post announces two prices for a pairwise trade, one for sale of a currency (ask) for another currency, and one for purchase (bid) of a currency for another currency. Under the assumption of contestable markets, there is Bertrand competition among managers at the stage of entering the market (see Howitt, 2005, for a similar formulation). Thus, the manager of a trading post surviving the competition offers the bid and ask prices that are just sufficient to cover the fixed costs of setting up the trading post, given the buyers and sellers of the currency pair in which the trading post operates. These prices then represent the equilibrium nominal exchange rates for each currency pair.

With N countries and trading posts for each pair of currencies, there are $N(N-1)/2$ possible trading posts. But with each trading post incurring fixed costs, in principle this can be improved upon by using one currency as an intermediate, and trading twice, buying the intermediate, or 'vehicle' currency, and then selling it to obtain the currency required for purchasing the desired goods. When one currency plays the role of a 'vehicle', then only $N-1$ trading posts need to exist in order to facilitate trade between all countries.

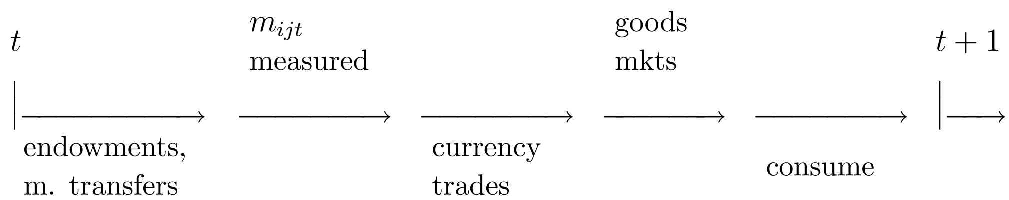
With fixed costs of setting up trading posts, there can be many Nash equilibria that differ from each other in the number of active posts. To see this, suppose that an agent believes that no (or only a few) other agents will go to a particular trading post. Then trading at that post will not be sufficient to cover the fixed cost, and so the agent will have no incentive to bring a currency to buy or sell at that trading post. In this case, the trading post will remain inactive.

2.3. Timing of Events

The timing of events is as follows. At the beginning of a period, agents receive unspent cash balances in each currency. They receive their income from last period sales of their endowment, in their own currency, plus a currency transfer from the monetary authorities. In total, this leaves them m_{ijt}. Agents then visit the trading posts of their choice in

[4] In reality of course, currency traders do not just trade one currency for another. But there are clear limits on the number of exchange possibilities that exist. Few commercial currency exchanges are willing to buy or sell much more than about a half dozen currencies. Moreover, bid ask spreads are typically higher for smaller currencies. The use of trading posts allows us a simplified way to handle the frictions inherent in currency trading.

order to exchange currencies. After currency exchange at trading posts, they hold m'_{ijt} of each currency. After the currency trading is over, they visit the goods market, with each household dividing into a shopper and a seller. At the end of the period, the households consumes all the goods purchased. The following illustration describes the timing:



We will suppress the time subscript t whenever possible and use the subscript $\pm z$ to stand for $t \pm z$, where $z \geq 1$. Denote the (normalized) nominal exchange rate for a buyer of currency j at a post kj as s^a_{kj}. This is the normalized currency k 'ask' price of one unit of currency j, at the post kj.[5] Likewise, for a seller of currency j, at trading post kj, the exchange rate is s^b_{kj}, which is the 'bid' price of currency j in terms of currency k. Clearly, $s^a_{kj} \geq s^b_{kj}$ is required for trading post kj to be viable.

Let f^{kj}_{ik} be the amount of currency k (normalized by the total stock of currency k) brought to the post kj by the representative country i household. Because households cannot short on currencies at any post, $f^{kj}_{ik} \geq 0$ for all i, k, j.[6]

3. Symmetric Trading Equilibrium

In this section, we describe a configuration where there is a trading post open for all pairs of currencies. In total, there are $N(N-1)/2$ posts open. Households of each country can then engage in direct currency trade in order to obtain the currency required to purchase any country's good. We describe an equilibrium of this setup as a Symmetric Trading Equilibrium (STE).

3.1. Household Choices

Consider an arbitrary country i and let us examine the decision problem of a representative household in country i. For given money holdings, the household chooses a sequence $\{h_{it}\}^{\infty}_{t=0}$, where $h_i = \left((c_{ij})^N_{j=1}, (f^{ij}_{ii})_{i<j}, (f^{ji}_{ii})_{i>j}, (m'_{ij})^N_{j=1}, (m_{ij(+1)})^N_{j=1}\right)$, to maximize U^i subject to the following constraints:

$$m_{ii} = \frac{1}{\gamma_i}\left[m'_{ii(-1)} - n_i p_{i(-1)} c_{ii(-1)} + p_{i(-1)}\right] + \tau_i \tag{3.1}$$

[5] The normalization implies multiplying the nominal exchange rate by the currency j money stock, and dividing by the currency k money stock. This means that permanent differences in money growth across countries k and j do not affect s_{kj}.

[6] The post kj is said to be active if at least one side of the post has a positive amount of currency, i.e., if $\left(\sum^N_{i=1} f^{kj}_{ik}\right) + \left(\sum^N_{i=1} f^{kj}_{ij}\right) > 0$.

$$m_{ij} = \frac{1}{\gamma_j}\left[m'_{ij(-1)} - n_j p_{j(-1)} c_{ij(-1)}\right], \quad j \neq i, \tag{3.2}$$

$$m'_{ii} = m_{ii} - \sum_{j>i} f_{ii}^{ij} - \sum_{j<i} f_{ii}^{ji}, \tag{3.3}$$

$$m'_{ij} = m_{ij} + \frac{1}{s_{ij}^a} f_{ii}^{ij}, \quad i < j, \tag{3.4}$$

$$m'_{ij} = m_{ij} + s_{ji}^b f_{ii}^{ji}, \quad i > j, \tag{3.5}$$

$$m'_{ij} \geq n_j p_j c_{ij}, \text{ all } j. \tag{3.6}$$

Equation (3.1) describes the dynamics of domestic cash balances and (3.2) the dynamics of the balances of foreign currencies. For the domestic currency, holdings at the beginning of the period consist of left-over currency in the last period, sales of goods in the last period, or monetary transfers. Note that the household spends $n_i p_i c_{ii}$ on all domestic goods (where p_i is the normalized price of good i), but receives income only from its own endowment p_i. Money growth γ_i is applied to the money carried over from the last period because $m'_{ii(-1)}$ and $p_{i(-1)}$ are normalized by last period's money stock. For a foreign currency $j \neq i$, holdings at the beginning of the period consist entirely of the left-over currency in the last period, as described in (3.2).

The household then visits the $N-1$ currency trading posts ij (for $i < j$) and ji (for $i > j$), supplying respectively f_{ii}^{ij} and f_{ii}^{ji} to these posts, as described in (3.3). Recall that m is measured immediately before currency trades and m' is measured immediately after currency trades. At the ij trading post ($i < j$), the household pays the 'ask' price for currency j, and receives f_{ii}^{ij}/s_{ij}^a units of currency j in return. At the ji trading post ($i > j$), the household receives the 'bid' price for its sale of currency i, and gets $s_{ji}^b f_{ii}^{ji}$ units of currency j. These constraints are described in (3.4) and (3.5). In addition, the cash in advance constraint (3.6) must be satisfied for all consumption of each country's goods.

We first examine the optimal choices of households, taking exchange rates as given, and then look at equilibrium exchange rates which ensure that trading posts are viable in an STE. To proceed, assume that all cash-in-advance constraints are binding.[7] This means that households have no foreign currency left over at the beginning of a period, and they hold the entire stock of domestic currency. That is, $m_{ij} = 0$ for all $j \neq i$ and so $m_{ii} = 1/n_i$. The households must visit all trading posts in order to ensure that they can consume all goods.

In Appendix A, we show that optimal choices for household i give the conditions:

$$\text{for } j > i: \quad s_{ij}^a p_j c_{ij} = p_i c_{ii}, \tag{3.7}$$

$$\text{for } j < i: \quad p_j c_{ij} = s_{ji}^b p_i c_{ii}. \tag{3.8}$$

[7] Conditions under which this will be confirmed are given below.

Because the household holds no foreign currency across periods, consumption of a foreign good j must be financed entirely by the amount of currency j that the household purchases in the current period. That is, $f_{ii}^{ij}/s_{ij}^{a} = n_j p_j c_{ij}$ for $j > i$, and $f_{ii}^{ji} s_{ji}^{b} = n_j p_j c_{ij}$ for $j < i$. Also, all purchases of foreign currencies in the period must come from holdings of domestic currency at the beginning of the period. Therefore, using (3.3) together with these conditions, we get:

$$\frac{1}{n_i} = m_{ii} = n_i p_i c_{ii} + \sum_{j>i} s_{ij}^{a} n_j p_j c_{ij} + \sum_{j<i} \frac{1}{s_{ji}^{b}} n_j p_j c_{ij}. \tag{3.9}$$

Now, substituting the first-order conditions for consumption into (3.9), we have:

$$p_i c_{ii} = \frac{1}{n_i}, \tag{3.10}$$

$$f_{ii}^{ij} = \frac{n_j}{n_i},\ j > i; \quad f_{ii}^{ji} = \frac{n_j}{n_i},\ j < i. \tag{3.11}$$

Thus, households bring more of their total cash balances to trading posts offering the currency of larger countries.

3.2. Trading Posts and Exchange Rate Determination

There is a firm at each trading post ij. The firm sets prices s_{ij}^{a} and s_{ij}^{b} so as to just break even, after it incurs the fixed cost ϕ in good i and ϕ in good j. The firm must pay these fixed costs with currency. Hence, the firm must hold currency i in the (normalized) amount $p_i\phi$ and currency j in the amount $p_j\phi$.

As a result, exchange rates in trading post ij must satisfy two conditions. The first condition, determining the ask price of currency j, is:

$$s_{ij}^{a}\left[n_j f_{jj}^{ij} - p_j\phi\right] = n_i f_{ii}^{ij}, \tag{3.12}$$

This is explained as follows. In an STE, trading post ij receives total currency j payments of $n_j f_{jj}^{ij}$ (since only country j agents hold currency j at the beginning of each period in this equilibrium), and must hold currency $p_j\phi$ to pay the good j fixed costs of setting up the trading post. It receives $n_i f_{ii}^{ij}$ deliveries of currency i from country i residents. It must set the ask price of currency j that country i residents will pay so that its holdings of currency j, in excess of its fixed costs, are all paid out to country i households. From this condition, s_{ij}^{a} exactly satisfies this property.

In a similar manner, to determine the bid price, s_{ij}^{b}, the trading post must satisfy the condition that deliveries of currency i made by country i households, less required currency holdings of $p_i\phi_i$, must equal the deliveries of currency j by country j residents. This condition is:

$$s_{ij}^{b} n_j f_{jj}^{ij} = n_i f_{ii}^{ij} - p_i\phi. \tag{3.13}$$

¿From the fact that all cash in advance constraints bind, in conjunction with market clearing, we have that $m_i = 1 = n_i p_i$, so that $p_i = 1/n_i$, for all i. Using this in (3.12) and (3.13), and substituting the solutions for the currency trades f_{ii}^{ij}, we get (for $i < j$):

$$s_{ij}^{aSTE} = \frac{n_j}{n_i - \phi/n_j}; \quad s_{ij}^{bSTE} = \frac{n_j - \phi/n_i}{n_i}. \tag{3.14}$$

Bilateral (normalized) nominal exchange rates are proportional to the relative size of the countries, adjusted for transactions costs. The bigger is country j relative to i, the greater is the total demand for currency j by country i residents, leading to a higher cost of j. We impose the restriction $\phi < n_i n_j$, for all i, j, so that these solutions are meaningful.

The above results, together with (3.7) and (3.8), lead to the following statements: (a) the bid-ask spread at trading post ij under STE is:

$$\left(\frac{s_{ij}^a}{s_{ij}^b}\right)^{STE} = \left(1 - \frac{\phi}{n_i n_j}\right)^{-2} > 1; \tag{3.15}$$

(b) Consumption levels under STE are:

$$c_{ii} = 1. \tag{3.16}$$

$$c_{ij} = 1 - \frac{\phi}{n_i n_j}, \text{ all } j \neq i. \tag{3.17}$$

The equilibrium bid-ask spread reflects the presence of trading costs. The bid-ask spread will be higher, the smaller the countries i and j, since this implies that a smaller volume of total currency is brought by both buyers and sellers to the ij trading post.

Of each type of good endowed to a country i, each domestic resident of the country consumes one unit, and so total consumption of this good by domestic residents is n_i (< 1). In contrast, of each type of good endowed to a foreign country j $(\neq i)$, a resident of country i consumes less than one unit and so total consumption of each foreign good by country i residents is less than n_i. The presence of trading costs in the currency market introduces an endogenous home bias in consumption. Given the form of preferences and the trading cost technology, the STE has the property that the fixed costs of setting up the ij trading post are fully borne by households of country i and j. The fixed costs in terms of good j (i) are borne by country i (j).[8]

How does country size affect the outcome of the STE? From (3.17) above, we see that consumption is higher if the trade involves a larger country. Take the example where $n_1 = n$, and $n_i = n'$ for all $i > 1$ (since $\sum n_i = 1$ we must have $n' = (1-n)/(N-1)$).

[8] To see how this is consistent with market clearing, note that for each individual good in country j there is an amount $1 - (N-1)\phi/n_j$ available for consumption, which is equal to the endowment less the cost of setting up $N-1$ trading posts, averaged over the number of goods in the country. Total consumption is $\sum_{j=1}^{N} n_i c_{ij}$. Substituting the solutions for consumption above, it can be established that this equals the available endowment.

Assume also that $n_1 > n'$. Hence, $c_{1i} = c_{i1} = 1 - \frac{\phi}{n_1 n'}$ for $i > 1$. In addition, $c_{1i} > c_{ij}$, $i, j > 1$. Consumption is higher if the trade involves a larger country. Intuitively, c_{1j} is higher than c_{ij}, because country 1 has more residents sharing the fixed good j cost of setting up trading post $1j$ than country i has to share the fixed good j cost of post ij. Likewise, c_{i1} is higher than c_{ij} because the good 1 fixed cost of setting up trading post $1i$ is spread among more goods than the good j cost of setting up trading post ij (or ji, if $i > j$). In this example, since $c_{1j} > c_{ij}$ for all $i, j > 1$, we may also conclude that country 1 residents have higher welfare than other countries. Because of its size, country 1 receives higher consumption of all other country's goods, whereas all other countries receive higher consumption of only country 1's good.

Note that consumption in the STE is independent of home or foreign country money growth. Money is neutral, and there are no international 'spillovers' of monetary policy.

Finally, we check that the cash in advance constraints indeed bind. Using the first order conditions above, it is easy to establish that cash in advance constraints for each currency i will bind in a steady state if $\gamma_i > \beta$.

4. Currency 1 as a Vehicle

Now assume that currency 1 serves as the vehicle currency. In a VCE (Vehicle Currency Equilibrium) currency 1 has active trading posts with all other currencies, but there are no bilateral posts except those with currency 1. This reduces the total number of trading posts from $N(N-1)/2$ to $N-1$. We call country 1 the VC country or the center country and other countries the peripheral countries.

4.1. Households' Decisions

In a VCE, residents of all other countries $i > 1$ must engage in two foreign exchange transactions in order to consume goods other than their own or country 1's good. This means that, from the time of their decision to consume an additional unit of these goods, they must wait one period for consumption to take place. To obtain other peripheral country currencies $j \neq i, 1$, a household in a peripheral country i ($\neq 1$) must carry a positive amount of the vehicle currency between periods. That is, $m_{i1} > 0$ for all $i \neq 1$. As a result, the total holdings of currency 1 by country 1 residents must be lower than the entire stock of currency 1, i.e., $m_{11} < 1/n_1$. Because the peripheral countries hold currency 1 between two adjacent periods, the cash in advance constraint on currency 1 does not bind for these countries. In contrast, for the VC country, the cash in advance constraint on currency 1 binds under the same conditions as in the STE. Also, as before, the cash in advance constraints on all non-vehicle currencies bind for all countries. Thus, $m_{ij} = 0$ for all $i \neq j$ and $j \neq 1$, and $m_{ii} = 1/n_i$ for all $i \neq 1$.

The decision problem facing country 1 is identical to that described above, because country 1 has active trading posts with all other countries. For country $i > 1$, the dynamics of money holdings are still given by (3.1) and (3.2), and the cash in advance constraints by (3.6). However, the other constraints are modified as follows:

$$m'_{ii} = m_{ii} - f_{ii}^{1i}, \tag{4.1}$$

$$m'_{i1} = m_{i1} - \sum_{j \notin \{i,1\}} f_{i1}^{1j} + s_{1i}^{b} f_{ii}^{1i}, \tag{4.2}$$

$$m'_{ij} = m_{ij} + \frac{1}{s_{1j}^{a}} f_{i1}^{1j}, \quad j \notin \{i,1\}, \tag{4.3}$$

$$m_{i1} \geq \sum_{j \neq i} f_{i1}^{1j}. \tag{4.4}$$

Constraint (4.1) says that the only domestic currency i that the household spends in the currency market is that brought to the $1i$ post. The household's holding of the vehicle currency coming out of the foreign exchange market is described by (4.2). This comprises its initial holding of vehicle currency m_{i1}, less its purchases of other peripheral currencies, made with vehicle currency, i.e. $\sum_{j \notin \{i,1\}} f_{i1}^{1j}$, plus new purchases of vehicle currency, $s_{1i}^{b} f_{ii}^{1i}$. The constraint (4.3) gives the household's holdings of other non-vehicle currency $j \notin \{1, i\}$ after the currency exchange. The household uses the vehicle currency to exchange for such a non-vehicle currency at the $1j$ post, and the amount of the vehicle currency that the household brings to the post is f_{i1}^{1j}. Finally, (4.4) requires that the total amount of the vehicle currency that the household brings into the ij posts should not exceed the amount that the household has when it enters the period. We may call this constraint the 'vehicle currency constraint'. It prevents the household from short sales in vehicle currency, since $m_{i1} \geq 0$ must always hold. The vehicle currency constraint binds, provided $\gamma_1 > \beta$.

In Appendix A, we show that the optimal choices of a peripheral country i household yield the following conditions:

$$p_i c_{ii} = \frac{1}{s_{1i}^{b}} p_1 c_{i1} \tag{4.5}$$

$$s_{1i}^{b} p_i c_{ii} = \frac{\gamma_{1(+1)} s_{1j(+1)}^{a}}{\beta} p_{j(+1)} c_{ij(+1)}, \; j \notin \{i,1\}. \tag{4.6}$$

The condition (4.5) characterizes the trade-off between consuming good 1 and the domestic good, which is the same as before. For each country $i > 1$, the relative price of good 1 is $p_1/(s_{1i}^{b} p_i)$. But the trade-off involved between consumption of the domestic good and another peripheral country good is quite different. Sacrificing one unit of the domestic good gives p_i in domestic currency, and hence $s_{1i}^{b} p_i$ in currency 1 when converted at the $1i$ trading post. This can only be converted into a country j's ($j \notin \{i, 1\}$) currency in next period's foreign exchange trading session. In the next period, each dollar of currency 1 can obtain $1/[\gamma_{1(+1)} s_{1j(+1)}^{a} p_{j(+1)}]$ units of good j. Equating the costs and benefits in utility terms, and discounting, gives condition (4.6).

There are three aspects of the vehicle currency equilibrium, relative to the STE, that affect the decisions of peripheral countries. First, to consume other peripheral goods, they must undertake two foreign exchange transactions, accepting the bid price of their own currency i in terms of currency 1, and paying the ask price of currency $j \notin \{1, i\}$ in terms of currency 1. Second, the transaction involves a delay, which is costly because agents discount future utility. Finally, it also involves a cost due to country 1 money growth, as country 1 inflation will reduce the real value of their currency 1 money holdings over time.

As in the previous section, only residents of country $i \neq 1$ hold currency i between periods. Thus, $m_{ii} = 1/n_i$ and $p_i = 1/n_i$ for all $i \neq 1$, as before. Also, a country i's holdings of currency i are equal to the sum of expenditures on goods. However, because the expenditures on other peripheral countries' goods occur with a one period delay, as explained above, the condition (3.9) needs to be modified. In Appendix A, it is shown that:

$$m_{ii} = 1/n_i = n_i p_i c_{ii} + \frac{1}{s^b_{1i}} \left(n_1 p_1 c_{i1} + \sum_{j \neq 1,i} \gamma_{1(+1)} s_{1j(+1)} n_j p_{j(+1)} c_{ij(+1)} \right). \tag{4.7}$$

Using (4.5) and (4.6), we can establish that:

$$c_{ii} = \frac{1}{\delta_i}, \tag{4.8}$$

$$f^{1i}_{ii} = \frac{1}{n_i} - n_i p_i c_{ii} = \frac{1}{n_i} - \frac{1}{\delta_i}, \tag{4.9}$$

$$f^{1j}_{i1} = \frac{\beta n_j}{n_i \delta_i} \left(\frac{s^b_{1i(-1)}}{\gamma_1} \right), \ j \notin \{i, 1\}, \tag{4.10}$$

$$m_{i1} = \sum_{j \notin \{i,1\}} f^{1j}_{i1} = \frac{\beta(1 - n_i - n_1)}{n_i \delta_i} \frac{s^b_{1i(-1)}}{\gamma_1}. \tag{4.11}$$

where $\delta_i \equiv n_i + n_1 + \beta(1 - n_i - n_1)$.

Expression (4.8) shows that for $\beta < 1$, a peripheral country i consumes a higher share of its own good than under STE, since trading off consumption of good i for good $j \notin \{1, i\}$ involves waiting one period, and future consumption is discounted. Condition (4.9) says that whatever country i ($\neq 1$) does not spend on its home good, it brings to the $1i$ trading post to obtain currency 1. For all feasible values of β the household brings a larger volume of domestic currency to the $1i$ trading post under VCE than under STE. For instance, in the case $\beta = 1$, the household spends a fraction n_i of its total domestic money balances on domestic goods, and brings the rest, $1 - n_i$, to the $1i$ post.

Condition (4.10) gives the amount of currency 1 brought to the $1j$ trading post ($j \neq i$). Recall that in the STE country i residents bring n_j/n_i of their own currency to the ij trading post (i.e. condition 3.11). But in the VCE, the amount of currency 1 brought to the $1j$ post by country $i \neq 1, j$, will depend on discounting, country 1 money growth, and the previous period's bid rate at which currency i was sold. We can establish that (4.10) is below n_j/n_i for all values of $\beta \leq 1$ and $\gamma_1 \geq 1$.

The condition (4.11), which is just the sum over j of (4.10), gives the total amount of currency 1 that country i holds at the beginning of the period.

For country 1, optimal consumption is chosen in the same manner as under the STE:

$$s^a_{1i} p_i c_{1i} = p_1 c_{11}, \text{ for } i \neq 1. \tag{4.12}$$

As a vehicle currency, currency 1 will be held by residents of all countries. This means that, compared to the STE, it is no longer true that $m_{11} = 1/n_1$. In fact, since $n_1 m_{11} + \sum_{i \neq 1} n_i m_{i1} = 1$, using (4.11), it must be the case that normalized holdings of currency 1 by country 1 residents are:

$$m_{11} = \frac{1}{n_1}\left(1 - \frac{\beta}{\gamma_1}\sum_{i \neq 1}\frac{(1 - n_i - n_1)s^b_{1i(-1)}}{\delta_i}\right) \tag{4.13}$$

Country 1's consumption of goods may be written as

$$c_{11} = \frac{m_{11}}{p_1},\ c_{1i} = \frac{m_{11}}{s^a_{1i} p_i},\ \text{ for } i \neq 1. \tag{4.14}$$

The amount of currency 1 brought to the $1i$ post by a country 1 household is:

$$f^{1i}_{11} = n_i m_{11}, \tag{4.15}$$

which must be less than the equivalent measure under STE, since $m_{11} < 1/n_1$.

To compute the price level, p_1, notice that the cash in advance constraint on currency 1 binds for country 1. Using this fact and the fact $\tau_1 = (\gamma_1 - 1)/\gamma_1 n_1$, we rewrite the constraint (3.1) for $i = 1$ as follows:

$$n_1 p_1 = 1 - \gamma_{1(+1)}\left[1 - n_1 m_{11(+1)}\right]. \tag{4.16}$$

Thus, country 1's normalized price level is influenced by the holdings of currency 1 by all other countries.

4.2. Trading Posts with a Vehicle Currency

We now determine exchange rates under the VCE. In each period, country i residents in total bring $n_i f^{1i}_{ii}$ to the $1i$ post. At the $1i$ post, currency 1 is supplied by country 1, in the amount $n_1 f^{1i}_{11}$, and by each of the other peripheral countries $j \notin \{i, 1\}$, in the amount $n_j f^{1i}_{j1}$. Then, the ask and bid prices of currency i are determined by:

$$s^a_{1i}\left[n_i f^{1i}_{ii} - \phi p_i\right] = n_1 f^{1i}_{11} + \sum_{j \notin \{i,1\}} n_j f^{1i}_{j1}, \tag{4.17}$$

$$s^b_{1i} n_i f^{1i}_{ii} = n_1 f^{1i}_{11} + \sum_{j \notin \{i,1\}} n_j f^{1i}_{j1} - p_1 \phi. \tag{4.18}$$

We focus on a steady state where γ_1 is constant over time. Then, all real variables and all normalized nominal variables are constant over time. In the steady state, the above conditions in the currency market and the condition (4.13) yield the following proposition:

Proposition 4.1. *Under the VCE, ask and bid exchange rates for trading posts* $1i$, $i > 1$, *may be written as:*

$$s_{1i}^{bVCE} = \delta_i \left[D_i - (1 - n_1 m_{11}) E_i \right], \tag{4.19}$$

$$s_{1i}^{aVCE} = \delta_i \frac{(\delta_i - n_i) \left[D_i - (1 - n_1 m_{11}) E_i \right] + p_1 \phi}{\delta_i (1 - \phi / n_i) - n_i} \tag{4.20}$$

where

$$D_i \equiv \frac{(1 - \frac{\beta}{\gamma_1}) n_i - \frac{\phi}{n_1} + \frac{\beta n_i}{\gamma_1 n_1} \left(1 - \frac{(N-1)\phi}{n_1} \right)}{\delta_i - n_i + (\beta / \gamma_1) n_i},$$

$$E_i \equiv \frac{(1 - \frac{\beta}{\gamma_1})) n_i - \frac{\gamma_1 \phi}{n_1} + \frac{\beta n_i}{n_1} \left(1 - \frac{(N-1)\phi}{n_1} \right)}{\delta_i - n_i + (\beta / \gamma_1) n_i}.$$

Proof: See Appendix A.

The solutions (4.19) and (4.20) require knowledge of m_{11} and p_1. From (4.13) and (4.19) we can calculate m_{11} as given by:

$$1 - n_1 m_{11} = \frac{\sum_{i \neq 1} (1 - n_i - n_1) D_i}{\gamma_1 / \beta + \sum_{i \neq 1} (1 - n_i - n_1) E_i}. \tag{4.21}$$

Then, (4.16) determines p_1.

The full expressions for s_{1i}^{bVCE} and s_{1i}^{aVCE} are quite complicated. In order to develop the intuition behind the solutions, we begin by focusing on some special cases.

4.3. Some Special Cases

Case A: $\{n = 1/N,\ \gamma_1 = 1,\ \beta \to 1\}$. In this case, all countries are of equal size, country 1 money growth is zero, and the discount factor tends to unity. For this case, the only difference in the opportunity set of peripheral country agents and residents of the VC is that the former must engage in indirect trading.

Since countries are of equal size, S_{1i}^{bVCE} and S_{1i}^{aVCE} are independent of i. Then we can write (4.17) as:

$$s^{bVCE} \frac{(N-1)}{N} = \frac{m_{11}}{N^2} + \frac{(1 - m_{11}/N)}{N-1} - p_1 \phi$$

$$= \frac{1}{N} \left(1 - s^{bVCE} \frac{(N-2)(N-1)}{N} \right) + s^{bVCE} \frac{(N-2)}{N} - N\phi \left(1 - s^{bVCE} \frac{(N-2)(N-1)}{N} \right) \tag{4.22}$$

The first line is explained as follows. The supply of peripheral currency i to the $1i$ trading post originates with the demand of country i households for non-i goods, which equals their money holdings $n_i m_{ii}$ $(= 1)$ times the measure of non-i goods, which is $1 - n_i = (N-1)/N$.

Country i sellers then receive the bid price s_{1i}^{bVCE} per unit of currency. The demand for currency i comes from residents of country 1 and country $j \notin \{1, i\}$. First, country 1 residents' total nominal demand for goods is $n_1 m_{11} = m_{11}/N$, and thus their demand for country i goods is $n_i n_1 m_{11} = m_{11}/N^2$. Second, residents of each peripheral country $j \notin \{1, i\}$ exchange currency 1 for currency i. In total, the amount of currency 1 held by peripheral countries is equal to $1 - m_{11}/N$, so the amount per country is $(1 - m_{11}/N)/(N - 1)$. An amount $1/(N-2)$ of this is spent on currency i, but there are $N-2$ such countries. Hence, $(1 - m_{11}/N)/(N-1)$ represents the total spending on currency i coming from peripheral countries. However, the supply of currency 1 to the $1i$ market is reduced by the amount $p_1\phi$, which is the amount of currency 1 that needs to be held by the $1i$ trading post manager, to cover the fixed cost of setting up the post.

The second line of (4.22) comes from expanding the definitions of m_{11} and p_1 from (4.13) and (4.16). Note that there is a simultaneity here in that both the supply *and* demand for peripheral currency depends on the equilibrium bid price under VCE. Intuitively, the equilibrium bid price determines how much of currency 1 can be taken on to the next trading post.

After re-arranging (4.22), we obtain the solution for s^{bVCE} as:

$$s^{bVCE} = \frac{(1 - \phi N^2)}{(N-2)(N-1)(1-\phi N^2)/N + 1}. \tag{4.23}$$

This exchange rate is lower than (3.14). Thus the VCE equilibrium pushes down exchange rates for the peripheral countries. Both the demand and supply for currency i at the trading post $1i$ rise in the VCE, relative to the STE. But demand rises by less than supply, since the increase in the demand for i by peripheral countries (bringing currency 1 from last period) is partly offset by a lower demand for i from the residents of country 1, the vehicle currency country, given that their money holdings are lower.

The value of s^{aVCE} in case A is:

$$s^{aVCE} = \frac{s^{bVCE}}{(1 - N^2\phi)\,\Omega_A(N)}, \tag{4.24}$$

where

$$\Omega_A(N) = \frac{N - 1 - N^2\phi}{N - 1 - (N-2)\,N^2\phi} > 1 - N^2\phi.$$

Comparing (4.24) with (3.15), we see that the bid-ask spread is lower under the case A VCE than under the STE, for all feasible values of ϕ. Intuitively, greater trade volume on both sides of the foreign exchange market pushes down spreads.

Case B: $\{n = 1/N\}$. This case is more general than Case A. While the case restricts all countries to be of equal size, it leaves the discount factor and the rate of country 1 money growth to be arbitrary. In this case, we can write the bid-ask spread as:

$$\left(\frac{s^a}{s^b}\right)^{VCE} = \frac{\Omega_B(\gamma_1)}{(1 - \phi_1 N^2)^2}, \tag{4.25}$$

where

$$\Omega_B(\gamma_1) = \left(1 - \phi_1 N^2\right) \frac{\beta(N-2) + 1 - \beta(N-2)N\phi_1\left(N - 1 + \frac{1}{\gamma_1}\right)}{[\beta(N-2) + 2 - (\beta(N-2)+2)N^2\phi]} < 1.$$

Note that $\Omega'_B(\gamma_1) > 0$. Again, the bid-ask spread is smaller than under STE, but the spread is increasing in money growth. Higher country 1 money growth reduces a peripheral country's currency deliveries to each trading post in a VCE, thus reducing trading volume and bidding up spreads. But it is still the case that $\lim_{\gamma_1 \to \infty} \Omega_B(\gamma_1) < 1$. Money growth can not generate a spread higher than that in the STE.

4.4. Efficiency and Resource Allocation with a Vehicle Currency

The VCE reduces the resources needed to operate the exchange, relative to STE, and hence raises available world resources for consumption. Each peripheral country now sets up just one trading post. With less resources used up in trading posts, there are more of all goods $i > 1$ available for consumption, and the same amount of good 1. For large N, this efficiency gain can be substantial. But at the same time, the vehicle currency introduces an asymmetry into the allocation of world resources. In this section, we analyze the nature of the global gains from a vehicle currency, as well as the asymmetric gains achieved by the vehicle currency country.

Again, we begin with some special cases.

Case A: $\{n = 1/N, \gamma_1 = 1, \beta \to 1\}$

In this case, the efficiency gains from the VCE are easy to illustrate. In the STE, each country's net output of each of its goods is $1 - \phi N(N-1)$ (the endowment less the cost of setting up $N-1$ trading posts, divided by the number of goods in the country; $1/N$). In a VCE, net output of each centre country good is unchanged, since it must set up $N-1$ trading posts still. But output of each good of each peripheral country is now $1 - \phi N$, since only one trading post is set up, for each country.

Although output of each peripheral country good is larger, the benefits of the VCE go disproportionately to VC country residents. For Case A, we may show that:

$$c_{11}^{VCE} = 1, \tag{4.26}$$

$$c_{1i}^{VCE} = \Omega_A(N) \geq 1, \; i > 1, \tag{4.27}$$

where Ω_A is defined following (4.24). Country 1's consumption of the home good is the same as in STE. Consumption of all other country's goods differs from (3.17), however. It is easy to see that $c_{1i}^{VCE} > c_{1i}^{STE}$. Moreover, from (4.27), $\Omega_A(3) = 1$, and $\Omega'_A(N) > 0$, so that $c_{1i}^{VCE} \geq 1$. Since c_{11} is unchanged, and c_{1i} is higher, the VC country is unambiguously better off than in the STE.

For the peripheral countries, we may establish that:

$$c_{ii}^{VCE} = 1 \tag{4.28}$$

$$c_{i1}^{VCE} = (1 - \phi N^2) \tag{4.29}$$

$$c_{ij}^{VCE} = (1 - \phi N^2)\Omega_A(N). \quad (4.30)$$

For the peripheral country, consumption of the domestic good and country 1 good is the same as in the STE. Consumption of *other* peripheral countries differs however. From (4.30), since $(1 - \phi N^2) < 1$, we must have $c_{ij}^{VCE} < c_{1j}^{VCE}$, $i > 1, i \neq j$. Thus, the gain from VCE for peripheral countries is lower than that of the VC country. Comparing (4.29) and (4.30) with (4.26) and (4.27), we can see that in equilibrium, all the transactions costs of setting up trading posts are borne by the peripheral countries. Thus the good 1 cost of setting up the $1i$ trading post is borne by country i, given (4.29), and $c_{11}^{VCE} = 1$. But also, the good j cost of the $1j$ trading post is borne by country i, given (4.30). In fact, since $\Omega_A(N) > 1$, for $N > 3$, the VC country consumes more than the average endowment of peripheral goods, so that in a VCE, the peripheral countries incur *more* than the full amount of the transactions costs.

Does this mean that peripheral countries are worse off? The answer is no, because, while they bear all the transactions costs, the overall transactions costs are far lower in VCE than in STE, and the transactions cost saving is increasing in N. From (4.30), we know that $c_{ij}^{VCE} \geq c_{ij}^{STE}$, with strict inequality for $N > 3$. Because $c_{ii}^{VCE} = c_{ii}^{STE}$ and $c_{i1}^{VCE} = c_{i1}^{STE}$, and for $N = 3$, $c_{ij}^{VCE} = c_{ij}^{STE}$, then for the case of three countries, peripheral countries are exactly as well off in VCE as in STE. But for $N > 3$, $c_{ij}^{VCE} > c_{ij}^{STE}$, and welfare is higher under VCE.

The higher is N, the greater is the transaction cost saving due to the vehicle currency. Country 1's consumption of peripheral goods may be written as $c_{1i}^{VCE} = p_1/(s_{1i}^a p_i)c_{11}$. Since c_{11} is constant in this special case, a rise in c_{1i}^{VCE} is equivalent to country 1 receiving a higher terms of trade, or a lower relative price of the peripheral good. We may write $p_1/(s_{1i}^a p_i) = \Omega_A(N)$. This is greater than the analogous price under STE, which is $1 - \phi N^2$. For the peripheral countries, consumption of other peripheral country goods is written as $c_{ij}^{VCE} = s_{1i}^b p_i/(s_{1j}^a p_j)c_{ii}$. Since c_{ii} is constant, the increase in consumption of other peripheral country goods, relative to the STE, comes about only if there is a fall in their relative price, $(s_{1j}^a p_j)/s_{1i}^b p_i$. In case A, $s_{1i}^b p_i/(s_{1j}^a p_j) = (1 - \phi N^2)\Omega_A(N) \geq 1 - \phi N^2$. Thus, the existence of a vehicle currency effectively improves the terms of trade for *all countries.* Nevertheless, the gains for country 1 exceed those for peripheral countries. Country 1 has to trade only once in order to consume any good, while peripheral countries must trade twice. Even without time discounting or money growth, this leads the terms of trade gains to be lower for the peripheral country, relative to the VC country. In addition, as we have noted, for $N = 3$, *all the gains* go to the VC country.

Case C: $\{\gamma_1 = 1,\ \beta \to 1,\ n_1 = n,\ n_i = (1 - n)/(N - 1),\ i > 1\}$.

We use this case to illustrate how the level and distribution of welfare gains from a VCE change with the VC country's size. In this case, country 1 can have a different size from peripheral countries. For instance, if $n > 1/N$, then $n_i < 1/N$ for all $i > 1$, which implies that the VC country is larger than all peripheral countries.

The solution in this case can be shown as follows. First, we find that $c_{11}^{VCE} = c_{11}^{STE}$, $c_{ii}^{VCE} = c_{ii}^{STE}$, and $c_{i1}^{VCE} = c_{i1}^{STE}$, $i > 1$, as in case A. So again, VCE only makes a difference for consumption of peripheral country goods for country 1, and consumption of non-domestic peripheral goods for the countries $i > 1$. Solving, we find that:

$$c_{1i}^{VCE} = \Omega_C(n, N), \tag{4.31}$$

$$c_{ij}^{VCE} = \left[1 - \frac{\phi(N-1)}{n(1-n)}\right] \Omega_C(n, N). \tag{4.32}$$

where

$$\Omega_C(n, N) = \left[1 - \phi\frac{(N-1)^2}{(1-n)(n+N-2)}\right] \Big/ \left[1 - \phi\frac{(N-1)(N-2)}{n(n+N-2)}\right].$$

We may use these solutions to construct the values $c_{1j}^{VCE} - c_{1j}^{STE}$, and $c_{ij}^{VCE} - c_{ij}^{STE}$, measuring the degree to which the VC country and the peripheral countries gain from the VCE, relative to STE. Using the solutions (4.31) and (3.17), we may show that:

$$c_{1j}^{VCE} - c_{1j}^{STE} = \frac{\phi\rho}{1-\phi\rho}\left[2 - \phi\frac{(N-1)}{(1-n)n}\right],$$

where $\rho \equiv \frac{(N-2)(N-1)}{n(n+N-2)} < 1$. Under the feasibility condition $\phi\frac{(N-1)}{(1-n)n} < 1$, this difference in consumption is always positive. Thus, the VC country always gains, whatever its relative size.

However, the peripheral countries do not always gain. We may obtain:

$$c_{ij}^{VCE} - c_{ij}^{STE} = \phi\rho_1\left[n(N-4+3n) - \phi(N-1)(N-3)\right]$$

where

$$\rho_1 \equiv \left(\frac{N-1}{1-n}\right)^2 \Big/ \left[n(n+N-2)(1-\phi\rho)\right] > 0.$$

It is possible to have $c_{ij}^{VCE} < c_{ij}^{STE}$. If this occurs, then peripheral countries must lose as a result of the VCE. Take the case $N = 3$ as an example. Recall that in case A, with $N = 3$ (and $n = 1/N$), then $c_{ij}^{VCE} - c_{ij}^{STE} = 0$. But here, with $N = 3$, the expression inside the square parentheses is $n(3n-1) < 0$, so if $n < 1/3$ we have $c_{ij}^{VCE} - c_{ij}^{STE} < 0$, and the peripheral country is worse off in the VCE. The intuition is easy to see. In the case $N = 3$ before, the peripheral countries were indifferent between the VC and STE. The costs of indirect trade were just offset by the gains from shutting down trading posts. But with $N = 3$ and $n < 1/3$, the costs of indirect trade exceed the gains from fewer trading posts, since using the vehicle currency involves trading through a smaller market with higher transactions costs. Thus, a VCE where the vehicle currency country is smaller than the average sized country may reduce welfare for peripheral countries.

We may also explore the way in which the gains from the VCE change in response to changes in country size. In Appendix A, it is shown that:

$$\left.\frac{d(c_{1j}^{VCE} - c_{1j}^{STE})}{dn}\right|_{n=1/N} < 0, \quad \left.\frac{d(c_{ij}^{VCE} - c_{ij}^{STE})}{dn}\right|_{n=1/N} > 0.$$

Thus, the consumption gains for the VC country are negatively related to its size. In the STE, a rise in country 1's size has a large effect on country 1's consumption of all goods $j > 1$, as described in above. But in the VCE, the increase in country 1's size has a smaller impact, because each trading post has more currency j on the other side. A marginal increase in the size of the vehicle currency economy has a diluted impact on its consumption of other goods in the VCE relative to STE.

By contrast, for peripheral countries, the gain goes in the opposite direction. A rise in the relative size of country 1 will reduce c_{ij}^{STE}, since each peripheral country becomes relatively smaller. But in the VCE, the negative impact of a rise in n is diminished, because country i is purchasing country j's good via the $1i$ and $1j$ currency markets. Hence, while the VC country size tends to lower gains for the VC country itself, it will raise gains for peripheral countries.

Case D: $\{\beta \to 1,\, n = n_i = 1/N\}$.

We use this case to examine the impact of country 1 money growth, again assuming no time discounting, and all countries being of equal size. We may derive the consumption of country 1 in a VCE as:

$$c_{11}^{VCE} = \frac{N[1 + (\gamma_1 - 1)(N-1)(1 - \phi(N-2))]}{N(2\gamma_1 - 1) + 2(1 - \gamma_1)}, \tag{4.33}$$

$$c_{1j}^{VCE} = [1 + (\gamma_1 - 1)(N-1)(1 - \phi(N-2))]\Omega_D(\gamma_1). \tag{4.34}$$

where

$$\Omega_D(\gamma_1) = \frac{1 - \phi\frac{N^2}{N-1}}{\gamma_1 - \phi\frac{N(N-2)}{N-1}(\gamma_1(N-1)+1)}.$$

Country 1 money growth affects allocations in the VCE because it represents a tax on peripheral country holders of currency 1. Both c_{11}^{VCE} and c_{1j}^{VCE} from (4.33) and (4.34) are increasing in γ_1, although $\Omega_D'(\gamma_1) < 0$. Since, under STE, allocations are independent of monetary policy, clearly the gains to VCE for country 1 are increasing in γ_1.

Analogously, we can derive the consumption for peripheral countries under VCE as:

$$c_{ii} = 1, \tag{4.35}$$

$$c_{i1} = \frac{(1 - \phi N^2)\gamma_1 N}{N(2\gamma_1 - 1) + 2(1 - \gamma_1)}, \tag{4.36}$$

$$c_{ij} = (1 - \phi N^2)\Omega_D(\gamma_1). \tag{4.37}$$

Country 1 money growth reduces peripheral country consumption of both good 1 and all other peripheral country goods. From (4.37), we see that $\lim_{\gamma_1 \to \infty} c_{ij} = 0$, since country 1 inflation progressively erodes the usefulness of the vehicle currency in exchange. Then consumption of good j goes only to residents of country j and country 1. We note also that, even though the financing for consumption of good 1 does not require peripheral country residents to hold currency 1 over time, their consumption of the vehicle currency good is eroded by money growth in the vehicle currency country. This happens because higher money growth reduces the demand for currency $i > 1$ coming from residents of all other peripheral countries, since it reduces the value of these agents currency 1 holdings. This pushes down the exchange rate that country i residents receive in the $1i$ trading post, reducing their terms of trade. In this way, money growth has both a direct and an indirect effect on peripheral country welfare.

Case D assumes $\beta \to 1$. In fact, the results just illustrated hold for general $\beta \leq 1$, but are more cumbersome to show. Nevertheless, we may state the following proposition, which is proved in Appendix B.

Proposition 4.2. *Under the assumption that $n_i = n = 1/N$, $i > 1$, the VCE satisfies the following features: (i) s^a_{1i}/s^b_{1i} is increasing in γ_1; (ii) c_{i1} ($i \neq 1$) is decreasing in γ_1, but c_{ii} is independent of γ_1; (iii) c_{ij} ($j \neq i, 1$) is decreasing in γ_1; (iv) c_{11} is increasing in γ_1; (v) c_{1i} is increasing in γ_1.*

4.5. Welfare Comparison

We now move to the general model, taking into account money growth, country size, time discounting, and variation in the number of countries. We examine the welfare gains from a vehicle currency, relative to the STE. We calibrate the model as follows. Although it is reasonable to assume that the carrying time period of vehicle currency is relatively small, the function of a vehicle currency extends across a number of different frequencies.[9] We set $\beta = 0.99$, to match a quarterly trading frequency. The value of the gross money growth rate γ_1 is taken from the US CPI growth rate over 1980-2006, which was 0.9 percent at a quarterly frequency. Thus we set $\gamma_1 = 1.009$.

There is a large literature on the measurement of transactions costs involved in foreign exchange trading. In Emerson et al. (1992), estimates of the gains to a single currency in Europe, using a survey of different measurement approaches, suggested that the reduction in transactions costs would be 0.4 percent of EU GDP. More direct estimates of transactions costs have been obtained from observed bid-ask spreads (e.g. Glassman (1987). Bid-ask spreads in large foreign exchange markets are typically much smaller, in the order of .08 percent (e.g. Huang and Stoll 1997). Aliber et al. (2000) criticize the use of bid-ask spreads and instead argue for using quoted data from foreign exchange futures. Their estimate of the equivalent transaction cost is 0.05 percent.

[9]This represents a compromise between different perspectives on the use of a vehicle currency. For some financial traders, the holding period of currency might be hours or days, while for other exporters or importers using vehicle currency to facilitate ongoing transactions, the time period would be significantly longer. More generally however, the need to hold either vehicle currency cash or liquid assets in order to facilitate trade might impose a cost over a much longer horizon. Since our model is based on currency use for commodity trade, we use a quarterly frequency. With much higher frequencies, the model implies that VC inflation rates can be very high without affecting the usefulness of the vehicle currency. The quantitative estimates of the benefits of a vehicle currency, relative to STE, are not sensitive to the frequency chosen, however.

From our perspective, the use of observed bid-ask spreads to measure transactions costs may be misleading. In our model, average transactions costs depend on volume, and hence on whether a vehicle currency exists. Because foreign exchange markets are already dominated by a vehicle currency, bid-ask spreads from such markets are not likely to give an adequate measure of the costs that would be borne in alternative trading structures. Given this uncertainty, we report results for a range of alternative values of ϕ, beginning with a basemark value for ϕ implied by the lowest of the above estimates, i.e. $\phi = 0.0005$. We also report results for a range of values of N, the number of countries, and n, the relative size of the vehicle currency country. Following case C above, we assume that all peripheral countries are of equal size, so that $n_i = (1-n)/(N-1)$, for all $i = 2, ..., N$.

$r_{ij}= c_{ij}^{VC1}/c_{ij}^{STE}$: country i's consumption of country j goods in VCE relative to STE

We compare the allocations received under the VCE with those of the STE. Define the consumption ratio between the STE and VCE as:

$$r_{ij} = c_{ij}^{VC1}/c_{ij}^{STE} \text{ for all } i, j \in \{1, 2, ..., N\}. \tag{4.38}$$

As a welfare measure we compute the uniform increase in the consumption of all goods that an agent would require, in the STE, to make her indifferent between STE and VCE. We denote this as dc_i, and compute this separately for agents of country 1 and country $j > 1$. We also compute average world welfare, which is defined as:

$$U^W = \sum_{i=1}^{N} n_i u_i \tag{4.39}$$

where the weights n_i on individual country utility reflect the population of each country. Changes in U^W are translated into uniform increases in world consumption, which we denote dc_W.

Figures 1a and 1b illustrate the relative consumption ratios r_{11}, r_{1j}, r_{i1}, r_{ii}, and r_{ij}, $i, j > 1$, $i \neq j$. The horizontal axis depicts the relative size of the VC country. The Figures assumes $N = 10$. Therefore, $n = 0.1$ represents a symmetric point where all countries are of equal size. For $n > 0.1$ ($n < 0.1$), country 1 is relatively larger (smaller) than all other countries. The Figure shows that the main effects of a vehicle currency are to increase consumption of peripheral country goods, both by country 1 and by other peripheral countries. At the symmetric point $n = 0.1$, c_{1j} ($j > 1$) is 16 percent higher under the VCE than in the STE, while c_{ij} ($i \neq j$, i, $j > 1$) is 3 percent higher. c_{11} is 6 percent higher than under STE. By contrast, c_{ii} is only slightly higher, since this differs across equilibria only due to time discounting (see 4.8), and the discount factor is very close to unity in this calibration. c_{i1} is slightly lower under VCE relative to STE.[10]

Figure 1 also illustrates the impact of the relative size of country 1. As country 1 gets larger relative to the rest of the world, both c_{11} and c_{1j} fall, while c_{ij} rises. Thus, country 1 tends to lose, as it gets larger, while peripheral countries tend to gain. This is consistent with the discussion above. Under the STE, a rise in n involves a fall in the relative size of the periphery, which raises average cost of trading and reduces the gains from trade with

[10]Output of good 1 in a VCE is lower than that of peripheral countries, because good 1 is used to cover transactions costs for N-1 trading posts, while good $i > 1$ is just used for 1 post.

one another. By contrast, trading through the large vehicle country currency involves a gain via a reduction in average trading costs, and this gain is greater, the larger is the vehicle currency country.

Figure 2 translates the results directly into welfare equivalent measures. The vertical axis represents the consumption benefit of the VCE, dc_i, for country 1, and for the peripheral countries, and for a measure of average world utility given by (4.39). For the baseline calibration with $N = 10$ and $n = 0.1$, the welfare gains to a vehicle currency are heavily weighted towards the centre country. It gains the equivalent of 15 percent of consumption, while the peripheral country gains represent only 3 percent of consumption. But the gains are very sensitive to country size. If the centre country is larger - say $n = .25$ (approximately the US share of world GDP), then the welfare gains are much closer - 5 percent for country 1 and 3.2 percent for the peripheral countries. As n rises above 0.3, the gains for peripheral countries exceed those of the VC.

Figure 2 is based on a highly conservative estimate of the transaction cost of international currency exchange. If we use a higher estimate (based on the bid-ask spreads measured in Huang and Stoll (1998)) of $\phi = 0.001$, the welfare gains to a vehicle currency are much larger. Note that this is still a very small transaction cost, one tenth of 1 percent of GDP. Figure 3 shows the results using this estimate. In the baseline case of $n = 0.1$, the centre country consumption gain is 24 percent, and the peripheral countries gain 5 percent. If in this case we use the higher estimate of $n = 0.25$, then the peripheral countries gain exceeds that of the centre country.

These welfare gains are extremely large, relative to standard estimates of gains from the public finance literature.[11] What accounts for the large size of the benefits? The key feature of the VCE is that, for a relatively large number of countries, it leads to a dramatic reduction in the number of trading posts, and hence greatly reduces the overall costs of transactions. With $N = 10$, and $n = 0.1$, in the STE each country must set up $N - 1$ trading posts. The costs of setting up a trading post must be recouped equally from each agent's endowment, so the total cost undergone per agent is $\phi(N - 1)/n_i$. With $n_i = n = 1/N$, this means that output of each good in country i is $(1 - \phi(N - 1)N)$. For the calibration used in Figures 1 and 2, this implies that trading costs reduce output by 2.7 percent. By contrast, in the VCE, for a peripheral country, only one trading post must be formed. Output per good then is $(1 - \phi N)$, and transactions costs reduce output by only 0.3 percent. Even though individual transactions costs are very small, the overall cost can be very large when summed across a large number of bilateral trading posts. The aggregate welfare benefits are then obviously tied directly to the size of ϕ and the number of countries.

Figure 4 follows through on this logic. We illustrate the welfare gains as a function of the number of countries, N, assuming equal country size, so that $n = 1/N$, using the baseline estimates for all other parameters. From the discussion above, we know that peripheral countries do not benefit at all if there is zero discounting, zero money growth, and $N = 3$.

[11] Note however that the counterfactual involved is not necessarily equivalent to a policy change, since the move from STE to VCE is not chosen by governments. In addition, we could argue that the STE allocation is not a historically observed outcome.

Thus, for $N = 3$, $\beta < 1$, and $\gamma_1 > 1$, peripheral countries are *worse* off in a VCE. Thus, in the baseline calibration, peripheral countries only gain from a VCE if N is above a critical level. For Figure 3, a vehicle country is beneficial to the peripheral countries only for $N \geq 6$. But then as N rises above this, the welfare gains rise exponentially. While the efficiency gains from a vehicle currency are clearly higher for large N, an interesting feature of these gains is that for peripheral countries, the gains may not be monotonic in N. Figure 4 illustrates this effect by showing the gains to VCE for a higher rate of country 1 inflation. In this case, country 1 gains are higher, not surprisingly. But also, for small N, peripheral country gains may be *falling* in N initially. The intuition for this negative effect of N is that increasing the number of countries makes each country more open, because it consumes approximately $1 - 1/N$ of total goods as imports. This means that in the VCE, it is more exposed to the inflation tax of country 1, while in the STE this has no effect. Hence, beginning at $N = 3$, an increase in N may reduce welfare for a peripheral country initially, relative to STE. But as N rises further, the benefits of reduced transactions costs take over, and the gains are increasing in N.

Note that while the VCE offers welfare gains for the world economy, the distribution of gains depends on the money growth rate of country 1. Figure 5 illustrates the gains in the baseline calibration, except setting $\gamma_1 = 1$. In this case the gain to each peripheral country is larger, and the gain to the VC country falls from 15 percent to 9 percent. Thus 6 percent of the welfare gain in the baseline case is due to the monetary policy followed by the VC. Note that the overall *world* welfare gain is relatively independent of γ_1. The gain for the VC country is offset closely by the losses to peripheral countries.

How high can γ_1 increase before it eliminates the gains for the peripheral countries? This will depend upon both N and n. For a large number of countries, and a VC country which is large relative to others, there are still gains to a vehicle currency even for high rates of VC money growth. Figure 6 shows the gains to peripheral countries, for various levels of γ_1. When $n = 0.1$ (VC country equal size), peripheral gains from the VC are eliminated at $\gamma_1 = 1.036$. But if $n = 0.2$, there are still gains to peripheral countries for $\gamma_1 < 1.044$. Thus, VC country inflation rates can be very high before eliminating the welfare gains to a vehicle currency.

Nevertheless, the above result raises questions about the degree to which the VCE itself is sustainable in face of high centre country money growth. Moreover, in assessing the benefits to a vehicle currency, there is a clear trade-off between the rate of inflation in the VC country and the size of the VC country. In the next section, we explore the question of sustainability of a vehicle currency, and show how it relates to this trade-off.

5. Robustness of the Vehicle Currency Equilibrium

We have shown that there may be large welfare gains to a vehicle currency equilibrium. But we did not show how a vehicle currency arises, or which currency will play the role of a vehicle currency. Because of the trading technology and the existence of fixed costs, there are many equilibria in the model. Such multiplicity is inevitable when there are fixed costs of organizing the currency exchange. If some bilateral markets are not open, then no individual trading firm has an incentive to incur a fixed cost in order to trade in that market, since, with no customers, it will perceive that there are no profits to be gained. This multiplicity is robust to the refinements of trembling hands by a small measure of agents or of evolutionary stability.[12]

Given this characteristic of trading posts technologies with fixed costs, we must explore the robustness of a vehicle currency equilibrium through alternative approaches than the standard evaluation of Nash equilibria. In order for a deviation from any equilibrium to have aggregate consequences, it must be undertaken by a large number of agents. In this section we examine whether the VC equilibrium is robust to deviations undertaken by all agents within a country. One way to think of this national deviation is as an implicit policy choice by national governments.

We focus on two types of deviations from a VCE. First, we examine the impact of a bilateral deviation, in which all households in two countries choose to trade their currencies directly, but maintain the use of the vehicle currency in trading with all other countries. We then evaluate a deviation in which all households in all peripheral countries switch to using a different currency as the vehicle currency.

5.1. Bilateral Deviations

Let us first consider a bilateral deviation by two countries, say, country 2 and country 3. Suppose that all households in the two countries deviate to trade their own two currencies directly. Other countries do not participate in the 23 post. Moreover, countries 2 and 3 still supply their domestic currencies to trade for currency 1 and use currency 1 to get other peripheral currencies. However, country 2 does not use currency 1 to buy currency 3, and country 3 does not use currency 1 to buy currency 2.

Denote $I = \{1, 2, 3\}$. For a country $i \notin I$, the decision problem is the same as in the VCE characterized in the previous section, because all currency posts which the country participated before are still active after the above deviation. Since the decision problems of a household in country 2 and of a household in country 3 are images of one another, we only formulate the problem for country 2.

With the deviation, a household in country 2 faces the following constraints involving currencies 1, 2 and 3:

$$m'_{22} = m_{22} - f^{12}_{22} - f^{23}_{22}, \quad m'_{23} = m_{23} + \frac{1}{s^a_{23}} f^{23}_{22},$$

$$m'_{2j} = m_{2j} + \frac{1}{s^a_{1j}} f^{1j}_{21},\ j \notin I, \quad m'_{21} = m_{21} - \sum_{j \notin I} f^{1j}_{21} + s^b_{12} f^{12}_{22},$$

$$\sum_{j \notin I} f^{1j}_{21} \leq m_{21}.$$

[12]For example, if a small measure of agents from any two countries exchange their domestic currencies directly in the VCE constructed above, they will make a loss as the amount of currencies brought into that post will not be sufficient to cover the fixed trading cost. Similarly, if a small measure of agents deviate to using a different currency as the vehicle currency, they will make a loss.

Other constraints that the household faces, such as the cash in advance constraints in the goods markets, are the same as those in the previous section.

Because country 2 still needs currency 1 to exchange for other non-I currencies, the cash in advance constraint on currency 1 in the goods market does not bind for country 2, as in the previous section. All other cash in advance constraints bind. Then, the household's optimal choices yield:

$$p_2 c_{22} = \frac{1}{s_{12}^b} p_1 c_{21} = s_{23}^a p_3 c_{23} = \frac{\gamma_{1(+1)} s_{1j(+1)}^a}{\beta s_{12}^b} p_{j(+1)} c_{2j(+1)}, \ j \notin I.$$

As before, $m_{22} = 1/n_2$, $m_{j2} = 0$ ($j \neq 1, 2$), and $p_2 = 1/n_2$. Adding up country 2's spending of currency 2, invoking stationarity, and substituting the first-order conditions for c yields:

$$c_{22} = \frac{1}{\delta_2^d}.$$

where $\delta_2^d = n_2 + n_1 + n_3 + \beta(1 - n_1 - n_2 - n_3)$. The household's consumption levels of other goods can be calculated accordingly. Also, for $j \notin I$, the household's optimal decisions on the quantities of currency trade yield:

$$f_{22}^{12} = \frac{1}{n_2} - p_2 n_2 c_{22} - n_3 s_{23}^a p_3 c_{23} = \frac{n_1 + \beta(1 - n_1 - n_2)}{n_2 \delta_2^d}, \tag{5.1}$$

$$f_{22}^{23} = \frac{n_3}{n_2 \delta_2^d}$$

$$f_{21}^{1j} = n_j s_{1j}^a p_j c_{2j} = \frac{\beta}{\gamma_1} \frac{s_{12}^b n_j}{n_2 \delta_2^d}$$

$$m_{21} = \sum_{j \notin I} f_{21}^{1j} = \frac{\beta}{\gamma_1} \frac{(1 - n_1 - n_2 - n_3) s_{12}^b}{n_2 \delta_2^d}. \tag{5.2}$$

At the 23 post, bid/ask prices satisfy $f_{22}^{23}/s_{23}^a = f_{33}^{23} - \frac{\phi}{n_3}$ and $s_{23}^b f_{33}^{23} = f_{22}^{23} - \frac{\phi}{n_2}$. The solutions are:

$$s_{23}^b = \left(\frac{n_3}{\delta_2^d} - \frac{\phi}{n_2}\right)\left(\frac{n_2}{\delta_2^d}\right)^{-1} \tag{5.3}$$

$$s_{23}^a = \left(\frac{n_2}{\delta_2^d} - \frac{\phi}{n_3}\right)^{-1} \frac{n_3}{\delta_2^d} \tag{5.4}$$

The bid-ask spread at the 23 post is smaller than that in the STE, provided $N > 3$. This is because, when $\beta < 1$, countries 2 and 3 will assign a higher fraction of their budget to each other's good than they will to other peripheral country goods, given that the consumption of those other goods requires a delay in consumption.

In the analysis below, $j \notin I$ unless it is specified otherwise. To compute exchange rates at the 12 post and the 13 post after the deviation by countries 2 and 3, we count the total amount of currency 1 that is held by the peripheral countries at the beginning of a period as follows:

$$1 - n_1 m_{11} = n_2 m_{21} + n_3 m_{31} + \sum_{j \notin I} n_j m_{j1}.$$

At the 12 post, bid/ask prices satisfy the following conditions:

$$s_{12}^a \left(f_{22}^{12} - \frac{\phi}{n_2} \right) = f_{11}^{12} + \sum_{j \notin I} f_{j1}^{12} \tag{5.5}$$

$$s_{12}^b f_{22}^{12} = f_{11}^{12} + \sum_{j \notin I} f_{j1}^{12} - p_1 \phi. \tag{5.6}$$

At the 13 post, the conditions are analogous. At the $1j$ post ($j \notin I$), the conditions are:

$$s_{1j}^a \left(f_{jj}^{1j} - \frac{\phi}{n_j} \right) = f_{11}^{1j} + f_{21}^{1j} + f_{31}^{1j} + \sum_{j \notin I \cup \{j\}} f_{i1}^{1j} \tag{5.7}$$

$$s_{1j}^b f_{jj}^{1j} = f_{11}^{1j} + f_{21}^{1j} + f_{31}^{1j} + \sum_{j \notin I \cup \{j\}} f_{i1}^{1j} - p_1 \phi. \tag{5.8}$$

These equations determine the exchange rate at each post involving currency 1.

Is the deviation profitable for countries 2 and 3? In general, in order to assess this question we need to compare utility levels in a deviating equilibrium, relative to the VCE. But in the special case where $\beta \to 1$, and $\gamma_1 = 1$, we may use the property that a bilateral deviation by countries 2 and 3 leaves *unchanged* both the relative prices and consumption of all goods $i \notin \{2, 3\}$ by all countries $i = 1, ..., N$. This means that in assessing the benefits from a deviation to a bilateral trade for countries 2 and 3, we can simply look at the change in consumption of goods 2 and 3. Moreover, from (5.3) and (5.4), note that evaluated at $\beta = 1$, the bilateral exchange rates between currencies 2 and 3 are identical to those in the STE. This means that in the case $\beta \to 1$, and $\gamma_1 = 1$, $c_{23}^{DEV} = c_{23}^{STE}$. This implies that the conditions under which a bilateral deviation by countries 2 and 3 is beneficial to these countries are *equivalent* to the conditions that welfare of the peripheral countries under VCE is lower than that under STE (again in case $\beta \to 1$, and $\gamma_1 = 1$).

We may summarize this in the following proposition:

Proposition 5.1. *In the case $\beta \to 1$ and $\gamma_1 = 1$, there are no gains to deviating to a bilateral trading arrangement when $n \geq 1/N$. When in addition to $n \geq 1/N$, $N > 3$, the deviating countries are strictly worse off.*

Again, we note that this condition may fail when n is too small, for the same reason that the VCE may lead to lower welfare than under STE. In addition, the result implies that, under this case, when considering a bilateral deviation, each country's welfare calculation is exactly aligned with average welfare for all peripheral countries. A bilateral deviation is only desirable individually when it is desirable in the aggregate.

To gain another perspective on the effect of a bilateral deviation, we can compare the direct exchange of currency 2 for currency 3 and the indirect exchange through the vehicle currency. With the direct exchange, a household in country 2 gets $1/s_{23}^a$ units of currency 3 for each unit of currency 2. With the indirect exchange, one unit of currency 2 returns s_{12}^b units of currency 1 in the current period, which the household can use to exchange for s_{12}^b/s_{13}^a next period. In the absence of discounting and money growth, the indirect exchange through the vehicle currency gives a higher payoff to a household in country 2 than the direct exchange if and only if $s_{12}^b/s_{13}^a > 1/s_{23}^a$, or $s_{23}^a s_{12}^b/s_{13}^a > 1$. It turns out that this condition holds if and only if the gain from VC is negative.[13]

In the more general case where $\beta < 1$ and $\gamma_1 \geq 1$, a bilateral deviation has implications for consumption of all goods. Moreover, individual incentives are no longer aligned with aggregate welfare. But even then, the main impact of a bilateral deviation is on the consumption of the goods of the deviating countries, by the deviating countries themselves, and if γ_1 is large, by the VC country, since in the latter case, a deviation implies that it loses some inflation tax revenue. For the deviating countries, the switch to bilateral trade reduces the inflation tax embodied in trade using the vehicle currency, and as a result, consumption of the deviating partners good may rise, so long as n is relatively small. But if country 1 is large enough, the benefit from avoiding the inflation tax is offset by the higher transactions costs of trading bilaterally, relative to going through the cheaper vehicle currency.

Figure 7 illustrates the welfare gains from remaining in VCE, relative to a bilateral deviation, for the deviating countries. This is compared to the general welfare gain from the VCE, relative to STE, as calculated in the previous section. Under the calibration behind the Figure, we see that there is no gain to a deviation for any value of n shown in the Figure, even if country 1 is disproportionately small relative to other countries.[14] Likewise, as we saw above, there is a positive aggregate welfare gain to a VCE relative to STE.

We saw in the previous section that the threshold quarterly rate of inflation which eliminates the gain to the VCE for equal size countries was 3.6 percent. But the analogous threshold for a bilateral deviation is substantially lower. The reason is that two countries individually can pursue a bilateral deviation and avoid the inflation tax in their mutual trade, without giving up the benefits of a vehicle currency in trading with all other countries. Thus, at any value for n and N, the maximum value of γ_1 that eliminates a bilateral deviation is smaller than the value that eliminates gains from a vehicle currency for all the peripheral countries together. Figure 8 illustrates this relationship in the form of a trade-off between γ_1 and n that just eliminates the incentive to undertake a bilateral deviation from VCE, and the analogous trade off for values that just eliminate gains from a VCE to the peripheral countries. Take the case where the VC country is 25 percent of world GDP.

[13]This partial equilibrium assessment is appropriate only in the case where $\beta \to 1$ and $\gamma_1 = 1$. When these conditions do not apply, then the bilateral deviation will change bid-ask spreads on bilateral trades other than the 23 trade.

[14]Note that there is always a value for n small enough to warrant a bilateral deviation. For this Figure, in order to gain from a deviation, we need $n < 0.006$.

Then the peripheral countries still gain from the VC even for (quarterly) inflation rates of 5 percent. But in order to avoid a bilateral deviation, inflation rates must be no higher than 3 percent.

5.2. The Introduction of the Euro

The discussion above showed that when countries are of equal size, an increase in the number of countries implies that each individual country specializes in a narrower range of goods, which increases the costs of bilateral trade, thereby increasing the gains to a vehicle currency. This discussion was based on the assumption of a one-to-one relationship between currencies and countries. But if some countries join a single currency area, then the number of currencies as measured by N will fall, and the economic size of the currency area will equal the sum of the measure of goods produced in the member countries. What impact does this have on the incentives of peripheral countries who remain outside the currency area to engage in a bilateral deviation from currency 1 as a vehicle currency? This relates to the question of the sustainability of the US dollar as an international currency in the presence of the euro, as discussed in the introduction.

To explore this question, we take the same example as before, but assume that $K \geq 2$ countries, from $j = (N + 1 - K), ..., N$, join a single currency area, eliminating the transactions cost of monetary trade within the area. This reduces the number of separate currencies in the world economy from N to $N + 1 - K$. But it also increases the economic size of the $N + 1 - K$ currency area.[15]

We calibrate as before so that initially $N = 10$, and assume that all peripheral countries are of equal size $n_i = (1 - n)/(N - 1)$. Let $K = 3$, so that the number of currencies falls from 10 to 8, and the size of the 8th 'country' is now $3(1-n)/(N-1)$. Now we ask, what is the incentive for a peripheral country $i \notin \{1, N+1-K\}$ and for the $N+1-K$ currency area to undertake a bilateral deviation in order to trade currency directly with each other rather than indirectly, using currency 1 as the vehicle currency. This trade-off now differs for country i and country $N + 1 - K$, since they do not have equivalent incentives for a bilateral deviation. In both cases, the trade-off shifts dramatically downwards as shown in Figure 9, indicating that the creation of a single currency area substantially increases the incentive to engage in a bilateral deviation, both for remaining peripheral currencies, outside the new single currency area, and for the new single currency area itself. As in Figure 7, Figure 9 shows the gain from remaining in the VCE, relative to deviating to bilateral currency trade, for a peripheral country and for a member of the single currency

[15] In this analysis, we take the currency area as a given institution. The determination of the number and size of currency areas represents a separate question that cannot be addressed without enhancing the model. This is because according to the assumptions made here, a currency area removes the transactions cost of monetary exchange, and for a peripheral country, there is no gain to having a separate currency. Thus, all countries $j = 2..N$ would wish to join the currency area. One interpretation of the experiment here is that there are some set-up costs to a currency area that are not modeled explicitly, and that the only efficient currency area is that among the K countries. For instance, if governments had unpredictable spending demands which required seigniorage revenue, then they would have to balance the needs for funds against the reduced transactions costs from joining a currency area.

area, as a function of the size of the VCE country. For n less than 0.3, this gain is negative. In other words, there is a gain from deviating from the VCE both for a member of the single currency area, and for a peripheral country, unless the vehicle currency country is 30 percent of world GDP. An equivalent interpretation is that the maximum rate of inflation that the VCE country can sustain without triggering a deviation, shifts sharply downwards. In comparison with Figure 8 where all peripheral countries are of equal size, the maximum rate of quarterly inflation that the VCE country can sustain when $n = 0.25$, is now only 0.4 percent.

This example suggests that the dominance of a vehicle currency is limited in a very natural way by the opening up of an outside single currency area, both because it increases the economic size of non-VC currency economies, and because it reduces the total number of currencies in existence. In separate ways, both effects increase the incentive to abandon a vehicle currency.

5.3. Choice among Vehicle Currencies

The calculations in previous two subsections can be interpreted as measures of the restrictions imposed on the monetary policy of the vehicle currency in order to avoid bilateral deviations among peripheral countries. If these conditions fail, then of course all countries would have an incentive for a bilateral deviation. In this subsection, we perform another robustness check on the vehicle currency equilibrium. We explore the consequences of a switch from one vehicle currency to a second vehicle currency, where the switch is undertaken jointly by all peripheral countries in unison.

First ignore country size differences, and assume that $n_i = 1/N$ for all $i = 1, ..., N$. We wish to compare the welfare from one vehicle currency equilibrium with an alternative vehicle currency. Assume initially that currency N is the vehicle currency, and denote this equilibrium VCN. Now compare this with another equilibrium where another currency is chosen as the vehicle currency. Without loss of generality, assume this is currency 1, and denote this equilibrium as $VC1$. Since countries are identical in all respects except money growth rates, the only source of welfare difference between $VC1$ and VCN arises from differences in γ_1 and γ_N.

To compare the two equilibria, we recall the following properties of the vehicle currency equilibrium from the previous sections: when currency N is the vehicle currency i, c_{iN} $(i \neq N)$ is decreasing in γ_N, (ii) c_{ii} is independent of γ_N; and (iii) c_{ij} $(j \neq i, N)$ decreases in γ_N.

Together these properties imply that all countries $j = 2, ..., N$ will gain from the switch to $VC1$ if and only if $\gamma_1 < \gamma_{N.}$. This follows because by property (i), $c_{i1}^{VC1} > c_{iN}^{VCN}$, and by property (iii) $c_{iN}^{VC1} > c_{i1}^{VCN}$ and $c_{ij}^{VC1} > c_{ij}^{VCN}$.

Now consider country 1. Fix a good produced in country $i \neq N, 1$. We have:

$$\frac{c_{1i}^{VC1}}{c_{1i}^{VCN}} > \frac{c_{1i}^{VC1}}{\left[c_{1i}^{VCN}\right]_{\gamma_N=\gamma_1}} = \frac{\gamma_1/\beta + (\gamma_1 - 1)(N-2)\left[1-(N-1)N\phi\right]}{1-N^2\phi}.$$

The inequality follows from property (iii) above, and the assumption $\gamma_N > \gamma_1$, while the equality follows from re-arranging terms. The last expression is increasing in γ_1 and it is greater than one when $\gamma_1 = 1$. Thus, $c_{1i}^{VC1}/c_{1i}^{VCN} > 1$ for all $\gamma_1 \geq 1$.

We may also verify that $c_{1N}^{VC1} > c_{1N}^{VCN}$ for all $\gamma_1 \geq 1$. However, it is not necessarily true that $c_{11}^{VC1} > c_{11}^{VCN}$, even when $\gamma_1 \geq 1$; since more of good 1 is used up in transactions costs in $VC1$ than in VCN. However, given the logarithmic utility function, country 1's utility gain from consumption of good N will offset any losses from consumption of good 1, in utility terms, when comparing $VC1$ with VCN. That is, $(c_{1N}c_{11})^{VC1} > (c_{1N}c_{11})^{VCN}$.[16] Given this, we may conclude with the following proposition regarding the robustness of a vehicle currency with respect to joint deviations:

Proposition 5.2. *Assume $n = 1/N$, and $\gamma_1 < \gamma_N$. Every peripheral country i ($\neq N, 1$) is strictly better off in $VC1$ than in VCN. If $\gamma_1 \geq 1$, then country 1 is strictly better off in $VC1$ than in VCN. Therefore, VCN is not robust to a joint deviation to $VC1$ by the peripheral countries together. On the other hand, if $\gamma_1 > \gamma_N$, then VCN is robust to the joint deviation.*

Figure 10 illustrates the trade-off (in the more general case where $n \neq 1/N$ along the lines of Figure 8, except now representing the incentive for countries $i = 2, ..., N-1$ to deviate from the VCN and adopt currency 1 as the vehicle currency. Again, we assume here that $n_i = (1-n)/(N-1)$, and again we assume that $N = 10$. The three loci represent respectively, values of γ_N equal to 1, 1.009, (as in the baseline calibration), and 1.02. If country N follows a policy of complete price stability ($\gamma_N = 1$), then, even if country 1 represents a large fraction of the world economy e.g. $n = 0.5$, VCN is robust to a joint deviation by all peripheral countries, so long as $\gamma_1 > 1.01$.

Thus, we again find that the option of deviating, where here it is a joint deviation to a new vehicle currency, may place tight restrictions on the monetary policy of the VC country required to ensure sustainability of the VCE.

5.4. Co-existence of Vehicle Currencies

So far we have discussed only cases where one vehicle currency was used by all peripheral countries. But there is some evidence of 'currency blocs', in which certain geographic regions adopt regionally dominant currencies for intra-regional trade, but use alternative currencies for inter-regional trade. For instance, EU countries not in the euro area have begun to trade with one another in euro (Papademos, 2006), while it is well known that in

[16] To see this, note that

$$\begin{aligned}\frac{(c_{1N}c_{11})^{VC1}}{(c_{1N}c_{11})^{VCN}} &> \frac{(c_{1N}c_{11})^{VC1}}{\left[(c_{1N}c_{11})^{VCN}\right]_{\gamma_N=\gamma_1}} \\ &= \frac{\left[1+\beta(1-\frac{1}{\gamma_1})(N-2)(1-(N-1)N\phi)\right]^2}{\left[\beta(N-2)+1-\beta(N-1+\frac{1}{\gamma_1})(N-2)N\phi\right]}\frac{[\beta(N-2)+1-(\beta(N-2)+2)N\phi]}{[1-N^2\phi]}\end{aligned}$$

The last expression is an increasing function of γ_1 and its value at $\gamma_1 = 1$ is greater than one. Thus, $(c_{1N}c_{11})^{VC1} > (c_{1N}c_{11})^{VCN}$ for all $\gamma_1 \geq 1$.

Asia, the US dollar is the widely accepted trade currency (McKinnon and Schnabl, 2003). Is it possible to have multiple vehicle currencies exist within the modeling structure here? We briefly discuss this by way of a simple example in which there are hypothetically two vehicle currencies, and define the sense in which both vehicle currencies can co-exist.

Say that currencies 1 and 2 are both vehicle currencies. Assume that all trading posts $1i$ and $2i$, for $i = 3, ..., N$ are open. In addition, just to make the example easier, assume that all peripheral countries $i = 3, ..., N$ are of equal size. Since all trading posts between 1 , 2, and all peripheral countries are open, a peripheral country i may obtain currency $j \neq i$ through the $1i$ and $1j$ posts, or the $2i$ and $2j$ posts. Then it is easy to see that generically, only one vehicle currency will be used. This is because the choice of whether to use vehicle currency 1 or vehicle currency 2 depends on a comparison of the cost of obtaining currency j through currency 1, which is $\gamma_1 s^a_{1j}/s^b_{1i}$, relative to obtaining currency j through currency 2, which is $\gamma_2 s^a_{2j}/s^b_{2i}$. If $\gamma_1 s^a_{1j}/s^b_{1i} < \gamma_2 s^a_{2j}/s^b_{2i}$, then no peripheral countries will use currency 2 as a vehicle currency. If $\gamma_1 s^a_{1j}/s^b_{1i} > \gamma_2 s^a_{2j}/s^b_{2i}$, then the opposite applies. Hence, the coexistence of two vehicle countries can only be supported in the knife-edge case where $\gamma_1 s^a_{1j}/s^b_{1i} = \gamma_2 s^a_{2j}/s^b_{2i}$.

In this example, therefore, there can be only one vehicle currency, if we define a vehicle currency as one which has open trading posts with all other currencies. But it is possible to have 'local' vehicle currencies in the following sense. Take the example again where currencies 1 and 2 are vehicle currencies. Instead of all posts $1i$ and $2i$ being open, however, assume that currency 1 has active trading posts only with currencies $I_1 = \{3, ..., N/2\}$, (assuming N is even), while currency 2 has active posts only with currencies $I_2 = \{N/2 + 1, ..., N\}$. In this case agents in peripheral countries $i \in I_1$ use currency 1 to purchase currency $j \in I_1$, $j \neq 1, 2, i$, and similarly agents in I_2 will use currency 2 to obtain other peripheral currencies in I_2. But, since there are no trading posts $1i$, $i \in I_2$ or $2i$, $i \in I_1$, agents in peripheral countries must trade in *both* vehicle currencies in order to trade currencies between I_1 and I_2. For instance, in order for agent $i \in I_1$ to purchase goods of country $j \in I_2$, she must first purchase currency 1. Then, in the next period, she will trade currency 1 for currency 2 at the 12 trading post. Finally, in the period *after* that, she obtains currency j at the $2j$ trading post, and consumes good j.

Clearly this equilibrium with local vehicle currencies is robust to individual deviations, since there is only a single channel within which to affect all money trades. Hence, the two vehicle currencies can co-exist so long as they do not overlap within regional sub-groupings. For brevity however, we defer a full analysis of this case to a future paper.

6. Conclusions

This paper has developed a model in which a globally acceptable currency can function as a medium of exchange among countries, facilitating international trade, and economizing on resources when trading currencies requires costly transactions technologies. By eliminating the need to set up bilateral currency trading posts among all possible countries, a vehicle country reduces the average cost of currency trade. But the cost savings are distributed unevenly, with the center country gaining disproportionately. With a small number of countries, peripheral countries will be worse off with a vehicle currency relative to a symmetric trading equilibrium. But the gains from a vehicle currency may be substantial when there are a large number of countries and currencies, and when the centre country is large relative to peripheral countries. Even with many countries, however, these gains are eroded by higher rates of inflation in the VC country. If inflation in the center country goes to high, then our robustness analysis suggests that the use of the vehicle currency will collapse.

The model could be extended in a number of ways. We could allow for uncertainty in money growth and output levels. In this case, the risk-hedging properties of a vehicle currency would be important, in addition to its exchange use. We could also do a more explicit welfare analysis of monetary policy, assuming a social planner that weights each countries utility and can make monetary transfers across countries. We leave these issues for future research.

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Appendix

A. Derivations for Sections 3 and 4

First, we derive (3.7) and (3.8). Let the current-value Lagrangian multiplier be λ_{ii} for (3.1), λ_{ij} for (3.2), η_{ii} for (3.3), η_{ij} for (3.4) and (3.5), and ψ_{ij} for (3.6). With the logarithmic utility function, the first-order conditions for c_{ij} and m'_{ij} yield the following result for all i and j:

$$\frac{1}{p_j c_{ij}} = \frac{\beta}{\gamma_{j(+1)}}\lambda_{ij(+1)} + \psi_{ij} = \eta_{ij}, \tag{A.1}$$

where the subscript +1 indicates the next period. The first-order conditions for f_{ii}^{ij} $(i < j)$ and f_{ii}^{ji} $(i > j)$ yield:

$$\eta_{ii} = \frac{1}{s_{ij}^a}\eta_{ij}\ (i < j); \quad \eta_{ii} = s_{ji}^b \eta_{ij}\ (i > j). \tag{A.2}$$

Dividing (A.1) for $j \neq i$ by the condition for $j = i$, and using (A.2), we obtain (3.7) and (3.8).

Second, we derive (4.5) and (4.6). Let the current-value Lagrangian multiplier be η_{ii} for (4.1), η_{ij} for (4.3), η_{i1} for (4.2), and μ_{i1} for (4.4). As in the STE, the multiplier is λ_{ii} for (3.1), λ_{ij} for (3.2), and ψ_{ij} for (3.6). It is easy to verify that the first-order conditions for c_{ij} and m'_{ij} yield the same equation, (A.1), as in the STE. The first-order conditions for f_{ii}^{1i} and f_{i1}^{1j} are as follows:

$$\eta_{ii} = s_{1i}^b \eta_{i1}, \quad i \neq 1, \tag{A.3}$$

$$\eta_{i1} + \mu_{i1} = \eta_{ij}/s_{1j}^a, \quad j \neq i, 1. \tag{A.4}$$

The envelope conditions for m_{ij} are:

$$\lambda_{ij} = \eta_{ij}\ (j \neq 1); \qquad \lambda_{i1} = \eta_{i1} + \mu_{i1}. \tag{A.5}$$

Substituting the last condition into (A.4) yields $\eta_{ij} = s_{1j}^a \lambda_{i1}$ for all $j \neq i, 1$. Dividing (A.1) for $j = i$ by (A.1) for $j = 1$, and using (A.3), we obtain (4.5).

To establish (4.6), we show that $\psi_{i1} = 0$ for all $i \neq 1$. Suppose, to the contrary, that $\psi_{i1} > 0$. Then, $m'_{i1} = n_1 p_1 c_{i1}$, and so $m_{i1(+1)} = 0$ by (3.2). With (4.4), this further implies $f_{i1(+1)}^{1j} = 0$ for all $j \neq i$. That is, the household will have no foreign currency in the next period. As a result, consumption of foreign goods will be zero. This is not optimal since the marginal utility of such consumption is infinite when consumption is zero.

Since $\psi_{i1} = 0$, (A.1) implies $\lambda_{i1(+1)} = \eta_{i1}\gamma_{1(+1)}/\beta$. Then, for all $j \neq i, 1$, we have:

$$\eta_{ij(+1)} = s_{1j(+1)}^a \lambda_{i1(+1)} = \frac{\gamma_{1(+1)}}{\beta} s_{1j(+1)}^a \eta_{i1} = \frac{\gamma_{1(+1)}}{\beta}\left(\frac{s_{1j(+1)}^a}{s_{1i}^b}\right)\eta_{ii}.$$

The first equality comes from a result derived above, the second equality is obvious, and the last equality comes from (A.3). Now, dividing (A.1) for $j \neq i, 1$ in the next period by (A.1) for $j = i$ in the current period, and using the above condition, we get (4.6).

Third, we derive the results (4.7) – (4.11). For (4.7), consider a household in a country $i \neq 1$. Notice that the household spends the domestic currency in the current period to acquire currency 1 and to purchase domestic goods. Part of currency 1 that the household acquires today is spent on country 1 goods. The rest will be spent in *the next period* to purchase other peripheral currencies which, in turn, will be spent on goods of these peripheral countries. Thus, the household's holdings of domestic currency at the beginning of the period, $m_{ii} = 1/n_i$, are equal to the sum of three types of expenditures of the household: the current expenditure on domestic goods, the current expenditure on country 1 goods, and the expenditure in the next period on goods of other peripheral countries. The three terms on the right-hand side of (4.7) are the amounts of these expenditures.

Substituting 4.5) and (4.6), we obtain (4.8). The result (4.9) comes from the fact that the household spends all domestic currency on domestic goods and on acquiring the vehicle currency. The result (4.10) comes from the constraint $f_{i1}^{1j} = s_{1j}^a m'_{ij} = s_{1j}^a n_j p_j c_{ij}$ for $j \neq i, 1$. The result (4.11) comes from (4.4).

Finally, we prove Proposition 4.1 by deriving (4.19) and (4.20). For (4.19), substitute f and p_1 from the (4.9), (4.10), (4.11), and (4.16) into (4.18). We get:

$$\begin{aligned} & \left(n_i - \tfrac{\gamma_1 \phi_1}{n_1}\right)(1 - n_1 m_{11}) - \left[n_i - \tfrac{\phi_1}{n_1} + \tfrac{\beta n_i}{\gamma_1} \textstyle\sum_{j\neq 1} \tfrac{s_{1j}^b}{\delta_j}\right] \\ = & \left(\tfrac{1}{\gamma_1} - 1\right) \beta (1 - n_i - n_1) \tfrac{s_{1i}^b}{\delta_i} - \left[n_1 + \tfrac{\beta}{\gamma_1}(1 - n_1)\right] \tfrac{s_{1i}^b}{\delta_i}. \end{aligned}$$

Summing over $i \neq 1$ and using (4.13) reversely, we have:

$$\sum_{i\neq 1} \frac{s_{1i}^b}{\delta_i} = (1 - n_1 m_{11}) \left[1 - \frac{\gamma_1}{n_1}\left(1 - \frac{(N-1)\phi_1}{n_1}\right)\right] + \frac{1}{n_1}\left(1 - \frac{(N-1)\phi_1}{n_1}\right) - 1.$$

Substituting this result into the left-hand side of the previous equation yields (4.19).

For (4.20), use (4.18) to rewrite (4.17) as follows:

$$s_{1i}^a = \frac{n_i f_{ii}^{1i} s_{1i}^b + \phi p_1}{n_i f_{ii}^{1i} - \phi p_i}.$$

Substituting f_{ii}^{1i} from (4.9) and s_{1i}^b from (4.19) yields (4.20).

B. Proof of Proposition 4.2

Let $n = 1/N$. Then the formulas following (4.19) and (4.20) can be simplified as follows:

$$D = \frac{1 + \frac{\beta}{\gamma_1}(N-1)}{1 + \beta(N - 1 + \frac{1}{\gamma_1})}(1 - N^2 \phi_1)$$

$$E = \frac{(1-\gamma_1)\left(1 - \frac{\beta}{\gamma_1}\right) + \left[\beta(N-1) + \gamma_1\right](1 - N^2\phi_1)}{1 + \beta(N - 1 + \frac{1}{\gamma_1})}$$

Using these, the solutions for s^b_{1i} and s^a_{1i}/s^a_{1i} may be written

$$\frac{s^b_{1i}}{\beta+(1-\beta)2/N} = \frac{N(1-N^2\phi)}{\beta(N-1)(N-2)(1-N^2\phi)+N-\beta(\frac{1}{\gamma_1}-1)(N-2)}$$

$$\frac{s^a_{1i}}{s^b_{1i}} = \frac{\beta(N-2)+1-\beta(N-1+\frac{1}{\gamma_1})(N-2)N\phi}{(1-N^2\phi)\left[\beta(N-2)+1-(\beta(N-2)+2)N\phi\right]} \tag{B.1}$$

The solutions for m_{11} and p_1 are

$$1-n_1m_{11} = \frac{1-N^2\phi_1}{\gamma_1(1-N^2\phi_1)+\frac{1}{N-1}\left[\frac{\gamma_1 N}{\beta(N-2)}-(1-\gamma_1)\right]}$$

$$p_1 = \frac{N\left[N-\beta(\frac{1}{\gamma_1}-1)(N-2)\right]}{\beta(N-1)(N-2)(1-N^2\phi_1)+N-\beta(\frac{1}{\gamma_1}-1)(N-2)}$$

Then, using these solutions in the VCE formulas for consumption (4.5), (4.6), and (4.8), we get, for $i\neq 1$ and $j\neq i,1$, we have:

$$c_{ii} = \frac{1}{\beta+(1-\beta)2/N} \tag{B.2}$$

$$c_{i1} = N\frac{1-N^2\phi_1}{N-\beta\left(\frac{1}{\gamma_1}-1\right)(N-2)} \tag{B.3}$$

$$c_{ij} = \frac{\beta\left(1-N^2\phi_1\right)\left[\beta(N-2)+1-(\beta(N-2)+2)N\phi_j\right]}{\gamma_1\left[\beta+(1-\beta)2/N\right]\left[\beta(N-2)+1-\beta(N-1+\frac{1}{\gamma_1})(N-2)N\phi_1\right]}. \tag{B.4}$$

For country 1, consumption is:

$$c_{11} = N\frac{\beta\left(1-\frac{1}{\gamma_1}\right)(N-2)\left[1-(N-1)N\phi_1\right]+1}{N-\beta(\frac{1}{\gamma_1}-1)(N-2)} \tag{B.5}$$

$$c_{1i} = N\frac{\left[1+\beta(1-\frac{1}{\gamma_1})(N-2)(1-(N-1)N\phi_1)\right]}{\left[\beta(N-2)+1-\beta(N-1+\frac{1}{\gamma_1})(N-2)N\phi_1\right]} \times\frac{\left[\beta(N-2)+1-(\beta(N-2)+2)N\phi_i\right]}{\beta(N-2)+2} \tag{B.6}$$

Then, using (B.1), (B.2)-(B.4) and (B.5)-(B.6), parts (i)-(v) of Proposition can be verified.

Figure 1a

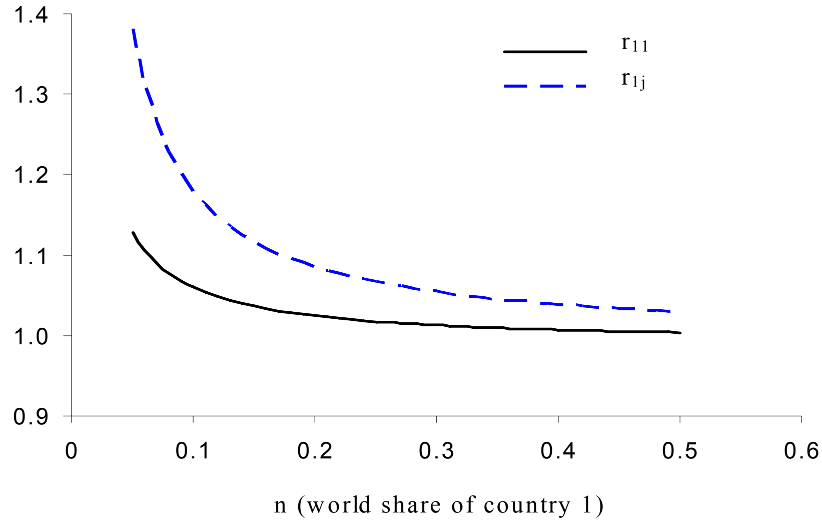
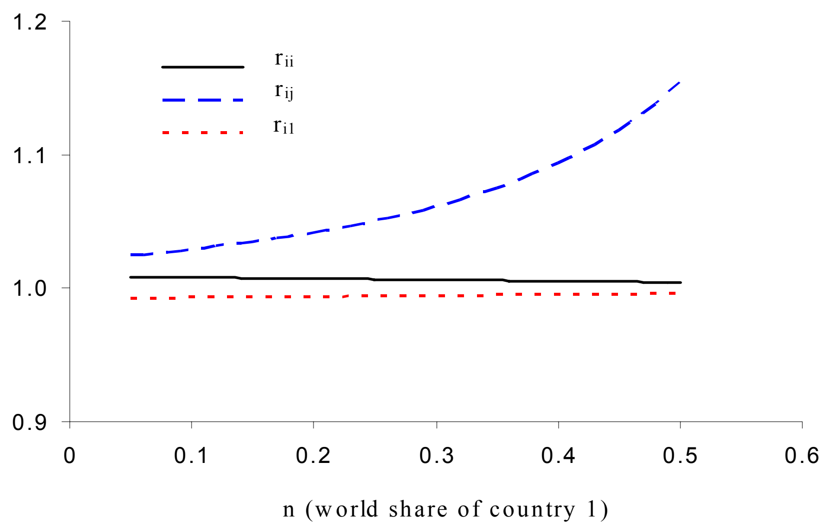
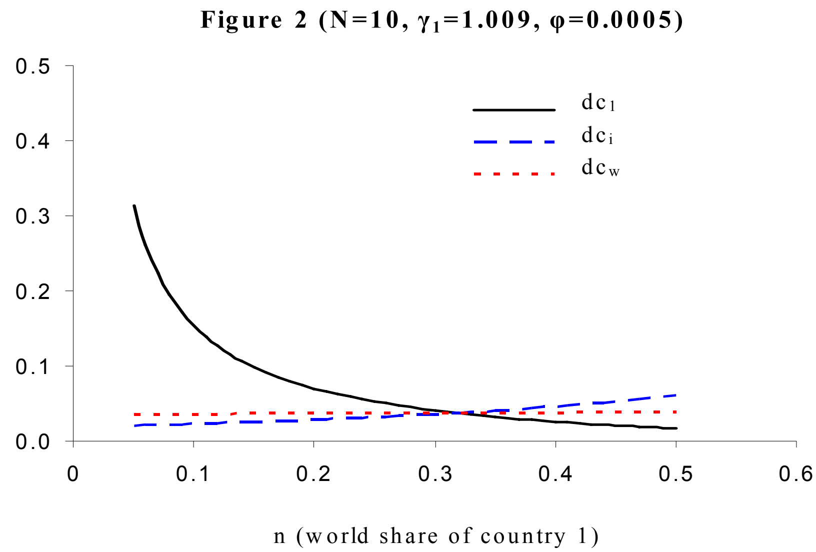


Figure 1b

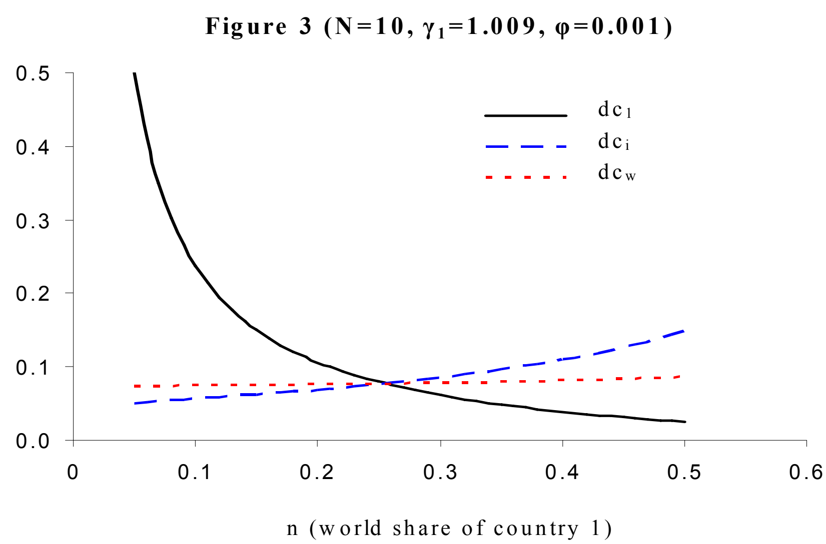


$r_{ij}= c_{ij}^{VC1}/c_{ij}^{STE}$: country i's consumption of country j's goods in VCE relative to STE

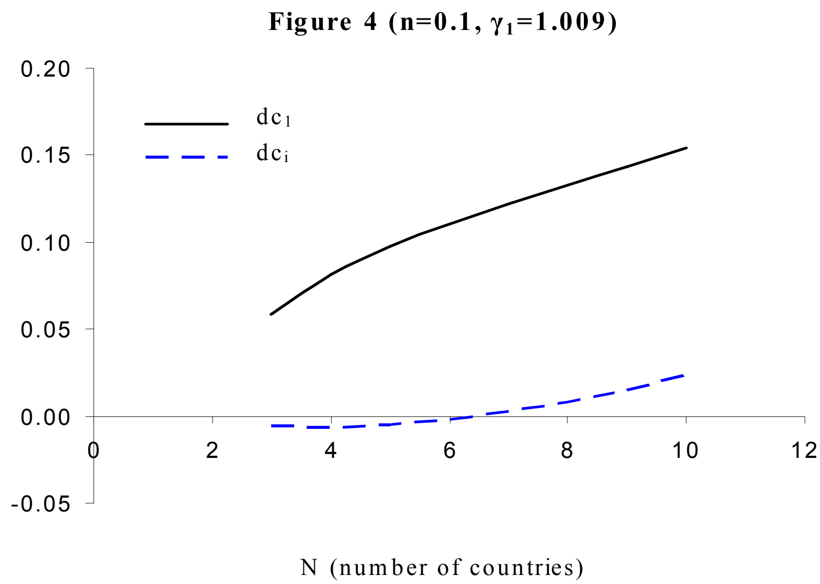
Figure 1. Relative consumption levels in VCE to STE



dc_1, dc_i, dc_w: equivalent consumption changes for country 1, i and the world (w)
Figure 2. Gains to VCE with a transaction cost $\phi = 0.0005$

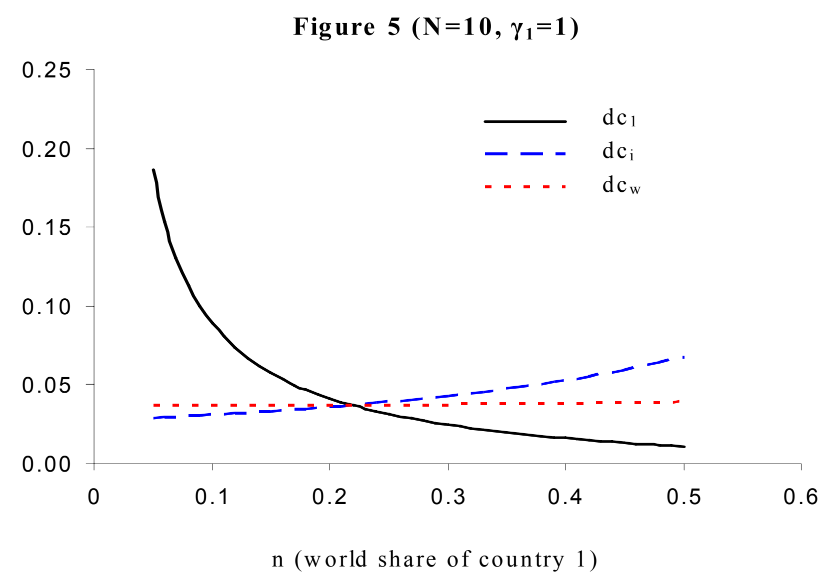


dc_1, dc_i, dc_w: equivalent consumption changes for country 1, i and the world (w)
Figure 3. Gains to VCE with a transaction cost $\phi = 0.001$



dc_1, dc_i: equivalent consumption changes for country 1 and i

Figure 4. Gains to VCE as a function of N (with $n = 1/N$ and $\gamma_1 = 1.009$)



dc_1, dc_i, dc_w: equivalent consumption changes for country 1, i and the world (w)

Figure 5. Gains to VCE with VC money growth $\gamma_1 = 1$

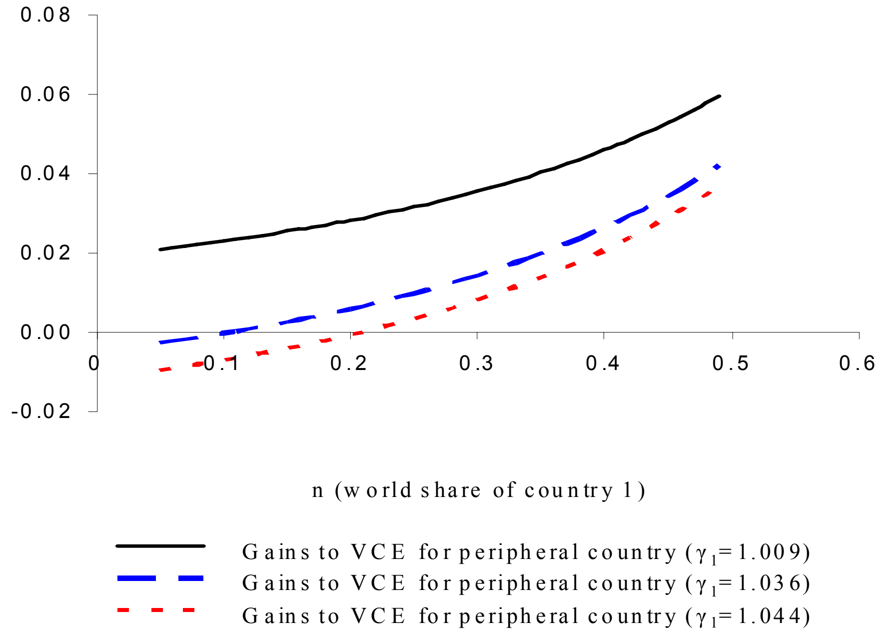


Figure 6. Gains to VCE for peripheral countries for three levels of γ_1

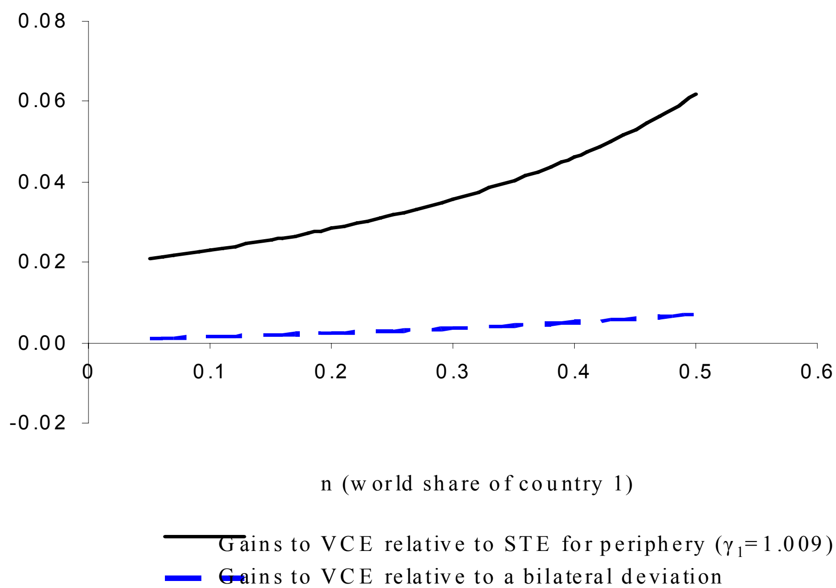


Figure 7. Gains to VCE relative to a bilateral deviations

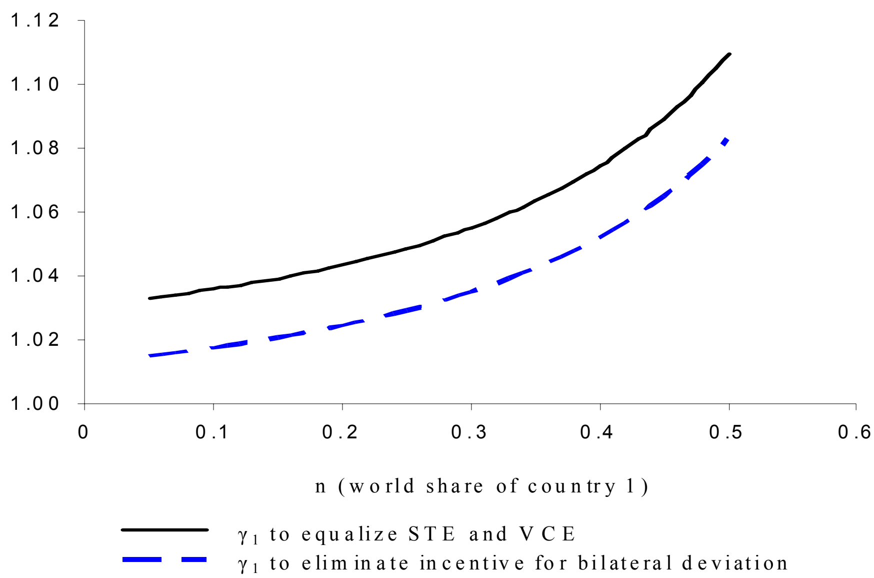


Figure 8. Threshold levels of VC money growth γ_1 below which VCE sustains deviations to STE or bilateral deviations

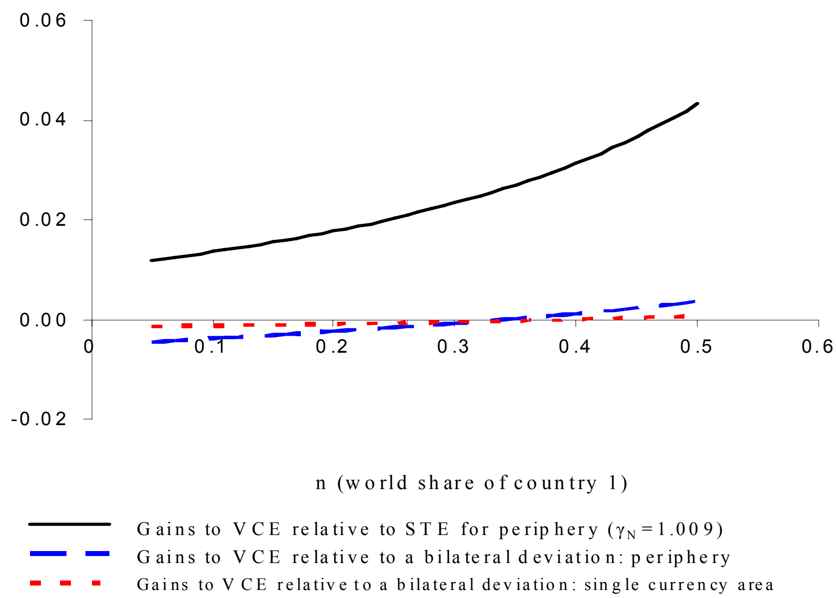


Figure 9. Gains to VCE relative to a bilateral deviation: peripheral countries and a single currency area

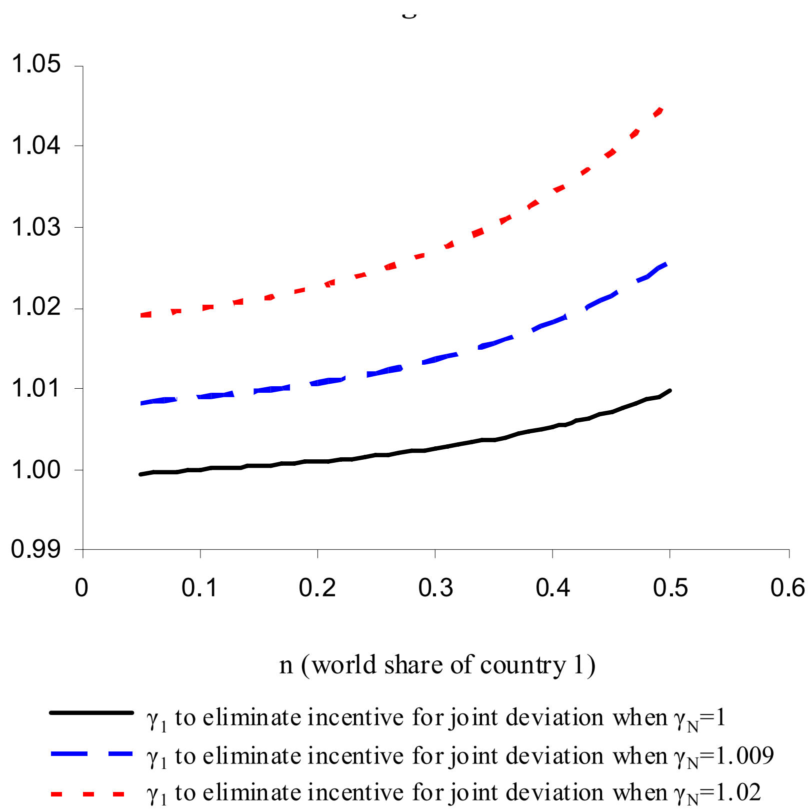


Figure 10. Threshold levels of money growth γ_1 above which peripheral countries do not switch from currency N as the VC to currency 1