ISSN 1936-5330

THE FEDERAL RESERVE BANK *of* KANSAS CITY
ECONOMIC RESEARCH DEPARTMENT

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October 2008
RWP 08-03

RESEARCH WORKING PAPERS

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Abstract: We investigate implications of search and matching frictions in the labor market for inflation targeting interest rate policy in terms of equilibrium stability. When the interest rate is set in response to past or present inflation, determinacy of equilibrium is ensured similarly to comparable previous studies with frictionless labor markets. In stark contrast to these studies, indeterminacy is very likely if the interest rate is adjusted in response solely to expected future inflation. This is due to a vacancy channel of monetary policy that stems from the labor market frictions and renders inflation expectations self-fulfilling. The indeterminacy can be overcome once the interest rate is adjusted in response also to output or the unemployment rate or if the policy contains interest rate smoothing. When E-stability is adopted as an equilibrium selection criterion, a unique E-stable fundamental rational expectations equilibrium is generated under active, but not too strong, policy responses only to expected future inflation. This suggests that the problem is not critical from the perspective of learnability of the fundamental equilibrium.

Keywords: Labor market search and matching frictions; inflation targeting; indeterminacy; vacancy channel of monetary policy; E-stability

JEL classification: E24; E52

The authors are grateful for discussions and comments from Miguel Casares, Kai Christoffel, Daniele Coen-Pirani, John Duffy, Huberto Ennis, Marvin Goodfriend, Michael Krause, Thomas Lubik, Bennett McCallum, and Alexander Wolman, as well as participants at the 2007 Midwest Macroeconomics Meeting, EEA-ESEM 2007, 2008 North American Summer Meeting of the Econometric Society, 2008 Learning Week Conference, Bank for International Settlements, Bank of Canada, European Central Bank, Federal Reserve Bank of Kansas City, Federal Reserve Bank of Richmond, Portland State University, University of Notre Dame, and West Virginia University. Any remaining errors are the sole responsibility of the authors. The paper has been written while the second author was visiting the Research Department of the Federal Reserve Bank of Richmond. He thanks the Richmond Fed for a stimulating research environment. The views expressed herein are those of the authors and should not be interpreted as those of the Bank of Japan, the Federal Reserve Bank of Kansas City, the Federal Reserve Bank of Richmond or the Federal Reserve System.

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1 Introduction

There has recently been a surge of interest in the role of labor market search and matching frictions along the lines of Mortensen and Pissarides (1994) in dynamic stochastic general equilibrium models with sticky prices.[1] Employment adjustment takes time and place at the extensive margin, giving rise to equilibrium unemployment, and wages are determined by Nash bargaining between workers and firms. These features are in stark contrast with Walrasian competitive labor markets, which have been used in the monetary policy literature.[2]

In this paper we examine implications of the labor market search and matching frictions for inflation targeting interest rate policy in terms of equilibrium stability. We consider three policy specifications, each of which adjusts the interest rate in response solely to either past inflation (backward-looking), present inflation (current-looking) or expected future inflation (forward-looking). We show that the current-looking and the backward-looking policies ensure (local) determinacy of rational expectations equilibrium (REE) under similar conditions to those obtained in comparable previous studies with frictionless labor markets, such as Bullard and Mitra (2002) and Woodford (2003). Determinacy is guaranteed under active policy responses to current inflation or under active, but not too strong, ones to past inflation. In stark contrast to the previous studies, we find that the forward-looking policy is very likely to induce indeterminacy, and thus makes excessively volatile REE possible. This finding is critical because actual central banks, inflation targeting ones in particular, are concerned about expected future inflation rather than actual past or present inflation.

Why does the forward-looking policy render REE indeterminate? Passive policy does so, due to the weakness of the conventional aggregate demand channel of monetary policy, as in line with the previous studies. Active policy also induces indeterminacy in the presence of a vacancy channel of monetary policy that stems from the labor market search and matching frictions. This is in stark contrast with the previous studies and it occurs because the vacancy channel makes inflation expectations self-fulfilling. The labor market frictions result in firms' sluggish adjustment in employment and hence output. Specifically, firms' reduction in vacancy posting, induced by dampened consumption demand following a rise in the real interest rate, decreases the level of employment available for production in current and future periods. Thus, the real

[1]See e.g. Christoffel and Kuester (2008), Krause and Lubik (2007), Krause, Lopez-Salido and Lubik (2008), Ravenna and Walsh (2008), Sveen and Weinke (2008), Trigari (2008), Walsh (2005), Van Zandweghe (2007), among others.

[2]Some recent exceptions are Blanchard and Galí (2008), Faia (2008) and Thomas (2008), who study optimal monetary policy in the presence of frictional labor markets.

interest rate rise lowers future output supply. At the same time, such a rate rise also prompts households to substitute current with future consumption, so that firms expect consumption demand to recover in future periods after its current decline. From this expected rise in future demand and the diminished future supply, firms anticipate a strong expansion of future vacancy posting, which raises expected future real marginal cost and hence expected future inflation. Therefore, the vacancy channel leads a rise in the real interest rate to increase expected future inflation. This renders inflation expectations self-fulfilling under sufficiently strong active policy responses solely to expected future inflation, thereby inducing indeterminacy of REE.

Actual labor markets are characterized by search and matching frictions, and likewise much evidence suggests that monetary policy in major economies has been forward-looking especially since 1979 (e.g. Clarida, Galí and Gertler, 1998). Yet the actual economy has not exhibited excessive volatility in recent decades as the vacancy channel leads to predict.[3] We examine two possible explanations. The first one is interest rate smoothing or interest rate policy adjustment for output or the unemployment rate in addition to expected future inflation. We then find that the policy adjustment for current output can overcome the indeterminacy as long as a long run version of the Taylor principle is satisfied: in the long run the nominal interest rate should be raised by more than the increase in inflation. With a policy adjustment for expected future output, this amelioration of the problem is limited to mild policy responses, since a strong policy response to expected future output causes indeterminacy as in line with previous studies with frictionless labor markets.[4] The intuition for the amelioration is that indeterminacy is induced by the vacancy channel of the forward-looking policy, in which a rise in the real interest rate stemming from inflationary expectations increases expected future inflation and hence such expectations become self-fulfilling. But, the policy adjustment for current or expected future output subdues the real interest rate rise because output falls as a consequence of such a rate rise. With an interest rate policy reaction to the unemployment rate we find that the forward-looking policy brings about determinacy when it satisfies an associated long run version of the Taylor principle. This is because in our model the unemployment rate changes proportionally to fluctuations in production, so that the policy reaction to the unemployment rate yields almost the same result as the one to current output. Finally, we consider interest rate smoothing and find that it helps the forward-looking policy generate determinacy. Such smoothing implies

[3] On the contrary, there is ample evidence of a moderation of U.S. economic aggregates since 1984.

[4] The upper bound on the policy coefficient on expected future output that guarantees determinacy is induced by the demand channel of monetary policy, as mentioned later.

policy responses to lagged interest rates and hence makes the forward-looking policy respond also to current and past inflation like the current-looking and the backward-looking policies, thereby ameliorating the indeterminacy problem. These results provide an additional argument in favor of flexible interest rate policy instead of strict inflation targeting, and the policies thus constitute prescriptions for the indeterminacy.

Next, we consider expectational (or E-)stability as an REE selection criterion and examine whether the forward-looking policy generates a unique E-stable fundamental REE even in cases of indeterminacy.[5] As Evans and Honkapohja (2001) show in a broad class of linear stochastic models, if a fundamental REE is E-stable, it is least-squares learnable, i.e. stable under least-squares learning. Therefore, E-stability is an essential condition for any REE to be regarded as plausible, as stressed by McCallum (2003).[6] We find that a unique E-stable fundamental REE is generated under active, but not too strong, policy responses solely to expected future inflation. This is in stark contrast with Bullard and Mitra (2002), who show that the Taylor principle (i.e. active policy) is a necessary and sufficient condition for the unique E-stable REE in the absence of the labor market search and matching frictions. The presence of such frictions makes the Taylor principle no longer a sufficient condition. Since the interval of the policy coefficient on expected future inflation that generates the unique E-stable REE is wide enough to contain all empirically relevant values, our E-stability result suggests that the indeterminacy problem induced by the forward-looking policy is not critical from the perspective of E-stability or least-squares learnability of fundamental REE.

The findings above are based on our benchmark model in which consumption preferences are standard, job destruction is exogenous and hiring is instantaneous. These findings remain unchanged qualitatively even when we introduce habit formation in consumption preferences or when we consider an alternative labor market specification in which jobs are also endogenously destroyed and new hires become productive in the subsequent period, which is a more conventional one used in previous studies such as Trigari (2008), Walsh (2005) and Krause and Lubik (2007).

[5]Throughout the paper, "fundamental" refers to Evans and Honkapohja's (2001) minimal state variable (MSV) solutions to linear RE models so as to distinguish them from McCallum's (1983) original MSV solution. We do not examine E-stability of non-fundamental REE such as sunspot equilibria, which may exist in cases of indeterminacy. For E-stability analysis of these REE, see e.g. Honkapohja and Mitra (2004), Carlstrom and Fuerst (2004) and Evans and McGough (2005), who all use associated models with frictionless labor markets. We leave E-stability analysis of non-fundamental REE in our model for future work.

[6]McCallum (2003) argues that in cases of indeterminacy there may be a unique REE that is E-stable and thus least-squares learnable, whereas a determinate REE that is E-unstable and thus not least-squares learnable is arguably not a plausible candidate for equilibrium that could be observed in the actual economy.

Among related literature, Burda and Weder (2002), Giammarioli (2003), and Krause and Lubik (2004) analyze equilibrium determinacy in real business cycle models with labor market search and matching frictions, yielding no implication for monetary policy. Zanetti (2006) investigates monetary policy implications using a sticky price model in which wages and employment are determined via simultaneous Nash bargaining, but such a model involves no search and matching frictions. To our knowledge, the present paper is the first to examine monetary policy implications of labor market search and matching frictions in terms of determinacy and E-stability of REE.

The remainder of the paper proceeds as follows. Section 2 presents an optimizing model with sticky prices and labor market search and matching frictions. Section 3 analyzes determinacy of REE under three alternative specifications of inflation targeting interest rate policy. Section 4 considers prescriptions for the indeterminacy problem induced by the forward-looking policy. Section 5 assesses the problem from the perspective of E-stability. Section 6 contains some robustness analysis in which habit formation is introduced in consumption preferences or a more conventional labor market specification is used. Finally, Section 7 concludes.

2 A model with labor market search and matching frictions

Our model is an optimizing model with sticky prices and labor market search and matching frictions. This model is in line with recent business cycle studies, such as Christoffel and Kuester (2008), Krause and Lubik (2007), Krause, Lopez-Salido and Lubik (2008), Ravenna and Walsh (2008), Sveen and Weinke (2008), Trigari (2008), Walsh (2005), and Van Zandweghe (2007). But, it is in stark contrast to recent monetary policy studies with competitive labor markets in that employment adjustment is costly and takes place at the extensive margin, which gives rise to equilibrium unemployment, and wages are determined by Nash bargaining.

The model economy consists of four types of agents: households, perfectly competitive wholesale firms, monopolistically competitive retail firms, and a monetary authority.[7] We describe each in turn.

[7]As in recent monetary policy studies, we assume that fiscal policy is 'Ricardian', i.e. it appropriately accommodates consequences of monetary policy for the government budget constraint. We thus leave hidden the government budget constraint and fiscal policy. For recent analyses of equilibrium determinacy under non-Ricardian fiscal policy and interest rate policy, see e.g. Benhabib, Schmitt-Grohé and Uribe (2001), Benhabib and Eusepi (2005), Linnemann (2006) and Kurozumi (2005).

2.1 Households

In the economy there is a continuum of households. To avoid distributional issues, we assume as in Andolfatto (1996) and Merz (1995) that employed and unemployed households pool consumption. Thus, we can consider the presence of a representative household. This household purchases C_t consumption goods, supplies one unit of labor inelastically, and holds B_t nominal one-period bonds, which earn the gross nominal interest rate R_t in the subsequent period. The household chooses consumption and bond holdings so as to maximize expected lifetime utility

$$E_0 \sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\sigma}}{1-\sigma} e^{g_t}$$

under the budget constraint

$$P_t C_t + B_t \leq P_t D_t + B_{t-1} R_{t-1},$$

where $\beta \in (0,1)$ is a discount factor, $\sigma > 0$ measures (relative) risk aversion, g_t is a preference shock, D_t is real income that consists of monopoly profits from retail firms, rents related to labor market frictions from wholesale firms, either a wage w_t from employment or a benefit b when unemployed, minus a lump-sum transfer to finance unemployment benefits. The disutility from employment is normalized to zero. Consumption $C_t = [\int_0^1 C_t(i)^{(\epsilon-1)/\epsilon} di]^{\epsilon/(\epsilon-1)}$ is a composite of differentiated goods produced by retail firms, with an elasticity of substitution $\epsilon > 1$. Thus, cost-minimizing demand for good i is given by $C_t(i) = [P_t(i)/P_t]^{-\epsilon} C_t$, where the aggregate price index satisfies

$$P_t = \left[\int_0^1 P_t(i)^{\epsilon-1} di \right]^{\frac{1}{\epsilon-1}}. \tag{1}$$

The optimality conditions for consumption and bond holdings are given by

$$\lambda_t = C_t^{-\sigma} e^{g_t}, \quad \lambda_t = \beta E_t \lambda_{t+1} \frac{R_t}{\pi_{t+1}},$$

where λ_t / P_t is the Lagrange multiplier on the budget constraint and $\pi_t = P_t / P_{t-1}$ is the gross inflation rate. These conditions yield the consumption Euler equation

$$C_t^{-\sigma} e^{g_t} = \beta E_t C_{t+1}^{-\sigma} e^{g_{t+1}} \frac{R_t}{\pi_{t+1}}. \tag{2}$$

2.2 Wholesale firms

Wholesale firms use labor as the only input in production and sell homogeneous goods at a price P_t^w to retail firms in a perfectly competitive market. The labor market is characterized by search and matching frictions. The population size is normalized to one. The time line of period t is as follows. At the beginning of the period there are n_{t-1} matches between workers and wholesale firms. Then, a proportion $\rho \in (0,1)$ of the existing matches is exogenously destroyed, thus job destruction equals ρn_{t-1}.[8] These workers join the pool of searching workers, so that the measure of search unemployment is

$$u_t = 1 - (1-\rho)n_{t-1}. \tag{3}$$

Next, u_t searching workers and v_t vacancies participate in the matching market, giving rise to m_t new matches (i.e. job creation), a number increasing in search unemployment and vacancies according to a constant returns to scale technology $m_t = \psi u_t^{\xi} v_t^{1-\xi}$, where $\psi > 0$ and $\xi \in (0,1)$ is the search elasticity of matches. We assume as in Blanchard and Galí (2008) and Ravenna and Walsh (2008) that new matches become productive instantaneously.[9] Thus, the number of worker-firm matches that produce in period t is given by

$$n_t = (1-\rho)n_{t-1} + m_t, \tag{4}$$

where the change in employment is equal to the difference between job creation and job destruction. Then, the unemployment rate is

$$U_t = 1 - n_t. \tag{5}$$

Each worker-firm match produces one unit of wholesale goods in every period, so that aggregate production of the wholesale sector is

$$y_t = n_t. \tag{6}$$

The ratio

$$\theta_t = \frac{v_t}{u_t} \tag{7}$$

measures the tightness of the labor market. An unmatched wholesale firm's probability to fill a vacancy (i.e. the firm matching rate) is

$$q_t \equiv \frac{m_t}{v_t} = \psi \theta_t^{-\xi}, \tag{8}$$

[8]The exogenous job destruction rate is empirically supported by Hall (2006) and Shimer (2007), who argue that the job separation rate explains only a small fraction of fluctuations in the unemployment rate.

[9]In Section 6.2 we analyze a more conventional timing in which new matches become productive in the subsequent period and contemporaneous employment adjustment takes place only via job destruction.

which rises when the labor market becomes slack. A searching worker's probability to find a job (i.e. the worker matching rate) is

$$p_t \equiv \frac{m_t}{u_t} = \psi \theta_t^{1-\xi}, \tag{9}$$

which is increasing in labor market tightness.

Job creation is costly for wholesale firms, which must pay a fixed cost $\gamma > 0$ each period they post vacancies. This cost gives rise to a joint surplus from a match, which is split between the matched worker and firm through Nash bargaining. To determine a wage that gives the worker his/her share of the bargain, it is convenient to consider asset values of matched and unmatched workers and firms. The asset value of a matched firm, F_t^m, is the sum of real net revenue that accrues to the firm in the current period and the discounted present value of this firm in the next period. The match is dissolved with probability ρ, so that the value of a matched firm is given by

$$F_t^m = z_t - w_t + E_t \beta_{t,t+1}[(1-\rho)F_{t+1}^m + \rho F_{t+1}^u],$$

where $\beta_{t,t+j} = \beta^j \lambda_{t+j}/\lambda_t$ is the stochastic discount factor, $z_t = P_t^w/P_t$ is the real price of wholesale goods, and F_t^u is the asset value of an unmatched firm in period t. An unmatched firm pays the vacancy posting cost and is matched with probability q_t. Since new matches become productive instantaneously, the value of an unmatched firm is given by

$$F_t^u = -\gamma + q_t F_t^m + (1-q_t) E_t \beta_{t,t+1} F_{t+1}^u.$$

Free entry in the matching market drives the asset value of an unmatched firm to zero in equilibrium. Combining these firm asset values yields a job creation condition that makes the expected cost of a match equal its expected value

$$\frac{\gamma}{q_t} = z_t - w_t + E_t \beta_{t,t+1}(1-\rho)\frac{\gamma}{q_{t+1}}.$$

The asset value of a matched (unmatched) worker is the wage (unemployment benefit) plus the expected present discounted value of this worker's employment status in the next period

$$\begin{aligned} W_t^m &= w_t + E_t \beta_{t,t+1} \left\{ [1-\rho(1-p_{t+1})] W_{t+1}^m + \rho(1-p_{t+1}) W_{t+1}^u \right\}, \\ W_t^u &= b + E_t \beta_{t,t+1} \left\{ p_{t+1} W_{t+1}^m + (1-p_{t+1}) W_{t+1}^u \right\}. \end{aligned}$$

The Nash bargaining outcome $\eta F_t^m = (1-\eta)(W_t^m - W_t^u)$, where $\eta \in (0,1)$ is the worker's share of the surplus (i.e. the worker bargaining power), then results in the wage equation

$$w_t = \eta \left[z_t + E_t \beta_{t,t+1}(1-\rho) p_{t+1} \frac{\gamma}{q_{t+1}} \right] + (1-\eta)\, b.$$

The worker is compensated for a fraction η of firm revenue and the hiring cost that the firm expects to save thanks to the match. In addition, the worker is compensated for a fraction $1-\eta$ of the forgone unemployment benefit. Substituting for the wage, the job creation condition becomes

$$\frac{\gamma}{q_t} = (1-\eta)(z_t - b) + E_t\beta_{t,t+1}(1-\rho)\,(1-\eta\, p_{t+1})\,\frac{\gamma}{q_{t+1}}. \tag{10}$$

2.3 Retail firms

There is a continuum of retail firms $i \in [0,1]$, each of which produces one unit of differentiated good i from one unit of wholesale goods and sells a quantity $Y_t(i)$ of good i to households in a monopolistically competitive market. Cost minimization implies that each retail firm's real marginal cost is equal to the wholesale goods' real price z_t. Then, facing households' demand $Y_t(i) = C_t(i) = [P_t(i)/P_t]^{-\epsilon}\,C_t$, each retail firm chooses its profit-maximizing price subject to Calvo (1983) and Yun (1996) style price stickiness. That is, each period a fraction α of retail firms does not reoptimize price and instead adjusts it for steady state inflation π, while the remaining fraction $1-\alpha$ of firms faces the problem

$$\max_{P_t(i)}\; E_t \sum_{j=0}^{\infty} \alpha^j \beta_{t,t+j}\left[\frac{P_t(i)\pi^j}{P_{t+j}} - z_{t+j}\right]\left[\frac{P_t(i)\pi^j}{P_{t+j}}\right]^{-\epsilon} C_{t+j}.$$

The optimality condition for price setting is

$$P_t(i) = \frac{\epsilon}{\epsilon-1}\,\frac{E_t\sum_{j=0}^{\infty}(\alpha\pi^{-\epsilon})^j\beta_{t,t+j}P_{t+j}^{\epsilon}C_{t+j}z_{t+j}}{E_t\sum_{j=0}^{\infty}(\alpha\pi^{1-\epsilon})^j\beta_{t,t+j}P_{t+j}^{\epsilon-1}C_{t+j}}. \tag{11}$$

If prices are perfectly flexible (i.e. $\alpha = 0$), (11) reduces to $P_t(i) = [\epsilon/(\epsilon-1)]P_t z_t$, which shows that $1/z = \epsilon/(\epsilon-1)$ is the steady state markup of each retail firm's price over its marginal cost. In the presence of price stickiness, the firm's actual markup differs from, but tends toward, the steady state markup.

2.4 Monetary authority

The monetary authority conducts inflation targeting policy that adjusts the interest rate in response solely to either past, present or expected future inflation

$$R_t = R\left(\frac{E_t\pi_{t+j}}{\pi}\right)^{\phi_\pi}, \quad j = -1, 0, 1, \tag{12}$$

where R is the steady state nominal interest rate and ϕ_π is a non-negative policy coefficient on inflation. These three policy specifications are referred to as, respectively, backward-looking ($j=-1$), current-looking ($j=0$) and forward-looking ($j=1$) in what follows.

2.5 Equilibrium and calibration

A rational expectations equilibrium (REE) is a set of processes for all the endogenous variables satisfying (1)−(12), the aggregate resource constraint $y_t = Y_t + \gamma v_t$, and the market clearing condition $Y_t(i) = C_t(i)$ for each retail good $i \in [0,1]$, which implies $Y_t = \Delta_t C_t$, where $\Delta_t \equiv \int_0^1 [P_t(i)/P_t]^{-\epsilon} di$ measures relative price dispersion across retail firms. Log-linearizing these equilibrium conditions around the steady state and rearranging the resulting equations yields

$$\hat{\theta}_t = \hat{v}_t + \frac{1-u}{u}\hat{n}_{t-1}, \tag{13}$$

$$\hat{n}_t = (1-\rho)\hat{n}_{t-1} + \rho\Big(\hat{v}_t - \xi\hat{\theta}_t\Big), \tag{14}$$

$$\xi\hat{\theta}_t = \chi\hat{z}_t + \beta(1-\rho)(\xi-\eta p)E_t\hat{\theta}_{t+1} - \beta(1-\rho)(1-\eta p)\Big(\hat{R}_t - E_t\hat{\pi}_{t+1}\Big), \tag{15}$$

$$\hat{n}_t = s_c\hat{C}_t + s_v\hat{v}_t, \tag{16}$$

$$\hat{C}_t = E_t\hat{C}_{t+1} - \sigma^{-1}\Big(\hat{R}_t - E_t\hat{\pi}_{t+1}\Big) + \sigma^{-1}(g_t - E_t g_{t+1}), \tag{17}$$

$$\hat{\pi}_t = \beta E_t\hat{\pi}_{t+1} + \kappa\hat{z}_t, \tag{18}$$

$$\hat{R}_t = \phi_\pi E_t\hat{\pi}_{t+j}, \quad j = -1, 0, 1, \tag{19}$$

where $\chi \equiv z(1-\eta)/(\gamma/q) > 0$, $s_v \equiv \gamma v/y$ is the steady state vacancy creation share of production, $s_c \equiv 1 - \gamma v/y$ is the steady state consumption share of production, and $\kappa \equiv (1-\alpha)(1-\alpha\beta)/\alpha > 0$ is the real marginal cost elasticity of inflation. The preference shock, g_t, is assumed to follow a stationary first order autoregressive process with a parameter $\rho_g \in (-1,1)$ and a white noise ε_t

$$g_t = \rho_g g_{t-1} + \varepsilon_t. \tag{20}$$

In the presence of search and matching frictions in the labor market, firms' adjustment in employment and output is persistent and as a consequence, the transmission mechanism of monetary policy consists of a vacancy channel in addition to an aggregate demand channel which is the only channel in the absence of the labor market frictions. One point here is that these two channels have opposing effects on inflation. To see this, consider the effect of a rise in the real interest rate. Then, households reduce their current consumption according to the Euler equation (17). This decreases retail firms' current output because of monopolistic competition and hence dampens these firms' current demand for wholesale goods. In response to this dampened demand, wholesale firms reduce current vacancy posting and hence the current labor market becomes slack via (13), which lowers retail firms' current real marginal cost via the job creation condition (15) because the labor market slackness decreases the hiring cost and hence wholesale goods' price.[10] Consequently, current inflation is reduced via the Phillips curve (18).[11] This is the aggregate demand channel of monetary policy, through which a higher real interest rate lowers current inflation.

The vacancy channel of monetary policy, on the contrary, leads a rise in the real interest rate to increase expected future and current inflation. A real interest rate rise implies that consumption demand is expected to recover in future periods after its current decline, and hence expected future demand for wholesale goods increases above its current level. As explained just above, the rate rise also reduces wholesale firms' current vacancy posting and hence lowers the level of employment available for production in the subsequent periods. Then, facing the expected recovery in future demand, wholesale firms anticipate a strong expansion of future vacancy posting and hence a tightened future labor market, which in turn raises expected future real marginal cost via the next-period job creation condition.[12] Thus, expected future and current inflation is raised via the Phillips curve. This is the vacancy channel of monetary policy that stems from the labor market frictions. As shown later, this vacancy channel induces a possibility that inflation expectations become self-fulfilling and the REE is indeterminate if active interest rate policy has sufficiently strong responses solely to expected future inflation.

The ensuing analysis uses a realistic calibration of model parameters to illustrate conditions for determinacy and E-stability. Our baseline calibration for the quarterly model is summarized in Table 1. The discount factor $\beta = 0.99$, the risk aversion $\sigma = 1$, the substitution elasticity $\epsilon = 10$ yielding a steady state markup of $1/z = 1.11$, and the probability of no price optimization $\alpha = 0.67$ as in line with recent literature such as Woodford (2003). The labor market parameters are the worker bargaining power $\eta = 0.5$ following most of the literature on labor market search and matching, the search elasticity of matches $\xi = 0.4$ based on the empirical estimates of Blanchard and Diamond (1989), the firm matching rate $q = 0.7$ and the job destruction rate $\rho = 0.1$ taken from den Haan, Ramey and Watson (2000), the steady state unemployment rate $U = 1 - n$ of six percent as in Walsh (2005), and the vacancy posting cost $\gamma = 0.06$ consistent

[10] The real interest rate rise also reduces the labor market tightness directly by lowering the expected value of a match in the job creation condition (15). But, under realistic calibrations, this effect is weak enough to ensure procyclical real marginal cost.

[11] This transmission can also be explained from the perspective of the wholesale goods market. This market is perfectly competitive, and wholesale goods' real price is equal to retail firms' real marginal cost because such goods are the only input in retail firms' production. Therefore, the dampened current demand for wholesale goods drives current real marginal cost downward.

[12] Again, this can be explained from the perspective of the wholesale goods market. The expected rise in future demand for wholesale goods drives retail firms' expected future real marginal cost upward.

with the value implied by the steady state with endogenous job destruction in Walsh (2005). The steady state relationships then imply values for the remaining parameters: p, u, s_c, s_v.

3 Equilibrium determinacy under interest rate policy

In the model presented above, we examine implications of the labor market search and matching frictions for inflation targeting interest rate policy in terms of determinacy of REE.

3.1 Forward-looking policy

We begin with the case of the forward-looking policy, i.e. $j = 1$ in (12). The system of log-linearized equilibrium conditions (13)−(20) can be reduced to a system of the form

$$E_t x_{t+1} = A x_t + B g_t, \tag{21}$$

where $x_t = [\hat{\pi}_t \ \hat{C}_t \ \hat{n}_{t-1}]'$ and the coefficient matrix A is given in Appendix A.[13] In this system the two variables $\hat{\pi}_t$ and $\hat{C}_t$ are non-predetermined, while the remaining one, $\hat{n}_{t-1}$, is predetermined. Therefore, determinacy of REE is generated if and only if the coefficient matrix A has exactly one eigenvalue inside the unit circle and the other two outside the unit circle.[14] We thus obtain the following proposition using Proposition C.2 of Woodford (2003).

Proposition 1 *The forward-looking policy, i.e. $j = 1$ in (12), ensures determinacy of REE if and only if either of the following two cases is satisfied.*

Case I: (22), (23), *and* (24) *or* (25) *hold.*

$$\phi_\pi > 1, \tag{22}$$

$$\phi_\pi < 1 + \frac{2\chi\sigma a_1(1+\beta)}{\kappa\{s_c(2u-\rho)[\xi+\beta(1-\rho)(\xi-\eta p)] - 2\beta\sigma a_1(1-\rho)(1-\eta p)\}}, \tag{23}$$

$$b_3(b_3 - b_1) + b_2 > 1 \quad or \quad |b_1| > 3, \tag{24}$$

$$\phi_\pi > 1 + \frac{\chi\sigma[\rho(1-\xi) - s_v]}{\kappa(1-\rho)\{s_c(\xi-\eta p) - \sigma(1-\eta p)[\rho(1-\xi) - s_v]\}}, \tag{25}$$

where $a_1 = \rho(1-\xi)[u - s_v(1-u)] - s_v(2u-\rho)$ *and* b_i, $i = 1, 2, 3$, *are given in Appendix B.*

Case II: *the two strict inequalities opposite to* (22) *and* (23) *hold and* (24) *or the strict inequality opposite to* (25) *holds.*

Proof See Appendix B. ■

[13] The form of the vector B is omitted since it is not needed in what follows.

[14] To be precise, this condition is sufficient for determinacy but only generically necessary. Throughout the paper, consideration of non-generic boundary cases is omitted.

We illustrate the conditions for determinacy with the baseline calibration. Determinacy obtains for a very narrow interval of the policy coefficient on expected future inflation, $1 < \phi_\pi < 1.16$. Therefore, the forward-looking policy is very likely to render the REE indeterminate. Note that Case I is the empirically relevant condition for determinacy, since Case II cannot obtain with realistic calibrations of the model parameters including the baseline one. The lower bound of the determinacy interval is of course given by Taylor principle (22), while the upper bound is induced by the first inequality of (24), which limits the policy coefficient on expected future inflation very severely. This is in stark contrast to Proposition 4 of Bullard and Mitra (2002) and Proposition 4.5 of Woodford (2003), which show that in the absence of the labor market search and matching frictions the forward-looking policy ensures determinacy if and only if it satisfies the Taylor principle but its response to expected future inflation is not too strong, $1 < \phi_\pi < 1 + 2(1 + \beta)/\kappa = 25$ under the baseline calibration.[15]

What causes the forward-looking policy to induce indeterminacy of REE? Indeterminacy is induced by any inflation coefficient less than one, due to the weakness of the demand channel of monetary policy. That is, passive policy makes the REE indeterminate, as in line with previous monetary policy literature. It is also induced by any inflation coefficient greater than 1.16 in the presence of the vacancy channel of monetary policy that stems from the labor market search and matching frictions. As noted above, this vacancy channel leads a rise in the real interest rate to increase expected future inflation and therefore makes inflation expectations self-fulfilling if the forward-looking policy has sufficiently strong responses solely to expected future inflation. Consequently, the REE fails to be determinate. Only if the policy coefficient lies in the very narrow interval of (1,1.16), the effect of the vacancy channel is negligible and hence a determinate REE is generated.

The indeterminacy result is robust with respect to the model parameters. As sensitivity analysis, we consider how the upper bound on the policy coefficient on expected future inflation changes for alternative values of parameters that determine the effect of the vacancy channel of monetary policy. As noted above, two factors give rise to the vacancy channel: the sluggish adjustment in output due to the labor market frictions and the expected recovery of future consumption after a rise in the real interest rate. Thus, the effect of the vacancy channel depends on the sluggishness of the labor market holding back the adjustment in future output

[15] In this policy coefficient interval, κ is the real marginal cost elasticity of inflation, but not the output (gap) elasticity of inflation, which is given by κ/σ and appears in the determinacy interval of the policy coefficient given in Proposition 4 of Bullard and Mitra (2002) and Proposition 4.5 of Woodford (2003).

and on the strength of the recovery of expected future consumption in response to real interest rate changes.[16] In the labor market, we can see from the labor market tightness (13) and the employment motion law (14) that the dynamics of employment and output are determined by the proportion of separations ρ, the steady state unemployment rate U, and the search elasticity of matches ξ. A small ρ implies via (14) that changes in current output persist strongly into the future and that current vacancies have a small effect on employment creation. Therefore, a smaller ρ makes the labor market more sluggish and hence the indeterminacy problem is worsened. For a small value of $\rho = 0.07$ (e.g. Merz, 1995) the interval of the inflation coefficient for which determinacy is ensured becomes $1 < \phi_\pi < 1.13$, while for a large value of $\rho = 0.15$ (e.g. Andolfatto, 1996) it widens to $1 < \phi_\pi < 1.20$. A large U reduces the employment coefficient $(1-u)/u$ in (13) and thus, in combination with (14), employment has a more persistent effect on expected future employment. As a consequence, the determinacy interval becomes narrower for a larger steady state unemployment rate; e.g. if $U = 0.12$ (0.03) this interval is $1 < \phi_\pi < 1.10$ (1.29).[17] A reduction of ξ dampens the firm matching rate's response to changes in labor market tightness. This increases the sluggishness of the labor market by making expected future employment more sensitive to current employment via (13) and (14). But it also raises the proportion of newly matched vacancies and dampens the rise of the expected cost of a match in response to a tightening labor market. For a small value of $\xi = 0.235$ (Hall, 2005) the determinacy interval narrows to $1 < \phi_\pi < 1.11$, while for a large value of $\xi = 0.5$ (e.g. Krause, Lopez-Salido and Lubik, 2008) it becomes $1 < \phi_\pi < 1.26$. As for the strength of the recovery of expected future consumption in response to real interest rate changes, it is determined entirely by households' degree of risk aversion σ, the inverse of which measures the intertemporal substitution elasticity of consumption. A smaller degree of risk aversion makes consumption movements more sensitive to real interest rate changes, resulting in a strong expected growth of future consumption after a real interest rate rise. Consequently, the determinacy interval becomes narrower for a smaller σ; e.g. if $\sigma = 0.16$ (Woodford, 2003) and $\sigma = 5$ (McCallum and Nelson, 1999), the REE is determinate for $1 < \phi_\pi < 1.03$ and $1 < \phi_\pi < 1.35$, respectively.

[16] The other parameter that determines the effect of the vacancy channel is the probability of not optimizing price α, which measures price stickiness. A smaller α increases the real marginal cost elasticity of inflation $\kappa = (1-\alpha)(1-\alpha\beta)/\alpha$, which strengthens the vacancy channel effect and hence the indeterminacy problem deteriorates; e.g. if $\alpha = 0.5$ (0.8), the determinacy interval is $1 < \phi_\pi < 1.05$ (1.50).

[17] The determinacy interval for alternative unemployment rates is obtained by keeping the job finding rate p at its baseline value implied by $U = 0.06$, to isolate the effect of U on the dynamics. The change in p implied by a change in U would have an opposing effect on the length of the determinacy interval.

3.2 Current-looking and backward-looking policies

We turn next to the current-looking and the backward-looking policies, i.e. $j = 0$ and $j = -1$ in (12). These policies yield, respectively, third and fourth order characteristic equations for the systems' coefficient matrices and then determinacy requires that exactly two solutions to these equations be outside the unit circle and the others lie inside the unit circle. To our knowledge, there is no general result about conditions under which fourth order equations have such solutions, and thus we investigate the backward-looking policy numerically. For the current-looking policy we obtain the following proposition.

Proposition 2 *The current-looking policy, i.e.* $j = 0$ *in* (12), *ensures determinacy of REE if and only if either of the following two cases is satisfied.*

Case I: (22), (26), *and* (27) *or* (28) *hold.*

$$\phi_\pi > -1 - \frac{2\chi\sigma a_1(1+\beta)}{\kappa\{s_c(2u-\rho)[\xi+\beta(1-\rho)(\xi-\eta p)] - 2\beta\sigma a_1(1-\rho)(1-\eta p)\}}, \tag{26}$$

$$c_3(c_3 - c_1) + c_2 > 1 \quad or \quad |c_1| > 3, \tag{27}$$

$$-1 > \frac{\chi\sigma[\rho(1-\xi) - s_v]}{\kappa(1-\rho)\{s_c(\xi-\eta p) - \sigma(1-\eta p)[\rho(1-\xi) - s_v]\}}, \tag{28}$$

where a_1 *is given in Proposition 1 and* c_i, $i = 1, 2, 3$, *are given in Appendix C.*

Case II: *the two strict inequalities opposite to* (22) *and* (26) *hold and* (27) *or the strict inequality opposite to* (28) *holds.*

Proof See Appendix C. ■

Under the baseline calibration, the current-looking policy guarantees determinacy of REE if and only if Taylor principle (22) is satisfied, i.e. $\phi_\pi > 1$. In that calibrated case, (26) and the first inequality of (27) are satisfied, such that (22) generates determinacy. The backward-looking policy ensures determinacy as long as it meets the Taylor principle but its response to past inflation is not too strong, $1 < \phi_\pi < 10.4$ under the baseline calibration; otherwise, it induces indeterminacy for $0 \leq \phi_\pi < 1$ and makes the REE explosive for $\phi_\pi \geq 10.4$. These results are robust with respect to any realistic value of each model parameter and are in line with Bullard and Mitra (2002) and Woodford (2003) who consider the case of a frictionless labor market.[18]

[18] For unrealistically large values of the risk aversion σ, active current-looking and backward-looking policies induce indeterminacy; e.g. in the case of $\sigma = 15$, the current-looking policy with $2.16 \leq \phi_\pi \leq 91.66$ and the backward-looking policy with $4.91 \leq \phi_\pi \leq 19.08$ render REE indeterminate.

4 Prescriptions for the indeterminacy problem

We have shown that the forward-looking policy is very likely to render the REE indeterminate due to the vacancy channel of monetary policy that stems from the labor market search and matching frictions. In this section we consider three prescriptions for this indeterminacy problem. Specifically, we examine the following generalization of the forward-looking policy.[19]

$$R_t = R^{1-\phi_R}(R_{t-1})^{\phi_R}\left(\frac{E_t\pi_{t+1}}{\pi}\right)^{\phi_\pi}\left(\frac{E_tY_{t+k}}{Y}\right)^{\phi_Y}\left(\frac{1-U_t}{1-U}\right)^{\phi_U}, \quad k=0,1, \tag{29}$$

where ϕ_R, ϕ_Y, ϕ_U are non-negative policy coefficients on the lagged interest rate, current or expected future output and the unemployment rate. The log-linearization of (29) is given by

$$\hat{R}_t = \phi_R\hat{R}_{t-1} + \phi_\pi E_t\hat{\pi}_{t+1} + \phi_Y E_t\hat{Y}_{t+k} - \phi_U\hat{U}_t, \quad k=0,1, \tag{30}$$

where $\hat{U}_t = U_t - U$. The first prescription is the policy adjustment for current or expected future output in addition to expected future inflation. Second, we consider interest rate smoothing, i.e. the policy response to the lagged interest rate. These two are motivated by empirical studies such as Clarida, Galí and Gertler (1998, 2000) and Orphanides (2004), who use them as a good description of actual monetary policy conducted in industrialized countries. The last prescription is the policy reaction to the unemployment rate. Blanchard and Galí (2008) and Faia (2008) find that interest rate policy with responses to inflation and unemployment rates can approximate well optimal policy responses to shocks in a sticky price model with labor market frictions.

4.1 Policy response to output

We first investigate whether the policy response to current or expected future output as well as expected future inflation, i.e. $\phi_R = \phi_U = 0$ in (29), can resolve the indeterminacy problem induced by the forward-looking policy.

In the case of the policy response to expected future output, $k = 1$ in (29), the system consisting of (13)−(18), (20) and (30) can be reduced to a system of the same form as (21) with a different coefficient matrix A given in Appendix A. Analyzing this coefficient matrix yields the following proposition.

[19] This generalization includes, as the special case in which $\phi_x = \tilde{\phi}_x(1-\phi_R)$ for $x = \pi, Y, U$,

$$R_t = (R_{t-1})^{\phi_R}\left[R\left(\frac{E_t\pi_{t+1}}{\pi}\right)^{\tilde{\phi}_\pi}\left(\frac{E_tY_{t+k}}{Y}\right)^{\tilde{\phi}_Y}\left(\frac{1-U_t}{1-U}\right)^{\tilde{\phi}_U}\right]^{1-\phi_R}.$$

Proposition 3 *Suppose $\phi_Y \neq \sigma$. If the forward-looking policy responds also to expected future output, i.e. $k = 1$, $\phi_R = \phi_U = 0$ in (29), it ensures determinacy of REE if and only if either of the following two cases is satisfied.*

Case I: (31), (32), *and* (33) *or* (34) *hold.*

$$\phi_\pi + \frac{\chi(1-\beta)[u(1-\xi)-\xi(s_v/s_c)]}{\kappa[\xi-\beta(1-\rho)(\xi-\eta p)]}\phi_Y \; > \; 1, \tag{31}$$

$$\phi_\pi \; < \; 1 + \frac{\chi a_1(1+\beta)(2\sigma-\phi_Y)}{\kappa\{s_c(2u-\rho)[\xi+\beta(1-\rho)(\xi-\eta p)]-2\beta\sigma a_1(1-\rho)(1-\eta p)\}}, \tag{32}$$

$$d_3(d_3-d_1)+d_2 \; > \; 1 \quad or \quad |d_1| \; > \; 3, \tag{33}$$

$$\phi_\pi \; > \; 1 + \frac{\chi[\rho(1-\xi)-s_v](\sigma-\phi_Y)}{\kappa(1-\rho)\{s_c(\xi-\eta p)-\sigma(1-\eta p)[\rho(1-\xi)-s_v]\}}, \tag{34}$$

where a_1 is given in Proposition 1 and d_i, $i = 1, 2, 3$, are given in Appendix D.

Case II: *the two strict inequalities opposite to* (31) *and* (32) *hold and* (33) *or the strict inequality opposite to* (34) *holds.*

Proof See Appendix D. ■

Like Bullard and Mitra (2002), Woodford (2003) and Kurozumi and Van Zandweghe (2008), (31) can be given the following economic interpretation. By equilibrium conditions (13)−(18), each percentage point of permanently higher inflation implies a permanent increase in output of $\chi(1-\beta)[u(1-\xi)-\xi(s_v/s_c)]/\{\kappa[\xi-\beta(1-\rho)(\xi-\eta p)]\}$ percentage points. Hence the left-hand side of (31) shows the long run rise in the nominal interest rate by policy (30) with $\phi_R = \phi_U = 0$ for each percentage point permanent increase in the inflation rate. Therefore, (31) can be interpreted as the long run version of the Taylor principle: in the long run the nominal interest rate should be raised by more than the increase in inflation.

Figure 1 illustrates a region of policy coefficients on expected future inflation and output (ϕ_π, ϕ_Y) that generate determinacy under the baseline calibration. Case I is the empirically relevant condition for determinacy, since Case II cannot obtain under realistic calibrations of the model parameters including the baseline one. We can see that moderate policy adjustment for expected future output ameliorates the indeterminacy problem. The lower bound on the inflation coefficient ϕ_π is given by Taylor principle (31). The first inequality of condition (33) yields an upper bound on the inflation coefficient and allows a much wider determinacy interval of the inflation coefficient as the output coefficient ϕ_Y increases. However, once it increases beyond a certain threshold given by the intersection of (32) and (33), the determinacy interval becomes narrower due to (32), which imposes the upper bound on the inflation and output coefficients. This upper bound is induced by the demand channel of monetary policy because we can see the corresponding one in Figure 3 of Bullard and Mitra (2002) who examine the case of a frictionless labor market, in which monetary policy contains only the demand channel and the upper bound limits the determinacy region more severely. The intuition for this amelioration of the indeterminacy problem is as follows. Indeterminacy is induced by the vacancy channel of the forward-looking policy, in which a rise in the real interest rate stemming from inflationary expectations increases expected future inflation. But, the policy response to expected future output subdues such a rate rise because this output falls as a consequence of the rate rise. Hence, an expected recovery of future consumption following the real interest rate rise is subdued and the expected need for more future vacancies is prevented. Therefore, determinacy is generated.

In the case of the policy response to current output, $k = 0$ in (29), a similar analysis of the system's coefficient matrix A given in Appendix A yields the following proposition.

Proposition 4 *If the forward-looking policy responds also to current output, i.e.* $k = 0$*,* $\phi_R = \phi_U = 0$ *in* (29)*, it ensures determinacy of REE if and only if either of the following two cases is satisfied.*

Case I: (31)*,* (35)*, and* (36) *or* (25) *hold.*

$$\phi_\pi \ < \ 1 + \frac{\chi a_1 (1+\beta)(2\sigma + \phi_Y)}{\kappa\{s_c(2u-\rho)[\xi + \beta(1-\rho)(\xi - \eta p)] - 2\beta\sigma a_1 (1-\rho)(1-\eta p)\}}, \tag{35}$$

$$e_3(e_3 - e_1) + e_2 \ > \ 1 \quad \text{or} \quad |e_1| \ > \ 3, \tag{36}$$

where a_1 *is given in Proposition 1 and* e_i*,* $i = 1, 2, 3$*, are given in Appendix E.*

Case II: *the two strict inequalities opposite to* (31) *and* (35) *hold and* (36) *or the strict inequality opposite to* (25) *holds.*

Proof See Appendix E. ■

If the forward-looking policy responds also to current output, the long run version of the Taylor principle yields the same inequality as (31) in the case of the policy response to expected future output. Figure 2 shows a determinacy region of policy coefficients on expected future inflation and current output under the baseline calibration. Note that Case I is the empirically relevant condition for determinacy because of (35) and Taylor principle (31), the latter of which gives the lower bound on the inflation coefficient. The upper bound on the inflation coefficient is induced by the first inequality in (36) for any output coefficient less than a certain threshold given by the intersection of (35) and (36),[20] while it is induced by (35) for any output coefficient greater than this threshold. This is in contrast with the case of the policy response to expected future output, in which the determinacy interval narrows once the output coefficient increases beyond the threshold specified before. Thus, the policy response to current output is a much better prescription for the indeterminacy problem than the one to expected future output. Intuitively, this amelioration of the indeterminacy problem arises because current output falls as a consequence of a rise in the real interest rate stemming from inflationary expectations and hence the policy response to this output subdues such a rate rise. Therefore, the policy response to current output prevents the inflationary expectations from becoming self-fulfilling and REE from being indeterminate.

4.2 Policy response to unemployment

We turn next to the second prescription, the forward-looking policy with responses to the unemployment rate, i.e. $\phi_R = \phi_Y = 0$ in (29). Analyzing the system's coefficient matrix A given in Appendix A yields the following necessary and sufficient condition for determinacy.

Proposition 5 *If the forward-looking policy responds also to the unemployment rate, i.e.* $\phi_R = \phi_Y = 0$ *in* (29), *it ensures determinacy of REE if and only if either of the following two cases is satisfied.*

Case I: (37), (38), *and* (39) *or* (25) *hold.*

$$\phi_\pi + \frac{\chi u(1-\beta)(1-\xi)(1-U)}{\kappa[\xi-\beta(1-\rho)(\xi-\eta p)]}\phi_U \; > \; 1, \tag{37}$$

$$\phi_\pi \; < \; 1 + \frac{\chi(1+\beta)[2\sigma a_1 + u\rho(1-\xi)(1-U)\phi_U]}{\kappa\{s_c(2u-\rho)[\xi+\beta(1-\rho)(\xi-\eta p)] - 2\beta\sigma a_1(1-\rho)(1-\eta p)\}}, \tag{38}$$

$$f_3(f_3 - f_1) + f_2 \; > \; 1 \quad or \quad |f_1| \; > \; 3, \tag{39}$$

where a_1 *is given in Proposition 1 and* f_i, $i = 1, 2, 3$, *are given in Appendix F.*

Case II: *the two strict inequalities opposite to* (37) *and* (38) *hold and* (39) *or the strict inequality opposite to* (25) *holds.*

Proof See Appendix F. ■

Percent changes in production are reflected to a very large extent in percentage point changes in the unemployment rate because of the relation $\hat{U}_t = -(1-U)\hat{n}_t$, where the steady

[20] This intersection appears at an inflation coefficient greater than five, so that the upper bound induced by (35) does not appear in Figure 2.

state unemployment rate is a very small number, e.g. $U = 0.06$ under the baseline calibration. Thus, the policy response to the unemployment rate yields almost the same determinacy result as the one to current output, since output also largely reflects production. The intuition for determinacy is also the same. Figure 3 illustrates the determinacy region of policy coefficients on expected future inflation and current unemployment rates under the baseline calibration. As is the case with the policy response to current output, (37) can be interpreted as the long run version of the Taylor principle and provides the lower bound on the inflation coefficient, while (38) and the first inequality of (39) induce the upper bound. Therefore, the policy response to the unemployment rate, as well as the one to current output, is a better prescription for the indeterminacy problem.

4.3 Interest rate smoothing

Finally, we consider whether interest rate smoothing can help the forward-looking policy generate determinacy of REE, i.e. $\phi_Y = \phi_U = 0$ in (29). It seems hard to analytically examine determinacy with this policy specification, since interest rate smoothing leads to a fourth order characteristic equation for a system's coefficient matrix, which has two predetermined variables, and hence determinacy requires that two solutions are outside the unit circle and the remaining two lie inside the unit circle. There seems to be no general result about conditions for that and thus we numerically investigate determinacy.

Figure 4 shows the determinacy region of policy coefficients of inflation and interest rate smoothing under the baseline calibration. The long run version of the Taylor principle yields $\phi_\pi > 1 - \phi_R$, which provides the lower bound on the inflation coefficient. We can see that a sufficiently high degree of interest rate smoothing of $\phi_R = 0.3$ or more brings about determinacy as long as the long run Taylor principle is met. The intuition for determinacy is that interest rate smoothing implies the policy responses to lagged interest rates and hence makes the forward-looking policy respond also to current and past inflation like the current-looking and the backward-looking policies, which are likely to generate determinacy. Thus, determinacy is guaranteed with interest rate smoothing. In sum, the forward-looking policy with sufficiently strong interest rate smoothing is also a better prescription for the indeterminacy problem.

5 E-stability analysis of the indeterminacy problem

In this section we assess the indeterminacy problem induced by the forward-looking policy from the perspective of E-stability. Specifically, we examine whether the forward-looking policy generates a unique E-stable fundamental REE even in cases of indeterminacy.[21] Following the literature, our E-stability analysis is based on the so-called "Euler equation" approach suggested by Honkapohja, Mitra and Evans (2003): the rational expectations operator E_t is replaced with a possibly non-rational one $\hat{E}_t$ in the system of (13)−(20). This system can be reduced to a system of the form

$$F\tilde{x}_t = G\hat{E}_t\tilde{x}_{t+1} + H\hat{n}_{t-1} + Jg_t, \tag{40}$$

where $\tilde{x}_t = [\hat{\pi}_t \; \hat{C}_t \; \hat{n}_t]'$ and the coefficient matrices F, G, H are given in Appendix G.[22] Then, fundamental RE solutions to system (40) are given by

$$\tilde{x}_t = \bar{c} + \bar{\Phi}\hat{n}_{t-1} + \bar{\Gamma}g_t, \tag{41}$$

where the coefficient matrices are determined by

$$\bar{c} \;=\; 0_{3\times1}, \quad H \;=\; (F - G\bar{\Phi}[0\;0\;1])\bar{\Phi}, \quad \bar{\Gamma} \;=\; \{F - G\bar{\Phi}[0\;0\;1] - \rho_g G\}^{-1}J.$$

Note that $\bar{\Gamma}$ is uniquely determined given a $\bar{\Phi}$, but $\bar{\Phi}$ is not generally uniquely determined, which induces multiplicity of fundamental REE.

Following Section 10.2 of Evans and Honkapohja (2001), we analyze E-stability of fundamental REE.[23] Corresponding to fundamental RE solutions (41), all agents are assumed to be endowed with a perceived law of motion (PLM) of $\tilde{x}_t$

$$\tilde{x}_t = c + \Phi\hat{n}_{t-1} + \Gamma g_t. \tag{42}$$

[21] Recall that in this paper we refer to Evans and Honkapohja's (2001) MSV solutions to linear RE models as fundamental and do not undertake E-stability analysis of non-fundamental REE.

[22] The form of the vector J is omitted, since it is not needed in what follows.

[23] System (40) contains a predetermined variable $\hat{n}_{t-1}$, so that we can consider two learning environments, which are studied respectively in Section 10.2 and 10.3 of Evans and Honkapohja (2001). One environment allows agents to use current endogenous variables in expectation formation, whereas the other does not. In this paper we present only E-stability analysis with the latter environment, as in Bullard and Mitra (2002). This is because any inflation coefficient that generates a unique E-stable fundamental REE in the latter environment does so in the former one, as Kurozumi (2006) shows in the absence of the labor market frictions. An intuition for this is that in forming future expectations, agents have more information by the current endogenous variables and hence E-stability is more likely in the former environment than in the latter one. Another reason for our focus on the latter environment is that the former induces a problem with simultaneous determination of the expectations and current endogenous variables, which is critical to equilibrium under non-rational expectations as indicated by Evans and Honkapohja (2001) and Bullard and Mitra (2002).

Using a forecast from the PLM and the relation $\hat{n}_t = [0\ 0\ 1]\tilde{x}_t$ to substitute $\hat{E}_t\tilde{x}_{t+1}$ out of (40) leads to an actual law of motion (ALM) of $\tilde{x}_t$

$$\begin{aligned} \tilde{x}_t &= F^{-1}G(I + \Phi[0\ 0\ 1])c + F^{-1}(G\Phi[0\ 0\ 1]\Phi + H)\hat{n}_{t-1} \\ &\quad + F^{-1}\{G(\Phi[0\ 0\ 1]\Gamma + \rho_g\Gamma) + J\}g_t \end{aligned} \tag{43}$$

provided that F is invertible. Here, I denotes a conformable identity matrix. Then, a mapping T from the PLM (42) to the ALM (43) can be defined by

$$\begin{aligned} T(c, \Phi, \Gamma) &= \big(F^{-1}G(I + \Phi[0\ 0\ 1])c,\ F^{-1}(G\Phi[0\ 0\ 1]\Phi + H), \\ &\qquad F^{-1}\{G(\Phi[0\ 0\ 1]\Gamma + \rho_g\Gamma) + J\}\big)\,. \end{aligned}$$

For a fundamental RE solution $(\bar{c}, \bar{\Phi}, \bar{\Gamma})$ to be E-stable, the matrix differential equation

$$\frac{d}{d\tau}(c, \Phi, \Gamma) = T(c, \Phi, \Gamma) - (c, \Phi, \Gamma)$$

must have local asymptotic stability at the solution, where τ denotes a notional time. Then, we have

$$\begin{aligned} DT_c(c, \Phi) &= F^{-1}G(I + \Phi[0\ 0\ 1]), \\ DT_\Phi(\Phi) &= F^{-1}G([0\ 0\ 1]\Phi I + \Phi[0\ 0\ 1]), \\ DT_\Gamma(\Phi, \Gamma) &= F^{-1}G(\rho_g I + \Phi[0\ 0\ 1]). \end{aligned}$$

Therefore, it follows that a fundamental RE solution $(\bar{c}, \bar{\Phi}, \bar{\Gamma})$ is E-stable if and only if all eigenvalues of three matrices, $DT_c(\bar{c}, \bar{\Phi})$, $DT_\Phi(\bar{\Phi})$, $DT_\Gamma(\bar{\Phi}, \bar{\Gamma})$, have real parts less than one. We summarize this result in the following lemma.

Lemma 1 *Suppose that the coefficient matrix F is invertible. A fundamental RE solution to the system of* (13)−(20) *with the forward-looking policy, i.e.* $j = 1$ *in* (12)*, is E-stable if and only if all eigenvalues of three matrices,* $F^{-1}G(\varphi I + \bar{\Phi}[0\ 0\ 1])$*,* $\varphi = 1$*,* ρ_g*,* $\bar{\Phi}_3$*, have real parts less than one, where* $\bar{\Phi}_3$ *is the third element of the RE solution vector* $\bar{\Phi}$*.*

With this lemma we investigate E-stability of fundamental REE numerically, since it seems impossible to analytically solve the matrix equation for $\bar{\Phi}$ in fundamental RE solutions (41) and thus to obtain explicit conditions for E-stability. As pointed out by McCallum (1998), distinct fundamental REE are obtained for different orderings of stable generalized eigenvalues of the matrix pencil for system (40).[24]

[24]In cases of indeterminacy, the baseline calibration shows order one or two indeterminacy and hence two or three distinct fundamental REE.

The E-stability analysis shows that in the presence of the labor market search and matching frictions, the forward-looking policy generates a unique E-stable fundamental REE if the policy response to expected future inflation lies in one of two intervals, which both satisfy the Taylor principle: $1 < \phi_\pi < 7.25$ and $\phi_\pi > 25.06$.[25] Only the policy response to expected future inflation in these intervals succeeds in guiding temporary equilibria under non-rational expectations toward the unique E-stable REE. Because the first interval is wide enough to contain all empirically relevant values, the result suggests that the indeterminacy problem induced by the forward-looking policy is not critical from the perspective of E-stability or least-squares learnability of fundamental REE.

This result is a generalization of Bullard and Mitra (2002), who examine the case of a frictionless labor market to show that the forward-looking policy generates a unique E-stable fundamental REE if and only if it meets the Taylor principle. In the presence of the labor market search and matching frictions, the vacancy channel emerges and reduces the guiding effect of the demand channel. As a consequence, multiple fundamental REE are E-stable if the policy response to expected future inflation lies in the intermediate one between the two intervals of inflation coefficients that generate the unique E-stable REE.

6 Robustness analysis

In this section we analyze the robustness of our results obtained with the benchmark model by introducing habit formation in consumption preferences or considering an alternative labor market specification that is a more conventional one in previous studies.

6.1 Habit formation in consumption preferences

As noted before, the vacancy channel of monetary policy causes the forward-looking policy to induce indeterminacy of REE because output supply recovers sluggishly relative to expected future consumption demand after a tightening of the policy. This suggests that the indeterminacy problem might be less severe when habit formation in consumption preferences is taken into account, since such preferences imply that consumption demand adjusts sluggishly to real

[25] The price stickiness changes these intervals quantitatively. For instance, if $\alpha = 0.5$ (0.8), an inflation coefficient in the interval of $1 < \phi_\pi < 3.05$ (20.93) or $\phi_\pi > 8.89$ (77.71) generates a unique E-stable fundamental REE.

interest rate changes.[26] Thus, in this subsection we assume that the period t utility is given by

$$\frac{(C_t - hC_{t-1})^{1-\sigma}}{1-\sigma} e^{g_t},$$

where $h \in [0, 1)$ measures (internal) habit persistence in consumption and the benchmark model examined above is contained as the special case of $h = 0$. Then, the representative household's optimality condition for consumption becomes

$$\lambda_t = (C_t - hC_{t-1})^{-\sigma} - \beta h(E_t C_{t+1} - hC_t)^{-\sigma}.$$

As predicted, indeterminacy becomes less likely with a large h. For instance, in the case of $h = 0.8$, the interval of the policy response to expected future inflation for which determinacy is ensured widens to $1 < \phi_\pi < 1.25$.[27] Yet, this interval is still very narrow and hence the indeterminacy problem remains.[28] The prescriptions for this problem examined above are still effective for guaranteeing determinacy. Also, when E-stability is adopted as an REE selection criterion, a unique E-stable fundamental REE is generated by active, but not too strong, responses to expected future inflation,[29] suggesting that the indeterminacy problem is not critical from the perspective of E-stability of fundamental REE.

6.2 Alternative labor market specification

In the benchmark model we assume that job destruction is exogenous and hiring is instantaneous. Previous studies, such as Trigari (2008), Walsh (2005) and Krause and Lubik (2007), however, use a distinct labor market specification in which jobs are also endogenously destroyed and new hires become productive in the subsequent period. This specification implies that contemporaneous employment adjustment takes place only via job destruction, rather than only via job creation as in the benchmark model.[30] In this subsection we use the alternative labor

[26] In a sticky price model with internal habit formation in consumption preferences, Walsh (2005) shows that the labor market search and matching frictions affect the dynamics of real marginal cost to the effect of augmenting the persistence in output and inflation. This is not the case in the absence of the habit formation as pointed out by Krause and Lubik (2007).

[27] Introducing habit persistence into the model with frictionless labor market of Bullard and Mitra (2002) and Woodford (2003) shifts up the upper bound of the determinacy interval for the policy coefficient from 25 ($h = 0$) to 1,862 ($h = 0.8$).

[28] The current-looking and the backward-looking policies guarantee determinacy of REE under similar conditions to those obtained with the benchmark model.

[29] This interval of policy responses becomes narrower with larger h. There is no second interval of very large policy responses to expected future inflation for which a unique E-stable fundamental REE is generated as in the benchmark model.

[30] This specification of the labor market corresponds to early evidence that job loss rates rise strongly during recessions. More recently, Shimer (2007) and Hall (2006) argue that estimates of job finding rates account for

market specification to examine the robustness of our results obtained with the benchmark model.

Each worker-firm match surviving in period t produces a_t goods, which is a job-specific productivity level that is drawn from a distribution F with a positive support. There is a threshold level of job productivity, denoted by $\tilde{a}_t$, below which matches are discontinued. Specifically, at the beginning of the period, a fraction ρ_x of existing matches is destroyed exogenously, and so is the share of remaining matches that fall below the productivity threshold. Thus, the rate of job destruction is

$$\rho_t = \rho_x + (1 - \rho_x)F(\tilde{a}_t)$$

and the measure of search unemployment is

$$u_t = 1 - (1 - \rho_t)n_{t-1}.$$

We assume that newly formed matches become productive only in the subsequent period. Then, the aggregate production of the wholesale sector becomes

$$y_t = (1 - \rho_t)n_{t-1}H(\tilde{a}_t),$$

where $H(\tilde{a}_t) = E[a|a > \tilde{a}_t] = \int_{\tilde{a}_t}^{\infty} a \; dF(a)/[1 - F(\tilde{a}_t)]$, and the employment including hired, but non-productive, workers evolves according to

$$n_t = (1 - \rho_t)n_{t-1} + m_t.$$

As before, the asset values of a matched firm and worker can be used to obtain the job creation condition and the wage equation from Nash bargaining. The asset value of a matched firm is the current real net revenue plus the expected continuation value of the match. An unmatched firm pays the vacancy posting cost and produces in the next period with probability $(1 - \rho_{t+1})q_t$. Combining these firm asset values yields the job creation condition

$$\frac{\gamma}{q_t} = E_t\beta_{t,t+1}(1 - \rho_{t+1}) \left[z_{t+1}H(\tilde{a}_{t+1}) - \overline{w}_{t+1}(\tilde{a}_{t+1}) + \frac{\gamma}{q_{t+1}} \right],$$

where $\overline{w}(\cdot)$ is the average wage which is defined below. The asset value of a matched (unmatched) worker is the wage (unemployment benefit) plus the expected present discounted

much of the changes in the unemployment rate, a finding that motivates the constant separation rate in many recent search and matching models. But this view is also contested; e.g. Elsby, Michaels and Solon (2008) find that inflows and outflows of unemployment are both important in explaining cyclical unemployment variation. The results in this subsection indicate that this debate is not relevant for the question about indeterminacy of REE with interest rate policy.

value of this worker's employment status in the next period. The Nash bargaining outcome then results in the wage equation

$$w_t(a_t) = \eta\left(z_t a_t + p_t \frac{\gamma}{q_t}\right) + (1-\eta)\, b.$$

Thus, the average wage is

$$\overline{w}_t(\tilde{a}_t) \equiv \int_{\tilde{a}_{t+1}}^{\infty} \frac{w_t(a)}{1-F(\tilde{a}_{t+1})} dF(a) = \eta\left[z_t H(\tilde{a}_t) + p_t \frac{\gamma}{q_t}\right] + (1-\eta)\, b.$$

Substituting the wage equation, the job creation condition becomes

$$\frac{\gamma}{q_t} = E_t \beta_{t,t+1}(1-\rho_{t+1})\left\{(1-\eta)[z_{t+1}H(\tilde{a}_{t+1}) - b] + (1-\eta\, p_{t+1})\,\frac{\gamma}{q_{t+1}}\right\}.$$

Finally, the threshold value $\tilde{a}_t$ is determined by $F^m(\tilde{a}_t) = 0$, or equivalently,

$$\tilde{a}_t = \frac{1}{z_t}\left(b - \frac{1-\eta\, p_t}{1-\eta}\frac{\gamma}{q_t}\right).$$

Log-linearizing these labor market conditions around the steady state and rearranging the resulting equations yields

$$\hat{y}_t = \varepsilon_{H,\tilde{a}}\hat{\tilde{a}}_t + \hat{n}_{t-1} - \frac{\rho}{1-\rho}\hat{\rho}_t, \tag{44}$$

$$\hat{\rho}_t = \frac{\rho-\rho_x}{\rho}\varepsilon_{F,\tilde{a}}\hat{\tilde{a}}_t, \tag{45}$$

$$\hat{\theta}_t = \hat{v}_t + \frac{1-u}{u}\left(\hat{n}_{t-1} - \frac{\rho}{1-\rho}\hat{\rho}_t\right), \tag{46}$$

$$\hat{n}_t = (1-\rho)\left(\hat{n}_{t-1} - \frac{\rho}{1-\rho}\hat{\rho}_t\right) + \rho\left(\hat{v}_t - \xi\hat{\theta}_t\right), \tag{47}$$

$$\begin{aligned} \xi\hat{\theta}_t = {} & \tilde{\chi}\left(E_t\hat{z}_{t+1} + \varepsilon_{H,\tilde{a}}E_t\hat{\tilde{a}}_{t+1}\right) - \left(\hat{R}_t - E_t\hat{\pi}_{t+1}\right) \\ & + \beta(1-\rho)(\xi-\eta p)E_t\hat{\theta}_{t+1} - \frac{\rho}{1-\rho}E_t\hat{\rho}_{t+1}, \end{aligned} \tag{48}$$

$$\hat{\tilde{a}}_t = -\frac{(\gamma/q)(\xi-\eta p)}{(1-\eta)b - (\gamma/q)(1-\eta p)}\hat{\theta}_t - \hat{z}_t, \tag{49}$$

$$\hat{y}_t = s_c \hat{C}_t + s_v \hat{v}_t, \tag{50}$$

where $\tilde{\chi} \equiv \beta(1-\rho)H(\tilde{a})\chi > 0$, and $\varepsilon_{H,\tilde{a}}$ and $\varepsilon_{F,\tilde{a}}$ are respectively the steady state productivity elasticity of H and F at the threshold $\tilde{a}$. Because current production now depends on past rather than present matching activity, the expected value of a match depends on expected future real marginal cost in the job creation condition (48). The log-linearized equilibrium conditions are now given by (44)−(50), the consumption Euler equation (17), the Phillips curve (18) and interest rate policy (19).

With the alternative labor market specification, we numerically investigate determinacy of REE. Following Walsh (2005), it is assumed that idiosyncratic productivity shocks are drawn from a log-normal distribution and are serially uncorrelated with zero mean and a variance of 0.13^2. The exogenous job destruction rate is assumed equal to $\rho_x = 0.068$ following den Haan, Ramey and Watson (2000). With these assumptions the value of the vacancy posting cost is determined by the model's steady state relationships and is equal to $\gamma = 0.06$. Under the baseline calibration, the current-looking and the backward-looking policies guarantee determinacy of REE under similar conditions to those obtained with the benchmark model. The forward-looking policy generates a determinate REE for any inflation coefficient in the interval of $1 < \phi_\pi < 1.21$, which changes little from the one obtained with the benchmark model. Thus, the indeterminacy problem with the forward-looking policy remains.

Intuitively, this problem arises because changes in employment and output are persistent, such that the monetary transmission mechanism contains a vacancy channel in addition to the demand channel as in the benchmark model. Although the labor market specification examined here is more complex than the one in the benchmark model, we can see from (44) and (45) that contemporaneous adjustment of production to a decrease in consumption demand occurs via a rise in the separation rate. But this also destroys employment available for production in the subsequent period in (47). To meet an expected recovery of future consumption demand, firms expand current vacancy creation to have new matches in production in the following period. Thus, the rise in the separation rate and the rise in vacancies have opposing effects on the labor market tightness in (46) and on future employment. However, to the extent that a strong recovery of future consumption is expected, the current labor market tightens. This is associated via the job creation condition (48) with a rise in expected future real marginal cost. Thus, initial inflation expectations can become self-fulfilling.

Regarding the prescriptions for the indeterminacy problem induced by the forward-looking policy, each policy response to current or expected future output, the unemployment rate, or the lagged interest rate, in addition to expected future inflation, yields a very similar determinacy region as in the benchmark model illustrated in Figures 1−4 respectively. Thus, these prescriptions remain effective for overcoming the indeterminacy problem. When considering E-stability as an REE selection criterion, the forward-looking policy with an inflation coefficient in the intervals of $1 < \phi_\pi < 14.83$ and $\phi_\pi > 22.41$ generates a unique E-stable fundamental REE.

In sum, even when we introduce habit formation in consumption preferences or even when we consider the more conventional labor market specification, the findings obtained with the benchmark model remain unchanged.

7 Concluding remarks

We have examined implications of search and matching frictions in the labor market for inflation targeting interest rate policy in terms of equilibrium stability. Such labor market frictions cause sluggish adjustment of production capacity to changes in demand. As a consequence, a rise in the real interest rate increases expected future real marginal cost and hence expected future inflation. Therefore, indeterminacy is likely under forward-looking policy, which adjusts the interest rate in response solely to expected future inflation. However, this indeterminacy can be overcome if the policy adjusts the interest rate in response also to output or the unemployment rate or if it contains interest rate smoothing. Further, if E-stability is adopted as an equilibrium selection criterion, the forward-looking policy generates a unique E-stable fundamental rational expectations equilibrium under active, but not so strong or extremely strong, responses to expected future inflation. These findings are robust even when we introduce habit formation in consumption preferences or when we consider a more conventional labor market specification used in previous studies.

Appendix

A Coefficient matrices in systems of form (21)

In the case of the forward-looking policy, i.e. $j = 1$ in (12), the coefficient matrix A of system (21) is given by

$$A = [A_{ij}] = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{11}(\phi_\pi - 1)/\sigma & 1 + A_{12}(\phi_\pi - 1)/\sigma & A_{13}(\phi_\pi - 1)/\sigma \\ 0 & A_{32} & A_{33} \end{bmatrix}, \tag{51}$$

where

$$\begin{aligned}
A_{11} &= \frac{\chi}{\beta\,\Omega_1}[\,\rho(1-\xi) - s_v],\\
A_{12} &= -\frac{\kappa}{\beta\,\Omega_1}\left\{s_c\xi - \beta(1-\rho)(\xi - \eta p)\left[s_c - A_{32}\left(1 - \frac{\rho}{u} + s_v\frac{1-u}{u}\right)\right]\right\},\\
A_{13} &= \frac{\kappa}{\beta\,\Omega_1}[\,\xi - \beta(1-\rho)(1-\eta p)A_{33}],\quad A_{32} = \frac{s_c\rho(1-\xi)}{\rho(1-\xi) - s_v},\quad A_{33} = \frac{s_v\,[\rho(1-\xi) - (1-\rho\xi/u)]}{\rho(1-\xi) - s_v},\\
\Omega_1 &= \chi[\,\rho(1-\xi) - s_v] + \Omega_0(\phi_\pi - 1),\quad \Omega_0 = \kappa(1-\rho)\{(1-\eta p)[\rho(1-\xi) - s_v] - (\xi - \eta p)s_c/\sigma\}.
\end{aligned}$$

Let A^0_{ij} refer to the elements A_{ij} under the forward-looking policy. Then, for the current-looking policy, i.e. $j = 0$ in (12), the system's coefficient matrix A is the same as (51), except $A_{1j} = \tilde{A}_{1j}$, where $\tilde{A}_{11} = (A^0_{11}\Omega_1 - \Omega_0\phi_\pi)/(\Omega_1 - \Omega_0\phi_\pi)$, $\tilde{A}_{12} = A^0_{12}\Omega_1/(\Omega_1 - \Omega_0\phi_\pi)$, $\tilde{A}_{13} = A^0_{13}\Omega_1/(\Omega_1 - \Omega_0\phi_\pi)$, and $A_{21} = (\phi_\pi - \tilde{A}_{11})/\sigma$, $A_{22} = 1 - \tilde{A}_{12}/\sigma$, and $A_{23} = -\tilde{A}_{13}/\sigma$. If the forward-looking policy responds also to expected future output, i.e. $k = 1$, $\phi_R = \phi_U = 0$ in (29), the system's coefficient matrix A is the same as (51), except $A_{11} = A^0_{11}(1 - \Omega_2)$, $A_{12} = A^0_{12}(1 - \Omega_2) - \Omega_2\sigma/(\phi_\pi - 1)$, $A_{13} = A^0_{13}(1 - \Omega_2)$, $A_{2j} = A^0_{2j}\sigma/(\sigma - \phi_Y)$, where $\Omega_2 = \Omega_0\phi_Y(\phi_\pi - 1)/\{\chi[\rho(1-\xi) - s_v](\sigma - \phi_Y) + \Omega_0\sigma(\phi_\pi - 1)\}$. Alternatively, when the forward-looking policy responds also to current output, i.e. $k = 0$, $\phi_R = \phi_U = 0$ in (29), the system's coefficient matrix A is the same as (51), except $A_{12} = A^0_{12} - (\Omega_0/\Omega_1)\phi_Y$ and $A_{22} = A^0_{22} + \phi_Y/\sigma$. Finally, when the forward-looking policy responds also to the unemployment rate, i.e. $\phi_R = \phi_Y = 0$ in (29), the system's coefficient matrix A is the same as (51), except $A_{12} = A^0_{12} - (\Omega_0/\Omega_1)A_{32}\phi_Y$, $A_{13} = A^0_{13} - (\Omega_0/\Omega_1)A_{33}\phi_Y$, $A_{22} = A^0_{22} + A_{32}\phi_Y$, and $A_{23} = A^0_{23} + A_{33}\phi_Y$.

B Proof of Proposition 1

For the system's coefficient matrix A given in Appendix A, we can show that its three eigenvalues are the solutions to the cubic equation

$$\mu^3 + b_1\mu^2 + b_2\mu + b_3 = 0,$$

where $b_1 = -1 - A^0_{11} - A_{33} - A^0_{12}(\phi_\pi - 1)/\sigma,\ b_2 = A^0_{11} + (1 + A^0_{11})A_{33} + (A^0_{12}A_{33} - A^0_{13}A_{32})(\phi_\pi - 1)/\sigma,$ and $b_3 = -A^0_{11}A_{33}$.

Because determinacy of equilibrium obtains if and only if the coefficient matrix A has exactly one eigenvalue inside the unit circle and the other two outside the unit circle, it follows that the necessary and sufficient condition for determinacy is that exactly two solutions to the cubic equation above are outside the unit circle and one is inside the unit circle. By Proposition C.2 of Woodford (2003), this is the case if and only if either of the following two cases is satisfied.

(Case 1) $\quad b_1 + b_2 + b_3 < -1,\ b_1 - b_2 + b_3 > 1;$

(Case 2) $\quad b_1 + b_2 + b_3 > -1,\ b_1 - b_2 + b_3 > 1,\ b_3(b_3 - b_1) + b_2 - 1 > 0$ *or* $|b_1| > 3$.

Then, because Ω_1 is a common denominator in these inequalities, (Case 2) can be reduced to (22)−(25) if $\Omega_1 > 0$ and to (24) and the strict opposite of (22), (23) and (25) if $\Omega_1 < 0$. Likewise, (Case 1) can be reduced to (25) and the strict opposite of (22) and (23) if $\Omega_1 > 0$ and to (22) and (23) and the strict opposite of (25) if $\Omega_1 < 0$.

C Proof of Proposition 2

For the system's coefficient matrix A given in Appendix A, we can show that its three eigenvalues are the solutions to the cubic equation $\mu^3 + c_1\mu^2 + c_2\mu + c_3 = 0$, where $c_1 = b_1 - (1 + A^0_{11} - A^0_{12}/\sigma)\Omega_0\phi_\pi/(\Omega_1 - \Omega_0\phi_\pi) + A^0_{12}\phi_\pi/\sigma,\ c_2 = b_2 + (A^0_{11} - 1)(1 + A_{33})\Omega_0\phi_\pi/(\Omega_1 - \Omega_0\phi_\pi) - (A^0_{12}A_{33} - A^0_{13}A_{32})[1 + \Omega_0/(\Omega_1 - \Omega_0\phi_\pi)]\phi_\pi/\sigma - A^0_{12}\Omega_1\phi_\pi/[(\Omega_1 - \Omega_0\phi_\pi)\sigma],\ c_3 = b_3 - A^0_{11}A_{33}\Omega_0\phi_\pi/(\Omega_1 - \Omega_0\phi_\pi) + (A^0_{12}A_{33} - A^0_{13}A_{32})\Omega_1\phi_\pi/[(\Omega_1 - \Omega_0\phi_\pi)\sigma]$, A^0_{1j} and A_{3j} are given in Appendix A, and b_i, $i = 1, 2, 3$, are given in Appendix B. The remainder of the proof proceeds in the same way, mutatis mutandis, as in Appendix B.

D Proof of Proposition 3

For the system's coefficient matrix A given in Appendix A, we can show that its three eigenvalues are the solutions to the cubic equation $\mu^3 + d_1\mu^2 + d_2\mu + d_3 = 0$, where $d_1 = b_1 + A^0_{11}\Omega_2 + [1 + A^0_{12}(\phi_\pi - 1)/\sigma](\Omega_2\sigma - \phi_Y)/(\sigma - \phi_Y),\ d_2 = b_2 - A^0_{11}A_{33}\Omega_2 - [A^0_{11} + A_{33} + (A^0_{12}A_{33} - A^0_{13}A_{32})(\phi_\pi - 1)/\sigma](\Omega_2\sigma - \phi_Y)/(\sigma - \phi_Y),\ d_3 = b_3 + A^0_{11}A_{33}(\Omega_2\sigma - \phi_Y)/(\sigma - \phi_Y)$, A^0_{1j} and A_{3j} are given in Appendix A, and b_i, $i = 1, 2, 3$, are given in Appendix B. The remainder of the proof proceeds in the same way, mutatis mutandis, as in Appendix B.

E Proof of Proposition 4

For the system's coefficient matrix A given in Appendix A, we can show that its three eigenvalues are the solutions to the cubic equation $\mu^3 + e_1\mu^2 + e_2\mu + e_3 = 0$, where $e_1 = b_1 - \beta A_{11}^0 \phi_Y/\sigma$, $e_2 = b_2 + A_{11}^0(1 + \beta A_{33})\phi_Y/\sigma$, $e_3 = b_3(1 + \phi_Y/\sigma)$, A_{11}^0 and A_{33} are given in Appendix A, and b_i, $i = 1, 2, 3$, are given in Appendix B. The remainder of the proof proceeds in the same way, mutatis mutandis, as in Appendix B.

F Proof of Proposition 5

For the system's coefficient matrix A given in Appendix A, we can show that its three eigenvalues are the solutions to the cubic equation $\mu^3 + f_1\mu^2 + f_2\mu + f_3 = 0$, where $f_1 = b_1 - \beta A_{11}^0 A_{32}(1 - U)\phi_U/\sigma$, $f_2 = b_2 + A_{11}^0 A_{32}(1 - U)\phi_U/\sigma$, $f_3 = b_3$, A_{11}^0 and A_{32} are given in Appendix A, and b_i, $i = 1, 2, 3$, are given in Appendix B. The remainder of the proof proceeds in the same way, mutatis mutandis, as in Appendix B.

G Coefficient matrices in system (40)

The coefficient matrices F, G, H of system (40) are given by

$$F = \begin{bmatrix} A_{11}^0 & A_{12}^0 & 0 \\ 0 & 1 & 0 \\ 0 & A_{32} & -1 \end{bmatrix}, \quad G = \begin{bmatrix} 1 & 0 & 0 \\ -(\phi_\pi - 1)/\sigma & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad H = \begin{bmatrix} -A_{13}^0 \\ 0 \\ -A_{33} \end{bmatrix}.$$

where A_{1j}^0 and A_{3j} are given in Appendix A.

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Table 1: Baseline calibration for our quarterly model

β	discount factor	0.99
σ	risk aversion	1
ϵ	elasticity of substitution between retail goods	10
α	probability of not reoptimizing price	0.67
η	worker bargaining power	0.5
ξ	search elasticity of matches	0.4
q	firm matching rate	0.7
ρ	job destruction rate	0.1
U	steady state unemployment rate	0.06
γ	flow cost of a vacancy	0.06
ρ_g	autoregressive coefficient for preference shocks	0.35

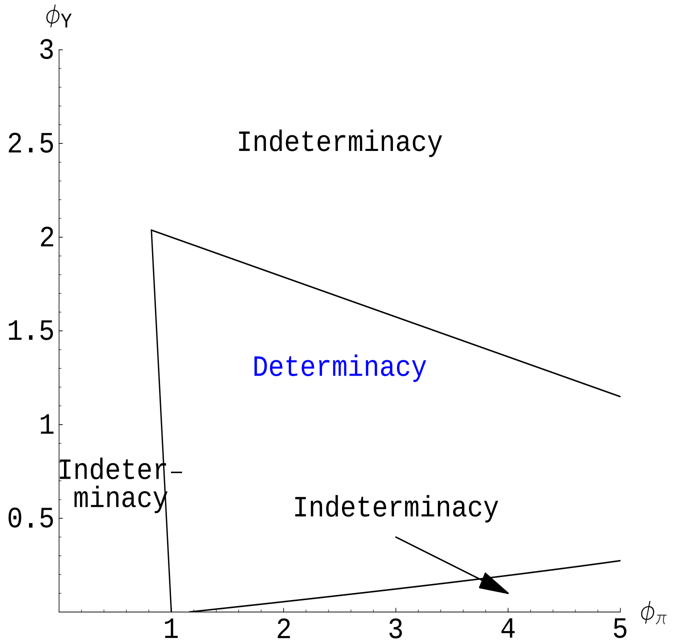

Figure 1: Determinacy region of interest rate policy coefficients on expected future inflation and expected future output

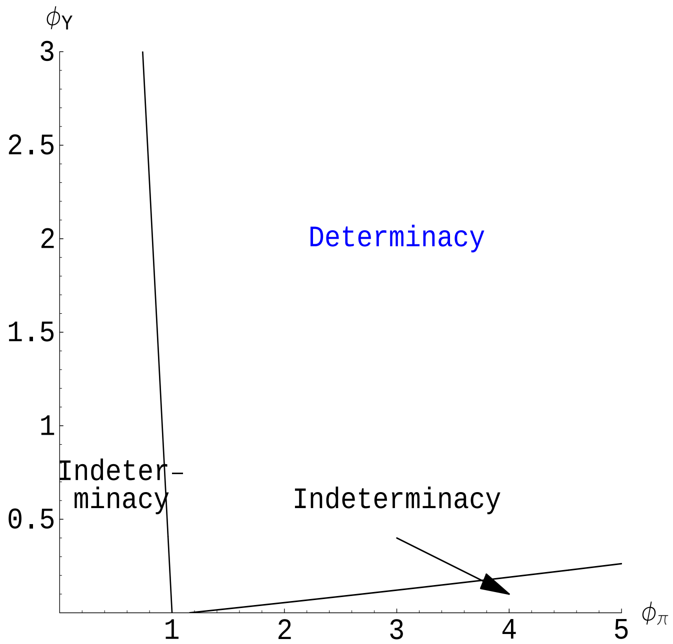

Figure 2: Determinacy region of interest rate policy coefficients on expected future inflation and current output

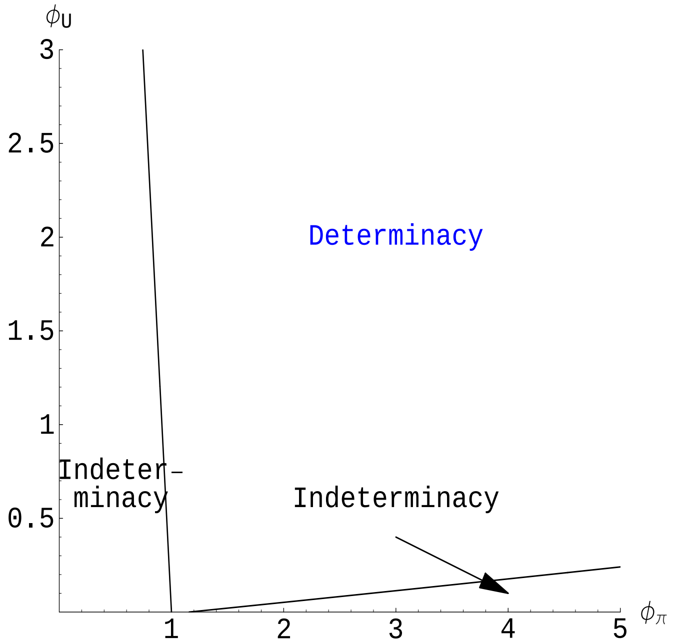

Figure 3: Determinacy region of interest rate policy coefficients on expected future inflation and unemployment

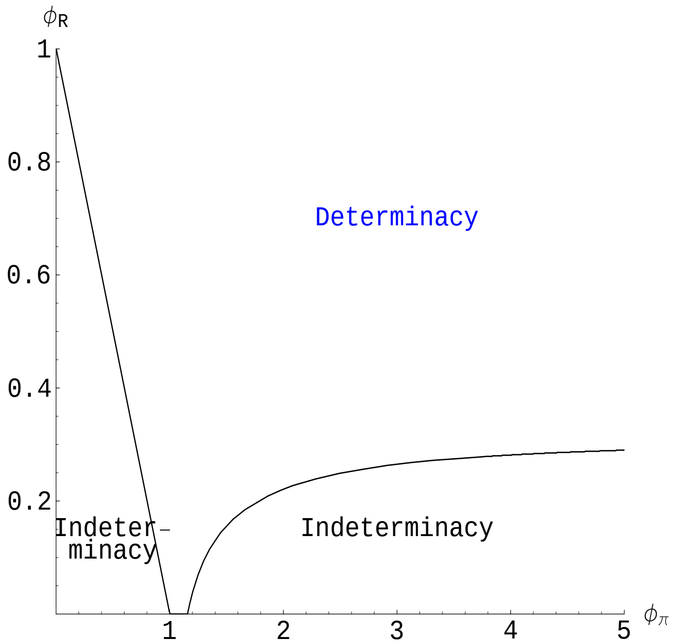

Figure 4: Determinacy region of interest rate policy coefficients on expected future inflation and interest rate smoothing