Inflation Persistence, Inflation Targeting and the Great Moderation

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There is growing evidence that the empirical Phillips curve within the US has changed significantly since the early 1980's. In particular, inflation persistence has declined sharply. The paper demonstrates that this decline is consistent with a standard Dynamic New Keynesian (DNK) model in which: (i) the variability of technology shocks has declined, and (ii) the central bank more aggressively responds to inflation.

Key words: inflation persistence, Phillip's curve, mark-up shocks, inflation targeting.

JEL code: E31, E37, E47

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1. Introduction.

There is growing evidence that the time series behavior of US inflation is changing. The most notable change is the decline in inflation persistence. For example, in a reduced form model Cogley (2005) and Cogley and Sargent (2007) show that inflation persistence has declined dramatically and that this decline occurred around 1983.

But the decline in inflation persistence is not the only change. There is evidence suggesting that the slope of the Phillip's curve may also have declined. To make these statements more concrete, a typical empirical Phillips curve is estimated by

$$\pi_t = a + \beta_1(L)\pi_{t-1} + \beta_2 gap_t \tag{1}$$

where π_t denotes inflation, $\beta_1(L)$ is a distributed lag, and gap_t denotes the output gap. The sum of the distributed lag coefficients is referred to as inflation persistence and the slope of the Phillips curve is given by β_2. Both of these appear to have declined. These drops have troubled some policymakers because they imply that the costs to lowering inflation have likely risen.

This paper uses a dynamic stochastic general equilibrium model to explain the decline in inflation persistence, and to a lesser extent the slope of the Phillips curve. Our hypothesis is that there has not been a change in the structural Phillips curve. Assuming a constant Phillips curve, we see how well the model can match the time series behavior of these estimated coefficients. Our results suggest that the change in Fed policy and the dramatic drop in the variability of total factor productivity (an important factor behind the so called "great moderation") may be responsible for the changes in inflationary

dynamics. In particular, we show that (1) a change in the aggressiveness of the central bank response to inflation (a more aggressive Taylor rule), and (2) a decrease in the relative variance of technology shocks are qualitatively consistent with the changed coefficients.

The paper is organized as follows. We first examine a standard DNK model and derive analytical expressions for inflation persistence and the gap coefficient. We argue that if mark-up shocks become relatively more important, inflation persistence will decline. Furthermore, if the central bank becomes more aggressive in fighting inflation, inflation persistence will also decline. We then calibrate our model and argue that a more aggressive Taylor rule in conjunction with a decrease in the relative variance of technology shocks (assuming that the variance of mark-up shocks did not change) is quantitatively consistent with the changed coefficients. We then present some sensitivity analysis.

2. Analytical Results.

This section uses a fairly standard DNK model to demonstrate how the model's regression coefficients are affected by the relative variability of mark-up shocks and the degree of aggressiveness in the central bank's response to inflation. Production is linear in labor with a random and auto-correlated productivity shock given by θ_t, while household preferences are separable in consumption and labor with risk-aversion coefficient of σ and Frisch labor supply elasticity of $1/v$. The Calvo pricing equation assumes that there is indexing to past inflation as in Christiano, Eichenbaum, and Evans (2005). This introduces a backward element into the Calvo equation. (See also Woodford

(2003) for details.) The central bank follows a simple Taylor rule

with $R_t = \tau \pi_t + \tau_{gap} gap_t$ with $\tau \geq 1$.[1] We consider more general Taylor rules in the next section.

The familiar DNK model can be expressed as

$$\tau \pi_t + \tau_{gap} gap_t - \pi_{t+1} = M(gap_{t+1} - gap_t) + P(\rho - 1)\theta_t + Q(\rho_g - 1)g_t \qquad (2)$$

$$\pi_t(1 + \beta) = \beta E_t \pi_{t+1} + \pi_{t-1} + \lambda gap_t + \varepsilon_t^\pi \qquad (3)$$

where

$$M \equiv \frac{\sigma}{c_{ss}},$$

$$P \equiv \frac{(1+v)\sigma}{(\sigma + v c_{ss})},$$

$$Q \equiv \frac{-\sigma v g_{ss}}{\sigma + v c_{ss}} g_t,$$

$$\theta_t = \rho \theta_{t-1} + \varepsilon_t^\theta$$

$$g_t = \rho_g g_{t-1} + \varepsilon_t^g,$$

and E_t denotes the expectations operator, c_{ss} denotes steady-state consumption, g_t denotes government spending, and ε_t^π denotes a mark-up shock. We follow the convention of assuming that the mark-up shocks are iid (see Fuhrer (2006), Roberts (2006), and the empirical evidence in Adam and Billi (2006)). Note that technology and government spending shocks enter into (2) symmetrically. The empirical evidence reported below suggests that the autocorrelation of the technology and government

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[1] We assume $\tau > 1$ because as we show in the appendix the size of τ_{gap} does not affect the necessary condition for determinacy. This is because of the Christiano et al. (2005) assumption that the Calvo curve is indexed to lagged inflation.

spending processes are comparable. For simplicity in our analytical results we focus only on technology shocks to the Fisher equation (2).[2]

Econometric estimation of the system (2)-(3) is reminiscent of the standard identification problems with simultaneous equations. To the extent that the data is generated by Fisher equation shocks in (2), the inflation-gap data will reveal a positive relationship between inflation and the gap as in the Phillips curve (3). Conversely, to the extent that mark-up shocks are more important, the data will reveal a negative relationship between inflation and the gap as in (2). Hence, a key issue in the analysis is the relative variability of shocks to the Fisher equation (2) versus shocks to the Phillips curve (3).

The importance of mark-up shocks versus Fisher equation shocks is also affected by the nature of monetary policy. An increase in how aggressively the central bank fights inflation will increase the relative importance of mark-up shocks so that the data will reveal the Fisher equation and a negative relationship between inflation and the gap. Solving (2)-(3) forward we have

$$\pi_t = \sum_{j=0}^{\infty} \left(\frac{1}{\tau}\right)^{j+1} E_t \left(M\Delta gap_{t+j+1} + \theta_{t+j} \right) \tag{2a}$$

$$\pi_t = \pi_{t-1} + \left(\frac{\lambda}{\mu}\right) \sum_{j=0}^{\infty} \beta^j E_t gap_{t+j} + \mu^{-1} \varepsilon_t^\pi , \tag{3a}$$

where (2a) assumes $\tau_{gap} = 0$. The Fisher equation shocks are weighted by $1/\tau$, but the mark-up shocks are not. Hence, a more aggressive monetary policy tends to directly reduce the effect that Fisher shocks have on inflation and the gap so that the inflation-gap data will be primarily driven by mark-up shocks. Similarly, if the monetary policy rule

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[2] We do not consider monetary shocks, but they would also enter into (2) in a symmetric fashion.

offsets technology shocks then this would also increase the relative importance of mark-up shocks.[3]

We will now formally prove these conjectures. We begin the analysis with some results on the equilibrium decision rules and then turn to the implications for the regression coefficients. Remarkably, the iid mark-up shocks and the form of the Calvo equation with past indexation imply that the way the gap responds to lagged inflation and the mark-up shock are identical.

Proposition 1: The decision rules are given by

$$\pi_t = a\pi_{t-1} + a\varepsilon_t^\pi + c\theta_t \tag{4}$$

$$gap_t = a\gamma\pi_{t-1} + a\gamma\varepsilon_t^\pi + d\theta_t \tag{5}$$

where $a < 1$, $\gamma \equiv \dfrac{\tau - a}{M(a-1) - \tau_{gap}} = \dfrac{-(1-a)(1-\beta a)}{\lambda a} < 0$,

$$\frac{c}{d} = \frac{\lambda}{1 + \beta - \beta(a + \rho)}$$

$$d = \frac{(1-\rho)P}{M(\rho-1) - \tau_{gap} + \dfrac{\lambda[\gamma(M + \tau_{gap}) + \rho]}{1 + \beta - \beta(a + \rho)}}$$

As $\tau \to \infty$, $a \to 0$, $\gamma \to -\infty$, $(a\tau) \to \dfrac{(M + \tau_{gap})}{\lambda}$, and $(a\gamma) \to \dfrac{-1}{\lambda}$. Similarly, as $\tau_{gap} \to \infty$, $a \to 1$, $\gamma \to 0$. Note that as $\rho \to 1$, c and d both converge to zero.

Proof: See Appendix.

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[3] For example, if the shocks are observable optimal monetary policy implies that the policy rule would directly respond to Fisher-equation shocks (eg., $R_t = \tau\pi_t + P(\rho-1)\theta_t$) in order to keep the real rate of interest at its "natural" rate in response to technology shocks (again, see Woodford (2003)). In this case we have a model in which inflation and the output gap respond only to mark-up shocks so that the model data would behave as if there were no technology or government spending shocks.

Suppose that an econometrician runs an OLS regression like (1) on the data that comes out of this model. The simple structure of the theoretical model implies that only one lag of inflation is needed in the estimates. Hence, we will consider an OLS regression of the form:

$$\pi_t = \beta_1 \pi_{t-1} + \beta_2 gap_t.$$ (6)

Although the variances of the underlying shocks have no effect on the (linear) decision rule coefficients in Proposition 1, these variances will have an important effect on the OLS coefficients. Our key analytical results are summarized in the following proposition and its three corollaries.

Proposition 2: The OLS coefficients in equation (6) are given by:

$$\beta_1 = a[1 - \gamma\beta_2] + \rho d\Omega\left[\frac{c}{d} - \beta_2\right]$$

$$\beta_2 = \frac{cd(c\rho^2\Omega - 1 + a\rho) - \gamma a^2(1 - a\rho)rv}{d^2(c\rho^2\Omega - 1 + a\rho) - \gamma^2 a^2(1 - a\rho)rv}$$

where

$$\Omega \equiv \frac{\sigma_{\pi\theta}^2}{\sigma_\pi^2} = \frac{c(1 - a^2)}{(1 - a\rho)a^2 rv + c^2(1 + a\rho)}$$

$$rv = \frac{\text{var}(\varepsilon^\pi)}{\text{var}(\theta)} = (1 - \rho^2)\frac{\text{var}(\varepsilon^\pi)}{\text{var}(\varepsilon^\theta)}.$$

Proof: See Appendix.

We have the following corollaries.

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Corollary 1: Suppose there are only technology shocks ($rv = 0$). Then we have

$$\beta_1 = a[1 - \gamma \frac{c}{d}], \quad \beta_2 = \frac{c}{d}.$$

As $\tau \to \infty$,

$$\beta_1 \to \frac{1}{1 + \beta(1 - \rho)} > 0$$

$$\beta_2 \to \frac{\lambda}{1 + \beta(1 - \rho)} > 0.$$

Proof: The expressions for the OLS coefficients come from simple algebra. As for the limiting results, Proposition 1 implies that as $\tau \to \infty$, $\frac{c}{d} \to \frac{\lambda}{1 + \beta(1 - \rho)}$, and $-a\gamma \to \frac{1}{\lambda}$

QED.

Corollary 2: Suppose there are only mark-up shocks ($rv \to \infty$). Then we have

$$\beta_1 = 0, \quad \beta_2 = 1/\gamma < 0.$$

As $\tau \to \infty$, $\beta_2 \to 0$ (from below).

Proof: By inspection.

Corollary 3: Suppose that both shocks are operative (rv is nonzero and finite). As $\tau \to \infty$ ($\gamma \to -\infty$),

$$\beta_1 \to \frac{P^2 \lambda^2 (1 - \rho)^2 \rho}{\left(P^2 \lambda^2 (1 - \rho)^2 + (M + \tau_{gap})^2 rv\right)}.$$
$$\beta_2 \to 0.$$

As $\tau_{gap} \to \infty$ ($\gamma \to 0$),

$\beta_1 \to 1$, and

$$\beta_2 \to \frac{c}{d} = \frac{\lambda}{1 - \beta\rho}.$$

Proof: See Appendix.

Notice that since λ is typically estimated to be quite small, Corollary 3 implies that as τ goes to infinity β_1 is essentially zero. Thus inflation persistence will be nearly zero for either a strict inflation-targeting central bank or an economy with only mark-up shocks ($rv \to \infty$). Conversely, a gap-targeting central bank ($\tau_{gap} \to \infty$) results in inflation persistence of unity, the lag coefficient in the decision rule (4). Hence a more aggressive Taylor Rule can have different effects on measured inflation persistence depending on which policy parameter has increased, τ or τ_{gap}.

A summary of these Corollaries is presented in Table 1. Note that as $rv \to 0$ Corollary 3 does not converge to Corollary 1. With a central bank that reacts infinitely-aggressively to inflation, there is a discontinuity between an economy with rv = 0 and an economy with a very small amount of mark-up shocks.

Table 1 confirms the earlier intuition suggesting two possible explanations for the decline in the size of the estimated OLS coefficients: (i) a decline in the variance of technology shocks (and thus an increase in the relative importance of mark-up shocks), and (ii) a more aggressive monetary policy response to inflation (a larger τ). But are these qualitative results quantitatively relevant? In the next section, we provide some numerical results that suggest that the answer is "yes."

3. Numerical Results.

In this section we present two sets of results on the quantitative effect of changes in monetary policy and changes in relative variances. Our first results are a quantitative extension of the closed-form expressions in Proposition 2. Our second set of results simulate the model by feeding into it estimates of the actual shock process hitting the US economy since 1960 and then estimating the implied OLS coefficients.

First we need to discuss calibration. We choose parameter values that are within the standard range in the literature: $\beta = 0.99$, $\nu = 1.0$, $\sigma = 1.0$, and $\lambda = 0.104$ (implying an average of 5 quarters between price changes). We consider two time periods, 1960-1979 and 1983-2004, where the break is suggested by the well-known change in monetary policy, see Clarida, Gali and Gertler (2000). We discard the middle years because of the significant uncertainty regarding the policy rule during this time period.

The shock processes for technology[4] and government spending[5] are estimated for these two sample periods. The results are shown in Table 2. The autocorrelation of both shocks is comparable so that we essentially have one shock in the Fisher equation. (For the closed-form results we abstract from government spending shocks.) Time variation in the estimated AR(1) coefficients is small so we treat them as being constant over the

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[4] The technology series was kindly provided by Peter Rupert. Capital includes both equipment and structures, but excludes residential investment. The series is logged and HP-filtered with a smoothing parameter 1600.

[5] In the model government spending is the share of government expenditures in total output. Since our model does not include capital, we choose to measure government spending as government spending divided by the sum of government spending and total consumption expenditures. This data is obtained from the BEA. The series are also logged and HP-filtered with smoothing parameter 1600.

sample and impose a common autoregressive coefficient from the entire sample of $\rho =$ 0.75.[6]

The relative variance (rv) of mark-up shocks is crucial to our analysis. From Proposition 2 we have

$$rv = \frac{\text{var}(\varepsilon^{\pi})}{\text{var}(\theta)} = (1 - \rho^2) \frac{\text{var}(\varepsilon^{\pi})}{\text{var}(\varepsilon^{\theta})}.$$

Although Table 2 provides evidence on the changing variability of technology shocks, there is no corresponding direct evidence on mark-up shocks. Instead we will assume that the variability of mark-up shocks is constant over time. Later we calibrate this variance so that the model matches the level of inflation persistence (0.26) in the data over the period 1983-2004.

Even with an assumed constant variance of mark-up shocks, the dramatic decline in the variance of total factor productivity (productivity shocks) since 1983 implies that the relative variability of mark-up shocks increased. Table 2 shows that the standard deviation of productivity shocks decreased from, 0.0087 to 0.0048, implying that the relative variance of mark-up shocks increased by a factor of 3.29.

Figures 1-2 present numerical results calculated using the closed form expressions in Proposition 2. The figures document the effect of changes in rv and change in the policy rule on the measured regression coefficients. We assume there are no fiscal shocks and that technology shocks have an autocorrelation of $\rho = 0.75$ (the estimated value over the entire sample). We begin by assuming that monetary policy follows the simple Taylor rule assumed in the last section. Figures 1-2 considers several different levels of τ, where

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[6] A standard Andrews (1993) supremum test for a break in the AR(1) coefficient of technology at unknown break point cannot reject the null hypothesis of structural stability at the 10 percent level. The same result holds for government spending.

$\tau = 1$ is a natural lower bound as lower values imply equilibrium indeterminacy. We also

consider the effect of a response to the output gap ($\tau_{gap} = 1$).

The qualitative pictures in Figures 1-2 are as anticipated: more aggressive

monetary policy and/or a higher relative variance of the mark-up shocks, puts downward

pressure on both estimated coefficients. The quantitative effect can be quite large. For

example, a change in τ from 1.0 to 1.5 typically cuts the estimated inflation persistence

by more than half. As the relative variance approaches infinity so that mark-up shocks

dominate the system, inflation persistence goes to zero for all of these rules. More

aggressive monetary policy has a similar effect. When $\tau =10$, inflation persistence and the

output gap coefficients are L-shaped functions of the relative variance of mark-up shocks:

A small amount of mark-up shocks makes inflation persistence drop close to zero. As τ

gets large, these coefficients drop discontinuously to zero for arbitrarily small levels of

mark-up shocks. Adding the output gap to the Taylor rule has only a modest effect on the

behavior of inflation persistence, but dramatically reduces the measured gap coefficient

pushing it to counterfactually negative levels.

These figures allow for a multitude of experiments and can provide a sense of the

quantitative size of the effects. For example, if monetary policy in the pre-1979 period is

given by $\tau =1.0$ and rv = 0.30 in that period (as would be implied by our later calibration),

inflation persistence would be over 0.8. If monetary policy remained the same and rv

increased to 0.97 (given the lower variability of technology shocks), inflation persistence

falls to 0.63. Conversely, if rv remained at 0.30 in both periods, and only monetary policy

changed from $\tau =1.0$ to a more aggressive regime ($\tau = 1.5$ and $\tau_{gap} = 1.0$), then measured

inflation persistence would have declined to 0.38. Taken together, if the increase in rv is

matched with a more aggressive monetary policy ($\tau = 1.5$ and $\tau_{gap} = 1.0$), then persistence falls to 0.16.

To illustrate how well our benchmark model does in quantitatively replicating the data we need shock processes and a more realistic monetary policy rule. Because of problems with equilibrium indeterminacy for the earlier (1960-79) period we assume that monetary policy is given by $R_t = \tau \pi_t$ with $\tau = 1.001$. For the 1983-2004 period we assume a variety of monetary policy rules of the form

$$i_t = \rho_i i_{t-1} + (1 - \rho_i)(\tau \pi_t + \tau_{gap} gap_t + \tau_\Delta \Delta gap_t). \qquad (7)$$

Our benchmark calibration uses Mehra's (2002) estimates: $\rho_i = 0.8$, $\tau = 1.6$, $\tau_{gap} = 0$, $\tau_\Delta = 0.62$. Mehra (2002) presents evidence that changes in the gap are more important than the level of the gap in explaining monetary policy. But we later report different permutations of this rule to get a sense of the importance of each parameter. Responding to the change in the gap turns out to be important since it gets rid of the large counterfactual decline in the gap coefficient in the post-1983 period.

As for shocks, we feed into the model's decision rules the actual technology and government spending shocks from the data. Agents perceive these exogenous series to evolve according to AR(1) processes with persistence of 0.75 and 0.84, respectively which were the estimated values over the 1960-2004 period (see Table 2). The mark-up shocks are drawn from a normal distribution with a constant variance.

This constant variance is chosen so that the model matches the 1983-2004 inflation persistence we see in the data. We need a standard deviation of mark-up shocks

of 0.0072 in order for inflation persistence to match the 0.26 value observed in the data.[7]

Our calibration therefore implies that rv = 0.97 in the post-1983 period. Since the

standard deviation of technology was about 80 percent larger pre-1979, our assumption

that the variability of mark-up shocks was constant across time implies that rv = 0.30

prior to 1979. Hence, our benchmark calibration is that rv = 0.30 pre-1979, and rv = 0.97

post 1983.[8]

Figures 3-4 present the average OLS coefficients generated by the model across

1000 simulations of the model where the mark-up shock for each simulation is drawn

from a distribution as reported above. This gives rise to error bounds around the model.

We report the data along with the 2.5 and 97.5 percentile bounds. For ease of comparison

the estimated inflation persistence and gap coefficients from the data are also reported.

Both the model-based and real-data estimates are 40-quarter rolling regressions. The

simulated data shows a fall in persistence from about 0.75 in the 1980's to around 0.2 in

the new millennium. The timing of this drop coincides well with the data. Inflation

persistence in the data falls even further to zero at the turn of the century, but the

estimated inflation persistence is within the error bands from the simulations almost all

the time.[9]

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[7] Inflation persistence is defined to be the sum of the coefficients on the 4 lags of inflation. Similar to Cogley (2005) and Cogley and Sargent (2007), we correct this regression to account for the likely change in the Fed's implicit long-term inflation target over time. We proxy for long-term inflation by smoothing the inflation data with an HP filter (assuming a HP-filter smoothing parameter of 1600).

[8] The absolute size of rv is somewhat arbitrary and depends upon the way in which we write the Calvo equation (3). For example, one natural alternative is to divide (3) by $(1+\beta)$. In this case rv is scaled down by a factor of about four, $(1+\beta)^2$, and the scales in the figures are correspondingly transformed.

[9] There are also error bounds around the estimated coefficients in the data. We choose not to report them in this picture, but they are considerably tighter than the error band around the model.

Figures 5-6 provide some sensitivity analysis to the general Taylor rule (7) above. Rule 1 is once again $\tau = 1$ and everything else set to zero. Rule 2 is the benchmark rule without a response to the gap or the change in the gap: $\rho_i = 0.8$, $\tau = 1.6$, $\tau_{gap} = 0$, $\tau_\Delta = 0$. Rule 3 adds a response to the level of the gap: $\rho_i = 0.8$, $\tau = 1.6$, $\tau_{gap} = 0.62$, $\tau_\Delta = 0$. Rule 4 is the benchmark rule without inertia: $\rho_i = 0$, $\tau = 1.6$, $\tau_{gap} = 0$, $\tau_\Delta = 0.62$. Rule 5 is the benchmark Mehra Taylor rule estimates: $\rho_i = 0.8$, $\tau = 1.6$, $\tau_{gap} = 0$, $\tau_\Delta = 0.62$.

These results are given in figures 5-6. All of these rules imply that inflation persistence and the gap coefficient fall as the relative variance of mark-up shocks increases, although the magnitude of the declines depends upon the particular rule. We can use these Figures to get a sense of what inflation persistence and the output gap coefficient would look like if we thought about calibrating to these alternative rules assuming that pre-1979 monetary policy is still given by $\tau = 1$. Table 3 conducts these experiments. Suppose that monetary policy in the post-1983 period is given by one of the rules in the first column of Table 3. We then choose rv in the post-1983 period to match the inflation persistence in the actual post-1983 data. Given the decline in the variance of measured technology shocks, this then implies a lower rv for the pre-1979 period. The remaining columns in Table 3 report the implied OLS coefficients for each of the subperiods.

 Our benchmark calibration has inflation persistence declining from 0.82 to 0.26. The bottom rows decompose this drop into two other possibilities. If monetary policy were given by our benchmark calibration in both sub-periods but the variability of technology shocks had dropped, inflation persistence would have fallen from 0.47 to 0.26. Similarly, if the variability of technology shocks had not changed but monetary

policy had changed, inflation persistence would have fallen from 0.62 to 0.26. The fall in inflation persistence is similar across all the other rules and falls from a little greater than 0.8 to 0.26.

The behavior of the gap coefficients, however, is noticeably different. Our benchmark has the gap coefficient falling a little from 0.03 to -0.12. In the data the gap coefficient remains virtually unchanged (0.013 versus 0.076 in the latter period). Calibrating to rule 4, the benchmark rule without inertia (rv=0.45 for post-1983 and 0.14 pre-1983), produces nearly an identical drop for inflation persistence as does the benchmark rule, but the gap coefficient (which already drops too much in our base-line calibration) now drops even more (0.16 in the pre-79 period and -0.21 in the post-83 period). Similar results obtain when the post 1983 policy is described by a reaction to the level of the gap instead of the change in the gap (as in rule 3), but now the drop in the output gap coefficient is larger yet (0.12 to -0.33). A policy rule with inertia that does not react to either the gap or the change in the gap (rule 2) looks very similar to our base-line calibration where the central bank reacts to the change in the gap in the latter period but the gap coefficient in the early period is larger.

It is easy to see that the model has a much harder time matching the gap coefficient. But the gap is most likely subject to measurement error to a much larger extent than the inflation data. Additionally, Table 3 indicates that the gap coefficient is much more heavily influenced by the nature of the policy rule. Changes in the policy rule, for example associated with Greenspan's appointment in 1987, could change the gap coefficient significantly while inflation persistence would be little affected. It is also encouraging that the model that best matches the data is our benchmark rule that was

calibrated to an estimated policy rule by Mehra (2002). It is important to point out, however, that our choice of a rule that reacted to the change in the gap versus the level of the gap is important.[10] Evidence, however, suggests that Fed policy is best described by a rule that reacts to the change in the gap and theoretically such a rule more closely characterizes optimal monetary policy as discussed in Walsh (2003).

As a final form of sensitivity analysis, we have also considered the case in which there is external habit formation in preferences (indexed by $h \in [0,1]$). This introduces the lagged output gap into both the Fisher equation (2) and the Phillips curve (1). As before, our calibration strategy is to choose the level of rv in the post-1983 period to match the measured inflation persistence in this same period. Interestingly, this calibration strategy is not possible if the habit parameter is large, eg., h > 0.3. That is, for large values of h, the OLS inflation persistence coefficient (as a function of rv) becomes asymptotic at a level exceeding the desired calibration of 0.26. This experiment is possible for lower values of the habit parameter. For example, with h = 0.2, the calibration implies that inflation persistence would fall from 0.73 to 0.26 across the two periods, while the gap coefficient would slightly decline from -0.09 to -0.11. This is only modestly different from our benchmark results without habit.

The inability to calibrate with habit persistence is because our estimated Phillips curve (6) is misspecified. Habit persistence implies that the lag of consumption should be in the Calvo equation. Therefore, we have also investigated the possibility of altering the OLS regression (6) to include a lag of the gap. In this case, the calibration strategy can be implemented for much higher levels of habit. For example, with h = 0.7, the calibration

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[10] We have assumed that mark-up shocks are iid.. The effect of responding to the change in the gap, versus the level of the gap, is actually reversed if mark-up shocks are extremely persistent. In this case, inflation persistence only falls if monetary policy reacts to the level of the gap (or doesn't react to the gap at all).

implies that inflation persistence would fall from 0.79 to 0.26 across the two periods, while the sum of the gap coefficients would sharply decline from 0.30 to 0.19.

4. Conclusion

Inflation persistence has dropped markedly since the early 1980's. Policymakers are faced with different ways to interpret such data. The evidence could reflect changes in the underlying nature of price setting, such as the degree of forward vs. backward looking agents. Such changes are important to policymakers as they impact the monetary transmission mechanism. Instead, we present a model in which the structural equations are constant over time, except for the monetary policy rule itself. We combine plausible time variation of policy with a decrease in the volatility of technology shocks (one of the factors behind the great moderation) and replicate the observed time series properties of inflation. Hence, the structural model of price setting may very well be stable even if reduced form inflation persistence displays large instability.

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Appendix.

Proposition 1: The decision rules are given by

$$\pi_t = a\pi_{t-1} + a\varepsilon_t^\pi + c\theta_t \qquad\qquad (4)$$

$$gap_t = a\gamma\pi_{t-1} + a\gamma\varepsilon_t^\pi + d\theta_t \qquad\qquad (5)$$

where $a < 1$, $\gamma \equiv \dfrac{\tau - a}{M(a-1) - \tau_{gap}} = \dfrac{-(1-a)(1-\beta a)}{\lambda a} < 0,$

$$\frac{c}{d} = \frac{\lambda}{1 + \beta - \beta(a+\rho)}$$

$$d = \frac{(1-\rho)P}{M(\rho-1) - \tau_{gap} + \dfrac{\lambda[\gamma(M+\tau_{gap}) + \rho]}{1 + \beta - \beta(a+\rho)}}$$

As $\tau \to \infty$, $a \to 0$, $\gamma \to -\infty$, $(a\tau) \to \dfrac{(M+\tau_{gap})}{\lambda}$, and $(a\gamma) \to \dfrac{-1}{\lambda}$. Similarly when $\tau_{gap} \to \infty$, $a \to 1$, $\gamma \to 0$. Note that if $\rho = 1$, then $c = d = 0$.

Proof: The model is given by (2) and (3) where we abstract from government spending shocks without loss of generality (assuming there are comparable auto-correlation coefficients). Using (3) to solve for gap_t, we can use the method of undetermined coefficients and expression (2) to derive the characteristic polynomial of the system:

$$f(x) \equiv M\beta x^3 - [\lambda + \beta(M + \tau_{gap}) + M(1+\beta)]x^2 + [\tau\lambda + M + (1+\beta)(M + \tau_{gap})]x - (\tau_{gap} + M) = 0$$

For $\tau > 1$, it is straightforward to show that we have equilibrium determinacy (one root in the unit circle, two roots outside). Henceforth, we will let "a" denote the unique root in the unit circle. Without loss of generality, we can then define γ such that aγ is the

corresponding coefficient for marginal cost where equation (2)

implies $\gamma \equiv \dfrac{\tau - a}{M(a-1) - \tau_{gap}}$.   The decision rules can thus be written as

$$\pi_t = a\pi_{t-1} + b\varepsilon_t^{\pi} + c\theta_t$$

$$gap_t = a\gamma\pi_{t-1} + e\varepsilon_t^{\pi} + d\theta_t$$

The second equality for γ comes from using the method of undetermined coefficients on

(3) (isolating the π_{t-1} term) and solving for γ.

Isolating the b and e coefficients from from equation (2) we have

$$b(\tau - a) + \tau_{gap}e = M(ab\gamma - e).$$

Given γ above this is satisfied with e = bγ. As for the ratio a/b, equation (3) implies:

$$b(1 + \beta) = \beta ab + b\lambda\gamma + 1.$$

If we multiply this by a/b, and compare it to f(a), we have that b/a = 1. Similarly, the

method-of-undetermined-coefficients calculation yields the coefficient on the technology

shock. The limiting values come from the cubic f(a). Note that if we divide through by

τ, and then take the limit as τ gets large, we have f(x) = λx so that we have a unique stable

root at a = 0. If we define q = aτ, we can then define a cubic $g(q) \equiv f(q/\tau)$. As τ goes

to infinity, we get that a finite root of g is $q \to \dfrac{(M + \tau_{gap})}{\lambda}$. This then determines the

limiting value of aτ. The limiting value of aγ is determined by substituting the above

decision rules into equation 2 and isolating the π_{t-1} term. To show that as $\tau_{gap} \to \infty$,

$a \to 1$ notice that as $\tau_{gap} \to \infty$, we get that $f \to -\beta x^2 + (1 + \beta)x - 1$, which has a root of

unity. QED.

Proposition 2: The OLS coefficients in equation (13) are given by:

$$\beta_1 = a[1 - \gamma\beta_2] + \rho d\Omega\left[\frac{c}{d} - \beta_2\right]$$

$$\beta_2 = \frac{cd(c\rho^2\Omega - 1 + a\rho) - \gamma a^2(1 - a\rho)rv}{d^2(c\rho^2\Omega - 1 + a\rho) - \gamma^2 a^2(1 - a\rho)rv}$$

where

$$\Omega \equiv \frac{\sigma_{\pi\theta}^2}{\sigma_\pi^2} = \frac{c(1 - a^2)}{(1 - a\rho)a^2 rv + c^2(1 + a\rho)}$$

$$rv = \frac{\text{var}(\varepsilon^\pi)}{\text{var}(\theta)} = (1 - \rho^2)\frac{\text{var}(\varepsilon^\pi)}{\text{var}(\varepsilon^\theta)}.$$

Proof: The OLS coefficients are given by the solution to

$$\frac{\min}{\beta_1, \beta_2} E \frac{(\pi_t - \beta_1\pi_{t-1} - \beta_2 gap_t)^2}{2}.$$

The optimization conditions include:

$$E(\pi_t - \beta_1\pi_{t-1} - \beta_2 gap_t)\pi_{t-1} = 0$$

$$E(\pi_t - \beta_1\pi_{t-1} - \beta_2 gap_t)gap_t = 0$$

We can then replace π_t and gap_t by the decision rules in Proposition 1. Tedious algebra then yields the general expressions for the OLS coefficients.
QED

Corollary 3: As $\tau \to \infty$ ($\gamma \to -\infty$),

$$\beta_1 \to \frac{P^2\lambda^2(1-\rho)^2\rho}{\left(P^2\lambda^2(1-\rho)^2 + (M + \tau_{gap})^2 rv\right)}.$$

$$\beta_2 \to 0.$$

As $\tau_{gap} \to \infty$ ($\gamma \to 0$),

$$\beta_1 \to 1.$$
$$\beta_2 \to \frac{c}{d} = \frac{\lambda}{1 - \beta\rho}.$$

Proof: For the limiting cases as $\tau \to \infty$, we have that $a \to 0$ and $\gamma \to -\infty$. Proposition 1

implies that

$$\beta_2 = \frac{\lambda^2 cd(c\rho^2\Omega - 1)}{\lambda^2 d^2(c\rho^2\Omega - 1) + rv} = 0$$

$$\beta_1 = \rho c\Omega = \frac{\rho c^2}{a^2 rv + c^2} = \frac{\rho}{\left(\dfrac{a}{c}\right)^2 rv + 1}$$

Using the results of Proposition 1 we have that

$$\gamma d \to \frac{(1-\rho)P(1+\beta-\beta\rho)}{\lambda(M + \tau_{gap})}$$

so that

$$\frac{a}{c} = \frac{\gamma a}{\gamma d} \frac{d}{c} \to \frac{-(M + \tau_{gap})}{(1-\rho)P\lambda}.$$

Substituting this back into the earlier expression we have that

$$\beta_1 \to \frac{P^2\lambda^2(1-\rho)^2\rho}{\left(P^2\lambda^2(1-\rho)^2 + (M + \tau_{gap})^2 rv\right)}.$$

When $\tau_{gap} \to \infty$ the proof is straight forward since $a \to 1$ and $\gamma \to 0$.

QED.

Table 1
OLS Coefficients

$$\pi_t = \beta_1 \pi_{t-1} + \beta_2 gap_t$$

	β_1	β_2
Cost push shocks only	0	$1/\gamma < 0$ As $\tau \to \infty$ or $\tau_{gap} \to \infty$, converges to 0.
Technology shocks only	$a[1 - \gamma \dfrac{c}{d}]$ As $\tau \to \infty$, converges to $\dfrac{1}{1 + \beta(1-\rho)}$. As $\tau_{gap} \to \infty$, converges to 1.	$\dfrac{c}{d}$. As $\tau \to \infty$, converges to $\dfrac{\lambda}{1 + \beta(1-\rho)} > 0$. As $\tau_{gap} \to \infty$, converges to $\dfrac{\lambda}{1 - \beta\rho} > 0$.

Table 2
Estimation of shock processes

	1960-79	1983-2004
Technology		
Autocorrelation	0.76	0.71
Std. deviation of innovation	0.0087	0.0048
Government		
Autocorrelation	0.87	0.75
Std. deviation of innovation	0.0027	0.0017

Table 3
Alternative Experiments[*]

$$\pi_t = \beta_1 \pi_{t-1} + \beta_2 gap_t$$

Experiment:	β₁, pre-1979	β₁, post-1983 (calibrated)	β₂, pre-1979	β₂, post-1983
Rule 2: $\rho_i = 0.8$, $\tau = 1.6$, $\tau_{gap} = 0$, $\tau_\Delta = 0$	0.86	0.26	0.15	-0.12
Rule 3: $\rho_i = 0.8$, $\tau = 1.6$, $\tau_{gap} = 0.62$, $\tau_\Delta = 0$	0.85	0.26	0.12	-0.33
Rule 4: $\rho_i = 0$, $\tau = 1.6$, $\tau_{gap} = 0$, $\tau_\Delta = 0.62$	0.86	0.26	0.16	-0.21
Rule 5: $\rho_i = 0.8$, $\tau = 1.6$, $\tau_{gap} = 0$, $\tau_\Delta = 0.62$ (Benchmark)	0.82	0.26	0.03	-0.12
Pre-1979: $\tau = 1$, rv = 0.97 Post-1983: Rule 5, rv = 0.97	0.64	0.26	-0.26	-0.12
Pre-1979: Rule 5, rv = 0.30 Post-1983: Rule 5, rv = 0.97	0.47	0.26	0.01	-0.12
Data	0.62	0.26	0.013	0.076

[*]The pre-1979 policy rule is $\tau = 1$ unless otherwise noted. Rule 5 is the benchmark policy rule. The general form of the policy rule is given by

$$i_t = \rho_i i_{t-1} + (1 - \rho_i)(\tau \pi_t + \tau_{gap} gap_t + \tau_\Delta \Delta gap_t).$$
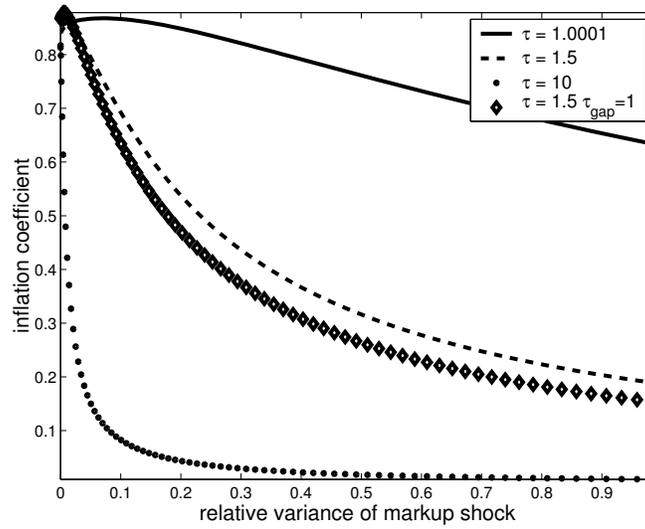
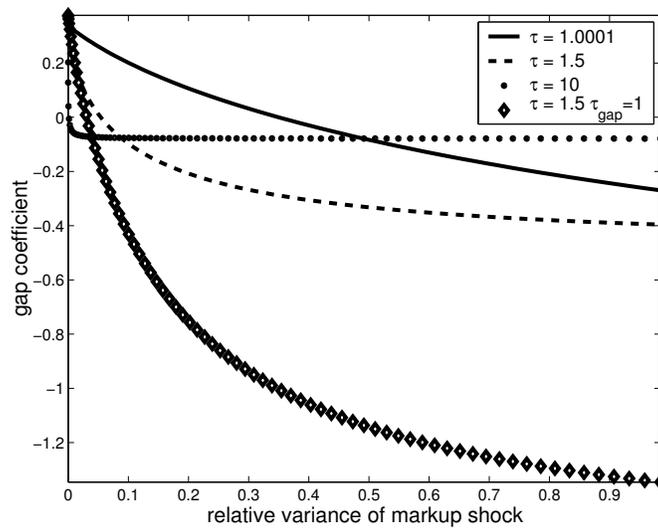Figure 1: Model closed form prediction for lagged inflation coefficient

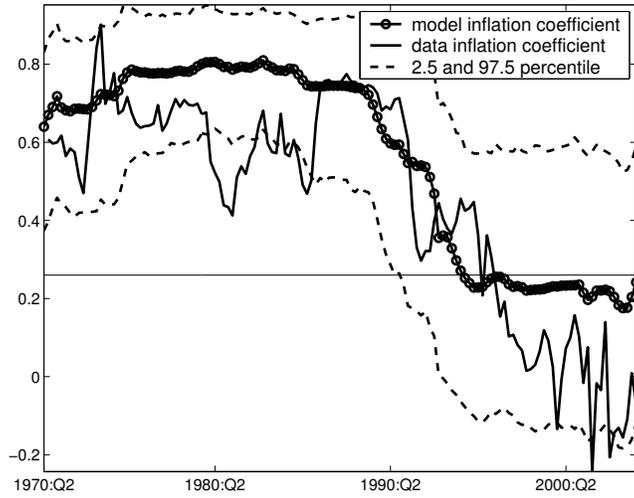Figure 2: Model closed form prediction for gap coefficient.

Figure 3: Inflation coefficient. 40 quarter rolling regressions: model predictions vs. data. Percentiles based on 1000 simulations of model data generated by random draws of i.i.d. markup shocks and estimated series for technology and government spending shocks.
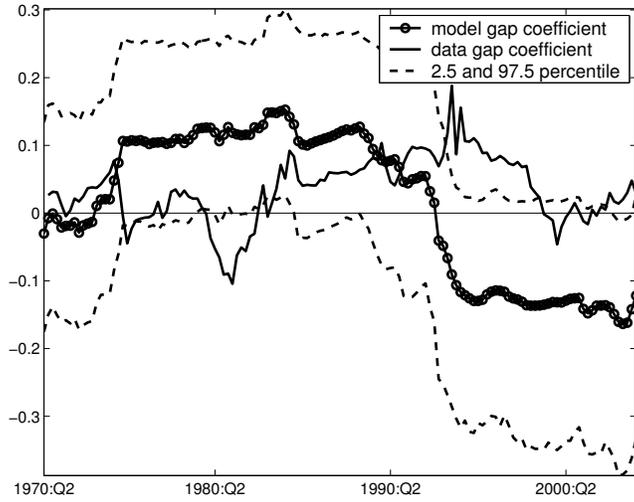
Figure 4: Gap coefficient. 40 quarter rolling regressions: model predictions vs. data. Percentiles based on 1000 simulations of model data by random draws of i.i.d. markup shocks and estimated series for technology and government spending shocks
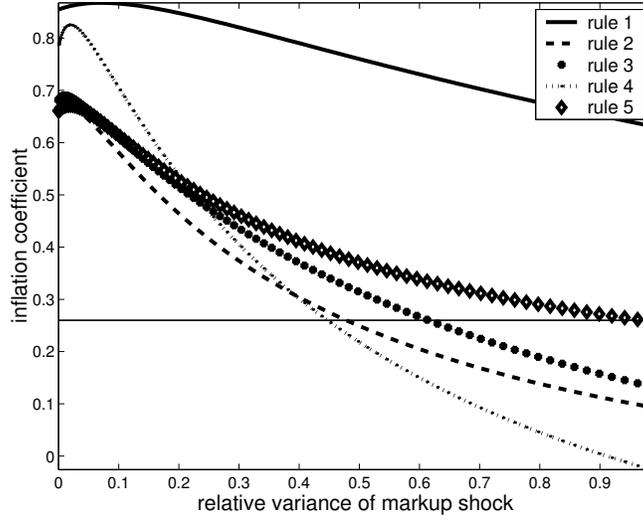
Figure 5: Model closed form prediction for lagged inflation coefficient. The horizontal lines correspond to inflation persistence of 0.26. Taylor rule is of the form: $i_t = \rho_i i_{t-1} + (1-\rho)(\tau\pi_t + \tau_{gap}gap_t + \tau_\Delta \Delta gap_t)$. Rule 1: $\tau = 1.0001$, all other zero. Rule 2: $\tau = 1.6$, $\rho_i = 0.8$, all other zero. Rule 3: $\tau = 1.6$, $\tau_{gap} = 0.62$, $\rho_i = 0.8$, all other zero. Rule 4: $\tau = 1.6$, $\tau_\Delta = 0.62$, all other zero. Rule 5: $\tau = 1.6$, $\rho_i = 0.8$, $\tau_\Delta = 0.62$, all other zero. (Mehra rule)
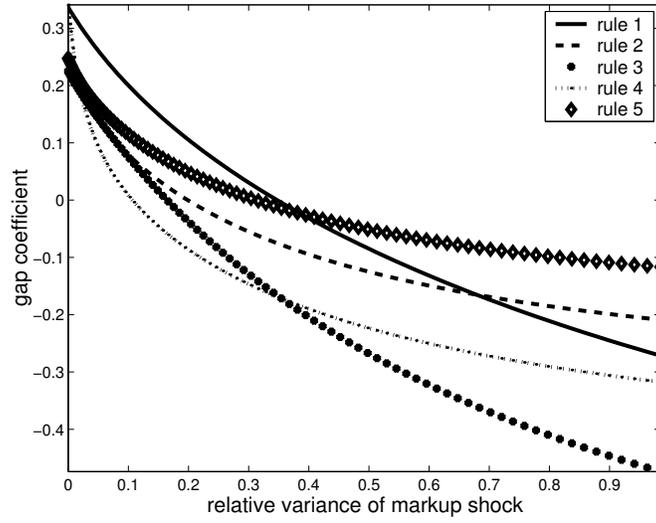
Figure 6: Model closed form prediction for gap coefficient. For a description of rules, see Figure 6.