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Timing and Real Indeterminacy in Monetary Models

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An increasingly common approach to the theoretical analysis of monetary policy is to ensure that a proposed policy does not introduce real indeterminacy and thus sunspot fluctuations into the model economy. Policy is typically conducted in terms of directives for the nominal interest rate. This paper uses a discrete-time money-in-the-utility function model to demonstrate how seemingly minor modifications in the trading environment result in dramatic differences in the policy restrictions needed to ensure real determinacy. These differences arise because of the differing pricing equations for the nominal interest rate.

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1. Introduction.

An increasingly common approach to the theoretical analysis of monetary policy is to ensure that a proposed policy does not introduce real indeterminacy and thus sunspot fluctuations into the model economy. Policy is typically conducted in terms of directives for the nominal interest rate. For example, a simple Taylor (1993) type rule posits that the central bank conducts policy according to the following rule: $R_t = A \cdot (\pi_{t+1})^{\tau}$ for $\tau \geq 0$, where R_t and π_{t+1} denote the (gross) nominal interest and inflation rate (between t and t+1).[1] That is, the central bank varies the nominal rate in relation to movements in inflation with an elasticity of τ. In this context, an important policy question is what restrictions on τ are needed to ensure real determinacy.

A standard result in the literature is that a necessary condition for real determinacy is $\tau > 1$ (the sufficient condition is for τ to exceed one, but to not be too large). See for example, Bernanke and Woodford (1997) and Clarida, Gali, Gertler (1998). These analyses are all reduced-form sticky price models, where the underlying structural model is a money-in-the-utility function (MIUF) model with a zero cross-partial between consumption and real balances.[2] In sharp contrast, Carlstrom and Fuerst (1999a) analyze a flexible price, cash-in-advance (CIA) model and demonstrate that a necessary and sufficient condition for determinacy is $\tau < 1$.[3] The motivation for this paper is to help

[1] This corresponds to targeting the expected inflation rate, a policy consistent with the practice of central banks with explicit inflation targets. In contrast, Taylor's (1993) original rule has the central bank responding to current and past inflation rates. As we demonstrate this difference is critical.
[2] A more complete analysis of these conditions is provided by Benhabib, Schmitt-Grohe, and Uribe (1998). But as argued below, their continuous time analysis ignores the central issue of this paper.
[3] Carlstrom and Fuerst (1999a) consider interest rate rules in which the nominal rate responds to the real rate with an elasticity of γ, and find that a necessary and sufficient condition for determinacy is $\gamma < ½$. There is a one-to-one mapping between a policy rule in terms of the real rate and a policy rule in terms of the inflation rate, $\tau = \gamma/(\gamma-1)$, so that $\gamma < ½$ implies $\tau < 1$.

reconcile these differing results.

The resolution does not come solely from the sticky price vs. flexible price assumption. For example, if we dropped the stick price assumption from the Clarida et al (1998) environment, then any $\tau \neq 1$ yields determinacy. Surprisingly, the major difference comes from an even more fundamental assumption—which money balances enter the utility function? The traditional MIUF assumption and that assumed by Clarida et al is that end-of-period balances enter the functional. But a direct extension of a typical cash-in-advance economy suggests that the money the household has left after leaving the bond market and before entering the goods market is more appropriate. Remarkably this distinction is critical for questions of determinacy.

We analyze a MIUF endowment economy under differing assumptions about which money balances enter the utility function.[4] The first timing convention we analyze is a direct extension of typical CIA timing. That is, the money available to satisfy consumption needs is the money the household has left after leaving the bond market but *before* entering the goods market. In contrast, we also analyze "cash-when-I'm-done" (CWID) timing where end-of-period money balances (net of current income and current consumption) enter the utility functional. These differing assumptions lead to different pricing equations for the nominal interest rate. In a model in which the central bank operates monetary policy via the nominal interest rate these differences have important effects on the conditions for determinacy.

We then extend this analysis to a model with production. Surprisingly, a standard

[4] We utilize a MIUF model because of its generality. Feenstra (1986) demonstrates that any transactions cost (TC) economy can be written as a MIUF economy. Similarly, a shopping-time (ST) model can be rewritten as a MIUF economy. Finally, cash-in-advance (CIA) models are extreme versions of MIUF and

production model with capital generates the same conditions for determinacy irrespective of how money affects utility. That is, the conditions for determinacy are identical to the endowment economy model where consumption and real cash-balances are separable. Thus for CIA timing the $\tau < 1$ rule of Carlstrom and Fuerst (1998) continues to hold. With CWID timing, however, the model is determinate for all $\tau \neq 1$.

These results suggest that as monetary theorists we must be more careful in writing down the basics of our models. Relatedly, it suggests that there are concerns with continuous-time analyses that simply sweep this fundamental timing issue under the rug, i.e., in a continuous time model the time interval between bond and goods market transactions collapses to zero. Since these indeterminacy issues arise for any discrete but arbitrarily small time period this resolution of the timing question is artificial.

The paper proceeds as follows. Section 2 lays out the basic environment. Section 3 presents the determinacy results for different modeling assumptions. Section 4 adds production to the model. Section 5 takes a brief look at a sticky-price model, and Section 6 concludes.

2. A MIUF Economy.

The economy consists of numerous infinitely-lived households with preferences given by

$$\sum_{t=0}^{\infty} \beta^t U(c_t, A_t/P_t),$$

where c_t and A_t/P_t denote consumption and real money balances, respectively. *The key issue is what measure of money appears in the utility function*. We will turn to this

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TC economies. Thus TC, ST, and CIA models imply particular functional forms for the MIUF economy.

shortly. The household begins the period with M_t cash balances and B_{t-1} holdings of nominal bonds. Before proceeding to the goods market, the household visits the financial market where it carries out bond trading and receives a cash transfer of X_t from the monetary authority. Hence, before entering goods trading, the household has cash balances given by $M_t + X_t + B_{t-1}R_{t-1} - B_t$, where R_{t-1} denotes the nominal interest rate from t-1 to t.

After engaging in goods trading, the household ends the period with cash balances given by the intertemporal budget constraint.

$$M_{t+1} = M_t + X_t + B_{t-1}R_{t-1} - B_t - P_tc_t + P_ty_t, \qquad (1)$$

We now turn to the central issue of the paper—what money balances aid in contemporaneous transactions, i.e., what is A_t? The existing literature contains two prominent choices[5]:

Model 1: CIA Timing

$$A_t = M_t + X_t + B_{t-1}R_{t-1} - B_t \qquad (2)$$

Model 2: CWID Timing

$$A_t = M_{t+1} = M_t + X_t + B_{t-1}R_{t-1} - B_t - P_tc_t + P_ty_t \qquad (3)$$

Model 1 assumes that what matters for time-t transactions is the money with which one enters the time-t goods market, i.e., cash held in advance of goods market trading. This timing assumption is always used in models with strict CIA constraints (eg., Lucas (1982), and Lucas and Stokey (1987)), but is typically not used in MIUF

[5] The working-paper version of this paper also considers a related CIA model in which time-t goods market trading precedes time-t bond market trading so that $A_t = M_t + X_t$. As noted in Farmer's text (1993), it is quite easy to get real indeterminacy in such a model. This timing seems artificial since agents would prefer

environments.

In contrast, the traditional MIUF approach is to assume Model 2 timing, i.e., that end-of-period balances, $A_t = M_{t+1}$, enter into the utility function.[6] It is very difficult to justify CWID timing on theoretical grounds. Using end-of-period money implies that money at the *beginning of t+1* reduces transactions costs in *period t*. Equivalently, Model 2 implies that what matters for transactions purposes is the money you leave the goods market with, *net of current consumption and current income.* That is, what aids in current transactions is the money I leave the supermarket with *not* the money I entered the market with.

With CWID timing current income is included as part of current money balances. This violates Clower's (1967) dictum that "money buys goods, and goods buy money, but goods do not buy goods." One can imagine trading environments in which this violation is possible. But it is very difficult to defend the subtraction of current consumption from current money balances.

To see these differences more starkly, consider a Leontief MIUF specification so that optimal behavior is given by the constraint $P_t c_t = A_t$. In the case of Model 1 timing, the substitution of A_t yields the standard CIA constraint, $P_t c_t = M_t + X_t$, where we have dropped the bond trading for simplicity. But in the case of CWID timing this substitution yields $P_t c_t = M_t + X_t + P_t y_t - P_t c_t$. This is a peculiar transactions constraint.[7]

to have the bond market precede the goods market.

[6] In fact, Model 2 is typically used in all monetary models (MIUF, TC, ST) except CIA models.

[7] This constraint is so peculiar that it is natural to ask why the profession ever made Model 2 its choice for MIUF, TC, and ST models. One explanation suggested by Ben Eden is that when Patinkin (1965) and others first wrote down MIUF models, for mathematical simplicity the profession used a finite horizon model as a proxy for an infinite-horizon model. In order for money to be held in the final period, the

We now present the different bond-pricing and money demand and equations that arise from these two timing conventions.[8]

Model 1: CIA Timing ($A_t = M_t + X_t + B_{t-1}R_{t-1} - B_t$).

$$[U_m(t)+U_c(t)]/P_t = R_t\beta\,[U_m(t+1)+U_c(t+1)]/P_{t+1} \tag{4}$$

$$U_m(t)/U_c(t) = (R_t\text{-}1) \tag{5}$$

Model 2: CWID Timing ($A_t = M_t + X_t + B_{t-1}R_{t-1} - B_t - P_tc_t + P_ty_t$).

$$U_c(t)/P_t = R_t\beta U_c(t+1)/P_{t+1} \tag{6}$$

$$U_m(t)/U_c(t) = (R_t\text{-}1)/R_t \tag{7}$$

Note the fundamental differences between the Fisher equations in Models 1 and 2 (equations (4) and (6)). In Model 1, if the household liquidates a bond for cash with the intention of increasing current consumption this bond trade increases the utility from current consumption $U_c(t)$ *and* current liquidity $U_m(t)$. In contrast, in Model 2, a bond sale that is used for consumption purchases has no effect on current liquidity so that only $U_c(t)$ enters the bond equation. In effect, Model 2 assumes that a marginal bond sale can directly finance an increment to consumption. This seems to violate the essential nature of a monetary economy.

In Model 2, therefore, we get the standard Fisherian decomposition that results

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modeler had to assume that the household had preferences over end-of-the-world money. Instead of simply tacking on an additional utility functional, a more symmetric choice was to simply slide the entire sequencing backwards so that time T+1 money was in the time T utility functional.

[8] Equation 4-5 (6-7) can be derived from substituting 2 (3) into utility and maximizing subject to (1) with respect to c_t, B_t, and M_{t+1}. The Lagrangian associated with (1) was then substituted out.

from an entirely real model. Namely the real rate is given by the ratio of the marginal utilities of consumption. But with CIA timing, the fact that this is a *monetary* economy is paramount. The real rate of interest now depends on the marginal utilities of consumption *and* real cash balances, i.e., the utility of \$1 is more than its ability to purchase consumption but also its liquidity value. These differences in real rate determination imply that changes in real cash balances can directly influence the real rate in Model 1, while they do so only indirectly (via the cross partial U_{cm}) in Model 2.

Notice that in Model 1 we can use the money demand relationship (5) to rewrite the Fisher equation (4) as

$$U_c(t)/P_t = R_{t+1}\beta U_c(t+1)/P_{t+1}. \tag{8}$$

Hence, one manifestation of our CIA timing is that one can use the traditional Fisher expression (equation 6) but with the nominal rate scrolled forward one period.[9] This difference has important effects on issues of equilibrium determinacy. The remainder of the paper illustrates these differences. Our focus will be on interest rate rules.[10]

3. Real Indeterminacy in an Endowment Economy with $U_{cm} = 0$.

Suppose that the central bank conducts policy according to the following rule:

$$R_t = A(\pi_{t+1})^{\tau}, \tag{9}$$

where $\tau \geq 0$, $A \equiv R_{ss}\pi_{ss}^{-\tau}$, and R_{ss} and π_{ss} are the steady-state values. That is, the central bank varies the nominal rate in relation to movements in expected inflation with an elasticity of τ. Under such a policy we get the standard result that there is nothing to pin

[9] In a model with a strict CIA constraint the Fisher equation is of the form given by (6). This arises because the implicit Leontief transactions technology implies that $U_m = 0$ in equilibrium, ie., if $m = c$, additional real balances have no effect on the ability to transact, while if $m < c$, it is impossible to transact.

[10] These timing issues also have a dramatic effect on the conditions for real indeterminacy when central

down the initial $\pi_t \equiv P_t / P_{t-1}$. This is a pure nominal or price level indeterminacy which in this flexible price economy has no effect on real behavior. Instead our focus is on real indeterminacy—is the path for expected inflation, the nominal interest rate, and thus *real* cash balances pinned down by the interest rate policy? If not, then we will conclude that the economy suffers from real indeterminacy and thus sunspot fluctuations.[11, 12]

How could real indeterminacy arise? Consider the following scenario. Suppose that there is an increase in expected inflation of 1%. The central bank responds by increasing the nominal rate by τ% thus producing a decline in real cash balances (because of the higher nominal rate) and a change in the real interest rate of $(\tau - 1)$%. This expected inflation change, therefore, can only be self-fulfilling if the movement in real cash balances produces a movement in the real rate of $(\tau-1)$%. Since real interest rate determination is different in the two models, whether or not this is possible will depend crucially upon the timing assumption.

CWID Timing:

Suppose that utility is separable and we normalize $U_c(t)$ in this endowment economy to unity. In Model 2, this implies that the real rate of interest is constant so that real indeterminacy is not possible (except for $\tau = 1$). To be specific, substituting the interest rate rule into the Fisher equation for CWID timing implies the following:

$$\beta A \pi_{t+1}^{\tau} = \pi_{t+1} . \tag{10}$$

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banks utilize money growth rules.

[11] With an endowment economy and separable preferences this indeterminacy has no effect on consumption but does affect utility (through real cash balances), so we classify this as real indeterminacy.

[12] As is well known, in some monetary models there are also the possibility of equilibria with self-fulfilling hyperinflations in which real balances go to zero in the limit. We ignore these non-stationary equilibria as they are easily eliminated if the central bank stands ready to redeem fiat money at an arbitrarily small but nonzero real value. Our focus is on *stationary* sunspot equilibria.

Notice that while π_t is free (nominal indeterminacy) future inflation and nominal interest rates are nailed down (when $\tau \neq 1$). (Since $\beta A = \pi_{ss}^{1-\tau}$, $\pi_{t+j} = \pi_{ss}$ for all $j \geq 1$.) Hence, with Model 2 and separable preferences there is never real indeterminacy except for the borderline case ($\tau = 1$).[13] The key is that today's real interest rates are not affected by an increase in expected inflation. The extreme borderline case does produce indeterminacy since nominal interest rates and expected inflation move one-for-one so that real interest rates in this case are not affected by changes in expected inflation.

CIA Timing:

The story is quite different in Model 1. Even with separable utility (so that $U_c(t)$ is constant) the real rate is still variable since from (4) the marginal utility of money ($U_m(t)$) affects the real rate of interest. This feedback to the real rate of interest creates the possibility of sunspots. Turning to the details, substituting the assumed monetary reaction function (9) into (8) for CIA timing yields the following:

$$\beta A \pi_{t+2}^{\tau} = \pi_{t+1}. \qquad (11)$$

Once again we always have nominal indeterminacy since π_t is free. For real determinacy, we need π_{t+1} and hence R_t to be pinned down. This arises if and only if the above mapping is explosive or if $\tau < 1$. In contrast, $\tau > 1$ generates real indeterminacy since all choices of π_{t+1} converge to π_{ss}.

The intuition is as follows: Suppose there is a sunspot-driven increase in expected inflation of 1%. Given the policy rule (9) this increases the nominal rate by τ% and the real rate of interest by ($\tau-1$)%. The increase in the nominal rate leads to a decline in real

[13] Real indeterminacy could arise if R_t depended on π_{t+k} for $k \geq 2$.

cash balances, which in turn increases the real rate (since U is concave in m). When $\tau > 1$ the initial sunspot-driven increase in expected inflation is equivalent to an increase in the real rate so that the circle is complete and there is real indeterminacy.

The above analysis made a very special, and implausible, assumption. Namely that utility is separable between money and consumption. The next section generalizes the above analysis by adding production to the above endowment economy. Remarkably this general model generates the exact same conditions for indeterminacy as derived above.[14]

4. Real Indeterminacy in a Production Economy with $U_{cm} \neq 0$.

Carlstrom and Fuerst (1999a) examine a standard real business cycle model with production in which money is added with a CIA constraint. They demonstrate that indeterminacy arises if and only if $\tau > 1$, the same condition derived above for CIA timing in an endowment economy and separable preferences.[15] This section illustrates that the nature of this result does not depend on a strict CIA constraint but merely on CIA-timing. The key is that with some fairly standard assumptions a model with production will generate the same conditions for indeterminacy as a model where $U_{cm} = 0$. This equivalence will be true for both the CWID- and CIA-timing MIUF models.

Assume that preferences are separable and linear in labor (L) and given by

$$U(c, m, 1-L) \equiv V(c, m) - AL,$$

and that production takes the standard Cobb-Douglas form:

$y = K^{\alpha} L^{1-\alpha}$ with a constant depreciation rate of δ.

[14] The working paper version of this paper derives the conditions for indeterminacy in the endowment economy when utility is not separable between real money and consumption.

[15] This "1" result is exactly true for linear leisure. For more general preferences the numerical differences are trivial.

The additional Euler equations for labor choice (12) and capital accumulation (13) are familiar:

$$\frac{U_L(t)}{U_c(t)} = f_L(t) \tag{12}$$

$$U_c(t) = \beta U_c(t+1)[f_K(t+1) + (1-\delta)]\,. \tag{13}$$

$$c_t = K_t^{\alpha} L_t^{1-\alpha} + (1-\delta)K_t - K_{t+1}\,. \tag{14}$$

These Euler equations are common across the two models because consumption and output enter symmetrically in both models. Real money balances indirectly enter both of these marginal conditions via the cross partials (U_{cm}) of the utility function. As a result the behavior of the nominal interest rate (and hence real balances) typically distorts the economy's behavior relative to an otherwise standard real business cycle (RBC) model.

We now state our principle result:

Proposition 1: Assume that preferences are separable and linear in leisure, U(c,m,1-L) = V(c,m) – AL, and that the production technology is CRS and Cobb-Douglas. Then with CIA timing a necessary and sufficient condition for determinacy is $\tau < 1$; with CWID timing, the equilibrium is determinate for all values of $\tau \neq 1$.

Proof: See the Appendix.

Although the proof of the proposition exploits the linearity in labor preferences, this assumption is theoretically convenient but computationally irrelevant. For example, if instead of linear leisure there was a constant labor supply elasticity of 0.1 then with plausible calibrations the bounds for determinacy are largely unchanged in both models. The model with CIA timing is determinate if and only if $\tau < 1.004$, while a search for a τ

that produces indeterminacy in the model with CWID timing proved futile. The assumption that leisure is separable in utility also proved to have no quantitative importance.

It is important to note that these conditions for determinacy are identical to an endowment economy with $U_{cm} = 0$. The additional restrictions on U_c implied by endogenous labor choice and capital accumulation cause the model to behave *as if* $U_{cm} = 0$. For example, consider a model without capital and with constant returns to labor ($\alpha = 0$). In this case linear leisure and (12) implies that U_c is constant!

With capital and CRS Cobb-Douglas technology this basic logic carries through. The key is that (given the above assumptions) we can rewrite (12) and then substitute (12) into (13) to obtain.

$$U_c(t) = \frac{x_t^{\alpha}}{(1-\alpha)}. \tag{15}$$

$$x_t^{\alpha} = \alpha\beta x_{t+1} + \beta(1-\delta)x_{t+1}^{\alpha},\ where\ x_t = \frac{L_t}{K_t}. \tag{16}$$

The equilibrium marginal utility of consumption is not directly affected by real money because it is entirely determined by the capital-labor ratio. This implies that the marginal product of capital is unaffected by changes in real money balances. Linearity implies that if $U_{cm}>0$ an increase in real money balances increases the marginal utility of consumption which elicits more production until the marginal utility of consumption returns to where it was before the increase in real money balances.

5. Timing and Sticky Prices.

This paper's analysis has been conducted in the context of flexible price models.

However, these timing concerns can arise in sticky price models as well. One example will illustrate this point.

Clarida, Gali, and Gertler (1998) analyze a sticky-price monetary model and conclude that a necessary condition for real determinacy is $\tau > 1$ – the exact opposite condition for determinacy in a flexible price economy with CIA timing!

Assuming constant potential output, their log-linearized system of equations is

$$\tilde{y}_t = -\sigma[\tilde{R}_t - \tilde{\pi}_{t+1}] + \tilde{y}_{t+1} \tag{17}$$

$$\tilde{\pi}_t = \lambda\tilde{y}_t + \beta\tilde{\pi}_{t+1} \tag{18}$$

$$\tilde{R}_t = \tau\tilde{\pi}_{t+1} \tag{19}$$

where tildes denote log-deviations from steady-state, λ is the slope of the Phillips curve, and σ is the intertemporal elastiticity of substitution. Equation (17) is the Fisher equation with CWID timing and $U_{cm} = 0$ (with no investment consumption equals output); equation (18) is the Phillips curve, and equation (19) is the forward-looking interest rate rule.

Unlike the flexible-price model, the Phillips curve implies that for there to be real determinacy π_t must be pinned down. Straightforward calculations imply that there is real determinacy if and only if

$$1 < \tau < \frac{2(\beta+1)+\sigma\lambda}{\sigma\lambda}.$$

For reasonable calibrations, the upper bound is quite high, about 14, so that the basic conclusion is that a τ greater than unity will achieve determinacy. This is the Clarida, Gali, and Gertler (1998) result.

With CIA timing, however, equation (8) implies that we must replace (17) with

$$\tilde{y}_t = -\sigma[\tilde{R}_{t+1} - \tilde{\pi}_{t+1}] + \tilde{y}_{t+1}. \qquad (20)$$

To analyze the conditions for determinacy substitute (18)-(19) into (20) to eliminate output and the nominal rate:

$$(\sigma\lambda\tau + \beta)\pi_{t+2} - (1 + \sigma\lambda + \beta)\pi_{t+1} + \pi_t = 0. \qquad (21)$$

For real determinacy we need both roots of this equation to lie outside the unit circle.[16] A sufficient (but not a necessary) condition for all forward-looking rules to be indeterminate is for $\sigma\lambda+\beta \geq 1$. Estimates of λ are extremely difficult to come by. According to Clarida, Gali, Gertler, estimates for λ range from 0.05 to 1.22. Assuming that β=0.99 then even if we take the lowest plausible estimates for λ the model will always be indeterminate unless $\sigma < 0.2$.

Recall that the essential difference between CWID-timing and CIA-timing is that the nominal interest rate in the latter is scrolled forward one period. This implies that the conditions for determinacy with *CIA timing* and a *current*-looking interest rate rule are identical to those for *CWID timing* and a *forward*-looking interest rate rule. Thus if we used CIA timing in Kerr and King's (1996) model, there would be determinacy for $\tau > 1$ (unless τ is too large) since their policy rule depends on current inflation. Carlstrom and Fuerst (1999b), however, demonstrate that if we add investment to this model then an aggressive *backward*-looking Taylor rule is necessary for real determinacy.

6. Conclusion.

Hippocrates advised the doctor to do no harm. This minimal advice is equally important to the central banker. In particular, a necessary condition for good monetary

[16] Because prices are sticky both π_t and π_{t+1} must be pinned down – there is no pure nominal indeterminacy in a sticky price model.

policy is that it does not introduce sunspot fluctuations into the real economy. This paper has demonstrated that the class of policies that are "good" in this regard depends on basic assumptions about the modeling environment. Hence, a central conclusion of this analysis is that we need to think much more carefully about basic modeling assumptions when writing down monetary models. A lot depends on apparently trivial assumptions.

These timing issues are of more than academic interest. Central banks that have adopted explicit inflation targeting all use inflation forecasts as an integral part of their decision-making. This corresponds to the forward-looking Taylor rule analyzed here. In a model with sticky prices and CWID timing, there is no problem as long as $\tau > 1$. In contrast, with CIA timing, such a policy is potentially disastrous since it introduces real indeterminacy and sunspot fluctuations into the real economy. This suggests that central banks should use either current or backward-looking Taylor rules.

Appendix

Proposition 1: Assume that preferences are separable and linear in leisure, U(c,m,1-L) = V(c,m) – AL, and that the production technology is CRS and Cobb-Douglas. Then with CIA timing a necessary and sufficient condition for determinacy is $\tau < 1$; with CWID timing, the equilibrium is determinate for all values of $\tau \neq 1$.

Proof: The proof for both models begins with the following conditions (15-16) from the text:

$$U_c(t) = \frac{x_t^{\alpha}}{(1-\alpha)}, \; where\; x_t = \frac{L_t}{K_t} \qquad \text{(A1)}$$

$$x_t^{\alpha} = \alpha\beta x_{t+1} + \beta(1-\delta)x_{t+1}^{\alpha} \qquad \text{(A2)}$$

Notice that (A1) implies that U_c depends only on x, so that real balances, m, depend only on c and x.

CIA timing: Defining $z_t = U_c(t) + U_m(t)$, and using the fact that m depends only on c and x, the budget constraint can be written as

$$K_{t+1} = K_t x_t^{1-\alpha} + (1-\delta)K_t - c(x_t, z_t) \qquad \text{(A3)}$$

Substituting the money demand equation (5) into (A1)

$$\frac{z_t}{R_t} = \frac{x_t^{\alpha}}{(1-\alpha)}, \; where\; x_t = \frac{L_t}{K_t} \qquad \text{(A4)}$$

Substituting the monetary policy rule (9) into the Fisher equation (4) yields

$$R_t = R_{ss}\left(\frac{z_t}{z_{t+1}}\right)^{\frac{\tau}{\tau-1}}$$

Substituting this into (A4) gives

$$R_{ss} z_t^{\frac{1}{1-\tau}} z_{t+1}^{\frac{\tau}{1-\tau}} = \frac{x_t^{\alpha}}{1-\alpha}. \tag{A5}$$

Equations (A2), (A3) and (A5) can be written as

$x_{t+1} = F(x_t)$

$K_{t+1} = G(x_t, z_t, K_t)$

$z_{t+1} = H(x_t, z_t)$

The three Eigenvalues of the characteristic matrix are

$$e_1 = F_x = \frac{\alpha}{1-\beta(1-\alpha)(1-\delta)} < 1,\ e_2 = G_K = \frac{1-\beta(1-\alpha)(1-\delta)}{\alpha\beta} > 1,\ e_3 = H_z = \frac{1}{\tau}.$$

F_x and G_k are the two Eigenvalues for the canonical real business cycle model. Since there is only one predetermined variable, for the economy to be determinate two Eigenvalues need to lie outside the unit circle. Therefore the economy is determinate if and only if $\tau < 1$.

CWID timing: Assume $\tau \neq 1$. Substituting the labor equation (A1) and the monetary policy rule (9) into the Fisher equation (6) gives

$$R_t = \left(\frac{x_t^{\alpha}}{x_{t+1}^{\alpha}}\right)^{\frac{\tau}{\tau-1}}. \tag{A6}$$

Define $z_t = U_c(t) - U_m(t)$. From the money demand equation (7) this yields

$$z_t = \frac{U_c}{R_t}. \tag{A7}$$

(A7) therefore becomes

$$z_t = \frac{\left(\frac{x_t^{\alpha}}{(1-\alpha)}\right)^{\frac{1}{\tau-1}} \left(\frac{x_{t+1}^{\alpha}}{(1-\alpha)}\right)^{\frac{\tau}{\tau-1}}}{R_{ss}}$$

Substituting this into (A3) gives

$x_{t+1} = F(x_t)$

$K_{t+1} = G(x_t, x_{t+1}, K_t)$

The Eigenvalues are

$$e_1 = F_x = \frac{\alpha}{1-\beta(1-\alpha)(1-\delta)} < 1, \quad e_2 = G_K = \frac{1-\beta(1-\alpha)(1-\delta)}{\alpha\beta} > 1.$$

Since there is one predetermined variable the system is always determinate. If $\tau = 1$, the nominal rate drops out of the Fisher equation so that the counterpart to (A7) is $x_{t+1} = x_t$. But now R_t and thus z_t (from (A6)) are entirely free, so that we have real indeterminacy.

QED

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