Modeling Fiscal Matters

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October 2010

Kansas City Fed Forecasting Workshop

The Messages

- Key aspects in modeling fiscal policy:

  1. expectations
  2. long-lasting dynamics
  3. information (fiscal foresight)
  4. interactions with monetary policy
  5. nonlinearity
  6. uncertainty

Recent Macro Policies

- Monetary and fiscal policy responses to recession and financial crisis of 2007-2009 have been unusual aggressive
- United States, Japan, China, many European countries employed large “discretionary” fiscal stimulus packages
- Many central banks have driven interest rates to near zero and engaged in unconventional operations that have exploded their balance sheets
- This lecture pulls together those key features of fiscal policy to address potential consequences of these actions
- Draws on Leeper-Plante-Traum (2010), Leeper-Walker-Yang (2010), Davig-Leeper (2010), Bi (2009), Bi-Leeper (2010)

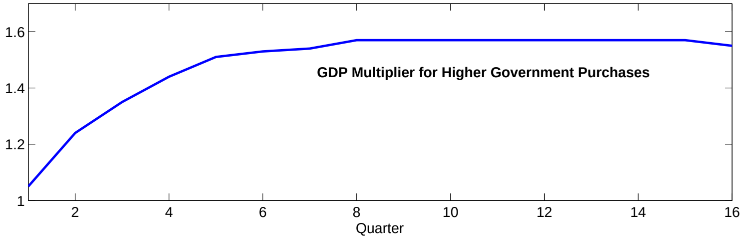
The Messages

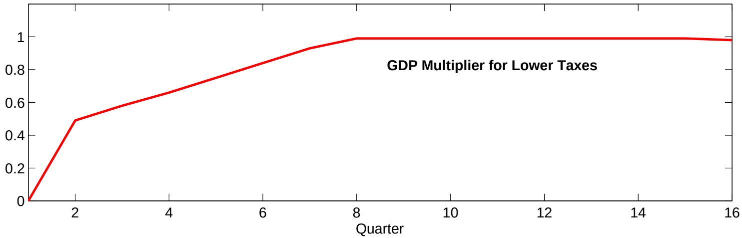
- Estimates of fiscal stimulus depend strongly on
  - how stimulus is implemented—tax cuts (which taxes); spending increases (which spending)
  - *how* and *when* the private sector expects the resulting debt expansion will be financed
  - whether the stimulus occurs gradually, so agents have fiscal foresight
  - how monetary policy behaves—whether it is active or passive
- Unfortunately, many of these considerations play little role in government projections of impacts of fiscal stimulus

The U.S. Example

- American Reinvestment and Recovery Act: $787 Billion (5 % GDP)
- Financed with new government debt issuance
- Rationale provided by paper by Romer-Bernstein reporting
  - multipliers for permanent 1% of GDP increase in G and decrease in T
  - forecasts of unemployment rate with and with stimulus
  - claim GDP will be 3.7% higher; 3.6 million new jobs

Romer-Bernstein Multipliers





Permanent Fiscal Shocks

Some Questions

- What economic models underlie the multipliers?
- Are the numbers reproducible?
- Why consider *permanent* changes when the Act makes transitory changes?
- What are the consequences of the stimulus for government debt?
- What are the repercussions of significantly higher debt?
- Will the debt run-up be sustained or retired?
- At what level will debt stabilize?
- How will policies adjust in the future to either sustain or retire debt?
- What assumptions about current and future monetary policy are embedded in the multipliers?

Some Answers from Obama Administration

Some Answers from Economic Research

- Four models of fiscal policy

1. Neoclassical growth model I (Leeper-Plante-Traum)
   - fiscal detail: 3 taxes rates, G consumption, transfers
   - estimated to U.S. data
2. Neoclassical growth model II (Leeper-Walker-Yang)
   - fiscal detail: 2 tax rates, G consumption, G investment, transfers
   - time-to-build in government infrastructure $\Rightarrow$ foresight
   - estimated to U.S. data
3. New Keynesian model (Davig-Leeper)
   - monetary & fiscal policy with regime switching in policies
   - calibrated to U.S. data
4. Model of sovereign debt default (Bi)
   - stochastic Laffer curve & fiscal limit
   - nonlinear risk premia

Some Answers from Economic Research

- There is also a ton of VAR evidence on multipliers
- Variety of identification schemes
  - restrictions on elasticities and timing (Blanchard-Perotti)
  - restrictions on signs of impulse responses (Mountford-Uhlig)
- Caldara & Kamps show fiscal VARs are generically unidentified: ultimately, identification achieved by *ad hoc* additional restrictions
- Joonyoung Kim is finding that two fresh kinds of restrictions have bite
  1. intertemporal government budget constraint
  2. combined with sources of fiscal financing
- The presumed death of VARs may be premature

Neoclassical Growth Model I

- Conventional except for specification of policy behavior
  - tax rules

$$
\begin{aligned}
\hat{\tau}_t^k &= \varphi_k \hat{Y}_t + \gamma_k \hat{B}_{t-1} + \phi_{kl} u_t^l + \phi_{kc} u_t^c + u_t^k \\
\hat{\tau}_t^l &= \varphi_l \hat{Y}_t + \gamma_l \hat{B}_{t-1} + \phi_{lk} u_t^k + \phi_{lc} u_t^c + u_t^l \\
\hat{\tau}_t^c &= \phi_{kc} u_t^k + \phi_{lc} u_t^l + u_t^c
\end{aligned}
$$

  - spending rules

$$
\begin{aligned}
\hat{G}_t &= -\varphi_g \hat{Y}_t - \gamma_g \hat{B}_{t-1} + u_t^g \\
\hat{Z}_t &= -\varphi_Z \hat{Y}_t - \gamma_Z \hat{B}_{t-1} + u_t^z
\end{aligned}
$$

hats are log-deviations, u's are AR(1) with innovations $N(0,1)$

Growth Model I: Results

- Data like to have many instruments adjust to stabilize debt
- Multipliers tend not to be very large
  - caveat: with certain monetary policies, multipliers can be *much* larger
- Short-run and long-run multipliers can be very different
- Source of financing can matter a lot, especially at longer horizons
- Both speed at which debt stabilized and size of automatic stabilizers—φ's—matter for fiscal impacts
- Takes many years to establish present-value budget balance—20 or more

Fiscal Multipliers

- A common measure [Blanchard-Perotti (2002), Romer-Bernstein (2009)]

$$\text{Impact Multiplier}(k) = \frac{\Delta Y_{t+k}}{\Delta G_t}$$

- Sweeps dynamics of fiscal variables under the rug
- Present value multiplier [Mountford and Uhlig]

$$\text{Present Value Multiplier}(k) = \frac{E_t \sum_{j=0}^{k} \prod_{i=0}^{j} (1 + r_{t+i})^{-j} \, \Delta Y_{t+k}}{E_t \sum_{j=0}^{k} \prod_{i=0}^{j} (1 + r_{t+i})^{-j} \, \Delta G_{t+k}}$$

Growth Model I: Multipliers

Capital Tax Present-Value Multipliers

Variable	1 quarter	10 quarters	∞
$\frac{\text{PV}(\Delta Y)}{\text{PV}(\Delta T^k)}$	-0.18	-0.33	-0.72
$\frac{\text{PV}(\Delta C)}{\text{PV}(\Delta T^k)}$	-0.076	-0.11	-0.47

Labor Tax Present-Value Multipliers

Variable	1 quarter	10 quarters	∞
$\frac{\text{PV}(\Delta Y)}{\text{PV}(\Delta T^l)}$	-0.19	-0.19	-0.21
$\frac{\text{PV}(\Delta C)}{\text{PV}(\Delta T^l)}$	-0.17	-0.29	-0.37

All fiscal instruments respond to debt

Growth Model I: Multipliers

Capital Tax Present-Value Multipliers

Variable	1 quarter	10 quarters	∞
$\frac{\text{PV}(\Delta Y)}{\text{PV}(\Delta T^k)}$	-0.18	-0.33	-0.72
	-0.14	-0.18	-3.70
$\frac{\text{PV}(\Delta C)}{\text{PV}(\Delta T^k)}$	-0.076	-0.11	-0.47
	-0.10	-0.18	-0.83

Labor Tax Present-Value Multipliers

Variable	1 quarter	10 quarters	∞
$\frac{\text{PV}(\Delta Y)}{\text{PV}(\Delta T^l)}$	-0.19	-0.19	-0.21
	-0.14	-0.04	0.92
$\frac{\text{PV}(\Delta C)}{\text{PV}(\Delta T^l)}$	-0.17	-0.29	-0.37
	-0.19	-0.34	0.06

Only capital and labor taxes respond to debt (red)

Growth Model I: Multipliers

Government Spending Present-Value Multipliers

Variable	1 quarter	10 quarters	∞
$\frac{\text{PV}(\Delta Y)}{\text{PV}(\Delta G)}$	0.64	0.33	0.03
$\frac{\text{PV}(\Delta C)}{\text{PV}(\Delta G)}$	-0.26	-0.35	-0.60

Transfers Present-Value Multipliers

Variable	1 quarter	10 quarters	∞
$\frac{\text{PV}(\Delta Y)}{\text{PV}(\Delta Z)}$	-0.02	-0.28	-0.59
$\frac{\text{PV}(\Delta C)}{\text{PV}(\Delta Z)}$	0.01	0.13	0.12

All fiscal instruments respond to debt

Growth Model I: Multipliers

Government Spending Present-Value Multipliers

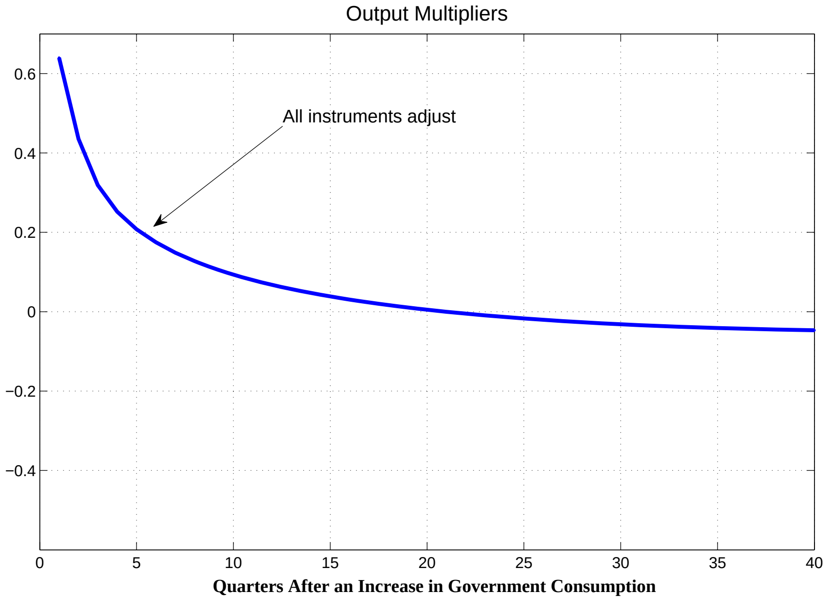
Variable	1 quarter	10 quarters	∞
$\frac{PV(\Delta Y)}{PV(\Delta G)}$	0.64	0.33	0.03
	0.59	0.14	−0.99
$\frac{PV(\Delta C)}{PV(\Delta G)}$	−0.26	−0.35	−0.60
	−0.24	−0.27	−0.89

Transfers Present-Value Multipliers

Variable	1 quarter	10 quarters	∞
$\frac{PV(\Delta Y)}{PV(\Delta Z)}$	−0.02	−0.28	−0.59
	−0.07	−0.33	−1.40
$\frac{PV(\Delta C)}{PV(\Delta Z)}$	0.01	0.13	0.12
	0.04	0.14	−0.38

Only capital and labor taxes respond to debt (red)

Government Spending Multipliers



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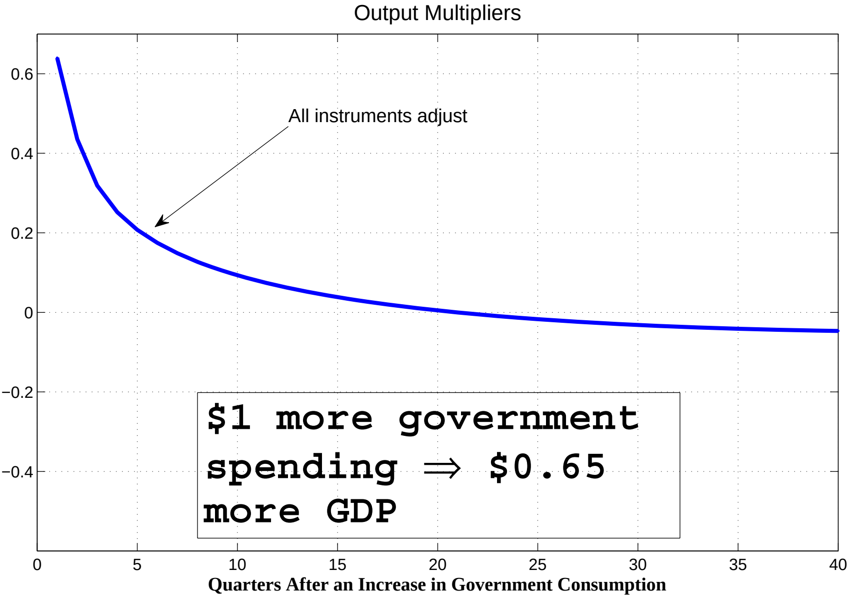

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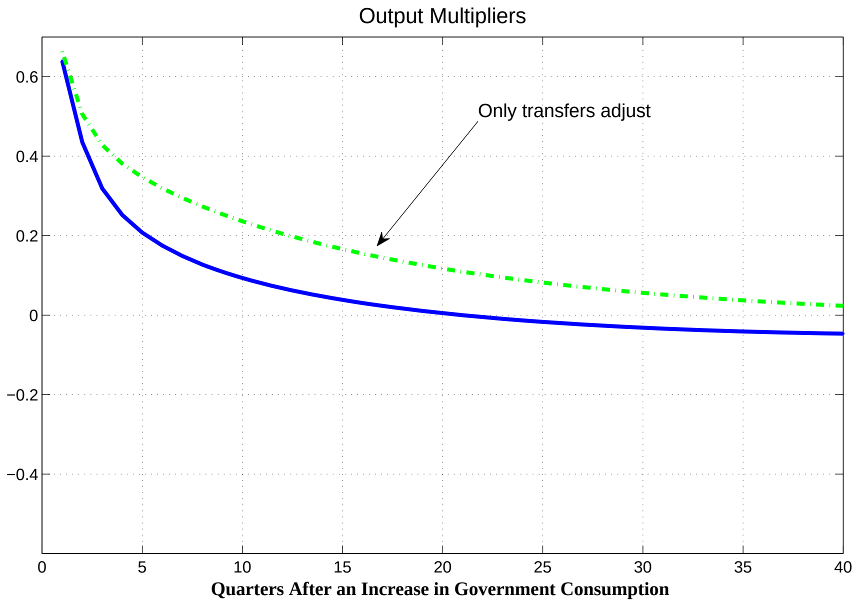

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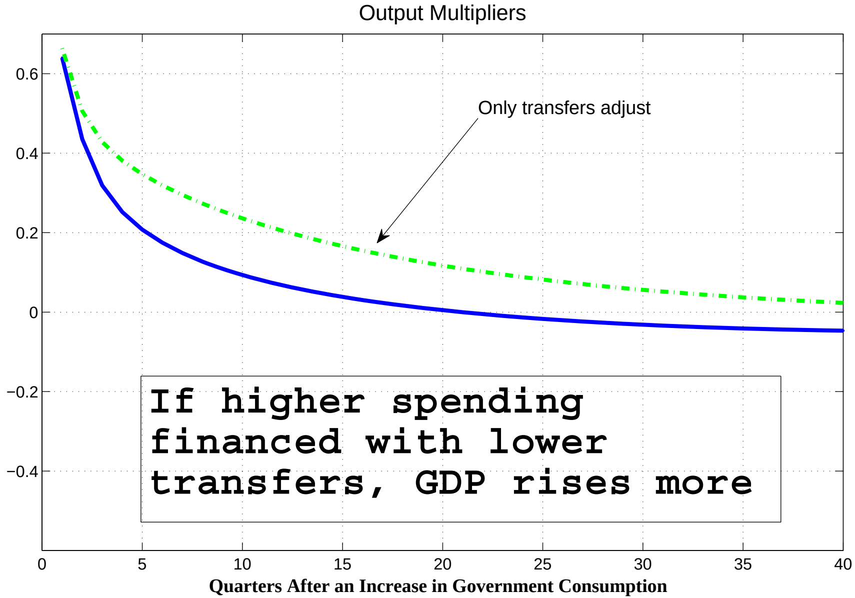

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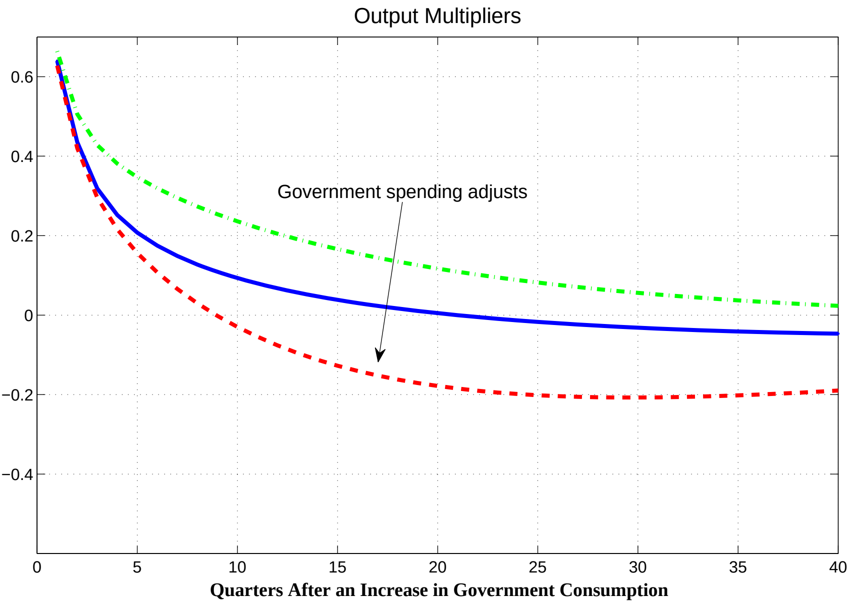

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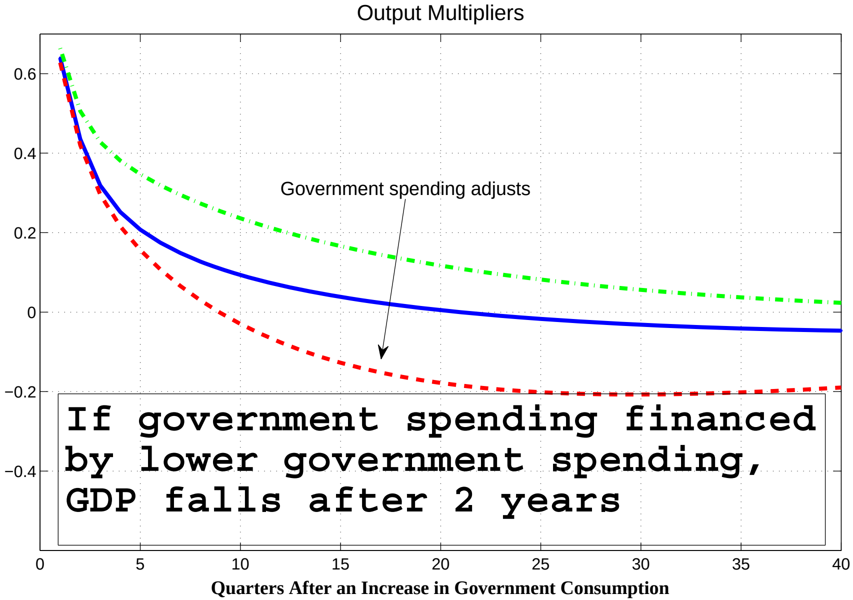

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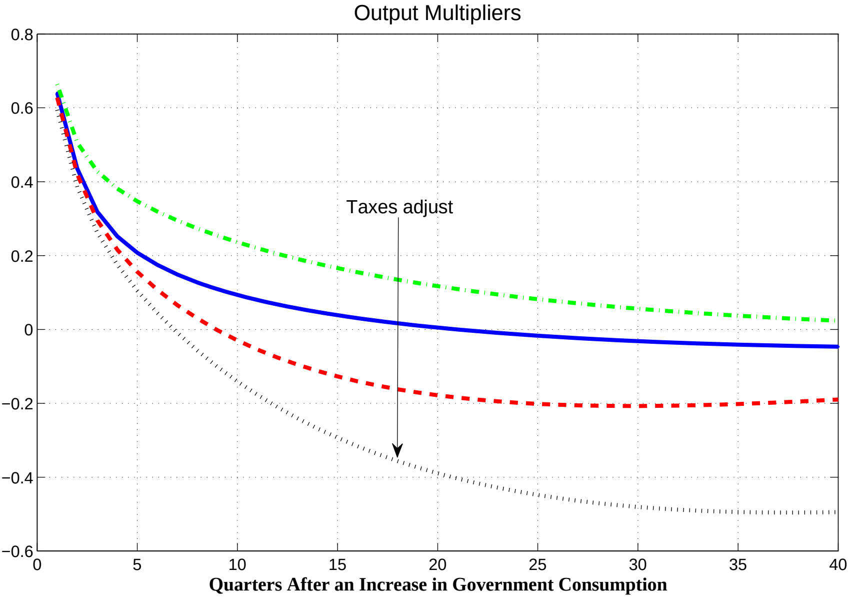

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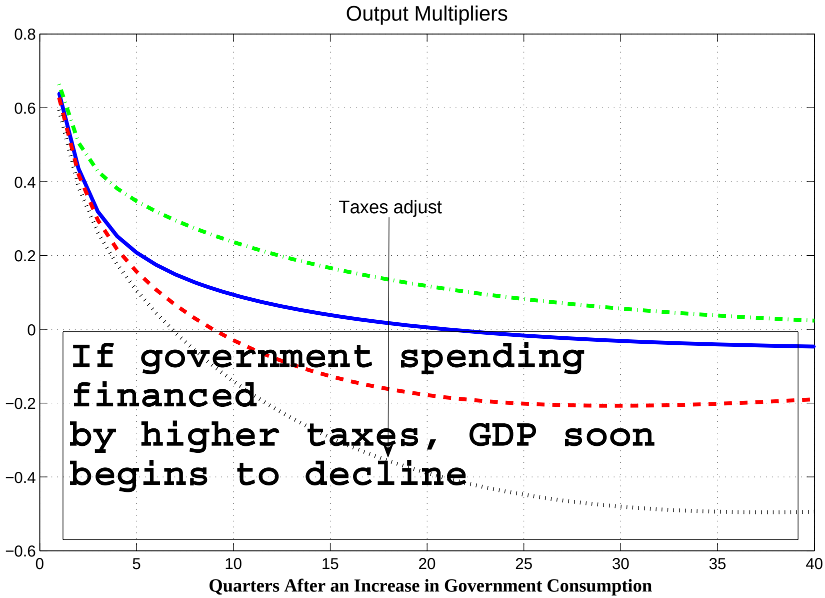


Speed of Fiscal Adjustment

- Obama administration has pledged to cut deficit in half within 4 years
- Echoing Europe, where cuts are actually occurring
- Done in response to outcries about fiscal "unsustainability"
- Use estimated model to answer: What are the implications for effectiveness of fiscal stimulus of slowing down or speeding up fiscal adjustments?
  - slowing down pushes adjustments into future
  - rational agents discount those more heavily
  - speeding up brings them forward
- Changes in the timing of fiscal adjustments can alter the government spending multipliers in important ways

Speed of Adjustment of Fiscal Instruments

- Modify fiscal rules to vary responsiveness to debt
  - tax rules

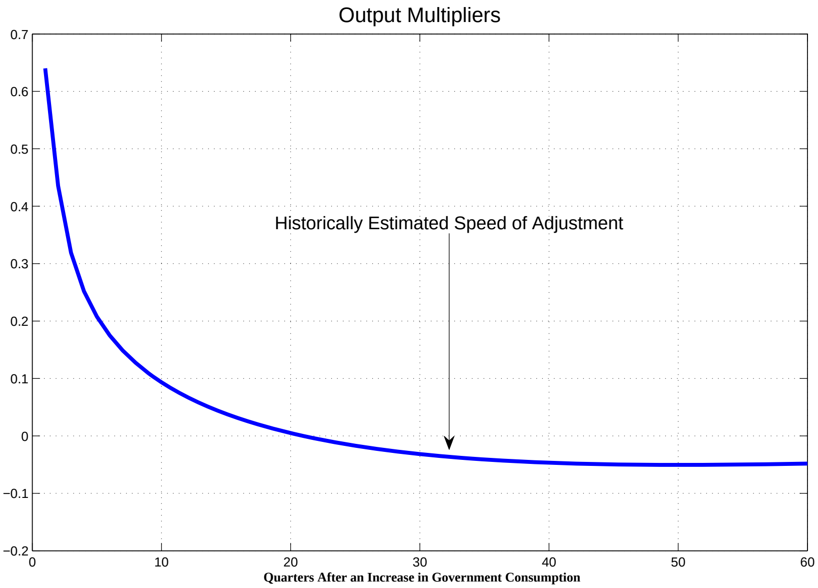
$$\begin{aligned}\hat{\tau}_t^k &= \varphi_k \hat{Y}_t + \mu\gamma_k \hat{B}_{t-1} + \phi_{kl} u_t^l + \phi_{kc} u_t^c + u_t^k \\ \hat{\tau}_t^l &= \varphi_l \hat{Y}_t + \mu\gamma_l \hat{B}_{t-1} + \phi_{lk} u_t^k + \phi_{lc} u_t^c + u_t^l \\ \hat{\tau}_t^c &= \phi_{kc} u_t^k + \phi_{lc} u_t^l + u_t^c\end{aligned}$$

  - spending rules

$$\begin{aligned}\hat{G}_t &= -\varphi_g \hat{Y}_t - \mu\gamma_g \hat{B}_{t-1} + u_t^g \\ \hat{Z}_t &= -\varphi_Z \hat{Y}_t - \mu\gamma_Z \hat{B}_{t-1} + u_t^z\end{aligned}$$

vary μ to speed up or slow down adjustment

Government Spending Multipliers



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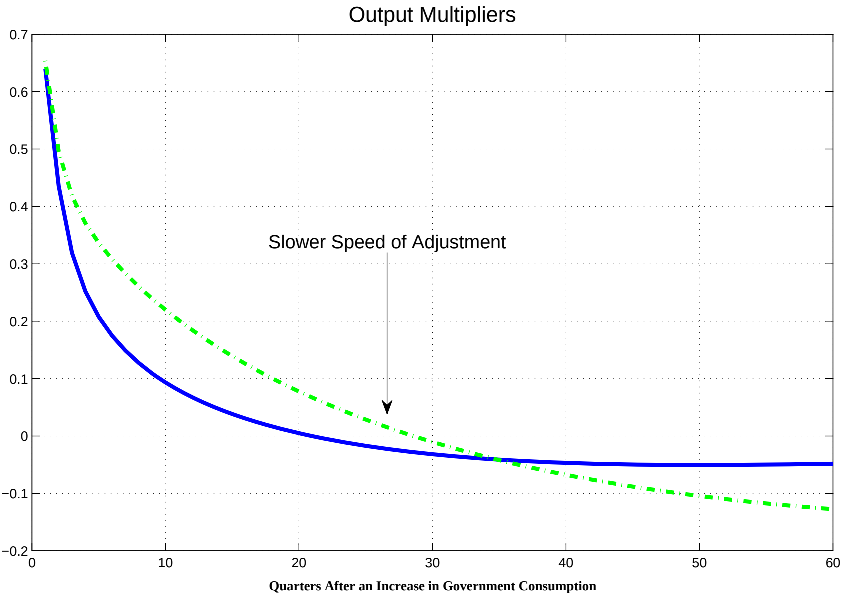

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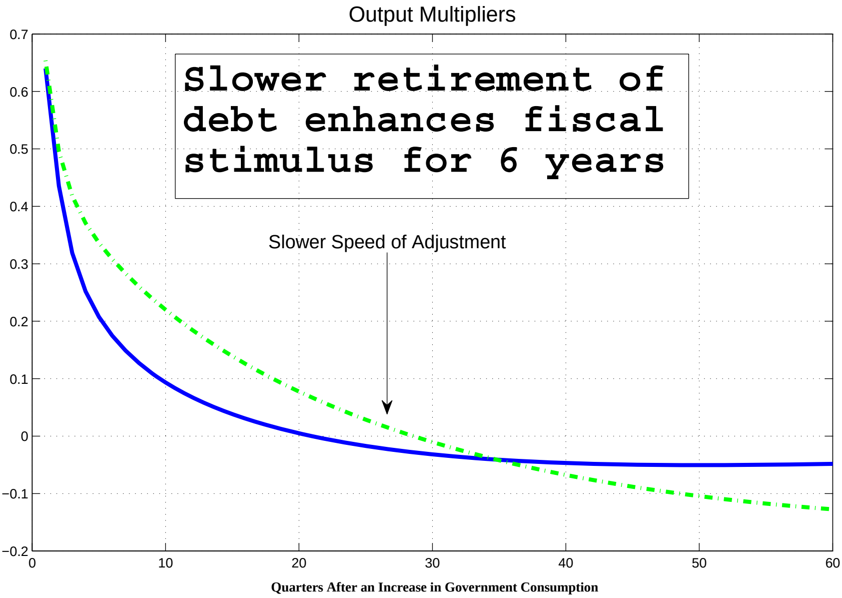

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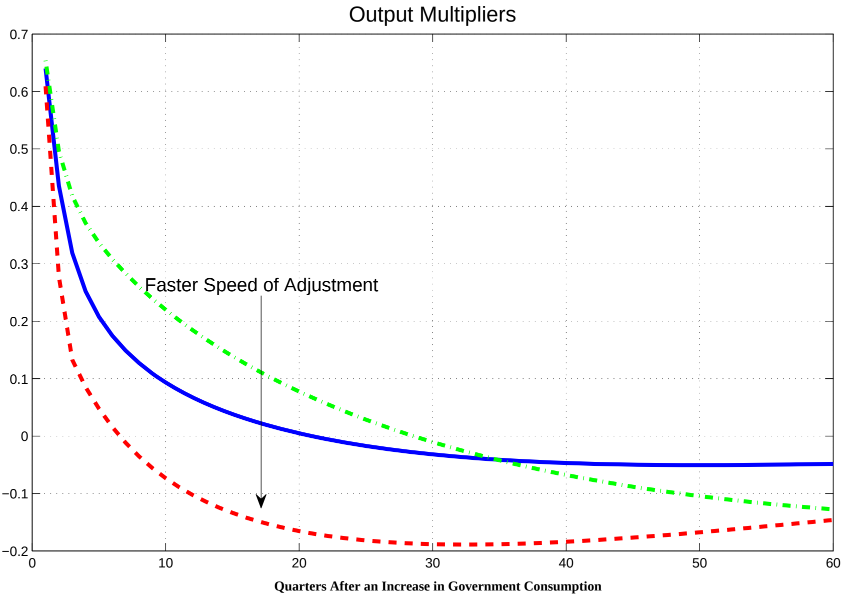

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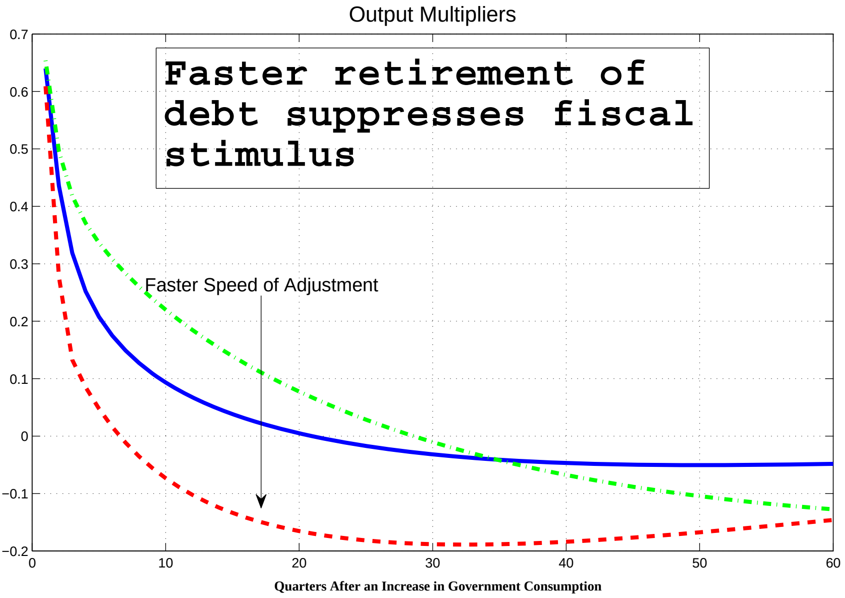
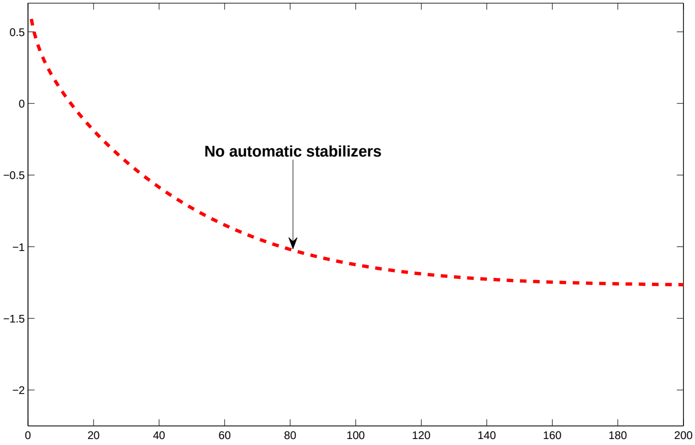
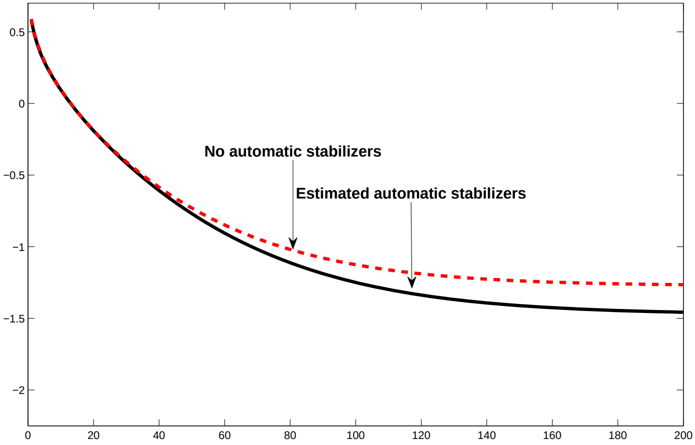


Strength of Automatic Stabilizers



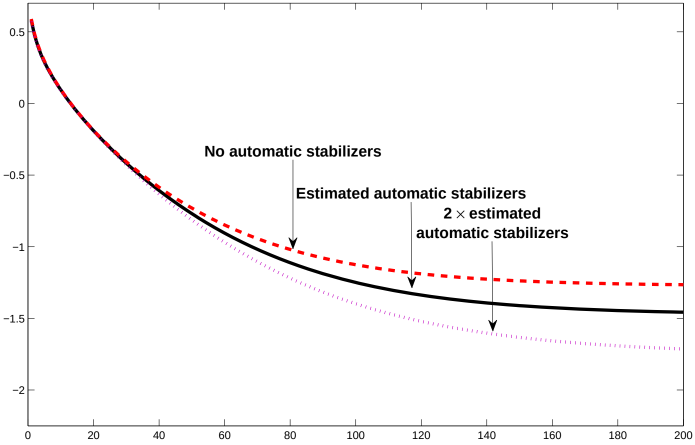
Present-value G multipliers for output: varying φ's

Strength of Automatic Stabilizers



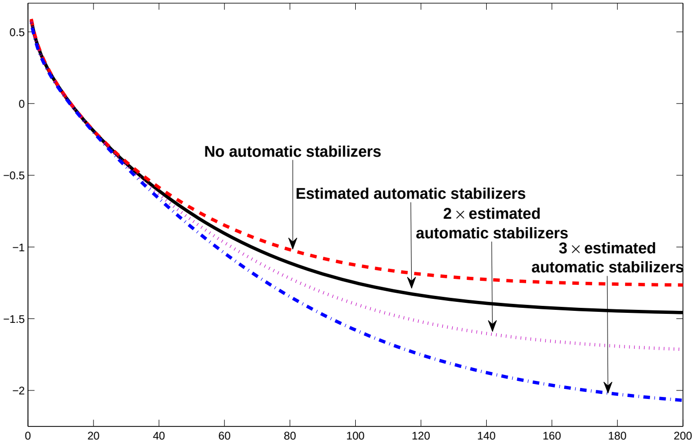
Present-value G multipliers for output: varying φ's

Strength of Automatic Stabilizers



Present-value G multipliers for output: varying φ's

Strength of Automatic Stabilizers



Present-value G multipliers for output: varying φ's

Neoclassical Growth Model II

- In U.S. and Europe, heavy emphasis on government infrastructure spending
- Similar in structure to previous model; two important extensions
  - introduction of productive government investment G^I
  - introduction of time-to-build in government capital
- Distinguish between "budget authority" and "outlays"
  - "authority" occurs first, giving total spending and planned path of "outlays"
  - implementation delays modeled with time-to-build

Implementation Delays: Example

Estimated costs for highway construction in Title XII of the American Recovery and Reinvestment Act of 2009

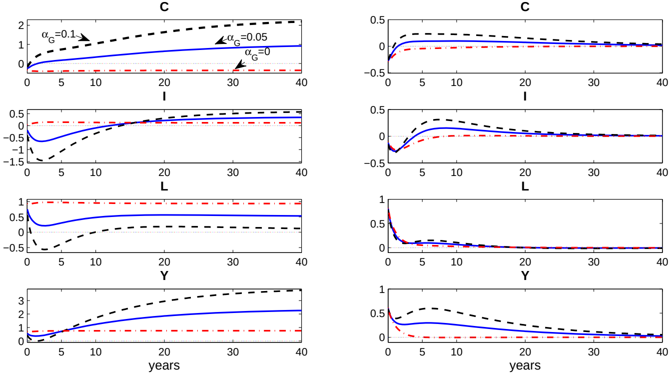
	2009	2010	2011	2012	2013	2014	2015	2016	Tota
Budget Authority	27.5	0	0	0	0	0	0	0	27.5
Estimated Outlay	2.75	6.875	5.5	4.125	3.025	2.75	1.925	.55	27.5

Billions of dollars. Source: Congressional Budget Office

Modeling Government Investment

- Aggregate production
$$Y_t = A\left(u_t K_{t-1}\right)^{\alpha_K}\left(L_t\right)^{\alpha_L}\left(K_{t-1}^G\right)^{\alpha_G}$$
  - α_G critical ($\alpha_G = 0 \Rightarrow$ unproductive)
- A_t^I: budget authorization; N quarters to complete project
- Law of motion for public capital
$$K_t^G = \left(1-\delta_G\right)K_{t-1}^G + A_{t-N+1}^I$$
  - budget authorization process an AR(1)
- Government investment implemented at t (outlaid)
$$G_t^I = \sum_{n=0}^{N-1}\phi_n A_{t-n}^I,$$
  - $\sum_{n=0}^{N-1}\phi_n = 1$; ϕ's are outlay rates

Role of Government Productivity

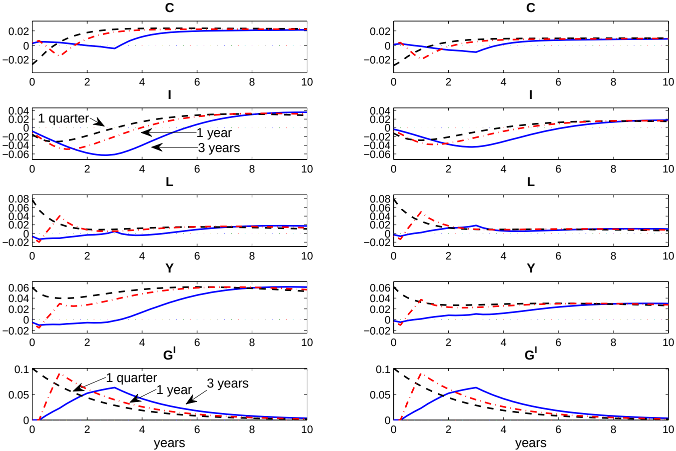


Permanent shock

Temporary shock

No implementation delays and lump-sum financing

Implementation Delays and Foresight



$\alpha_G = 0.1$ $\alpha_G = 0.05$

With implementation delays

New Keynesian Model

- Two key distortions that given monetary policy real effects:
  - monopolistic competition
  - sluggish price adjustment
- Elastic labor supply; inelastic capital
- Transmission mechanism of MP: real interest rates
- Transmission mechanism of FP: real interest rates & wealth effects
- Integrate monetary and fiscal policy
  - interest rate rule for MP
  - exogenous process for government spending
  - lump-sum taxes

New Keynesian Model

- Estimate switching rules for monetary & tax policy
- Embed rules in calibrated model
- Four possible policy regimes:
  1. Active MP/Passive FP
  2. Passive MP/Active FP
  3. Passive MP/Passive FP
  4. Active MP/Active FP
- With fixed regime: Passive/Passive $\Rightarrow$ indeterminacy
- With fixed regime: Active/Active $\Rightarrow$ non-existence
- Can study consequences of periodically visiting those forbidden regimes
- Focus on effects of *unproductive* G

U.S. Policy Responses to Recession

- Unusually aggressive **joint** policy response
  - federal funds rate near zero bound since Dec ’08
  - Fed’s balance sheet has more than doubled: $800 billion to $2.5 *trillion*
  - $125 billion tax refund in ’08 and $787 billion stimulus package in ’09
  - deficit is 13% of GDP now; debt will rise from 40% to 80% of GDP over the decade; may reach 277% by 2040
- Objective of stimulus is to create jobs by increasing consumption demand, labor demand, employment

The Modeling Effort

- Model two aspects of the policy response
  1. **joint** monetary and fiscal policy effort
  2. current aggressive policies not likely to continue indefinitely
- Use standard new Keynesian model with monetary and fiscal policy regime change
- Bottom-line: government spending multipliers can be large or small, depending on policy regime
- Simulate effects of American Recovery and Reinvestment Act under alternative policy assumptions

Government Spending: Crowd Out or In?

- Policy
  - Romer-Bernstein: output multiplier ≈ 1.5 and very persistent
  - CBO: stimulus makes recession less severe and shorter lived
- Research
  - no professional consensus that higher G raises private C
  - RBC or standard new Keynesian models $\Rightarrow G$ crowds out C
  - empirical evidence mixed, but favors crowding in

Policy Regimes

- Since the late 1940s, U.S. monetary & fiscal policies have fluctuated among:
  - Active MP $\Rightarrow$ Taylor principle holds
  - Passive MP $\Rightarrow$ Taylor principle not satisfied
  - Passive FP $\Rightarrow$ PV of taxes = PV of G
  - Active FP $\Rightarrow$ PV of taxes $<$ PV of G
- Current policy: passive MP & active FP

Why Policy Regime Matters

- Following an increase in G...
  1. Passive MP allows the real interest rate to fall in response to higher expected inflation
  2. Active FP diminishes the negative wealth effect induced by higher taxes
- Both of these increase the stimulative effect of government spending
- These do not happen under the usual active MP/passive FP regime
- A natural & relevant way to get large G multipliers

Monetary Policy Rule Estimates

- The monetary policy rule is

$$r_t = \alpha_0(S_t^M) + \alpha_\pi(S_t^M)\pi_t + \alpha_y(S_t^M)y_t + \sigma_r(S_t^M)\varepsilon_t^r$$

- S_t^M follows a four-state Markov chain
  - reaction coefficients and shock volatility switch independently
- Monetary policy breaks into regimes with
  - A strong response to inflation (active): $\alpha_\pi = 1.29$
  - A weak response to inflation (passive): $\alpha_\pi = .53$

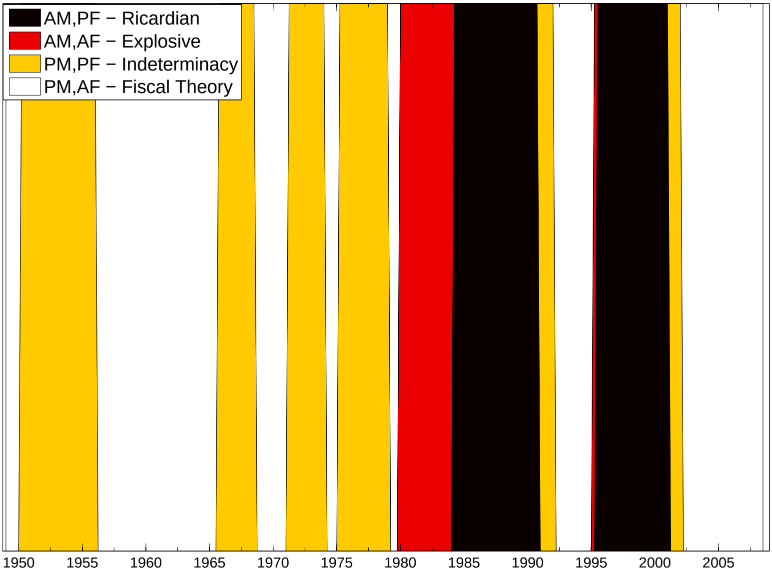
Fiscal Policy Rule Estimates

- The fiscal policy rule is

$$\tau_t = \gamma_0(S_t^F) + \gamma_b(S_t^F)b_{t-1} + \gamma_y(S_t^F)y_t + \gamma_g(S_t^F)G_t + \sigma_\tau(S_t^F)\varepsilon_t^\tau$$

- S_t^F follows a two-state Markov chain
- Fiscal policy breaks into regimes with
  - Taxes rise in response to debt (passive): $\gamma_b = .07$
  - Taxes fall in response to debt (active): $\gamma_b = -.025$

U.S. Monetary and Fiscal Regimes



Model Setup

- We use a basic New Keynesian model with variable government purchases
  - fixed capital; elastic labor supply; Calvo price rigidities
- Unproductive government spending financed via:
  - lump-sum taxes; one-period nominal bonds; seigniorage revenues
- Government purchases follow AR(1) (for now...)
- Government demands goods in same proportion as private sector

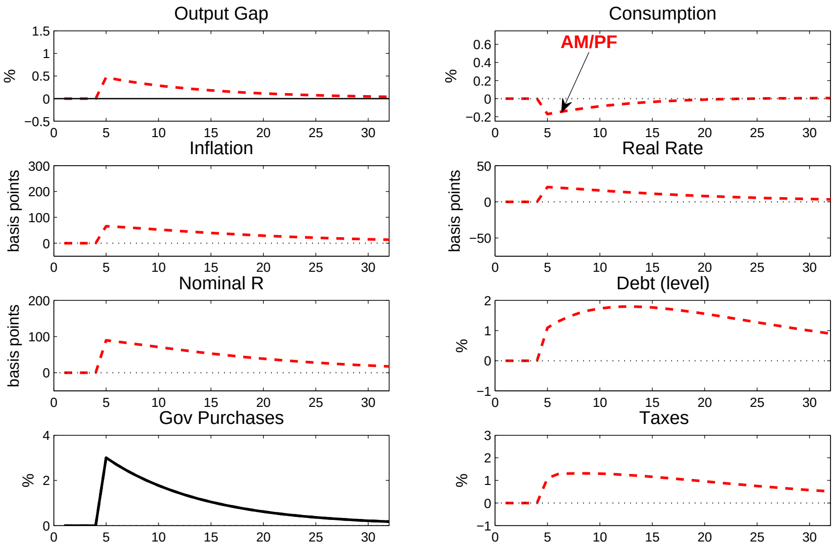
Perspective on Transmission of G

- The ubiquitous **Intertemporal Equilibrium Condition** holds in all regimes

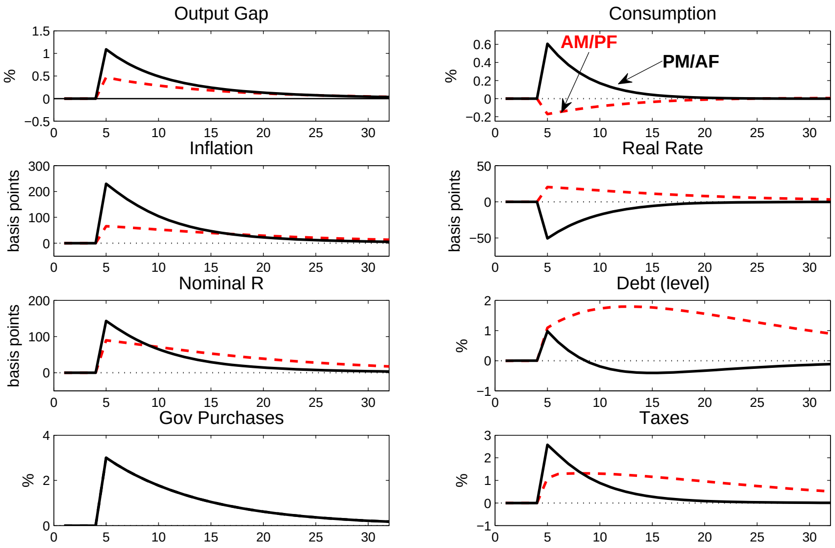
$$\frac{M_{t-1} + (1 + r_{t-1})B_{t-1}}{P_t} = E_t \sum_{T=t}^{\infty} \left[q_{t,T} \left(\tau_T - G_T + \frac{r_T}{1 + r_T} \frac{M_T}{P_T} \right) \right]$$

- A government liabilities valuation equation
- Higher path for G *without an equivalent higher path for* τ lowers the present value of primary surpluses
  - creates an imbalance—at initial prices—between the value of debt and its expected backing
- Equilibrium restored via a higher path of P, which is consistent with firms raising prices

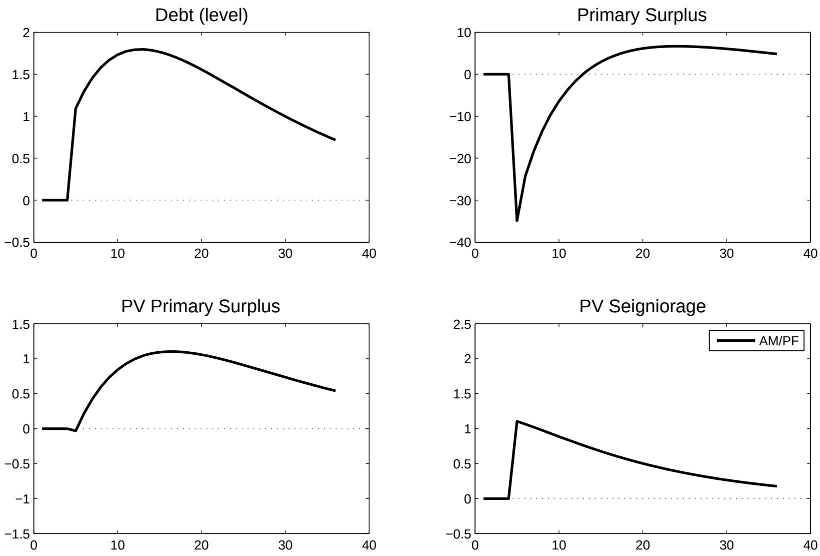
Higher G: Active MP / Passive FP



Higher G: Passive MP / Active FP



Intertemporal Adjustments



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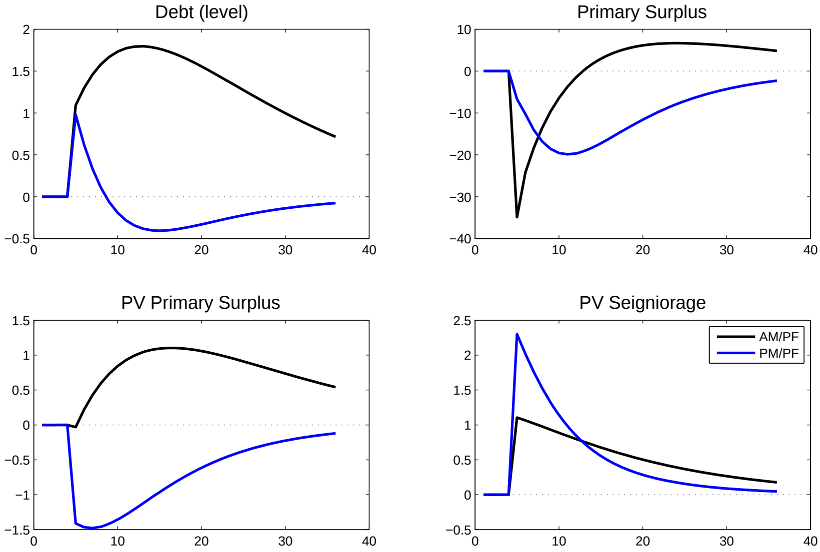
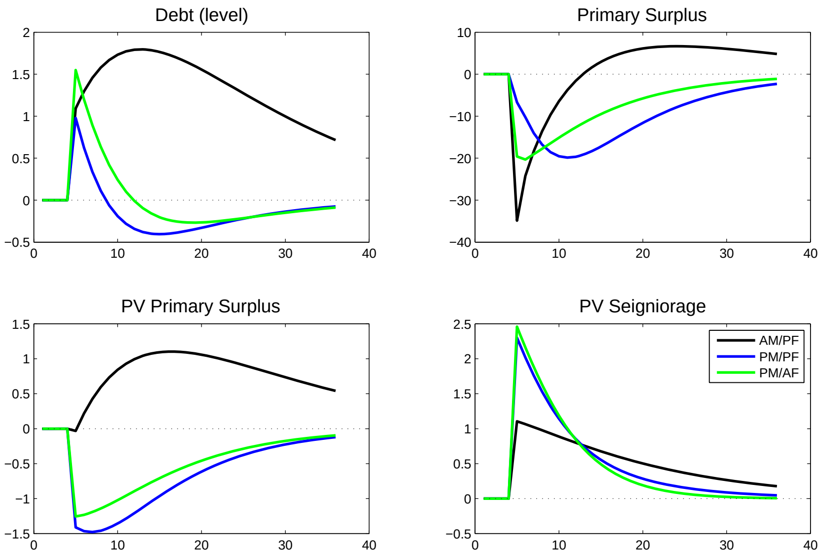


Intertemporal Adjustments



Present Value Multipliers

	$\frac{PV(\Delta Y)}{PV(\Delta G)}$ after			
Regime	5 quarters	10 quarters	25 quarters	∞
AM/PF	0.79	0.80	0.84	0.86
PM/PF	1.64	1.51	1.39	1.37
PM/AF	1.72	1.58	1.40	1.36

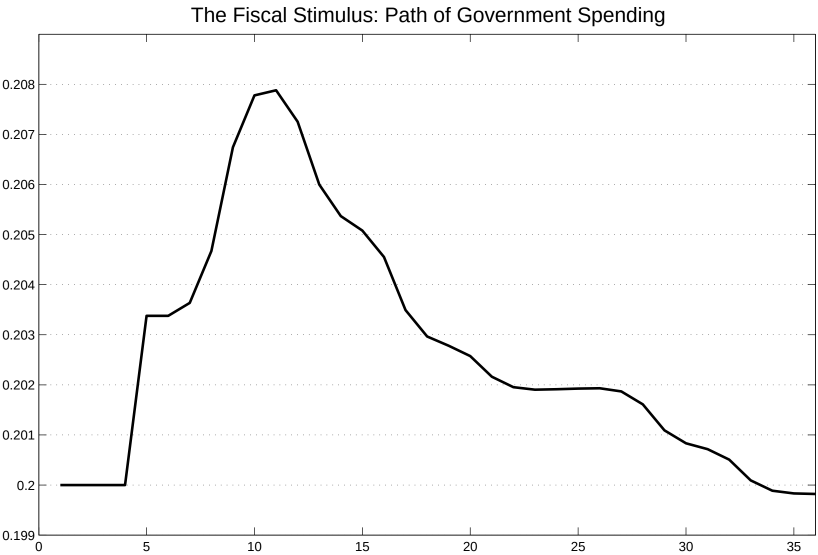
Table: Note: $\frac{PV(\Delta C)}{PV(\Delta G)} = \frac{PV(\Delta Y)}{PV(\Delta G)} - 1$

- Values greater than unity imply a positive consumption response to increases in G

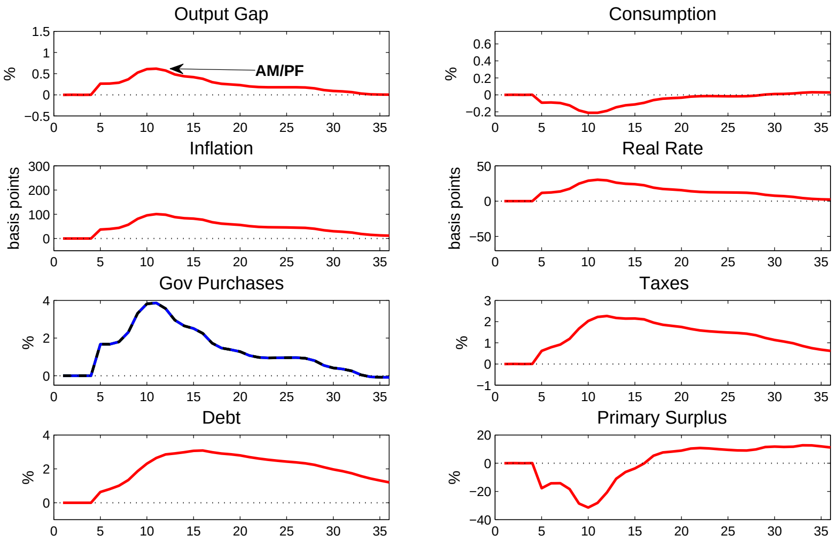
Simulating Stimulus: The 2009 ARRA

- The 2009 ARRA includes around $350 billion in spending on infrastructure, energy, healthcare, etc.
- $144 billion in federal transfers to state and local governments
  - Following Romer and Bernstein assume 60 percent is devoted to new spending
- We use the same path for additional G as Cogan, Cwik, Taylor, Wieland
- Simulate under different monetary-fiscal combinations

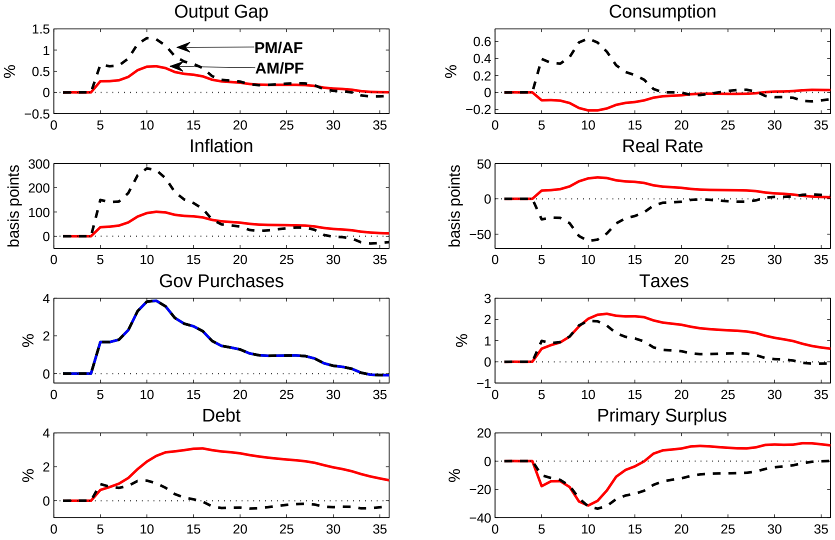
The ARRA's Path for G



2009 ARRA: AM/PF



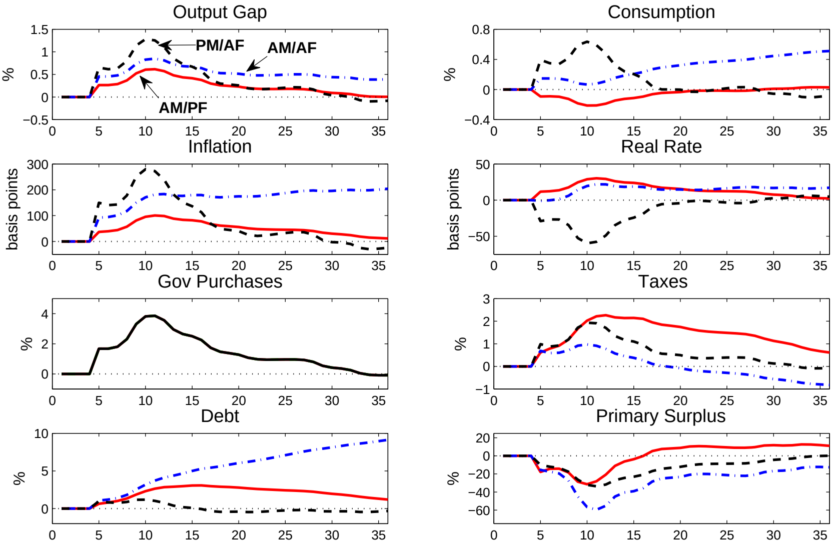
2009 ARRA: AM/PF & PM/AF



A Risky Game of Chicken

- What if, as inflation begins to rise, the Fed switches to an active stance (from PM/AF)?
- This is a very real possibility when there is no coordination between MP & FP
- Then there are two unstable relationships:
  - inflation due to the active MP
  - debt due to the active FP
- In a fixed AM/AF regime, there would be no equilibrium
- With switching, so long as you are sufficiently far from the “fiscal limit,” there is a build up of debt
- And persistently higher inflation because MP has lost control of inflation

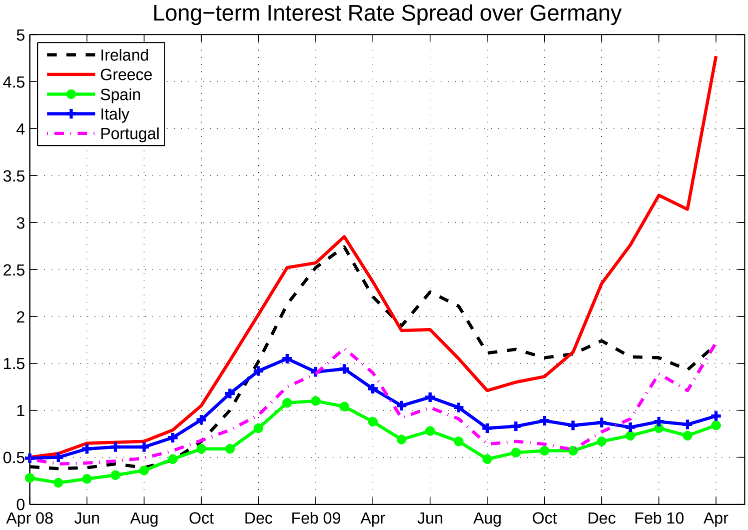
The 2009 ARRA: Active/Active



Nonlinearity & Fiscal Policy

- Fiscal limits are country specific:
  - depend on government size, degree of countercyclical fiscal policy, political risk, and shock processes
- Risk premia are nonlinear in level of government debt
- Long-term bonds can provide early warning
- Fiscal reforms can significantly shift distribution of fiscal limits

Recent Sovereign Risk Premia



A Model

Exogenous technology and government spending:

$$\ln \frac{A_t}{A} = \rho^u \ln \frac{A_{t-1}}{A} + \varepsilon_t^A \qquad \varepsilon_t^A \sim \mathcal{N}(0, \sigma_A^2)$$
$$\ln \frac{g_t}{g} = \rho^e \ln \frac{g_{t-1}}{g} + \varepsilon_t^g \qquad \varepsilon_t^g \sim \mathcal{N}(0, \sigma_g^2)$$

Household problem:

$$\max \qquad E_0 \sum_{t=0}^{\infty} \beta^t u\left(c_t, L_t\right)$$

$$s.t. \qquad A_t(1-\tau_t)(1-L_t) + z_t - c_t = b_t q_t - \underbrace{(1-\Delta_t)b_{t-1}}_{b_t^d}$$

FOC:

$$\frac{u_L(t)}{u_c(t)} = A_t\left(1-\tau_t\right)$$
$$q_t = \beta E_t\left[(1-\Delta_{t+1})\frac{u_c(t+1)}{u_c(t)}\right]$$

A Model

Government budget:

$$\tau_t A_t (1 - L_t) + b_t q_t = g_t + z_t + \underbrace{(1 - \Delta_t) b_{t-1}}_{b_t^d}$$

- Unenforceable bond contract:

$$\Delta_t = \begin{cases} 0 & \text{if } b_{t-1} < b_t^* \text{ with } b_t^* \sim \mathcal{N}(b^*, \sigma_b^2) \\ \delta & \text{if } b_{t-1} \geq b_t^* \end{cases}$$

- Debt-stabilizing tax rule:

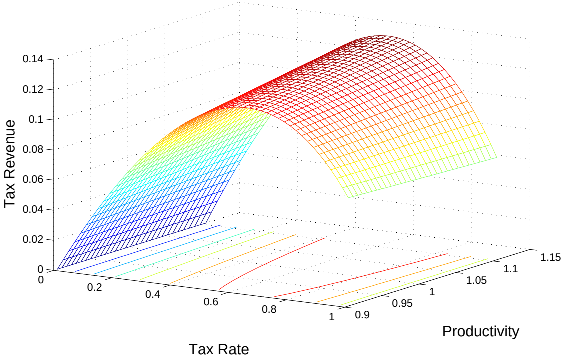
$$\tau_t - \tau = \gamma \left(b_t^d - b \right)$$

- Countercyclical lump-sum transfers:

$$\ln \frac{z_t}{z} = -\zeta^z \ln \frac{A_t}{A}$$

Dynamic Laffer Curve

$$T_t = \tau_t A_t (1 - L_t)$$
$$=> T^{max}(A, g) = \mathcal{T}(\tau^{max}(A, g); A, g)$$



Fiscal Limit

Fiscal limit: maximum sustainable level of government debt

$$\mathcal{B}^* = E_0 \sum_{t=0}^{\infty} \underbrace{\frac{u_c^{max}(t)}{u_c^{max}(0)}}_{\text{discount rate}} \underbrace{\theta_t}_{\text{political risk}} \underbrace{\left(T_t^{max} - g_t - z_t\right)}_{\text{future max fiscal surplus}}$$

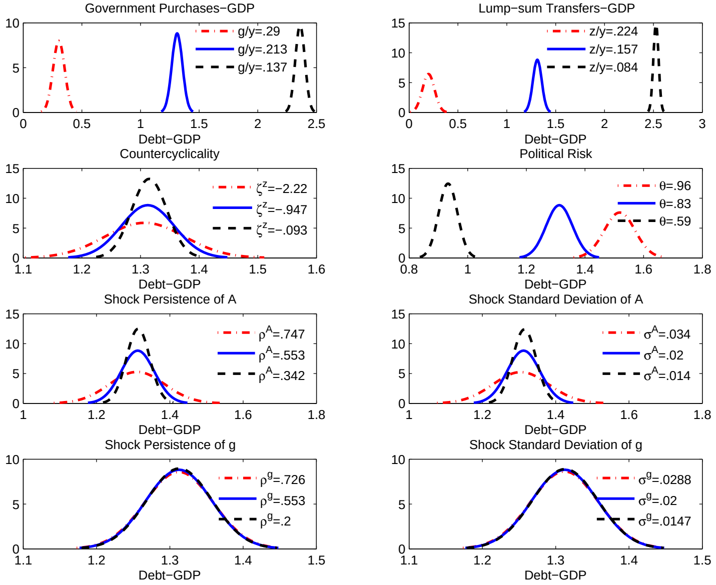
The distribution depends on:

- Government size: g/y and z/y
- Countercyclical lump-sum transfers: ζ^z
- Political risk: $0 < \theta_t \leq 1$ (ICRG index)
  Standard & Poor's (2008): "*stability*, *predictability*, and *transparency* of a country's political institutions are important considerations..."
- Shock processes

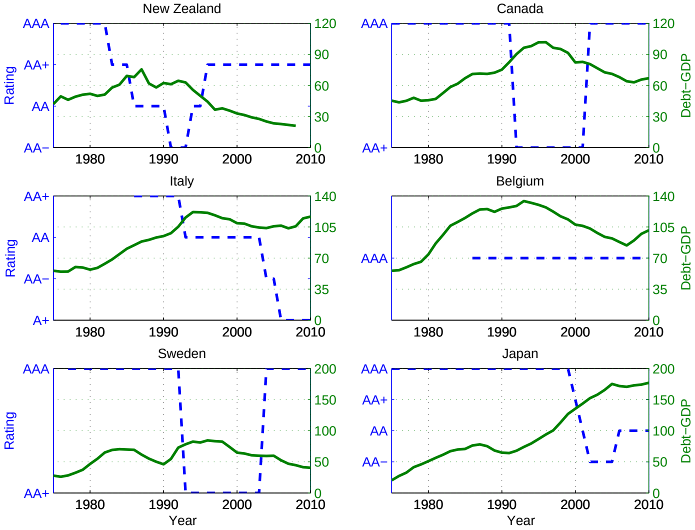
MCMC simulation:

- Simulate N paths to approximate $\mathcal{N}(b^*, \sigma_b^2)$.

Fiscal limit: Simulation



Fiscal limit: Data



Nonlinear solution

Monotone mapping method (Coleman (1991), Davig (2004)):

$$q_t = \beta E_t\Big((1-\Delta_{t+1})\frac{u_c(t+1)}{u_c(t)}\Big) \quad (1)$$

$$\frac{b_t^d + g_t + z(\psi_t) - \tau(\psi_t)A_t\Big(1 - L(\psi_t)\Big)}{f^b(\psi_t)}$$
$$= \beta E_t\left\{\Big(1 - \Delta(f^b(\psi_t), b_{t+1}^*)\Big)\frac{u_c(f^b(\psi_t), A_{t+1}, g_{t+1}, b_{t+1}^*)}{u_c(\psi_t)}\right\} \quad (2)$$

- Grid points of 3-dimension state space, $\psi_t = (b_t^d, g_t, A_t)$, using Tauchen (1991)
- Initial guess of the decision rule $f_0^b(.)$ ($b_t = f_0^b(\psi_t)$)
- Update the decision rule $f_i^b(.)$ by iterating over equation (2) until it converges $(\epsilon = 1e-8)$

  Numerical integration: Newton-Cotes formulas.

Calibration

- Default scheme: A higher uncertainty of fiscal limits implies higher δ

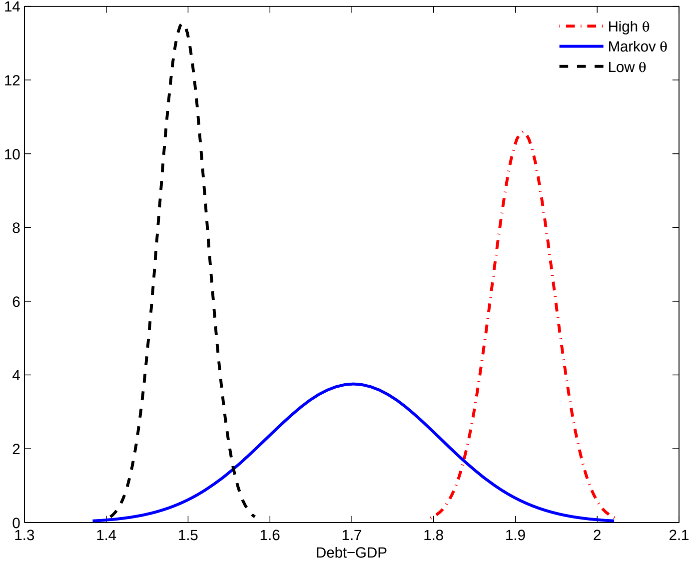
$$\Delta_t = \begin{cases} 0 & \text{if } b_{t-1} < b_t^* \\ \delta \equiv \frac{2\sigma_b}{b^*} & \text{if } b_{t-1} \geq b_t^* \end{cases} \qquad (b_t^* \sim \mathcal{N}(b^*, \sigma_b^2))$$

- Calibrate to Greece (1971 - 2007):

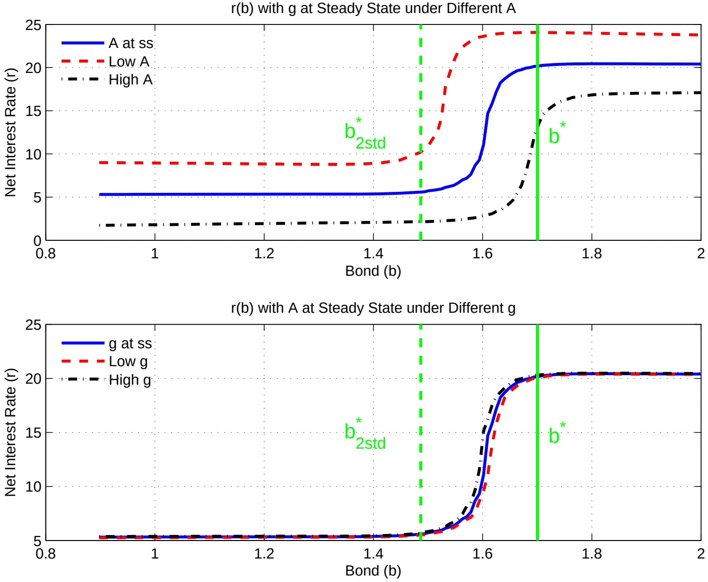
τ^L	γ	z/y	ζ^z	g/y	ρ^g	σ^g
0.32	0.42	0.134	-0.45	0.167	0.426	0.0294
θ_H	θ_L	p	β	L	ρ^A	σ^A
0.78	0.61	$1/13$	0.95	0.75	0.45	0.0328

  - Markov switching θ_t: $\theta_t \in \{\theta_H, \theta_L\}$ with $p_{LL} = p_{HH} = p$

Fiscal Limit: Greece



Decision Rule: $R(b^d, A, g)$

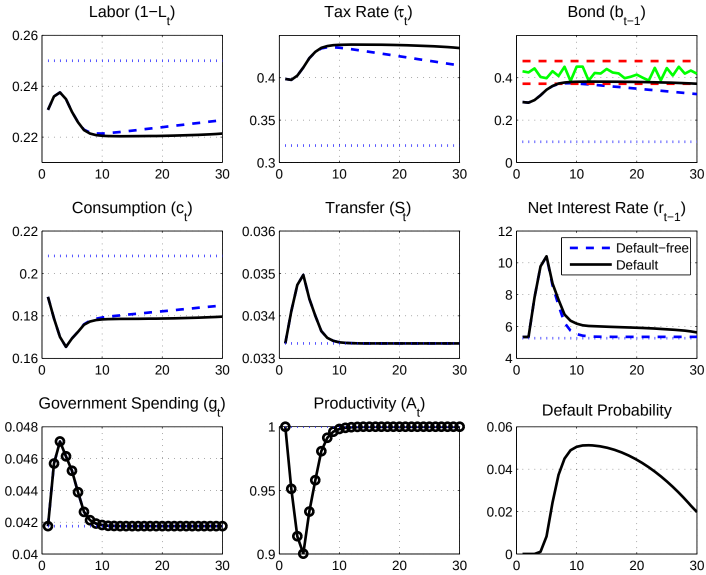


Simulation: A Severe Recession

- Given the paths of A_t and g_t.
- At each period, the effective fiscal limit (b_t^*, green line) is drawn from the approximated distribution.
- The paths of $c_t, L_t, \tau_t, b_t, r_t$ are determined by equilibrium conditions.

	t=1	t=2	t=3	t=4	t=5	t= 6
A_t	-4.88%	-8.61%	-9.97%	-6.67%	-4.21%	-1.92%
g_t/y_t	20.35%	21.68%	21.81%	21.08%	20.29%	19.52%

Nonlinear Simulation



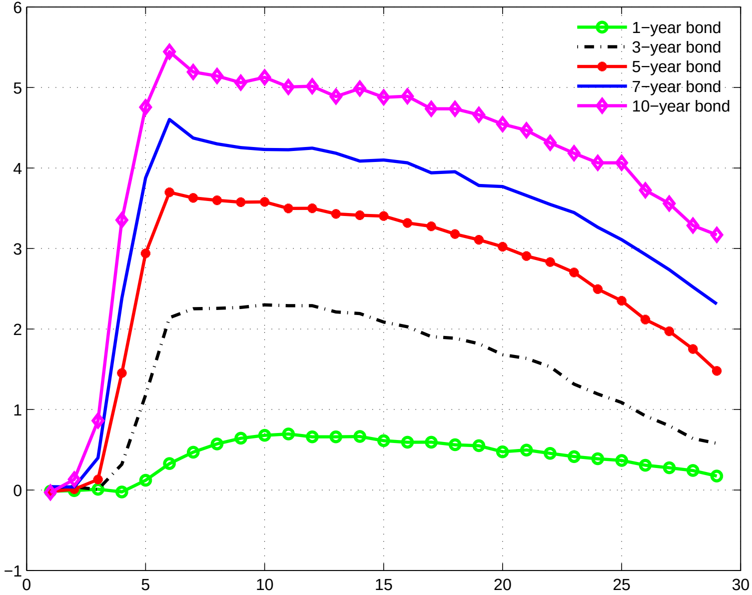
Long-term Bonds

- Price of long-term bond with maturity n:

$$
\begin{aligned}
Q_t^n &= \beta^n E_t \left((1 - \Delta_{t+n}) \frac{u_c(t+n)}{u_c(t)} \right) \\
r_t^{n\Delta} &= \frac{1}{Q_t^n} - \frac{1}{Q_t^{nf}}
\end{aligned}
$$

- Solution: finite-element method

Simulation: Long-Term Bonds



Wrap Up

- Modeling fiscal matters calls for substantial extensions to and modifications of existing DSGE models
    1. long-run issues: linearizing around "steady state"?
    2. nonstationarity: linearizing around "steady state"?
    3. nonlinearity: linearizing around "steady state"?
    4. nonnormality: linearizing around "steady state"?
- May be the death of Dynare