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This paper analyzes the restrictions necessary to ensure that the interest rate policy rule used by the central bank does not introduce real indeterminacy into the economy. It conducts this analysis in a flexible price economy and a sticky price model that satisfies the natural rate hypothesis. A necessary and sufficient condition for real determinacy in the sticky price model is for there to be nominal and real determinacy in the corresponding flexible price model. This arises if and only if the Taylor rule responds aggressively to lagged inflation rates.

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I. Introduction.

The celebrated Taylor (1993) rule posits that the central bank uses a fairly simple interest rate rule when conducting monetary policy. This rule is a reaction function linking movements in the nominal interest rate to movements in inflation and other endogenous variables. Recently there has been a considerable amount of interest in ensuring that such rules do no harm. The problem is that by following a rule in which the central bank responds to endogenous variables, it increases the likelihood that the central bank introduces real indeterminacy and sunspot equilibria into an otherwise determinate economy. These sunspot fluctuations are welfare-reducing and can potentially be quite large.

The recent literature on determinacy and Taylor rules is voluminous including Benhabib, Schmitt-Grohe and Uribe (2001a,b), Bernanke and Woodford (1997), Carlstrom and Fuerst (2000,2001ab), Clarida, Gali, Gertler (2000), Dupor (2001) and Kerr and King (1996). These papers typically analyze either an economy with perfectly flexible prices or impose Calvo-style (1983) staggered pricing on the economy. One peculiarity of the Calvo pricing assumption is that it does not satisfy the natural rate hypothesis (NRH). That is, the central bank can permanently increase output by engineering an ever-increasing inflation rate.

The novelty of the current paper is to focus on a sticky price model that does satisfy the NRH. This paper demonstrates that in a NRH model a necessary and sufficient condition for real determinacy is for the corresponding flexible price economy to have *both* real and nominal determinacy. This implies that in a NRH model a necessary and sufficient condition for real determinacy is for the monetary authority to react aggressively to *past* movements in inflation. This result is in contrast to a Calvo style model in which current-looking interest rate rules are typically determinate. The paper also demonstrates that the sunspot equilibria that arise in NRH models are typically learnable in the sense of E-stability.

II. The Basic Model

The economy consists of numerous households and firms. Since we are concerned with issues of determinacy without loss of generality we limit the discussion to a deterministic model. As is well known, if the deterministic dynamics are not unique, then it is possible to construct sunspot equilibria in the model economy. We make two further simplifying assumptions. First, we follow the convention of abstracting from capital accumulation as the point we wish to make is independent of the investment margin.[1] Second, we assume linear preferences over leisure. The results of Carlstrom and Fuerst (2000,2001b) suggest that this comes with little loss of generality.

Households are identical and infinitely-lived with preferences over consumption, real money balances and leisure given by

$$\sum_{t=0}^{\infty} \beta^t U(c_t,a_t,1-L_t),$$

where β is the personal discount rate, c_t is consumption, $a_t \equiv A_t/P_t$ is real cash balances available for transactions during time t, P_t is the price level, and $1-L_t$ is leisure. The utility function is given by $U(c,1-L) \equiv V(c,a) - L$.

[1] Carlstsrom and Fuerst (2000,2001b) and Dupor (2001) explore the effects of investment spending on the determinacy conditions.

The household begins the period with M_t cash balances and B_{t-1} holdings of nominal bonds. Before proceeding to the goods market, the household visits the financial market where it carries out bond trading and receives a cash transfer of $M_t^s(G_t - 1)$ from the monetary authority where M_t^s denotes the per capita money supply and G_t is the gross money growth rate, $G_t \equiv M^s_{t+1}/M^s_t$. Hence, before entering goods trading, the household has nominal cash balances given by

$$A_t \equiv M_t + M_t^s(G_t - 1) + B_{t-1}R_{t-1} - B_t,$$

where R_{t-1} denotes the gross nominal interest rate from t-1 to t. Notice that following Carlstrom and Fuerst (2000a) we utilize cash-in-advance (CIA) timing.[2] After engaging in goods trading, the household ends the period with cash balances given by the intertemporal budget constraint

$$M_{t+1} = A_t + P_t w_t L_t - P_t c_t + \Pi_t,$$

where the real wage is given by w_t and Π_t denotes the profit flow from firms.

As for firm behavior, we follow Yun (1996) and utilize a model of imperfect competition in the intermediate goods market. Final goods production in this economy is carried out in a perfectly competitive industry that utilizes intermediate goods in production. Intermediate goods firms are monopolist producers of differentiated intermediate goods. Each intermediate firm hires labor from households and utilizes a linear production function denoted by f(L) = L. Imperfect competition implies that factor payments are distorted, $w_t = z_t$ where z_t is marginal cost.

The model's equilibrium conditions are:

$$\frac{U_L(t)}{U_c(t)} = z_t \tag{1}$$

$$\frac{U_c(t) + U_m(t)}{P_t} = \beta R_t \left\{ \frac{U_c(t+1) + U_m(t+1)}{P_{t+1}} \right\} \tag{2}$$

$$\frac{U_c(t) + U_m(t)}{U_c(t)} = R_t \tag{3}$$

$$c_t = L_t \tag{4}$$

Using the assumed functional forms, and inserting the labor margin (1) and money demand curve (3) into the Fisher equation (2) yields:

$$\tilde{R}_{t+1} - \tilde{\pi}_{t+1} = \tilde{z}_{t+1} - \tilde{z}_t \tag{5}$$

where $\pi_t \equiv P_t/P_{t-1}$ and the tildes denote log deviations from the steady-state. The system is defined by the behavior of marginal cost and the nominal interest rate.

The behavior of marginal cost is determined by our assumptions on how the intermediate goods are priced. Below we consider two possibilities: a Calvo (1983) model of forever stickiness, and a NRH model with finite stickiness.

As for the nominal rate, we assume a central bank reaction function where the current nominal interest rate is a function of inflation:

$$R_t = R_{ss}\left(\frac{\pi_{t+i}}{\pi_{ss}}\right)^{\tau}, \textit{ where } \tau \geq 0,\ R_{ss} = \frac{\pi_{ss}}{\beta}. \tag{6}$$

[2] The alternative to CIA timing is cash-when-I'm-done-timing where $A_t = M_{t+1}$.

We consider three variations of this simple rule: i=-1 is a backward-looking rule, i=0 is a current-looking rule, and i=1 is a forward-looking rule. Under any such interest rate policy the money supply responds endogenously to be consistent with the interest rate rule. It is this endogeneity that leads to the possibility of indeterminacy.

III. Equilibrium Determinacy.

There are two types of indeterminacy that may arise. First, there is *nominal indeterminacy*—are the initial values of the price level and other nominal variables pinned down? In our notation this corresponds to the question of whether $\pi_t \equiv P_t/P_{t-1}$ is determined (where t is the initial time period). This nominal indeterminacy is of no consequence in and of itself, but is important only if it leads to real indeterminacy. By *real indeterminacy*, we mean a situation in which the behavior of one or more real variables is not pinned down by the model. This arises when the model does not pin down the behavior of the nominal interest rate and/or marginal cost.

Real Indeterminacy with Forever Stickiness.

Following Calvo (1983), assume that each period a fraction of firms get to set a new price, while the remaining fraction must charge the previous period's price times steady-state inflation. This probability of a price change is constant across time and is independent of how long it has been since any one firm has last adjusted its price. Yun (1996) demonstrates that in this case we have the following "Phillips curve"

$$\tilde{\pi}_t = \lambda \tilde{z}_t + \beta \tilde{\pi}_{t+1} . \quad (7)$$

Along an arbitrary deterministic path for inflation $\tilde{z}_t$ need never equal zero, ie., z_t need never equal $\bar{z} < 1$, the steady-state monopolistic competition distortion. This pricing arrangement does not satisfy the NRH as there are deterministic inflation paths that will keep $\tilde{z}_t > 0$ and thus permanently stimulate output. Carlstrom and Fuerst (2000) report the following determinacy conditions for this model.

Proposition 1: Suppose that prices are set in a Calvo fashion (7) *and that monetary policy is given by (6). Under the mild assumption* $\beta+\lambda \geq 1$, *all forward-looking rules (i = 1) are indeterminate. With a current-looking interest rate rule (i=0) there is real determinacy if and only if*

$$1 < \tau < \frac{2(\beta + 1) + \lambda}{\lambda}.$$

With a backward-looking interest rate rule (i=-1) there is real determinacy if and only if $\tau > 1$.

Note that an aggressive (but not too aggressive) current-looking rule is determinate in this environment.[3]

Real Indeterminacy with Finite Stickiness.

One disadvantage of the Calvo model is that violates the NRH. This is especially troublesome for issues of determinacy: An equilibrium is determinate if perturbations from the equilibrium path lead to explosive inflation dynamics, but surely the Calvo pricing arrangement would not continue to hold along such a path. Calvo pricing implies that there is some poor firm along these hyperinflationary paths that has never adjusted prices despite the fact that prices have increased a million-fold! In contrast, a NRH model implies that at some finite date all firms will have adjusted their prices. This section demonstrates that the determinacy conditions in a NRH model of finite stickiness are more stringent than in a Calvo model of forever stickiness. This result that forever

[3] With investment, however, all current looking rules would be indeterminate.

stickiness is different from finite stickiness is analogous to the folk theorem in game theory: a game that lasts for a finite but known period of time is fundamentally different from a game that lasts forever.

Assume that the counterpart to (7) is

$$\tilde{\pi}_t = \lambda \tilde{z}_t + \sum_{i=1}^{N} \kappa_i E_{t-i} \tilde{\pi}_t, \text{ where } \sum_{i=1}^{N} \kappa_i = 1, \tag{8}$$

where κ_i denotes the fraction of firms that set their time t prices in period t-i and N is the number of periods for which prices are sticky. There are many stories that could motivate a similar Phillips curve including Fischer (1977), McCallum (1994), and Mankiw and Reis (2001). During the first N periods z_t need not equal $\bar{z}$ as the economy begins with some established prices. But in contrast to Calvo (1983), this stickiness is finite: $z_{t+j} = \bar{z}$ for all $j \geq N$.

From t+N onwards, therefore, the deterministic version of the model is isomorphic to the flexible price model with a constant income tax rate of $1-\bar{z}$. This implies that a necessary condition for real determinacy is that the corresponding flexible price model be determinate for real variables. But this flexible price real determinacy is only necessary. For sufficiency we also need the corresponding flexible price economy to have *nominal* determinacy. This is because we need an extra condition to pin down behavior.

To demonstrate this result, split time into the first N periods (periods t to t+N-1) in which some prices are predetermined, and the rest of time in which the deterministic dynamics are flexible-price. The Fisher equation is given by

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See Carlstrom and Fuerst (2000).

$$\tilde{z}_{t+j} + \left[\tilde{R}_{t+1+j} - \tilde{\pi}_{t+1+j}\right] = \tilde{z}_{t+j+1} \quad \text{for j = 0,1,...N-2,} \tag{9a}$$

$$\tilde{z}_{t+N-1} + \left[\tilde{R}_{t+N} - \tilde{\pi}_{t+N}\right] = 0 \quad \text{and} \tag{9b}$$

$$\tilde{R}_{t+1+j} - \tilde{\pi}_{t+1+j} = 0 \quad \text{for all } j \geq N. \tag{9c}$$

Suppose policy is given by a current or backward rule (the argument for a forward rule is similar). The pricing equation (8) provides N restrictions on the 2N variables $\tilde{z}_{t+j}$ *and* $\tilde{\pi}_{t+j}$ for j = 0,1…N-1. After period N-1 this pricing equation is irrelevant. Equation (9a) provides an additional N-1 restrictions. Equation (9b) provides one additional restriction on $\tilde{z}_{t+N-1}$, but also introduces another free variable $\tilde{\pi}_{t+N}$. The only possible restriction on $\tilde{\pi}_{t+N}$ would come from (9c), the flexible price world that dawns at time t+N. If there is nominal determinacy in this flexible price world then, by definition, $\tilde{\pi}_{t+N}$ is determined and we have real determinacy in this sticky price model. But (9c) implies that there is both nominal and real determinacy if and only if policy is backward-looking with $\tau > 1$. In summary, we have the following:

Proposition 2: Suppose that prices are set as in (8) and that monetary policy is given by (6). All forward-looking (i = 1) and current-looking rules (i = 0) are subject to real indeterminacy. With a backward-looking interest rate rule (i=-1) there is real determinacy if and only if $\tau > 1$. More generally, in a world with finite stickiness, there is real determinacy if and only if there is nominal determinacy in the corresponding flexible price economy.

In a model with Calvo's forever stickiness, indeterminacy can be avoided by responding aggressively to current movements in inflation. In contrast, in a NRH model with finite stickiness any current-based policy is necessarily subject to sunspot fluctuations. The only way to avoid indeterminacy is to respond aggressively to past inflation.

IV. Learning.

The implicit premise in the previous analysis is that central banks should avoid rules that create indeterminacy because of the possibility of sunspot fluctuations. But is this argument reasonable? Or are these sunspot equilibria simply an intellectual curiosity with no empirical relevance? This is of course not a simple question to answer, but one mode of exploration is to ask if the equilibria are fragile under learning dynamics. We follow Evans and Honkapohja (2001) and interpret "learning" as E-stability, so that an equilibrium is "fragile" if it is not E-stable.[4]

Within a Calvo model, Honkapohja and Mitra (2001) demonstrate that the sunspot equiliria are not E-stable for forward- or current-looking Taylor rules. Carlstrom and Fuerst (2001c), however, utilize an alternative assumption about money demand timing (CIA-timing) and show that there is a parameter range in which sunspots under a forward rule are E-stable. In sharp contrast to these fairly narrow results for the Calvo model, this section demonstrates that sunspot equilibria in the NRH model are almost always E-stable. This difference between the Calvo and NRH models arises because the NRH model includes lagged expectations and these lagged expectations produce a class of RE sunspot equilibria that are more likely to be E-stable.

To examine E-stability we must introduce expectations operators. In the case of one-period stickiness (N=1), and a forward-based Taylor rule, the equilibrium condition is given by the stochastic counterpart to (9a):

[4] E-stability typically implies that least-squares learning converges to the rational expectations equilibrium.

$$E_t(\tau\tilde{\pi}_{t+2} - \tilde{\pi}_{t+1})\lambda + \pi_t - E_{t-1}\pi_t - (1-\rho)u_t = 0 \qquad (10)$$

where u_t denotes some additive fundamental shock to the pricing equation (8) with $u_t = \rho u_{t-1} + \varepsilon_t$, and ε_t is a mean-zero, iid shock. There exist sunspot equilibria of the form

$$\tilde{\pi}_t = \lambda\gamma s_t + \gamma s_{t-1} + f_1 u_{t-1} + f_2 \varepsilon_t \qquad (11)$$

where s_t is an iid, mean-zero sunspot shock, γ is an arbitrary constant, and the f_i's are unique (assuming $\rho \neq 0$). Posit a perceived law of motion (PLM) that is of the same form:

$$\tilde{\pi}_t = a_1 s_t + b_1 s_{t-1} + c_1 u_{t-1} + d_1 \varepsilon_t \, .$$

Using this PLM to replace all expectations in (10), we solve for the actual law of motion (ALM) given by

$$\tilde{\pi}_t = \lambda b_1 s_t + b_1 s_{t-1} + [c_1(1 + \lambda\rho - \tau\lambda\rho^2) + \rho(1-\rho)]u_{t-1} + [c_1(\lambda - \tau\lambda\rho) + (1-\rho)]\varepsilon_t \, .$$

We now have the implicit mapping from PLM to ALM. The stability matrix for this mapping has eigenvalues $(0, 1, 1+\lambda\rho-\tau\lambda\rho^2, 0)$. For E-stability we need all of these to be less than one (because γ is arbitrary, one of the eigenvalues corresponding to the sunspots can be unity). Hence, we have E-stability of the equilibria if and only if $\tau\rho > 1$. Note that this restriction arises from the fundamental shocks so that the bound does nothing to avoid the E-stability of the stationary sunspot equilibria.

In the case of a current rule, the sunspot equilibria are again of the form (11), and the comparable stability condition is $\rho\lambda(1-\tau) < 0$. As before this bound arises from the fundamental shocks, so that the sunspot equilibria are E-stable.

In contrast, in the case of a backward rule, the sunspot equilibria are of the form

$$\tilde{\pi}_t = \tau\tilde{\pi}_{t-1} + \lambda\gamma s_t + \gamma s_{t-1} + f_1 u_{t-1} + f_2 \varepsilon_t \quad (12)$$

with sunspots arising if and only if $\tau < 1$. The eigenvalues of the stability matrix are ($\tau\lambda+1,0, 1,1+\lambda\rho, 0$) implying that the equilibria of the form (12) are not E-stable. The fundamental equilibrium, however, is of the form

$$\tilde{\pi}_t = g_1 u_{t-1} + g_2 \varepsilon_t$$

and is E-stable if and only if $\tau > \rho$. Hence, a backward Taylor rule with $\tau > 1$ ensures determinacy and E-stability of the fundamental equilibrium.

V. Conclusion.

The central issue of this paper is to identify the restrictions on the Taylor interest rate rule needed to ensure real determinacy in a model that satisfies the NRH. In such a model all forward and current-looking interest rate rules are subject to real indeterminacy and the resulting sunspot equilibria are E-stable. In contrast, an aggressive ($\tau > 1$) backward-looking rule ensures determinacy and E-stability of the fundamental equilibrium. This result generalizes to other models that satisfy the NRH. For example, Carlstrom and Fuerst (1999) examine a limited participation model and reach exactly the same conclusions on the determinacy of an aggressive backward-looking Taylor rule. The analysis in Section III suggests that this result will extend to any model with a finite nominal rigidity.

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