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Rotation Designs and Composite Estimation in Sample Surveys
Part 1. Motivating Their Use

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Abstract

One can think of a rotation design as a compromise between a complete sample overlap and taking independent samples. Each extreme has advantages and disadvantages. By using a rotation design, one hopes to realize some of the variance reduction of the complete sample overlap, while reducing its excess burden. In this paper, we start by motivating the use of a rotation design and composite estimation to improve the estimator of current level of a parameter, θ_t, then look at compositing to improve the estimator of change, $\theta_t - \theta_{t-1}$. Some consideration is then given to doing both: estimating level and change simultaneously. Finally, we briefly discuss other practical issues that influence the choice of designs and estimators, including generalizing the estimators, panel conditioning, cost, the mode of data collection, and respondent burden.

Key Words: Repeated sampling, panel surveys, change over time, panel conditioning, internal consistency.

Introductory Note

This paper is the first of two on rotation designs and composite estimators. In this paper, we provide motivation for using composite estimators when the conditions are favorable by looking at several very simple rotation designs. The purpose here is not to find the one design and estimator that are optimal, even if that could be defined. Rather, we consider various parameters and circumstances that can broaden the options and lead to several good but differing strategies. A second paper will review the use of composite estimation in several surveys conducted by the U.S. Census Bureau and Statistics Canada. About half of the material in the two papers comes from a presentation made by the author a few years ago at a Census Bureau seminar.

1. Repeated Measurements and Rotation Designs

In a number of surveys conducted by government statistical agencies, sample units are canvassed more than once in a specified rotation design. For example, in a monthly survey, households might be interviewed several times over a number of months and then retired from the sample, while being replaced by another set of units. Alternatively, companies might rotate in and out of sample every third month over a period of years. One can think of a rotation design as a compromise between a complete sample overlap, where the units remain in sample indefinitely or for an extended period of time, and taking independent samples, where the responding units are contacted only once. Each extreme has advantages and disadvantages.

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[1] *This report is released to inform interested parties of research and to encourage discussion. The views expressed are those of the author and not necessarily those of the U.S. Census Bureau.*

With a complete sample overlap, the variance of an estimator of change can be reduced greatly if there is a strong positive correlation between estimates at consecutive points in time. Further, some costs of data collection might be incurred only when contacting a household or business the first time. Yet, by interviewing the same unit repeatedly, one may risk increasing nonresponse due to a potentially heavier burden. By using a rotation design, one hopes to realize some of the variance reduction of the complete sample overlap, while reducing some of its excess burden.

When taking repeated measurements within a set rotation design, one can sometimes develop more efficient estimators by applying composite estimation. That is, by taking a linear (or other) combination of the statistics from the current and prior sampling periods, one may reduce the variance of current estimators of level or change or both.

We start by briefly reviewing a small part of the literature on rotation designs and composite estimators, and go on to describe a few government surveys that use such designs. We then motivate the use of repeated measurements and composite estimation to improve the estimator of current level of a parameter, θ_t. Next, we look at compositing to improve the estimator of change, $\theta_t - \theta_{t-1}$. Some consideration is then given to doing both: estimating level and change simultaneously. Finally, we briefly discuss other practical issues that influence our choice of designs and estimators. These issues include the relative importance of the types of estimates desired, the correlation over time, generalizing the estimators, panel conditioning, cost, the mode of data collection, and respondent burden.

2. Some Literature on Rotation Designs and Composite Estimation

In his paper, Eckler (1955) discussed methods to construct an efficient rotation design. He derived optimal sampling schemes and their minimum-variance estimators first with one-level designs, and then for designs with two or more levels. (See the next section for a definition of the term "level" with rotation designs.) There is some discussion of earlier papers by Jessen (1942) and Patterson (1950).

Woodruff (1963) presented two methods as applied specifically to the Census Bureau's monthly trade surveys. To reduce variances for estimates of current monthly level and month-to-month change, he applied composite estimation to the estimates from a design with 12 rotation groups (then in use). What further distinguishes this paper is his procedure to address outlying or influential sample observations in an unbiased way by altering the rotation pattern and the survey weights for the influential sample units. In Rao and Graham (1964), the authors developed optimal results for measuring current level and change under a general rotation design, including the number of consecutive visits to the sample units, the coefficients of the optimal composite estimators, explicit formulae for the resulting variances, and the reduction in variance realized.

Gurney and Daly (1965) considered a multivariate approach to finding optimal estimators under rotation designs, and applied the results to two Census Bureau surveys, the Current Population Survey (CPS) and the Business Reports Survey. The authors compared several estimators using correlation coefficients as estimated from prior survey results. They also described and studied effects due to "time-in-sample bias" (briefly discussed here in Section 7). In her paper, Bailar (1975) presented evidence of time-in-sample bias in the CPS. She then studied in some depth its effects on composite estimates.

Wolter (1979) derived minimum variance estimators and their associated variances under specific rotations designs and several realistic correlation structures. He applied his results to the Census Bureau's Monthly Retail Trade Survey. Cantwell (1990) generalized the variance formulae for composite estimators of current level, change over time, and averages for any repeated survey in a wide set of realistic rotation designs. These generalized formulae can be applied easily to most of the government surveys described in the next section.

In the Wiley volume, Kasprzyk, Kalton, Duncan, and Singh (1989), the authors presented some of the research then being done on various aspects of survey designs with rotation groups. The topics include rotation designs, determining and evaluating more efficient estimators, related cognitive aspects, and issues with longitudinal panels.

3. Some Important Examples of Rotation Designs in Government Surveys

In this section, we describe briefly the rotation designs of several continuing government surveys. Most are household surveys, but two are surveys of business entities. We don't mean to imply that the set described here is in any way typical or representative of government surveys here or elsewhere, only that we are more familiar with them. Diagrams depicting the rotation designs of most of these surveys are found in Tables 6 through 11 in the Appendix. In this paper, we don't attempt to describe the subject matter of the surveys or their data products. References for such information can be found on-line.

For its Labour Force Survey, Statistics Canada uses a "6-in-then-out" rotation design. The housing units in a given rotation group are interviewed for six consecutive months, and are then dropped from the sample permanently. In any month, the people in six rotation groups are interviewed. See Table 6 in the Appendix. The Labour Force Survey of the Australia Bureau of Statistics has a very similar "8-in-then-out" design; see Table 7. Composite estimation of some type is used with each of these designs. See Singh, Kennedy, and Wu (2001); Gambino, Kennedy, and Singh (2001); and Bell (2001).

In the United States, characteristics on the labor force are measured through the Current Population Survey. While the survey is sponsored and the labor force statistics published by the Bureau of Labor Statistics, the survey data are collected by the Census Bureau in a 4-8-4 rotation design. See Table 8. The housing units in a rotation group are contacted in four consecutive months, then out of sample for the next eight months, and then contacted for another four months. Thus, a specific household is interviewed eight times over a period of 16 months. In any month, there are eight rotation groups in sample for interview. The design allows for a 75% overlap of the housing units from any one month to the next, as well as a 50% overlap of units from any one month to the same month a year later. This provides for efficient estimators of change from month to month and from year to year.

The three rotation patterns just described are examples of "one-level" designs. By this, we mean that the units in the survey provide information on only one period of time (e.g., a month) during the interview. An example of a multi-level survey is the Consumer Expenditure Survey, sponsored by the Bureau of Labor Statistics and conducted by the Census Bureau. In one interview, a person selected for sample is asked to report expenditures for the prior three months. In the next month, people in the next rotation group respond for the prior three months, and so forth. See Table 9. This describes a three-level design. In the expenditure survey, the units in any rotation group are contacted every third month for a total of five interviews, before they are retired from the sample and replaced with another group.

The National Crime Victimization Survey, sponsored by the Bureau of Justice Statistics and conducted by the Census Bureau, uses a similar six-level rotation design. Respondents are interviewed every sixth month for seven interviews, and report victimizations and other incidents for the prior six months. It might be noted that although the expenditure and crime surveys use rotation designs, neither incorporates composite estimation.

Another example of a multi-level rotation design is found in the Survey of Program and Participation (SIPP); see Table 10. The design for a SIPP sample or "panel" is different from those described earlier in that only four rotation groups continue in sample over time. After the four groups are interviewed a first time (called "Wave 1") one at a time over four months, no new rotation group comes into sample. The four groups are interviewed a second time (Wave 2) over the next four months. In all, each group is interviewed a total of eight or 12 times, and then interviewing for the panel is finished. The number of waves--8, 12, or something else--has varied with different SIPP panels depending on the budget and other relevant issues.

Finally, we present a rotation design used until 1997 on businesses, the Monthly Retail and Wholesale Trade Surveys conducted by the Census Bureau. These surveys used a "three-group two-level" rotation design. See Table 11. In any month, the sample firms in one of the three rotation groups were canvassed and asked to report their sales (and, possibly, inventories) for the most recent *two* months at the same time. The following month, the next group went through the same procedure; similarly for the third group in the third month. The process was then repeated; all firms in each group rotated in and out of sample every third month for five years.

4. Compositing to Improve the Estimator of Level, θ_t

To get an idea of how repeated sampling in a survey can increase the accuracy in estimating population parameters, consider the following simplified example. Suppose we estimate the mean or total of a population at time t, θ_t, by the function $\hat{\theta}_t$, such that Var($\hat{\theta}_t$) = σ_t^2. To estimate $\theta_t - \theta_{t-1}$, one can consider $\hat{\theta}_t - \hat{\theta}_{t-1}$. If $\rho_{t-1,t}$ represents the correlation between the estimates $\hat{\theta}_t$ and $\hat{\theta}_{t-1}$, then

$$\mathrm{Var}(\hat{\theta}_t - \hat{\theta}_{t-1}) = \sigma_t^2 + \sigma_{t-1}^2 - 2\rho_{t-1,t}\,\sigma_t\,\sigma_{t-1} \tag{1}$$

If the samples and procedures that produce the estimates at times t and t−1 are conducted independently, then $\rho_{t-1,t} = 0$ and the variance of the difference is $\sigma_t^2 + \sigma_{t-1}^2$. But in a rotation design, one can construct the samples so that they overlap at times t and t−1. The result is that some or all of the respondents are in sample at both times, usually causing $\rho_{t-1,t}$ to be greater than 0. This leads to a decrease in the variance of the difference above.

But what about the estimate of θ_t, $\hat{\theta}_t$? It's not so clear how the sample overlap can decrease the variance of an estimator of *current level* (mean or total) at time t. We usually want the estimator to be unbiased for the parameter, θ_t, that is, the expected value of the estimator should only reflect the mean or total at time t. Here, we'll look at a simple example to see how the same sample overlap and correlation between time periods can be used to increase the efficiency of an estimator of change *and* level.

For simplicity, suppose we employ a two-group one-level rotation design, where (1) the units in each rotation group are interviewed in two consecutive time periods, perhaps months; (2) each rotation group is of the same size; (3) in any month, one group responds for the first time, and another for the second and final time; and (4) the data and any estimators derived from non-overlapping rotation groups are mutually independent. Table 1 demonstrates the scheme.

Table 1. Unbiased Monthly Estimates in a Rotation Design with Two Groups Each Time Period

Month	Rotation Group						
	A	B	C	D	E	F	G
1	$x_{1,A}$						
2	$x_{2,A}$	$x_{2,B}$					
3		$x_{3,B}$	$x_{3,C}$				
4			$x_{4,C}$	$x_{4,D}$			
5				$x_{5,D}$	$x_{5,E}$		
6					$x_{6,E}$	$x_{6,F}$	
7						$x_{7,F}$	$x_{7,G}$

For a group J in sample at time t, in order to estimate the total (or mean) of some characteristic, θ_t, one can sum the weighted responses of the units in the group, where the weights are the inverses of the inclusion probabilities, yielding $x_{t,J}$. To simplify the examples, we will assume throughout that, if there are *m* rotation groups in sample in month t, then the sample weights are multiplied by *m*, so that each $x_{t,J}$ is unbiased for θ_t. Denote the variance of $x_{t,J}$ by σ_t^2; and the correlation between the estimates $x_{t,J}$ and $x_{t-1,L}$ by $\rho_{t-1,t}$, if J = L and group J is in sample at times t and t−1; and 0, otherwise. Let us assume that σ_t^2 and $\rho_{t,\,t-1}$ are constant across months at σ^2 and ρ, respectively. This is done to simplify what follows, but may be very reasonable in many practical repeated surveys if the rotation groups retain a constant size of sample.

To estimate the total for, say, month 4, θ_4, one can use the simple estimator

$$\hat{Y}_4 = (1/2)(x_{4,C} + x_{4,D}) \tag{2}$$

It is unbiased, and has a variance of $(1/2)\sigma^2$. We will develop a composite estimator to estimate θ_4 by combining information from month 4 with information from prior months. One might start with a simple convex combination

$$\hat{Y}_4^{CE} = (1 - k)\,\hat{Y}_4 + k\,\hat{Z}_4\,, \quad 0 < k \le 1 \tag{3}$$

where $\hat{Y}_4$ is given in (2), and $\hat{Z}_4$ is derived from month 4 *and* earlier month(s). How do we do this while keeping $\hat{Z}_4$ unbiased? One way is to start with $\hat{Y}_3 = (1/2)(x_{3,B} + x_{3,C})$, an unbiased estimator for θ_3, and add $\hat{\Delta}_{3,4} = (x_{4,C} - x_{3,C})$, an estimator of change from month 3 to 4:

$$\hat{Z}_4 = \hat{Y}_3 + \hat{\Delta}_{3,4} = (1/2)(x_{3,B} + x_{3,C}) + (x_{4,C} - x_{3,C}) \tag{4}$$

Note that $E(\hat{Z}_4) = \theta_4$; thus $E(\hat{Y}_4^{CE}) = \theta_4$. We can rewrite $\hat{Y}_4^{CE}$ as

$$\hat{Y}_4^{CE} = (1 - k)\hat{Y}_4 + k\hat{Z}_4 = (1 - k)\hat{Y}_4 + k(\hat{Y}_3 + \hat{\Delta}_{3,4})$$

$$= (1 - k)(1/2)(x_{4,C} + x_{4,D}) + k\,[(1/2)(x_{3,B} + x_{3,C}) + (x_{4,C} - x_{3,C})]$$

$$= (1/2)(1 - k)\,x_{4,D} + (1/2)(1 + k)\,x_{4,C} - (1/2)k\,x_{3,C} + (1/2)k\,x_{3,B} \tag{5}$$

For example, suppose $k = .2$. Then $\hat{Y}_4^{CE} = .4\,x_{4,D} + .6\,x_{4,C} - .1\,x_{3,C} + .1\,x_{3,B}$, and $E(\hat{Y}_4^{CE}) = \theta_4$. The coefficients of the $x_{4,J}$ terms sum to 1, while the coefficients of the $x_{3,J}$ terms sum to 0, maintaining the unbiasedness of the composite estimator. What does this compositing do to the variance of the estimator, relative to the simpler estimator, $\hat{Y}_4$?

If $k = .2$ and $\rho = .8$,

$$\mathrm{Var}(\hat{Y}_4^{CE}) = \{.16 + .36 + .01 + .01 - .12\,\rho\}\,\sigma^2$$

$$= \{.52 + .02 - .096\}\,\sigma^2 = .444\,\sigma^2 \tag{6}$$

Note: Throughout the examples, we will insert $\rho = .8$ to demonstrate gains that may be possible when there is a strong correlation between the rotation group estimates, $x_{t,J}$ and $x_{t-h,J}$, based on the same sample units. In some surveys and with some characteristics, such as the state of being employed in the Current Population Survey, this high correlation is realistic. With other characteristics, a realistic value of ρ might be closer to .5 (being unemployed), .2, or even 0. Recall that the variance of the simple unbiased estimator, $\hat{Y}_4$, is $.5\sigma^2$. Why do we see a decrease in the variance from including prior terms whose coefficients sum to 0? In fact, without considering the covariance term (equivalently, if ρ = .0), applying unequal coefficients, .4 and .6, to the terms $x_{4,D}$ and $x_{4,C}$, respectively, increases the variance from $.50\sigma^2$ to $.52\sigma^2$. Including two terms, .1 $x_{3,C}$ and .1 $x_{3,B}$, adds another $.02\sigma^2$ to the variance. But the negative covariance term more than makes up for this additional variance if ρ > 1/3 (with these coefficients).

To see it theoretically and in general, consider the definition of the correlation between the two terms, $x_{4,C}$ and $x_{3,C}$:

$$\rho = \frac{\text{Cov}(x_{4,C}, x_{3,C})}{\sigma^2} = \frac{E[(x_{4,C} - \theta_4)(x_{3,C} - \theta_3)]}{\sigma^2} \tag{7}$$

If $x_{4,C}$ and $x_{3,C}$ are strongly positively correlated (ρ > 0), when $x_{4,C}$ is larger than its mean, θ_4, $x_{3,C}$ *tends* to be larger than its mean, θ_3, and vice versa. By putting opposite signs in front of $x_{4,C}$ and $x_{3,C}$, a "large" (or small) value of one tends to adjust in the proper direction for a large (or small) value of the other. This cancellation, at least in expectation, brings down the variance of the combination below that of the simple estimator, $\hat{Y}_4$, if ρ is large enough.

The question arises: How much can one decrease the variance of the estimator in equation (5) by selecting k optimally? With

$$\hat{Y}_4^{CE} = (1/2)(1 - k)\, x_{4,D} + (1/2)(1 + k)\, x_{4,C} - (1/2)k\, x_{3,C} + (1/2)k\, x_{3,B}$$

it follows directly that

$$\text{Var}(\hat{Y}_4^{CE}) = (1/2)[1 + 2k^2 - \rho k(1+k)]\,\sigma^2 \tag{8}$$

The value of k that minimizes this expression is

$$k_{min} = \frac{\rho}{4 - 2\rho} \tag{9}$$

Inserting this value of k into (8) yields a minimum variance of

$$\text{Var}_{k,min}(\hat{Y}_4^{CE}) = \frac{16 - 16\rho + 2\rho^2 + \rho^3}{2\,(4 - 2\rho)^2}\,\sigma^2 \tag{10}$$

For example, if $\rho = .8$, the minimizing k is $k_{min} = 1/3$, producing a composite estimator

$$\hat{Y}_4^{CE} = (1/3)\, x_{4,D} + (2/3)\, x_{4,C} - (1/6)k\, x_{3,C} + (1/6)k\, x_{3,B} \tag{11}$$

whose variance is $Var_{k,min}(\hat{Y}_4^{CE}) = (13/30)\,\sigma^2 \approx .4333\,\sigma^2$

For the rotation scheme depicted in Table 1, we started with a composite estimator of the form found in equation (3) or (5). But, even within the family of unbiased linear estimators, there is no need to restrict the form. One can make the composite more general by extending our options to the family of two-parameters estimators. We can define the estimator as

$$\hat{Y}_4^{CE} = (1 - \alpha)\, x_{4,D} + \alpha\, x_{4,C} - \beta\, x_{3,C} + \beta\, x_{3,B}\,, \tag{12}$$

where we typically want α and β to satisfy $1/2 \le \alpha \le 1$, and $\beta \ge 0$. The special case where $\alpha = (1/2)(1+k)$ and $\beta = (1/2)k$ leads to equation (5) . We restrict the parameters for the following reasons. There are two terms, $x_{4,D}$ and $x_{4,C}$, whose coefficients will add to 1 to maintain unbaisedness. We want (1) the two coefficients to be fairly close to 1/2 to keep their sum of squares small, and (2) to place more weight on the component $x_{4,C}$ to take advantage of the covariance term that results from its dependence with $x_{3,C}$. We wish to make β positive ($-\beta$ negative) so that the covariance term just mentioned will be less than 0.

Can we choose α and β to minimize the variance of the composite estimator in (12)? The variance can be expressed as

$$Var(\hat{Y}_4^{CE}) = [\,1 - 2\alpha + 2\alpha^2 + 2\beta^2 - 2\rho\alpha\beta\,]\,\sigma^2 \tag{13}$$

Minimizing α and β as a pair, we obtain

$$\alpha_{min,level} = \frac{2}{4-\rho^2}\,, \quad \beta_{min,level} = \frac{\rho}{4-\rho^2} = (1/2)\,\rho\,\alpha_{min,level} \tag{14}$$

With the minimizing values of α and β, the variance of the composite estimator is

$$Var_{\alpha,\beta,min}(\hat{Y}_4^{CE}) = \frac{2-\rho^2}{4-\rho^2}\,\sigma^2 = (1 - \alpha_{min,level})\,\sigma^2 \tag{15}$$

When $\rho = .8$, the optimal parameters are $\alpha \approx .5952$ and $\beta \approx .2381$; they produce a composite estimator whose variance is $.4048\sigma^2$, about 19% below that of the simple estimator, $.5\sigma^2$, in (2).

5. Compositing to Improve the Estimate of Change, $\theta_t - \theta_{t-1}$

Suppose one wants to estimate the difference in the total (or mean) between two consecutive periods, $\theta_t - \theta_{t-1}$. For simplicity, we will estimate the difference from month 3 to month 4, $\theta_4 - \theta_3$, using unbiased estimates from rotation groups, $x_{t,J}$, similar to (sometimes the same as) what was shown in Table 1. We consider four cases, as defined by the sample rotation scheme, as well as the estimator applied.

Case 1. There is no overlap among the rotation groups from month to month.

Case 2. One of two rotation groups overlap from month to month. We do *not* use composite estimation.

Case 3. One of two rotation groups overlap from month to month. But we *do* use composite estimation.

Case 4. There is total overlap of the rotation groups.

Case 1. No overlap among the rotation groups from month to month.

This situation is depicted in Table 2. Note that we could just as easily combine rotation groups A and B into one; similarly for groups C and D, E and F, etc. However, for the cases studied in this paper, we will assume that all $x_{t,J}$'s are based on a common sample size and have the same variance, σ^2. This will facilitate comparisons among the four cases.

Table 2. No Overlap from One Month to the Next

Month	Rotation Group						
	A	B	C	D	E	F	G
1							
2							
3	$x_{3,A}$	$x_{3,B}$					
4			$x_{4,C}$	$x_{4,D}$			
5					$x_{5,E}$	$x_{5,F}$	
6							$x_{6,G}$
7							

We want an estimator of change from month t-1 to month t, here, month 3 to 4, that is unbiased. A simple estimator, analogous to $\hat{Y}_4 - \hat{Y}_3$ in the prior section, is

$$\hat{D}_{3,4}^{(1)} = (1/2)(x_{4,C} + x_{4,D}) - (1/2)(x_{3,A} + x_{3,B}) \qquad (16)$$

The estimator's superscript, (1), refers to the case under discussion, while the subscripts denote the relevant months. It is easy to see that $\hat{D}^{(1)}_{3,4}$ is unbiased for $\theta_4 - \theta_3$, and that

$$\text{Var}\left(\hat{D}^{(1)}_{3,4} \right) = 4\,(1/2)^2\,\sigma^2 = \sigma^2 \qquad (17)$$

Case 2. One of two rotation groups overlap from month to month. We do not use composite estimation.

The sampling scheme here is the same as was described in the prior section when estimating level; thus, Table 3 looks the same as Table 1.

Table 3. A Rotation Design with 50% Overlap (Same as Table 1)

Month	Rotation Group A	B	C	D	E	F	G
1	$x_{1,A}$						
2	$x_{2,A}$	$x_{2,B}$					
3		$x_{3,B}$	$x_{3,C}$				
4			$x_{4,C}$	$x_{4,D}$			
5				$x_{5,D}$	$x_{5,E}$		
6					$x_{6,E}$	$x_{6,F}$	
7						$x_{7,F}$	$x_{7,G}$

But here, we will consider an estimator *without* compositing and assess its attributes. Define

$$\hat{D}^{(2)}_{3,4} = (1/2)(x_{4,C} + x_{4,D}) - (1/2)(x_{3,B} + x_{3,C}) \qquad (18)$$

This estimator looks much like $\hat{D}^{(1)}_{3,4}$. However, the sample design here differs, providing for a 50% overlap of the sample from one month to the next. Once again, $E(\hat{D}^{(2)}_{3,4}) = \theta_4 - \theta_3$. But the variance of the estimator includes a component for the covariance:

$$\text{Var}\left(\hat{D}^{(2)}_{3,4} \right) = (4 - 2\rho)(1/2)^2\,\sigma^2 = (1 - .5\rho)\,\sigma^2 \qquad (19)$$

which is less than $\text{Var}(\hat{D}^{(1)}_{3,4}) = \sigma^2$ if $\rho > 0$. For $\rho = .8$, $\text{Var}(\hat{D}^{(2)}_{3,4}) = .6\sigma^2$. Note that this 40% decrease in the variance derives from the sampling scheme with a 50% overlap of units, even though compositing was *not* used in the estimation.

Case 3. One of two rotation groups overlap from month to month. We <u>do use</u> composite estimation.

The rotation design for this case is the same as that for Case 2; again, Table 3 applies. But now we let composite estimation take advantage of the 50% sample overlap. Starting with an estimator of level for month 4, we recall the composite estimator from the prior section in (12):

$$\hat{Y}_4^{CE} = (1 - \alpha)\, x_{4,D} + \alpha\, x_{4,C} - \beta\, x_{3,C} + \beta\, x_{3,B}, \text{ with } 1/2 \le \alpha \le 1, \beta \ge 0 \qquad (20)$$

The composite estimator for level in month 3, $\hat{Y}_3^{CE}$, is defined analogously. Although different sets of parameters, α and β, could be used for the two estimators of level, for simplicity and internal consistency, for now we'll consider only the case where the sets are the same. (More is said about internal consistency in Section 6.) This leads to a composite estimator for the difference, $\theta_4 - \theta_3$:

$$\hat{D}_{3,4}^{(3,CE)} = \hat{Y}_4^{CE} - \hat{Y}_3^{CE}$$

$$= (1 - \alpha)\, x_{4,D} + \alpha\, x_{4,C} + (\alpha - \beta - 1)\, x_{3,C} + (\beta - \alpha)\, x_{3,B} + \beta\, x_{2,B} - \beta\, x_{2,A} \qquad (21)$$

The estimator is unbiased for $\theta_4 - \theta_3$. After a bit of algebra, its variance can be expressed as

$$\text{Var}\left(\hat{D}_{3,4}^{(3,CE)} \right) = 2\{ (1 - \alpha)^2 + (\alpha-\beta)^2 + \beta(1+\beta) + \rho\, [(\alpha - \beta)^2 - \alpha] \}\, \sigma^2 \qquad (22)$$

The values of the parameters that minimize this variance are

$$\alpha_{\text{min,diff}} = \frac{3+3\rho+\rho^2}{2(3+2\rho)}, \quad \beta_{\text{min,diff}} = \frac{\rho\,(2+\rho)}{2(3+2\rho)} \qquad (23)$$

When $\rho = .8$, $\alpha_{\text{min,diff}} \approx .6565$ and $\beta_{\text{min,diff}} \approx .2435$, leading to a minimum variance in (22) of

$$\text{Var}_{\alpha,\beta,\text{min}}\left(\hat{D}_{3,4}^{(3,CE)} \right) \approx .4053\sigma^2 \qquad (24)$$

This decrease in the variance is almost 60% below that of the simple estimator when there is no overlap of units. It is also more than 32% below that of the simple estimator in Case 2 where there was a 50% overlap in sample units, but composite estimation was <u>not</u> allowed.

Case 4. Total overlap of the rotation groups.

Case 4 is shown in Table 4. We could equivalently combine the two groups from the same month into one group having twice as many units, and represent $x_{t,J}$ and $x_{t,L}$ by one estimator, as was possible with Case 1. However, we continue to keep them separate, so that all $x_{t,J}$'s will be based on the same sample size and the same variance, σ^2.

With a total overlap of the sample units, there is no need or desire to use a composite estimator; with 100% overlap in the sample, compositing doesn't help to decrease the variance when estimating the difference. Similar to Case 1, a simple difference of averages, analogous to $\hat{Y}_4 - \hat{Y}_3$ in the prior section, produces

$$\hat{D}^{(4)}_{3,4} = (1/2)(x_{4,A} + x_{4,B}) - (1/2)(x_{3,A} + x_{3,B}) \tag{25}$$

Unbiasedness is obtained, $E(\hat{D}^{(4)}_{3,4}) = \theta_4 - \theta_3$, and the variance is easily computed:

$$\mathrm{Var}(\hat{D}^{(4)}_{3,4}) = [4 - 2(2)\rho](1/2)^2\sigma^2 = (1 - \rho)\sigma^2 \tag{26}$$

If $\rho = .8$, the variance is $.2\sigma^2$, much lower than in Case 1--with a different sample design--and a substantial improvement over the results in Cases 2 and 3 that used a 50% overlap in the sample.

Table 4. A Rotation Design with 100% Overlap

Month	Rotation Group	
	A	B
1	$x_{1,A}$	$x_{1,B}$
2	$x_{2,A}$	$x_{2,B}$
3	$x_{3,A}$	$x_{3,B}$
4	$x_{4,A}$	$x_{4,B}$
5	$x_{5,A}$	$x_{5,B}$
6	$x_{6,A}$	$x_{6,B}$
7	$x_{7,A}$	$x_{7,B}$

6. Compositing to Improve the Estimates of Level *and* Change

Variance of level vs. variance of change

When comparing approaches, the results often depend on several conditions: what type (if any) of sample or rotation design is allowed or feasible; whether the correlation between consecutive estimates is high or low; and whether one wants to estimate current level or change. As might be expected, estimators that perform well or best under one set of circumstances need not do as well under others. Thus, a composite estimator may beat a simple estimator in one setting, but not in others. To illustrate with the simple examples discussed above, we compare the estimators from the prior sections according to the circumstances.

For each of Cases 1 to 4, the sample design and the form of the estimator are provided in Table 5, along with the variances (divided by σ^2) of the estimators of monthly level and month-to-month change. Case 3 is divided into three possibilities according to the selection of the parameters α and β. For Cases 1, 2a, and 4, the simple estimator is used, as in equations (16), (18), and (25). For completeness, the corresponding values of α (.5) and β (0) have been inserted into the table.

In this simplified example, by using a rotation design with sample overlap of 50% and a composite estimator, one can reduce the variance of the estimators of level and change, compared to Case 1 with no sample overlap. The optimal set of values α and β for minimizing the variance of level, Case 3b, .5952 and .2381, respectively, differs from the set that minimizes the variance for change, Case 3c, .6565 and .2435. But the differences in the parameters are small, as are the differences in the variances for level and change. Using either set, with the same 50% sample overlap, compared to the simple average (Case 2a), one can reduce the variance of level by more than 17%, and the variance of change by more than 32%.

Table 5. Variances of Different Unbiased Linear Estimators (divided by the common σ^2)

Case	Sample Design	Estimator: Composite?	Form	α	β	Var (level)	Var (diff)
1	No overlap	No	Simple average	(.5)	(0)	.5000	1.0000
2a	50% overlap	No	Simple average	(.5)	(0)	.5000	.6000
2b [1]	50% overlap	No	α, β to minimize Var(estim. of diff.)	.8333	.8333	.7222	.3333
3a	50% overlap	Yes	k = .2: α=.6 and β=.1	.6	.1	.4440	.4800
3b	50% overlap	Yes	α, β to minimize Var(estim. of level)	.5952	.2381	.4048	.4240
3c	50% overlap	Yes	α, β to minimize Var(estim. of diff.)	.6565	.2435	.4118	.4052
4	100% overlap	No	Simple average	(.5)	(0)	.5000	.2000

[1] Case 2b is included for comparison, but is discussed further below.

By changing the design to a complete overlap, Case 4, one sees a considerable decrease in the variance of change, over 50%, compared to Cases 3b and 3c, at the expense of a moderate increase in the variance of level, over 20%. Although we will discuss it only briefly in the next section, implementing a total sample overlap as in Case 4 brings in other considerations, such as a potentially heavier measure of response burden per unit in sample, which could affect the response rate.

Internal Consistency of the estimators

A completely different aspect of estimation is the internal consistency of estimators of current level and change (difference). In this context, rather than the usual statistical definition, we say that the estimators of level and change are "internally consistent" if the estimator for $\theta_t - \theta_{t-1}$ is equal to the difference of the separate estimators for θ_t and θ_{t-1}, in the same way one might desire that the estimator of a sum of variables is equal to the sum of the separate variable estimators. For many surveys, consistency is required as policy, as it is generally preferred by data users.

Why does the issue arise here? When considering Case 2, where there was an overlap of one out of two rotation groups but we did not use composite estimation, one could broaden the alternatives. Consider the set of estimators defined as

$$\hat{D}_{3,4}^{(2b)} = [(1 - \alpha)\, x_{4,D} + \alpha\, x_{4,C}] - [\beta\, x_{3,C} + (1 - \beta)\, x_{3,B}], \text{ with } 1/2 \le \alpha, \beta < 1 \quad (27)$$

The estimator is unbiased for $\theta_4 - \theta_3$. Its variance is

$$\text{Var}(\hat{D}_{3,4}^{(2b)}) = 2 (1 - \alpha + \alpha^2 - \beta + \beta^2 - \rho\, \alpha\, \beta)\, \sigma^2 \quad (28)$$

which is minimized for

$$\alpha_{min,diff} = \beta_{min,diff} = \frac{1}{2 - \rho} \quad (29)$$

The minimized variance is

$$\text{Var}_{\alpha,\beta,min}(\hat{D}_{3,4}^{(2b)}) = \frac{2(1-\rho)}{2-\rho}\, \sigma^2 \quad (30)$$

which can be smaller than that for other estimators considered above. For example, if $\rho = .8$, optimal values of $\alpha_{min,diff}$ and $\beta_{min,diff}$ are 5/6, and the minimized variance is $(1/3)\sigma^2$. But the estimator then becomes

$$\hat{D}_{3,4}^{(2b)} = [(1/6)\, x_{4,D} + (5/6)\, x_{4,C}] - [(5/6)\, x_{3,C} + (1/6)\, x_{3,B}] \quad (31)$$

The estimator for level will not be internally consistent with the estimator for change. In addition, the variance of the estimator of level will be undesirably high. Using the value of $\alpha_{min,diff}$, one can show that

$$\text{Var}_{min}(\hat{Y}_4^{CE}) = \text{Var}_{min}((1 - \alpha)\, x_{4,D} + \alpha\, x_{4,C}) = \frac{2 - 2\rho + \rho^2}{(2 - \rho)^2}\, \sigma^2 \qquad (32)$$

which is equal to (13/18)σ^2 or .7222σ^2, when ρ = .8.

On the other hand, if internal consistency is not an important characteristic of the estimators, one can use a simple estimator for level, as in equation (2),

$$\hat{Y}_4 = (1/2)(x_{4,C} + x_{4,D})$$

and, to estimate change, an estimator like

$$\hat{D}_{3,4}^{(2b)} = [(1/6)\, x_{4,D} + (5/6)\, x_{4,C}] - [(5/6)\, x_{3,C} + (1/6)\, x_{3,B}] \qquad (33)$$

Within the same sample design--a 50% overlap of the sample units--and without applying composite estimation, one retains a fairly good estimator of level (variance, .5σ^2) and an excellent estimator for change (variance, .3333 σ^2), if the correlation coefficient is .8. However, consistency of the estimators can be a lot to give up, and is usually a requirement of government data releases.

7. Other Issues to Consider with Rotation Designs and Composite Estimators

In the last several sections, we've presented a few simple rotation designs and various estimators. The idea was to motivate their use and to give a glimpse of some of the statistical gains that might be achieved. In practice, there are many other aspects of the design and implementation that must be evaluated before seriously considering a rotation design. What follows are some of the more basic ones.

Relative Importance of the Types of Estimates Desired. As discussed in prior sections, the types of estimates desired are a consideration when selecting what sample design to implement, whether to use composite estimation (if appropriate), and what estimator to use. The relative importance of level, change, and averages over time is a major factor. Typically, if other conditions are about the same, when measuring change is more important than measuring level or averages over time, a heavier overlap of the sample units may be preferred. When averages are desired, overlap generally is less desirable. But many other factors come into play, including those that affect survey operations, costs, and response.

Correlation Over Time. In the analysis above, results are presented in terms of the correlation, ρ, between estimators from a rotation group whose units respond in consecutive months. The selection of an optimal design or estimator depends on the true value of ρ, which is usually unknown and may change over time. However, we can often estimate ρ based on data collected previously, especially in continuing surveys.

In the examples given in this paper, the correlation coefficient was set to .8 to demonstrate potential gains under composite estimation when the rotation group estimates from one month to the next are highly correlated. However, for many characteristics, ρ may be smaller, such as when measuring the number of victimizations for a given group. In such cases, there may be only limited reductions in variance using composite estimation.

Generalizing the Estimators. In the study above, the composite estimators for a given month used only data collected in the current month and the month immediately prior. But there is no need to limit the number of periods used in a composite estimator. Note, however, that the reduction in variance from incorporating rotation groups further back in time tends to diminish with time. For a given set of periods with their associated estimators, $x_{t,J}$, one can generalize the coefficients further or entertain other options. See Gurney and Daly (1965) for an exposition of general linear combinations of the period estimators. In fact, although we have looked only at additive estimators--those in which the measure of change is added to the estimator of level--one could use a multiplicative estimator, in which the estimator of level is multiplied by the ratio of change, as was formerly done with the U.S. monthly trade surveys (Wolter 1979).

Panel Conditioning. In the simplified examples seen here, we examined and compared the variances of estimators to provide a quick statistical assessment. However, using a rotation design can introduce bias into the estimators. It sometimes happens that respondents or (when applicable) interviewers change their response tendencies or behavior after one or more contacts in a rotation design, an effect referred to as panel conditioning. For example, the respondent might realize that certain answers will elicit additional extra questions, and try to avoid such answers. Similarly, a field representative might not ask part of the designated question, figuring that the respondent has heard it before.

Panel conditioning can lead to "time-in-sample bias." A thorough discussion of this problem in household surveys can be found in Bailar (1975). Cantwell and Caldwell (1998) describe and measure "differential response bias" in the U.S. monthly trade surveys. This phenomenon was studied earlier by Waite (1974) in the retail and wholesale trade industries. When reporting sales for a given month *k*, the reports when month *k* has just been completed (the "current month" in Table 11) tend to be biased downward relative to reports for month *k* from respondents reporting a month later (the "prior month"). The result can lead to undesirable performance of the estimators, and was a reason for changing the design of the monthly trade surveys in 1997.

In fact, as with any survey, the issue of "recall" can influence the estimators, especially in a multi-level rotation design. In household surveys, a person often provides all or most of the responses from knowledge or recall, without reference to records. The length of time between the point of interest and the interview can affect the respondent's ability to recall the information requested. Recall errors may increase with the time lapse. Yet, in economic surveys, providing additional time between the occurrence and reporting of business transactions can improve the reliability of the data. This follows because respondents in economic surveys tend to rely more (or exclusively) on written or electronic records, rather than on their memory, and the extra time increases the chance that the record will be complete or (if necessary) corrected.

Cost. In the introduction, we implied that cost was an important consideration in the selection of a sampling design. Specifically, some costs need not be repeated with every interview, but are only incurred with the initial interview. For example, when introducing the respondent to the survey, the data collection agency might explain the authority for taking the survey, what the agency's goals are, how the respondent will be contacted, and what to expect. An interviewer may have to locate the housing unit or establishment to conduct the first interview. It may take some effort to initiate the processing for sample data records, or to record basic administrative data on the sample unit. In a rotation design, some of these costs may be decreased or eliminated in subsequent contacts.

Mode of Data Collection. Related to the cost of the survey is the method by which the interviews or contacts are completed. In some household surveys that use a rotation design, the first interview is conducted in person--which is generally more expensive--so that the interviewer can develop a rapport with the respondent or collect additional information. Later interviews may be conducted over the telephone by a local interviewer or someone in a centralized telephone facility. This will generally eliminate some travel and may reduce the cost of those interviews. However, in economic surveys, all interviews may be conducted by mail, telephone, fax, or a combination, eliminating the cost of travel associated with in-person interviews.

Respondent Burden. A survey's total burden over all respondents is sometimes measured as the product of the total number of interviews and the average interview length. A government survey organization may be required to limit the level of total burden. For an individual respondent, the amount of burden can be affected by the length of the interviews, the number, and perhaps even their frequency over a set period of time. When considering design options, one should investigate how burden on the individual respondents affects the response rates and the quality of the data collected.

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Appendix

Table 6. Canadian Labour Force Survey: 6-In-Then-Out Rotation Design
The number in the table indicates which interview is taking place for that group.

Month	ROTATION GROUPS 6 Groups Interviewed Each Month									
	A	B	C	D	E	F	G	H	I	J
1	1									
2	2	1								
3	3	2	1							
4	4	3	2	1						
5	5	4	3	2	1					
6	6	5	4	3	2	1				
7		6	5	4	3	2	1			
8			6	5	4	3	2	1		
9				6	5	4	3	2	1	
10					6	5	4	3	2	1
11						6	5	4	3	2
12							6	5	4	3
13								6	5	4
14									6	5
15										6
16										

Appendix

Table 7. Australian Labour Force Survey: 8-In-Then-Out Rotation Design
The number in the table indicates which interview is taking place for that group.

Month	ROTATION GROUPS 8 Groups Interviewed Each Month									
	A	B	C	D	E	F	G	H	I	J
1	1									
2	2	1								
3	3	2	1							
4	4	3	2	1						
5	5	4	3	2	1					
6	6	5	4	3	2	1				
7	7	6	5	4	3	2	1			
8	8	7	6	5	4	3	2	1		
9		8	7	6	5	4	3	2	1	
10			8	7	6	5	4	3	2	1
11				8	7	6	5	4	3	2
12					8	7	6	5	4	3
13						8	7	6	5	4
14							8	7	6	5
15								8	7	6
16									8	7
17										8
18										

Appendix

Table 8. U.S. Current Population Survey: 4-8-4 Rotation Design
The number in the table indicates which interview is taking place for that group.

Month	ROTATION GROUPS (8 Groups Interviewed Each Month)																
	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R
1	1																	
2	2	1																
3	3	2	1															
4	4	3	2	1														
5		4	3	2	1													
6			4	3	2	1												
7				4	3	2	1											
8					4	3	2	1										
9						4	3	2	1									
10							4	3	2	1								
11								4	3	2	1							
12									4	3	2	1						
13	5									4	3	2	1					
14	6	5									4	3	2	1				
15	7	6	5									4	3	2	1			
16	8	7	6	5									4	3	2	1		
17		8	7	6	5									4	3	2	1	
18			8	7	6	5									4	3	2	1
19				8	7	6	5									4	3	2
20					8	7	6	5									4	3
21						8	7	6	5									4
22							8	7	6	5								
23								8	7	6	5							

24									8	7	6	5						

Appendix

Table 9. Consumer Expenditure Survey: 5-Group, 3-Level Rotation Design
The number in the table indicates which interview is taking place for that group.

Month	ROTATION GROUPS (5 Groups Interviewed Each Month)																	
	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R
1																		
2	1																	
3		1																
4			1															
5	2			1														
6		2			1													
7			2			1												
8	3			2			1											
9		3			2			1										
10			3			2			1									
11	4			3			2			1								
12		4			3			2			1							
13			4			3			2			1						
14	5			4			3			2			1					
15		5			4			3			2			1				
16			5			4			3			2			1			
17				5			4			3			2			1		
18					5			4			3			2			1	
19						5			4			3			2			1
20							5			4			3			2		
21								5			4			3			2	
22									5			4			3			2
23										5			4			3		
24											5			4			3	

Appendix

Table 10. Survey of Income and Program Participation: 4-Group, 4-Level Rotation Design

Month	ROTATION GROUPS One Group Interviewed Each Month: 1	2	3	4
1	Wave 1[1] Interview in month 5			
2		Wave 1 Interview in month 6		
3			Wave 1 Interview in month 7	
4				Wave 1 Interview in month 8
5	Wave 2 Interview in month 9			
6		Wave 2 Interview in month 10		
7			Wave 2 Interview in month 11	
8				Wave 2 Interview in month 12
9	Wave 3 Interview in month 13			
10		Wave 3 Interview in month 14		
11			Wave 3 Interview in month 15	
12				Wave 3 Interview in month 16
13	Wave 4 Interview in month 17			
14		Wave 4 in month 18		
15			Wave 4 month 19	
16				Wave 4
. . .	. . .	. . .	. . .	. . .

[1] Depending on the panel, there have typically been 8 or 12 waves of interviewing conducted on persons in sample. For each wave, respondents are asked to report income and activities for each of the four reference months.

Appendix

Table 11. U.S. Monthly Retail and Wholesale Trade Surveys (*Formerly Used*): 3-Group, 2-Level Rotation Design

Month	ROTATION GROUP[1,2] One Group Interviewed Each Month		
	1	2	3
1	current	prior[3]	
2	-	current[3]	prior
3	prior	-	current
4	current	prior	-
5	-	current	prior
6	prior	-	current
7	current	prior	-
8	-	current	prior
9	prior	-	current
10	current	prior	-
11	-	current	prior
12	prior	-	current
13	current	prior	-
14	-	current	prior
15	prior	-	current
. . .	. . .	. . .	. . .

[1] These rotation groups were actually called "panels" by analysts working on the survey. We stay with the term "rotation group" to avoid confusion.

[2] In addition, units selected with probability 1 report every month.

[3] In Month 3, Rotation Group 2 reports for Months 1 (prior month) and 2 (current month); in Month 4, Group 3 reports for Months 2 (prior month) and 3 (current month); and so forth.