Technical Note: A Monetary Model of the Exchange Rate with Informational Frictions*

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Abstract

This technical note is developed as a mathematical companion to the paper 'A Monetary Model of the Exchange Rate with Informational Frictions'. It contains three basic calculations. First, I include a succinct discussion of the optimization problem for households, firms and financial intermediaries. I also derive the equilibrium conditions of the model. Second, I compute the zero-inflation, zero-current account (deterministic) steady state. Third, I describe the approach of log-linearizing the equilibrium conditions around the steady state. Simultaneously, I explain the system of equations that constitutes the basis for the paper.

Commentary is provided whenever necessary, but it remains limited given the scope of this note. The algebra is also abbreviated whenever possible, as long as it does not obscure the main derivations.

JEL Classification: F31, F37, F41

KEY WORDS: Asymmetric Information, Rational Expectations Equilibrium, Bilateral Exchange Rates, Fiat Money, Financial Intermediation

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*This is the companion technical note for the paper 'A Monetary Model of the Exchange Rate with Informational Frictions'. The extended version of the paper is available as Dallas Fed GMPI WP #2. All remaining errors are mine alone. Please report any mistakes, typos, and inconsistencies you find. The views expressed in this note do not necessarily reflect those of the Federal Reserve Bank of Dallas or the Federal Reserve System.

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A Introduction

Here, I present the derivation of the (symmetric) equilibrium conditions and the deterministic steady state characterizing the two-country economy in my paper 'A Monetary Model of the Exchange Rate with Informational Frictions'. I also study the linearization of the resulting system of equations. Since the model is built around two (almost-)symmetric countries, my analysis is substantially tilted towards the problem of the home country unless otherwise noted.

B The Equilibrium Conditions

The equilibrium conditions of the model pin down the policy rules used by rational households to determine their consumption-savings optimal path, by firms to decide their optimal pricing policy, and by financial intermediaries to solve their optimal portfolio allocation problem. These equations are complemented with a pair of stable money demand and labor supply functions per each household, and a resource constraint that summarizes the budgetary constraints of all the economic agents in the economy.

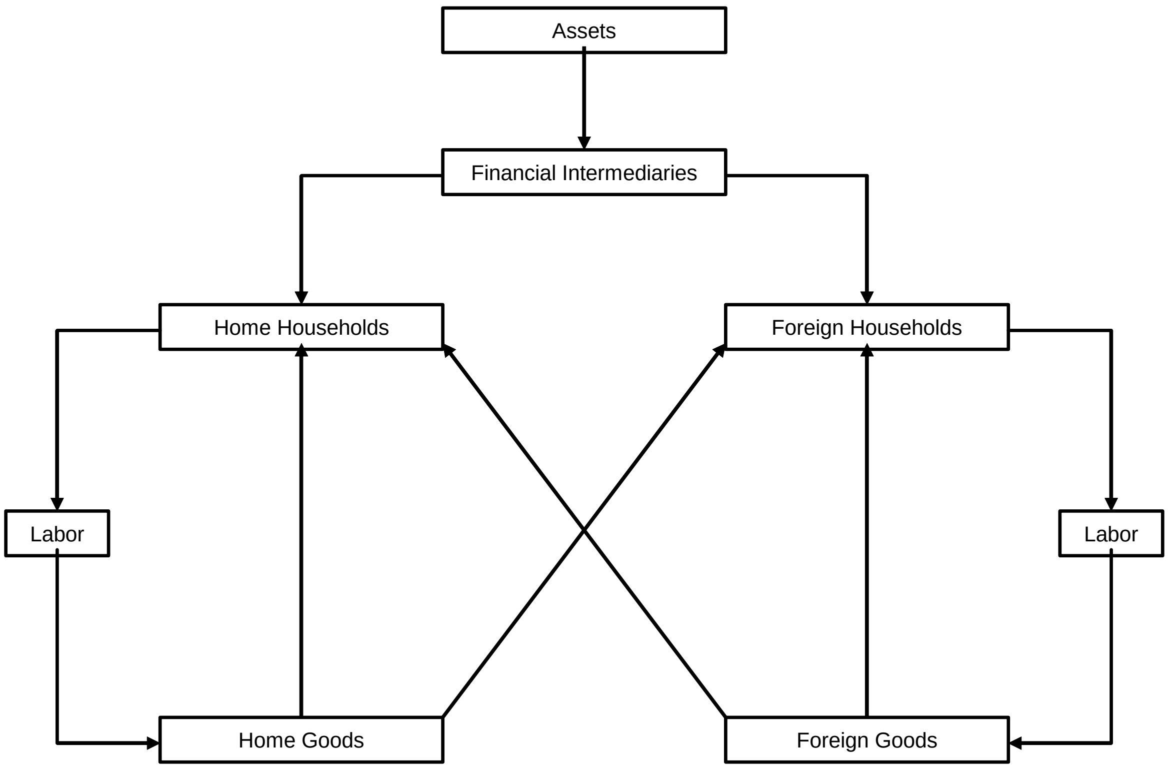

Figure 1. The Structure of the Economy

B.1 The Representative Household's Problem

The Optimization in the Home Country. The maximization problem of the domestic household j in the home country is expressed as,

$$\sum_{\tau=0}^{\infty} \beta^{\tau} \mathbb{E}_t \left\{ \exp\left(-\xi_{t+\tau}\right) \left[\frac{1}{1-\gamma} \left(C_{j,t+\tau} - bC_{t+\tau-1}\right)^{1-\gamma} + \frac{\chi}{1-\gamma} \left(\frac{M^d_{j,t+\tau}}{P_{t+\tau}}\right)^{1-\gamma} - \frac{\kappa}{1+\varphi} \left(L^s_{j,t+\tau}\right)^{1+\varphi} \right] \right\} +$$
$$+ \sum_{\tau=0}^{\infty} \mathbb{E}_t \left\{ \lambda_{j,t+\tau} \left[B_{j,t+\tau-1} + M^d_{j,t+\tau-1} + W_{t+\tau} L^s_{j,t+\tau} + \Pi_{j,t+\tau} + TR_{j,t+\tau} - P_{t+\tau} C_{j,t+\tau} - \frac{B_{j,t+\tau}}{\exp(i_{t+\tau})} - M^d_{j,t+\tau} \right] \right\},$$

where $\lambda_{j,t}$ refers to the Lagrange multiplier, and the expectations operator $\mathbb{E}_t\{\cdot\} \equiv \mathbb{E}\{\cdot \mid \mathcal{H}_t\}$ is conditional on a given information set $\mathcal{H}_t$. Individual consumption, $C_{j,t}$, money demand, $M^d_{j,t}$, demand for domestic bonds, $B_{j,t}$, and labor supply, $L^s_{j,t}$, are 'effectively' chosen at time t by the household itself. The habit component is observable at time t because all information up to time $t-1$ is public, but is treated as an externality. The current domestic preference shock, ξ_t, is also observable.

Households select their optimal strategy at time t prior to the opening of the markets. Even though consumption prices[1], P_t, wages, W_t, and interest rates, i_t, are not observable, households understand that the information in $\mathcal{H}_t$ allows them to generate 'imperfect' predictions of them. Naturally, this could affect their choices. In fact, if second-order effects are negligible (as it will be assumed whenever log-linearizing the equilibrium conditions) and information is common and perfect (that is, the realization of all shocks is observable by all agents up to time t prior to making a decision), then they may be able to anticipate correctly the prices that will prevail in each market before it opens up (perfect foresight).

For tractability, I assume that the households pre-commit to a pair of stable money demand and labor supply rules, i.e.

$$\chi \left(\frac{M^d_{j,t}}{P_t \left(C_{j,t} - bC_{t-1} \right)} \right)^{-\gamma} = \frac{\exp(i_t) - 1}{\exp(i_t)}, \tag{1}$$

$$\frac{W_t}{P_t} = \kappa \left(C_{j,t} - bC_{t-1} \right)^{\gamma} \left(L^s_{j,t} \right)^{\varphi}. \tag{2}$$

These rules correspond to the first-order conditions of the domestic households' problem for $M^d_{j,t}$ and $L^s_{j,t}$ in the standard model. This means that households pre-commit to operate in the money and labor markets as price-takers. Relaxing this characterization would be conceptually straightforward, but adds more complexity to the algebraic derivation of the model without significatively altering its qualitative results. In this sense, I can re-interpret the households' problem as an optimization program with random bond prices, and focus on the effect that information has on the more relevant 'consumption-savings' margin.

Taking (1) and (2) as given, the first-order necessary conditions for the home country are reduced to,

$$\begin{aligned} C_{j,t} &: \quad \exp(-\xi_t) \left(C_{j,t} - bC_{t-1} \right)^{-\gamma} - \lambda_{j,t} P_t \leq 0, \quad = 0 \text{ if } C_{j,t} > 0, \\ B_{j,t} &: \quad -\lambda_{j,t} \mathbb{E}_t \frac{1}{\exp(i_t)} + \beta \mathbb{E}_t \lambda_{j,t+1} \leq 0, \quad = 0 \text{ if } B_{j,t} > 0. \end{aligned}$$

After some simple algebra, I derive the following equilibrium condition for an interior solution,

$$B_{j,t} : \mathbb{E}_t \frac{1}{\exp(i_t)} = \beta \mathbb{E}_t \left[\exp\left(-\Delta \xi_{t+1} \right) \left(\frac{C_{j,t+1} - bC_t}{C_{j,t} - bC_{t-1}} \right)^{-\gamma} \frac{P_t}{P_{t+1}} \right], \tag{3}$$

which restates the standard first-order condition replacing the price of the domestic bond with its expected value. The budget constraint is satisfied with equality *ex post*.

The Optimization in the Foreign Country. The maximization problem of the foreign household j can be characterized in a similar fashion. Analogously, I start with a pre-commitment to,

$$M^{d*}_t \quad : \quad \chi \left(\frac{M^{d*}_{j,t}}{P^*_t \left(C^*_{j,t} - bC^*_{t-1} \right)} \right)^{-\gamma} = 1 - \mathbb{E}_t \frac{1}{\exp(i^*_t)}, \tag{4}$$

$$L^{s*}_t \quad : \quad \frac{W^*_t}{P^*_t} = \kappa \left(C^*_{j,t} - bC^*_{t-1} \right)^{\gamma} \left(L^{s*}_{j,t} \right)^{\varphi}, \tag{5}$$

[1] In fact, the household takes as given the prices $P_t(h)$ for all varieties $h \in [0, n]$ and $P_t(f)$ for all varieties $f \in (n, 1]$.

and I obtain the following first-order condition for an interior solution,

$$B_t^* : \mathbb{E}_t \frac{1}{\exp\left(i_t^*\right)} = \beta \mathbb{E}_t \left[\exp\left(-\Delta \xi_{t+1}^*\right) \left(\frac{C_{j,t+1}^* - bC_t^*}{C_{j,t}^* - bC_{t-1}^*} \right)^{-\gamma} \frac{P_t^*}{P_{t+1}^*} \right]. \tag{6}$$

The budget constraint is satisfied with equality *ex post*.

The Demand Curves. The home and foreign consumption bundles of the household, $C_{j,t}^H$, $C_{j,t}^F$, $C_{j,t}^{H*}$ and $C_{j,t}^{F*}$, are aggregated by means of a CES index as,

$$\begin{aligned} C_{j,t}^H &= \left[\left(\frac{1}{n}\right)^{\frac{1}{\theta}} \int_0^n C_{j,t}(h)^{\frac{\theta-1}{\theta}} dh \right]^{\frac{\theta}{\theta-1}}, \; C_{j,t}^F = \left[\left(\frac{1}{1-n}\right)^{\frac{1}{\theta}} \int_n^1 C_t(f)^{\frac{\theta-1}{\theta}} df \right]^{\frac{\theta}{\theta-1}}, \\ C_{j,t}^{H*} &= \left[\left(\frac{1}{n}\right)^{\frac{1}{\theta}} \int_0^n C_{j,t}^*(h)^{\frac{\theta-1}{\theta}} dh \right]^{\frac{\theta}{\theta-1}}, \; C_{j,t}^{F*} = \left[\left(\frac{1}{1-n}\right)^{\frac{1}{\theta}} \int_n^1 C_t^*(f)^{\frac{\theta-1}{\theta}} df \right]^{\frac{\theta}{\theta-1}}, \end{aligned}$$

while aggregate consumption, $C_{j,t}$ and $C_{j,t}^*$, is defined with another CES index as,

$$\begin{aligned} C_{j,t} &= \left[n^{\frac{1}{\sigma}} \left(C_{j,t}^H\right)^{\frac{\sigma-1}{\sigma}} + (1-n)^{\frac{1}{\sigma}} \left(C_{j,t}^F\right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}, \\ C_{j,t}^* &= \left[n^{\frac{1}{\sigma}} \left(C_{j,t}^{H*}\right)^{\frac{\sigma-1}{\sigma}} + (1-n)^{\frac{1}{\sigma}} \left(C_{j,t}^{F*}\right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}. \end{aligned}$$

The domestic and foreign households are also pre-committed to a standard allocation across varieties and bundles. Given the structure of preferences, the solution to the sub-utility maximization problem implies that the households' demands for each variety are given by,

$$C_{j,t}(h) = \frac{1}{n} \left(\frac{P_t(h)}{P_t^H} \right)^{-\theta} C_{j,t}^H, \; C_{j,t}^*(h) = \frac{1}{n} \left(\frac{P_t^*(h)}{P_t^{H*}} \right)^{-\theta} C_{j,t}^{H*}, \text{ if } h \in [0,n], \tag{7}$$

$$C_{j,t}(f) = \frac{1}{1-n} \left(\frac{P_t(f)}{P_t^F} \right)^{-\theta} C_{j,t}^F, \; C_{j,t}^*(f) = \frac{1}{1-n} \left(\frac{P_t^*(f)}{P_t^{F*}} \right)^{-\theta} C_{j,t}^{F*}, \text{ if } f \in (n,1], \tag{8}$$

while the demands for the bundles of home and foreign goods are simply equal to,

$$C_{j,t}^H = n \left(\frac{P_t^H}{P_t} \right)^{-\sigma} C_{j,t}, \; C_{j,t}^{H*} = n \left(\frac{P_t^{H*}}{P_t^*} \right)^{-\sigma} C_{j,t}^*, \tag{9}$$

$$C_{j,t}^F = (1-n) \left(\frac{P_t^F}{P_t} \right)^{-\sigma} C_{j,t}, \; C_{j,t}^{F*} = (1-n) \left(\frac{P_t^{F*}}{P_t^*} \right)^{-\sigma} C_{j,t}^*. \tag{10}$$

Because the problem of all domestic households is identical (symmetric), I drop the subscript j from now on. Similarly for the foreign households.

The Price Indexes. Under standard results on functional separability, I infer that the price indexes corresponding to the specified structure of preference are,

$$P_t = \left[n\left(P_t^H\right)^{1-\sigma} + (1-n)\left(P_t^F\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}, \tag{11}$$

$$P_t^H = \left[\frac{1}{n}\int_0^n P_t(h)^{1-\theta} dh\right]^{\frac{1}{1-\theta}}, \tag{12}$$

$$P_t^F = \left[\frac{1}{1-n}\int_n^1 P_t(f)^{1-\theta} df\right]^{\frac{1}{1-\theta}}. \tag{13}$$

Home and foreign households have identical tastes and, therefore, their respective price indexes are symmetric, i.e.

$$P_t^* = \left[n\left(P_t^{H*}\right)^{1-\sigma} + (1-n)\left(P_t^{F*}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}, \tag{14}$$

$$P_t^{H*} = \left[\frac{1}{n}\int_0^n P_t^*(h)^{1-\theta} dh\right]^{\frac{1}{1-\theta}}, \tag{15}$$

$$P_t^{F*} = \left[\frac{1}{1-n}\int_n^1 P_t^*(f)^{1-\theta} df\right]^{\frac{1}{1-\theta}}. \tag{16}$$

B.2 The Representative Firm's Problem

The problem of the firm becomes dynamic in the presence of nominal price rigidities à la Calvo (1983). Following G. Benigno (2004), I allow for differences in the 'degree of price stickiness' across producer locations and between domestic and foreign markets (i.e. $\alpha^H \neq \alpha^{H*} \neq \alpha^F \neq \alpha^{F*}$).

The Optimization in the Home Country. Whenever technologies are linear in labor, the optimization problem in the domestic and foreign markets can be easily separated for any given firm. This fact turns out to be very convenient to compute the optimal pricing rules. Let me define the intertemporal marginal rate of substitution (IMRS) as,

$$\beta^\tau \Xi_{t,t+\tau} \equiv \beta^\tau \exp\left(-\left(\xi_{t+\tau} - \xi_t\right)\right)\left(\frac{C_{t+\tau} - bC_{t+\tau-1}}{C_t - bC_{t-1}}\right)^{-\gamma}\frac{P_t}{P_{t+\tau}},$$

for any $\tau \geq 0$. Then, the maximization problem of firm h in the domestic market discounted on the basis of the domestic household IMRS is expressed as,

$$\sum\nolimits_{\tau=0}^{\infty}\left(\beta\alpha^H\right)^\tau \mathbb{E}_t\left\{\Xi_{t,t+\tau}\widetilde{Y}_{t,t+\tau}^d(h)\left[\widetilde{P}_t(h) - \frac{W_{t+\tau}}{\exp\left(a_{t+\tau}\right)}\right]\right\},$$

while in the foreign market is given by,

$$\sum\nolimits_{\tau=0}^{\infty}\left(\beta\alpha^{H*}\right)^\tau \mathbb{E}_t\left\{\Xi_{t,t+\tau}\widetilde{Y}_{t,t+\tau}^{d*}(h)\left[S_{t+\tau}\widetilde{P}_t^*(h) - \frac{W_{t+\tau}}{\exp\left(a_{t+\tau}\right)}\right]\right\},$$

where $\widetilde{Y}_{t,t+\tau}^d(h)$ and $\widetilde{Y}_{t,t+\tau}^{d*}(h)$ refer to the demand functions for a fixed time t price, and the expectations operator $\mathbb{E}_t\{\cdot\} \equiv \mathbb{E}\{\cdot \mid \mathcal{F}_t\}$ is conditional on a given information set $\mathcal{F}_t$. The prices of the good in each market, $\widetilde{P}_t(h)$ and $\widetilde{P}_t^*(h)$, are 'effectively' chosen at time t by the firm itself. The current domestic productivity shock, a_t, is observable, and all information up to time $t-1$ is public.

Firms select their optimal pricing strategy at time t prior to the opening of the markets. Even though consumption prices[2], P_t, wages, W_t, and interest rates, i_t, are not observable, firms understand that the information in $\mathcal{F}_t$ allows them to generate 'imperfect' predictions of them. Naturally, in this context, their forecast of wages has implications for their optimal choices and the dynamics of prices. For tractability, I assume that firms pre-commit to satisfy the consumer's demand derived in $(7)-(10)$ at the fixed prices until they can re-optimize again, i.e.

$$\begin{aligned}\widetilde{Y}^{d}_{t,t+\tau}(h) &= \left(\frac{\widetilde{P}_t(h)}{P^H_{t+\tau}}\right)^{-\theta}\left(\frac{P^H_{t+\tau}}{P_{t+\tau}}\right)^{-\sigma} C_{t+\tau},\\ \widetilde{Y}^{d*}_{t,t+\tau}(h) &= \left(\frac{\widetilde{P}^*_t(h)}{P^{H*}_{t+\tau}}\right)^{-\theta}\left(\frac{P^{H*}_{t+\tau}}{P^*_{t+\tau}}\right)^{-\sigma} C^*_{t+\tau},\end{aligned}$$

for any $\tau \geq 0$.

The first-order necessary conditions for the problem of a monopolistic competitor can be expressed as follows,

$$\begin{aligned}\widetilde{P}_t(h) \quad &: \quad \sum_{\tau=0}^{\infty}\left(\beta\alpha^H\right)^{\tau}\left[\mathbb{E}_t\left\{\Xi_{t,t+\tau}\widetilde{Y}^{d}_{t,t+\tau}(h)\right\} - \theta\mathbb{E}_t\left\{\Xi_{t,t+\tau}\widetilde{Y}^{d}_{t,t+\tau}(h)\left(\frac{\widetilde{P}_t(h)-\frac{W_{t+\tau}}{\exp(a_{t+\tau})}}{\widetilde{P}_t(h)}\right)\right\}\right] \leq 0,\\ &\qquad = 0 \text{ if } \widetilde{P}_t(h) > 0,\\ \widetilde{P}^*_t(h) \quad &: \quad \sum_{\tau=0}^{\infty}\left(\beta\alpha^{H*}\right)^{\tau}\left[\mathbb{E}_t\left\{\Xi_{t,t+\tau}\widetilde{Y}^{d*}_{t,t+\tau}(h)\,S_{t+\tau}\right\} - \theta\mathbb{E}_t\left\{\Xi_{t,t+\tau}\widetilde{Y}^{d*}_{t,t+\tau}(h)\left(\frac{S_{t+\tau}\widetilde{P}^*_t(h)-\frac{W_{t+\tau}}{\exp(a_{t+\tau})}}{\widetilde{P}^*_t(h)}\right)\right\}\right] \leq 0,\\ &\qquad = 0 \text{ if } \widetilde{P}^*_t(h) > 0.\end{aligned}$$

After some algebra, this would give me the following equilibrium conditions for an interior solution,

$$\widetilde{P}_t(h) \quad : \quad \widetilde{P}_t(h) = \frac{\theta}{\theta-1}\frac{\sum_{\tau=0}^{\infty}\left(\beta\alpha^H\right)^{\tau}\mathbb{E}_t\left\{\Xi_{t,t+\tau}\widetilde{Y}^{d}_{t,t+\tau}(h)\left(\frac{W_{t+\tau}}{\exp(a_{t+\tau})}\right)\right\}}{\sum_{\tau=0}^{\infty}\left(\beta\alpha^H\right)^{\tau}\mathbb{E}_t\left\{\Xi_{t,t+\tau}\widetilde{Y}^{d}_{t,t+\tau}(h)\right\}}, \tag{17}$$

$$\widetilde{P}^*_t(h) \quad : \quad \widetilde{P}^*_t(h) = \frac{\theta}{\theta-1}\frac{\sum_{\tau=0}^{\infty}\left(\beta\alpha^{H*}\right)^{\tau}\mathbb{E}_t\left\{\Xi_{t,t+\tau}\widetilde{Y}^{d*}_{t,t+\tau}(h)\left(\frac{W_{t+\tau}}{\exp(a_{t+\tau})}\right)\right\}}{\sum_{\tau=0}^{\infty}\left(\beta\alpha^{H*}\right)^{\tau}\mathbb{E}_t\left\{\Xi_{t,t+\tau}\widetilde{Y}^{d*}_{t,t+\tau}(h)\,S_{t+\tau}\right\}}, \tag{18}$$

which correspond to the standard first-order conditions for the problem of firm h. The consumer's demand is fully satisfied *ex post* in every period at the chosen prices.

The Optimization in the Foreign Country. The maximization problem of the foreign firm f in the domestic and foreign markets can be characterized in a similar fashion. Analogously, I start with a pre-commitment to,

$$\begin{aligned}\widetilde{Y}^{d}_{t,t+\tau}(f) &= \left(\frac{\widetilde{P}_t(f)}{P^F_{t+\tau}}\right)^{-\theta}\left(\frac{P^F_{t+\tau}}{P_{t+\tau}}\right)^{-\sigma} C_{t+\tau},\\ \widetilde{Y}^{d*}_{t,t+\tau}(f) &= \left(\frac{\widetilde{P}^*_t(f)}{P^{F*}_{t+\tau}}\right)^{-\theta}\left(\frac{P^{F*}_{t+\tau}}{P^*_{t+\tau}}\right)^{-\sigma} C^*_{t+\tau},\end{aligned}$$

[2] In reality, each firm 'knows' what the average firm not allowed to re-set prices does, because all information up to $t-1$ is public. If the firm was allowed to re-set prices, it would also 'know' the optimal price of its variety. In a symmetric equilibrium, this implies that all producers in each country have enough information to infer the price of the bundle of all their varieties in the domestic and foreign market.

and I obtain the following first-order conditions for an interior solution,

$$\widetilde{P}_t(f) \quad : \quad \widetilde{P}_t(f) = \frac{\theta}{\theta-1} \frac{\sum_{\tau=0}^{\infty} \left(\beta\alpha^F\right)^{\tau} \mathbb{E}_t \left\{ \Xi^*_{t,t+\tau} \widetilde{Y}^d_{t,t+\tau}(f) \left(\frac{W^*_{t+\tau}}{\exp\left(a^*_{t+\tau}\right)} \right) \right\}}{\sum_{\tau=0}^{\infty} \left(\beta\alpha^F\right)^{\tau} \mathbb{E}_t \left\{ \Xi^*_{t,t+\tau} \widetilde{Y}^d_{t,t+\tau}(f) \frac{1}{S_{t+\tau}} \right\}}, \tag{19}$$

$$\widetilde{P}^*_t(f) \quad : \quad \widetilde{P}^*_t(f) = \frac{\theta}{\theta-1} \frac{\sum_{\tau=0}^{\infty} \left(\beta\alpha^{F*}\right)^{\tau} \mathbb{E}_t \left\{ \Xi^*_{t,t+\tau} \widetilde{Y}^{d*}_{t,t+\tau}(f) \left(\frac{W^*_{t+\tau}}{\exp\left(a^*_{t+\tau}\right)} \right) \right\}}{\sum_{\tau=0}^{\infty} \left(\beta\alpha^{F*}\right)^{\tau} \mathbb{E}_t \left\{ \Xi^*_{t,t+\tau} \widetilde{Y}^{d*}_{t,t+\tau}(f) \right\}}, \tag{20}$$

where the expectations operator $\mathbb{E}_t\{\cdot\} \equiv \mathbb{E}\{\cdot \mid \mathcal{F}^*_t\}$ is conditional on a given information set $\mathcal{F}^*_t$, and the foreign IMRS is,

$$\beta^{\tau}\Xi^*_{t,t+j} \equiv \beta^{\tau} \exp\left(-\left(\xi^*_{t+j} - \xi^*_t\right)\right) \left(\frac{C^*_{t+j} - bC^*_{t+j-1}}{C^*_t - bC^*_{t-1}} \right)^{-\gamma} \frac{P^*_t}{P^*_{t+j}}.$$

The demand in each market is fully satisfied *ex post* every period at the chosen prices.

B.3 The Representative Financial Intermediary's Problem

The maximization problem of the financial intermediary z is expressed as,

$$\mathbb{E}_t \left\{ X_{z,t+1} - \exp(i_t) X_{z,t} - \frac{\lambda}{2} \left(S_t B^F_{z,t}\right)^2 \right\},$$

where $\lambda_{z,t}$ refers to the Lagrange multiplier, and the expectations operator $\mathbb{E}_t\{\cdot\} \equiv \mathbb{E}\{\cdot \mid \mathcal{I}_t\}$ is conditional on a given information set $\mathcal{I}_t$. Notice that the net position in foreign assets is 'effectively' chosen at time t by the intermediary itself. Intermediaries observe the money supply shock in both countries, m_t and m^*_t, and all information up to time $t-1$ is public.

Intermediaries select their optimal strategy at time t prior to the opening of the markets. Even though consumption prices, P_t and P^*_t, wages, W_t and W^*_t, interest rates, i_t and i^*_t, and nominal exchange rates, S_t, are not observable, intermediaries understand that the information in $\mathcal{I}_t$ allows them to generate 'imperfect' predictions of them. Naturally, their forecast of the nominal interest rate spread and the depreciation of the nominal exchange rate will affect their portfolio allocation as well as the 'instrumented' levels of international borrowing and lending (between the home and foreign country).

For tractability, I assume that intermediaries pre-commit to satisfy the intertemporal budget constraint with equality, i.e.

$$X_{z,t+1} = \exp(i_t) X_{z,t} + S_t B^F_{z,t} \left[\frac{S_{t+1}}{S_t} - \exp\left(i_t - i^*_t\right) \right],$$

where $X_{z,t}$ denotes the wealth of intermediary z at time t, and $B^F_{z,t}$ is the net foreign asset position of intermediary z at time t. The first-order necessary condition of the intermediary can be expressed as follows,

$$B^F_{z,t} : \mathbb{E}_t \left\{ -\lambda S_t B^F_{z,t} + \frac{S_{t+1}}{S_t} - \exp\left(i_t - i^*_t\right) \right\} \leq 0, \quad = 0 \text{ if } B^F_{z,t} > 0.$$

After some re-arranging, this would give me the following equilibrium condition for an interior solution,

$$B^F_{z,t} : \mathbb{E}_t \left\{ \frac{S_{t+1}}{S_t} - \exp\left(i_t - i^*_t\right) - \lambda S_t B^F_{z,t} \right\} = 0, \tag{21}$$

which is similar to the uncovered interest parity equation with the addition of a risk premium term linked to the net foreign asset position and the replacement of the interest rate differential by its expected value. The intertemporal budget constraint is satisfied with equality *ex post*. Because the problem of all financial intermediaries is identical, I drop the subscript z from now on.

B.4 The Resource Constraint

In this model, the control variables for households are money demand, labor supply and bond-holdings (savings). Given a choice of controls, consumption is determined from the budget constraint of the household. Consumption, therefore, adjusts *ex post* to ensure that demand and supply equate in all markets. The control variable for the financial intermediaries is their net foreign asset position, while wealth adjusts *ex post* to guarantee that the intertemporal budget constraint is always satisfied. The control variables for firms are the prices they charge for their own variety in the domestic and foreign markets for as long as the no-reoptimization spell lasts, while *ex post* output supply adjusts to guarantee that *ex post* the consumer's demand in every market is satisfied at the chosen prices.

This discussion is needed here because in a model with incomplete and asymmetric information I need to distinguish between *ex ante* decisions and *ex post* realizations. Agents either choose their controls optimally or pre-commit to a rule before the markets open. Their decisions depend on how they forecast the price system for the economy, and the error involved in that. Then, markets open and prices clear them. Hence, consumption for households, wealth for financial intermediaries and output for firms should adjust *ex post* to ensure that individual and aggregate constraints are always enforced. In other words, *ex post* the budget constraints have to be satisfied in each state of the world that the economy reached.

Aggregating the *ex post* budget constraint (under equality) for every household in the home country, I get that

$$\frac{B_t}{\exp(i_t)} + M_t^d = B_{t-1} + M_{t-1}^d + W_t L_t^s + \Pi_t + TR_t - P_t C_t.$$

Using the domestic money market clearing condition, i.e. $M_t^d = \exp(m_t)$, and the domestic government budget constraint, i.e. $TR_t = \exp(m_t) - \exp(m_{t-1})$, it follows that

$$\frac{B_t}{\exp(i_t)} = B_{t-1} + W_t L_t^s + \Pi_t - P_t C_t.$$

The profits of the firm distributed to shareholders are,

$$\Pi_t = \frac{1}{n}\int_0^n \left[P_t(h)\, nC_t(h) + S_t P_t^*(h)(1-n) C_t^*(h)\right] dh - W_t \frac{1}{n}\int L_t^d(h)\, dh.$$

The labor market and goods market clearing conditions require that $L_t^s = \frac{1}{n}\int L_t^d(h)\, dh$ and $Y_t^s(h) = nC_t(h) + (1-n) C_t^*(h)$, respectively. In a symmetric equilibrium $L_t^s = L_t^d(h)$. Then, if I combine this with the demand curves described in (7) and (9), I can write that

$$\begin{aligned}
\frac{B_t}{\exp(i_t)} &= B_{t-1} + \frac{1}{n}\int_0^n \left[P_t(h)\, nC_t(h) + S_t P_t^*(h)(1-n) C_t^*(h)\right] dh - P_t C_t \\
&= B_{t-1} + \left[\frac{1}{n}\int_0^n \left(\frac{P_t(h)}{P_t^H}\right)^{1-\theta} dh\right] P_t^H \left(\frac{P_t^H}{P_t}\right)^{-\sigma} nC_t + \\
&\quad + \left[\frac{1}{n}\int_0^n \left(\frac{P_t^*(h)}{P_t^{H*}}\right)^{1-\theta} dh\right] S_t P_t^{H*} \left(\frac{P_t^{H*}}{P_t^*}\right)^{-\sigma} (1-n) C_t^* - P_t C_t,
\end{aligned}$$

where B_t is the per capita domestic household's bond-holdings.

Using the price sub-indexes in (12) and (15), it is easy to prove that

$$\frac{1}{n}\int_0^n \left(\frac{P_t(h)}{P_t^H}\right)^{1-\theta} dh = \frac{1}{n}\int_0^n \left(\frac{P_t^*(h)}{P_t^{H*}}\right)^{1-\theta} dh = 1,$$

so I can express the resource constraint as follows,

$$\begin{aligned}\frac{B_t}{\exp(i_t)} &= B_{t-1} + P_t^H \left(\frac{P_t^H}{P_t}\right)^{-\sigma} nC_t + S_t P_t^{H*} \left(\frac{P_t^{H*}}{P_t^*}\right)^{-\sigma} (1-n) C_t^* - P_t C_t \\ &= B_{t-1} + P_t \left[\left(\frac{P_t^H}{P_t}\right)^{1-\sigma} nC_t + RS_t \left(\frac{P_t^{H*}}{P_t^*}\right)^{1-\sigma} (1-n) C_t^*\right] - P_t C_t,\end{aligned}$$

where the real exchange rate is defined as $RS_t \equiv \frac{S_t P_t^*}{P_t}$. This holds true in per capita terms independently of whether the prices are *flexible* or *sticky*.

Let me denote B_t^H the net domestic asset position of the financial intermediaries, i.e. $B_t^H = X_{t+1} - S_{t+1} B_t^F$. I assume that bonds are in zero-net supply, therefore $B_t = -B_t^H$. Moreover, it must hold that the net value of investment flows of the financial intermediaries into the foreign bond market (expressed in units of the domestic currency) must be the reciprocal of the net value of investment flows coming into the domestic bond market, i.e.

$$\left[\frac{1}{\exp(i_t)} B_t^H - B_{t-1}^H\right] = -S_t \left[\frac{1}{\exp(i_t^*)} B_t^F - B_{t-1}^F\right].$$

Hence, I obtain the following resource constraint,

$$\frac{1}{\exp(i_t^*)} B_t^{RF} = \frac{S_t}{S_{t-1}} \frac{P_{t-1}}{P_t} B_{t-1}^{RF} + \left[\left(\frac{P_t^H}{P_t}\right)^{1-\sigma} nC_t + RS_t \left(\frac{P_t^{H*}}{P_t^*}\right)^{1-\sigma} (1-n) C_t^* - C_t\right], \tag{22}$$

where $B_t^{RF} \equiv \frac{S_t B_t^F}{P_t}$ is the real value of the net foreign asset position of a representative intermediary. Equation (22) is the so-called *current account* equation.

C The Deterministic Steady State

By construction, the unconditional mean of the shocks driving the model is zero[3], i.e.

$$\mathbb{E}\theta_t = (\mathbb{E}m_t, \mathbb{E}m_t^*, \mathbb{E}\xi_t, \mathbb{E}\xi_t^*, \mathbb{E}a_t, \mathbb{E}a_t^*)^T = \mathbf{0}^T.$$

I require that all shocks be evaluated at their unconditional mean in steady state. I also conjecture the existence of a (symmetric) deterministic steady state in which prices, consumption and the nominal exchange rate are constant, i.e.

$$\begin{aligned}P_{t+1} &= P_t = \overline{P},\ P_{t+1}^* = P_t^* = \overline{P}^*, \\ C_{t+1} &= C_t = \overline{C},\ C_{t+1}^* = C_t^* = \overline{C}^*, \\ S_{t+1} &= S_t = \overline{S}.\end{aligned}$$

Then, I characterize the zero-inflation steady state as follows:

Given my conjecture and the Euler equations in (3) and (6), it follows that the steady state interest rate in both countries is identical and equal to the inverse of the rate of time-preference,

$$\beta = \exp\left(-\overline{i}\right) = \exp\left(-\overline{i}^*\right).$$

Here I used the assumption that $\mathbb{E}\xi_t = \mathbb{E}\xi_t^* = 0$. From the firm's first-order conditions in (17) – (20), I

[3] Notice that I not only require that $\mathbb{E}\gtreqqless\xi_t = \mathbb{E}\gtreqqless\xi_t^* = 0$. I also impose that $\mathbb{E}\xi_t = \mathbb{E}\xi_t^* = 0$.

obtain that the law of one-price (LOOP) holds in steady state, i.e.

$$\overline{\widetilde{P}}(h) = \overline{\widetilde{P}}^*(h)\overline{S} = \frac{\theta}{\theta-1}\overline{W}, \tag{23}$$

$$\frac{\overline{\widetilde{P}}(f)}{\overline{S}} = \overline{\widetilde{P}}^*(f) = \frac{\theta}{\theta-1}\overline{W}^*. \tag{24}$$

Here I used the assumption that $\mathbb{E}a_t = \mathbb{E}a_t^* = 0$. Notice that the pricing policy corresponds to the solution of the monopolistic competitor's problem under flexible prices. In other words, prices are set by the Dixit-Stiglitz monopolistic price-setting rule in steady state, and firms charge a mark-up over marginal costs. Moreover, profits per unit of output in steady state are proportional to steady state wages, i.e. $\left(\frac{1}{\theta-1}\right)\overline{W}$ and $\left(\frac{1}{\theta-1}\right)\overline{W}^*$ respectively.

I write the steady state price sub-indexes in equations (12) – (13) and (15) – (16) as follows,

$$\overline{P}^H = \overline{\widetilde{P}}(h),\ \overline{P}^{H*} = \overline{\widetilde{P}}^*(h), \tag{25}$$

$$\overline{P}^F = \overline{\widetilde{P}}(f),\ \overline{P}^{F*} = \overline{\widetilde{P}}^*(f), \tag{26}$$

and,

$$\overline{P}^H = \overline{P}^{H*}\overline{S},\ \overline{P}^F = \overline{P}^{F*}\overline{S}.$$

Using the price indexes in equations (11) and (14) evaluated at their steady state, I also infer that the real exchange rate should be equal to one,

$$\overline{P} = \overline{SP}^* \Rightarrow \overline{RS} \equiv \frac{\overline{SP}^*}{\overline{P}} = 1.$$

From the portfolio allocation rule obtained in (21), I know that the (nominal and real) net foreign asset position of the financial intermediaries in steady state is zero, i.e. $\overline{B}^F = \overline{B}^{RF} = 0$, if $\lambda \neq 0$.

It follows from the current account in equation (22) that in steady state,

$$\begin{aligned}\overline{C} &= \left(\frac{\overline{P}^H}{\overline{P}}\right)^{1-\sigma} n\overline{C} + \left(\frac{\overline{P}^{H*}}{\overline{P}^*}\right)^{1-\sigma}(1-n)\overline{C}^* \\ &= \left(\frac{\overline{P}^H}{\overline{P}}\right)^{1-\sigma}\left[n\overline{C} + (1-n)\overline{C}^*\right].\end{aligned}$$

From the steady state price index implied by (11) it is possible to re-write the current account as,

$$\overline{C}^* = \left(\frac{\overline{P}^F}{\overline{P}}\right)^{1-\sigma}\left[n\overline{C} + (1-n)\overline{C}^*\right],$$

so I infer that the ratio of the consumption level across countries can be expressed as a function of the ratio of domestic good prices relative to foreign good prices in steady state,

$$\frac{\overline{C}}{\overline{C}^*} = \left(\frac{\overline{P}^H}{\overline{P}^F}\right)^{1-\sigma}. \tag{27}$$

The labor market and the goods market clearing conditions imply that in steady state,

$$\begin{aligned}
\overline{L}^{s} &= \overline{L}^{d}(h) = \overline{Y}^{s}(h), \\
\overline{Y}^{s}(h) &= n\overline{C}(h) + (1-n)\overline{C}^{*}(h), \\
\overline{L}^{s*} &= \overline{L}^{d*}(h) = \overline{Y}^{s}(f), \\
\overline{Y}^{s}(f) &= n\overline{C}(f) + (1-n)\overline{C}^{*}(f).
\end{aligned}$$

Here I employed the assumptions that technologies are linear in labor and $\mathbb{E}a_t = \mathbb{E}a_t^* = 0$. Using the demand equations in $(7)-(10)$, I argue that for any variety $h \in [0,n]$ and $f \in (n,1]$ it must hold true in steady state that,

$$\overline{L}^{s} = n\overline{C}(h) + (1-n)\overline{C}^{*}(h) = \left(\frac{\overline{P}^{H}}{\overline{P}}\right)^{-\sigma}\left[n\overline{C} + (1-n)\overline{C}^{*}\right], \tag{28}$$

$$\overline{L}^{s*} = n\overline{C}(f) + (1-n)\overline{C}^{*}(f) = \left(\frac{\overline{P}^{F}}{\overline{P}}\right)^{-\sigma}\left[n\overline{C} + (1-n)\overline{C}^{*}\right]. \tag{29}$$

The labor supply equations in (2) and (5) evaluated at their steady state values give me that,

$$\begin{aligned}
\frac{\overline{W}}{\overline{P}} &= \kappa(1-b)^{\gamma}\left(\overline{C}\right)^{\gamma}\left(\overline{L}^{s}\right)^{\varphi}, \\
\frac{\overline{W}^{*}}{\overline{P}^{*}} &= \kappa(1-b)^{\gamma}\left(\overline{C}^{*}\right)^{\gamma}\left(\overline{L}^{s*}\right)^{\varphi}.
\end{aligned}$$

Hence, using $(23)-(29)$ I obtain that real wages satisfy these pair of conditions,

$$\frac{\theta-1}{\theta}\frac{\overline{P}^{H}}{\overline{P}} = \frac{\overline{W}}{\overline{P}} = \kappa(1-b)^{\gamma}\left(\overline{C}\right)^{\gamma}\left(\left(\frac{\overline{P}^{H}}{\overline{P}}\right)^{-\sigma}\left[n\overline{C} + (1-n)\overline{C}^{*}\right]\right)^{\varphi}, \tag{30}$$

$$\frac{\theta-1}{\theta}\frac{\overline{P}^{F}}{\overline{P}} = \frac{\overline{W}^{*}}{\overline{P}^{*}} = \kappa(1-b)^{\gamma}\left(\overline{C}^{*}\right)^{\gamma}\left(\left(\frac{\overline{P}^{F}}{\overline{P}}\right)^{-\sigma}\left[n\overline{C} + (1-n)\overline{C}^{*}\right]\right)^{\varphi}. \tag{31}$$

After some re-arranging, it immediately follows that

$$\frac{\overline{P}^{H}}{\overline{P}^{F}} = \left(\frac{\overline{C}}{\overline{C}^{*}}\right)^{\gamma}\left(\frac{\overline{P}^{H}}{\overline{P}^{F}}\right)^{-\varphi\sigma}.$$

If I use (27) now to replace the ratio $\frac{\overline{C}}{\overline{C}^{*}}$, I get that

$$\frac{\overline{P}^{H}}{\overline{P}^{F}} = \left(\frac{\overline{P}^{H}}{\overline{P}^{F}}\right)^{\gamma(1-\sigma)-\varphi\sigma}.$$

Therefore, as long as $\gamma(1-\sigma) - \varphi\sigma \neq 1$, this equality can only hold if $\overline{P}^{H} = \overline{P}^{F}$. As a result, I immediately infer that in a symmetric steady state prices and consumption satisfy these equalities,

$$\begin{aligned}
\overline{P} &= \overline{P}^{H} = \overline{P}^{F}, \ \overline{P}^{*} = \overline{P}^{H*} = \overline{P}^{F*}, \\
\overline{ToT} &= \overline{ToT}^{*} = \overline{T} = \overline{T}^{*} = \overline{S} = 1, \\
\overline{C}(h) &= \overline{C}(f) = \overline{C}, \ C^{*}(h) = C^{*}(f) = \overline{C}^{*}, \ \overline{C} = \overline{C}^{*}.
\end{aligned}$$

This means that perfect international *risk-sharing* holds in steady state, and hence consumption is fully equalized across countries.

Using the fact that $\overline{P} = \overline{P}^{H}$ combined with (30) allows me to derive the following characterization of steady state consumption,

$$\overline{C} = \overline{C}^{*} = \left[\kappa (1-b)^{\gamma} \left(\frac{\theta}{\theta - 1}\right)\right]^{-\frac{1}{\gamma+\varphi}}. \tag{32}$$

Notice how consumption in steady state is related to the mark-up charged by firms, i.e. $\frac{\theta}{\theta-1}$, the habit formation parameter, i.e. b, and the preference weight on labor disutility, i.e. κ. This function is decreasing and convex in the mark-up. Notice also that the steady state output and labor equilibrium levels are identical to the right-hand side in (32) because it follows that,

$$\overline{L}^{s} = \overline{L}^{s*} = \overline{Y}^{s}(h) = \overline{Y}^{s}(f) = \overline{C} = \overline{C}^{*}.$$

I evaluate the money demand equations in (1) and (4) at their steady state values and obtain that,

$$\overline{PC} = \overline{P}^{*}\overline{C}^{*} = \frac{1}{1-b}\left(\frac{1-\beta}{\chi}\right)^{\frac{1}{\gamma}}. \tag{33}$$

Here I employed the fact that money markets clear and $\mathbb{E}m_t = \mathbb{E}m_t^{*} = 0$. This equation implies that a version of the quantity theory of money holds in steady state. The left-hand side appears expressed in terms of nominal consumption. However, in steady state consumption equals output in each country, and this is consistent with the quantity theory. The right-hand side should include the velocity of money times the money supply. By construction, money supply is equal to one (since $\mathbb{E}m_t = \mathbb{E}m_t^{*} = 0$) in steady state. Therefore, the right-hand side of (33) corresponds to the steady state 'money velocity' by analogy. Money velocity is related to the rate of time preference, i.e. β, which is also the inverse gross nominal interest rate in steady state, to the habit formation parameter, i.e. b, and to the preference weight on utility from real balances, i.e. χ.

Substituting (32) in (33) I infer that the steady state consumption price index is equal to,

$$\overline{P} = \overline{P}^{*} = \kappa^{\frac{1}{\gamma+\varphi}} \chi^{-\frac{1}{\gamma}} (1-b)^{\frac{-\varphi}{\gamma+\varphi}} (1-\beta)^{\frac{1}{\gamma}} \left(\frac{\theta}{\theta-1}\right)^{\frac{1}{\gamma+\varphi}}. \tag{34}$$

Furthermore, combining (23) – (26) with (34) and $\overline{P} = \overline{P}^{H} = \overline{P}^{F*} = \overline{P}^{*}$, I also obtain that the steady state wages are equal to,

$$\overline{W} = \overline{W}^{*} = \kappa^{\frac{1}{\gamma+\varphi}} \chi^{-\frac{1}{\gamma}} (1-b)^{\frac{-\varphi}{\gamma+\varphi}} (1-\beta)^{\frac{1}{\gamma}} \left(\frac{\theta}{\theta-1}\right)^{\frac{1-\gamma-\varphi}{\gamma+\varphi}}. \tag{35}$$

This pins down the entire steady state of the model and satisfies my initial conjecture. Equations (34) and (35) both depend on the rate of time preference, i.e. β, the habit formation parameter, i.e. b, the mark-up charged by firms, i.e. $\frac{\theta}{\theta-1}$, and the preference weight on labor disutility and on utility from real balances, i.e. κ and χ respectively. Consumption prices are clearly increasing in the mark-up, while wages are decreasing whenever $\gamma + \varphi > 1$. Notice also that the steady state interest rate, β^{-1}, only affects prices and wages, but not consumption or output in steady state.

Let me assume for the sake of argument that this parameter is an instrument that can be 'chosen' by a central bank, or alternatively that the unconditional mean of money supply can be 'set' to be different than one. Both specifications produce similar results. My calculations suggest that the steady state outcome is consistent with the notion of long-run neutrality of money. Monetary variables have only effects on the price and wage levels, but not on production. However, in the short-run the presence of nominal rigidities (as well as other frictions) in the model means that monetary variables can have real effects that have to be accounted for.

D The Linearization of the Equilibrium Conditions

I linearize the equilibrium conditions around the deterministic zero-inflation, zero-current account steady state (see also King *et al.*, 1988). I approximate all variables in logs, except the real net foreign asset position of financial intermediaries (in levels) (e.g., P. Benigno, 2001, and Thoenissen 2003). I denote $\widehat{x}_t \equiv \ln X_t - \ln \overline{X}$ the deviation of a variable in logs from its steady state, and $\widehat{X}_t \equiv \frac{X_t - \overline{X}}{\overline{C}}$ the deviation of a variable in levels from its steady state relative to steady state consumption (or output).

The approximation technique of each equilibrium condition is applied as follows:

(i) Let me assume the equilibrium condition can be expressed as $X_{it} = H_i\left(X_{1t}, ..., X_{qt}\right)$, and X_{qt} must be approximated in levels. First, I re-express the equation in logs $\ln X_{it} = \ln H_i\left(e^{\ln X_{1t}}, ..., e^{\ln X_{q-1t}}, X_{qt}\right)$. Second, I take a first-order approximation in the log and level variables around the steady state, i.e.

$$\begin{aligned} \ln X_{it} \approx\ & \ln \overline{X}_i + \sum\nolimits_{j=1}^{q-1} \frac{1}{H_i\left(e^{\ln \overline{X}_1}, ..., e^{\ln \overline{X}_{q-1}}, \overline{X}_q\right)} \frac{\partial H_i\left(e^{\ln \overline{X}_1}, ..., e^{\ln \overline{X}_{q-1}}, \overline{X}_q\right)}{\partial X_{jt}} e^{\ln \overline{X}_j}\left(\ln X_{jt} - \ln \overline{X}_j\right) + \\ & + \frac{1}{H_i\left(e^{\ln \overline{X}_1}, ..., e^{\ln \overline{X}_{q-1}}, \overline{X}_q\right)} \frac{\partial H_i\left(e^{\ln \overline{X}_1}, ..., e^{\ln \overline{X}_{q-1}}, \overline{X}_q\right)}{\partial X_{qt}} \left(X_{qt} - \overline{X}_q\right), \end{aligned}$$

or simply,

$$\widehat{x}_{it} \approx \sum\nolimits_{j=1}^{q-1} \frac{\partial H_i\left(\overline{X}_1, ..., \overline{X}_q\right)}{\partial X_{jt}} \frac{\overline{X}_j}{H_i\left(\overline{X}_1, ..., \overline{X}_q\right)} \widehat{x}_{jt} + \frac{\partial H_i\left(\overline{X}_1, ..., \overline{X}_q\right)}{\partial X_{qt}} \frac{\overline{C}}{H_i\left(\overline{X}_1, ..., \overline{X}_q\right)} \widehat{X}_{qt},$$

assuming that $\overline{X}_i = H_i\left(\overline{X}_1, ..., \overline{X}_q\right) \neq 0$. In this sense, each log variable on the right-hand side of the previous equation is simply weighted by its steady state elasticity.

(ii) Let me assume the equilibrium condition can be expressed as $G_k\left(X_{1t}, ..., X_{qt}\right) = H_k\left(X_{1t}, ..., X_{qt}\right)$, and X_{qt} must be approximated in levels. First, I re-express both equations in logs $\ln G_k\left(e^{\ln X_{1t}}, ..., e^{\ln X_{q-1t}}, X_{qt}\right) = \ln H_k\left(e^{\ln X_{1t}}, ..., e^{\ln X_{q-1t}}, X_{qt}\right)$. Second, I take a first-order approximation in the log and level variables around the steady state, i.e.

$$\begin{aligned} & \ln G_k\left(e^{\ln \overline{X}_1}, ..., e^{\ln \overline{X}_q}\right) + \sum\nolimits_{j=1}^{q-1} \frac{1}{G_k\left(e^{\ln \overline{X}_1}, ..., e^{\ln \overline{X}_{q-1}}, \overline{X}_q\right)} \frac{\partial G_k\left(e^{\ln \overline{X}_1}, ..., e^{\ln \overline{X}_{q-1}}, \overline{X}_q\right)}{\partial X_{jt}} e^{\ln \overline{X}_j}\left(\ln X_{jt} - \ln \overline{X}_j\right) + \\ & + \frac{1}{G_k\left(e^{\ln \overline{X}_1}, ..., e^{\ln \overline{X}_{q-1}}, \overline{X}_q\right)} \frac{\partial G_k\left(e^{\ln \overline{X}_1}, ..., e^{\ln \overline{X}_{q-1}}, \overline{X}_q\right)}{\partial X_{qt}} \left(X_{qt} - \overline{X}_q\right) \\ \approx\ & \ln H_k\left(e^{\ln \overline{X}_1}, ..., e^{\ln \overline{X}_q}\right) + \sum\nolimits_{j=1}^{q-1} \frac{1}{H_k\left(e^{\ln \overline{X}_1}, ..., e^{\ln \overline{X}_{q-1}}, \overline{X}_q\right)} \frac{\partial H_k\left(e^{\ln \overline{X}_1}, ..., e^{\ln \overline{X}_{q-1}}, \overline{X}_q\right)}{\partial X_{jt}} e^{\ln \overline{X}_j}\left(\ln X_{jt} - \ln \overline{X}_j\right) + \\ & + \frac{1}{H_k\left(e^{\ln \overline{X}_1}, ..., e^{\ln \overline{X}_{q-1}}, \overline{X}_q\right)} \frac{\partial H_k\left(e^{\ln \overline{X}_1}, ..., e^{\ln \overline{X}_{q-1}}, \overline{X}_q\right)}{\partial X_{qt}} \left(X_{qt} - \overline{X}_q\right), \end{aligned}$$

or simply,

$$\begin{aligned}
&\sum\nolimits_{j=1}^{q-1} \frac{\partial G_k\left(\overline{X}_1,...,\overline{X}_q\right)}{\partial X_{jt}} \frac{\overline{X}_j}{G_k\left(\overline{X}_1,...,\overline{X}_q\right)} \widehat{x}_{jt} + \frac{\partial G_k\left(\overline{X}_1,...,\overline{X}_q\right)}{\partial X_{qt}} \frac{\overline{C}}{G_k\left(\overline{X}_1,...,\overline{X}_q\right)} \widehat{X}_{qt} \\
\approx \quad &\sum\nolimits_{j=1}^{q-1} \frac{\partial H_k\left(\overline{X}_1,...,\overline{X}_q\right)}{\partial X_{jt}} \frac{\overline{X}_j}{H_k\left(\overline{X}_1,...,\overline{X}_q\right)} \widehat{x}_{jt} + \frac{\partial H_k\left(\overline{X}_1,...,\overline{X}_q\right)}{\partial X_{qt}} \frac{\overline{C}}{H_k\left(\overline{X}_1,...,\overline{X}_q\right)} \widehat{X}_{qt},
\end{aligned}$$

assuming that $G_k\left(\overline{X}_1,...,\overline{X}_q\right) = H_k\left(\overline{X}_1,...,\overline{X}_q\right) \neq 0$.

(iii) Let me assume the equilibrium condition can be expressed as $G_k\left(X_{1t},...,X_{qt}\right) = H_k\left(X_{1t},...,X_{qt}\right)$, X_{qt} must be approximated in levels, and $G_k\left(\overline{X}_1,...,\overline{X}_q\right) = H_k\left(\overline{X}_1,...,\overline{X}_q\right) = 0$. Then, the equilibrium condition can still be approximated as follows,

$$\begin{aligned}
&\sum\nolimits_{j=1}^{q-1} \frac{\partial G_k\left(\overline{X}_1,...,\overline{X}_q\right)}{\partial X_{jt}} \overline{X}_j \widehat{x}_{jt} + \frac{\partial G_k\left(\overline{X}_1,...,\overline{X}_q\right)}{\partial X_{qt}} \overline{C} \widehat{X}_{qt} \\
\approx \quad &\sum\nolimits_{j=1}^{q-1} \frac{\partial H_k\left(\overline{X}_1,...,\overline{X}_q\right)}{\partial X_{jt}} \overline{X}_j \widehat{x}_{jt} + \frac{\partial H_k\left(\overline{X}_1,...,\overline{X}_q\right)}{\partial X_{qt}} \overline{C} \widehat{X}_{qt}.
\end{aligned}$$

This formula is particularly useful to log-linearize the resource constraint.

(iv) If the equilibrium condition contains leads and lags of certain variables, I approximate the equation around them too. If the equation contains expectations, then I use the fact that the expectation of the log is approximately equal to the log of the expectation.

More details on the log-linearization are provided as needed in the rest of this technical note.

D.1 The *CPI*, the *RS* and the *RP* Equations

Let me start by describing the log-linearization of the domestic and foreign consumption-based price indexes defined in (11) and (14). In steady state, it must hold that $\overline{P}^H = \overline{P}^F$ and $\overline{P}^{H*} = \overline{P}^{F*}$. Hence, taking a first-order approximation of the price indexes I derive the *CPI* equations as,

$$\begin{aligned}
\widehat{p}_t &\approx n\widehat{p}_t^H + (1-n)\,\widehat{p}_t^F, && (36)\\
\widehat{p}_t^* &\approx n\widehat{p}_t^{H*} + (1-n)\,\widehat{p}_t^{F*}. && (37)
\end{aligned}$$

I define the domestic and foreign inflation rates in deviations as $\widehat{\pi}_t \equiv \widehat{p}_t - \widehat{p}_{t-1}$ and $\widehat{\pi}_t^* \equiv \widehat{p}_t^* - \widehat{p}_{t-1}^*$, respectively.

I define the real exchange rate as $RS_t \equiv \frac{S_t P_t^*}{P_t}$ and the relative prices in the home and foreign country respectively as $T_t \equiv \frac{P_t^F}{P_t^H}$ and $\left(T_t^*\right)^{-1} \equiv \frac{P_t^{F*}}{P_t^{H*}}$. Therefore, it immediately follows that the *RS* equation is approximately equal to,

$$\begin{aligned}
\widehat{rs}_t &= \widehat{s}_t + \widehat{p}_t^* - \widehat{p}_t && (38)\\
&\approx \widehat{s}_t + \left(n\widehat{p}_t^{H*} + (1-n)\,\widehat{p}_t^{F*}\right) - \left(n\widehat{p}_t^H + (1-n)\,\widehat{p}_t^F\right).
\end{aligned}$$

And, the *RP* equations take the form of,

$$\begin{aligned}
\widehat{t}_t &= \widehat{p}_t^F - \widehat{p}_t^H, && (39)\\
\widehat{t}_t^* &= -\left(\widehat{p}_t^{F*} - \widehat{p}_t^{H*}\right). && (40)
\end{aligned}$$

I denote world relative prices, $\widehat{t}_t^W$, and the discrepancy on relative prices across countries, $\widehat{t}_t^R$, as follows,

$$\begin{aligned}
\widehat{t}_t^W &\equiv n\widehat{t}_t - (1-n)\,\widehat{t}_t^*,\\
\widehat{t}_t^R &\equiv \widehat{t}_t + \widehat{t}_t^*.
\end{aligned}$$

Hence, the RP equations can also be expressed in terms of $\widehat{t}_t^W$ and $\widehat{t}_t^R$ as,

$$\begin{aligned}
-\widehat{t}_t^* &= \widehat{t}_t^W - n\widehat{t}_t^R, \\
\widehat{t}_t &= \widehat{t}_t^W + (1-n)\,\widehat{t}_t^R.
\end{aligned}$$

I define domestic and foreign terms of trade respectively as $ToT_t \equiv \frac{P_t^H}{S_t P_t^{H*}} \frac{P_t^F}{P_t^H}$ and $ToT_t^* \equiv \frac{1}{ToT_t} = \frac{S_t P_t^{F*}}{P_t^F} \frac{P_t^{H*}}{P_t^{F*}}$. Then, I can infer that,

$$\begin{aligned}
\widehat{tot}_t &\equiv \left(\widehat{p}_t^H - \widehat{s}_t - \widehat{p}_t^{H*}\right) + \widehat{t}_t, \\
\widehat{tot}_t^* &= -\widehat{tot}_t = \left(\widehat{s}_t + \widehat{p}_t^{F*} - \widehat{p}_t^F\right) + \widehat{t}_t^*,
\end{aligned}$$

which in turn allows me to re-write the real exchange rate as,

$$\begin{aligned}
\widehat{rs}_t &\approx n\widehat{t}_t - (1-n)\,\widehat{t}_t^* - \left[\widehat{p}_t^F - \widehat{s}_t - \widehat{p}_t^{H*}\right] \\
&= \widehat{t}_t^W - \widehat{tot}_t.
\end{aligned}$$

Relative prices and terms of trade represent different measures of the value expressed in units of the local currency of imported goods in terms of the domestic good. The relative prices indicate the cost of replacing one unit of imports bought in the 'local market' with one unit of the domestically-produced good. The terms of trade ratio, in turn, expresses the cost of replacing one unit of imports with one unit of exports to the 'foreign market'. These calculations show that movements in the real exchange rate can be thought as the result of differences between world relative prices and domestic terms of trade.

These formulas and definitions are very useful in the derivation of the linearized equilibrium conditions of the model.

D.2 The Demand-Side of the Economy

D.2.1 The Investment-Savings Equations: IS^H and IS^F

To obtain the IS^H and IS^F equations I need to log-linearize the Euler equations in (3) and (6) around the steady state, i.e.

$$\begin{aligned}
\mathbb{E}\left[\underbrace{\exp\left(-\Delta\xi_{t+1}\right)\left(\frac{C_{t+1} - bC_t}{C_t - bC_{t-1}}\right)^{-\gamma} \frac{P_t}{P_{t+1}}}_{\equiv F_{t+1}} - \underbrace{\exp\left(-i_t\right)\frac{1}{\beta}}_{\equiv G_{t+1}} \mid \mathcal{H}_t\right] &= 0, \\
\mathbb{E}\left[\underbrace{\exp\left(-\Delta\xi_{t+1}^*\right)\left(\frac{C_{t+1}^* - bC_t^*}{C_t^* - bC_{t-1}^*}\right)^{-\gamma} \frac{P_t^*}{P_{t+1}^*}}_{\equiv F_{t+1}^*} - \underbrace{\exp\left(-i_t^*\right)\frac{1}{\beta}}_{\equiv G_{t+1}^*} \mid \mathcal{H}_t^*\right] &= 0.
\end{aligned}$$

In steady state holds that $\overline{f} = \overline{g}$ and $\overline{f}^* = \overline{g}^*$, then the log-linearization around the steady state can be expressed as,

$$\begin{aligned}
\mathbb{E}\left[\widehat{f}_{t+1} - \widehat{g}_{t+1} \mid \mathcal{H}_t\right] &\approx 0, \\
\mathbb{E}\left[\widehat{f}_{t+1}^* - \widehat{g}_{t+1}^* \mid \mathcal{H}_t^*\right] &\approx 0,
\end{aligned}$$

where the approximations are as follows,

$$
\begin{aligned}
\widehat{f}_{t+1} &\equiv -\Delta\widehat{\xi}_{t+1} - \gamma\left(\frac{1}{1-b}\right)\Delta\widehat{c}_{t+1} + \gamma\left(\frac{b}{1-b}\right)\Delta\widehat{c}_t - \widehat{\pi}_{t+1}, \\
\widehat{f}^*_{t+1} &\equiv -\Delta\widehat{\xi}^*_{t+1} - \gamma\left(\frac{1}{1-b}\right)\Delta\widehat{c}^*_{t+1} + \gamma\left(\frac{b}{1-b}\right)\Delta\widehat{c}^*_t - \widehat{\pi}^*_{t+1}, \\
\widehat{g}_{t+1} &\equiv -\widehat{i}_t,\ \widehat{g}^*_{t+1} \equiv -\widehat{i}^*_t.
\end{aligned}
$$

Then, I obtain the following system of two linearized Euler equations,

$$
\frac{\gamma}{1-b}\mathbb{E}\left[\Delta\widehat{c}_{t+1} - b\Delta\widehat{c}_t \mid \mathcal{H}_t\right] \approx \mathbb{E}\left[\widehat{i}_t - \widehat{\pi}_{t+1} \mid \mathcal{H}_t\right] - \mathbb{E}\left[\Delta\widehat{\xi}_{t+1} \mid \mathcal{H}_t\right], \tag{41}
$$

$$
\frac{\gamma}{1-b}\mathbb{E}\left[\Delta\widehat{c}^*_{t+1} - b\Delta\widehat{c}^*_t \mid \mathcal{H}^*_t\right] \approx \mathbb{E}\left[\widehat{i}^*_t - \widehat{\pi}^*_{t+1} \mid \mathcal{H}^*_t\right] - \mathbb{E}\left[\Delta\widehat{\xi}^*_{t+1} \mid \mathcal{H}^*_t\right], \tag{42}
$$

which are conventionally denoted the IS^H and IS^F equations, respectively. The *ex ante* Fisher equation requires that the model-based implicit real interest rate be equal to,

$$
\begin{aligned}
\widehat{r}_t &\equiv \mathbb{E}\left[\widehat{i}_t - \widehat{\pi}_{t+1} \mid \mathcal{H}_t\right], \\
\widehat{r}^*_t &\equiv \mathbb{E}\left[\widehat{i}^*_t - \widehat{\pi}^*_{t+1} \mid \mathcal{H}^*_t\right].
\end{aligned}
$$

The growth rate of consumption adjusted by the effect of the habit is proportional to this measure of the real interest rate and the preference-shock on consumption. The constant of proportionality is a function of the habit parameter, b, and the coefficient of relative risk aversion, γ. Using the linearized consumption price indexes in (36) – (37), I obtain that

$$
\frac{\gamma}{1-b}\mathbb{E}\left[\Delta\widehat{c}_{t+1} - b\Delta\widehat{c}_t \mid \mathcal{H}_t\right] \approx \mathbb{E}\left[\widehat{i}_t - n\widehat{\pi}^H_{t+1} - (1-n)\,\widehat{\pi}^F_{t+1} \mid \mathcal{H}_t\right] - \mathbb{E}\left[\Delta\widehat{\xi}_{t+1} \mid \mathcal{H}_t\right], \tag{43}
$$

$$
\frac{\gamma}{1-b}\mathbb{E}\left[\Delta\widehat{c}^*_{t+1} - b\Delta\widehat{c}^*_t \mid \mathcal{H}^*_t\right] \approx \mathbb{E}\left[\widehat{i}^*_t - n\widehat{\pi}^{H*}_{t+1} - (1-n)\,\widehat{\pi}^{F*}_{t+1} \mid \mathcal{H}^*_t\right] - \mathbb{E}\left[\Delta\widehat{\xi}^*_{t+1} \mid \mathcal{H}^*_t\right], \tag{44}
$$

where $\widehat{\pi}^H_{t+1} \equiv \widehat{p}^H_{t+1} - \widehat{p}^H_t$ and $\widehat{\pi}^F_{t+1} \equiv \widehat{p}^F_{t+1} - \widehat{p}^F_t$. This is the version of the IS^H and IS^F equations that I use in the paper.

Finally, using equations (39) – (40) for relative prices I can also re-write the IS^H and IS^F equations as,

$$
\begin{aligned}
\frac{\gamma}{1-b}\mathbb{E}\left[\Delta\widehat{c}_{t+1} - b\Delta\widehat{c}_t \mid \mathcal{H}_t\right] &\approx \mathbb{E}\left[\widehat{i}_t - \widehat{\pi}^H_{t+1} - (1-n)\,\Delta\widehat{t}_{t+1} \mid \mathcal{H}_t\right] - \mathbb{E}\left[\Delta\widehat{\xi}_{t+1} \mid \mathcal{H}_t\right], \\
\frac{\gamma}{1-b}\mathbb{E}\left[\Delta\widehat{c}^*_{t+1} - b\Delta\widehat{c}^*_t \mid \mathcal{H}^*_t\right] &\approx \mathbb{E}\left[\widehat{i}^*_t - \widehat{\pi}^{F*}_{t+1} - n\Delta\widehat{t}^*_{t+1} \mid \mathcal{H}^*_t\right] - \mathbb{E}\left[\Delta\widehat{\xi}^*_{t+1} \mid \mathcal{H}^*_t\right].
\end{aligned}
$$

However, in the literature it is common to rely on the world IS equation, IS^W, and the relative IS equation, IS^R. These two equations describe the dynamics of world consumption defined as a weighted average of domestic and foreign consumption, i.e. $\widehat{c}^W_t \equiv n\widehat{c}_t + (1-n)\,\widehat{c}^*_t$, and the dynamics of relative consumption, i.e. $\widehat{c}^R_t \equiv \widehat{c}_t - \widehat{c}^*_t$. They are sufficient to describe domestic and foreign consumption because it easily follows that,

$$
\begin{aligned}
\widehat{c}_t &\equiv \widehat{c}^W_t + (1-n)\,\widehat{c}^R_t, \\
\widehat{c}^*_t &\equiv \widehat{c}^W_t - n\widehat{c}^R_t.
\end{aligned}
$$

Their dynamics can be derived by computing $n\cdot(43)+(1-n)\cdot(44)$ and $(43)-(44)$, respectively. Nonetheless, for my purposes they only obscure the analysis. In part, because the dynamics for $\widehat{c}^W_t$ and $\widehat{c}^R_t$ involve the decisions of two distinct types of agents with different information sets. Hence, I stick with the standard IS^H and IS^F.

D.3 The Monetary-Side of the Economy

D.3.1 The Money Market Equations: MM^H and MM^F

To obtain the MM^H and MM^F equations I need to log-linearize the money demand equations in (1) and (4) around the steady state, i.e.

$$\begin{aligned}
\underbrace{\chi\left(\frac{M_t^d}{P_t\left(C_t-bC_{t-1}\right)}\right)^{-\gamma}}_{\equiv F_t} &= \underbrace{\frac{\exp\left(i_t\right)-1}{\exp\left(i_t\right)}}_{\equiv G_t}, \\
\underbrace{\chi\left(\frac{M_t^{d*}}{P_t^*\left(C_t^*-bC_{t-1}^*\right)}\right)^{-\gamma}}_{\equiv F_t^*} &= \underbrace{\frac{\exp\left(i_t^*\right)-1}{\exp\left(i_t^*\right)}}_{\equiv G_t^*}.
\end{aligned}$$

In steady state holds that $\overline{f}=\overline{g}$ and $\overline{f}^*=\overline{g}^*$, then the log-linearization around the steady state can be expressed as,

$$\begin{aligned}
\widehat{f}_t &\approx \widehat{g}_t, \\
\widehat{f}_t^* &\approx \widehat{g}_t^*,
\end{aligned}$$

where the approximations are as follows,

$$\begin{aligned}
\widehat{f}_t &\equiv -\gamma\widehat{m}_t^d+\gamma\widehat{p}_t+\frac{\gamma}{1-b}\left[\widehat{c}_t-b\widehat{c}_{t-1}\right], \\
\widehat{f}_t^* &\equiv -\gamma\widehat{m}_t^{d*}+\gamma\widehat{p}_t^*+\frac{\gamma}{1-b}\left[\widehat{c}_t^*-b\widehat{c}_{t-1}^*\right], \\
\widehat{g}_t &\equiv \frac{\beta}{1-\beta}\widehat{i}_t,\ \widehat{g}_t^*\equiv\frac{\beta}{1-\beta}\widehat{i}_t^*.
\end{aligned}$$

Then, I obtain a system of two stable money demand functions à la Cagan. In equilibrium, the per capita money demand must be equal to the per capita money supply, i.e.

$$\begin{aligned}
\widehat{m}_t^d &= \widehat{m}_t, \\
\widehat{m}_t^{d*} &= \widehat{m}_t^*.
\end{aligned}$$

Therefore, the following equations fully characterize the money market clearing conditions,

$$\widehat{m}_t-\widehat{p}_t \approx \frac{1}{1-b}\left[\widehat{c}_t-b\widehat{c}_{t-1}\right]-\frac{1}{\gamma}\left(\frac{\beta}{1-\beta}\right)\widehat{i}_t, \tag{45}$$

$$\widehat{m}_t^*-\widehat{p}_t^* \approx \frac{1}{1-b}\left[\widehat{c}_t^*-b\widehat{c}_{t-1}^*\right]-\frac{1}{\gamma}\left(\frac{\beta}{1-\beta}\right)\widehat{i}_t^*. \tag{46}$$

Real balances are cointegrated with the nominal interest rate, with aggregate consumption (instead of aggregate output as in Cagan's money demand functions) and with the consumption habit. The demand for real balances is a function of the habit parameter, b, and the coefficient of relative risk aversion, γ. Using (36) – (37) I obtain the version of the money market equations that I use in the paper,

$$\widehat{m}_t-n\widehat{p}_t^H-(1-n)\,\widehat{p}_t^F \approx \frac{1}{1-b}\left[\widehat{c}_t-b\widehat{c}_{t-1}\right]-\frac{1}{\gamma}\left(\frac{\beta}{1-\beta}\right)\widehat{i}_t, \tag{47}$$

$$\widehat{m}_t^*-n\widehat{p}_t^{H*}-(1-n)\,\widehat{p}_t^{F*} \approx \frac{1}{1-b}\left[\widehat{c}_t^*-b\widehat{c}_{t-1}^*\right]-\frac{1}{\gamma}\left(\frac{\beta}{1-\beta}\right)\widehat{i}_t^*, \tag{48}$$

which are conventionally denoted the MM^H and MM^F equations, respectively.

Taking the difference between MM^H and MM^F and using the definition of the real exchange rate is easy to derive an expression for the nominal interest rate spread across countries,

$$\widehat{i}_t^R \approx \gamma \left(\frac{1-\beta}{\beta}\right) \left[-\widehat{m}_t^R + \frac{1}{1-b}\left[\widehat{c}_t^R - b\widehat{c}_{t-1}^R\right] + \widehat{s}_t - \widehat{rs}_t\right],$$

where $\widehat{i}_t^R \equiv \widehat{i}_t - \widehat{i}_t^*$, $\widehat{c}_t^R \equiv \widehat{c}_t - \widehat{c}_t^*$, and $\widehat{m}_t^R \equiv \widehat{m}_t - \widehat{m}_t^*$. This implies that the interest rate differential is inversely proportional to the money supply differential. It also depends on consumption, consumption habit, and the differences in consumption prices across countries measured as,

$$\widehat{p}_t^R \equiv \widehat{p}_t - \widehat{p}_t^* = \widehat{s}_t - \widehat{rs}_t.$$

D.4 The Supply-Side of the Economy

Given the structure of the (symmetric) equilibrium, it suffices to explore the pricing rule of a representative firm within each country in order to derive the AS equations of the model.

D.4.1 The Optimal Pricing Equation for the Domestic Firm

Log-Linearization of the Home Market FOC. I can re-write the optimal pricing equation in (17) as follows,

$$\mathbb{E}\left\{\sum_{\tau=0}^{\infty} \left(\beta\alpha^H\right)^\tau \Xi_{t,t+\tau}^R \widetilde{Y}_{t,t+\tau}^d(h) \left[\frac{\widetilde{P}_t(h)}{P_{t+\tau}^H} - \frac{\theta}{\theta-1}\left(\frac{W_{t+\tau}}{P_{t+\tau}\exp\left(a_{t+\tau}\right)}\right)\frac{P_{t+\tau}}{P_{t+\tau}^H}\right] \mid \mathcal{F}_t\right\} = 0,$$

where

$$\Xi_{t,t+\tau}^R \equiv \exp\left(-\left(\xi_{t+\tau} - \xi_t\right)\right)\left(\frac{C_{t+\tau} - bC_{t+\tau-1}}{C_t - bC_{t-1}}\right)^{-\gamma}\frac{P_{t+\tau}^H}{P_{t+\tau}}.$$

Under a symmetric equilibrium it holds true that $L_t^s = L_t^d(h) = \frac{Y_t^s(h)}{\exp(a_t)}$. The second equality follows from the assumption that technologies are linear in labor. I use this equilibrium condition as well as the domestic labor supply equation in (2) to re-write the pricing equation as,

$$\mathbb{E}\left\{\sum_{\tau=0}^{\infty} \left(\beta\alpha^H\right)^\tau \Xi_{t,t+\tau}^R \widetilde{Y}_{t,t+\tau}^d(h) \left[\begin{array}{c} \frac{\widetilde{P}_t(h)}{P_{t+\tau}^H} - \\ -\frac{\theta\kappa}{\theta-1}\left(C_{t+\tau} - bC_{t+\tau-1}\right)^\gamma \left(\widetilde{Y}_{t,t+\tau}^s(h)\right)^\varphi \exp\left(\left(-1-\varphi\right)a_{t+\tau}\right)\frac{P_{t+\tau}}{P_{t+\tau}^H}\end{array}\right] \mid \mathcal{F}_t\right\} = 0,$$

where $\widetilde{Y}_{t,t+\tau}^s(h)$ denotes the aggregate supply of firm h at time $t+\tau$ conditional on having their prices fixed since period t. Naturally, the market clearing condition for variety h requires that[4],

$$\begin{aligned}\widetilde{Y}_{t,t+\tau}^s(h) &= n\widetilde{Y}_{t,t+\tau}^d(h) + (1-n)\widetilde{Y}_{t,t+\tau}^{d*}(h) \\ &= n\left(\frac{\widetilde{P}_t(h)}{P_{t+\tau}^H}\right)^{-\theta}\left(\frac{P_{t+\tau}^H}{P_{t+\tau}}\right)^{-\sigma} C_{t+\tau} + (1-n)\left(\frac{\widetilde{P}_t^*(h)}{P_{t+\tau}^{H*}}\right)^{-\theta}\left(\frac{P_{t+\tau}^{H*}}{P_{t+\tau}^*}\right)^{-\sigma} C_{t+\tau}^*,\end{aligned}$$

where the second equality identifies the aggregate demand based on the demand constraints of the firm.

[4] *Sensu stricto* G. Benigno's (2004) framework cannot be viewed as a pure double-signal Calvo model (where each firm receives one signal to re-optimize in the domestic market and another signal in the foreign market). Benigno discounts the fact that firms may receive a signal to re-optimize in one market but not the other, so he approximates the symmetric equilibrium with the behavior of a representative firm that re-optimizes in both markets simultaneously. For tractability and consistency, I use the same approach here. That explains the characterization of the market clearing condition.

Let me define the following pair of functions,

$$
\begin{aligned}
F^H_{t+\tau} &\equiv \frac{\widetilde{P}_t(h)}{P^H_{t+\tau}}, \\
G^H_{t+\tau} &\equiv \frac{\theta\kappa}{\theta-1}\left(C_{t+\tau}-bC_{t+\tau-1}\right)^{\gamma}\left(\widetilde{Y}^s_{t,t+\tau}(h)\right)^{\varphi}\exp\left(\left(-1-\varphi\right)a_{t+\tau}\right)\frac{P_{t+\tau}}{P^H_{t+\tau}},
\end{aligned}
$$

then the first-order condition for price-setting becomes simply,

$$
\mathbb{E}\left\{\sum\nolimits_{\tau=0}^{\infty}\left(\beta\alpha^H\right)^{\tau}\Xi^R_{t,t+\tau}\widetilde{Y}^d_{t,t+\tau}(h)\left[F^H_{t+\tau}-G^H_{t+\tau}\right]\mid\mathcal{F}_t\right\}=0.
$$

In steady state holds that $\overline{f}^H=\overline{g}^H$, then the log-linearization around the steady state can be expressed as,

$$
\mathbb{E}\left\{\sum\nolimits_{\tau=0}^{\infty}\left(\beta\alpha^H\right)^{\tau}\left[\widehat{f}^H_{t+\tau}-\widehat{g}^H_{t+\tau}\right]\mid\mathcal{F}_t\right\}\approx 0,
$$

where the approximations are as follows,

$$
\begin{aligned}
\widehat{f}^H_{t+\tau} &\equiv \widehat{\widetilde{p}}_t(h)-\widehat{p}^H_{t+\tau}, \\
\widehat{g}^H_{t+\tau} &\equiv \frac{\gamma}{1-b}\left(\widehat{c}_{t+\tau}-b\widehat{c}_{t+\tau-1}\right)+\varphi\widehat{\widetilde{y}}^s_{t,t+\tau}(h)-\left(1+\varphi\right)\widehat{a}_{t+\tau}+\left(\widehat{p}_{t+\tau}-\widehat{p}^H_{t+\tau}\right), \\
\widehat{\widetilde{y}}^s_{t,t+\tau}(h) &= n\left[-\theta\left(\widehat{\widetilde{p}}_t(h)-\widehat{p}^H_{t+\tau}\right)-\sigma\left(\widehat{p}^H_{t+\tau}-\widehat{p}_{t+\tau}\right)+\widehat{c}_{t+\tau}\right]+ \\
&\quad +\left(1-n\right)\left[-\theta\left(\widehat{\widetilde{p}}^*_t(h)-\widehat{p}^{H*}_{t+\tau}\right)-\sigma\left(\widehat{p}^{H*}_{t+\tau}-\widehat{p}^*_{t+\tau}\right)+\widehat{c}^*_{t+\tau}\right].
\end{aligned}
$$

Using equations (39) − (40) for relative prices and equations (36) − (37) for the price indexes, I can alternatively re-write the optimal pricing equation as,

$$
\mathbb{E}\left\{\sum\nolimits_{\tau=0}^{\infty}\left(\beta\alpha^H\right)^{\tau}\left[\begin{array}{c}
\left(1+n\varphi\theta\right)\left(\widehat{\widetilde{p}}_t(h)-\widehat{p}^H_{t+\tau}\right)+\left(1-n\right)\varphi\theta\left(\widehat{\widetilde{p}}^*_t(h)-\widehat{p}^{H*}_{t+\tau}\right)- \\
-\left(1-n\right)\left(\left(1+n\varphi\sigma\right)\widehat{t}_{t+\tau}-\left(1-n\right)\varphi\sigma\widehat{t}^*_{t+\tau}\right)- \\
-\frac{\gamma}{1-b}\left(\widehat{c}_{t+\tau}-b\widehat{c}_{t+\tau-1}\right)-\varphi\widehat{c}^W_{t+\tau}+\left(1+\varphi\right)\widehat{a}_{t+\tau}
\end{array}\right]\mid\mathcal{F}_t\right\}\approx 0,
$$

where

$$
\begin{aligned}
\widehat{p}^H_{t+\tau}-\widehat{p}_{t+\tau} &= -\left(1-n\right)\widehat{t}_{t+\tau}, \\
\widehat{p}^{H*}_{t+\tau}-\widehat{p}^*_{t+\tau} &= \left(1-n\right)\widehat{t}^*_{t+\tau},
\end{aligned}
$$

and world consumption is $\widehat{c}^W_{t+\tau}\equiv n\widehat{c}_{t+\tau}+\left(1-n\right)\widehat{c}^*_{t+\tau}$. Notice that I can re-express the price indexes $\widehat{p}^H_{t+\tau}$ and $\widehat{p}^{H*}_{t+\tau}$ respectively as $\widehat{p}^H_{t+\tau}=\widehat{p}^H_t+\sum\nolimits_{i=1}^{\tau}\widehat{\pi}^H_{t+i}$ and $\widehat{p}^{H*}_{t+\tau}=\widehat{p}^{H*}_t+\sum\nolimits_{i=1}^{\tau}\widehat{\pi}^{H*}_{t+i}$. Hence, the optimal pricing equation becomes,

$$
\begin{aligned}
&\left(1+n\varphi\theta\right)\left(\widehat{\widetilde{p}}_t(h)-\widehat{p}^H_t\right)+\left(1-n\right)\varphi\theta\left(\widehat{\widetilde{p}}^*_t(h)-\widehat{p}^{H*}_t\right) \\
\approx\; &\left(1-\beta\alpha^H\right)\mathbb{E}\left\{\sum\nolimits_{\tau=0}^{\infty}\left(\beta\alpha^H\right)^{\tau}\left[\begin{array}{c}
\left(1+n\varphi\theta\right)\left(\sum\nolimits_{i=1,\tau\geq 1}^{\tau}\widehat{\pi}^H_{t+i}\right)+\left(1-n\right)\varphi\theta\left(\sum\nolimits_{i=1,\tau\geq 1}^{\tau}\widehat{\pi}^{H*}_{t+i}\right)+ \\
+\left(1-n\right)\left(\left(1+n\varphi\sigma\right)\widehat{t}_{t+\tau}-\left(1-n\right)\varphi\sigma\widehat{t}^*_{t+\tau}\right)+ \\
+\frac{\gamma}{1-b}\left(\widehat{c}_{t+\tau}-b\widehat{c}_{t+\tau-1}\right)+\varphi\widehat{c}^W_{t+\tau}-\left(1+\varphi\right)\widehat{a}_{t+\tau}
\end{array}\right]\mid\mathcal{F}_t\right\}.
\end{aligned}
\tag{49}
$$

This expression shows compactly that optimal pricing in the domestic market depends on the optimal price set in the foreign market. This is due to the fact that prices at home and abroad determine the aggregate demand the firm faces, and therefore have an impact on domestic labor supply and wages.

Log-Linearization of the Foreign Market FOC. I can re-write the optimal pricing equation in (18) as follows,

$$\mathbb{E}\left\{\sum_{\tau=0}^{\infty}\left(\beta\alpha^{H*}\right)^{\tau}\Xi^{R}_{t,t+\tau}\widetilde{Y}^{d*}_{t,t+\tau}(h)\left[\frac{\widetilde{P}^{*}_{t}(h)}{P^{H*}_{t+\tau}}\frac{S_{t+\tau}P^{H*}_{t+\tau}}{P^{H}_{t+\tau}}-\frac{\theta}{\theta-1}\left(\frac{W_{t+\tau}}{P_{t+\tau}\exp\left(a_{t+\tau}\right)}\right)\frac{P_{t+\tau}}{P^{H}_{t+\tau}}\right]\mid\mathcal{F}_{t}\right\}=0,$$

where

$$\Xi^{R}_{t,t+\tau}\equiv\exp\left(-\left(\xi_{t+\tau}-\xi_{t}\right)\right)\left(\frac{C_{t+\tau}-bC_{t+\tau-1}}{C_{t}-bC_{t-1}}\right)^{-\gamma}\frac{P^{H}_{t+\tau}}{P_{t+\tau}}.$$

Under a symmetric equilibrium it holds true that $L^{s}_{t}=L^{d}_{t}(h)=\frac{Y^{s}_{t}(h)}{\exp(a_{t})}$. The second equality follows from the assumption that technologies are linear in labor. I use this equilibrium condition as well as the domestic labor supply equation in (2) to re-write the pricing equation as,

$$\mathbb{E}\left\{\sum_{\tau=0}^{\infty}\left(\beta\alpha^{H*}\right)^{\tau}\Xi^{R}_{t,t+\tau}\widetilde{Y}^{d*}_{t,t+\tau}(h)\left[\begin{array}{c}\frac{\widetilde{P}^{*}_{t}(h)}{P^{H*}_{t+\tau}}\frac{S_{t+\tau}P^{H*}_{t+\tau}}{P^{H}_{t+\tau}}-\\ -\frac{\theta\kappa}{\theta-1}\left(C_{t+\tau}-bC_{t+\tau-1}\right)^{\gamma}\left(\widetilde{Y}^{s}_{t,t+\tau}(h)\right)^{\varphi}\exp\left(\left(-1-\varphi\right)a_{t+\tau}\right)\frac{P_{t+\tau}}{P^{H}_{t+\tau}}\end{array}\right]\mid\mathcal{F}_{t}\right\}=0,$$

where $\widetilde{Y}^{s}_{t,t+\tau}(h)$ denotes the aggregate supply of firm h at time $t+\tau$ conditional on having their prices fixed since period t. Naturally, the market clearing condition for variety h requires that,

$$\begin{aligned}\widetilde{Y}^{s}_{t,t+\tau}(h) &= n\widetilde{Y}^{d}_{t,t+\tau}(h)+(1-n)\widetilde{Y}^{d*}_{t,t+\tau}(h)\\ &= n\left(\frac{\widetilde{P}_{t}(h)}{P^{H}_{t+\tau}}\right)^{-\theta}\left(\frac{P^{H}_{t+\tau}}{P_{t+\tau}}\right)^{-\sigma}C_{t+\tau}+(1-n)\left(\frac{\widetilde{P}^{*}_{t}(h)}{P^{H*}_{t+\tau}}\right)^{-\theta}\left(\frac{P^{H*}_{t+\tau}}{P^{*}_{t+\tau}}\right)^{-\sigma}C^{*}_{t+\tau},\end{aligned}$$

where the second equality identifies the aggregate demand based on the demand constraints of the firm.

Let me define the following pair of functions,

$$\begin{aligned}F^{H*}_{t+\tau} &\equiv \frac{\widetilde{P}^{*}_{t}(h)}{P^{H*}_{t+\tau}}\frac{S_{t+\tau}P^{H*}_{t+\tau}}{P^{H}_{t+\tau}},\\ G^{H*}_{t+\tau} &\equiv \frac{\theta\kappa}{\theta-1}\left(C_{t+\tau}-bC_{t+\tau-1}\right)^{\gamma}\left(\widetilde{Y}^{s}_{t,t+\tau}(h)\right)^{\varphi}\exp\left(\left(-1-\varphi\right)a_{t+\tau}\right)\frac{P_{t+\tau}}{P^{H}_{t+\tau}},\end{aligned}$$

then the first-order condition for price-setting becomes simply,

$$\mathbb{E}\left\{\sum_{\tau=0}^{\infty}\left(\beta\alpha^{H*}\right)^{\tau}\Xi^{R}_{t,t+\tau}\widetilde{Y}^{d*}_{t,t+\tau}(h)\left[F^{H*}_{t+\tau}-G^{H*}_{t+\tau}\right]\mid\mathcal{F}_{t}\right\}=0.$$

In steady state holds that $\overline{f}^{H*}=\overline{g}^{H*}$, then the log-linearization around the steady state can be expressed as,

$$\mathbb{E}\left\{\sum_{\tau=0}^{\infty}\left(\beta\alpha^{H*}\right)^{\tau}\left[\widehat{f}^{H*}_{t+\tau}-\widehat{g}^{H*}_{t+\tau}\right]\mid\mathcal{F}_{t}\right\}\approx 0,$$

where the approximations are as follows,

$$\begin{aligned}\widehat{f}^{H*}_{t+\tau} &\equiv \widehat{\widetilde{p}}^{*}_{t}(h)-\widehat{p}^{H*}_{t+\tau}+\left(\widehat{s}_{t+\tau}+\widehat{p}^{H*}_{t+\tau}-\widehat{p}^{H}_{t+\tau}\right),\\ \widehat{g}^{H*}_{t+\tau} &\equiv \frac{\gamma}{1-b}\left(\widehat{c}_{t+\tau}-b\widehat{c}_{t+\tau-1}\right)+\varphi\widehat{\widetilde{y}}^{s}_{t,t+\tau}(h)-(1+\varphi)\widehat{a}_{t+\tau}+\left(\widehat{p}_{t+\tau}-\widehat{p}^{H}_{t+\tau}\right),\\ \widehat{\widetilde{y}}^{s}_{t,t+\tau}(h) &= n\left[-\theta\left(\widehat{\widetilde{p}}_{t}(h)-\widehat{p}^{H}_{t+\tau}\right)-\sigma\left(\widehat{p}^{H}_{t+\tau}-\widehat{p}_{t+\tau}\right)+\widehat{c}_{t+\tau}\right]+\\ &\quad+(1-n)\left[-\theta\left(\widehat{\widetilde{p}}^{*}_{t}(h)-\widehat{p}^{H*}_{t+\tau}\right)-\sigma\left(\widehat{p}^{H*}_{t+\tau}-\widehat{p}^{*}_{t+\tau}\right)+\widehat{c}^{*}_{t+\tau}\right].\end{aligned}$$

Using equations (39) − (40) for relative prices and equations (36) − (37) for the price indexes, I can alternatively re-write the optimal pricing equation as,

$$\mathbb{E}\left\{\sum_{\tau=0}^{\infty}\left(\beta\alpha^{H*}\right)^{\tau}\left[\begin{array}{c} n\varphi\theta\left(\widehat{\widetilde{p}}_{t}\left(h\right)-\widehat{p}_{t+\tau}^{H}\right)+\left(1+\left(1-n\right)\varphi\theta\right)\left(\widehat{\widetilde{p}}_{t}^{*}\left(h\right)-\widehat{p}_{t+\tau}^{H*}\right)- \\ -\left(1-n\right)\left(\left(1+n\varphi\sigma\right)\widehat{t}_{t+\tau}-\left(1-n\right)\varphi\sigma\widehat{t}_{t+\tau}^{*}\right)- \\ -\frac{\gamma}{1-b}\left(\widehat{c}_{t+\tau}-b\widehat{c}_{t+\tau-1}\right)-\varphi\widehat{c}_{t+\tau}^{W}+\left(1+\varphi\right)\widehat{a}_{t+\tau}+ \\ +\left(\widehat{s}_{t+\tau}+\widehat{p}_{t+\tau}^{H*}-\widehat{p}_{t+\tau}^{H}\right) \end{array}\right]\mid\mathcal{F}_{t}\right\}\approx 0,$$

where

$$\begin{aligned} \widehat{p}_{t+\tau}^{H}-\widehat{p}_{t+\tau} &= -\left(1-n\right)\widehat{t}_{t+\tau}, \\ \widehat{p}_{t+\tau}^{H*}-\widehat{p}_{t+\tau}^{*} &= \left(1-n\right)\widehat{t}_{t+\tau}^{*}, \end{aligned}$$

and world consumption is $\widehat{c}_{t+\tau}^{W}\equiv n\widehat{c}_{t+\tau}+\left(1-n\right)\widehat{c}_{t+\tau}^{*}$. Notice that I can re-express the price indexes $\widehat{p}_{t+\tau}^{H}$ and $\widehat{p}_{t+\tau}^{H*}$ respectively as $\widehat{p}_{t+\tau}^{H}=\widehat{p}_{t}^{H}+\sum_{i=1}^{\tau}\widehat{\pi}_{t+i}^{H}$ and $\widehat{p}_{t+\tau}^{H*}=\widehat{p}_{t}^{H*}+\sum_{i=1}^{\tau}\widehat{\pi}_{t+i}^{H*}$. Hence, the optimal pricing equation becomes,

$$\begin{aligned} & n\varphi\theta\left(\widehat{\widetilde{p}}_{t}\left(h\right)-\widehat{p}_{t}^{H}\right)+\left(1+\left(1-n\right)\varphi\theta\right)\left(\widehat{\widetilde{p}}_{t}^{*}\left(h\right)-\widehat{p}_{t}^{H*}\right) \\ \approx\ & \left(1-\beta\alpha^{H*}\right)\mathbb{E}\left\{\sum_{\tau=0}^{\infty}\left(\beta\alpha^{H*}\right)^{\tau}\left[\begin{array}{c} n\varphi\theta\left(\sum_{i=1,\tau\geq 1}^{\tau}\widehat{\pi}_{t+i}^{H}\right)+\left(1+\left(1-n\right)\varphi\theta\right)\left(\sum_{i=1,\tau\geq 1}^{\tau}\widehat{\pi}_{t+i}^{H*}\right)+ \\ +\left(1-n\right)\left(\left(1+n\varphi\sigma\right)\widehat{t}_{t+\tau}-\left(1-n\right)\varphi\sigma\widehat{t}_{t+\tau}^{*}\right)+ \\ +\frac{\gamma}{1-b}\left(\widehat{c}_{t+\tau}-b\widehat{c}_{t+\tau-1}\right)+\varphi\widehat{c}_{t+\tau}^{W}-\left(1+\varphi\right)\widehat{a}_{t+\tau}- \\ -\left(\widehat{s}_{t+\tau}+\widehat{p}_{t+\tau}^{H*}-\widehat{p}_{t+\tau}^{H}\right) \end{array}\right]\mid\mathcal{F}_{t}\right\}. \end{aligned} \tag{50}$$

This expression shows compactly that optimal pricing in the foreign market depends also on the optimal price set in the domestic market. This is due to the fact that prices at home and abroad determine the aggregate demand the firm faces, and therefore have an impact on domestic labor supply and wages.

D.4.2 The Optimal Pricing Equation for the Foreign Firm

Log-Linearization of the Home Market FOC. I can re-write the optimal pricing equation in (19) as follows,

$$\mathbb{E}\left\{\sum_{\tau=0}^{\infty}\left(\beta\alpha^{F}\right)^{\tau}\Xi_{t,t+\tau}^{R*}\widetilde{Y}_{t,t+\tau}^{d}\left(f\right)\left[\frac{\widetilde{P}_{t}\left(f\right)}{P_{t+\tau}^{F}}\frac{P_{t+\tau}^{F}}{S_{t+\tau}P_{t+\tau}^{F*}}-\frac{\theta}{\theta-1}\left(\frac{W_{t+\tau}^{*}}{P_{t+\tau}^{*}\exp\left(a_{t+\tau}^{*}\right)}\right)\frac{P_{t+\tau}^{*}}{P_{t+\tau}^{F*}}\right]\mid\mathcal{F}_{t}^{*}\right\}=0,$$

where

$$\Xi_{t,t+\tau}^{R*}\equiv\exp\left(-\left(\xi_{t+\tau}^{*}-\xi_{t}^{*}\right)\right)\left(\frac{C_{t+\tau}^{*}-bC_{t+\tau-1}^{*}}{C_{t}^{*}-bC_{t-1}^{*}}\right)^{-\gamma}\frac{P_{t+\tau}^{F*}}{P_{t+\tau}^{*}}.$$

Under a symmetric equilibrium it holds true that $L_{t}^{s*}=L_{t}^{d*}\left(f\right)=\frac{Y_{t}^{s*}\left(f\right)}{\exp\left(a_{t}^{*}\right)}$. The second equality follows from the assumption that technologies are linear in labor. I use this equilibrium condition as well as the foreign labor supply equation in (5) to re-write the pricing equation as,

$$\mathbb{E}\left\{\sum_{\tau=0}^{\infty}\left(\beta\alpha^{F}\right)^{\tau}\Xi_{t,t+\tau}^{R*}\widetilde{Y}_{t,t+\tau}^{d}\left(f\right)\left[\begin{array}{c} \frac{\widetilde{P}_{t}\left(f\right)}{P_{t+\tau}^{F}}\frac{P_{t+\tau}^{F}}{S_{t+\tau}P_{t+\tau}^{F*}}- \\ -\frac{\theta\kappa}{\theta-1}\left(C_{t+\tau}^{*}-bC_{t+\tau-1}^{*}\right)^{\gamma}\left(\widetilde{Y}_{t,t+\tau}^{s*}\left(f\right)\right)^{\varphi}\exp\left(\left(-1-\varphi\right)a_{t+\tau}^{*}\right)\frac{P_{t+\tau}^{*}}{P_{t+\tau}^{F*}} \end{array}\right]\mid\mathcal{F}_{t}^{*}\right\}=0,$$

where $\widetilde{Y}^{s*}_{t,t+\tau}(f)$ denotes the aggregate supply of firm f at time $t+\tau$ conditional on having their prices fixed since period t. Naturally, the market clearing condition for variety f requires that[5],

$$\begin{aligned}
\widetilde{Y}^{s*}_{t,t+\tau}(f) &= n\widetilde{Y}^{d}_{t,t+\tau}(f) + (1-n)\,\widetilde{Y}^{d*}_{t,t+\tau}(f) \\
&= n\left(\frac{\widetilde{P}_t(f)}{P^F_{t+\tau}}\right)^{-\theta}\left(\frac{P^F_{t+\tau}}{P_{t+\tau}}\right)^{-\sigma} C_{t+\tau} + (1-n)\left(\frac{\widetilde{P}^*_t(f)}{P^{F*}_{t+\tau}}\right)^{-\theta}\left(\frac{P^{F*}_{t+\tau}}{P^*_{t+\tau}}\right)^{-\sigma} C^*_{t+\tau},
\end{aligned}$$

where the second equality identifies the aggregate demand based on the demand constraints of the firm.

Let me define the following pair of functions,

$$\begin{aligned}
F^F_{t+\tau} &\equiv \frac{\widetilde{P}_t(f)}{P^F_{t+\tau}}\frac{P^F_{t+\tau}}{S_{t+\tau}P^{F*}_{t+\tau}}, \\
G^F_{t+\tau} &\equiv \frac{\theta\kappa}{\theta-1}\left(C^*_{t+\tau}-bC^*_{t+\tau-1}\right)^{\gamma}\left(\widetilde{Y}^{s*}_{t,t+\tau}(f)\right)^{\varphi}\exp\left((-1-\varphi)\,a^*_{t+\tau}\right)\frac{P^*_{t+\tau}}{P^{F*}_{t+\tau}},
\end{aligned}$$

then the first-order condition for price-setting becomes simply,

$$\mathbb{E}\left\{\sum\nolimits_{\tau=0}^{\infty}\left(\beta\alpha^F\right)^{\tau}\Xi^{R*}_{t,t+\tau}\widetilde{Y}^{d}_{t,t+\tau}(f)\left[F^F_{t+\tau}-G^F_{t+\tau}\right] \mid \mathcal{F}^*_t\right\} = 0.$$

In steady state holds that $\overline{f}^F = \overline{g}^F$, then the log-linearization around the steady state can be expressed as,

$$\mathbb{E}\left\{\sum\nolimits_{\tau=0}^{\infty}\left(\beta\alpha^F\right)^{\tau}\left[\widehat{f}^F_{t+\tau}-\widehat{g}^F_{t+\tau}\right] \mid \mathcal{F}^*_t\right\} \approx 0,$$

where the approximations are as follows,

$$\begin{aligned}
\widehat{f}^F_{t+\tau} &\equiv \widehat{\widetilde{p}}_t(f) - \widehat{p}^F_{t+\tau} - \left(\widehat{s}_{t+\tau}+\widehat{p}^{F*}_{t+\tau}-\widehat{p}^F_{t+\tau}\right), \\
\widehat{g}^F_{t+\tau} &\equiv \frac{\gamma}{1-b}\left(\widehat{c}^*_{t+\tau}-b\widehat{c}^*_{t+\tau-1}\right)+\varphi\widehat{\widetilde{y}}^{s*}_{t,t+\tau}(f)-(1+\varphi)\,\widehat{a}^*_{t+\tau}+\left(\widehat{p}^*_{t+\tau}-\widehat{p}^{F*}_{t+\tau}\right), \\
\widehat{\widetilde{y}}^{s*}_{t,t+\tau}(f) &= n\left[-\theta\left(\widehat{\widetilde{p}}_t(f)-\widehat{p}^F_{t+\tau}\right)-\sigma\left(\widehat{p}^F_{t+\tau}-\widehat{p}_{t+\tau}\right)+\widehat{c}_{t+\tau}\right]+ \\
&\quad +(1-n)\left[-\theta\left(\widehat{\widetilde{p}}^*_t(f)-\widehat{p}^{F*}_{t+\tau}\right)-\sigma\left(\widehat{p}^{F*}_{t+\tau}-\widehat{p}^*_{t+\tau}\right)+\widehat{c}^*_{t+\tau}\right].
\end{aligned}$$

Using equations (39) – (40) for relative prices and equations (36) – (37) for the price indexes, I can alternatively re-write the optimal pricing equation as,

$$\mathbb{E}\left\{\sum\nolimits_{\tau=0}^{\infty}\left(\beta\alpha^F\right)^{\tau}\left[\begin{array}{c}
(1+n\varphi\theta)\left(\widehat{\widetilde{p}}_t(f)-\widehat{p}^F_{t+\tau}\right)+(1-n)\,\varphi\theta\left(\widehat{\widetilde{p}}^*_t(f)-\widehat{p}^{F*}_{t+\tau}\right)+ \\
+n\left(n\varphi\sigma\widehat{t}_{t+\tau}-(1+(1-n)\,\varphi\sigma)\,\widehat{t}^*_{t+\tau}\right)- \\
-\frac{\gamma}{1-b}\left(\widehat{c}^*_{t+\tau}-b\widehat{c}^*_{t+\tau-1}\right)-\varphi\widehat{c}^W_{t+\tau}+(1+\varphi)\,\widehat{a}^*_{t+\tau}- \\
-\left(\widehat{s}_{t+\tau}+\widehat{p}^{F*}_{t+\tau}-\widehat{p}^F_{t+\tau}\right)
\end{array}\right] \mid \mathcal{F}^*_t\right\} \approx 0,$$

where

$$\begin{aligned}
\widehat{p}^F_{t+\tau}-\widehat{p}_{t+\tau} &= n\widehat{t}_{t+\tau}, \\
\widehat{p}^{F*}_{t+\tau}-\widehat{p}^*_{t+\tau} &= -n\widehat{t}^*_{t+\tau},
\end{aligned}$$

[5] *Sensu stricto* G. Benigno's (2004) framework cannot be viewed as a pure double-signal Calvo model (where each firm receives one signal to re-optimize in the domestic market and another signal in the foreign market). Benigno discounts the fact that firms may receive a signal to re-optimize in one market but not the other, so he approximates the symmetric equilibrium with the behavior of a representative firm that re-optimizes in both markets simultaneously. For tractability and consistency, I use the same approach here. That explains the characterization of the market clearing condition.

and world consumption is $\widehat{c}^{W}_{t+\tau} \equiv n\widehat{c}_{t+\tau} + (1-n)\,\widehat{c}^{*}_{t+\tau}$. Notice that I can re-express the price indexes $\widehat{p}^{F}_{t+\tau}$ and $\widehat{p}^{F*}_{t+\tau}$ respectively as $\widehat{p}^{F}_{t+\tau} = \widehat{p}^{F}_{t} + \sum_{i=1}^{\tau} \widehat{\pi}^{F}_{t+i}$ and $\widehat{p}^{F*}_{t+\tau} = \widehat{p}^{F*}_{t} + \sum_{i=1}^{\tau} \widehat{\pi}^{F*}_{t+i}$. Hence, the optimal pricing equation becomes,

$$
\begin{aligned}
&(1+n\varphi\theta)\left(\widehat{\widetilde{p}}_{t}(f) - \widehat{p}^{F}_{t}\right) + (1-n)\,\varphi\theta\left(\widehat{\widetilde{p}}^{*}_{t}(f) - \widehat{p}^{F*}_{t}\right) \\
\approx\quad &\left(1-\beta\alpha^{F}\right)\mathbb{E}\left\{\sum_{\tau=0}^{\infty}\left(\beta\alpha^{F}\right)^{\tau}\left[\begin{array}{c}(1+n\varphi\theta)\left(\sum_{i=1,\tau\geq 1}^{\tau}\widehat{\pi}^{F}_{t+i}\right) + (1-n)\,\varphi\theta\left(\sum_{i=1,\tau\geq 1}^{\tau}\widehat{\pi}^{F*}_{t+i}\right) - \\ -n\left(n\varphi\sigma\widehat{t}_{t+\tau} - (1+(1-n)\,\varphi\sigma)\,\widehat{t}^{*}_{t+\tau}\right) + \\ +\frac{\gamma}{1-b}\left(\widehat{c}^{*}_{t+\tau} - b\widehat{c}^{*}_{t+\tau-1}\right) + \varphi\widehat{c}^{W}_{t+\tau} - (1+\varphi)\,\widehat{a}^{*}_{t+\tau} + \\ +\left(\widehat{s}_{t+\tau} + \widehat{p}^{F*}_{t+\tau} - \widehat{p}^{F}_{t+\tau}\right)\end{array}\right] \mid \mathcal{F}^{*}_{t}\right\}.
\end{aligned}
\tag{51}
$$

This expression shows compactly that optimal pricing in the domestic market depends also on the optimal price set in the foreign market. This is due to the fact that prices at home and abroad determine the aggregate demand the firm faces, and therefore have an impact on foreign labor supply and wages.

Log-Linearization of the Foreign Market FOC. I can re-write the optimal pricing equation in (20) as follows,

$$
\mathbb{E}\left\{\sum_{\tau=0}^{\infty}\left(\beta\alpha^{F*}\right)^{\tau}\Xi^{R*}_{t,t+\tau}\widetilde{Y}^{d*}_{t,t+\tau}(f)\left[\frac{\widetilde{P}^{*}_{t}(f)}{P^{F*}_{t+\tau}} - \frac{\theta}{\theta-1}\left(\frac{W^{*}_{t+\tau}}{P^{*}_{t+\tau}\exp\left(a^{*}_{t+\tau}\right)}\right)\frac{P^{*}_{t+\tau}}{P^{F*}_{t+\tau}}\right] \mid \mathcal{F}^{*}_{t}\right\} = 0,
$$

where

$$
\Xi^{R*}_{t,t+\tau} \equiv \exp\left(-\left(\xi^{*}_{t+\tau} - \xi^{*}_{t}\right)\right)\left(\frac{C^{*}_{t+\tau} - bC^{*}_{t+\tau-1}}{C^{*}_{t} - bC^{*}_{t-1}}\right)^{-\gamma}\frac{P^{F*}_{t+\tau}}{P^{*}_{t+\tau}}.
$$

Under a symmetric equilibrium it holds true that $L^{s*}_{t} = L^{d*}_{t}(f) = \frac{Y^{s*}_{t}(f)}{\exp(a^{*}_{t})}$. The second equality follows from the assumption that technologies are linear in labor. I use this equilibrium condition as well as the foreign labor supply equation in (5) to re-write the pricing equation as,

$$
\mathbb{E}\left\{\sum_{\tau=0}^{\infty}\left(\beta\alpha^{F*}\right)^{\tau}\Xi^{R*}_{t,t+\tau}\widetilde{Y}^{d*}_{t,t+\tau}(f)\left[\begin{array}{c}\frac{\widetilde{P}^{*}_{t}(f)}{P^{F*}_{t+\tau}} - \\ -\frac{\theta\kappa}{\theta-1}\left(C^{*}_{t+\tau} - bC^{*}_{t+\tau-1}\right)^{\gamma}\left(\widetilde{Y}^{s*}_{t,t+\tau}(f)\right)^{\varphi}\exp\left((-1-\varphi)\,a^{*}_{t+\tau}\right)\frac{P^{*}_{t+\tau}}{P^{F*}_{t+\tau}}\end{array}\right] \mid \mathcal{F}^{*}_{t}\right\} = 0,
$$

where $\widetilde{Y}^{s*}_{t,t+\tau}(f)$ denotes the aggregate supply of firm f at time $t+\tau$ conditional on having their prices fixed since period t. Naturally, the market clearing condition for variety f requires that,

$$
\begin{aligned}
\widetilde{Y}^{s*}_{t,t+\tau}(f) &= n\widetilde{Y}^{d}_{t,t+\tau}(f) + (1-n)\,\widetilde{Y}^{d*}_{t,t+\tau}(f) \\
&= n\left(\frac{\widetilde{P}_{t}(f)}{P^{F}_{t+\tau}}\right)^{-\theta}\left(\frac{P^{F}_{t+\tau}}{P_{t+\tau}}\right)^{-\sigma}C_{t+\tau} + (1-n)\left(\frac{\widetilde{P}^{*}_{t}(f)}{P^{F*}_{t+\tau}}\right)^{-\theta}\left(\frac{P^{F*}_{t+\tau}}{P^{*}_{t+\tau}}\right)^{-\sigma}C^{*}_{t+\tau},
\end{aligned}
$$

where the second equality identifies the aggregate demand based on the demand constraints of the firm.

Let me define the following pair of functions,

$$
\begin{aligned}
F^{F*}_{t+\tau} &\equiv \frac{\widetilde{P}^{*}_{t}(f)}{P^{F*}_{t+\tau}}, \\
G^{F*}_{t+\tau} &\equiv \frac{\theta\kappa}{\theta-1}\left(C^{*}_{t+\tau} - bC^{*}_{t+\tau-1}\right)^{\gamma}\left(\widetilde{Y}^{s*}_{t,t+\tau}(f)\right)^{\varphi}\exp\left((-1-\varphi)\,a^{*}_{t+\tau}\right)\frac{P^{*}_{t+\tau}}{P^{F*}_{t+\tau}},
\end{aligned}
$$

then the first-order condition becomes simply,

$$\mathbb{E}\left\{\sum_{\tau=0}^{\infty}\left(\beta\alpha^{F*}\right)^{\tau}\Xi_{t,t+\tau}^{R*}\widetilde{Y}_{t,t+\tau}^{d*}\left(f\right)\left[F_{t+\tau}^{F*}-G_{t+\tau}^{F*}\right]\mid\mathcal{F}_{t}^{*}\right\}=0.$$

In steady state holds that $\overline{f}^{F*}=\overline{g}^{F*}$, then the log-linearization around the steady state can be expressed as,

$$\mathbb{E}\left\{\sum_{\tau=0}^{\infty}\left(\beta\alpha^{F*}\right)^{\tau}\left[\widehat{f}_{t+\tau}^{F*}-\widehat{g}_{t+\tau}^{F*}\right]\mid\mathcal{F}_{t}^{*}\right\}\approx 0,$$

where the approximations are as follows,

$$\begin{aligned}
\widehat{f}_{t+\tau}^{F*} &\equiv \widehat{\widetilde{p}}_{t}^{*}\left(f\right)-\widehat{p}_{t+\tau}^{F*},\\
\widehat{g}_{t+\tau}^{F*} &\equiv \frac{\gamma}{1-b}\left(\widehat{c}_{t+\tau}^{*}-b\widehat{c}_{t+\tau-1}^{*}\right)+\varphi\widehat{\widetilde{y}}_{t,t+\tau}^{s*}\left(f\right)-\left(1+\varphi\right)\widehat{a}_{t+\tau}^{*}+\left(\widehat{p}_{t+\tau}^{*}-\widehat{p}_{t+\tau}^{F*}\right),\\
\widehat{\widetilde{y}}_{t,t+\tau}^{s*}\left(f\right) &= n\left[-\theta\left(\widehat{\widetilde{p}}_{t}\left(f\right)-\widehat{p}_{t+\tau}^{F}\right)-\sigma\left(\widehat{p}_{t+\tau}^{F}-\widehat{p}_{t+\tau}\right)+\widehat{c}_{t+\tau}\right]+\\
&\quad+\left(1-n\right)\left[-\theta\left(\widehat{\widetilde{p}}_{t}^{*}\left(f\right)-\widehat{p}_{t+\tau}^{F*}\right)-\sigma\left(\widehat{p}_{t+\tau}^{F*}-\widehat{p}_{t+\tau}^{*}\right)+\widehat{c}_{t+\tau}^{*}\right].
\end{aligned}$$

Using equations (39) − (40) for relative prices and equations (36) − (37) for the price indexes, I can alternatively re-write the optimal pricing equation as,

$$\mathbb{E}\left\{\sum_{\tau=0}^{\infty}\left(\beta\alpha^{F*}\right)^{\tau}\left[\begin{array}{c} n\varphi\theta\left(\widehat{\widetilde{p}}_{t}\left(f\right)-\widehat{p}_{t+\tau}^{F}\right)+\left(1+\left(1-n\right)\varphi\theta\right)\left(\widehat{\widetilde{p}}_{t}^{*}\left(f\right)-\widehat{p}_{t+\tau}^{F*}\right)+\\ +n\left(n\varphi\sigma\widehat{t}_{t+\tau}-\left(1+\left(1-n\right)\varphi\sigma\right)\widehat{t}_{t+\tau}^{*}\right)-\\ -\frac{\gamma}{1-b}\left(\widehat{c}_{t+\tau}^{*}-b\widehat{c}_{t+\tau-1}^{*}\right)-\varphi\widehat{c}_{t+\tau}^{W}+\left(1+\varphi\right)\widehat{a}_{t+\tau}^{*}\end{array}\right]\mid\mathcal{F}_{t}^{*}\right\}\approx 0,$$

where

$$\begin{aligned}
\widehat{p}_{t+\tau}^{F}-\widehat{p}_{t+\tau} &= n\widehat{t}_{t+\tau},\\
\widehat{p}_{t+\tau}^{F*}-\widehat{p}_{t+\tau}^{*} &= -n\widehat{t}_{t+\tau}^{*},
\end{aligned}$$

and world consumption is $\widehat{c}_{t+\tau}^{W}\equiv n\widehat{c}_{t+\tau}+\left(1-n\right)\widehat{c}_{t+\tau}^{*}$. Notice that I can re-express the price indexes $\widehat{p}_{t+\tau}^{F}$ and $\widehat{p}_{t+\tau}^{F*}$ respectively as $\widehat{p}_{t+\tau}^{F}=\widehat{p}_{t}^{F}+\sum_{i=1}^{\tau}\widehat{\pi}_{t+i}^{F}$ and $\widehat{p}_{t+\tau}^{F*}=\widehat{p}_{t}^{F*}+\sum_{i=1}^{\tau}\widehat{\pi}_{t+i}^{F*}$. Hence, the optimal pricing equation becomes,

$$\begin{aligned}
& n\varphi\theta\left(\widehat{\widetilde{p}}_{t}\left(f\right)-\widehat{p}_{t}^{F}\right)+\left(1+\left(1-n\right)\varphi\theta\right)\left(\widehat{\widetilde{p}}_{t}^{*}\left(f\right)-\widehat{p}_{t}^{F*}\right) \qquad (52)\\
\approx\ & \left(1-\beta\alpha^{F*}\right)\mathbb{E}\left\{\sum_{\tau=0}^{\infty}\left(\beta\alpha^{F*}\right)^{\tau}\left[\begin{array}{c} n\varphi\theta\left(\sum_{i=1,\tau\geq 1}^{\tau}\widehat{\pi}_{t+i}^{F}\right)+\left(1+\left(1-n\right)\varphi\theta\right)\left(\sum_{i=1,\tau\geq 1}^{\tau}\widehat{\pi}_{t+i}^{F*}\right)-\\ -n\left(n\varphi\sigma\widehat{t}_{t+\tau}-\left(1+\left(1-n\right)\varphi\sigma\right)\widehat{t}_{t+\tau}^{*}\right)+\\ +\frac{\gamma}{1-b}\left(\widehat{c}_{t+\tau}^{*}-b\widehat{c}_{t+\tau-1}^{*}\right)+\varphi\widehat{c}_{t+\tau}^{W}-\left(1+\varphi\right)\widehat{a}_{t+\tau}^{*}\end{array}\right]\mid\mathcal{F}_{t}^{*}\right\}.
\end{aligned}$$

This expression shows compactly that optimal pricing in the foreign market depends also on the optimal price set in the domestic market. This is due to the fact that prices at home and abroad determine the aggregate demand the firm faces, and therefore have an impact on foreign labor supply and wages.

D.4.3 The Price Index Equations

The Price Index in the Home Market. There exists a relationship between the price indexes $\left(P_{t}^{H},P_{t}^{H*}\right)$ and the (symmetric) optimal pricing rule $\left(\widetilde{P}_{t}\left(h\right),\widetilde{P}_{t}^{*}\left(h\right)\right)$. In a symmetric equilibrium under sticky prices, the aggregate domestic-good price indexes in equations (12) and (13) can be expressed as follows[6],

$$P_t^H = \left[\alpha^H \left(P_{t-1}^H\right)^{1-\theta} + \left(1-\alpha^H\right)\left(\widetilde{P}_t(h)\right)^{1-\theta}\right]^{\frac{1}{1-\theta}},$$

$$P_t^{H*} = \left[\alpha^{H*} \left(P_{t-1}^{H*}\right)^{1-\theta} + \left(1-\alpha^{H*}\right)\left(\widetilde{P}_t^*(h)\right)^{1-\theta}\right]^{\frac{1}{1-\theta}}.$$

A proportion $\left(1-\alpha^H\right)$ of all domestic firms set the (symmetric) optimal price in the domestic market, $\widetilde{P}_t(h)$, after receiving a signal to re-optimize at time t. The remaining firms, in proportion of α^H, behave as if on average they had kept the previous period price unchanged. The lagged term reflects the aggregate behavior of all firms who cannot re-set prices in the domestic market. Similarly, a proportion $\left(1-\alpha^{H*}\right)$ of all domestic firms set the (symmetric) optimal price in the foreign market, $\widetilde{P}_t^*(h)$, after receiving a signal to re-optimize at time t; while a proportion α^{H*} leaves prices unchanged from the previous period.

In a zero-inflation, zero-current account steady state, it must hold that $\overline{P}^H = \overline{\widetilde{P}}(h)$ and $\overline{P}^{H*} = \overline{\widetilde{P}}^*(h)$. Hence, the log-linear approximation becomes simply,

$$\widehat{p}_t^H \approx \alpha^H \widehat{p}_{t-1}^H + \left(1-\alpha^H\right)\widehat{\widetilde{p}}_t(h),$$

$$\widehat{p}_t^{H*} \approx \alpha^{H*} \widehat{p}_{t-1}^{H*} + \left(1-\alpha^{H*}\right)\widehat{\widetilde{p}}_t^*(h).$$

A straightforward manipulation of these equations tell me that the domestic and foreign inflation rates on the bundle of domestic goods (i.e., $\widehat{\pi}_t^H = \widehat{p}_t^H - \widehat{p}_{t-1}^H$ and $\widehat{\pi}_t^{H*} = \widehat{p}_t^{H*} - \widehat{p}_{t-1}^{H*}$) could be approximately computed as,

$$\widehat{\pi}_t^H \approx \left(1-\alpha^H\right)\left[\widehat{\widetilde{p}}_t(h) - \widehat{p}_{t-1}^H\right],\ \widehat{\pi}_t^{H*} \approx \left(1-\alpha^{H*}\right)\left[\widehat{\widetilde{p}}_t^*(h) - \widehat{p}_{t-1}^{H*}\right],$$

which implies that the difference between the optimal pricing rules, $\widehat{\widetilde{p}}_t(h)$ and $\widehat{\widetilde{p}}_t^*(h)$, and the price indexes, $\widehat{p}_t^H$ and $\widehat{p}_t^{H*}$, is proportional to the inflation rate in logs, i.e.

$$\left[\widehat{\widetilde{p}}_t(h) - \widehat{p}_t^H\right] \approx \left(\frac{\alpha^H}{1-\alpha^H}\right)\widehat{\pi}_t^H,\ \left[\widehat{\widetilde{p}}_t^*(h) - \widehat{p}_t^{H*}\right] \approx \left(\frac{\alpha^{H*}}{1-\alpha^{H*}}\right)\widehat{\pi}_t^{H*}. \tag{53}$$

The Price Index in the Foreign Market. There exists a relationship between the price indexes $\left(P_t^F, P_t^{F*}\right)$ and the (symmetric) optimal pricing rule $\left(\widetilde{P}_t(f), \widetilde{P}_t^*(f)\right)$. In a symmetric equilibrium under sticky prices, the aggregate foreign-good price indexes in equations (15) and (16) can be expressed as follows[7],

$$P_t^F = \left[\alpha^F \left(P_{t-1}^F\right)^{1-\theta} + \left(1-\alpha^F\right)\left(\widetilde{P}_t(f)\right)^{1-\theta}\right]^{\frac{1}{1-\theta}},$$

$$P_t^{F*} = \left[\alpha^{F*} \left(P_{t-1}^{F*}\right)^{1-\theta} + \left(1-\alpha^{F*}\right)\left(\widetilde{P}_t^*(f)\right)^{1-\theta}\right]^{\frac{1}{1-\theta}}.$$

[6] As noted before, G. Benigno (2004) discounts the fact that firms may receive a signal to re-optimize in one market but not the other, so he approximates the symmetric equilibrium with the behavior of a representative firm that re-optimizes in both markets simultaneously. I use the same approach here. That explains why I do not distinguish between firms that re-optimize in one market only and firms that re-optimize in both markets.

[7] As noted before, G. Benigno (2004) discounts the fact that firms may receive a signal to re-optimize in one market but not the other, so he approximates the symmetric equilibrium with the behavior of a representative firm that re-optimizes in both markets simultaneously. I use the same approach here. That explains why I do not distinguish between firms that re-optimize in one market only and firms that re-optimize in both markets.

A proportion $\left(1-\alpha^{F}\right)$ of all foreign firms set the (symmetric) optimal price in the domestic market, $\widetilde{P}_t(f)$, after receiving a signal to re-optimize at time t. The remaining firms, in proportion of α^{F}, behave as if on average they had kept the previous period price unchanged. Similarly, a proportion $\left(1-\alpha^{F*}\right)$ of all foreign firms set the (symmetric) optimal price in the foreign market, $\widetilde{P}^{*}_t(f)$, after receiving a signal to re-optimize at time t; while a proportion α^{F*} leaves prices unchanged from the previous period.

In a zero-inflation, zero-current account steady state, it must hold that $\overline{P}^{F}=\overline{\widetilde{P}}(f)$ and $\overline{P}^{F*}=\overline{\widetilde{P}}^{*}(f)$. Hence, the log-linear approximation becomes simply,

$$\begin{aligned}
\widehat{p}^{F}_t &\approx \alpha^{F}\widehat{p}^{F}_{t-1}+\left(1-\alpha^{F}\right)\widehat{\widetilde{p}}_t(f),\\
\widehat{p}^{F*}_t &\approx \alpha^{F*}\widehat{p}^{F*}_{t-1}+\left(1-\alpha^{F*}\right)\widehat{\widetilde{p}}^{*}_t(f).
\end{aligned}$$

A straightforward manipulation of these equations tell me that the domestic and foreign inflation rates on the bundle of foreign goods (i.e., $\widehat{\pi}^{F}_t=\widehat{p}^{F}_t-\widehat{p}^{F}_{t-1}$ and $\widehat{\pi}^{F*}_t=\widehat{p}^{F*}_t-\widehat{p}^{F*}_{t-1}$) could be approximately computed as,

$$\widehat{\pi}^{F}_t\approx\left(1-\alpha^{F}\right)\left[\widehat{\widetilde{p}}_t(f)-\widehat{p}^{F}_{t-1}\right],\ \widehat{\pi}^{F*}_t\approx\left(1-\alpha^{F*}\right)\left[\widehat{\widetilde{p}}^{*}_t(f)-\widehat{p}^{F*}_{t-1}\right],$$

which implies that the difference between the optimal pricing rule, $\widehat{\widetilde{p}}_t(f)$ and $\widehat{\widetilde{p}}^{*}_t(f)$, and the price indexes, $\widehat{p}^{F}_t$ and $\widehat{p}^{F*}_t$, is proportional to the inflation rate in logs, i.e.

$$\left[\widehat{\widetilde{p}}_t(f)-\widehat{p}^{F}_t\right]\approx\left(\frac{\alpha^{F}}{1-\alpha^{F}}\right)\widehat{\pi}^{F}_t,\ \left[\widehat{\widetilde{p}}^{*}_t(f)-\widehat{p}^{F*}_t\right]\approx\left(\frac{\alpha^{F*}}{1-\alpha^{F*}}\right)\widehat{\pi}^{F*}_t. \tag{54}$$

D.4.4 The Aggregate-Supply Equations for the Domestic Firm: AS^{H} and AS^{H*}

In order to derive the dynamics of the inflation rates for the domestic bundle of goods, i.e. $\widehat{\pi}^{H}_t$ and $\widehat{\pi}^{H*}_t$, I need to manipulate further the linearized first-order conditions of the firm in (49) and (50). The solution involves some algebra, but it is otherwise conceptually straightforward and can be summarized in three steps. First, I use the result in equation (53) to replace the optimal prices $\widehat{\widetilde{p}}_t(h)$ and $\widehat{\widetilde{p}}^{*}_t(h)$, i.e.

$$\begin{aligned}
&(1+n\varphi\theta)\left(\tfrac{\alpha^{H}}{1-\alpha^{H}}\right)\widehat{\pi}^{H}_t+(1-n)\varphi\theta\left(\tfrac{\alpha^{H*}}{1-\alpha^{H*}}\right)\widehat{\pi}^{H*}_t\\
&\approx\left(1-\beta\alpha^{H}\right)\mathbb{E}\left\{\sum\nolimits_{\tau=0}^{\infty}\left(\beta\alpha^{H}\right)^{\tau}\left[\begin{array}{c}(1+n\varphi\theta)\left(\sum\nolimits_{i=1,\tau\geq1}^{\tau}\widehat{\pi}^{H}_{t+i}\right)+(1-n)\varphi\theta\left(\sum\nolimits_{i=1,\tau\geq1}^{\tau}\widehat{\pi}^{H*}_{t+i}\right)+\\+(1-n)\left((1+n\varphi\sigma)\widehat{t}_{t+\tau}-(1-n)\varphi\sigma\widehat{t}^{*}_{t+\tau}\right)+\\+\tfrac{\gamma}{1-b}\left(\widehat{c}_{t+\tau}-b\widehat{c}_{t+\tau-1}\right)+\varphi\widehat{c}^{W}_{t+\tau}-(1+\varphi)\widehat{a}_{t+\tau}\end{array}\right]\mid\mathcal{F}_t\right\},
\end{aligned}$$

and,

$$\begin{aligned}
&n\varphi\theta\left(\tfrac{\alpha^{H}}{1-\alpha^{H}}\right)\widehat{\pi}^{H}_t+(1+(1-n)\varphi\theta)\left(\tfrac{\alpha^{H*}}{1-\alpha^{H*}}\right)\widehat{\pi}^{H*}_t\\
&\approx\left(1-\beta\alpha^{H*}\right)\mathbb{E}\left\{\sum\nolimits_{\tau=0}^{\infty}\left(\beta\alpha^{H*}\right)^{\tau}\left[\begin{array}{c}n\varphi\theta\left(\sum\nolimits_{i=1,\tau\geq1}^{\tau}\widehat{\pi}^{H}_{t+i}\right)+(1+(1-n)\varphi\theta)\left(\sum\nolimits_{i=1,\tau\geq1}^{\tau}\widehat{\pi}^{H*}_{t+i}\right)+\\+(1-n)\left((1+n\varphi\sigma)\widehat{t}_{t+\tau}-(1-n)\varphi\sigma\widehat{t}^{*}_{t+\tau}\right)+\\+\tfrac{\gamma}{1-b}\left(\widehat{c}_{t+\tau}-b\widehat{c}_{t+\tau-1}\right)+\varphi\widehat{c}^{W}_{t+\tau}-(1+\varphi)\widehat{a}_{t+\tau}-\left(\widehat{s}_{t+\tau}+\widehat{p}^{H*}_{t+\tau}-\widehat{p}^{H}_{t+\tau}\right)\end{array}\right]\mid\mathcal{F}_t\right\}.
\end{aligned}$$

Second, I realize that these expressions take the form of a discounted present-value system that can be re-written as the forward-looking (no-bubble) solution of a pair of expectational difference equations. In other words, I can say that the equations of interest are,

$$
\begin{aligned}
&(1+n\varphi\theta)\left(\tfrac{\alpha^{H}}{1-\alpha^{H}}\right)\widehat{\pi}_{t}^{H}+(1-n)\,\varphi\theta\left(\tfrac{\alpha^{H*}}{1-\alpha^{H*}}\right)\widehat{\pi}_{t}^{H*}\\
&\approx\left(1-\beta\alpha^{H}\right)\mathbb{E}\left\{\left[\begin{array}{c}(1-n)\left((1+n\varphi\sigma)\,\widehat{t}_{t}-(1-n)\,\varphi\sigma\widehat{t}_{t}^{*}\right)+\\+\tfrac{\gamma}{1-b}\left(\widehat{c}_{t}-b\widehat{c}_{t-1}\right)+\varphi\widehat{c}_{t}^{W}-(1+\varphi)\,\widehat{a}_{t}\end{array}\right]\mid\mathcal{F}_{t}\right\}+\\
&+\beta\alpha^{H}\mathbb{E}\left\{\left(1-\beta\alpha^{H}\right)\sum\nolimits_{\tau=0}^{\infty}\left(\beta\alpha^{H}\right)^{\tau}\left[\begin{array}{c}(1+n\varphi\theta)\left(\widehat{\pi}_{t+1}^{H}+\sum\nolimits_{i=1,\tau\geq1}^{\tau}\widehat{\pi}_{t+1+i}^{H}\right)+\\+(1-n)\,\varphi\theta\left(\widehat{\pi}_{t+1}^{H*}+\sum\nolimits_{i=1,\tau\geq1}^{\tau}\widehat{\pi}_{t+1+i}^{H*}\right)+\\+(1-n)\left((1+n\varphi\sigma)\,\widehat{t}_{t+1+\tau}-(1-n)\,\varphi\sigma\widehat{t}_{t+1+\tau}^{*}\right)+\\+\tfrac{\gamma}{1-b}\left(\widehat{c}_{t+1+\tau}-b\widehat{c}_{t+\tau}\right)+\varphi\widehat{c}_{t+1+\tau}^{W}-(1+\varphi)\,\widehat{a}_{t+1+\tau}\end{array}\right]\mid\mathcal{F}_{t}\right\},
\end{aligned}
$$

and,

$$
\begin{aligned}
&(1+n\varphi\theta)\left(\tfrac{\alpha^{H}}{1-\alpha^{H}}\right)\widehat{\pi}_{t}^{H}+(1-n)\,\varphi\theta\left(\tfrac{\alpha^{H*}}{1-\alpha^{H*}}\right)\widehat{\pi}_{t}^{H*}\\
&\approx\left(1-\beta\alpha^{H}\right)\mathbb{E}\left\{\left[\begin{array}{c}(1-n)\left((1+n\varphi\sigma)\,\widehat{t}_{t}-(1-n)\,\varphi\sigma\widehat{t}_{t}^{*}\right)+\\+\tfrac{\gamma}{1-b}\left(\widehat{c}_{t}-b\widehat{c}_{t-1}\right)+\varphi\widehat{c}_{t}^{W}-(1+\varphi)\,\widehat{a}_{t}\end{array}\right]\mid\mathcal{F}_{t}\right\}+\\
&+\beta\alpha^{H}\mathbb{E}\left\{\left[\begin{array}{l}(1+n\varphi\theta)\,\widehat{\pi}_{t+1}^{H}+(1-n)\,\varphi\theta\widehat{\pi}_{t+1}^{H*}+\\+\mathbb{E}\left\{\sum\nolimits_{\tau=0}^{\infty}\frac{\left(\beta\alpha^{H}\right)^{\tau}}{\left(1-\beta\alpha^{H}\right)^{-1}}\left[\begin{array}{c}(1+n\varphi\theta)\left(\sum\nolimits_{i=1,\tau\geq1}^{\tau}\widehat{\pi}_{t+1+i}^{H}\right)+\\+(1-n)\,\varphi\theta\left(\sum\nolimits_{i=1,\tau\geq1}^{\tau}\widehat{\pi}_{t+1+i}^{H*}\right)+\\+(1-n)\left((1+n\varphi\sigma)\,\widehat{t}_{t+1+\tau}-(1-n)\,\varphi\sigma\widehat{t}_{t+1+\tau}^{*}\right)+\\+\tfrac{\gamma}{1-b}\left(\widehat{c}_{t+1+\tau}-b\widehat{c}_{t+\tau}\right)+\varphi\widehat{c}_{t+1+\tau}^{W}-(1+\varphi)\,\widehat{a}_{t+1+\tau}\end{array}\right]\mid\mathcal{F}_{t+1}\right\}\end{array}\right]\mid\mathcal{F}_{t}\right\},
\end{aligned}
$$

where I used the law of iterated expectations. Therefore, I obtain the following transformation of (49),

$$
\begin{aligned}
&(1+n\varphi\theta)\left(\frac{\alpha^{H}}{1-\alpha^{H}}\right)\widehat{\pi}_{t}^{H}+(1-n)\,\varphi\theta\left(\frac{\alpha^{H*}}{1-\alpha^{H*}}\right)\widehat{\pi}_{t}^{H*}\\
\approx\ &\left(1-\beta\alpha^{H}\right)\mathbb{E}\left\{\left[\begin{array}{c}(1-n)\left((1+n\varphi\sigma)\,\widehat{t}_{t}-(1-n)\,\varphi\sigma\widehat{t}_{t}^{*}\right)+\\+\tfrac{\gamma}{1-b}\left(\widehat{c}_{t}-b\widehat{c}_{t-1}\right)+\varphi\widehat{c}_{t}^{W}-(1+\varphi)\,\widehat{a}_{t}\end{array}\right]\mid\mathcal{F}_{t}\right\}+\\
&+\beta\alpha^{H}\mathbb{E}\left\{(1+n\varphi\theta)\left(\frac{1}{1-\alpha^{H}}\right)\widehat{\pi}_{t+1}^{H}+(1-n)\,\varphi\theta\left(\frac{1}{1-\alpha^{H*}}\right)\widehat{\pi}_{t+1}^{H*}\mid\mathcal{F}_{t}\right\}.
\end{aligned}
$$

Analogously, I derive the following expression for the other linearized first-order condition in (50),

$$
\begin{aligned}
&n\varphi\theta\left(\frac{\alpha^{H}}{1-\alpha^{H}}\right)\widehat{\pi}_{t}^{H}+(1+(1-n)\,\varphi\theta)\left(\frac{\alpha^{H*}}{1-\alpha^{H*}}\right)\widehat{\pi}_{t}^{H*}\\
\approx\ &\left(1-\beta\alpha^{H*}\right)\mathbb{E}\left\{\left[\begin{array}{c}(1-n)\left((1+n\varphi\sigma)\,\widehat{t}_{t}-(1-n)\,\varphi\sigma\widehat{t}_{t}^{*}\right)+\\+\tfrac{\gamma}{1-b}\left(\widehat{c}_{t}-b\widehat{c}_{t-1}\right)+\varphi\widehat{c}_{t}^{W}-(1+\varphi)\,\widehat{a}_{t}-\left(\widehat{s}_{t}+\widehat{p}_{t}^{H*}-\widehat{p}_{t}^{H}\right)\end{array}\right]\mid\mathcal{F}_{t}\right\}+\\
&+\beta\alpha^{H*}\mathbb{E}\left\{n\varphi\theta\left(\frac{1}{1-\alpha^{H}}\right)\widehat{\pi}_{t+1}^{H}+(1+(1-n)\,\varphi\theta)\left(\frac{1}{1-\alpha^{H*}}\right)\widehat{\pi}_{t+1}^{H*}\mid\mathcal{F}_{t}\right\}.
\end{aligned}
$$

Based on the definition of relative prices, the consumer price indexes and the real exchange rate in (36) − (40), I can derive the following relationship,

$$
\begin{aligned}
\widehat{s}_{t}+\widehat{p}_{t}^{H*}-\widehat{p}_{t}^{H} &= \widehat{rs}_{t}+\left(\widehat{p}_{t}^{H*}-\widehat{p}_{t}^{*}\right)-\left(\widehat{p}_{t}^{H}-\widehat{p}_{t}\right)\\
&\approx \widehat{rs}_{t}+(1-n)\left(\widehat{p}_{t}^{H*}-\widehat{p}_{t}^{F*}\right)-(1-n)\left(\widehat{p}_{t}^{H}-\widehat{p}_{t}^{F}\right)\\
&= \widehat{rs}_{t}+(1-n)\left(\widehat{t}_{t}^{*}+\widehat{t}_{t}\right),
\end{aligned}
$$

where the discrepancy on relative prices across countries is denoted $\widehat{t}_{t}^{R}\equiv\widehat{t}_{t}+\widehat{t}_{t}^{*}$.

Finally, I notice that the expressions I have derived so far are all functions of the expected inflation rate of the domestic bundle of goods in the home as well as the foreign market. That is,

$$
\begin{aligned}
& \left(1+n\varphi\theta\right)\left(\frac{\alpha^{H}}{1-\alpha^{H}}\right)\left[\widehat{\pi}_{t}^{H}-\beta\mathbb{E}\left\{\widehat{\pi}_{t+1}^{H}\mid\mathcal{F}_{t}\right\}\right] \\
\approx\ & \left(1-\beta\alpha^{H}\right)\mathbb{E}\left\{\left[\begin{array}{c}\left(1-n\right)\left(\left(1+n\varphi\sigma\right)\widehat{t}_{t}-\left(1-n\right)\varphi\sigma\widehat{t}_{t}^{*}\right)+\\ +\frac{\gamma}{1-b}\left(\widehat{c}_{t}-b\widehat{c}_{t-1}\right)+\varphi\widehat{c}_{t}^{W}-\left(1+\varphi\right)\widehat{a}_{t}\end{array}\right]\mid\mathcal{F}_{t}\right\}- \\
& -\left(1-n\right)\varphi\theta\left(\frac{\alpha^{H*}-\alpha^{H}}{1-\alpha^{H*}}\right)\widehat{\pi}_{t}^{H*}-\left(1-n\right)\varphi\theta\left(\frac{\alpha^{H}}{1-\alpha^{H*}}\right)\left[\widehat{\pi}_{t}^{H*}-\beta\mathbb{E}\left\{\widehat{\pi}_{t+1}^{H*}\mid\mathcal{F}_{t}\right\}\right],
\end{aligned}
$$

and,

$$
\begin{aligned}
& \left(1+\left(1-n\right)\varphi\theta\right)\left(\frac{\alpha^{H*}}{1-\alpha^{H*}}\right)\left[\widehat{\pi}_{t}^{H*}-\beta\mathbb{E}\left\{\widehat{\pi}_{t+1}^{H*}\mid\mathcal{F}_{t}\right\}\right] \\
\approx\ & \left(1-\beta\alpha^{H*}\right)\mathbb{E}\left\{\left[\begin{array}{c}\left(1-n\right)\left(\left(1+n\varphi\sigma\right)\widehat{t}_{t}-\left(1-n\right)\varphi\sigma\widehat{t}_{t}^{*}\right)+\\ +\frac{\gamma}{1-b}\left(\widehat{c}_{t}-b\widehat{c}_{t-1}\right)+\varphi\widehat{c}_{t}^{W}-\left(1+\varphi\right)\widehat{a}_{t}\end{array}\right]\mid\mathcal{F}_{t}\right\}- \\
& -\left(1-\beta\alpha^{H*}\right)\mathbb{E}\left\{\widehat{rs}_{t}+\left(1-n\right)\left(\widehat{t}_{t}^{*}+\widehat{t}_{t}\right)\mid\mathcal{F}_{t}\right\}- \\
& -n\varphi\theta\left(\frac{\alpha^{H}-\alpha^{H*}}{1-\alpha^{H}}\right)\widehat{\pi}_{t}^{H}-n\varphi\theta\left(\frac{\alpha^{H*}}{1-\alpha^{H}}\right)\left[\widehat{\pi}_{t}^{H}-\beta\mathbb{E}\left\{\widehat{\pi}_{t+1}^{H}\mid\mathcal{F}_{t}\right\}\right].
\end{aligned}
$$

Hence, in the third step I substitute out the inflation expectations to ensure that each equation is expressed as a function of the expectations in one market only. The substitution requires a bit of painful effort on the algebra. After replacing the foreign market expectations in the linearized pricing equation for the domestic market, it follows that the supply curve for the home good in the home market is given by,

$$
\begin{aligned}
\widehat{\pi}_{t}^{H}\ \approx\ & \beta\mathbb{E}\left\{\widehat{\pi}_{t+1}^{H}\mid\mathcal{F}_{t}\right\}+k_{\pi}^{H}\widehat{\pi}_{t}^{H}+k_{\pi^{*}}^{H}\widehat{\pi}_{t}^{H*}+k_{c}^{H}\mathbb{E}\left\{\frac{\gamma}{1-b}\left(\widehat{c}_{t}-b\widehat{c}_{t-1}\right)+\varphi\widehat{c}_{t}^{W}-\left(1+\varphi\right)\widehat{a}_{t}\mid\mathcal{F}_{t}\right\}+ \\
& +k_{rs}^{H}\mathbb{E}\left\{\widehat{rs}_{t}\mid\mathcal{F}_{t}\right\}+k_{t}^{H}\mathbb{E}\left\{\widehat{t}_{t}\mid\mathcal{F}_{t}\right\}+k_{t^{*}}^{H}\mathbb{E}\left\{\widehat{t}_{t}^{*}\mid\mathcal{F}_{t}\right\}, \qquad (55)
\end{aligned}
$$

where

$$
\begin{aligned}
k_{\pi}^{H} &\equiv -\left(\frac{\alpha^{H*}-\alpha^{H}}{\alpha^{H*}}\right)\left(\frac{\varphi\theta(1-n)}{1+\varphi\theta}\right)\varphi\theta n, \\
k_{\pi^{*}}^{H} &\equiv \left(\frac{\alpha^{H}-\alpha^{H*}}{\alpha^{H}}\right)\left(\frac{1-\alpha^{H}}{1-\alpha^{H*}}\right)\left(\frac{1+\varphi\theta(1-n)}{1+\varphi\theta}\right)\varphi\theta\left(1-n\right), \\
k_{c}^{H} &\equiv \left[\left(\frac{\left(1-\beta\alpha^{H}\right)\left(1-\alpha^{H}\right)}{\alpha^{H}}\right)\left(\frac{1+\varphi\theta(1-n)}{1+\varphi\theta}\right)-\left(\frac{\left(1-\beta\alpha^{H*}\right)\left(1-\alpha^{H}\right)}{\alpha^{H*}}\right)\left(\frac{\varphi\theta(1-n)}{1+\varphi\theta}\right)\right], \\
k_{rs}^{H} &\equiv \left(\frac{\left(1-\beta\alpha^{H*}\right)\left(1-\alpha^{H}\right)}{\alpha^{H*}}\right)\left(\frac{\varphi\theta(1-n)}{1+\varphi\theta}\right), \\
k_{t}^{H} &\equiv \left[\left(\frac{\left(1-\beta\alpha^{H}\right)\left(1-\alpha^{H}\right)}{\alpha^{H}}\right)\left(\frac{(1+\varphi\theta(1-n))(1+\varphi\sigma n)}{1+\varphi\theta}\right)-\left(\frac{\left(1-\beta\alpha^{H*}\right)\left(1-\alpha^{H}\right)}{\alpha^{H*}}\right)\left(\frac{\varphi\theta(1-n)(\varphi\sigma n)}{1+\varphi\theta}\right)\right]\left(1-n\right), \\
k_{t^{*}}^{H} &\equiv \left[\left(\frac{\left(1-\beta\alpha^{H*}\right)\left(1-\alpha^{H}\right)}{\alpha^{H*}}\right)\left(\frac{(1+\varphi\sigma(1-n))\varphi\theta(1-n)}{1+\varphi\theta}\right)-\left(\frac{\left(1-\beta\alpha^{H}\right)\left(1-\alpha^{H}\right)}{\alpha^{H}}\right)\left(\frac{(1+\varphi\theta(1-n))\varphi\sigma(1-n)}{1+\varphi\theta}\right)\right]\left(1-n\right).
\end{aligned}
$$

Substituting now the domestic market expectations in the linearized pricing equation for the foreign market, it follows that the supply curve for the home good in the foreign market is given by,

$$
\begin{aligned}
\widehat{\pi}_{t}^{H*}\ \approx\ & \beta\mathbb{E}\left\{\widehat{\pi}_{t+1}^{H*}\mid\mathcal{F}_{t}\right\}+k_{\pi}^{H*}\widehat{\pi}_{t}^{H}+k_{\pi^{*}}^{H*}\widehat{\pi}_{t}^{H*}+k_{c}^{H*}\mathbb{E}\left\{\frac{\gamma}{1-b}\left(\widehat{c}_{t}-b\widehat{c}_{t-1}\right)+\varphi\widehat{c}_{t}^{W}-\left(1+\varphi\right)\widehat{a}_{t}\mid\mathcal{F}_{t}\right\}+ \\
& +k_{rs}^{H*}\mathbb{E}\left\{\widehat{rs}_{t}\mid\mathcal{F}_{t}\right\}+k_{t}^{H*}\mathbb{E}\left\{\widehat{t}_{t}\mid\mathcal{F}_{t}\right\}+k_{t^{*}}^{H*}\mathbb{E}\left\{\widehat{t}_{t}^{*}\mid\mathcal{F}_{t}\right\}, \qquad (56)
\end{aligned}
$$

where

$$\begin{aligned}
k_{\pi}^{H*} &\equiv \left(\frac{\alpha^{H*}-\alpha^{H}}{\alpha^{H*}}\right)\left(\frac{1-\alpha^{H*}}{1-\alpha^{H}}\right)\left(\frac{1+\varphi\theta n}{1+\varphi\theta}\right)\varphi\theta n,\\
k_{\pi^*}^{H*} &\equiv -\left(\frac{\alpha^{H}-\alpha^{H*}}{\alpha^{H}}\right)\left(\frac{\varphi\theta n}{1+\varphi\theta}\right)\varphi\theta\left(1-n\right),\\
k_{c}^{H*} &\equiv \left[\left(\frac{\left(1-\alpha^{H*}\right)\left(1-\beta\alpha^{H*}\right)}{\alpha^{H*}}\right)\left(\frac{1+\varphi\theta n}{1+\varphi\theta}\right)-\left(\frac{\left(1-\beta\alpha^{H}\right)\left(1-\alpha^{H*}\right)}{\alpha^{H}}\right)\left(\frac{\varphi\theta n}{1+\varphi\theta}\right)\right],\\
k_{rs}^{H*} &\equiv -\left(\frac{\left(1-\alpha^{H*}\right)\left(1-\beta\alpha^{H*}\right)}{\alpha^{H*}}\right)\left(\frac{1+\varphi\theta n}{1+\varphi\theta}\right),\\
k_{t}^{H*} &\equiv \left[\left(\frac{\left(1-\alpha^{H*}\right)\left(1-\beta\alpha^{H*}\right)}{\alpha^{H*}}\right)\left(\frac{\varphi\sigma n(1+\varphi\theta n)}{1+\varphi\theta}\right)-\left(\frac{\left(1-\beta\alpha^{H}\right)\left(1-\alpha^{H*}\right)}{\alpha^{H}}\right)\left(\frac{\varphi\theta n(1+\varphi\sigma n)}{1+\varphi\theta}\right)\right]\left(1-n\right),\\
k_{t^*}^{H*} &\equiv \left[\left(\frac{\left(1-\beta\alpha^{H}\right)\left(1-\alpha^{H*}\right)}{\alpha^{H}}\right)\left(\frac{(\varphi\theta n)\varphi\sigma(1-n)}{1+\varphi\theta}\right)-\left(\frac{\left(1-\alpha^{H*}\right)\left(1-\beta\alpha^{H*}\right)}{\alpha^{H*}}\right)\left(\frac{(1+\varphi\theta n)(1+\varphi\sigma(1-n))}{1+\varphi\theta}\right)\right]\left(1-n\right).
\end{aligned}$$

Equations (55) and (56) are the version of the AS^H and AS^{H*} equations that I use in the paper. The evolution of prices in each market is not completely insulated from the inflation rate in the other market whenever the contract duration varies across markets, i.e. $\alpha^H \neq \alpha^{H*}$.

The Relative AS^H Equation. It follows from the two domestic aggregate supply curves in (55) and (56) that,

$$\begin{aligned}
\widehat{\pi}_t^H - \widehat{\pi}_t^{H*} \quad \approx \quad & \beta\mathbb{E}\left\{\widehat{\pi}_{t+1}^H - \widehat{\pi}_{t+1}^{H*} \mid \mathcal{F}_t\right\} + \left(k_{\pi}^H - k_{\pi}^{H*}\right)\widehat{\pi}_t^H + \left(k_{\pi^*}^H - k_{\pi^*}^{H*}\right)\widehat{\pi}_t^{H*} + \\
& + \left(k_c^H - k_c^{H*}\right)\mathbb{E}\left\{\frac{\gamma}{1-b}\left(\widehat{c}_t - b\widehat{c}_{t-1}\right) + \varphi\widehat{c}_t^W - \left(1+\varphi\right)\widehat{a}_t \mid \mathcal{F}_t\right\} + \\
& + \left(k_{rs}^H - k_{rs}^{H*}\right)\mathbb{E}\left\{\widehat{rs}_t \mid \mathcal{F}_t\right\} + \left(k_t^H - k_t^{H*}\right)\mathbb{E}\left\{\widehat{t}_t \mid \mathcal{F}_t\right\} + \left(k_{t^*}^H - k_{t^*}^{H*}\right)\mathbb{E}\left\{\widehat{t}_t^* \mid \mathcal{F}_t\right\}.
\end{aligned}$$

If $\alpha^H = \alpha^{H*}$, then the relative equation can be re-written as follows,

$$\widehat{\pi}_t^H - \widehat{\pi}_t^{H*} \approx \beta\mathbb{E}\left\{\widehat{\pi}_{t+1}^H - \widehat{\pi}_{t+1}^{H*} \mid \mathcal{F}_t\right\} + \frac{\left(1-\beta\alpha^H\right)\left(1-\alpha^H\right)}{\alpha^H}\left[\widehat{s}_t + \widehat{p}_t^{H*} - \widehat{p}_t^H\right],$$

which explains why pricing differences in the standard model are closely linked to the nominal exchange rate, and why the standard model has a hard time explaining the low degree of pass-through found in the data.

D.4.5 The Aggregate-Supply Equations for the Foreign Firm: AS^F and AS^{F*}

In order to derive the dynamics of the inflation rates for the foreign bundle of goods, i.e. $\widehat{\pi}_t^F$ and $\widehat{\pi}_t^{F*}$, I need to manipulate further the linearized first-order conditions of the firm in (51) and (52). The solution involves some algebra, but it is otherwise conceptually straightforward and can be summarized in three steps. First, I use the result in equation (54) to replace the optimal prices $\widehat{\widetilde{p}}_t(f)$ and $\widehat{\widetilde{p}}_t^*(f)$, i.e.

$$\begin{aligned}
& \left(1+n\varphi\theta\right)\left(\frac{\alpha^F}{1-\alpha^F}\right)\widehat{\pi}_t^F + \left(1-n\right)\varphi\theta\left(\frac{\alpha^{F*}}{1-\alpha^{F*}}\right)\widehat{\pi}_t^{F*} \\
& \approx \left(1-\beta\alpha^F\right)\mathbb{E}\left\{\sum\nolimits_{\tau=0}^{\infty}\left(\beta\alpha^F\right)^{\tau}\left[\begin{array}{c}
\left(1+n\varphi\theta\right)\left(\sum\nolimits_{i=1,\tau\geq 1}^{\tau}\widehat{\pi}_{t+i}^F\right) + \left(1-n\right)\varphi\theta\left(\sum\nolimits_{i=1,\tau\geq 1}^{\tau}\widehat{\pi}_{t+i}^{F*}\right) - \\
-n\left(n\varphi\sigma\widehat{t}_{t+\tau} - \left(1+\left(1-n\right)\varphi\sigma\right)\widehat{t}_{t+\tau}^*\right) + \\
+\frac{\gamma}{1-b}\left(\widehat{c}_{t+\tau}^* - b\widehat{c}_{t+\tau-1}^*\right) + \varphi\widehat{c}_{t+\tau}^W - \left(1+\varphi\right)\widehat{a}_{t+\tau}^* + \\
+\left(\widehat{s}_{t+\tau} + \widehat{p}_{t+\tau}^{F*} - \widehat{p}_{t+\tau}^F\right)
\end{array}\right] \mid \mathcal{F}_t^*\right\},
\end{aligned}$$

and,

$$
\begin{aligned}
& n\varphi\theta\left(\tfrac{\alpha^{F}}{1-\alpha^{F}}\right)\widehat{\pi}_{t}^{F}+\left(1+\left(1-n\right)\varphi\theta\right)\left(\tfrac{\alpha^{F*}}{1-\alpha^{F*}}\right)\widehat{\pi}_{t}^{F*} \\
& \approx\left(1-\beta\alpha^{F*}\right)\mathbb{E}\left\{\sum\nolimits_{\tau=0}^{\infty}\left(\beta\alpha^{F*}\right)^{\tau}\left[\begin{array}{c} n\varphi\theta\left(\sum\nolimits_{i=1,\tau\geq1}^{\tau}\widehat{\pi}_{t+i}^{F}\right)+\left(1+\left(1-n\right)\varphi\theta\right)\left(\sum\nolimits_{i=1,\tau\geq1}^{\tau}\widehat{\pi}_{t+i}^{F*}\right)- \\ -n\left(n\varphi\sigma\widehat{t}_{t+\tau}-\left(1+\left(1-n\right)\varphi\sigma\right)\widehat{t}_{t+\tau}^{*}\right)+ \\ +\frac{\gamma}{1-b}\left(\widehat{c}_{t+\tau}^{*}-b\widehat{c}_{t+\tau-1}^{*}\right)+\varphi\widehat{c}_{t+\tau}^{W}-\left(1+\varphi\right)\widehat{a}_{t+\tau}^{*}\end{array}\right]\mid\mathcal{F}_{t}^{*}\right\}.
\end{aligned}
$$

Second, I realize that these expressions take the form of a discounted present-value system that can be re-written as the forward-looking (no-bubble) solution of a pair of expectational of difference equations. In other words, I can say that the equations of interest are,

$$
\begin{aligned}
& \left(1+n\varphi\theta\right)\left(\tfrac{\alpha^{F}}{1-\alpha^{F}}\right)\widehat{\pi}_{t}^{F}+\left(1-n\right)\varphi\theta\left(\tfrac{\alpha^{F*}}{1-\alpha^{F*}}\right)\widehat{\pi}_{t}^{F*} \\
& \approx\left(1-\beta\alpha^{F}\right)\mathbb{E}\left\{\left[\begin{array}{c} -n\left(n\varphi\sigma\widehat{t}_{t}-\left(1+\left(1-n\right)\varphi\sigma\right)\widehat{t}_{t}^{*}\right)+ \\ +\frac{\gamma}{1-b}\left(\widehat{c}_{t}^{*}-b\widehat{c}_{t-1}^{*}\right)+\varphi\widehat{c}_{t}^{W}-\left(1+\varphi\right)\widehat{a}_{t}^{*}+ \\ +\left(\widehat{s}_{t}+\widehat{p}_{t}^{F*}-\widehat{p}_{t}^{F}\right)\end{array}\right]\mid\mathcal{F}_{t}^{*}\right\}+ \\
& +\beta\alpha^{F}\mathbb{E}\left\{\left(1-\beta\alpha^{F}\right)\sum\nolimits_{\tau=0}^{\infty}\left(\beta\alpha^{F}\right)^{\tau}\left[\begin{array}{c} \left(1+n\varphi\theta\right)\left(\widehat{\pi}_{t+1}^{F}+\sum\nolimits_{i=1,\tau\geq1}^{\tau}\widehat{\pi}_{t+1+i}^{F}\right)+ \\ +\left(1-n\right)\varphi\theta\left(\widehat{\pi}_{t+1}^{F*}+\sum\nolimits_{i=1,\tau\geq1}^{\tau}\widehat{\pi}_{t+1+i}^{F*}\right)- \\ -n\left(n\varphi\sigma\widehat{t}_{t+1+\tau}-\left(1+\left(1-n\right)\varphi\sigma\right)\widehat{t}_{t+1+\tau}^{*}\right)+ \\ +\frac{\gamma}{1-b}\left(\widehat{c}_{t+1+\tau}^{*}-b\widehat{c}_{t+\tau}^{*}\right)+\varphi\widehat{c}_{t+1+\tau}^{W}-\left(1+\varphi\right)\widehat{a}_{t+1+\tau}^{*}+ \\ +\left(\widehat{s}_{t+1+\tau}+\widehat{p}_{t+1+\tau}^{F*}-\widehat{p}_{t+1+\tau}^{F}\right)\end{array}\right]\mid\mathcal{F}_{t}^{*}\right\},
\end{aligned}
$$

and,

$$
\begin{aligned}
& \left(1+n\varphi\theta\right)\left(\tfrac{\alpha^{F}}{1-\alpha^{F}}\right)\widehat{\pi}_{t}^{F}+\left(1-n\right)\varphi\theta\left(\tfrac{\alpha^{F*}}{1-\alpha^{F*}}\right)\widehat{\pi}_{t}^{F*} \\
& \approx\left(1-\beta\alpha^{F}\right)\mathbb{E}\left\{\left[\begin{array}{c} -n\left(n\varphi\sigma\widehat{t}_{t}-\left(1+\left(1-n\right)\varphi\sigma\right)\widehat{t}_{t}^{*}\right)+ \\ +\frac{\gamma}{1-b}\left(\widehat{c}_{t}^{*}-b\widehat{c}_{t-1}^{*}\right)+\varphi\widehat{c}_{t}^{W}-\left(1+\varphi\right)\widehat{a}_{t}^{*}+ \\ +\left(\widehat{s}_{t}+\widehat{p}_{t}^{F*}-\widehat{p}_{t}^{F}\right)\end{array}\right]\mid\mathcal{F}_{t}^{*}\right\}+ \\
& +\beta\alpha^{F}\mathbb{E}\left\{\left[\begin{array}{l} \left(1+n\varphi\theta\right)\widehat{\pi}_{t+1}^{F}+\left(1-n\right)\varphi\theta\widehat{\pi}_{t+1}^{F*}+ \\ +\mathbb{E}\left\{\sum\nolimits_{\tau=0}^{\infty}\frac{\left(\beta\alpha^{F}\right)^{\tau}}{\left(1-\beta\alpha^{F}\right)^{-1}}\left[\begin{array}{c} \left(1+n\varphi\theta\right)\left(\sum\nolimits_{i=1,\tau\geq1}^{\tau}\widehat{\pi}_{t+1+i}^{F}\right)+ \\ +\left(1-n\right)\varphi\theta\left(\sum\nolimits_{i=1,\tau\geq1}^{\tau}\widehat{\pi}_{t+1+i}^{F*}\right)- \\ -n\left(n\varphi\sigma\widehat{t}_{t+1+\tau}-\left(1+\left(1-n\right)\varphi\sigma\right)\widehat{t}_{t+1+\tau}^{*}\right)+ \\ +\frac{\gamma}{1-b}\left(\widehat{c}_{t+1+\tau}^{*}-b\widehat{c}_{t+\tau}^{*}\right)+\varphi\widehat{c}_{t+1+\tau}^{W}-\left(1+\varphi\right)\widehat{a}_{t+1+\tau}^{*}+ \\ +\left(\widehat{s}_{t+1+\tau}+\widehat{p}_{t+1+\tau}^{F*}-\widehat{p}_{t+1+\tau}^{F}\right)\end{array}\right]\mid\mathcal{F}_{t+1}^{*}\right\}\end{array}\right]\mid\mathcal{F}_{t}^{*}\right\},
\end{aligned}
$$

where I used the law of iterated expectations. Therefore, I obtain the following transformation of (51),

$$
\begin{aligned}
& \left(1+n\varphi\theta\right)\left(\frac{\alpha^{F}}{1-\alpha^{F}}\right)\widehat{\pi}_{t}^{F}+\left(1-n\right)\varphi\theta\left(\frac{\alpha^{F*}}{1-\alpha^{F*}}\right)\widehat{\pi}_{t}^{F*} \\
\approx\; & \left(1-\beta\alpha^{F}\right)\mathbb{E}\left\{\left[\begin{array}{c} -n\left(n\varphi\sigma\widehat{t}_{t}-\left(1+\left(1-n\right)\varphi\sigma\right)\widehat{t}_{t}^{*}\right)+ \\ +\frac{\gamma}{1-b}\left(\widehat{c}_{t}^{*}-b\widehat{c}_{t-1}^{*}\right)+\varphi\widehat{c}_{t}^{W}-\left(1+\varphi\right)\widehat{a}_{t}^{*}+\left(\widehat{s}_{t}+\widehat{p}_{t}^{F*}-\widehat{p}_{t}^{F}\right)\end{array}\right]\mid\mathcal{F}_{t}^{*}\right\}+ \\
& +\beta\alpha^{F}\mathbb{E}\left\{\left(1+n\varphi\theta\right)\left(\frac{1}{1-\alpha^{F}}\right)\widehat{\pi}_{t+1}^{F}+\left(1-n\right)\varphi\theta\left(\frac{1}{1-\alpha^{F*}}\right)\widehat{\pi}_{t+1}^{F*}\mid\mathcal{F}_{t}^{*}\right\}.
\end{aligned}
$$

Analogously, I derive the following expression for the other linearized first-order condition in (52),

$$
\begin{aligned}
& n\varphi\theta\left(\frac{\alpha^{F}}{1-\alpha^{F}}\right)\widehat{\pi}_{t}^{F}+\left(1+\left(1-n\right)\varphi\theta\right)\left(\frac{\alpha^{F*}}{1-\alpha^{F*}}\right)\widehat{\pi}_{t}^{F*} \\
\approx\; & \left(1-\beta\alpha^{F*}\right)\mathbb{E}\left\{\left[\begin{array}{c} -n\left(n\varphi\sigma\widehat{t}_{t}-\left(1+\left(1-n\right)\varphi\sigma\right)\widehat{t}_{t}^{*}\right)+ \\ +\frac{\gamma}{1-b}\left(\widehat{c}_{t}^{*}-b\widehat{c}_{t-1}^{*}\right)+\varphi\widehat{c}_{t}^{W}-\left(1+\varphi\right)\widehat{a}_{t}^{*} \end{array}\right]\mid\mathcal{F}_{t}^{*}\right\}+ \\
& +\beta\alpha^{F*}\mathbb{E}\left\{n\varphi\theta\left(\frac{1}{1-\alpha^{F}}\right)\widehat{\pi}_{t+1}^{F}+\left(1+\left(1-n\right)\varphi\theta\right)\left(\frac{1}{1-\alpha^{F*}}\right)\widehat{\pi}_{t+1}^{F*}\mid\mathcal{F}_{t}^{*}\right\}.
\end{aligned}
$$

Based on the definition of relative prices, the consumer price indexes and the real exchange rate in (36) − (40), I can derive the following relationship,

$$
\begin{aligned}
\widehat{s}_{t}+\widehat{p}_{t}^{F*}-\widehat{p}_{t}^{F} &= \widehat{rs}_{t}+\left(\widehat{p}_{t}^{F*}-\widehat{p}_{t}^{*}\right)-\left(\widehat{p}_{t}^{F}-\widehat{p}_{t}\right) \\
&\approx \widehat{rs}_{t}-n\left(\widehat{p}_{t}^{H*}-\widehat{p}_{t}^{F*}\right)+n\left(\widehat{p}_{t}^{H}-\widehat{p}_{t}^{F}\right) \\
&= \widehat{rs}_{t}-n\left(\widehat{t}_{t}^{*}+\widehat{t}_{t}\right),
\end{aligned}
$$

where the discrepancy on relative prices across countries is denoted $\widehat{t}_{t}^{R}\equiv\widehat{t}_{t}+\widehat{t}_{t}^{*}$.

Finally, I notice that the expressions I have derived so far are all functions of the expected inflation rate of the foreign bundle of goods in the home as well as the foreign market. That is,

$$
\begin{aligned}
& \left(1+n\varphi\theta\right)\left(\frac{\alpha^{F}}{1-\alpha^{F}}\right)\left[\widehat{\pi}_{t}^{F}-\beta\mathbb{E}\left\{\widehat{\pi}_{t+1}^{F}\mid\mathcal{F}_{t}^{*}\right\}\right] \\
\approx\; & \left(1-\beta\alpha^{F}\right)\mathbb{E}\left\{\left[\begin{array}{c} -n\left(n\varphi\sigma\widehat{t}_{t}-\left(1+\left(1-n\right)\varphi\sigma\right)\widehat{t}_{t}^{*}\right)+ \\ +\frac{\gamma}{1-b}\left(\widehat{c}_{t}^{*}-b\widehat{c}_{t-1}^{*}\right)+\varphi\widehat{c}_{t}^{W}-\left(1+\varphi\right)\widehat{a}_{t}^{*} \end{array}\right]\mid\mathcal{F}_{t}^{*}\right\}+ \\
& +\left(1-\beta\alpha^{F}\right)\mathbb{E}\left\{\widehat{rs}_{t}-n\left(\widehat{t}_{t}^{*}+\widehat{t}_{t}\right)\mid\mathcal{F}_{t}^{*}\right\}+ \\
& -\left(1-n\right)\varphi\theta\left(\frac{\alpha^{F*}-\alpha^{F}}{1-\alpha^{F*}}\right)\widehat{\pi}_{t}^{F*}-\left(1-n\right)\varphi\theta\left(\frac{\alpha^{F}}{1-\alpha^{F*}}\right)\left[\widehat{\pi}_{t}^{F*}-\beta\mathbb{E}\left\{\widehat{\pi}_{t+1}^{F*}\mid\mathcal{F}_{t}^{*}\right\}\right],
\end{aligned}
$$

and,

$$
\begin{aligned}
& \left(1+\left(1-n\right)\varphi\theta\right)\left(\frac{\alpha^{F*}}{1-\alpha^{F*}}\right)\left[\widehat{\pi}_{t}^{F*}-\beta\mathbb{E}\left\{\widehat{\pi}_{t+1}^{F*}\mid\mathcal{F}_{t}^{*}\right\}\right] \\
\approx\; & \left(1-\beta\alpha^{F*}\right)\mathbb{E}\left\{\left[\begin{array}{c} -n\left(n\varphi\sigma\widehat{t}_{t}-\left(1+\left(1-n\right)\varphi\sigma\right)\widehat{t}_{t}^{*}\right)+ \\ +\frac{\gamma}{1-b}\left(\widehat{c}_{t}^{*}-b\widehat{c}_{t-1}^{*}\right)+\varphi\widehat{c}_{t}^{W}-\left(1+\varphi\right)\widehat{a}_{t}^{*} \end{array}\right]\mid\mathcal{F}_{t}^{*}\right\}- \\
& -n\varphi\theta\left(\frac{\alpha^{F}-\alpha^{F*}}{1-\alpha^{F}}\right)\widehat{\pi}_{t}^{F}-n\varphi\theta\left(\frac{\alpha^{F*}}{1-\alpha^{F}}\right)\left[\widehat{\pi}_{t}^{F}-\beta\mathbb{E}\left\{\widehat{\pi}_{t+1}^{F}\mid\mathcal{F}_{t}^{*}\right\}\right].
\end{aligned}
$$

Hence, in the third step I substitute out the inflation expectations to ensure that each equation is expressed as a function of the expectations in one market only. The substitution requires a bit of painful effort on the algebra. After replacing the foreign market expectations in the linearized pricing equation for the domestic market, it follows that the supply curve for the foreign good in the home market is given by,

$$
\begin{aligned}
\widehat{\pi}_{t}^{F} \approx\; & \beta\mathbb{E}\left\{\widehat{\pi}_{t+1}^{F}\mid\mathcal{F}_{t}^{*}\right\}+k_{\pi}^{F}\widehat{\pi}_{t}^{F}+k_{\pi^{*}}^{F}\widehat{\pi}_{t}^{F*}+k_{c}^{F}\mathbb{E}\left\{\frac{\gamma}{1-b}\left(\widehat{c}_{t}^{*}-b\widehat{c}_{t-1}^{*}\right)+\varphi\widehat{c}_{t}^{W}-\left(1+\varphi\right)\widehat{a}_{t}^{*}\mid\mathcal{F}_{t}^{*}\right\}+ \\
& +k_{rs}^{F}\mathbb{E}\left\{\widehat{rs}_{t}\mid\mathcal{F}_{t}^{*}\right\}+k_{t}^{F}\mathbb{E}\left\{\widehat{t}_{t}\mid\mathcal{F}_{t}^{*}\right\}+k_{t^{*}}^{F}\mathbb{E}\left\{\widehat{t}_{t}^{*}\mid\mathcal{F}_{t}^{*}\right\},
\end{aligned}
\tag{57}
$$

where

$$
\begin{aligned}
k_{\pi}^{F} &\equiv -\left(\tfrac{\alpha^{F*}-\alpha^{F}}{\alpha^{F*}}\right)\left(\tfrac{\varphi\theta(1-n)}{1+\varphi\theta}\right)\varphi\theta n,\\
k_{\pi^*}^{F} &\equiv \left(\tfrac{\alpha^{F}-\alpha^{F*}}{\alpha^{F}}\right)\left(\tfrac{1-\alpha^{F}}{1-\alpha^{F*}}\right)\left(\tfrac{\varphi\theta(1-n)(1+\varphi\theta(1-n))}{1+\varphi\theta}\right),\\
k_{c}^{F} &\equiv \left[\left(\tfrac{\left(1-\beta\alpha^{F}\right)\left(1-\alpha^{F}\right)}{\alpha^{F}}\right)\left(\tfrac{1+\varphi\theta(1-n)}{1+\varphi\theta}\right)-\left(\tfrac{\left(1-\beta\alpha^{F*}\right)\left(1-\alpha^{F}\right)}{\alpha^{F*}}\right)\left(\tfrac{\varphi\theta(1-n)}{1+\varphi\theta}\right)\right],\\
k_{rs}^{F} &\equiv \left(\tfrac{\left(1-\beta\alpha^{F}\right)\left(1-\alpha^{F}\right)}{\alpha^{F}}\right)\left(\tfrac{1+\varphi\theta(1-n)}{1+\varphi\theta}\right),\\
k_{t}^{F} &\equiv \left[\left(\tfrac{\left(1-\beta\alpha^{F*}\right)\left(1-\alpha^{F}\right)}{\alpha^{F*}}\right)\left(\tfrac{\varphi\theta(1-n)\varphi\sigma n}{1+\varphi\theta}\right)-\left(\tfrac{\left(1-\beta\alpha^{F}\right)\left(1-\alpha^{F}\right)}{\alpha^{F}}\right)\left(\tfrac{(1+\varphi\theta(1-n))(1+\varphi\sigma n)}{1+\varphi\theta}\right)\right]n,\\
k_{t^*}^{F} &\equiv \left[\left(\tfrac{\left(1-\beta\alpha^{F}\right)\left(1-\alpha^{F}\right)}{\alpha^{F}}\right)\left(\tfrac{(1+\varphi\theta(1-n))\varphi\sigma(1-n)}{1+\varphi\theta}\right)-\left(\tfrac{\left(1-\beta\alpha^{F*}\right)\left(1-\alpha^{F}\right)}{\alpha^{F*}}\right)\left(\tfrac{\varphi\theta(1-n)(1+\varphi\sigma(1-n))}{1+\varphi\theta}\right)\right]n.
\end{aligned}
$$

Substituting now the domestic market expectations in the linearized pricing equation for the foreign market, it follows that the supply curve for the foreign good in the foreign market is given by,

$$
\begin{aligned}
\widehat{\pi}_t^{F*} \approx\ & \beta\mathbb{E}\left\{\widehat{\pi}_{t+1}^{F*}\mid\mathcal{F}_t^*\right\}+k_{\pi}^{F*}\widehat{\pi}_t^{F}+k_{\pi^*}^{F*}\widehat{\pi}_t^{F*}+k_c^{F*}\mathbb{E}\left\{\frac{\gamma}{1-b}\left(\widehat{c}_t^*-b\widehat{c}_{t-1}^*\right)+\varphi\widehat{c}_t^W-(1+\varphi)\,\widehat{a}_t^*\mid\mathcal{F}_t^*\right\}+\\
&+k_{rs}^{F*}\mathbb{E}\left\{\widehat{rs}_t\mid\mathcal{F}_t^*\right\}+k_t^{F*}\mathbb{E}\left\{\widehat{t}_t\mid\mathcal{F}_t^*\right\}+k_{t^*}^{F*}\mathbb{E}\left\{\widehat{t}_t^*\mid\mathcal{F}_t^*\right\},
\end{aligned}
\tag{58}
$$

where

$$
\begin{aligned}
k_{\pi}^{F*} &\equiv \left(\tfrac{\alpha^{F*}-\alpha^{F}}{\alpha^{F*}}\right)\left(\tfrac{1-\alpha^{F*}}{1-\alpha^{F}}\right)\left(\tfrac{(1+\varphi\theta n)\varphi\theta n}{1+\varphi\theta}\right),\\
k_{\pi^*}^{F*} &\equiv -\left(\tfrac{\alpha^{F}-\alpha^{F*}}{\alpha^{F}}\right)\left(\tfrac{(\varphi\theta n)\varphi\theta(1-n)}{1+\varphi\theta}\right),\\
k_{c}^{F*} &\equiv \left[\left(\tfrac{\left(1-\beta\alpha^{F*}\right)\left(1-\alpha^{F*}\right)}{\alpha^{F*}}\right)\left(\tfrac{1+\varphi\theta n}{1+\varphi\theta}\right)-\left(\tfrac{\left(1-\beta\alpha^{F}\right)\left(1-\alpha^{F*}\right)}{\alpha^{F}}\right)\left(\tfrac{\varphi\theta n}{1+\varphi\theta}\right)\right],\\
k_{rs}^{F*} &\equiv -\left(\tfrac{\left(1-\beta\alpha^{F}\right)\left(1-\alpha^{F*}\right)}{\alpha^{F}}\right)\left(\tfrac{\varphi\theta n}{1+\varphi\theta}\right),\\
k_{t}^{F*} &\equiv \left[\left(\tfrac{\left(1-\beta\alpha^{F}\right)\left(1-\alpha^{F*}\right)}{\alpha^{F}}\right)\left(\tfrac{\varphi\theta n(1+\varphi\sigma n)}{1+\varphi\theta}\right)-\left(\tfrac{\left(1-\beta\alpha^{F*}\right)\left(1-\alpha^{F*}\right)}{\alpha^{F*}}\right)\left(\tfrac{(1+\varphi\theta n)\varphi\sigma n}{1+\varphi\theta}\right)\right]n,\\
k_{t^*}^{F*} &\equiv \left[\left(\tfrac{\left(1-\beta\alpha^{F*}\right)\left(1-\alpha^{F*}\right)}{\alpha^{F*}}\right)\left(\tfrac{(1+\varphi\theta n)(1+\varphi\sigma(1-n))}{1+\varphi\theta}\right)-\left(\tfrac{\left(1-\beta\alpha^{F}\right)\left(1-\alpha^{F*}\right)}{\alpha^{F}}\right)\left(\tfrac{(\varphi\theta n)\varphi\sigma(1-n)}{1+\varphi\theta}\right)\right]n.
\end{aligned}
$$

Equations (57) and (58) are the version of the AS^F and AS^{F*} equations that I use in the paper. The evolution of prices in each market is not completely insulated from the inflation in the other market whenever the contract duration varies across markets, i.e. $\alpha^F \neq \alpha^{F*}$.

The Relative AS^F Equation. It follows from the two foreign aggregate supply curves in (57) and (58) that,

$$
\begin{aligned}
\widehat{\pi}_t^{F}-\widehat{\pi}_t^{F*} \approx\ & \beta\mathbb{E}\left\{\widehat{\pi}_{t+1}^{F}-\widehat{\pi}_{t+1}^{F*}\mid\mathcal{F}_t^*\right\}+\left(k_{\pi}^{F}-k_{\pi}^{F*}\right)\widehat{\pi}_t^{F}+\left(k_{\pi^*}^{F}-k_{\pi^*}^{F*}\right)\widehat{\pi}_t^{F*}+\\
&+\left(k_c^{F}-k_c^{F*}\right)\mathbb{E}\left\{\frac{\gamma}{1-b}\left(\widehat{c}_t^*-b\widehat{c}_{t-1}^*\right)+\varphi\widehat{c}_t^W-(1+\varphi)\,\widehat{a}_t^*\mid\mathcal{F}_t^*\right\}+\\
&+\left(k_{rs}^{F}-k_{rs}^{F*}\right)\mathbb{E}\left\{\widehat{rs}_t\mid\mathcal{F}_t^*\right\}+\left(k_t^{F}-k_t^{F*}\right)\left\{\widehat{t}_t\mid\mathcal{F}_t^*\right\}+\left(k_{t^*}^{F}-k_{t^*}^{F*}\right)\left\{\widehat{t}_t^*\mid\mathcal{F}_t^*\right\}.
\end{aligned}
$$

If $\alpha^F = \alpha^{F*}$, then the relative equation can be re-written as follows,

$$
\widehat{\pi}_t^{F}-\widehat{\pi}_t^{F*}\approx\beta\mathbb{E}\left\{\widehat{\pi}_{t+1}^{F}-\widehat{\pi}_{t+1}^{F*}\mid\mathcal{F}_t^*\right\}+\frac{\left(1-\beta\alpha^F\right)\left(1-\alpha^F\right)}{\alpha^F}\left[\widehat{s}_t+\widehat{p}_t^{F*}-\widehat{p}_t^{F}\right],
$$

which explains why pricing differences in the standard model are closely linked to the nominal exchange rate, and why the standard model has a hard time explaining the low degree of pass-through found in the data.

D.5 The Financial-Side of the Economy

To obtain the *UIP* equation I need to log-linearize the first-order condition of the financial intermediary in (21) around the steady state, i.e.

$$\mathbb{E}\left[\underbrace{\frac{S_{t+1}}{S_t} - \lambda P_t B_t^{RF}}_{\equiv F_{t+1}} - \underbrace{\exp\left(i_t - i_t^*\right)}_{\equiv G_{t+1}} \mid \mathcal{I}_t\right] = 0,$$

where $B_t^{RF} \equiv \frac{S_t B_t^F}{P_t}$ is the net foreign asset position in real terms. In steady state holds that $\overline{f} = \overline{g}$ and $\overline{f}^* = \overline{g}^*$, then the log-linearization around the steady state can be expressed as,

$$\mathbb{E}\left[\widehat{f}_{t+1} - \widehat{g}_{t+1} \mid \mathcal{I}_t\right] \approx 0,$$

where the approximations are as follows,

$$\begin{aligned} \widehat{f}_{t+1} &\equiv \widehat{s}_{t+1} - \widehat{s}_t + 0 \cdot \widehat{p}_t + \lambda \overline{PC} \widehat{B}_t^{RF}, \\ \widehat{g}_{t+1} &\equiv \left(\widehat{i}_t - \widehat{i}_t^*\right). \end{aligned}$$

Using the steady state values for consumption and prices in (33) to substitute out $\overline{PC}$, I obtain the following *UIP* equation,

$$\mathbb{E}\left[\Delta \widehat{s}_{t+1} - \left(\widehat{i}_t - \widehat{i}_t^*\right) - \delta \widehat{B}_t^{RF} \mid \mathcal{I}_t\right] \approx 0, \tag{59}$$

where $\delta \equiv \lambda \frac{1}{1-b}\left(\frac{1-\beta}{\chi}\right)^{\frac{1}{\gamma}}$ 'measures' the cost of the bond-holdings. This is the version of the *UIP* equation that I use in the paper. Notice that the risk premium faced in the exchange market due to intermediation is linear in the real net foreign asset position, $\widehat{B}_t^{RF}$. The coefficient δ is a function of the habit parameter, b, the rate of time preference, β, the weight on utility from real balances, χ, and the financial intermediary's preference parameter, λ.

D.6 The *CA* Equation and the Current Account Balance

To obtain the *CA* equation I need to log-linearize the resource constraint in (22) around the steady state, i.e.

$$\underbrace{\frac{1}{\exp\left(i_t^*\right)} B_t^{RF} - \frac{S_t}{S_{t-1}} \frac{P_{t-1}}{P_t} B_{t-1}^{RF}}_{\equiv F_t} = \underbrace{\left(\frac{P_t^H}{P_t}\right)^{1-\sigma} n C_t + R S_t \left(\frac{P_t^{H*}}{P_t^*}\right)^{1-\sigma} (1-n) C_t^* - C_t}_{\equiv G_t},$$

where $B_t^{RF} \equiv \frac{S_t B_t^F}{P_t}$ is the net foreign asset position in real terms. The function G_t identifies the current account balance of trade (in goods) between the two countries in period t. In steady state holds that $\overline{f} = \overline{g}$ and $\overline{f}^* = \overline{g}^*$, then the log-linearization around the steady state can be expressed as,

$$\widehat{f}_t \approx \widehat{g}_t,$$

where the approximations are as follows,

$$\widehat{f}_t \equiv \beta\overline{C}\widehat{B}_t^{RF} - \overline{C}\widehat{B}_{t-1}^{RF} + 0\cdot\Delta\widehat{s}_t + 0\cdot\widehat{\pi}_t,$$
$$\widehat{g}_t \equiv (1-\sigma)\,n\overline{C}\left(\widehat{p}_t^H - \widehat{p}_t\right) + (1-\sigma)(1-n)\,\overline{C}\left(\widehat{p}_t^{H*} - \widehat{p}_t^*\right) + (1-n)\,\overline{C}\widehat{rs}_t + (n-1)\,\overline{C}\widehat{c}_t + (1-n)\,\overline{C}\widehat{c}_t^*.$$

where $\widehat{B}_t^{RF} \equiv \frac{S_t B_t^F}{P_t}\frac{1}{\overline{C}}$ represents the real per capita net foreign asset position, relative to domestic steady state consumption. Notice that the deterministic steady state of the model implies that the current account is zero and $\overline{B}^{RF} = 0$ (if $\lambda \neq 0$). This result affects the coefficients of the linearized equation $\widehat{f}_t$. Most notably, it means that the nominal exchange rate depreciation and the domestic CPI inflation rate drop out altogether.

Using the linearized price indexes in $(36)-(37)$ to substitute out $\widehat{p}_t$ and $\widehat{p}_t^*$, and the definition of relative prices in $(39)-(40)$, I obtain the following *CA* equation,

$$\beta\widehat{B}_t^{RF} \approx \widehat{B}_{t-1}^{RF} + (1-n)\left[(\sigma-1)\left(n\widehat{t}_t - (1-n)\,\widehat{t}_t^*\right) + \widehat{rs}_t - (\widehat{c}_t - \widehat{c}_t^*)\right], \tag{60}$$

where the world relative price is defined as $\widehat{t}_t^W \equiv n\widehat{t}_t - (1-n)\,\widehat{t}_t^*$. This is the version of the *CA* equation that I use in the paper. Therefore, the current account is a function of world relative prices, the real exchange rate and the consumption differential across countries. The only parameters that affect the current account are the population size, n, and the elasticity of intratemporal substitution between home and foreign bundles, σ. This is a special case of the expression derived by Thoenissen (2003).

E The Linearized Rational Expectations Model

The model I study in depth in my paper is described in four basic blocks. First, I combine the investment-savings equations in $(43)-(44)$ and the money market equations in $(47)-(48)$ to define the two *demand-side equations* of the economy as follows,

$$\begin{aligned}&\tfrac{\gamma}{1-b}\mathbb{E}\left[\Delta\widehat{c}_{t+1} - b\Delta\widehat{c}_t - \left(\tfrac{1-\beta}{\beta}\right)(\widehat{c}_t - b\widehat{c}_{t-1}) \mid \mathcal{H}_t\right]\\ &\approx -\mathbb{E}\left[n\widehat{\pi}_{t+1}^H + (1-n)\,\widehat{\pi}_{t+1}^F - \gamma\left(\tfrac{1-\beta}{\beta}\right)\left(n\widehat{p}_t^H + (1-n)\,\widehat{p}_t^F\right) \mid \mathcal{H}_t\right] - \mathbb{E}\left[\gamma\left(\tfrac{1-\beta}{\beta}\right)\widehat{m}_t + \Delta\widehat{\xi}_{t+1} \mid \mathcal{H}_t\right],\end{aligned} \tag{61}$$

$$\begin{aligned}&\tfrac{\gamma}{1-b}\mathbb{E}\left[\Delta\widehat{c}_{t+1}^* - b\Delta\widehat{c}_t^* - \left(\tfrac{1-\beta}{\beta}\right)(\widehat{c}_t^* - b\widehat{c}_{t-1}^*) \mid \mathcal{H}_t^*\right]\\ &\approx -\mathbb{E}\left[n\widehat{\pi}_{t+1}^{H*} + (1-n)\,\widehat{\pi}_{t+1}^{F*} - \gamma\left(\tfrac{1-\beta}{\beta}\right)\left(n\widehat{p}_t^{H*} + (1-n)\,\widehat{p}_t^{F*}\right) \mid \mathcal{H}_t^*\right] - \mathbb{E}\left[\gamma\left(\tfrac{1-\beta}{\beta}\right)\widehat{m}_t^* + \Delta\widehat{\xi}_{t+1}^* \mid \mathcal{H}_t^*\right].\end{aligned} \tag{62}$$

Second, I combine the aggregate-supply equations in $(55)-(58)$ with the definition of relative prices and the real exchange rate in $(38)-(40)$, to define the four *supply-side equations* of the economy as follows,

$$\begin{aligned}\widehat{\pi}_t^H &\approx \beta\mathbb{E}\left[\widehat{\pi}_{t+1}^H \mid \mathcal{F}_t\right] + k_\pi^H\widehat{\pi}_t^H + k_{\pi^*}^H\widehat{\pi}_t^{H*} + k_c^H\mathbb{E}\left[\tfrac{\gamma}{1-b}(\widehat{c}_t - b\widehat{c}_{t-1}) + \varphi\widehat{c}_t^W - (1+\varphi)\,\widehat{a}_t \mid \mathcal{F}_t\right] +\\ &+ \mathbb{E}\left[k_{rs}^H\widehat{s}_t - \left(k_t^H + nk_{rs}^H\right)\widehat{p}_t^H + \left(k_{t^*}^H + nk_{rs}^H\right)\widehat{p}_t^{H*} + \left(k_t^H - (1-n)\,k_{rs}^H\right)\widehat{p}_t^F - \left(k_{t^*}^H - (1-n)\,k_{rs}^H\right)\widehat{p}_t^{F*} \mid \mathcal{F}_t\right],\end{aligned} \tag{63}$$

$$\begin{aligned}\widehat{\pi}_t^{H*} &\approx \beta\mathbb{E}\left[\widehat{\pi}_{t+1}^{H*} \mid \mathcal{F}_t\right] + k_\pi^{H*}\widehat{\pi}_t^H + k_{\pi^*}^{H*}\widehat{\pi}_t^{H*} + k_c^{H*}\mathbb{E}\left[\tfrac{\gamma}{1-b}(\widehat{c}_t - b\widehat{c}_{t-1}) + \varphi\widehat{c}_t^W - (1+\varphi)\,\widehat{a}_t \mid \mathcal{F}_t\right] +\\ &+ \mathbb{E}\left[k_{rs}^{H*}\widehat{s}_t - \left(k_t^{H*} + nk_{rs}^{H*}\right)\widehat{p}_t^H + \left(k_{t^*}^{H*} + nk_{rs}^{H*}\right)\widehat{p}_t^{H*} + \left(k_t^{H*} - (1-n)\,k_{rs}^{H*}\right)\widehat{p}_t^F - \left(k_{t^*}^{H*} - (1-n)\,k_{rs}^{H*}\right)\widehat{p}_t^{F*} \mid \mathcal{F}_t\right],\end{aligned} \tag{64}$$

$$\begin{aligned}\widehat{\pi}_t^F &\approx \beta\mathbb{E}\left[\widehat{\pi}_{t+1}^F \mid \mathcal{F}_t^*\right] + k_\pi^F\widehat{\pi}_t^F + k_{\pi^*}^F\widehat{\pi}_t^{F*} + k_c^F\mathbb{E}\left[\tfrac{\gamma}{1-b}\left(\widehat{c}_t^* - b\widehat{c}_{t-1}^*\right) + \varphi\widehat{c}_t^W - (1+\varphi)\,\widehat{a}_t^* \mid \mathcal{F}_t^*\right] +\\ &+ \mathbb{E}\left[k_{rs}^F\widehat{s}_t - \left(k_t^F + nk_{rs}^F\right)\widehat{p}_t^H + \left(k_{t^*}^F + nk_{rs}^F\right)\widehat{p}_t^{H*} + \left(k_t^F - (1-n)\,k_{rs}^F\right)\widehat{p}_t^F - \left(k_{t^*}^F - (1-n)\,k_{rs}^F\right)\widehat{p}_t^{F*} \mid \mathcal{F}_t^*\right],\end{aligned} \tag{65}$$

$$\begin{aligned}\widehat{\pi}_t^{F*} &\approx \beta\mathbb{E}\left[\widehat{\pi}_{t+1}^{F*} \mid \mathcal{F}_t^*\right] + k_\pi^{F*}\widehat{\pi}_t^F + k_{\pi^*}^{F*}\widehat{\pi}_t^{F*} + k_c^{F*}\mathbb{E}\left[\tfrac{\gamma}{1-b}\left(\widehat{c}_t^* - b\widehat{c}_{t-1}^*\right) + \varphi\widehat{c}_t^W - (1+\varphi)\,\widehat{a}_t^* \mid \mathcal{F}_t^*\right] +\\ &+ \mathbb{E}\left[k_{rs}^{F*}\widehat{s}_t - \left(k_t^{F*} + nk_{rs}^{F*}\right)\widehat{p}_t^H + \left(k_{t^*}^{F*} + nk_{rs}^{F*}\right)\widehat{p}_t^{H*} + \left(k_t^{F*} - (1-n)\,k_{rs}^{F*}\right)\widehat{p}_t^F - \left(k_{t^*}^{F*} - (1-n)\,k_{rs}^{F*}\right)\widehat{p}_t^{F*} \mid \mathcal{F}_t^*\right],\end{aligned} \tag{66}$$

where the coefficients of composite parameters are given by,

$$
\begin{aligned}
k_{\pi}^{H} &\equiv -\left(\frac{\alpha^{H*}-\alpha^{H}}{\alpha^{H*}}\right)\left(\frac{\varphi\theta(1-n)}{1+\varphi\theta}\right)\varphi\theta n, \\
k_{\pi^*}^{H} &\equiv \left(\frac{\alpha^{H}-\alpha^{H*}}{\alpha^{H}}\right)\left(\frac{1-\alpha^{H}}{1-\alpha^{H*}}\right)\left(\frac{1+\varphi\theta(1-n)}{1+\varphi\theta}\right)\varphi\theta\left(1-n\right), \\
k_{c}^{H} &\equiv \Phi^{H}\left(\frac{1+\varphi\theta(1-n)}{1+\varphi\theta}\right)-\Phi_{H}^{H*}\left(\frac{\varphi\theta(1-n)}{1+\varphi\theta}\right), \\
k_{rs}^{H} &\equiv \Phi_{H}^{H*}\left(\frac{\varphi\theta(1-n)}{1+\varphi\theta}\right), \\
k_{t}^{H} &\equiv \left[\begin{array}{c} \Phi^{H}\left(\frac{(1-n)(1+\varphi\theta(1-n))(1+\varphi\sigma n)}{1+\varphi\theta}\right)- \\ -\Phi_{H}^{H*}\left(\frac{(1-n)^{2}\varphi\theta(\varphi\sigma n)}{1+\varphi\theta}\right) \end{array}\right], \\
k_{t^*}^{H} &\equiv \left[\begin{array}{c} \Phi_{H}^{H*}\left(\frac{(1-n)^{2}(1+\varphi\sigma(1-n))\varphi\theta}{1+\varphi\theta}\right)- \\ -\Phi^{H}\left(\frac{(1-n)^{2}(1+\varphi\theta(1-n))\varphi\sigma}{1+\varphi\theta}\right) \end{array}\right],
\end{aligned}
\qquad
\begin{aligned}
k_{\pi}^{H*} &\equiv \left(\frac{\alpha^{H*}-\alpha^{H}}{\alpha^{H*}}\right)\left(\frac{1-\alpha^{H*}}{1-\alpha^{H}}\right)\left(\frac{1+\varphi\theta n}{1+\varphi\theta}\right)\varphi\theta n, \\
k_{\pi^*}^{H*} &\equiv -\left(\frac{\alpha^{H}-\alpha^{H*}}{\alpha^{H}}\right)\left(\frac{\varphi\theta n}{1+\varphi\theta}\right)\varphi\theta\left(1-n\right), \\
k_{c}^{H*} &\equiv \Phi^{H*}\left(\frac{1+\varphi\theta n}{1+\varphi\theta}\right)-\Phi_{H*}^{H}\left(\frac{\varphi\theta n}{1+\varphi\theta}\right), \\
k_{rs}^{H*} &\equiv -\Phi^{H*}\left(\frac{1+\varphi\theta n}{1+\varphi\theta}\right), \\
k_{t}^{H*} &\equiv \left[\begin{array}{c} \Phi^{H*}\left(\frac{(1-n)n\varphi\sigma(1+\varphi\theta n)}{1+\varphi\theta}\right)- \\ -\Phi_{H*}^{H}\left(\frac{(1-n)n\varphi\theta(1+\varphi\sigma n)}{1+\varphi\theta}\right) \end{array}\right], \\
k_{t^*}^{H*} &\equiv \left[\begin{array}{c} \Phi_{H*}^{H}\left(\frac{(1-n)^{2}(\varphi\theta n)\varphi\sigma}{1+\varphi\theta}\right)- \\ -\Phi^{H*}\left(\frac{(1-n)(1+\varphi\theta n)(1+\varphi\sigma(1-n))}{1+\varphi\theta}\right) \end{array}\right],
\end{aligned}
$$

$$
\begin{aligned}
k_{\pi}^{F} &\equiv -\left(\frac{\alpha^{F*}-\alpha^{F}}{\alpha^{F*}}\right)\left(\frac{\varphi\theta(1-n)}{1+\varphi\theta}\right)\varphi\theta n, \\
k_{\pi^*}^{F} &\equiv \left(\frac{\alpha^{F}-\alpha^{F*}}{\alpha^{F}}\right)\left(\frac{1-\alpha^{F}}{1-\alpha^{F*}}\right)\left(\frac{\varphi\theta(1-n)(1+\varphi\theta(1-n))}{1+\varphi\theta}\right), \\
k_{c}^{F} &\equiv \Phi^{F}\left(\frac{1+\varphi\theta(1-n)}{1+\varphi\theta}\right)-\Phi_{F}^{F*}\left(\frac{\varphi\theta(1-n)}{1+\varphi\theta}\right), \\
k_{rs}^{F} &\equiv \Phi^{F}\left(\frac{1+\varphi\theta(1-n)}{1+\varphi\theta}\right), \\
k_{t}^{F} &\equiv \left[\begin{array}{c} \Phi_{F}^{F*}\left(\frac{n^{2}\varphi\theta(1-n)\varphi\sigma}{1+\varphi\theta}\right)- \\ -\Phi^{F}\left(\frac{n(1+\varphi\theta(1-n))(1+\varphi\sigma n)}{1+\varphi\theta}\right) \end{array}\right], \\
k_{t^*}^{F} &\equiv \left[\begin{array}{c} \Phi^{F}\left(\frac{n\varphi\sigma(1-n)(1+\varphi\theta(1-n))}{1+\varphi\theta}\right)- \\ -\Phi_{F}^{F*}\left(\frac{n\varphi\theta(1-n)(1+\varphi\sigma(1-n))}{1+\varphi\theta}\right) \end{array}\right],
\end{aligned}
\qquad
\begin{aligned}
k_{\pi}^{F*} &\equiv \left(\frac{\alpha^{F*}-\alpha^{F}}{\alpha^{F*}}\right)\left(\frac{1-\alpha^{F*}}{1-\alpha^{F}}\right)\left(\frac{(1+\varphi\theta n)\varphi\theta n}{1+\varphi\theta}\right), \\
k_{\pi^*}^{F*} &\equiv -\left(\frac{\alpha^{F}-\alpha^{F*}}{\alpha^{F}}\right)\left(\frac{(\varphi\theta n)\varphi\theta(1-n)}{1+\varphi\theta}\right), \\
k_{c}^{F*} &\equiv \Phi^{F*}\left(\frac{1+\varphi\theta n}{1+\varphi\theta}\right)-\Phi_{F*}^{F}\left(\frac{\varphi\theta n}{1+\varphi\theta}\right), \\
k_{rs}^{F*} &\equiv -\Phi_{F*}^{F}\left(\frac{\varphi\theta n}{1+\varphi\theta}\right), \\
k_{t}^{F*} &\equiv \left[\begin{array}{c} \Phi_{F*}^{F}\left(\frac{n^{2}\varphi\theta(1+\varphi\sigma n)}{1+\varphi\theta}\right)- \\ -\Phi^{F*}\left(\frac{n^{2}(1+\varphi\theta n)\varphi\sigma}{1+\varphi\theta}\right) \end{array}\right], \\
k_{t^*}^{F*} &\equiv \left[\begin{array}{c} \Phi^{F*}\left(\frac{n(1+\varphi\theta n)(1+\varphi\sigma(1-n))}{1+\varphi\theta}\right)- \\ -\Phi_{F*}^{F}\left(\frac{n\varphi\sigma(1-n)(\varphi\theta n)}{1+\varphi\theta}\right) \end{array}\right],
\end{aligned}
$$

$$
\begin{aligned}
\Phi_{H*}^{H} &\equiv \left(\frac{\left(1-\beta\alpha^{H}\right)\left(1-\alpha^{H*}\right)}{\alpha^{H}}\right), \\
\Phi_{F*}^{F} &\equiv \left(\frac{\left(1-\beta\alpha^{F}\right)\left(1-\alpha^{F*}\right)}{\alpha^{F}}\right),
\end{aligned}
\qquad
\begin{aligned}
\Phi_{H}^{H*} &\equiv \left(\frac{\left(1-\beta\alpha^{H*}\right)\left(1-\alpha^{H}\right)}{\alpha^{H*}}\right), \\
\Phi_{F}^{F*} &\equiv \left(\frac{\left(1-\beta\alpha^{F*}\right)\left(1-\alpha^{F}\right)}{\alpha^{F*}}\right),
\end{aligned}
$$

$$
\Phi^{i} \equiv \left(\frac{\left(1-\beta\alpha^{i}\right)\left(1-\alpha^{i}\right)}{\alpha^{i}}\right),\ i = H, H^{*}, F, F^{*},
$$

Third, I combine my version of the uncovered interest parity equation in (59), the price indexes in $(36)-(37)$ and the money market equations in $(47)-(48)$, to define the *financial-side equation* of the economy as follows,

$$
\begin{aligned}
&\mathbb{E}\left[\Delta\widehat{s}_{t+1}+\gamma\left(\frac{1-\beta}{\beta}\right)\left(\widehat{m}_{t}-\widehat{m}_{t}^{*}\right)-\delta\widehat{B}_{t}^{RF}\mid\mathcal{I}_{t}\right] \\
&\quad\approx\left(\frac{1-\beta}{\beta}\right)\mathbb{E}\left[\gamma\left(n\left(\widehat{p}_{t}^{H}-\widehat{p}_{t}^{H*}\right)+(1-n)\left(\widehat{p}_{t}^{F}-\widehat{p}_{t}^{F*}\right)\right)+\frac{\gamma}{1-b}\left(\widehat{c}_{t}-\widehat{c}_{t}^{*}-b\left(\widehat{c}_{t-1}-\widehat{c}_{t-1}^{*}\right)\right)\mid\mathcal{I}_{t}\right],
\end{aligned}
\tag{67}
$$

where $\delta\equiv\lambda\frac{1}{1-b}\left(\frac{1-\beta}{\chi}\right)^{\frac{1}{\gamma}}$. Forth, I combine the current account equation in (60) with the definition of relative prices and the real exchange rate in $(38)-(40)$, to define the *'international'-side equation* of the economy as follows,

$$
\beta\widehat{B}_{t}^{RF}\approx\widehat{B}_{t-1}^{RF}+(1-n)\left[\widehat{s}_{t}-n\sigma\widehat{p}_{t}^{H}+\left(1-(1-n)\sigma\right)\widehat{p}_{t}^{H*}-\left(1-n\sigma\right)\widehat{p}_{t}^{F}+\left(1-n\right)\sigma\widehat{p}_{t}^{F*}-\left(\widehat{c}_{t}-\widehat{c}_{t}^{*}\right)\right].
\tag{68}
$$

Let me denote $\widehat{z}_{t}=\left(\widehat{c}_{t},\widehat{c}_{t}^{*},\widehat{p}_{t}^{H},\widehat{p}_{t}^{H*},\widehat{p}_{t}^{F},\widehat{p}_{t}^{F*},\widehat{s}_{t},\widehat{B}_{t}^{RF}\right)^{T}$ the (8×1) vector of all relevant endogenous variables determined at time t. Let me denote $\widehat{\theta}_{t}=\left(\widehat{m}_{t},\widehat{m}_{t}^{*},\Delta\widehat{\xi}_{t},\Delta\widehat{\xi}_{t}^{*},\widehat{a}_{t},\widehat{a}_{t}^{*}\right)^{T}$ the (6×1) vector of exogenous variables at time t. The elements in $\widehat{z}_{t}$ and $\widehat{\theta}_{t}$ are expressed in deviations relative to their deterministic steady state values. In my paper 'A Monetary Model of the Exchange Rate with Informational Frictions' I study the model configured in equations (61) – (68). All other endogenous variables, like the real exchange rate or the CPI inflation rates, can be linearly approximated as functions of $\widehat{z}_t$. I conjecture that the solution $\widehat{z}_t$ to this system of equations takes the form of a VARMA process where the MA part depends on current and past realizations of $\widehat{\theta}_t$. Notice, however, that a solution may not exist for certain range of values in the parameter space. For more details on the solution method, read my paper or -even better- Christiano's (2002) paper.

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