Estimating DSGE Models with Observed Real-Time Expectation Data[∗]

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Abstract

We propose a novel method for estimating DSGE models when real-time information on expectations is available. We use the Survey of Professional Forecasters (SPF) data to estimate a medium-scale New Keynesian DSGE model using the proposed technique. Our results suggest that the unconditional forecasts of private agents differ from the expectations of the SPF and that private agents find it useful to exploit existing real-time exogenous information when forming their own expectations and making their decisions. However, given that the accuracy of conditioning information decreases over time, they rely only on a subset of that information. We also find that the economic implications of the model in which agents exploit exogenous expectation information substantially differ from those of the benchmark in which agents are not allowed to use off-model information. In particular, the model using SPF data outperforms the benchmark both in terms of fit and in terms of forecast performance.

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1 Introduction

New-generation dynamic stochastic general equilibrium (DSGE) models, have to a large extent, assuaged one of the frustrations macroeconomists used to face: designing a model that is both theoretically consistent and empirically relevant – in other words, a model that both allows the type of story-telling that policymakers require and also forecasts well.

But while the assumption of rational expectations underlying the foundations of DSGE models implies that agents systematically exploit all available information that has the potential of improving their forecasts, in standard DSGE models (e.g. Smets and Wouters (2007)) the parameters are estimated disregarding the possibility that the current knowledge of future events affects current decisions. In this paper we consider the estimation of the structural parameters of a DSGE model in an environment in which agents, in their decision-making process, exploit existing real-time information on expectations of future events.

There are at least two advantages in estimating models using the information set actually available to economic agents at the time they make their decisions. First, from an econometric point of view, real-time data potentially contains useful information about the location of the structural parameters to estimate. Secondly, the estimated parameters in turn affect both the economic implications and the forecasting performance of models.

If private agents' unconditional forecasts are aligned with some existing expectation information, and that the model at hand captures the behavior of the agents, then there is potentially no gain in estimating a model using expectation information. But there are reasons to believe that none of those two assumptions is satisfied. One of the reasons is that there are many institutions, alongside many central banks, who routinely publish forecasts about future developments. Those forecasts are heterogenous and are revised over time. A second reason has to do with the fact that for the same economy there can be many DSGE models yielding different predictions about the expectations of private agents.

Now if real-time information need not match the expectations of the agents or that the information merely represents an exogenous signal that agents take advantage of, a signal that is revised every period and therefore inaccurate, the next question is how to model such an environment. We exploit the conditional forecast technique of Maih (2010) in two important ways. First, we allow agents to instantaneously react to anticipated events to

occur in the future. This is done by solving the DSGE model under the assumption that the information set of the agents includes anticipated shocks. Secondly, the information on future events in then used to back out the distribution of those future shocks. This distribution is eventually fed to the filtering procedure to compute the likelihood function.

In our application, we use Survey of Professional Forecasters (SPF) data on inflation, output growth and consumption growth and re-estimate the Smets and Wouters (2007) model. A priori, there is nothing implying that SPF forecasts are better than the private agents' own forecasts. An issue to address then is to what extent the agents will want to rely on the SPF forecasts. Put differently, if we have conditioning information that is inaccurate, how much of it should we take into account. We address that issue by estimating the horizon up to which agents rely on existing information about the future.

This imparts a technical difficulty in that we now have a problem of mixed-variable estimation whereby there is a set of parameters defined over a continuous space and one discrete parameter. This is a challenging problem that can be addressed in many ways. One strategy would be fix the horizon or anticipation parameter and optimize over the continuous parameters and repeat this for all possible values of the anticipation parameter and finally pick the value of the anticipation parameter that maximizes the objective function. In our example, this would imply running 7 different estimations, which is feasible given that there is only one discrete parameter to optimize over and that the search space is small. But this strategy rapidly becomes problematic if one more or several discrete parameters are added as it would be the case if the anticipation parameter is shock specific (i.e. all the shocks no longer have the same anticipation horizon) or variable specific. We use a different strategy: we treat the anticipation parameter as continuous when guessing parameter values and round it to the nearest integer within its bounds before solving the model.

Previewing the results, the specification of the Smets and Wouters (2007) model estimated using SPF data nests its benchmark in which agents do not rely on exogenous information as a special case. In that model the anticipation parameter is 0. The estimated anticipation parameter in the augmented model is well above 0, suggesting that private agents do rely on SPF information. The values estimated for the economic parameters as well as the shock processes are significantly affected relative to the benchmark. All in all, our results suggest that allowing private agents to exploit SPF data improves the fit of the model and may qualitatively affect its economic implications. Ignoring SPF information, the estimated shocks appear to be a mix of structural shocks and expected shocks.

The rest of the paper briefly reviews the literature to better emphasize our contributions (section 2). Then we present the framework for estimating DSGE models combining historical and real-time data (section 3), before applying the technique presented to the Smets and Wouters (2007) model (section 4).

2 Related literature

We are not aware of any paper attempting to estimate a DSGE model using real time information on expectations. But nevertheless our work could be easily related to the literature estimating this class of models using expectation data. For instance DelNegro and Eusepi (2010) fit two DSGE models to US data using observations on inflation expectations. Their modeling strategy is to consider the expectations obtained from the Survey of Professional (SPF) Forecasters as being the expectations of private agents¹. In this paper, we take a different approach and simply treat expectations of the SPF as information that is available before private agents form their own expectations and make decisions. It is information that private agents can choose to take advantage of or to disregard. So rather than having expectation data in the measurement equation, our approach proceeds by conditioning the forecasts of private agents on the information set at their disposal. A special case of our approach, as we will show, is the case where the predictions of the SPF coincide with the predictions of private agents.

Our framework could be given different interpretations. First it could be interpreted as implying that the model is misspecified so that it is ill-equipped to capture the actual expectations of private agents and so, conditioning on that information may help improve the forecasts of other variables in the model. A second interpretation is that of a partial information. The SPF has some information about future shocks for which agents have a zeromean prior. In reality, the SPF itself does not forecast the future accurately. Nevertheless, the decision-making process of rational agents could be such

¹If the SPF expectations are those of the private agents, this potentially holds true not just for inflation, but also for other variables the SPF provides forecasts for.

that they find it profitable to listen to the "news" from the SPF to have a better idea about the shocks that possibly could hit the system in the future.

Naturally, perhaps, our framework is also related to the literature on news shocks like Fujiwara, Hirose, and Shintani (2009), Christiano, Ilut, Motto, and Rostagno (2008), to mention a few. In this literature, news shocks are typically modeled as ad-hoc additional lagged shocks in the shock processes of a DSGE model. One consequence is that lagged shocks at all horizons have the same impact on the expectations. In addition, in such a setup, anticipated shocks always materialize. The approach we follow in this respect is more flexible. Rather than changing the structural model, we solve the structural model under the assumption that agents may have some information about the future. In this sense events occurring at different horizons are discounted differently and anticipated events need not materialize.

Finally, as already hinted above, our approach is related to the conditional forecasting literature in VARs (see Doan, Litterman, and Sims (1984), Waggoner and Zha (1999), Andersson, Palmqvist, and Waggoner (2008)), in DSGE models (Christoffel, Coenen, and Warne (2007), Benes, Binning, and Lees (2008)) and using relative entropy (Robertson, Tallman, and Whiteman (2005)). But more specifically we follow Maih (2010), who develops conditional forecasting techniques for DSGE models, where the conditioning information is allowed to be a full distribution, a truncated density (soft conditions), or just a central tendency (hard conditions). But while Maih (2010), just as in all the aforementioned papers, considers an environment in which conditioning information occurs to agents for the first time, in this paper we assume that the information structure of the agents is the same from period to period. In particular, in each period the SPF, provides agents with real-time information about its forecasts of future events and updates those forecasts as time goes by. That is, we allow the views of the SPF about future events to change every period, hence the real time dimension.

With the exception of Waggoner and Zha (1999), none of those papers mentioned above attempts to estimate model parameters using conditional information. But since Waggoner and Zha (1999) use a VAR, in their case conditioning information occurs at the end of the sample only. In this paper we estimate model parameters in the context of a DSGE model, using information that is revised over time and also estimate the horizon up to which private agents use a conditioning information that is likely not to materialize but which may still be useful.

3 DSGE estimation using real-time information on expectations: the econometrics

The DSGE model is given by

$$
E_t \mathcal{F}_{\theta}(y_t, y_{t-1}, y_{t+1}, \varepsilon_t) = 0 \tag{1}
$$

with

$$
\varepsilon_t \sim N(0, I)
$$

In equation (1), E_t represents the expectation operator given the information at time t, y_t is an $n \times 1$ vector of endogenous variables, ε_t is a $r \times 1$ vector of exogenous variables, θ is the vector collecting all the structural parameters of the model and \mathcal{F}_{θ} is linear in its arguments.

In every period, the SPF provides private agents with information about what will happen in the next k periods. The information typically comes in the form of a density

$$
D_t Y_t \sim N(\mu_t, \Omega_t)
$$

where

$$
Y_t \equiv \left[y_t', y_{t+1}', ..., y_{t+k}' \right]
$$

Private agents interpret this information as implying that some non-zero mean shocks, which they otherwise would expect to have zero mean, will occur in the future. Next they use the Maih (2010) techniques to find the implied distribution of those shocks. Recognizing that the information is uncertain, the SPF is not clairvoyant, and that the reliability of the SPF information may decrease over time, they choose a horizon $s \leq k$, up to which they want to look into the future. In the end at time t , the information set of the agents is $I_t = \{ \theta, y_{t-1}, \varepsilon_t^t, \varepsilon_{t+1}^t, ..., \varepsilon_{t+s-1}^t \}$ where $\varepsilon_{t+j}^t \equiv E_t(\varepsilon_{t+j}), j =$ 1, 2, ..., s. In this case the solution of the model takes the form

$$
y_t = T(\theta) y_{t-1} + R(\theta) \eta_t \tag{2}
$$

where

$$
\eta_t \equiv \left[\left(\varepsilon_t^t \right)', \left(\varepsilon_{t+1}^t \right)', \dots, \left(\varepsilon_{t+s-1}^t \right)' \right]'
$$
\n(3)

and

$$
R \equiv [R_0, R_1, ..., R_{s-1}]
$$

When $s = 1$, we have the traditional framework.

3.1 From conditional information on endogenous to conditional information on shocks

Using the dynamics of the model, this can be written as

$$
Y_t = \ddot{T}y_{t-1} + \Phi \bar{\eta}_t
$$

where

$$
\dddot{T} \equiv \begin{bmatrix} T \\ T^2 \\ \vdots \\ T^k \end{bmatrix} \text{ and } \bar{\eta}_t \equiv \left[(\eta_t)' , (\eta_{t+1})', ..., (\eta_{t+k})' \right]'
$$

such that $\eta_t = S_t \bar{\eta}_t$.

Then $D_t Y_t \sim N(\mu_t, \Omega_t)$ implies

$$
D_t \Phi \bar{\eta}_t \sim N(\mu_t - D_t \ddot{T} y_{t-1}, \Omega_t)
$$

Following Maih (2010) we let $\bar{\eta}_t = M_{1t}\gamma_{1t} + M_{2t}\gamma_{2t}$, where M_{1t} is an orthonormal basis for the null space of the $q \times rsk$ matrix $D_t \Phi$ and M_{2t} is an orthonormal basis for the column space of $D_t\Phi$. In that case $D_t\Phi M_{1t} = 0$ and $D_t \Phi M_{2t}$ is invertible. This assumes that $D_t \Phi$ is of full rank q. $\gamma_{1t} \sim N(0, I)$ is a $(rsk - q) \times 1$ vector of disturbances that do not affect the restrictions, while the distribution of the $q \times 1$ vector γ_{2t} is given by

$$
\gamma_{2t} \sim N \left[\left(D_t \Phi M_{2t} \right)^{-1} \left(\mu_t - D_t \ddot{T} y_{t-1} \right), \left(D_t \Phi M_{2t} \right)^{-1} \Omega_t \left(\left(D_t \Phi M_{2t} \right)^{-1} \right)' \right]
$$

so that

$$
\bar{\eta}_t \sim N \left[M_{2t} \left(D_t \Phi M_{2t} \right)^{-1} \left(\mu_t - D_t \ddot{T} y_{t-1} \right), M_{2t} \left(D_t \Phi M_{2t} \right)^{-1} \Omega_t \left(\left(D_t \Phi M_{2t} \right)^{-1} \right)' M_{2t}' + M_{1t} M_{1t}' \right]
$$

implying that

$$
\bar{\eta}_t = M_{2t} \left(D_t \Phi M_{2t} \right)^{-1} \left(\mu_t - D_t \ddot{T} y_{t-1} \right) + \omega_t \xi_t \tag{4}
$$

where ω_t is such that

$$
\omega_t \omega'_t = M_{2t} \left(D_t \Phi M_{2t} \right)^{-1} \Omega_t \left(\left(D_t \Phi M_{2t} \right)^{-1} \right)' M'_{2t} + M_{1t} M'_{1t} \tag{5}
$$

and $\xi_t \sim N[0, I].$

3.2 Conditional state-space system

Using (4) and (2) we get the following solution for the restricted dynamics

$$
\begin{bmatrix} y_t \\ \bar{\eta}_t \end{bmatrix} = \begin{bmatrix} RS_t M_{2t} (D_t \Phi M_{2t})^{-1} \\ M_{2t} (D_t \Phi M_{2t})^{-1} \end{bmatrix} \mu_t + \begin{bmatrix} T - RS_t M_{2t} (D_t \Phi M_{2t})^{-1} D_t \ddot{T} & 0 \\ -M_{2t} (D_t \Phi M_{2t})^{-1} D_t \ddot{T} & 0 \end{bmatrix} \begin{bmatrix} y_{t-1} \\ \bar{\eta}_{t-1} \end{bmatrix} + \begin{bmatrix} RS_t \omega_t \\ \omega_t \end{bmatrix} \xi_t
$$

from which it is clear that the restrictions in future periods beyond the next one are taken into account.

When $\mu_t = D_t T y_{t-1}$ the mean of the information provided by the SPF matches the mean of the unconditional forecasts of the agents and $E(\bar{\eta}_t) = 0$. When $\Omega_t = D_t \Phi \Phi' D'_t$, $V(\bar{\eta}_t) = M_{1t} M'_{1t} + M_{2t} M'_{2t} = I$, the uncertainty of the restrictions of the SPF matches the uncertainty of the agents. This suggests a way of estimating Ω_t and μ_t when they are not available, but also shows that the case where the expectations of the SPF correspond to the forecasts of private agents is a special case of our framework. When both of the conditions are satisfied, then $\bar{\eta}_t \sim N[0, I]$ so that conditioning does not bring in any additional information. In that case estimating a model with $s > 1$ is equivalent to estimating a model with $s = 1$.

The state space model consists of a transition equation of the form

$$
\alpha_{t+1} = b_t + T_t \alpha_t + R_t \xi_{t+1}, \xi_{t+1} \sim N(0, I)
$$
\n(6)

where b_t , T_t and R_t are define below and the measurement equation

$$
y_t^* = Z_t \alpha_t + \epsilon_t, \ \epsilon_t \sim N(0, H_t)
$$

$$
\alpha_t \equiv \begin{bmatrix} y_t \\ z \end{bmatrix}
$$
 (7)

with

$$
\alpha_t \equiv \left\lfloor \begin{array}{c} y_t \\ \bar{\eta}_t \end{array} \right\rfloor
$$

3.3 Signal extraction and estimation

We define

$$
a_t \equiv E(\alpha_t | I_{t-1}), P_t \equiv V(\alpha_t | I_{t-1}), a_{t|t} \equiv E(\alpha_t | I_t), P_{t|t} \equiv V(\alpha_t | I_t)
$$

The likelihood function is computed using the Kalman filter for given measurements $\{y_t^*\}_{t=1}^n$, the sequence of restrictions $\{D_t, \mu_t, \Omega_t\}_{t=1}^n$, the state matrices $T, R, \{Z_t\}_{t=1}^n$ and initial conditions P_0 and a_0 , which can be chosen to be the unconditional mean and covariance matrix of the state vector α_t^2 .

Then for $t = 1, 2, ..., n$ we have the following recursions for the Kalman filter:

$$
v_{t} = y_{t}^{*} - Z_{t}a_{t}
$$
\n
$$
F_{t} = Z_{t}P_{t}Z_{t}' + H_{t}
$$
\n
$$
M_{t} = P_{t}Z_{t}'
$$
\n
$$
a_{t|t} = a_{t} + M_{t}F_{t}^{-1}v_{t}
$$
\n
$$
P_{t|t} = P_{t} - M_{t}F_{t}^{-1}M_{t}'
$$
\n
$$
T_{t} \equiv \left[T - RS_{t}M_{2t} (D_{t}\Phi M_{2t})^{-1} D_{t}T \right] 0 \right]
$$
\n
$$
b_{t} \equiv \left[RS_{t}M_{2t} (D_{t}\Phi M_{2t})^{-1} \right] \mu_{t}
$$
\n
$$
\omega_{t} = chol \left(M_{2t} (D_{t}\Phi M_{2t})^{-1} \Omega_{t} ((D_{t}\Phi M_{2t})^{-1})' M_{2t}' + M_{1t}M_{1t}' \right)
$$
\n
$$
R_{t} \equiv \left[RS_{t}\omega_{t} \right]
$$
\n
$$
a_{t+1} = b_{t} + T_{t}a_{t|t}
$$

$$
P_{t+1} = T_t P_{t|t} T'_t + R_t R'_t
$$

The likelihood function is useful for the estimation of the parameters and is given by:

$$
\log lik_t = -\frac{1}{2} \sum_{t=1}^{n} \left(\log |F_t| + p \log 2\pi + v_t' F_t^{-1} v_t \right)
$$

²An excellent exposition of the Kalman filter can be found in Durbin and Koopman (2001).

4 Application to the Smets and Wouters (2007) model

In this section, we apply the technique developed in the previous sections to the medium-scale New Keynesian model of Smets and Wouters (2007). One of the advantages of choosing the Smets and Wouters (2007) model is has been shown to fit the data well and in particular, its forecasting performance compares well with VARs. For the sake of brevity, only the set of log-linearized equations are presented in the appendix and with refer to Smets and Wouters (2007) for the details and derivations of those equations³.

For the variables used in the measurement equations of the Kalman filter we use the same data (spanning 1966Q1:2004Q4) as Smets and Wouters (2007), to which, again, we refer to for details. Besides those variables, we also use expectation data on (GDP growth, Inflation(GDP deflator) and consumption growth. The two first series are available 5 quarters in advance (from quarter 2 to 6) and span the period 1968Q4:2004Q4 in our estimations (2004Q4 is the date of the last observation in the Smets and Wouters (2007) data). The third is available for 6 quarters in advance (quarters 1 to 6) and span the period 1981Q3 to 2004Q4.

We also use the same prior distributions as Smets and Wouters for all the parameters, except the anticipation parameter for which we do not have a strong prior. The prior on this last parameter is thus taken to be uniform over the interval $[0,6]$, where 0 means that expectations from the SPF are not taken into account by private agents, which is the Smets and Wouters (SW) model. An anticipation parameter of 1 means that agents use only expectations for the period following the one they are in, while an anticipation parameter of 6 means they use all the 6 quarters of information available. All the prior distributions are given of table 2 (columns 2 and 3) for the economic parameters and table 3 (columns 2 and 3) for the shock processes (persistence and standard deviations).

Most of the empirical exercises we go through will consist in comparing the results of the SW model estimated using various specifications of conditional information, against the benchmark, in which agents do not make use of SPF information. In particular, we will compare the benchmark model against the model with the best conditioning information and then analyze

³The latex equations have been generated almost entirely automatically by dynare, based on the original code by Smets and Wouters (2007).

the effects of adding less accurate information. The reason for using various specifications of the conditioning information is because, as pointed out by Maih (2010), the forecasting performance of a model using conditional information depends on how well the model is good at capturing the actual correlations between the conditioning variables and the other variables in the system, but also on the accuracy of the conditioning information.

Table 1 reports the correlations between the expectations of inflation, GDP growth and consumption growth as provided by the SPF and the actual realization of those variables. We immediately see that the conditional information on both GDP growth and consumption growth is not as good as the conditioning information on inflation. Also, for all the three variables the accuracy of the forecasts provided by the SPF decreases with the length of the forecast horizon.

Conditioning on poor information is potentially detrimental for the forecast performance of the model in terms of the other variables in the system. And so, rather than conditioning on all the variables at the same time, we proceed to estimate 5 alternative models: a) the benchmark model without conditioning information, b) the model in which we condition on inflation only, which given the correlation results in table 1, will be the leading alternative model, c) the model in which we condition on inflation and GDP growth, d) a model in which we condition on inflation and consumption growth and e) a model in which we condition on inflation, GDP growth and consumption growth. As the SPF information does not come in the form of a multivariate density, we estimate the joint density of the restrictions to be the one generated by the model.

We begin by discussing the parameter estimates and the model fit in section 4.1, then we analyze the implications of the estimated parameters in terms of variance and historical decompositions (section 4.2) and then impulse responses (section 4.3). One of the benefits of our empirical strategy using real-time information is that we can also get smoothed estimates of the expected shocks, which we will use to see how agents combining the SW model with SPF information would have been surprised (section 4.4). We then move on to investigating the forecast performance of the alternative models (section 4.5). All the results are based on the posterior mode.

4.1 Parameter estimates and models fit

Tables 2 and 3 report the estimates of the economic parameters and those of the shock processes respectively. Column 5 reports the estimates of the benchmark model (SW), column 6 reports the estimates of the model conditioning on inflation only (Infl), column 7 reports the estimates of the model conditioning on inflation and output growth (Infl+GDP), column 8 reports the estimates of the model conditioning on inflation and consumption growth (Infl+Cons) and column 9 reports the estimates of the model conditioning on inflation, output growth and consumption growth (Infl+GDP+Cons).

Overall, the parameters appear to be very sensitive to the specification of conditioning information. One robust result emerges though, which is that the anticipation parameter (Anticipation) is estimated to be 2 for all the models using conditioning information. This parameter is well identified by the data as suggested by the curvature of the posterior kernel in that dimension at the posterior mode (see figure 1 for the Infl model). A value of 2 implies that private agents do rely on the exogenous expectation information provided by the SPF, but only for two quarters and this despite their having access to an additional 4 quarters of potentially useful information. This also implies that the expectations of the SPF do not correspond to the unconditional expectations of private agents in the SW model.

Besides the anticipation parameter, the economic parameters in table 2 can be divided into three main groups. The first group comprises the parameters pertaining to preferences. For those parameters, compared to the SW model, the Infl model implies a somewhat similar discount factor (β) (0.9986 against 0.9987), a negative and imprecisely estimated steady state of labor supply (l) , a higher steady-state elasticity of the capital adjustment cost function (φ) , a higher elasticity of intertemporal substitution (σ_c) , lower habit persistence (λ), a lower probability of not adjusting wages (ξ_w), a higher elasticity of labor supply with respect to real wages (σ_l) and a higher wage indexation to past inflation (ι_w) . Smets and Wouters (2007) note that relaxing their prior distributions, the degree of wage stickiness rises more than their benchmark estimate. Imposing additional conditional information significantly affects the values of the estimated parameters in this first group. In particular, the discount factor falls to about 0.9980 in both the Infl+Cons and Infl+GDP+Cons models, while the degree of wage stickiness increases substantially.

The second group comprises the parameters related to production. For

those parameters and compared to the SW model, the Infl model suggests a lower price stickiness parameter (ξ_p) , a higher indexation of current inflation to past inflation (ι_p) , a higher elasticity of the capital utilization adjustment cost function (ψ) , somewhat similar fixed costs in production (Φ) and capital share in production (α) and a slightly higher trend growth rate (γ). As for the parameters in the first group, forcing additional conditioning information on changes the values of the parameters in a significant manner. For instance, the trend growth rate that the Infl model gets above the benchmark (a feature that Smets and Wouters would see as desirable), falls substantially in the three other conditional models.

In the third group we include the monetary policy parameters. Relative to the SW model, the Infl model implies a somewhat lower response of interest rate to inflation (r_{π}) , a lower interest rate smoothing (ρ) , a substantially lower response of interest rate to deviations of the output gap from the flexprice output gap (r_y) , a lower response of inflation to changes in output growth $(r_{\Delta y})$ and a lower steady-state inflation target.

Turning to the estimates of the shock processes, in table 3, overall, the Infl specification implies higher persistence but lower standard deviations of the shocks. Notable exceptions are the persistence of the wage markup shock (respectively ρ_w), which is lower than in the benchmark.

The models can also be compared on their ability to fit the data. The Laplace approximation of the marginal data density (MDD) for the SW model is estimated to be (-922.39). For the conditional models, the Laplace approximation of the MDD hard to compute directly given that there is one parameter defined over a discrete space. We approximate the MDD by fixing the anticipation parameter at its estimated value and then re-estimate the other parameters. Starting at the previously found mode, the optimization algorithm does not move and so we get back the exact same parameters. Using that strategy, the MDD for all the conditional models appear to be significantly better than the benchmark (see table 2). So on this count, the data seem to prefer the conditional models.

Although the differences in MDDs across conditional models are not substantial, the Infl+Cons model appears to be the one with the best performance. Adding GDP expectation information decreases the MDD. Despite these results, we choose to focus on the Infl model in the light of the results on the accuracy of the conditional information.

4.2 Variance and historical decompositions

Table 4 reports the variance decompositions in forecast errors of GDP, Inflation and the Fed's funds rate at various horizons for the Infl model. In the short run, GDP is driven by government spending shocks and risk premium shocks, echoing the findings of Smets and Wouters (2007). However, our results suggest that another important driving force of GDP in the short run is the technology shock and not the investment-specific technology shock. The results also suggest that the importance of the neutral technology shock for GDP growth increases over time as well as that of wage markup shocks. But wage markup shocks do not significantly dominate productivity shocks as would be the case in the SW model.

In the short run, inflation is dominated by price and wage markup shocks but while the former dominates the latter in the short run, in the long run wage markup shocks become the chief mover of inflation.

In the short run, interest rates are explained by monetary policy and risk premium shocks, with the first one being the most important. In the medium to long run, wage markup shocks and to a lesser extent investment-specific technology shocks also become important for the variations of the interest rate. In the long run, investment specific technology, monetary policy and wage markup shocks are the driving forces of the Fed's funds rate.

The variance decompositions are consistent with the historical decompositions of inflation and output displayed in figure 10. Here we follow Smets and Wouters (2007) and bundle risk premium, investment-specific technology and government spending shocks into the group of demand shocks, while markup shocks comprise wage markup and price markup shocks. The history of inflation is clearly dominated by markup shocks, while the history of GDP growth fluctuations can be to a large extent ascribed to productivity shocks.

4.3 Impulse responses

Impulse responses (of output, consumption, investment, hours worked, real wages, inflation, interest rate and capacity utilization) to the various shocks of the models are displayed in figures 2 to 8. The blue line represents the impulse responses of a shock occurring in the current period and the responses to such unanticipated shocks are qualitatively in line with those implied by the SW model. In particular, a positive unanticipated shock to productivity (see figure 3) leads to an expansion of aggregate demand, output, real wages

and a fall in the number of hours. The subsequent fall in the real interest rate is not enough to prevent a fall in inflation.

When shocks are anticipated, however, as shown by the green line, the short-run response of hours, and of interest rate, is positive. Fujiwara, Hirose, and Shintani (2009) find the same results using a different methodology. Another example in which short-run responses are different depending on whether they are anticipated or not is given in figure 4, where following a positive government spending shock, output, interest rate and hours increase if the shock is unanticipated and decrease otherwise.

4.4 Evolution of beliefs

If agents are able to foresee future shocks to some extent, they will attempt to mitigate the effects of those shocks. In this section, we investigate the extent to which conditional information helps anticipate future shocks. This is done by analyzing smoothed estimates of η_t (see equation 3), which represent the evolution of beliefs in terms of shocks. Figure 9 depicts the evolution of the deviations of the expected shocks from their actual realizations in the Infl model. The differences are scaled by the standard deviations of the respective shocks.

Overall, the deviations are stationary around 0 but fluctuate considerably. And so, despite conditioning information on inflation, shock surprises are substantial. Wage markup shocks, which, in the variance decompositions, were found to be important drivers of the business cycle, are alongside investment-specific technology shocks are the shocks for which agents make the most predictions errors. Over the period analyzed, the most predictable shocks are the risk premium and the monetary policy shocks⁴. In fact the variance of the prediction errors on monetary policy and risk premium shocks have decrease considerably since the middle of the 80s.

It is not surprising that monetary policy and risk premium shocks prediction errors move closely together since in the SW model, risk premium shocks represent the wedge between the interest rate controlled by the central bank and the return on assets held by households. Under the estimated policy rule, monetary policy reacts strongly to inflation and if inflation is accurately predicted, monetary policy shocks surprises will tend to be small.

⁴It would be interesting to extend this analysis to include the latest financial crisis period and revisit this point.

A look at the correlation between actual inflation and its predictions by the SPF seems to support this idea (see table 1).

Finally we can also relate the forecast errors of figure 9 to the historical decompositions of figure 10. The sharp reduction in output growth of 1975 is related to miss-predictions of government spending and productivity shocks. The decline of output in 1980Q3 is associated with poor predictions of monetary policy, investment specific, government spending and risk premium shocks. Productivity shocks and wage markup shocks prediction errors can also be associated with the year 2000 recession.

4.5 Forecast performance

As argued earlier, one of the advantages for considering the Smets Wouters model for this exercise is that it has been shown to fit the data well. In particular, its forecasting performance compares well with VARs and BVARs. In this section, we are interested in investigating whether adding conditional information improves the forecast performance relative to the benchmark. As estimation is expensive, and especially so in the presence of parameters defined over a discrete space, we do not re-estimate the models. We keep the parameters constant in the computation of 8-step ahead forecasts for all the models and starting from the beginning of the sample. For the conditional models, when conditional information is available, we use 2 quarters of information only, which is the estimated anticipation horizon. It is important here to remember that for both inflation and output growth, conditioning information is available only at the second quarter, while information on consumption growth is available for both the first and the second quarters.

Figure 11 plots the ratios of the RMSFE for the benchmark model over the RMSFE for all the conditional models. Hence values above 1 imply a better performance of the conditional models and values below 1 imply a deterioration of the performance. By itself, inflation is very useful. The Infl model outperforms the benchmark not just for inflation, but also for GDP growth, investment growth, hours worked and the Fed's fund rate for all the horizons considered. The Infl model also outperforms the benchmark on consumption growth up to 6 quarters. So for those variables, there seems to be a systematic evidence that inflation information contributes to a consistent improvement in forecast accuracy. The Infl model is still competitive in terms of wage growth also as it is un-dominated by the benchmark. All in all, conditioning on inflation we are able to improve not only the medium-term

forecasts but also the short-term ones.

Adding expectation information on consumption, we see that the forecast performance deteriorates markedly in the Infl+Cons model for GDP growth and wage growth, especially in the medium term. It is remarkable that this is also the case for consumption growth but perhaps not surprising given that the consumption information is not very accurate. This stands in contrast with the results obtained using the MDD, for which the Infl+Cons model was the best model.

If instead of consumption growth information we add GDP growth information, the forecasts in the Infl+GDP model remain competitive but are not as good at the forecasts of the Infl model. It seems that bad information on consumption growth is more damaging than bad information on output growth if we compare the Infl+Cons model to the Infl+GDP+Cons model.

We here retrieve some of the results in Maih (2010), namely that conditioning does not always improve the forecasting performance and in some cases it might even deteriorate forecast accuracy. This depends, as said earlier, on the quality of the conditioning information, but also on how good the model is at capturing the correlations between the conditioning variables and the other variables of interest. As we saw, a good information on inflation does not necessarily translate into better forecasts for wage growth. This suggests that the correlation between inflation and wage growth is not well captured by the model and that fitting one variable sometimes comes at the expense of fitting one of several other variables. This final point emphasizes the usefulness of the conditional forecasting technique of Maih (2010) as a misspecification detection tool.

4.6 Robustness

[To be written]

5 Conclusions

[To be written]

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A Log-linearized Smets and Wouters (2007) model

Those equations are almost entirely generated automatically from the Smets and Wouters (2007) code using Dynare.

A.1 Cross-parameter restrictions

Steady state inflation

$$
\pi=1+\frac{\bar{\pi}}{100}
$$

Common quarterly gross trend growth rate of GDP, consumption, investment and wages

$$
\gamma=1+\frac{\bar{\gamma}}{100}
$$

Discount factor

$$
\beta = \frac{1}{1 + \frac{\chi_{\beta}}{100}}
$$

$$
\phi_p=\Phi
$$

where Φ is the fixed-cost parameter in the production function

$$
\bar{\beta} = \beta \, \gamma^{(-\sigma_c)}
$$

$$
cr = \frac{\pi}{\bar{\beta}}
$$

$$
crk = \beta^{(-1)}\gamma^{\sigma_c} - (1 - \tau)
$$

where τ is the depreciation of capital

$$
cw = \left(\frac{\alpha^{\alpha} (1 - \alpha)^{1 - \alpha}}{\Phi (crk)^{\alpha}}\right)^{\frac{1}{1 - \alpha}}
$$

$$
cikbar = 1 - \frac{1 - \tau}{\gamma}
$$

$$
cik = \gamma (cikbar)
$$

$$
clk = \frac{1 - \alpha}{\alpha} \frac{crk}{cw}
$$

where α is the capital share in production

 $cky = \Phi$ $(clk)^{\alpha -1}$

 $\mathit{ciy} = \mathit{cik} \, \times \mathit{cky}$

 $ccy = 1 - cg - ciy$

$$
crkky = crk \times cky
$$

$$
cwhlc = \frac{cky^{\frac{(1-\alpha)\frac{1}{\phi_w}}{\alpha}}(crk)}{1-cg-ciy}
$$

where ϕ_w is one plus the steady-state labor-market markup Steady-state nominal interest rate

$$
\bar{r} = 100 \ (cr - 1)
$$

A.2 Main equations

A.2.1 Sticky price economy

Definition of marginal costs

$$
mc_t = \alpha r_t^k + (1 - \alpha) w_t - \varepsilon_t^a \tag{8}
$$

Equilibrium condition for capacity utilization

$$
z_t = \frac{1}{\frac{\psi}{1-\psi}} r_t^k \tag{9}
$$

where ψ is a positive function of the elasticity of the capital utilization adjustment cost function normalized to lie between 0 and 1 and such that when $\psi = 1$, it is extremely costly to change the utilization of capital.

Equilibrium condition for the rental rate of capital

$$
r_t^k = w_t + l_t - k_t^s \tag{10}
$$

Definition of current capital services

$$
k_t^s = z_t + k_{t-1} \tag{11}
$$

Investment Euler equation

$$
i_t = \varepsilon_t^i + \frac{1}{1 + \gamma \bar{\beta}} \left(i_{t-1} + i_{t+1} \gamma \bar{\beta} + q_t \frac{1}{\varphi \gamma^2} \right)
$$
 (12)

where φ is the steady state elasticity of the capital adjustment cost function.

Arbitrage (Euler) equation for the real value of existing capital (q_t)

$$
q_{t} = (-r_{t}) + \pi_{t+1} + \varepsilon_{t}^{b} \frac{1}{\frac{1-\frac{\lambda}{\gamma}}{\sigma_{c}\left(1+\frac{\lambda}{\gamma}\right)}} + r_{t+1}^{k} \frac{crk}{1-\tau+crk} + q_{t+1} \frac{1-\tau}{1-\tau+crk} \quad (13)
$$

Consumption Euler equation

$$
c_t = \varepsilon_t^b + c_{t-1} \frac{\frac{\lambda}{\gamma}}{1 + \frac{\lambda}{\gamma}} + c_{t+1} \frac{1}{1 + \frac{\lambda}{\gamma}} + (l_t - l_{t+1}) \frac{(\sigma_c - 1) \ cwhlc}{\sigma_c \left(1 + \frac{\lambda}{\gamma}\right)} - (r_t - \pi_{t+1}) \frac{1 - \frac{\lambda}{\gamma}}{\sigma_c \left(1 + \frac{\lambda}{\gamma}\right)} \tag{14}
$$

Resource constraint

$$
y_t = \varepsilon_t^g + c_t (1 - cg - ciy) + i_t \, ciy + z_t \, (crk) \, cky \tag{15}
$$

Production function

$$
y_t = \Phi \left(\varepsilon_t^a + \alpha \, k_t^s + (1 - \alpha) \, l_t \right) \tag{16}
$$

Price Phillips curve

$$
\pi_t = \varepsilon_t^p + \frac{1}{1 + \iota_p \gamma \bar{\beta}} \left(\iota_p \pi_{t-1} + \pi_{t+1} \gamma \bar{\beta} + mc_t \frac{\frac{(1 - \xi_p)(1 - \xi_p \gamma \bar{\beta})}{\xi_p}}{1 + (\Phi - 1) \epsilon_p} \right)
$$
(17)

Wage Phillips curve

$$
w_t = \varepsilon_t^w + \frac{1}{1+\gamma\overline{\beta}} w_{t-1} + \frac{\gamma\overline{\beta}}{1+\gamma\overline{\beta}} w_{t+1} + \frac{\iota_w}{1+\gamma\overline{\beta}} \pi_{t-1} - \frac{1+\iota_w\gamma\overline{\beta}}{1+\gamma\overline{\beta}} \pi_t + \frac{\gamma\overline{\beta}}{1+\gamma\overline{\beta}} \pi_{t+1} + \frac{1}{1+(\phi_w-1)\epsilon_w} \frac{(1-\xi_w)(1-\xi_w\gamma\overline{\beta})}{\xi_w(1+\gamma\overline{\beta})} \left(\sigma_l l_t + c_t \frac{1}{1-\frac{\lambda}{\gamma}} - c_{t-1} \frac{\frac{\lambda}{\gamma}}{1-\frac{\lambda}{\gamma}} - w_t\right)
$$
\n
$$
(18)
$$

Monetary policy reaction function

$$
r_t = \pi_t r_\pi (1 - \rho) + (1 - \rho) r_y \left(y_t - y_t^f \right) + r_{\Delta y} \left(y_t - y_t^f - y_{t-1} + y_{t-1}^f \right) + \rho r_{t-1} + \varepsilon_{rt}
$$
\n(19)

Capital law of motion

$$
k_t = k_{t-1} \left(1 - cikbar \right) + cikbar \times i_t + \varepsilon_t^i \varphi \left(cikbar \right) \gamma^2 \tag{20}
$$

A.2.2 Flexible-price economy

Relations analogous to the sticky price economy apply for the flexible price economy

$$
\varepsilon_t^a = \alpha r_t^{kf} + (1 - \alpha) w_t^f \tag{21}
$$

$$
z_t^f = r_t^{kf} \frac{1}{\frac{\psi}{1-\psi}}
$$
\n⁽²²⁾

$$
r_t^{kf} = w_t^f + l_t^f - k_t^{sf}
$$
\n
$$
(23)
$$

$$
k_t^{sf} = z_t^f + k_{t-1}^f \tag{24}
$$

$$
i_t^f = \varepsilon_t^i + \frac{1}{1 + \gamma \bar{\beta}} \left(i_{t-1}^f + i_{t+1}^f \gamma \bar{\beta} + q_t^f \frac{1}{\varphi \gamma^2} \right) \tag{25}
$$

$$
q_t^f = (-rrf_t) + \varepsilon_t^b \frac{1}{\frac{1-\frac{\lambda}{\gamma}}{\sigma_c \left(1+\frac{\lambda}{\gamma}\right)}} + r_{t+1}^{kf} \frac{crk}{1-\tau+crk} + q_{t+1}^f \frac{1-\tau}{1-\tau+crk} \tag{26}
$$

$$
c_t^f = \varepsilon_t^b + c_{t-1}^f \frac{\frac{\lambda}{\gamma}}{1 + \frac{\lambda}{\gamma}} + c_{t+1}^f \frac{1}{1 + \frac{\lambda}{\gamma}} + \left(l_t^f - l_{t+1}^f\right) \frac{(\sigma_c - 1) \ cwhlc}{\sigma_c \left(1 + \frac{\lambda}{\gamma}\right)} - rrf_t \frac{1 - \frac{\lambda}{\gamma}}{\sigma_c \left(1 + \frac{\lambda}{\gamma}\right)}\tag{27}
$$

$$
y_t^f = \varepsilon_t^g + c_t^f \left(1 - cg - ciy\right) + i_t^f \, ciy + z_t^f \, (crk) \, cky \tag{28}
$$

$$
y_t^f = \Phi\left(\varepsilon_t^a + \alpha k_t^{sf} + (1 - \alpha) l_t^f\right) \tag{29}
$$

$$
w_t^f = l_t^f \sigma_l + c_t^f \frac{1}{1 - \frac{\lambda}{\gamma}} - c_{t-1}^f \frac{\frac{\lambda}{\gamma}}{1 - \frac{\lambda}{\gamma}}
$$
(30)

$$
k_t^f = k_{t-1}^f \left(1 - (cikbar)\right) + i_t^f \left(cikbar\right) + \varepsilon_t^i \left(cikbar\right) \varphi \gamma^2 \tag{31}
$$

A.3 Shock processes

Total factor productivity

$$
\varepsilon_t^a = \rho_a \, \varepsilon_{t-1}^a + \eta_t^a \tag{32}
$$

Risk premium

$$
\varepsilon_t^b = \rho_b \, \varepsilon_{t-1}^b + \eta_t^b \tag{33}
$$

Government spending

$$
\varepsilon_t^g = \rho_g \, \varepsilon_{t-1}^g + \eta_t^g + \eta_t^a \, \rho_{ga} \tag{34}
$$

Investment-specific technology

$$
\varepsilon_t^i = \rho_I \, \varepsilon_{t-1}^i + \eta_t^I \tag{35}
$$

Monetary policy

$$
\varepsilon_{rt} = \rho_r \, \varepsilon_{rt-1} + \eta_t^r \tag{36}
$$

Price markup

$$
\varepsilon_t^p = \rho_p \, \varepsilon_{t-1}^p + \eta_t^p - \mu_p \, \eta_{t-1}^p \tag{37}
$$

Wage markup

$$
\varepsilon_t^w = \rho_w \, \varepsilon_{t-1}^w + \eta_t^w - \mu_w \, \eta_{t-1}^w \tag{38}
$$

A.4 Measurement equations

GDP growth

$$
dIGDP_t = \bar{\gamma} + y_t - y_{t-1} \tag{39}
$$

Consumption growth

$$
dICONS_t = \bar{\gamma} + c_t - c_{t-1} \tag{40}
$$

Investment growth

$$
dINV_t = \bar{\gamma} + i_t - i_{t-1}
$$
\n⁽⁴¹⁾

Wage growth

$$
dIWAG_t = \bar{\gamma} + w_t - w_{t-1} \tag{42}
$$

Inflation

$$
dlP_t = \bar{\pi} + \pi_t \tag{43}
$$

Nominal interest rate

$$
FEDFUNDS_t = r_t + \bar{r}
$$
\n⁽⁴⁴⁾

Hours worked

$$
IHOURS_t = l_t + \bar{l} \tag{45}
$$

where \bar{l} is the steady-state hours worked

				h	
GDP growth	NA	0.2731 0.2570 0.2375 0.2321 0.0760			
Consumption growth $\vert 0.2063 \quad 0.2571 \quad 0.1453 \quad 0.2297 \quad 0.2136 \quad -0.0635$					
Inflation	N A		0.8221 0.7557 0.6741 0.5976 0.5340		

Table 1: Correlations between actual data and predictions of the Survey of Professional Forecasters for quarters 1 to 6

Table 2: Results from posterior maximization (economic parameters). Reported are the mode results for the benchmark model without conditioning information (SW), the models with conditioning information on inflation (Infl), on inflation and GDP growth (Infl+GDP), on inflation and consumption growth (Infl+Cons), Table 2: Results from posterior maximization (economic parameters). Reported are the mode results for the benchmark model without conditioning information (SW), the models with conditioning information on inflation (Infl), on inflation and GDP growth (Infl+GDP), on inflation and consumption growth (Infl+Cons), on inflation, GDP growth and Consumption growth $(\mathrm{In\ddot{H}+GDP+Cons})$ on inflation, GDP growth and Consumption growth (Infl+GDP+Cons)

 $\frac{1}{2}$

Figure 1: Check plot for the anticipation parameter

	TFP	Risk	Gov	Invest		Mon. pol price mkp	wage mkp		
	1-step ahead, in percent								
GDP	24.86	14.31	38.35	5.71	11.16	2.93	2.67		
Inflation	6.63	2.05	0.39	3.61	4.95	48.82	33.54		
Fed Funds rate	9.00	13.71	2.79	1.20	66.21	4.57	2.53		
	8-step ahead, in percent								
GDP	32.76	3.69	8.42	10.63	5.37	10.24	28.89		
Inflation	7.13	2.58	0.91	8.68	8.68	21.83	50.19		
Fed Funds rate	13.04	12.17	3.20	23.61	21.22	6.62	20.14		
	∞ -step ahead, in percent								
GDP	35.47	1.15	3.76	6.48	1.72	13.89	37.52		
Inflation	7.21	2.46	1.42	10.09	8.60	21.33	48.89		
Fed Funds rate	12.15	9.41	4.86	31.54	16.39	6.33	19.32		

Table 4: Variance decompositions for the model with conditioning information on Inflation

Figure 2: Impulse responses to a risk premium shock (unanticipated (blue), anticipated (green))

Figure 3: Impulse responses to a technology shock (unanticipated (blue), anticipated (green))

Figure 4: Impulse responses to a government spending shock (unanticipated (blue), anticipated (green))

Figure 5: Impulse responses to a an investment specific technology shock (unanticipated (blue), anticipated (green))

Figure 6: Impulse responses to a monetary policy shock (unanticipated (blue), anticipated (green))

Figure 7: Impulse responses to a price markup shock (unanticipated (blue), anticipated (green))

Figure 8: Impulse responses to a wage markup shock (unanticipated (blue), anticipated (green))

Figure 9: Scaled revisions of expected shocks at horizon 2

Figure 10: Historical decomposition of inflation and GDP growth. Following Smets and Wouters (2007), demand shocks include (risk premium, investment technology and government spending shocks); Markup shocks include (wage markup and price markup shocks). The variable denoted by rest refers to initial conditions. 40

Figure 11: Forecasting performance relative to the benchmark. Ratios of RMSFE of the benchmark over the RMSFE of alternative models