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Asset Prices, Nominal Rigidities, and Monetary Policy

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Should monetary policy respond to asset prices? This paper analyzes this question from the vantage point of equilibrium determinacy.

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1. Introduction.

Should monetary policy respond to asset prices? This is a classic question in monetary policy. This paper addresses this issue in the context of a general equilibrium model with nominal rigidities. Our focus is on equilibrium determinacy.

Bernanke and Gertler (1999,2001) address the efficacy of a central bank response to asset prices in the model outlined in Bernanke, Gertler and Gilchrist (2000). In their sticky price model a shock to asset prices increases aggregate demand and thus drives up the price level. Bernanke and Gertler conclude that there is no need for a direct central bank response to asset prices because a central bank that is responding to general price inflation is already responding to asset price movements. They state: "Policy should not respond to asset prices, except insofar as they signal changes in expected inflation."

Cecchetti, Genberg, Lipsky, and Wadhwani (2000), however, argue that central banks, at least in inflation targeting countries, should respond to asset prices: "[a] central bank concerned with hitting an inflation target at a given time horizon, and achieving as smooth a path as possible for inflation, is likely to achieve superior performance by adjusting its policy instruments not only to inflation (or to its inflation forecast) and the output gap, but to asset prices as well." [Page 2]

Whether or not the central bank can potentially make the economy better off by responding to asset prices in a judicious way seems to be jumping ahead of the game. Elsewhere we have argued (Carlstrom and Fuerst (2001a)) that the focus on monetary policy should not be in finding the optimal rule, but to first ensure that any proposed rule does no harm. The problem is that by following a rule in which the central bank responds to endogenous variables, the central bank may introduce real indeterminacy and sunspot equilibria into an otherwise determinate economy.[1] These sunspot fluctuations are welfare-reducing and can potentially be quite large.

Asset prices reflect market forecasts of current and future profitability. For a central bank responding to asset prices, this presents a potential problem from the perspective of equilibrium determinacy. Firm profitability is negatively related to costs of production. In a model with sticky prices, the underlying distortion is the level of marginal cost. As marginal cost falls, the price mark-up rises, implying greater monopoly power. Equilibrium determinacy is more likely if the central bank responds positively to the underlying distortion, i.e., raises the nominal rate with increases in marginal cost. But by responding positively to asset prices the central bank is responding negatively to marginal cost, thus making real equilibrium indeterminacy more likely.

The paper proceeds as follows. In the next two sections we lay out the benchmark sticky price model and demonstrate the possibility of equilibrium indeterminacy. One criticism of this benchmark model is that in response to a monetary tightening (an increase in the nominal rate), marginal cost falls so sharply that profits actually rise. To counter this criticism, we next add sticky wages to the basic model and demonstrate that equilibrium indeterminacy may still arise even if profits do not behave in a counterfactual fashion. Finally, we consider the alternative "cash-in-advance" money demand timing suggested by Carlstrom and Fuerst (2001a). We demonstrate that under this timing assumption equilibrium indeterminacy arises for a central bank targeting share prices even if prices are relatively

[1] The term "sunspot" is in one sense misleading since these shocks are accommodated by the money supply movements needed to support the interest rate policy. But we use the term since the central bank introduces real indeterminacy by responding to public expectations which can be driven by sunspots.

flexible. Even with fairly flexible prices, very modest share price targeting will produce indeterminacy in this model variant. The final section concludes.

2. The Benchmark Model.

The theoretical model consists of households and firms. We will discuss the decision problems of each in turn. Since we are interested in issues of local equilibrium determinacy we will restrict our analysis to a deterministic model.

2.a. Households.

Households are infinitely lived, discounting the future at rate β. Their period-by-period utility function is given by

$$U(C_t, L_t, \frac{M_{t+1}}{P_t}) \equiv \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{L_t^{1+\gamma}}{1+\gamma} + V(\frac{M_{t+1}}{P_t})$$

where $\sigma > 0$, $\gamma > 0$, V is increasing and concave, C_t denotes consumption, L_t denotes labor and $\frac{M_{t+1}}{P_t}$ denotes real cash balances that can facilitate time-t transactions. The household begins period t with M_t cash balances, B_{t-1} one-period nominal bonds that pay R_{t-1} gross interest, and N_{t-1} shares of stock that sell at price Q_t and pay dividend D_t. With W_t denoting the real wage, P_t the price level, and X_t the time-t monetary injection, the household's intertemporal budget constraint is given by

$$P_tC_t + P_tQ_tN_t + B_t + M_{t+1} \leq M_t + R_{t-1}B_{t-1} + P_tN_tD_t + P_tQ_tN_{t-1} + P_tW_tL_t + X_t,$$

The stock shares are shares in the ownership of firms, and the dividends are the corresponding profit flow.

We are assuming what Carlstrom and Fuerst (2001a) call "cash-when-I'm-done timing" (CWID). That is, the cash balances that the household has in its time-t utility functional are the cash balances the household has after carrying out time-t goods transactions. As a form of sensitivity analysis we will consider "cash-in-advance" (CIA) timing below.

The household's optimization conditions include the following:

$$C_t^{\sigma} L_t^{\gamma} = W_t \tag{1}$$

$$\frac{V'(M_{t+1}/P_t)}{C_t^{-\sigma}} = \frac{R_t - 1}{R_t} \tag{2}$$

$$C_t^{-\sigma} = R_t \beta C_{t+1}^{-\sigma} / \pi_{t+1} \tag{3}$$

$$C_t^{-\sigma}(Q_t - D_t) = \beta C_{t+1}^{-\sigma} Q_{t+1} \tag{4}$$

Using the bond equation (3) in the share price equation (4) we have the familiar asset price relationship:

$$(Q_t - D_t) = \beta \frac{Q_{t+1}\pi_{t+1}}{R_t}. \tag{5}$$

2.b. Firms.

The firms in the model utilize labor services L_t from households to produce the final good using the linear technology: $Y_t = L_t$. The firm is a monopolistic producer of these goods implying that labor will be paid below its marginal product. Let Z_t denote marginal cost so that we have $W_t = Z_t$. The variable Z_t is the monopoly distortion as it measures how far the firm's marginal product of unity differs from the real wage. In the case of perfectly flexible but monopolistic prices, $Z_t = Z$ is constant and less than unity. The smaller is Z, the greater is the monopoly power. In the case of sticky prices, Z_t is variable and moves in response to the real and nominal shocks hitting the economy. Yun (1996) demonstrates that in log deviations *nominal* price adjustment is given by:

$$\pi_t = \lambda z_t + \beta \pi_{t+1} \tag{6}$$

where π_t is time-t nominal price growth (as a deviation from steady-state nominal price growth) and lower case z_t denotes the log deviation from steady-state.

As noted above, the firm's profits are paid out as dividends to the shareholders. For simplicity we assume that the measure of firms is equal to the measure of households. Hence we have that the dividend payment is given by the profits of the typical firm: $D_t = (1 - Z_t)Y_t$

2.c. Equilibrium.

There are five markets in this theoretical model: the labor market, the goods market, the asset market, the bond market, and the money market. The respective market-clearing conditions include: $C_t = L_t$, $N_t = 1$, and $B_t = 0$. The money market clears with the household holding the per capita money supply intertemporally. In what follows we assume that monetary policy is defined by a path for the gross nominal interest rate R_t. In log deviations the monetary policy rule is given by

$$R_t = \tau \pi_t + \tau_q q_t , \tag{7}$$

where q_t denotes log deviations in the share price. The implied money supply behavior (the X_t process) is passive and can be backed out of the first order condition for money holdings.

2.d. Log-linearizing the model.

It is convenient to express the equilibrium in terms of log-deviations from the steady-state. We will use lower-case letters to denote log deviations from the steady-state.

$$\sigma c_t + \gamma l_t = w_t \tag{8}$$

$$\sigma(c_{t+1} - c_t) = R_t - \pi_{t+1} \tag{9}$$

$$q_t = (1-\beta)d_t + \beta[q_{t+1} - (R_t - \pi_{t+1})] \tag{10}$$

$$\pi_t = \lambda z_t + \beta \pi_{t+1} \tag{11}$$

$$w_t = z_t \tag{12}$$

$$d_t = c_t - \left(\frac{z}{(1-z)}\right) z_t \tag{13}$$

Equation (10) implies that share prices depend upon the discounted stream of all future dividends. As for dividends, (8) and (12)-(13) can be combined to yield

$$d_t = -A z_t, \tag{14}$$

where $A \equiv \left(\frac{z(1+\sigma+\gamma)-1}{(\sigma+\gamma)(1-z)}\right)$. For all plausible calibrations we have that A > 0. We will henceforth assume that this restriction holds. Since a positive innovation in the interest rate (a monetary contraction) will decrease marginal cost, the assumption of A > 0 implies that profits will increase with positive interest rate innovations. This counterfactual implication is a well-known critique of models in which sticky-prices are the only nominal rigidity. We will return to this issue below.

Under the policy rule (7), the central bank is responding *negatively* to the discounted stream of all future marginal costs. As noted earlier, this negative element to the rule suggests that equilibrium determinacy may be a problem.

3. Equilibrium Determinacy

3.a. Equilibrium determinacy with sticky prices.

We now consider the parameter restrictions on τ and τ_q that ensure determinacy of the equilibrium. After collapsing the system into three variables, we have the following:

$$\begin{pmatrix} -1 & \frac{-\sigma}{\sigma+\gamma} & 0 \\ \beta & 0 & 0 \\ \beta & 0 & \beta \end{pmatrix} \begin{pmatrix} \pi_{t+1} \\ z_{t+1} \\ q_{t+1} \end{pmatrix} = \begin{pmatrix} -\tau & \frac{-\sigma}{\sigma+\gamma} & -\tau_q \\ 1 & -\lambda & 0 \\ \beta\tau & (1-\beta)A & \beta\tau_q + 1 \end{pmatrix} \begin{pmatrix} \pi_t \\ z_t \\ q_t \end{pmatrix}$$

Note that if $\tau_q = 0$, the system separates so that q_t has no effect on π_t and z_t, although the latter two variables do of course affect share prices. It is straightforward to demonstrate that in this case $\tau > 1$ is necessary and sufficient for equilibrium determinacy.

The above dynamic system has three eigenvalues. For equilibrium determinacy all three must lie outside the unit circle. It is straightforward to demonstrate that one is equal to $1/\beta$. Let F(x) denote the remaining quadratic. It is given by:

$$F(x) \equiv x^2 - \frac{[(1+\lambda+\beta+\beta\tau_q)\sigma + \lambda\gamma]}{\beta\sigma} x + F_0$$

where

$$F_0 \equiv \frac{\tau_q[\sigma\beta - A(\sigma+\gamma)(1-\beta)] + [\sigma + \lambda\tau(\sigma+\gamma)]}{\beta\sigma}.$$

A necessary condition for determinacy is that F(-1) and F(1) be of the same sign. Since F is decreasing in this range, this is both a necessary and sufficient condition for determinacy. We thus have two cases, $F(-1) > F(1) > 0$ and $F(1) < F(-1) < 0$. These values are given by:

$$F(-1)=\frac{\tau_q[2\beta\sigma-(1-\beta)A(\sigma+\gamma)]+[2\sigma(1+\beta)+\lambda(\tau+1)(\sigma+\gamma)]}{\beta\sigma}$$

$$F(1)=\frac{-\tau_q(1-\beta)A(\sigma+\gamma)+\lambda(\sigma+\gamma)(\tau-1)}{\beta\sigma}$$

If F(-1) > 0 (this is the case for all reasonable calibrations), a necessary and sufficient condition for determinacy is that F(1) > 0. If F(-1) < 0 we always have determinacy. Combining we find that there is indeterminacy if and only if

$$\tau_q^p \equiv \frac{\lambda(\tau-1)}{(1-\beta)A} < \tau_q \quad \text{if } 2\beta\sigma > (1-\beta)A(\sigma+\gamma) \tag{15}$$

$$\tau_q^p < \tau_q < \frac{[2\sigma(1+\beta)+\lambda(\tau+1)(\sigma+\gamma)]}{[(1-\beta)A(\sigma+\gamma)-2\beta\sigma]} \quad \text{if } 2\beta\sigma < (1-\beta)A(\sigma+\gamma) \tag{16}$$

For reasonable parameter values, (15) is the relevant constraint. Hence, we have local indeterminacy for values of τ_q in excess of τ_q^p. There are two points worth noting about this bound. First it is proportional to τ - 1. Reacting aggressively enough to inflation ensures that responding to share prices will not create indeterminacy. Secondly as β approaches unity, indeterminacy is also not a problem. This can be understood by noting that when β=1 the impact of dividends drops out of the model (see (10)).

To analyze the likelihood of indeterminacy, consider the following parameter values: β = .99, σ = γ = 2, z = .9, λ = .019, and τ = 1.5. In this case we have that there is indeterminacy whenever τ_q >.109. Note that if prices are less sticky, say λ = .19, the bound increases to 1.09.[2]

[2] There is considerable uncertainty about the value of this parameter. Our baseline number of 0.019 comes from Woodford (2003), page 347.

Bullard and Mitra (2002) report the determinacy bound for a sticky price model in which the central bank responds to the output gap and inflation. In the current context their interest rate rule is given by

$$R_t = \tau\pi_t + \tau_z z_t \, .$$

Under this rule, the necessary and sufficient condition for determinacy is given by $\lambda(\tau-1)+(1-\beta)\tau_z > 0$. In comparison, (15) implies that the determinacy condition is $\lambda(\tau-1)-(1-\beta)A\tau_q > 0$. As emphasized above, targeting share prices is very similar to negatively responding to marginal cost. The amplification effect of the "A" term comes from the fact that dividends are a multiple of marginal cost (see (14)). Thus for the issue of equilibrium determinacy, reacting to share prices is equivalent to reacting to dividends.

3.b. Equilibrium determinacy with sticky wages.

We now consider the polar opposite model of nominal rigidity: flexible prices, but sticky nominal wages. Such a model may deliver differing determinacy results because firm profits, and thus share prices, move oppositely with the monopoly power of workers. Unlike the sticky price model, in a sticky wage model profits will fall with positive interest rate innovations. Given that profits are now responding in the empirically plausible direction, is indeterminacy still a possibility?

Following Erceg, Henderson and Levin (2000), we assume that households are monopolistic suppliers of labor and that nominal wages are adjusted in a Calvo-style (1983). In this case labor supply behavior is given by

$$C_t^{\sigma} L_t^{\gamma} = Z h_t W_t \, . \tag{17}$$

The variable Zh_t is the monopoly distortion as it measures how far the household's marginal rate of substitution is from the real wage. In the case of perfectly flexible but monopolistic wages, Zh_t = Zh is constant and less than unity. The smaller is Zh, the greater is the monopoly power. In the case of sticky wages, Zh_t is variable and moves in response to the real and nominal shocks hitting the economy. Erceg et al. (2000) demonstrate that in log deviations *nominal* wage adjustment is given by:

$$\pi_t^W = \lambda^W zh_t + \beta \pi_{t+1}^W \tag{18}$$

where π_t^W is time-t net nominal wage growth (in log deviations). In this case the deterministic system is given by (7), (18) and:

$$\sigma c_t + \gamma l_t = zh_t \tag{19}$$

$$\sigma(c_{t+1} - c_t) = R_t - \pi_{t+1} \tag{20}$$

$$q_t = (1-\beta)d_t + \beta[q_{t+1} - (R_t - \pi_{t+1})] \tag{21}$$

$$w_t - w_{t-1} = \pi_t^W - \pi_t \tag{22}$$

$$w_t = 0 \tag{23}$$

$$d_t = c_t \tag{24}$$

Note that $\pi_{t+j}^W = \pi_{t+j}, \; for \;\; j \geq 1$. Thus, scrolling the system forward one period we have

$$\sigma(c_{t+2} - c_{t+1}) = R_{t+1} - \pi_{t+2}$$

$$q_{t+1} = (1-\beta)c_{t+1} + \beta[q_{t+2} - (R_{t+1} - \pi_{t+2})]$$

$$\pi_{t+1} = \lambda^W zh_{t+1} + \beta\pi_{t+2}$$

$$c_{t+1} = \left(\frac{1}{(\sigma+\gamma)}\right) zh_{t+1}.$$

If this system is determinate at t+1 we can count equations and unknowns to show that the system at time t is also determinate. The system above is isomorphic to the case of sticky prices, with one key difference. Solving for dividends as a function of the monopoly distortion we have:

$$d_t = \left(\frac{1}{(\sigma+\gamma)}\right) zh_t. \tag{25}$$

In sharp contrast to the case of sticky prices, dividends now respond *positively* to movements in the nominal wage distortion (recall that a higher zh_t implies a decline in monopoly power by workers). This suggests that responding positively to share prices will not generate indeterminacy. Unlike the sticky price model, profits now fall with positive interest rate innovations.

In matrix form the system is identical to the sticky price model except for the term reflecting how dividends respond to zh_t, which is now negative. The model is given by

$$\begin{pmatrix} -1 & \frac{-\sigma}{\sigma+\gamma} & 0 \\ \beta & 0 & 0 \\ \beta & 0 & \beta \end{pmatrix} \begin{pmatrix} \pi^W_{t+2} \\ zh_{t+2} \\ q_{t+2} \end{pmatrix} = \begin{pmatrix} -\tau & \frac{-\sigma}{\sigma+\gamma} & -\tau_q \\ 1 & -\lambda^W & 0 \\ \beta\tau & \frac{-(1-\beta)}{(\sigma+\gamma)} & \beta\tau_q + 1 \end{pmatrix} \begin{pmatrix} \pi^W_{t+1} \\ zh_{t+1} \\ q_{t+1} \end{pmatrix}$$

with the characteristic equation

$$F(x) \equiv x^2 - \frac{[(1+\lambda^W + \beta + \beta\tau_q)\sigma + \lambda\gamma]}{\beta\sigma} x + F_0$$

where

$$F_0 \equiv \frac{\tau_q[\sigma\beta + (1-\beta)] + [\sigma + \lambda^W \tau(\sigma+\gamma)]}{\beta\sigma}.$$

Note that F is convex, F(0) > 0, and F'(1) < 0. A necessary and sufficient condition for determinacy is that F(1) be positive. It is given by:

$$F(1) = \frac{\tau_q(1-\beta) + \lambda^W(\sigma+\gamma)(\tau-1)}{\beta\sigma}$$

With $\tau > 1$, this is the case for all values of $\tau_q > 0$.

Hence, in a model with sticky nominal wages but perfectly flexible nominal prices, asset price targeting is not prone to indeterminacy. The reason is that in the case of sticky wages a central bank responding positively to asset prices is also responding positively to the underlying labor market distortion.

3.c. Equilibrium determinacy with sticky prices and wages.

The previous results suggest that we should examine the model with both forms of nominal stickiness. In this case, there are two distortions arising from the monopoly power of firms and the monopoly power of workers. The first distortion is proxied by marginal cost z_t, while the second is given by the mark-up of real wages over the household's marginal rate of substitution zh_t. The corresponding expression for dividends is given by:

$$d_t = \left(\frac{zh_t}{\sigma+\gamma} - Az_t\right) \tag{26}$$

Note how dividends are related to the underlying distortions. By responding to asset prices the central bank is responding positively to one distortion but negatively to the other.

There are two issues for this section. First, can real indeterminacy arise when both forms of nominal rigidity are present? Second, can real indeterminacy arise even if profits respond positively to a monetary expansion (a decline in the nominal rate)? We first turn to the determinacy analysis.

The dynamic system is now a 5th-order and is given by

$$A\begin{pmatrix} \pi_{t+1} \\ w_{t+1} \\ w_t \\ zh_{t+1} \\ q_{t+1} \end{pmatrix} = B\begin{pmatrix} \pi_t \\ w_t \\ w_{t-1} \\ zh_t \\ q_t \end{pmatrix}$$

where A and B are:

$$A \equiv \begin{pmatrix} -1 & \frac{-\sigma}{\sigma+\gamma} & \frac{\sigma}{\sigma+\gamma} & \frac{-\sigma}{\sigma+\gamma} & 0 \\ \beta & 0 & \lambda & 0 & 0 \\ \beta & \beta & -(1+\beta) & 0 & 0 \\ \beta & 0 & -(1-\beta)A & 0 & \beta \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

$$B \equiv \begin{pmatrix} -\tau & 0 & 0 & \frac{-\sigma}{\sigma+\gamma} & -\tau_q \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & -1 & -\lambda_W & 0 \\ \beta\tau & 0 & 0 & \frac{\beta-1}{\sigma+\gamma} & \beta\tau_q+1 \\ 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$

There is one state variable so that determinacy requires four explosive roots and one root within the unit circle. As before, one root is always $1/\beta > 1$, so that we are left with a quartic of the following form:

$$F(x) \equiv x^4 + F_3x^3 + F_2x^2 + F_1x + F_0$$

where F_0 (the product of the roots) and F_3 (the negative of the sum of the roots) have a simple form

$$F_0 \equiv \frac{\sigma + \tau_q(1-\beta+\beta\sigma)}{\beta^2\sigma} > 1$$

$$F_3 \equiv -\frac{2(1+\beta)+\beta\tau_q+\lambda+\lambda_W}{\beta} < 0\,.$$

We also have that $F_2 > 0$. For determinacy, we need one root in the unit circle. Since F is convex at zero, and F'''(1) > 0, we know that at least two roots lie to the right of one. Hence, a necessary and sufficient condition for determinacy is that F(1) and F(-1) are of the opposite sign. These are given by:

$$F(1) = \frac{(1-\tau)\lambda\lambda_W(\sigma+\gamma)+\tau_q(\lambda_W A(\sigma+\gamma)-\lambda)(1-\beta)}{\beta^2\sigma}$$

$$F(-1) = \frac{(\tau+1)\lambda\lambda_W(\sigma+\gamma)+2\sigma(1+\beta)[(\lambda+\lambda_W)+2(1+\beta)]+\tau_q B_q}{\beta^2\sigma}$$

where

$$B_q \equiv 2\beta\sigma[(\lambda+\lambda_W)+2(1+\beta)]+2(1-\beta^2)-(\lambda_W A(\sigma+\gamma)-\lambda)(1-\beta)\,.$$

If F(-1) < 0, then F(1) > 0. F(-1) can be negative only if $B_q < 0$. If $B_q > 0$, then F(-1) > 0, so that a necessary and sufficient condition for determinacy is F(1) < 0. Hence we have the following bounds for determinacy: If $B_q > 0$, then there is indeterminacy if and only if

$$(1-\tau)\lambda\lambda_W(\sigma+\gamma)+\tau_q(\lambda_W A(\sigma+\gamma)-\lambda)(1-\beta) > 0. \tag{27}$$

Assuming $\tau > 1$, we can express this as

$$\tau_q > \tau_q^{wp} \equiv \frac{\lambda\lambda_W(\sigma+\gamma)(\tau-1)}{(1-\beta)(A(\sigma+\gamma)\lambda_W-\lambda)}\,. \tag{28}$$

If $B_q < 0$, then there is indeterminacy if and only if

$$\tau_q^{wp} < \tau_q < \frac{(\tau+1)\lambda\lambda_W(\sigma+\gamma)+2\sigma(1+\beta)[(\lambda+\lambda_W)+2(1+\beta)]}{-B_q}. \tag{29}$$

Note that as λ_W goes to infinity, these bounds collapse to the previous bounds for the sticky price model (see (15) and (16)).

For plausible parameter values, the term B_q will be positive so that condition (27) is relevant. Note that $B_q > 0$ is a much weaker restriction than the corresponding restriction for sticky prices ($2\beta\sigma > (1-\beta)A(\sigma+\gamma)$). Once again this same bound also occurs if the central bank were to respond to dividends (26) instead of share prices. The τ_q bound in (28) is larger than the case of sticky prices and flexible wages but converges to it as $\lambda_W \to \infty$. As we will show below, however, the impact of sticky wages has only a marginal quantitative impact on the bound.

Let us now return to the profits issue. Can real indeterminacy arise even if profits respond positively to a monetary expansion (a decline in the nominal rate)? Suppose that the central bank's policy rule is given by

$$R_t = \tau\pi_t + \tau_q q_t + \eta_t$$

where η_t is an iid exogenous shock to the nominal rate. Assume that the model is at the steady-state, and consider an iid decrease in the nominal rate (η_t < 0) such that marginal cost increases by $z_t = 1$. For profits to not respond to this shock we need that $zh_t = A(\sigma+\gamma)z_t = A(\sigma+\gamma)$. The remaining equilibrium conditions include:

$$z_t - z_{t-1} = \pi_t^W - \pi_t \tag{30}$$

$$\pi_t^W = \lambda^W zh_t + \beta\pi_{t+1}^W \tag{31}$$

$$\pi_t = \lambda z_t + \beta\pi_{t+1}. \tag{32}$$

If we look at the difference in the price and wage adjustment equations (31)-(32) we have

$$\pi_t^W - \pi_t = (A\lambda^W(\sigma+\gamma)-\lambda)z_t + \beta(\pi_{t+1}^W - \pi_{t+1}).$$

Using (30) we have

$$1 = (A\lambda^W(\sigma+\gamma)-\lambda) + \beta(\rho_z - 1)$$

where $0 < \rho_z < 1$ denotes a stable root of the dynamic system and thus the decay rate in z_t. If there is determinacy there is only one such root. In the case of indeterminacy, there is more than one root to choose from. In this case we will choose the root that is closest to the single root when there is determinacy.

Rearranging we have that the zero profit condition is given by

$$1 + \lambda + \beta(1-\rho_z) = A\lambda^W(\sigma+\gamma).$$

If the left-hand side is greater (lesser) than the right-hand side, then profits increase (decrease) with a monetary expansion. Note the tension between the ability of profits to increase with a monetary expansion and indeterminacy. For profits to increase *and* for there to be indeterminacy we need $0 < A\lambda^W(\sigma+\gamma) - \lambda < 1 + \beta(1-\rho_z)$. This does suggest, however, that both can occur.

For example, consider the following numerical experiments. Woodford reports values for the nominal rigidities of $\lambda^W = .035$ and $\lambda = .019$.[3] Using these values, and $\beta = .99$, $\sigma = \gamma = 2$, $z = .9$, and $\tau = 1.5$, we have that determinacy requires $\tau_q < .11$. Note that λ_W has little effect on the τ_q bound: as we let $\lambda_W \to \infty$ this bound drops only to .109. However, the sticky nominal wages improves the profits prediction. Under this calibration, the profits condition is mildly positive: $1 + \lambda + \beta(1-\rho_z) - A\lambda^W(\sigma+\gamma) = .0019$.

Woodford (2003), however, argues for a much lower calibration for the preference coefficients: $\sigma = .16$, $\gamma = .47$ (the case of "strategic complementarity").[4] In this case the determinacy bound is $\tau_q < .145$, and profits respond strongly to a monetary expansion: $1+\lambda+\beta(1-\rho_z)-A\lambda^W(\sigma+\gamma)=1.05.$

3.d. Equilibrium determinacy with CIA Timing.

Carlstrom and Fuerst (2001a) have criticized the previous money demand timing convention as it reflects "cash-when-I'm-done timing" (CWID). That is, the cash balances that the household has in its time-t utility functional are the cash balances the household has after carrying out time-t goods transactions. As a form of sensitivity analysis we will now consider the more intuitive "cash-in-advance" (CIA) timing. We will restrict our discussion to the model with sticky prices and wages.

The households' period-by-period utility function is now given by

$$U(C_t, L_t, \frac{MM_t}{P_t}) \equiv \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{L_t^{1+\gamma}}{1+\gamma} + V(\frac{MM_t}{P_t})$$

where C_t denotes consumption, L_t denotes labor and $\frac{MM_t}{P_t}$ denotes real cash balances that can facilitate time-t transactions. The household begins period t with M_t cash balances, B_{t-1} one-period nominal bonds that pay R_{t-1} gross interest, and N_{t-1} shares of stock that sell at price Q_t and pay dividend D_t. The asset market opens at the beginning of the period leaving the household with the following cash balances for use in the goods market:

$$MM_t \equiv M_t + X_t + R_{t-1}B_{t-1} + P_tQ_tN_{t-1} - B_t - P_tN_t(Q_t - D_t).$$

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[3] The coefficients λ and λ_W are taken from Woodford (2003), page 347.

where P_t is the price level, and X_t denotes the time-t monetary injection. The household's intertemporal budget constraint is given by

$$P_tC_t + P_tQ_tN_t + B_t + M_{t+1} \le M_t + R_{t-1}B_{t-1} + P_tN_tD_t + P_tQ_tN_{t-1} + P_tW_tL_t + X_t,$$

The household's optimization conditions include the following:

$$C_t^{\sigma}L_t^{\gamma} = W_t \tag{33}$$

$$\frac{C_t^{-\sigma} + V'(m_t)}{C_t^{-\sigma}} = R_t \tag{34}$$

$$[C_t^{-\sigma} + V'(m_t)] = R_t\beta[C_{t+1}^{-\sigma} + V'(m_{t+1})]/\pi_{t+1} \tag{35}$$

$$[C_t^{-\sigma} + V'(m_t)](Q_t - D_t) = \beta[C_{t+1}^{-\sigma} + V'(m_{t+1})]Q_{t+1} \tag{36}$$

Compared to CWID timing, the key difference is in the Fisher equation (35). In sharp contrast to the case of CWID timing (see (3)), in the case of CIA timing the purchase of a bond at time-t lowers household time-t liquidity. Hence, even in the present case of separability between consumption and real balances (U_{cm} =0), the level of real balances has a direct effect on bond pricing. Seen another way, we can substitute the money demand equation into the bond equation and yield:

$$[C_t^{-\sigma}] = R_{t+1}\beta[C_{t+1}^{-\sigma}]/\pi_{t+1} \tag{37}$$

Using the bond equation in the share price equation we have

$$(Q_t - D_t) = \beta\frac{Q_{t+1}\pi_{t+1}}{R_t} \tag{38}$$

In comparison to CWID timing, the key difference is the reduced form Fisher equation, (37). However, arbitrage between bonds and shares yields the same pricing relationship for share prices (38).

[4] We have the case of strategic complements when σ+γ < 1. See Woodford (2003), page 165.

In the case of sticky prices and sticky wages, the dynamic system is given by

$$A\begin{pmatrix} \pi_{t+1} \\ w_{t+1} \\ w_t \\ zh_{t+1} \\ q_{t+1} \end{pmatrix} = B\begin{pmatrix} \pi_t \\ w_t \\ w_{t-1} \\ zh_t \\ q_t \end{pmatrix}$$

where A and B are:

$$A \equiv \begin{pmatrix} \tau-1 & \frac{-\sigma}{\sigma+\gamma} & \frac{\sigma}{\sigma+\gamma} & \frac{-\sigma}{\sigma+\gamma} & \tau_q \\ \beta & 0 & \lambda & 0 & 0 \\ \beta & \beta & -(1+\beta) & 0 & 0 \\ \beta & 0 & -(1-\beta)A & 0 & \beta \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

$$B \equiv \begin{pmatrix} 0 & 0 & 0 & \frac{-\sigma}{\sigma+\gamma} & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & -1 & -\lambda_W & 0 \\ \beta\tau & 0 & 0 & \frac{\beta-1}{\sigma+\gamma} & \beta\tau_q+1 \\ 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$

For simplicity, we will analyze this system for $\beta = 1$ and then present numerical results for $\beta < 1$. There is one state variable so that determinacy requires four explosive roots and one root within the unit circle. One root is always one which we will treat as explosive, so that we are left with a quartic of the form

$$F(x) \equiv x^4 + F_3x^3 + F_2x^2 + F_1x + F_0 .$$

As in the CWID model, $F'''(1) < 0$, $F_3 < 0$, $F_1 < 0$, $F_0 > 0$. But the sign of F_2 is ambiguous and is given by

$$F_2 \equiv 2\sigma(\lambda+\lambda_W+3) - \lambda\lambda_W(\tau-1)(\sigma+\gamma) + \tau_q[\lambda\lambda_W(\sigma+\gamma) + \sigma(\lambda+\lambda_W+3)].$$

For now let us assume that $F_2 > 0$ (so that all the roots have positive real parts). Analogous to the CWID proof, we need one root in the unit circle for determinacy, so that a necessary and sufficient condition for determinacy is

$$F(1) \equiv \frac{(\sigma+\gamma)\lambda\lambda_W[\tau_q-(\tau-1)]}{\sigma} < 0.$$

Assuming $F_2 > 0$, indeterminacy arises if and only if $\tau_q > (\tau-1)$. Note that if $F(1) > 0$, then we necessarily have $F_2 > 0$ and

$$F'(1) \equiv \frac{2(\sigma+\gamma)\lambda\lambda_W[\tau_q-(\tau-1)]+\sigma(\lambda+\lambda_W)}{\sigma} > 0$$

so that $\tau_q > (\tau-1)$ is sufficient for indeterminacy.[5] Estimates of τ are typically below 1.5, so that $\tau_q > .5$ implies indeterminacy. This is in sharp contrast to the CWID model where $\beta = 1$ implied that responding to share prices could never produce indeterminacy.

We can develop some intuition for these conflicting results by examining the two money demand models and the implied links between nominal interest rates and real share prices. In the case of CWID timing, the Fisher equation (9) and the share price equation (10) can be combined to yield:

$$q_t = \beta q_{t+1} + [(1-\beta)d_t - \beta\sigma\Delta c_{t+1}] \tag{39}$$

where $\Delta c_{t+1} \equiv c_{t+1} - c_t$. In the case of $\beta = 1$, we have that $q_t = \sigma c_t$. In this case, the nominal interest rate has an effect on share prices only through its effect on consumption and thus the real rate. Substituting the expression for asset prices into the policy rule we have

[5] For $\tau_q < (\tau-1)$ and $F_2 < 0$, indeterminacy can still arise if τ is sufficiently large. Carlstrom and Fuerst (2001) note that a CIA model with a current-based interest rate rule is isomorphic to a CWID model with a forward-based interest rate rule. Bullard and Mitra (2002) consider the latter and note that indeterminacy arises for inflation responses that are too aggressive.

$R_t = \tau\pi_t + \tau_q \sigma c_t$. Because β =1 this rule has the same determinacy conditions as the simpler rule where the central bank only responds to inflation, $R_t = \tau\pi_t$, implying that for τ > 1 we have determinacy for all $\tau_q \geq 0$.

Matters are much different in the case of the CIA model. The Fisher equation (37) can now be expressed as

$$R_t - \pi_{t+1} = \sigma\Delta c_{t+1} - \Delta R_{t+1} \tag{40}$$

The nominal rate directly enters this expression because (in this case of CIA timing) real money balances matter for bond pricing. The share price equation can be expressed as:

$$q_t = \beta q_{t+1} + (1-\beta)d_t - \beta\sigma\Delta c_{t+1} + \beta\Delta R_{t+1}.$$

Now we have an additional channel for indeterminacy: Since the current nominal rate directly affects the current share price in a negative direction, the central bank's response to inflation is muted. For example, in the case of β = 1 we have $q_t = \sigma c_t - R_t$, implying a Taylor rule of the form $R_t = \frac{\tau}{1+\tau_q}\pi_t + \frac{\tau_q}{1+\tau_q}\sigma c_t$. Once again because β =1 this rule has the same determinacy conditions as the simpler rule where the central bank only responds to inflation, $R_t = \frac{\tau}{1+\tau_q}\pi_t$. Responding to share prices indirectly lowers the central bank's response to inflation. If $\frac{\tau}{1+\tau_q} < 1$ or $\tau_q > \tau - 1$, indeterminacy arises even holding dividends fixed!

The assumption of β = 1 is convenient for the above derivations. But because β = 1 is equivalent to holding dividends fixed, this assumption completely closes off the channel for indeterminacy emphasized in the earlier sections with CWID timing. The actual bounds for determinacy tighten sharply as we move from β=1 to β = .99. The central bank is once again responding to dividends or negatively to marginal cost as in (26). For example, with $\sigma = \gamma = 2$, z = .9, $\lambda^W = .035$, $\lambda = .019$, and $\tau = 1.5$, we have that determinacy requires $\tau_q < .097$ (compared to .11 with CWID). Using Woodford's calibration for the preference coefficients, $\sigma = .16$, $\gamma = .47$, we have a share price bound of $\tau_q < .12$ (compared to .145 with CWID).[6]

The difference between the CIA and CWID bounds, however, becomes more significant as we move to a more flexible price/wage economy. Suppose we increase the nominal adjustment coefficients by a factor of ten: $\lambda^W = .35$, $\lambda = .19$. With $\beta = .99$, $\sigma = \gamma = 2$, z = .9, and $\tau = 1.5$, we have that determinacy requires $\tau_q < .35$ for the CIA model. In the case of CWID, the bound is $\tau_q < 1.1$. Using Woodford's preference calibration, $\sigma = .16$, $\gamma = .47$, the CIA bound is $\tau_q < .37$, while the CWID bound is $\tau_q < 1.45$.

4. Conclusion.

The celebrated Taylor rule (1993) posits that central bank behavior can be described by a fairly simple rule linking nominal rate movements to movements in inflation and output. This seminal paper has spawned a large literature concerned with issues of stability: under what situations can a Taylor-rule formulation of monetary policy create real indeterminacy and thus sunspot fluctuations in the model economy? See for example, Benhabib, Schmitt-Grohe and Uribe (2001), Bernanke and Woodford (1997), Carlstrom and Fuerst (2001a,2001b,2004), Clarida, Gali and Gertler (2000), and Kerr and King (1996).

The current paper extends this literature in a natural direction. Many have suggested that the central bank should adjust policy in response to asset price movements. If we put

[6] The conditions for profits are identical to the earlier case with CWID timing.

such a response into a Taylor rule, is equilibrium indeterminacy more or less likely? The answer appears to be "more." If wages and prices are quite sticky, as is typically calibrated, the possibility of indeterminacy is quantitatively relevant irrespective of money demand timing. With more flexible prices and wages, the possibility of indeterminacy is less likely with CWID timing, but remains quantitatively relevant for CIA timing.

In their defense of a central bank response to asset prices, Cecchetti, Genberg, Lipsky, and Wadhwani (2000) argue that "…reacting to asset prices in the normal course of policy-making will reduce the likelihood of asset price bubbles forming, thus reducing the risk of boom-bust investment cycles." [Page 2] Bubbles are of course non-fundamental movements in asset prices. The moral of this paper is almost the polar opposite of Cecchetti et al. By reacting to asset prices, the central bank can inadvertently introduce real indeterminacy and thus sunspot equilibria into the economy. Hence by trying to avoid bubbles, the central bank can inadvertently introduce non-fundamental movements into both asset prices and real activity.

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