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WORKER FLOWS AND JOB FLOWS: A QUANTITATIVE
INVESTIGATION

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# Worker Flows and Job Flows: A Quantitative Investigation[*]

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## Abstract

Worker flows and job flows behave differently over the business cycle. We investigate the sources of the differences by studying quantitative properties of a multiple-worker version of the search/matching model that features endogenous job separation and intra-firm wage bargaining. Our calibration incorporates micro- and macro-level evidence on worker and job flows. We show that the dynamic stochastic equilibrium of the model replicates important cyclical features of worker flows and job flow simultaneously. In particular, the model correctly predicts that hires from unemployment move countercyclically while the job creation rate moves procyclically. The key to this result is to allow for a large hiring flow that does not go through unemployment but is part of job creation, for which procyclicality of the job finding rate dominates its cyclicality. We also show that the model generates large volatilities of unemployment and vacancies when a worker's outside option is at 83% of aggregate labor productivity.

**JEL Classification**: E24, E32, J63, J64
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# 1 Introduction

Worker flows and job flows behave differently over the business cycle. It is well-known that gross worker flows between unemployment and employment are countercyclical: The separation flow into unemployment goes up during recessions; the hiring flow from the unemployment pool also rises because the increases in the separation flow raise the unemployment pool rapidly, thus increasing hires as well.[1] The behavior of job flows is different: Job destruction is countercyclical, whereas job creation is procyclical. Despite the clear difference in their cyclicality, there has been no attempt to reconcile it in the literature.

In the canonical model of labor search/matching, a worker-firm match that operates linear technology is taken to be the unit of analysis. While this traditional framework allows researchers to analyze worker transitions, it cannot be applied to the analysis of job flows because job flows are measured from establishment-level employment observations. In order to study both flows simultaneously, one needs a framework in which firms can hire multiple workers subject to the search friction.

In the previous literature, researchers have arbitrarily used either job flows or worker flows as the model's empirical counterparts. For example, Mortensen and Pissarides (1994), Cole and Rogerson (1999), and den Haan, Ramey, and Watson (2000) take job flows as the empirical counterpart, while, starting with Shimer (2005), the literature's focus has largely shifted to the cyclicality of transition rates between employment and unemployment. However, given that the two flows have different cyclical properties, the standard model is inherently unable to match the cyclicality of both flows. In this paper, we explore whether and how the search/matching model with multiple-worker firms can simultaneously explain the cyclicality of worker flows and job flows. Through this study, we can provide a fuller account of the U.S. labor market dynamics.

A recent paper by Elsby and Michaels (2008) develops a multiple-worker version of the matching model where production technology exhibits decreasing returns to scale and firms make endogenous hiring and separation decisions under the presence of the search friction. They adopt the intra-firm bargaining framework by Stole and Zwiebel (1996a,b) to determine wages, which naturally generalizes Nash bargaining often used in the standard model. Our model is based on theirs. However, their focus is on the model's ability to solve the well-known volatility puzzle by Shimer (2005), and their analysis mostly relies on steady-state comparative statics. They pay no attention to cross-sectional implications of the model nor the differences in the cyclicality of worker flows and job flows.[2] We extend their model in several important dimensions (which are discussed shortly) and solve for the stochastic dynamic equilibrium, applying a version of the algorithm developed by Krusell and Smith (1998).[3] Another recent paper by Cooper,

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[1]The job finding rate drops significantly during recessions, lowering hires, but this effect is dominated by the increase in the separation flow.

[2]Acemoglu and Hawkins (2006) also develop a similar model and examine its implications for the volatility puzzle. However, their model assumes exogenous separation, and they also consider only the steady-state comparative statics. Other papers that consider the multiple-worker-firm setting include Smith (1999), Cahuc and Wasmer (2001), and Cahuc, Marque, and Wasmer (2008). But they assume exogenous separation and focus on analytical properties of the model. Yashiv (2006), Rotemberg (2006), and Krause and Lubik (2007) examine quantitative properties of the large-firm model, but again, exogenous separation is assumed.

[3]Elsby and Michaels (2008) actually compute transition dynamics of their model. However, they assume,

Haltiwanger, and Willis (2007) considers a similar environment and the authors do examine quantitative properties of the model including aggregate dynamics as well as its cross-sectional implications. However, their focus is again on the volatility puzzle. Furthermore, wages are derived under the assumption that firms make a take-it-or-leave-it offer, implying that employed workers obtain zero surplus.[4]

This paper is also related to the long-standing literature on aggregate implications of the economy with a large number of heterogeneous establishments. We do not attempt to provide an exhaustive review of this literature. We instead focus on recent papers that are most relevant to our paper.[5] Campbell and Fisher (2000) analyze the effects of hiring and firing costs on the dynamics of job creation and destruction rates. They, however, assume the competitive labor market so that there is no unemployment in their model. Further, the aggregate uncertainty takes the form of shocks to the aggregate wage rate, which is exogenous to their model. Veracierto (2008) studies a similar environment but solves for the full stochastic general equilibrium in which all prices are endogenous. However, he focuses on the effects of firing taxes and again assumes the competitive labor market, so that no unemployment exists (see also Samaniego (2008)).

In summary, none of the existing papers attempts to match the behavior of both worker flows and job flows simultaneously. As mentioned above, the recent search/matching literature mostly focuses on worker transition rates, whereas the literature on heterogeneous establishments studies job flows. We take a step toward integrating the two branches of the literature.

As mentioned above, our model is based on Elsby and Michaels (2008) but differs from theirs in quantitatively important ways. First, we allow for exogenous worker turnover, which plays a critical role in our quantitative exercises. Second, we introduce the firing cost, which is incurred when the firm endogenously sheds its workers. We calibrate the model at weekly frequency and then construct quarterly job flows and monthly worker flows following the same procedures used by the BLS.

We show that the dynamic stochastic equilibrium of the model successfully replicates important cyclical features of worker flows and job flows. In particular, the model correctly predicts that hires from unemployment move countercyclically while job creation moves procyclically. An important assumption for achieving this result is that workers separated due to endogenous job destruction go to the unemployment pool. This creates countercyclical job destruction as well as the worker flow into unemployment. When the negative aggregate shock hits the economy, the separation rate into unemployment increases while the job finding rate drops, as is the case in the standard search/matching model. The hiring flow from the unemployment pool rises as a result of increasing unemployment in the face of the negative shock. The countercyclicality of worker flows between employment and unemployment is consistent with the empirical liter-

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among other restrictive assumptions, that the aggregate shock is nonstationary. In contrast, we use a stationary aggregate shock. This is important because we can then apply the Krusell-Smith algorithm originally developed to solve for a *stationary* equilibrium of heterogenous consumer models.

[4]Another limitation of Cooper, Haltiwanger, and Willis (2007) is that important parameters are estimated to match volatilities of unemployment and vacancies, and thus it is unclear whether their result arises from the model's internal magnification mechanism or not.

[5]Important earlier contributions in this area can be found in the references of the papers discussed in this paragraph.

ature.[6] However, an important observation to make is that the flow into unemployment takes up less than one-third of establishment-level total separations. We assume that the remaining part of separations occurs at a constant exogenous rate. The hiring flow associated with these exogenously separated workers moves procyclically because, for those workers, the movement of the job finding rate is the only factor affecting the hiring flow. Given the presence of the large procyclical hiring flow, job creation, which counts all hiring flows, becomes procyclical. This finding points to importance of heterogeneity in labor market flows. We underscore this point by showing that the model with either endogenous (countercyclical) separation only or exogenous (constant) separation only cannot replicate the cyclicality of both flows simultaneously.

We also find that the model is able to generate large volatilities of labor market variables. Our benchmark calibration implies that the outside benefit parameter is at 83% of average labor productivity.[7] This is much smaller than the value of 96% that is needed in Hagedorn and Manovskii (2008) to fully account for volatilities of unemployment and vacancies.[8] In the literature of the volatility puzzle, the model's volatility is often examined through the steady-state comparative statics. However, impulse response functions from our model reveal that the dynamic stochastic equilibrium implies large deviations from the steady state at least in the short run and that magnification of the shock largely come from such deviations.

In the next section, we review the business-cycle facts about job flows and worker flows. Section 3 lays out the model, and Section 4 provides some useful characterizations of the model. In Section 5, we exert an effort in calibrating the model as tightly as possible by referring to the employment growth distribution as well as first moments for worker flows and job flows. Section 6 gives a brief description of the computational method used to solve for the dynamic stochastic equilibrium of the model. The details on the algorithm are given in Appendix B. Section 7 discusses the main results of this paper and also points out some problems of the model. We then conduct the sensitivity analysis in Section 8 with respect to three alternative calibrations. We find that our main results are largely intact with respect to those alternative calibrations.

Having established that the model replicates key dynamic features of worker flows and job flows, Section 9 discusses three important applications using the model. First, we assess the extent to which different data collection procedures influence cyclicality of worker flows and job flows. Specifically, we calibrate and solve the model at weekly frequency and thus are able to assess the extent of time aggregation biases that may pertain to observed job flows and worker flows, which are, respectively, collected at quarterly and monthly frequencies. The second application focuses on the effects of shutting down the endogenous job separation decision on measured job destruction. Since job destruction is measured from net employment changes over a quarterly period, it can fluctuate reflecting firms' hiring decision (even with no endogenous job separation) and our exercise intends to measure how large this effect is. The issues addressed in these two applications are important to our understanding of the labor market dynamics,

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[6]See for example Fujita and Ramey (2006) and Fujita (forthcoming).

[7]Our calibration strategy leaves no degree of freedom for the outside option parameter.

[8]Elsby and Michaels (2008) emphasize the feature that downward sloping labor demand makes the surplus size endogenous in this environment and argue that the model does a better job of magnifying the shock. The idea is that even though the marginal surplus, which is important for magnification, is small, the average surplus can be large. The same intuition applies to our results as well.

especially because some researchers have posed skepticism on the usefulness of job flows as empirical measures of labor flows. Lastly, we examine the extent of nonlinearity and asymmetry in the aggregate dynamics of our model. Section 10 concludes the paper by offering promising future research avenues identified by our results.

# 2    Facts

In this section, we review the definitions of worker flows and job flows and then summarize the cyclical properties of those series.

## 2.1    Definitions

**Job flows.**    The job flow series are measured from the Business Employment Dynamics (BED) data, which are based on the administrative records of the Quarterly Census of Employment and Wages (QCEW). The coverage of the QCEW is very broad, representing 98% of employment on nonfarm payrolls. The administrative records are linked across quarters to provide a longitudinal history for each establishment. The linkage process allows the tracking of net employment changes at the establishment level, which in turn allows calculating net employment gains at opening and expanding establishments and net employment losses at closing and contracting establishments.

The measures of job flows were originally developed by Davis, Haltiwanger, and Schuh (1996): job creation (destruction) is defined as the sum of net employment gains (losses) over all establishments that expand (contract) or start up (shut down) between the two sampling dates. Since we are interested in business cycle fluctuations of the series, we use the series that trace net employment changes over a quarterly period. Normalizing creation and destruction by aggregate employment yields *rates* of job creation and destruction, respectively.[9] In this paper, we use the term "job flows" to represent "rates" unless otherwise specifically mentioned. Job flow series are one of the most widely-used measures to gauge the "churning" of the economy from the perspective of firms. The sample period of the job flow series starts at 1992Q3 and ends at 2008Q2.[10]

**Worker flows.**    Similar but different measures can be constructed based on changes in the labor market status of workers. The Current Population Survey (CPS) polls a large number of workers each month, ascertaining whether they are employed and, if nonemployed, whether or not they engaged in active job search activities (i.e., unemployed) over the preceding month. It is the official survey that underlies well-known statistics such as the unemployment rate and the employment-to-population ratio. While the CPS is designed to provide a snapshot of the U.S. labor market for each month, one can use its longitudinal component to obtain measures of worker

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[9]More precisely, average employment between the beginning and the end of the quarter is used for normalization.

[10]Unfortunately, the series only go back to the early 90s. Longer time series are available for the manufacturing sector. In terms of their cyclicality, job flows for the whole economy and the manufacturing sector behave similarly over the overlapping period. However, levels of job flows and their volatilities are significantly different.

flows. We use the flow series constructed by the BLS.[11] Our analysis focuses on gross worker flows and transition rates between employment and unemployment, although in the calibration section, we also discuss transitions into and out of the out-of-the-labor-force pool. Gross worker flows based on the CPS come from the comparison of the labor market status at each monthly survey. To be specific, transition rates between employment and unemployment are, respectively, measured by:

$$\frac{EU_{t-k,t}}{E_{t-k}} \text{ and } \frac{UE_{t-k,t}}{U_{t-k}},$$

where $EU_{t-k,t}$ ($UE_{t-k,t}$) refers to the number of workers who switch their labor market status from "employed" ("unemployed") to "unemployed" ("employed") between week $t-k$ and $t$. The value of $k$ takes either 4 or 5, depending on the calendar. We call the former the separation rate and call the latter the job finding rate. The numerators $EU_{t-k,t}$ and $UE_{t-k,t}$ are what we call worker flows.

## 2.2 Measurement in the Search/Matching Models

The literature has explored whether the search/matching model is able to replicate the business cycle features of the U.S. labor market. Because the model is silent about which data to refer to in evaluating the model's quantitative performance, some researchers have used job flows, whereas others have considered worker transition rates. For example, Mortensen and Pissarides (1994), Cole and Rogerson (1999), and den Haan, Ramey, and Watson (2000) all evaluate the model's performance with respect to job flows. On the other hand, probably starting with Shimer (2005), the literature's focus has shifted toward fluctuations of worker transition rates, in particular, the job finding rate of unemployed workers.

However, as we will see, job flows and worker flows behave quite differently over the business cycle, meaning that the canonical models of labor search/matching are inherently unable to explain both flows simultaneously. Furthermore, it seems misleading to evaluate the quantitative performance of the model that does not have a notion of "firm," with respect to the data measured from a firm's perspective.

## 2.3 Business Cycle Statistics

**Unimportance of entry and exit.** First, consider Figure 1 where we plot the time series of job flows in the private business sector. In the figure, we show not only the total rates of job creation and destruction but also their breakdowns into expansion, entry, contraction, and exit. The intention of this figure is to show unimportance of the extensive margins for the business-cycle fluctuations of job flows. According to the data, roughly 75% of total job flows come from expansion or contraction of the existing establishments. More important, cyclical fluctuations of total job flows are mostly accounted for by expansion or contraction. For instance, the correlation

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[11]The data are available at www.bls.gov/cps/cps_flows.htm. Fujita and Ramey (2006) also construct worker flow series that are comparable to the BLS series. The cyclicality of the two data sets is very similar. See Fujita and Ramey (2006) for data construction details and measurement issues in the CPS.
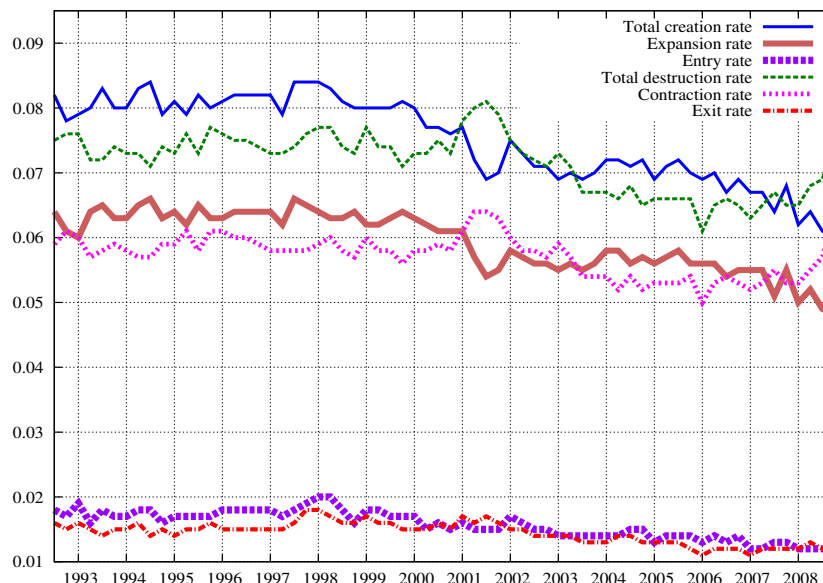
**Figure 1: Job creation and destruction rates**

**Notes**: The data are taken from the BLS Business Employment Dynamics and cover the private business sector. See Subsection 2.1 for definitions of the series.

between the total job creation (destruction) rate and the expansion (contraction) rate is higher than 0.95. This fact is important because our model developed in the next section does not feature extensive margins. The statistics below are thus calculated using the expansion rate and contraction rate. From here on, we use the terms "job creation rate" and "job destruction rate" to represent the expansion rate and contraction rate, respectively.

**Business-cycle statistics.** Table 1 characterizes the cyclicality of job flows and worker flows using standard business-cycle statistics. The original series are logged and then detrended by using the HP filter with smoothing parameter of 1,600. As mentioned above, original worker flows and transition rates are monthly series. We render them quarterly by simple averaging so that we can examine their cyclicality on an equal footing with job flow series. The real GDP series is used as a cyclical indicator to judge each variable's volatility and cyclicality. We can summarize the characteristics of the labor market flows as follows.

- The separation rate into unemployment is countercyclical and the job finding rate is procyclical.

- The job finding rate is somewhat more volatile than the separation rate.[12]

- The separation flow is somewhat more volatile than the hiring flow.

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[12]Shimer (2007) and Hall (2005a) argue that the separation rate into unemployment is roughly constant over the business cycle. Fujita and Ramey (2006, 2009); Fujita (forthcoming); Elsby, Michaels, and Solon (2009); Canova, Lopez-Salido, and Michelacci (2007); and Yashiv (2007) argue otherwise.

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**Table 1: Business Cycle Statistics for Worker Flows and Job Flows**

|  | Volatility | Relative volatility | Corr. with output |
|---|---|---|---|
| Worker flows | | | |
|     E to U | 0.067 | 5.987 | −0.694 |
|     U to E | 0.057 | 5.061 | −0.468 |
| Transition rates | | | |
|     Separation rate | 0.073 | 6.488 | −0.739 |
|     Job finding rate | 0.086 | 7.685 | 0.772 |
| Job flows | | | |
|     Creation rate | 0.028 | 3.099 | 0.472 |
|     Destruction rate | 0.035 | 3.838 | −0.398 |
| Stocks | | | |
|     Unemployment rate | 0.129 | 8.018 | −0.827 |
|     Vacancies | 0.141 | 8.785 | 0.875 |

**Notes**: The first column gives the standard deviation of each series that is logged and HP filtered with smoothing parameter of 1,600. The middle column gives the standard deviation of each variable relative to that of real GDP. Sample periods are as follows. Worker flows and transition rates: 1990Q1–2009Q2. Job flows: 1992Q3–2008Q4. Unemployment and vacancies: 1951Q1–2009Q2. Worker flows and transition rates are calculated from the the BLS labor flow series that are available at www.bls.gov/cps/cps_flows.htm. The separation and job finding rates are based on transition rates between employment and unemployment. The original monthly series are converted into quarterly series by time averaging. The sample period for real GDP is adjusted to match the sample period of each variable.

- The job destruction rate is countercylical and the job creation rate is procyclical.

- The job destruction rate is somewhat more volatile than the job creation rate.

- Worker flows are more volatile than job flows.

Table 1 also shows volatilities of the unemployment rate and vacancies. As is well known in the literature, these two variables are quite volatile when compared with volatility of labor productivity. The same is true with respect to output volatility. Lastly, a well-known fact about the cyclicality of unemployment and vacancies, i.e., Beveridge curve, can also be observed from each variable's correlation with output.

# 3    Model

The model presented here is derived from the one developed by Elsby and Michaels (2008). Our model, however, differs from theirs in three important ways. First, we allow for exogenous
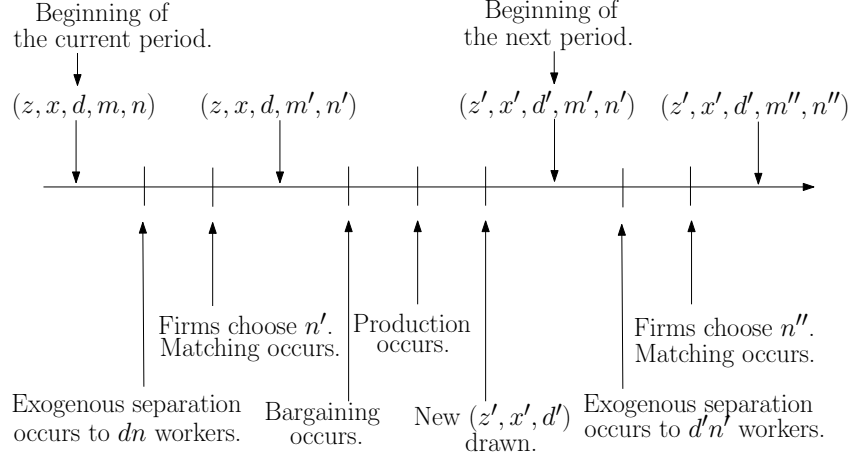
**Figure 2: Timing of events**

worker turnover, which turns out to play a critical role in our quantitative exercises. Second, we introduce the firing cost, which is incurred when a firm endogenously sheds its workers. Third, we specify idiosyncratic and aggregate productivity processes that are more general and plausible than theirs.

Time is discrete. There are two types of agents: firms and workers. Both are infinitely lived. The total measure of firms is normalized to one. The total measure of workers is denoted by $L$. The timing of events is summarized in Figure 2. Since we write down the model recursively, we drop time subscripts from all variables and follow the convention that primes and double-primes denote variables in the next period and the following period, respectively.

## 3.1 Firm

At the beginning of each period, a firm is characterized by $(x, d, n)$. The variable $x$ represents idiosyncratic productivity of a firm. The variable $d$ is the proportion of workers leaving the firm for exogenous reasons before the firm makes its employment decision. In other words, $d$ represents the size of *exogenous separations*. The variable $n$ is the number of workers employed at the firm. Let $m$ be the type distribution of firms. In addition, firms are affected by the aggregate productivity shock $z$. The aggregate state of the world is represented by $(z, m)$. We use $G_z(z'|z)$, $G_x(x'|x)$, and $G_d(d'|d)$ to represent the stochastic process of $z$, $x$, and $d$, respectively.

In each period, a firm (i) may lose part of its workers according to realization of the exogenous separation shock $d$, (ii) adjusts the number of workers (either by hiring or shedding), (iii) negotiates wage with its workers, and (iv) produces and pays the negotiated wage to the workers.

In the first stage, $dn$ workers leave the firm exogenously. The variable $d$ could be zero in which case no worker leaves the firm. Note that it is natural to assume that all firms lose workers at a constant rate every period, which is nested in our specification. However, as will be discussed in Section 5, our specification gives us a flexibility of matching a certain cross-sectional feature of the data. Since those workers are considered to leave the firm voluntarily, we assume that exogenous separation imposes no direct costs on the firm.

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In the next stage, the firm adjusts the size of employment. We assume that when the endogenous reduction of its employment size occurs, the firm incurs the "firing cost" $\tau$ per worker. Recall that the marginal cost of exogenous separation is zero. Therefore, with $\tau > 0$, a firm that desires to reduce its size of employment does not necessarily appeal to endogenous reduction of employment. Instead, it may choose to let its employment shrink through exogenous worker turnover. Hiring workers requires the firm to post vacancies. As is standard in the search/matching literature, it is assumed that it incurs the flow vacancy posting cost $\kappa$ for each vacancy posted. Each vacancy is filled with the job filling probability $q(z, m)$, which will be endogenized later. Because of the law of large numbers, the cost of hiring a worker turns out to be deterministic and $\frac{\kappa}{q(z,m)}$. Observe that the firm can always hire the exact number of workers it is willing to hire by taking into account the job filling probability.

In the production stage, the following production technology is available to all firms:

$$y = zxF(n'),\tag{1}$$

where $F' > 0$ and $F'' < 0$. A prime is attached to $n$ owing to our timing assumption. The negotiated wage is a function of both aggregate and individual states and thus is expressed as $w(z, x, d, m', n')$. The expected present discount value of the firm before the employment decision, $\Pi(z, x, d, m, n)$, can be represented as follows:

$$\Pi(z, x, d, m, n) = \max_{n' \geq 0} \left\{ zxF(n') - w(z, x, d, m', n')n' - \frac{\kappa}{q(z,m)} \max(n' - (1-d)n, 0) \right.$$

$$\left. -\tau \max((1-d)n - n', 0) + \beta \int \int \int \Pi(z', x', d', m', n') dG_d(d'|d) dG_x(x'|x) dG_z(z'|z) \right\},\tag{2}$$

where $m' = \Phi_m(z, m)$ is a law of motion of the type distribution of firms. The terms $\frac{\kappa}{q(z,m)} \max(n' - (1-d)n, 0)$ and $\tau \max((1-d)n - n', 0)$ capture the hiring and firing costs, respectively. Naturally, these costs are asymmetric. Notice also that workers at the same firm obtain the same wage $w(z, x, d, m', n')$. The Bellman equation above yields the optimal decision rule of the firm $n' = \phi_n(z, x, d, m, n)$.

## 3.2 Worker

Workers are engaged in one of two activities: producing or searching for a job. When working at the firm characterized by $(x, d, n')$ under the aggregate states $z$ and $m'$, the worker receives the bargained wage $w(z, x, d, m', n')$. In the following period, with probability $d'$, the worker separates from the firm exogenously. If exogenous separation does not occur in the next period, the worker is subject to the risk of endogenous separation. After separating from the firm, whether endogenously or exogenously, the worker starts looking for a job.

While the worker is looking for a job, he obtains the flow value $b$ per period. With probability $f(z', m')$, which we will characterize later, the worker will find a job and become employed in the next period. Let $W_e(z, x, d, m', n')$ and $W_s(z, m')$ be the values of being employed and looking

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for a job, respectively. The two values can be express by the following Bellman equations:

$$W_e(z, x, d, m', n') = w(z, x, d, m', n')$$

$$+ \beta \int \int \int \left( d' + (1 - d')\mathcal{I}_{n''<(1-d')n'} \frac{(1 - d')n' - n''}{(1 - d')n'} \right) W_s(z', m'')$$

$$+ (1 - d') \left( \mathcal{I}_{n''>(1-d')n'} + \mathcal{I}_{n''=(1-d')n'} + \mathcal{I}_{n''<(1-d')n'} \frac{n''}{(1 - d')n'} \right) W_e(z', x', d', m'', n'')$$

$$dG_d(d'|d)dG_x(x'|x)dG_z(z'|z), \quad (3)$$

$$W_s(z, m') = b + \beta \int (1 - f(z', m'))W_s(z', m'')dG_z(z'|z)$$

$$+ \beta \int \int f(z', m')W_e(z', x', d', m'', n'')dG_f(x', d', n'')dG_z(z'|z), \quad (4)$$

where $G_f(x', d', n'')$ represents the type distribution of hiring firms. $\mathcal{I}$ is an indicator function that takes the value 1 if the logical expression attached to it is true and takes the value zero otherwise. In Equation (3), the expression in front of $W_s(z', m'')$ summarizes all possibilities of separating from the firm. Specifically, the first term corresponds to the case with exogenous separation and the second term corresponds to the case in which the firm is reducing its workforce and the worker happens to be one of those workers. The expression in front of $W_e(z', x', d', m'', n'')$ summarizes all possibilities of staying with the firm. The first two terms represent the cases in which either the firm is expanding or inactive. The last term captures the possibility that the worker stays with the firm even though the firm is shrinking. In Equation (4), the second term on the right-hand side gives the expected value of failing to find a new job, and the third term gives the expected value of succeeding in a job search. Observe that the worker needs to take the expectation with respect to the firm type distribution as well as the aggregate shock.

## 3.3 Bargaining

Since production technology exhibits diminishing returns and the firm can employ multiple workers, bargaining is not as trivial as in the standard setting, which features bargaining between one worker and one firm. We adopt the bargaining solution proposed by Stole and Zwiebel (1996a,b), which naturally generalizes the Nash surplus sharing rule to the multiple-worker-firm setting. The bargaining outcome takes the form that total surplus is split between the firm and its workers according to the Shapley value of each agent.

Remember that since firms finished adjusting the number of workers at the timing of the wage negotiation, the hiring cost is sunk in the negotiation. The marginal surplus of a firm, which we denote $J(z, x, d, m', n')$, takes the following form:

$$J(z, x, d, m', n') = zxF'(n') - w(z, x, d, m', n') - w_n(z, x, d, m', n')n' + \beta D(z, x, d, m', n'), \quad (5)$$

where

$$D(z, x, d, n', m') = \int \int \int \Pi_n(z', x', d', m', n')dG_d(d'|d)dG_x(x'|x)dG_z(z'|z) \quad (6)$$

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is the expected marginal profit of the firm. The intra-firm bargaining results in the outcome that the marginal surplus is divided between the worker and the firm based on each party's bargaining weight. Letting $\eta$ be the bargaining power of the workers, the bargained wage $w(z, x, d, m', n')$ is implicitly characterized by the following rule:

$$(1 - \eta) \left[ W_e(z, x, d, m', n') - W_s(z, m') \right] = \eta \left[ J(z, x, d, m', n') + \tau \right]. \tag{7}$$

Note that the firing cost $\tau$ is added to the marginal value of the firm because the firm can avoid paying the firing cost by continuing the relationship (i.e., if the negotiation breaks down, the loss to the firm is $J(z, x, d, m', n') + \tau$).

## 3.4 Matching and Separation

Matching technology is characterized by an aggregate matching function $M = M(S, V)$, where $M$ is the number of new matches created, $S$ is the number of workers looking for a job, and $V$ is the number of vacancies posted. Notice that, when matching occurs, the type distribution of firms is represented by $m$. We can compute $S$ from $m$ and the total number of workers $L$ as follows:

$$S(m) = L - \int n \, dm. \tag{8}$$

When firms make a decision about hiring/firing, firms do not know $V$ a priori. Since knowing $V$ is crucial in forming expectation about the job filling rate $q(z, m)$, firms form expectation about $V$. We denote $V = \Phi_V(z, m)$ as the forecasting function used by all agents in the model economy to predict the number of vacancies posted when the aggregate state is $(z, m)$. In equilibrium, expected $V$ has to coincide with the realized $V$, which we denote $\widetilde{V}$. Using the optimal hiring/firing policy of firms, $\widetilde{V}$ can be computed as follows:

$$\widetilde{V} = \frac{\int \max(\phi_n(z, x, d, m, n) - (1 - d)n, 0) dm}{q(z, m)}. \tag{9}$$

Note that the max operator is used to count only the number of new jobs that firms create and that in order to predict $q(z, m)$ firms need to predict $V$.

The expected new matches created can be defined as follows:

$$M(z, m) = M(S(m), \Phi_V(z, m)). \tag{10}$$

The job finding probability $f(z, m)$ and the vacancy-filling probability $q(z, m)$ are, respectively, written as follows:

$$f(z, m) = \frac{M(z, m)}{S(m)}, \tag{11}$$

$$q(z, m) = \frac{M(z, m)}{\Phi_V(z, m)}. \tag{12}$$

The number of aggregate hires under the aggregate state $(z, m)$ is computed by:

$$\int \max(\phi_n(z, x, d, m, n) - (1 - d)n, 0)dm. \tag{13}$$

The number of aggregate exogenous separations is computed by:

$$\int dn \; dm. \tag{14}$$

The number of aggregate endogenous separations is computed by:

$$\int \max((1 - d)n - \phi_n(z, x, d, m, n), 0) \; dm. \tag{15}$$

Adding the two types of separations gives the number of aggregate total separations:

$$\int dn + \max((1 - d)n - \phi_n(z, x, d, m, n), 0) \; dm. \tag{16}$$

## 3.5 Equilibrium

We define the dynamic stochastic equilibrium of the economy as follows.

**Definition 1 (Recursive stationary equilibrium)**
*A recursive stationary equilibrium of the model economy consists of the value functions, $\Pi(z, x, d, m, n)$, $D(z, x, d, m', n')$, $W_e(z, x, d, m', n')$, $W_s(z, m')$, optimal decision rule $\phi_n(z, x, d, m, n)$, wage function $w(z, x, d, m', n')$, forecasting functions of the employment in the next period $\Phi_m(z, m)$, and number of vacancies $\Phi_V(z, m)$, such that:*

1. *Given forecasting functions and wage function, firms choose $\phi_n(z, x, d, m, n)$ optimally, and $\Pi(z, x, d, m, n)$ is the resulting value function, solving (2).*

2. *$D(z, x, d, m', n')$ is consistent with the optimal decision rule $\phi_n(z, x, d, m, n)$.*

3. *Given forecasting functions, wage function, and firms' optimal decision rules, $W_e(z, x, d, m', n')$ and $W_s(z, m')$ solve the Bellman equations (3) and (4), respectively.*

4. *$w(z, x, d, m', n')$ is the bargaining solution characterized by (7).*

5. *Forecasting function $\Phi_m(z, m)$ is consistent with the stochastic process of $x$ and $d$, and the optimal decision rule $\phi_n(z, x, d, m, n)$.*

6. *Forecasting function $\Phi_V(z, m)$ is consistent with the realized number of vacancies, which is implied by firms' optimal decision rule.*

# 4 Characterization

## 4.1 Optimal Hiring/Firing Rules

First, we characterize the optimal decision rule of firms and then the bargaining outcome. Even though the model does not have a simple analytical solution like the one derived by Elsby and Michaels (2008), the characterization greatly helps us solve the model numerically.

Using the recursive formulation of firms' expected present discount value of profits (5), the firm's optimal decision is characterized by the following first-order conditions with respect to $n'$:

$$zxF'(n') - w(z, x, d, m', n') - w_n(z, x, d, m', n')n'$$
$$- \frac{\kappa}{q(z, m)}\mathcal{I}_{n'>(1-d)n} + \tau\mathcal{I}_{n'<(1-d)n} + \beta D(z, x, d, m', n') = 0. \quad (17)$$

The indicator function is necessary because the marginal cost of adjusting $n'$ depends on whether (i) the firm is increasing employment after exogenous separation occurs $(n' > (1-d)n)$, or (ii) it is not changing the employment $(n' = (1-d)n)$, or (iii) it is shedding workers on top of exogenous separations $(n' < (1-d)n)$.

The first-order condition (17) is helpful in characterizing the optimal decision of the firms. First, notice that the only term that includes the current $n$ is the marginal adjustment costs of employment $\frac{\kappa}{q(z,m)}\mathcal{I}_{n'>(1-d)n}$ and $\tau\mathcal{I}_{n'<(1-d)n}$. This implies that the solution to the first-order condition is affected by the current $n$ only through the marginal adjustment costs. Since both $\frac{\kappa}{q(z,m)}$ and $\tau$ are positive, the left-hand side of the first-order condition in the case of $n' > (1-d)n$ can be obtained by shifting the left-hand side for $n' = (1-d)n$ downward, as in Figure 3. In the case of $n' < (1-d)n$, the left-hand side of the first-order condition is obtained by shifting the one for $n' = (1-d)n$ upward, again as in Figure 3. The solutions to the first-order condition corresponding to the cases of $n' > (1-d)n$ and $n' < (1-d)n$ are $\underline{n}^*$ and $\overline{n}^*$, respectively. Using $\underline{n}^*$ and $\overline{n}^*$, we can characterize the optimal decision rule $\phi_n(z, x, d, m, n)$ as follows:

$$zxF'(\overline{n}^*) - w(z, x, d, m', \overline{n}^*) - w_n(z, x, d, m', \overline{n}^*)\overline{n}^* + \tau + \beta D(z, x, d, m', \overline{n}^*) = 0, \quad (18)$$

$$zxF'(\underline{n}^*) - w(z, x, d, m', \underline{n}^*) - w_n(z, x, d, m', \underline{n}^*)\underline{n}^* - \frac{\kappa}{q(z, m)} + \beta D(z, x, d, m', \underline{n}^*) = 0. \quad (19)$$

Figure 4 presents the optimal decision rule for a given $(z, x, d, m)$. The optimal decision rule takes the form of the $(s, S)$ rule, with $[\underline{n}^*, \overline{n}^*]$ as the inactive region. When the current $n$ after exogenous separation $((1-d)n)$ is above $\overline{n}^*$, the firm reduces its employment to $\overline{n}^*$. When $(1-d)n$ is below $\underline{n}^*$, the firm increases its employment to $\underline{n}^*$. When $(1-d)n$ is in the inactive region, the firm lets its employment size decline with exogenous separation only. When the model does not feature exogenous separation, the diagonal line in Figure 4 becomes a 45-degree line.

In order to compute the optimal decision rule, we need the wage function, $w(z, x, d, m', n')$ and the firm's expected marginal value with respect to $n'$, $D(z, x, d, m', n')$. How can we compute $D(z, x, d, m', n')$? Using (6), the definition of $D(z, x, d, m', n')$, and the optimal decision rule that we just obtained, we can characterize the updating formula for $D(z, x, d, m', n')$ as follows:
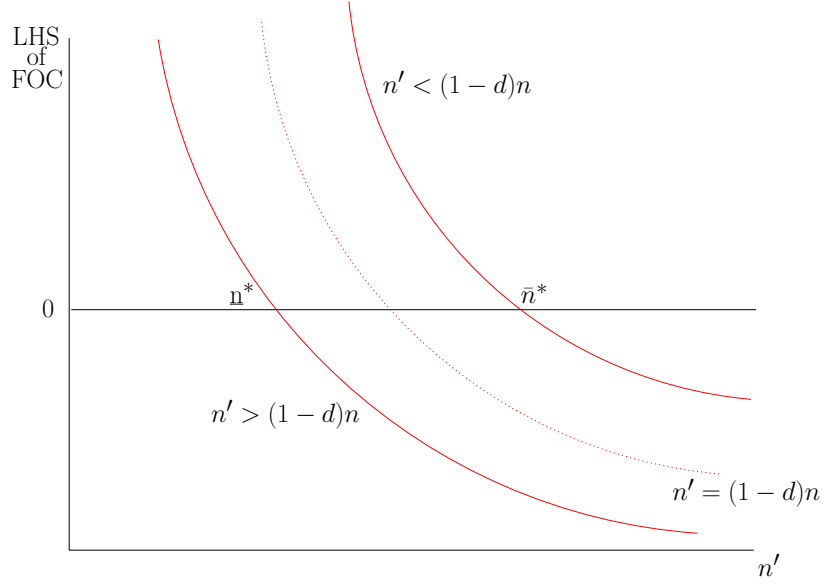
**Figure 3: First-order condition**

$$D(z, x, d, m', n') = \int \int \int \Pi_n(z', x', d', m', n') dG_d(d'|d) dG_x(x'|x) dG_z(z'|z), \tag{20}$$

where

$$\Pi_n(z', x', d', m', n') = (1 - d')$$

$$\begin{cases} -\tau & \text{if } \widetilde{n}' > \overline{n}^* \\ z'x'F'(\widetilde{n}') - w(z', x', d', m'', \widetilde{n}') - w_n(z', x', d', m'', \widetilde{n}')\widetilde{n}' + \beta D(z', x', d', m'', \widetilde{n}') & \text{if } \widetilde{n}' \in [\underline{n}^*, \overline{n}^*] \\ \frac{\kappa}{q(z', m')} & \text{if } \widetilde{n}' < \underline{n}^*, \end{cases}$$

and $m'' = \Phi_m(z', m')$, $\widetilde{n}' = (1 - d')n'$, $\overline{n}^*$ and $\underline{n}^*$ are characterized by equations (18), (19), respectively, for $(z', x', d', m')$. For the steady-state version of the model without exogenous separation and firing cost, Elsby and Michaels (2008) show that Equation (20) is a contraction mapping in $D$ and thus has a unique fixed point.

## 4.2 Bargaining Outcome

Combining (3), (4), (5), and (17) with the formula for the bargaining solution (7), we can obtain the following differential equation that governs the behavior of the bargained wage. See Appendix A.1 for derivation.

$$w(z, x, d, m', n') = (1 - \eta)b + \eta \left[ zxF'(n') - w_n(z, x, d, m', n')n' \right.$$

$$\left. + \tau \left( 1 - \beta + \beta \int d' \, dG_d(d'|d) \right) + \beta \int f(z', m') \left( \tau + \frac{\kappa}{q(z', m')} \right) dG_z(z'|z) \right]. \tag{21}$$

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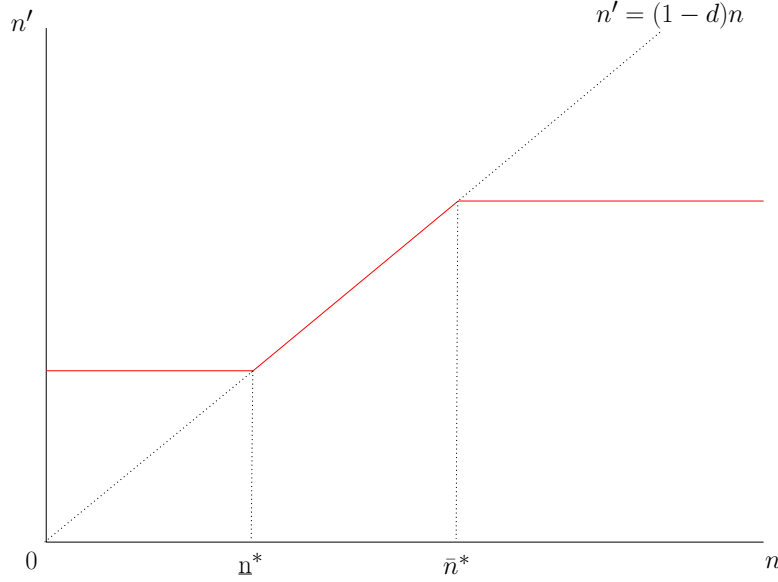
**Figure 4: Optimal $n'$**

If we further assume $F(n) = n^{\alpha}$, we can obtain the following closed form solution to the differential equation above.

$$w(z, x, d, m', n') = (1 - \eta)b + \eta \left[ \frac{zx\alpha n'^{\alpha - 1}}{1 - \eta(1 - \alpha)} \right.$$

$$\left. + \tau \left( 1 - \beta + \beta \int d' \, dG_d(d'|d) \right) + \beta \int f(z', m') \left( \tau + \frac{\kappa}{q(z', m')} \right) dG_z(z'|z) \right]. \quad (22)$$

See Appendix A.2 for derivation. Equations (21) and (22) generalize the wage equations derived by Elsby and Michaels (2008) to the case in which the firing cost and exogenous worker turnover are present.

# 5  Calibration

This section discusses the benchmark calibration of the model. The model is calibrated at weekly frequency. High frequency calibration is essential since it allows us to emulate the empirical measurement. With respect to the parameters for which we do not have tight identifying restrictions, we will conduct the sensitivity analysis in Section 8.

## 5.1  Functional Forms

We assume that the matching function takes the Cobb-Douglas form:

$$M = M(S, V) = \mu S^{\psi} V^{1-\psi}. \quad (23)$$

Let us denote the ratio of job seekers to vacancies (i.e., market tightness) by $\theta$. The production function for individual firms is assumed to take the simple functional form that exhibits decreasing returns to scale with $\alpha < 1$:

$$y = zxF(n) = zxn^\alpha. \tag{24}$$

Both the aggregate and idiosyncratic productivity shocks follow standard AR(1) processes:

$$z' = \rho_z z + \epsilon_z, \tag{25}$$

$$x' = \rho_x x + \epsilon_x, \tag{26}$$

where $\epsilon_z \sim N(0, \sigma_z^2)$, and $\epsilon_x \sim N(0, \sigma_x^2)$.[13] The exogenous separation probability $d$ is chosen to follow the following process:

$$d = \begin{cases} \widetilde{d} & \text{with probability } p_d, \\ 0 & \text{with probability } 1 - p_d, \end{cases} \tag{27}$$

where $p_d$ denotes the probability that exogenous separation occurs at each firm. Conditional on the firm being hit by the shock, each worker faces the separation probability of $\widetilde{d}$. As we will see later in this section, this process gives us the flexibility of matching a certain feature of the employment growth distribution.[14]

## 5.2    Measurement of Labor Market Flows

Measurement of worker flows and job flows plays an important role in our quantitative exercises. We need to address two important issues here.

First, job flows are measured from establishment-level net employment changes over a quarterly period as discussed in Section 2. Worker flows are measured from changes in the labor market status over a monthly period. The difference in frequency of data collection can cause differences in cyclical properties of job flows and worker flows. To deal with this issue, we solve the model at weekly frequency, and the simulated weekly observations are compiled in the same way as the BLS does. This way we can assess the extent to which differences in the cyclicality of the observed series are accounted for by the different measurement practices.

The second issue is that worker flows between unemployment and employment are only a part of all worker flows. In the literature, the attention has often been focused on this flow, mainly because researchers are interested in fluctuations in the unemployment rate. However, establishment-level data in general include all worker flows. More specifically, establishment-level total separations (hires) consist of three types of flows (i) separations into (hires from) the unemployment pool, (ii) separations into (hires from) the out-of-the-labor-force pool, and (iii)

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[13]The search/matching literature, including Elsby and Michaels (2008), often uses the memoryless process for idiosyncratic productivity. Adopting such a process often helps obtain an analytical solution while maintaining persistence of the idiosyncratic shock. However, this process is inconsistent with evolution of establishment-level productivities.

[14]An alternative is to assume $p_d = 1$, so that all firms shed workers at the same exogenous rate. Our specification nests this alternative but the model is not able to match the feature with $p_d = 1$.

separations into (hires from) other employers. Obviously, these three types of worker flows affect the behavior of job flows, which are measured from establishment-level net employment changes.

This paper's interest is to simultaneously account for the cyclicality of worker flows between employment and unemployment as well as job creation and destruction. To this end, we take the following strategy. First, we calibrate the model such that hires and separations that occur at the establishment level include all three types of flows. We then make the following three assumptions: (a) workers separated due to endogenous employment reduction go to the unemployment pool; (b) two other types of separations are lumped together into exogenous separation; and (c) all job seekers face the same job finding rate.

First, note that these assumptions entail the implication that the size of job seekers is expanded to accommodate hires that do not go through the unemployment pool. Importantly, we do so without introducing further heterogeneity into the model, such as participation and/or on-the-job search decisions.[15] We now discuss the three assumptions in detail. First, separations associated with employer-initiated contractions of establishment size are often labeled as "layoffs," and a plausible presumption is that those workers are more likely to go to the unemployment pool.[16] As shown, for example, by Fujita and Ramey (2009), the separation rate into unemployment is strongly countercyclical in the data and the behavior of endogenous separations generated from the model is consistent with this empirical observation.

As for the assumption (b), note first that the separation rate into the out-of-labor-force pool is nearly acyclical. Our assumption of the constant separation rate is consistent with this empirical evidence.[17] The remaining part of exogenous separations corresponds to the job-to-job flow. Note that there are no direct transitions from one job to another in our model. The model does, however, generate job-to-job transitions by way of time aggregation: The model is simulated at weekly frequency but the data are collected at monthly frequency. Those who separate and find jobs within a month show up as the job-to-job flow. The job-to-job transition rate in our model indeed exhibits procyclicality (as in the data) owing to the procyclical job finding rate.[18] Nagypál (2008) plots the monthly time series of the job-to-job transition rate calculated from the SIPP from the period of 1996 through 2003. According to the graph, it exhibits weak procyclicality as in our model.

The assumption (c) is made for the sake of simplicity. The assumption implies that we apply the job finding rate inferred from experience of unemployed workers to all separated workers. This simplifying assumption comes from the fact that there is no or limited evidence on the size of the pool of job seekers being out of the labor force or on the job. While there are many studies that emphasize the importance of on-the-job search, they provide no empirical evidence on the job finding rate for on-the-job seekers. For those who are out of the labor force, one could use

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[15]Previous prominent papers have also made similar assumptions. See, for example, Cole and Rogerson (1999), den Haan et al. (2000) and Cooper, Haltiwanger, and Willis (2007).

[16]A piece of supporting evidence is that, according to the "reasons for unemployment" data in the official CPS, of all workers who are unemployed due to "layoffs" or "quits," the fraction of the former is around 80%.

[17]We confirmed this by looking at the data constructed by Fujita and Ramey (2006), although they do not directly analyze the cyclicality of that flow in the paper.

[18]The model with on-the-job search tends to imply stronger procyclicality of the job-to-job transition rate (see for example Tasci (2007)). It is because of the procyclical job finding rate as well as the procyclical job-seeking activity. The latter mechanism is absent from our model but is rarely tested against the available evidence.

**Table 2: Benchmark Parameter Values: Weekly Calibration**

| Parameter | Value | Description |
|:---:|:---:|:---|
| $\psi$ | 0.5000 | Elasticity of matching function w.r.t. vacancies |
| $\alpha$ | 0.6700 | Curvature of production function |
| $\beta$ | 0.9990 | Time discount factor |
| $\eta$ | 0.7200 | Workers' bargaining power |
| $\mu$ | 0.1492 | Scale parameter of matching function |
| $\tau$ | 0.0894 | Firing cost |
| $p_d$ | 0.0406 | Prob. of firm-level exogenous separation shock |
| $\widetilde{d}$ | 0.2155 | Conditional prob. of exogenous separation for workers |
| $b$ | 0.4000 | Flow outside benefit (normalization) |
| $\rho_x$ | 0.9800 | Persistence of idiosyncratic shock |
| $\sigma_x$ | 0.0300 | Standard deviation of idiosyncratic shock |
| $\kappa$ | 0.0073 | Flow cost of posting a vacancy |
| $L$ | 11.472 | Labor force size |
| $\rho_z$ | 0.9957 | Persistence of aggregate shock |
| $\sigma_z$ | 0.0025 | Standard deviation of aggregate shock |

the pool of "want-a-job" workers. However, there is a good reason to believe that the flow from the out-of-labor force may not necessarily come from this pool of workers.[19] In any event, the pool of "want-a-job" workers is also strongly countercyclical; therefore, applying the same job finding rate to these workers does not change our results below in significant ways.

## 5.3 Parameter Values

Table 2 summarizes the parameters of the model. The weekly time discount factor is set to 0.999, which implies a quarterly interest rate of 1.2%. The elasticity of the matching function with respect to vacancies is set to 0.5. The available evidence on this parameter, summarized in Petrongolo and Pissarides (2001), varies widely across studies. The chosen value for the benchmark calibration is on the low side within the reasonable range. We later examine the sensitivity of our results with respect to a higher value. We do not have tight direct evidence on the bargaining power parameter. For the benchmark calibration, we simply use 0.72, the value used by Mortensen and Nagypál (2007). Again, the sensitivity with respect to the alternative value will be examined later.

Next, the curvature parameter of production is set to 0.67. The similar value is often used in the literature that looks at establishment-level employment dynamics using the same production technology (e.g., Campbell and Fisher (2000) and Cooper, Haltiwanger, and Willis (2007)). We

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[19]See Fujita and Ramey (2006) for details.

will conduct the sensitivity analysis along this dimension as well. The AR(1) coefficient of the aggregate productivity process is set such that quarterly first-order autocorrelation coincides with 0.95 ($\approx 0.9957^{12}$). The calibration of the standard deviation of the aggregate shock $\sigma_z$ is discussed later.

**Parameters set internally.** We target the following labor market statistics to select some of the remaining parameters. First, we target the monthly job finding rate of job seekers at 25%. This roughly corresponds to the historical average of the monthly transition rate from unemployment to employment.[20] Given the monthly-level target, we set the target for the weekly job finding rate $f(\theta)$ at 6.75%. The weekly job filling rate $q(\theta)$ is targeted at 33%, the value used by Ramey (2008), which is in turn based on the study by Barron, Berger, and Black (1997). These two target values for $f$ and $q$ pin down steady-state labor market tightness $\theta$ at 0.205. We can then calculate the scale parameter of the matching function $\mu$ through $\frac{f}{\theta^\psi}$.

The steady-state endogenous separation rate in the model, which, as discussed above, corresponds to the employment-to-unemployment transition rate. We set its target to 1.5% at monthly frequency, which roughly corresponds to its historical average. The steady-state total separation rate is targeted at 5% at monthly frequency. The JOLTS (Job Openings and Labor Turnover Survey) reports the monthly total separation rate, which is much smaller than 5%. However, as shown by Davis, Faberman, Haltiwanger, and Rucker (2008), the JOLTS data seriously underestimate the level of the total separation rate. These authors adjust the JOLTS series by using the more comprehensive BED data and show that the time-series average of the adjusted JOLTS data is about 5% from the period of January 2001 through December 2006. Given the target levels of the endogenous separation rate and the total separation rate, the exogenous separation rate is targeted at 3.5% ($= 5\% - 1.5\%$) at monthly frequency. Accordingly, the weekly level exogenous separation rate, $p_d\widetilde{d}$, is chosen to be 0.875% ($= \frac{3.5\%}{4}$). We will determine $p_d$ to be 0.0406 below, and $\widetilde{d}$ is thus set equal to 0.216.

The parameters for the idiosyncratic productivity process, $\rho_x$ and $\sigma_x$, and frequency of the exogenous separation shock $p_d$ are identified by using the establishment-level information on which the three parameters have strong influence. First, the aggregate job flow rate is aimed at around 8%, which roughly corresponds to the historical average of the private-sector job creation and destruction rates. Second, the average one-quarter persistence measure of the job creation rate is at around 0.7. This statistic is proposed by Davis, Haltiwanger, and Schuh (1996) and measures the percentage of newly-created jobs at time $t$ that remain filled at the next sampling date one quarter later. They report that the historical average of this measure for the manufacturing sector over the period of 1972Q2 through 1988Q4 is 0.678.[21] Lastly, we also use a piece of evidence on the employment growth distribution, which is reported by Davis, Faberman,

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[20]See Fujita and Ramey (2006) for the time-series behavior of the series. The level of the job finding rate does not take into account time aggregation error pointed out by Shimer (2007). But this issue is not relevant here because our data collection procedure is exactly the same as the one used by the BLS.

[21]Unfortunately, empirical evidence on this measure is available only for the manufacturing sector. The persistence measure of job destruction is defined similarly as the percentage of newly-destroyed jobs at time $t$ that do not reappear at the next sampling date. Davis, Haltiwanger, and Schuh (1996) report that job destruction persistence in manufacturing is 0.723 over the same period.

Haltiwanger, and Rucker (2008). Specifically, we target the fraction of establishments that have no employment change at 15.7%. Recall our assumption that at the establishment level, exogenous separation occurs only with probability $p_d$. This specification is adapted to match this statistic.[22] Assigning the three parameters to match the three statistics yield $\rho_x = 0.98$, $\sigma_x = 0.03$, and $p_d = 0.0406$.

The three parameters $b$, $\kappa$ and $L$ are determined as follows. First, note that one of the three parameters can be set at an arbitrary value as normalization. Accordingly we set $b$ equal to 0.4.[23] The remaining two parameters are set such that the model matches steady-state levels of labor market tightness $\theta$ and the endogenous separation rate given all other parameter values. Through this process, we obtain $\kappa = 0.0073$ and $L = 11.472$.

Finally, determining the remaining two parameters, $\sigma_z$, and $\tau$, we appeal to second moment properties of the model. First, we set $\sigma_z$ at the level (0.0025) that delivers the aggregate output volatility of roughly 2%. To see how $\tau$ is set, note that $\tau$ and the average hiring cost ($\frac{\kappa}{q}$) have strong influences on the firm's dynamic hiring/firing decision. The latter has already been pinned down and we thus choose $\tau$ to match the *relative* volatility of the job destruction rate and job creation rate. The implied level of this parameter (0.894) turns out to be quite low. It amounts to 25% of average *weekly* wage. This estimate is consistent with the empirical evidence that the firing cost in the U.S. is very low.[24]

## 5.4   Steady-State Properties

Before discussing the dynamic properties of the model, let us first look at the performance of the model in the steady state. Table 3 compares the model's steady-state values with corresponding target values. While we are unable to achieve an exact match due to the model's nonlinearity, the model delivers the steady-state values broadly in line with the empirical targets.

We also examine the model's cross-sectional implications. Table 4 compares the employment growth distributions based on the simulated data and the actual data. Recall that we select the parameter values (in particular, $p_d$) to match the fraction of establishments with no net employment change. The table shows that the model has difficulties in replicating some of the features of the empirical growth distribution. In particular, there are too many establishments in the model making large employment changes, i.e., more than a 20% increase or reduction of employment. We conjecture that this problem stems from the structure of the adjustment costs in our model: In the model, the labor adjustment costs are linear in the number of hiring

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[22]The case with $p_d = 1$ is unable to match the statistic because exogenous separations always result in declines in employment even when the firm does not actively change the employment size.

[23]This level by itself has no particular meaning in our paper. In the context of the volatility puzzle, what is relevant is its relative level to average labor productivity, which we will discuss shortly.

[24]Despite the empirical evidence, the literature on the effects of the firing cost on labor market dynamics typically uses much higher values. For example, Campbell and Fisher (2000) set the firing cost equal to 50% of the *quarterly* wage in their benchmark calibration. Hopenhayn and Rogerson (1993) and Veracierto (2008) also use similar values. The interpretation offered by Campbell and Fisher is that it corresponds to the cost of destroying the job position. It is thus possible that the firm incurs no cost of replacing workers for the same position but incurs a larger cost in getting rid of the position itself. However, our model does not have a distinction between worker turnover and job-position turnover. The correct interpretation of the firing cost in our model is thus the cost associated with worker turnover, which is empirically low.

**Table 3: Steady-State Implications: Benchmark Calibration**

|  | Data collection frequency | Empirical target | Model |
|---|---|---|---|
| Worker-side data |  |  |  |
|     Separation rate | Monthly | 0.015 | 0.014 |
|     Job finding rate | Monthly | 0.250 | 0.242 |
|     Unemployment rate | Monthly | 0.057 | 0.053 |
| Establishment-side data |  |  |  |
|     Total separation rate | Monthly | 0.050 | 0.045 |
|     Job flow rates | Quarterly | 0.080 | 0.081 |
|     Job flow persistence measure | Quarterly | 0.710 | 0.730 |
| $b/$(average labor productivity) | – | – | 0.830 |

**Notes**: The persistence measure in the data is taken from Davis, Haltiwanger, and Schuh (1996), which covers the manufacturing sector for 1972Q2-1988Q4. The total separation rate is taken from Davis et al. (2008), which is based on the JOLTS data. The model-based moments are calculated by aggregating one million establishment-level observations. To calculate the model based moments, we follow the same data-collection procedures as those used in actual surveys.

or firing. In reality, such large employment adjustments may also require an adjustment of the capital stock, which could incur large fixed costs. The model completely abstracts away from such considerations. Integrating the type of model considered in this paper with, for example, the model of lumpy investment is beyond the scope of this paper at this point and thus left for future research.

# 6 Computation

We solve the model numerically since there is no analytical solution. Our solution method is based on the *partial information approach* developed by Krusell and Smith (1998). We make use of their method because our model faces the same problem of having an infinite dimensional aggregate state variable, namely, the type distribution of heterogeneous firms $m$. The essence of the approach is to limit the information that agents in the model use to a finite set of statistics summarizing the type distribution and transform the original problem to a tractable approximated problem. The approach is implemented by replacing the large state variable by a finite set of statistics that summarize the type distribution. Unlike the problem solved by Krusell and Smith (1998), we found that simple log-linear functional form is not sufficient to achieve a high accuracy of the approximation. We overcame the problem by introducing higher-order terms in the approximate forecasting functions. Appendix B contains details about the computational algorithm.

Table 4: Employment Growth Distribution

| Growth rate interval | | | Empirical | Model |
|---|---|---|---|---|
| | > | −0.20 | 0.076 | 0.159 |
| −0.20 | to | −0.05 | 0.167 | 0.194 |
| −0.05 | to | −0.02 | 0.097 | 0.044 |
| −0.02 | to | 0.00 | 0.078 | 0.023 |
| No change | | | 0.157 | 0.157 |
| 0.00 | to | 0.02 | 0.080 | 0.020 |
| 0.02 | to | 0.05 | 0.100 | 0.045 |
| 0.05 | to | 0.20 | 0.169 | 0.169 |
| 0.20 | < | | 0.076 | 0.190 |

**Notes**: The table reports employment shares for indicated intervals of the quarterly employment growth rate in the BED micro data from 2001 to 2006 and in the steady state of the model. The empirical distribution is taken from Davis et al. (2008) and is based on continuously existing establishments. The model-based growth distribution is calculated from one million establishment-level observations.

# 7 Main Results

## 7.1 Business Cycle Statistics

Table 5 summarizes the main results. The table presents volatilities of variables of our interest (panel (a)), the ratio of each variable's volatility to that of output (panel (b)), and the correlation with output. The first column presents the empirical moments and the second column lists the model-based moments based on the benchmark calibration. The remaining columns are discussed in the later section. All series are first logged and HP filtered with smoothing parameter of 1,600.

The model-based moments are calculated from a large panel that consists of 6,000 weekly observations (after discarding the first 120 weekly observations) across one million establishment-level observations. Aggregate time series for each series is obtained by mimicking the data collection procedures used in actual surveys. Monthly worker flows and transition rates are converted into quarterly series by time averaging.

In this section, we first discuss the comparison of the empirical moments with the model-based moments under the benchmark calibration.

**Volatilities.** The first two columns of panels (a) and (b) compare volatilities of variables of our interest. Given that our calibration does not target volatilities of labor market variables, the model does a reasonably good job in this regard, even though our model generates somewhat

smaller volatilities than those of empirical data. On the other hand, the model is able to mimic the empirical features that (i) the job finding rate is more volatile than the separation rate, (ii) the separation flow is more volatile than the hiring flow, and (iii) worker flows are more volatile than job flows.[25]

The last two rows of panel (a) and (b) also show that the model generates fluctuations of unemployment and vacancies that are somewhat smaller yet roughly comparable to those in the data. In Table 5, we evaluate the model's performance, taking output as a cyclical indicator. Recall, however, the literature on the volatility puzzle looks at magnification of labor market variables relative to fluctuations in labor productivity. If we take labor productivity as a cyclical indicator, volatilities of the model are quite close to those in the data.[26] In the standard search/matching model, it is well-known that the outside option parameter $b$ plays a key role for volatilities of the model. In particular, Hagedorn and Manovskii (2008) show in their setting that when the outside option parameter is set to the level close to labor productivity, the model exhibits large magnification. Recall that our calibration strategy leaves no degree of freedom of assigning parameters to match volatilities of labor market variables. Our benchmark calibration implies the level of $b$ that is 83% of average labor productivity, which is substantially lower than Hagedorn and Manovskii's value (96%). The reason is that, as Elsby and Michaels (2008) emphasize, with downward sloping labor demand, average surplus can be relatively large even though marginal surplus is small. An important point to note here is that Elsby and Michaels' claim is based on the steady-state elasticities. That is, they match the elasticities calculated from the regressions of labor productivity on unemployment and vacancies. On the other hand, our results are based on the simulations of the stochastic dynamic equilibrium: We match unconditional volatilities of these variables.

**Correlations.** Panel (c) of Table 5 considers the correlation pattern with respect to aggregate output. While the model is short of matching the comovement pattern exactly, the overall comovement pattern is consistent with the data. First, observe that the model replicates the countercyclical separation rates into unemployment and the procyclical job finding rate. Second, the model captures the overall differences in the cyclicality of worker flows and job flows. That is, the model generates the procyclical job creation rate and countercyclical job destruction rate while maintaining countercyclicality of worker flows. The fact that the model is able to capture countercyclicality of separations and the job destruction rate is not surprising; it is a direct consequence of the countercyclical separation rate into unemployment.[27]

Why is the job creation rate procyclical? The presence of the large hiring flow governed by the exogenous component of separations plays an important role. Note that, in the model, there is a flow of workers that separates from their employer independently of the firm-level idiosyncratic shocks and aggregate shock and that this flow occurs at a constant rate of aggregate employment.

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[25]Recall that we calibrate the model so that volatilities of creation and destruction rates are roughly equal to each other.

[26]In our benchmark calibration, the relative volatilities of unemployment and vacancies are, respectively, 15.9 and 18.4. These figures are actually somewhat higher than comparable figures reported in the literature.

[27]Strictly speaking, the countercyclicality of the destruction rate is less clear because the link with the separation rate is not tight. We will come back to this issue shortly.

## Table 5: Comparison of Business Cycle Properties

| | Empirical | Benchmark | Alternative calibration | | |
|---|---|---|---|---|---|
| | | | $\alpha = 0.4$ | $\eta = 0.5$ | $\psi = 0.6$ |
| (a) Standard deviation | | | | | |
| E-to-U flow | 0.067 | 0.077 | 0.042 | 0.092 | 0.080 |
| U-to-E flow | 0.057 | 0.043 | 0.023 | 0.052 | 0.046 |
| Separation rate | 0.073 | 0.078 | 0.043 | 0.093 | 0.082 |
| Job finding rate | 0.086 | 0.084 | 0.046 | 0.098 | 0.079 |
| Job destruction rate | 0.035 | 0.058 | 0.031 | 0.069 | 0.058 |
| Job creation rate | 0.028 | 0.059 | 0.030 | 0.068 | 0.055 |
| Unemployment rate | 0.129 | 0.107 | 0.059 | 0.127 | 0.108 |
| Vacancies | 0.141 | 0.124 | 0.066 | 0.144 | 0.157 |
| (b) Relative standard deviation | | | | | |
| E-to-U flow | 5.987 | 3.910 | 3.148 | 4.321 | 4.143 |
| U-to-E flow | 5.061 | 2.206 | 1.738 | 2.452 | 2.355 |
| Separation rate | 6.844 | 3.957 | 3.197 | 4.386 | 4.237 |
| Job finding rate | 7.685 | 4.287 | 3.412 | 4.612 | 4.084 |
| Job destruction rate | 3.838 | 2.963 | 2.302 | 3.233 | 2.999 |
| Job creation rate | 3.099 | 2.980 | 2.262 | 3.209 | 2.867 |
| Unemployment rate | 8.018 | 5.444 | 4.434 | 5.974 | 5.580 |
| Vacancies | 8.785 | 6.318 | 4.973 | 6.778 | 8.136 |
| (c) Correlation with output | | | | | |
| E-to-U flow | -0.694 | -0.280 | -0.377 | -0.287 | -0.358 |
| U-to-E flow | -0.468 | -0.344 | -0.338 | -0.387 | -0.467 |
| Separation rate | -0.739 | -0.452 | -0.538 | -0.454 | -0.510 |
| Job finding rate | 0.772 | 0.970 | 0.987 | 0.965 | 0.971 |
| Job destruction rate | -0.398 | -0.257 | -0.344 | -0.249 | -0.288 |
| Job creation rate | 0.472 | 0.124 | 0.192 | 0.102 | 0.095 |
| Unemployment rate | -0.827 | -0.985 | -0.977 | -0.983 | -0.985 |
| Vacancies | 0.875 | 0.875 | 0.915 | 0.866 | 0.906 |

**Notes**: See notes to Table 1 for the calculation of the empirical moments. The model-based moments are calculated from a large panel that consists of 6,000 weekly observations (after discarding the first 120 weekly observations) across one million establishment-level observations. Aggregate time series for each series is obtained by mimicking the data collection procedures used in actual surveys. Monthly worker flows and transition rates are converted into quarterly series by time averaging. The 500 quarterly observations are first logged and HP filtered. Panel (b) gives the ratio of standard deviation of each variable to that of output.

However, because of the lower job finding rate in downturns, hires from the non-unemployed pool of job seekers go down. In other words, the flow associated with the exogenous component of separations is procyclical. Furthermore, as we saw in the calibration section, a share of the separation flow into unemployment is less than a third of total separations. The job creation rate, which in principle counts all hiring flows, then becomes procyclical.[28]

Next consider the cyclicality of unemployment and vacancies. The model generates strong countercyclicality of unemployment and strong procyclicality of vacancies. The latter result is remarkable in the sense that the standard search/matching models with the endogenous separation decision (such as the model of Mortensen and Pissarides (1994) or its DSGE version by den Haan, Ramey, and Watson (2000)) are unable to replicate the strong procyclicality of vacancies. In our model, the correlation between the two variables, which is not reported in Table 5, is highly negative at $-0.86$.

**Endogenous vs. exogenous separation.** We have argued above that in oder to match the cyclical behavior of job flows and worker flows simultaneously, it is essential to feature two types of flows: worker flows (i) that are mainly influenced by countercyclical separation and (ii) that are dominated by the procyclical job finding rate. In our model, the former flows are associated with endogenous separation, while the latter flows are associated with the exogenous part of separation. To appreciate this point more fully, we consider the following two hypothetical economies in which separation occurs either for the exogenous reasons only or for the endogenous reason only.[29]

Table 6 reports correlations of hires and separations with output in these two cases. The first column of the table also presents correlations from the benchmark model, which features both types of separations. Observe that the model with exogenous separation only generates hires and separations that are both procyclical. Similarly, the model with endogenous separation only generates countercyclical flows. The model that features both types of worker flows countercyclical separations and procyclical hires simultaneously. This last point underlies our main result that our model successfully matches the cyclical behavior of job flows.

## 7.2   Impulse Responses

Figure 5 plots impulse responses of the model to the 1% negative aggregate shock. Note that in the figure, we distinguish between the number of job creation and destruction (Figure 5 (b)) and rates of job creation and destruction (Figure 5 (c)). First consider Figure 5 (a) where worker flows between unemployment and employment are plotted. Note first that, in the impact period, the separation flow into and the hiring flow from unemployment move in the opposite direction. However, the decline in the hiring flow is only one third of the increase in the separation flow. This is because some of the initial declines within the quarter are already reversed toward the

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[28]Again, there is an issue of time aggregation associated with the quarterly definition of job flows, but we will examine this issue below.

[29]Note that when the model features exogenous separation only, the firing cost ($\tau$) is zero under the aforementioned assumption that only endogenous separation incurs the firing cost. For the endogenous-separation-only case, we also assume $\tau = 0$, to be consistent with the exogenous-separation-only case.

<div align="center">

**Table 6: Endogenous vs.Exogenous Separations**

</div>

|             | Benchmark | Exogenous only | Endogenous only |
|-------------|-----------|----------------|-----------------|
| Separations | -0.130    | 0.503          | -0.448          |
| Hires       | 0.228     | 0.382          | -0.456          |

**Notes**: The table reports correlation of separations and hires with output in the model. See notes to Table 5 for construction of the statistics. In all three, the steady-state total separation rate is set equal to 4.5% at monthly frequency. The benchmark model features both endogenous and exogenous separation, and "separations" and "hires" in this case include all worker flows.
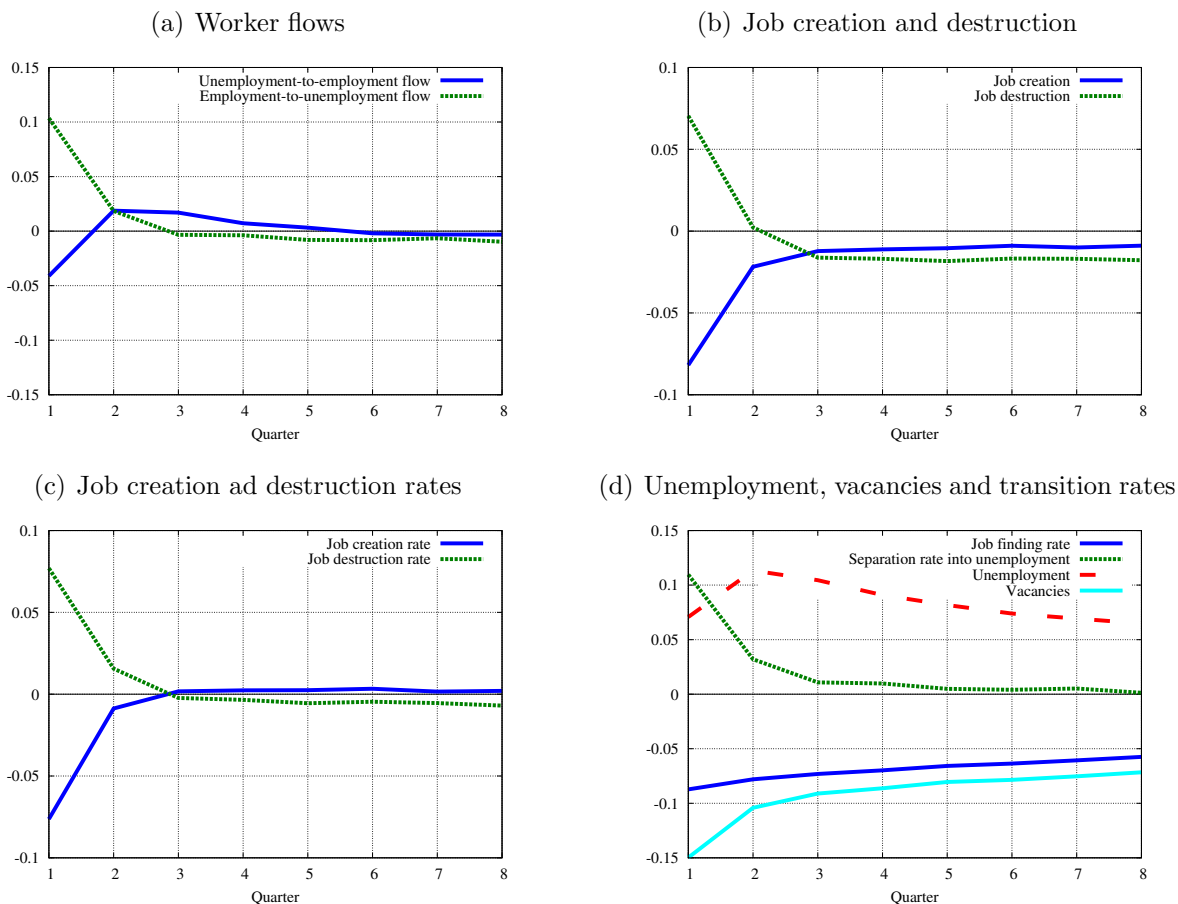
end of the quarter as the unemployment pool expands. In the second quarter, the initial decline in the hiring flow from unemployment is completely eliminated, reaching a higher level than the steady-state level. On the other hand, Figure 5 (b) shows that the initial decline in job creation is twice as large as the decline in the unemployment-to-employment flow and that the bounce-back of job creation from the second quarter on is much less pronounced.

0.59

Next, observe that the response of the stock of unemployment, shown in Figure 5 (d), exhibits a small hump. Note that the difference between separations and hires plotted in Figure 5 (a) corresponds to the change in unemployment. The behavior of worker flows implies that the changes in unemployment are positive only up to the second quarter. From the third quarter on, unemployment continues to decline.

We can draw two general important implications from the impulse responses. First, the analysis based on the dynamic stochastic equilibrium is quite different from the one based on the steady-state analysis. In the literature, magnification of the model is often inferred from the steady-state elasticity because "labor market turnover is fast." However, by definition, the steady-state analysis implies that labor market flows in the opposite directions are always equated. As is clear from the dynamics of the model shown in Figure 5, this condition is far from satisfied in the short run. In particular, the increase in unemployment in the impact quarter is largely explained by the difference between the hiring flow and separation flow.

Second, the impulse responses shown in the figure clearly show the lack of the propagation mechanism in the model. In particular, the effects on the endogenous separation rate mostly die away in two quarters; it reaches back to the steady-state level in the third quarter. Accordingly, the effects of the aggregate shock on the employment-to-unemployment flow as well as job destruction are concentrated in the first quarter. Empirically, the response of the separation rate and the associated flow is known to be much more persistent (see for example Fujita (forthcoming)). It is also known that the job destruction rate is more persistent in the data than in the model, even though the evidence is limited to the manufacturing job destruction rate (e.g., Davis and Haltiwanger (1999)). In other words, the model implausibly implies too much "cleansing effect" in the short run. Furthermore, the lack of persistence in the separation rate makes it dif-

**(a) Worker flows**

**(b) Job creation and destruction**

**(c) Job creation ad destruction rates**

**(d) Unemployment, vacancies and transition rates**

**Figure 5: Impulse responses in the baseline economy**

**Notes**: Plotted are responses to the $-1\%$ aggregate shock expressed as deviations from the steady-state levels. Job flows and job flow rates are calculated from the establishment-level net employment changes over a quarterly period. Quarterly worker flows, transition rates, unemployment, and vacancies are averages of monthly data. See also notes to Table 5.

ficult for the model to generate the empirically plausible unemployment response, which exhibits a much more pronounced hump-shape. Recall that in Subsection 5.4, we pointed out that our assumption of proportional labor adjustment costs may hinder the model's ability to match the tails of the employment growth distributions. We conjecture that the lack of persistence can be also traced to the assumption of proportional labor adjustment costs.

# 8  Sensitivity Analysis

In this section, we conduct the sensitivity analysis with respect to three alternative calibrations: (i) lower curvature parameter of the production function (i.e., 0.4), (ii) lower bargaining power

**Table 7: Parameter Values: Alternative Calibrations**

| Parameter | Alternative calibrations | | |
|:---:|:---:|:---:|:---:|
| | lower $\alpha$ | lower $\eta$ | higher $\psi$ |
| $\psi$ | 0.5000 | 0.5000 | **0.5900** |
| $\alpha$ | **0.4000** | 0.6700 | 0.6700 |
| $\beta$ | 0.9990 | 0.9990 | 0.9990 |
| $\eta$ | 0.7200 | **0.5000** | 0.7200 |
| $\mu$ | 0.1492 | 0.1492 | 0.1273 |
| $\tau$ | 0.1345 | 0.1564 | 0.0897 |
| $p_d$ | 0.0465 | 0.0406 | 0.0406 |
| $\widetilde{d}$ | 0.1882 | 0.2155 | 0.2158 |
| $b$ | 0.4000 | 0.4000 | 0.4000 |
| $\rho_x$ | 0.9900 | 0.9800 | 0.9800 |
| $\sigma_x$ | 0.0500 | 0.0300 | 0.0300 |
| $\kappa$ | 0.0222 | 0.0129 | 0.0074 |
| $L$ | 2.4327 | 8.1544 | 10.220 |
| $\rho_z$ | 0.9957 | 0.9957 | 0.9957 |
| $\sigma_z$ | 0.0330 | 0.0330 | 0.0330 |
| $b/$(ave. prod.) | 0.6350 | 0.7700 | 0.8300 |

of workers (i.e, 0.5), and (iii) higher elasticity of the matching function with respect to vacancies (i.e., 0.6). For each case, we re-calibrate the model following the same procedure as the benchmark calibration. In other words, the same moment conditions are maintained and thus each of the parameter changes also involves changes in other parameter values.[30] Table 7 presents all parameter values for these three cases.

## 8.1  Steady States

We find that steady-state properties of the model under these alternative calibrations are very close to those under the benchmark calibration. That is, under all three alternative calibrations we match the steady-state values targeted earlier with the benchmark calibration. We also obtain employment growth distributions that share basically the same properties as the one under the the benchmark calibration.[31] However, one dimension along which three calibrations differ is the ratio of the outside option parameter $b$ to average labor productivity (see the last row of Table 7). Remember that we do not have a degree of freedom to target this value. The implied ratios, therefore, arise as the endogenous outcomes of the calibrations. Naturally, the variations in the relative level of $b$ yield a different strength of magnification.

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[30]We, however, maintain the same parameter values for the aggregate TFP process.

[31]All steady-state results are available upon request.

## 8.2 Business Cycle Statistics

The last three columns of Table 5 give business cycle statistics under the three alternative calibrations. Overall, all three calibrations deliver a performance similar to the one under the benchmark calibration. One can notice, however, that volatilities of unemployment and vacancies tend to rise, moving from left to right. The changes in volatilities can be explained by the aforementioned differences in the relative level of $b$. First, lowering $\alpha$ underpredicts volatilities, since the calibration implied the level of $b$ that is 64% of average labor productivity. Note, however, that even with the low level of $b$, roughly 60% of the unemployment volatility can be explained. The calibration with lower bargaining power of workers generates larger volatilities than under the benchmark calibration. This is achieved even though this calibration implies a lower relative level of $b$. This suggests that giving lower bargaining power to workers raise volatilities for the same relative level of $b$, as is the case in the standard model.[32] The panel (c) of Table 5 presents correlations with output. While the comovement pattern varies under different calibrations, none of the key results from the benchmark calibration are overturned, suggesting the robustness of the model's quantitative property in this regard.

# 9 Applications

Having established that the model's quantitative properties are consistent with empirical observations, we now consider the following three experiments using the model. First, we examine the effects of time aggregation on the cyclicality of worker flows and job flows. The second experiment, which is related to the first one, examines the cyclicality of job flows, especially job destruction, when all separations are assumed to occur at an exogenous constant rate. Note that these two applications are very important for our understanding of the labor market dynamics, especially given that some researchers have posed skepticism on the usefulness of job flows as empirical measures of labor market churning.[33] Third, we analyze the extent of asymmetry and nonlinearity in the aggregate dynamics of the model.

## 9.1 Effects of Time Aggregation

One potential explanation for why job flows and worker flows behave differently could be traced to their different measurement practice. Specifically, job flows are measured from net employment changes over a quarterly period, whereas worker flows are measured from changes in workers' labor market status over a one-month period. Table 8 presents the business cycle statistics using quarterly averages of weekly job flows in comparison to those based on the actual definitions of job flows. Since job flows are identical to worker flows at weekly frequency, this comparison reveals the extent to which the measurement practice accounts for the differences in their cyclicality.

This table shows that the different data collection practice does not induce large systematic biases in the cyclicality of job flows. First, job flows based on weekly averages are less volatile

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[32]For the effect of changing the bargaining power parameter on the steady-state elasticity of market tightness, see for example Mortensen and Nagypál (2007).

[33]See, for example, Section 2.7 in Hall (2005b). Shimer (2007) also makes a similar point.

|  | Benchmark | Weekly average | Benchmark | Weekly average |
|---|---|---|---|---|
|  | Relative s.d. | | Corr. with output | |
| Job destruction rate | 2.963 | 2.390 | −0.257 | −0.294 |
| Job creation rate | 2.980 | 2.532 | 0.124 | 0.091 |

**Notes**: See notes to Table 5. For the numbers under "weekly average," we first obtain aggregate job creation and destruction rates at weekly frequency and then average them over the 12-week period. We use those "quarterly" series to calculate the reported statistics.

than the quarterly job flows, but the differences in volatilities are relatively small. Second, calculating job flows using weekly averages alters the correlation pattern with respect output only slightly. These results thus exclude the possibility that differences in cyclicality of worker flows and job flows are attributed to the data collection practice.

## 9.2 Job Destruction Rate with No Endogenous Separation

The second application entails usefulness of the job destruction rate as a measure of active employment reduction, which corresponds to endogenous job destruction in the model. Specifically, measured job destruction can be affected whenever an establishment reduces the stock of employment over a quarterly period, regardless of the cause of the employment change. Consider a case in which employment declines as a result of exogenous worker turnover that the firm decides not to replace immediately. This may be the optimal choice for the firm when laying off workers incurs large costs. If the firm follows this employment policy, measured job destruction loses its usefulness as a measure of active employment reduction. It is actually the decision of not hiring that causes job destruction to move.

To infer the upper-bound of this effect, we deliberately shut down the channel of endogenous job destruction to see how much the job destruction rate can fluctuate only through the channel of this no-replacement policy.[34] Table 9 presents the results from this exercise in comparison to the observed data and the benchmark model. Let us first start with panel (c) of the table. Interestingly, even without endogenous separation, the measured job destruction rate exhibits strong countercyclicality. This is because, during downturns, more establishments decide not to replace workers who left the establishment for exogenous reasons.

Consider now panel (b), which compares relative volatilities of the job destruction rate as well as other variables of interest. It shows that the volatility of the job destruction rate in the model

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[34]Although the model is based on the benchmark calibration, some adjustments of calibration are necessary. For example, we target the size of the total monthly separation rate at 5% as in the benchmark case. However, since all separations occur for exogenous reasons, we set $p_d \widetilde{d} = 0.0125$, instead of 0.00875. We then chose $p_d$ to achieve the fraction of establishments with no employment change at 0.157. This procedure results in $p_d = 0.115$ and $\widetilde{d} = 0.109$. Further, we reset the vacancy posting cost $\kappa$ at 0.035 to achieve the same level of $b$ relative to average labor productivity (83%) as in the benchmark calibration.

### Table 9: Experiment with No Endogenous Separation

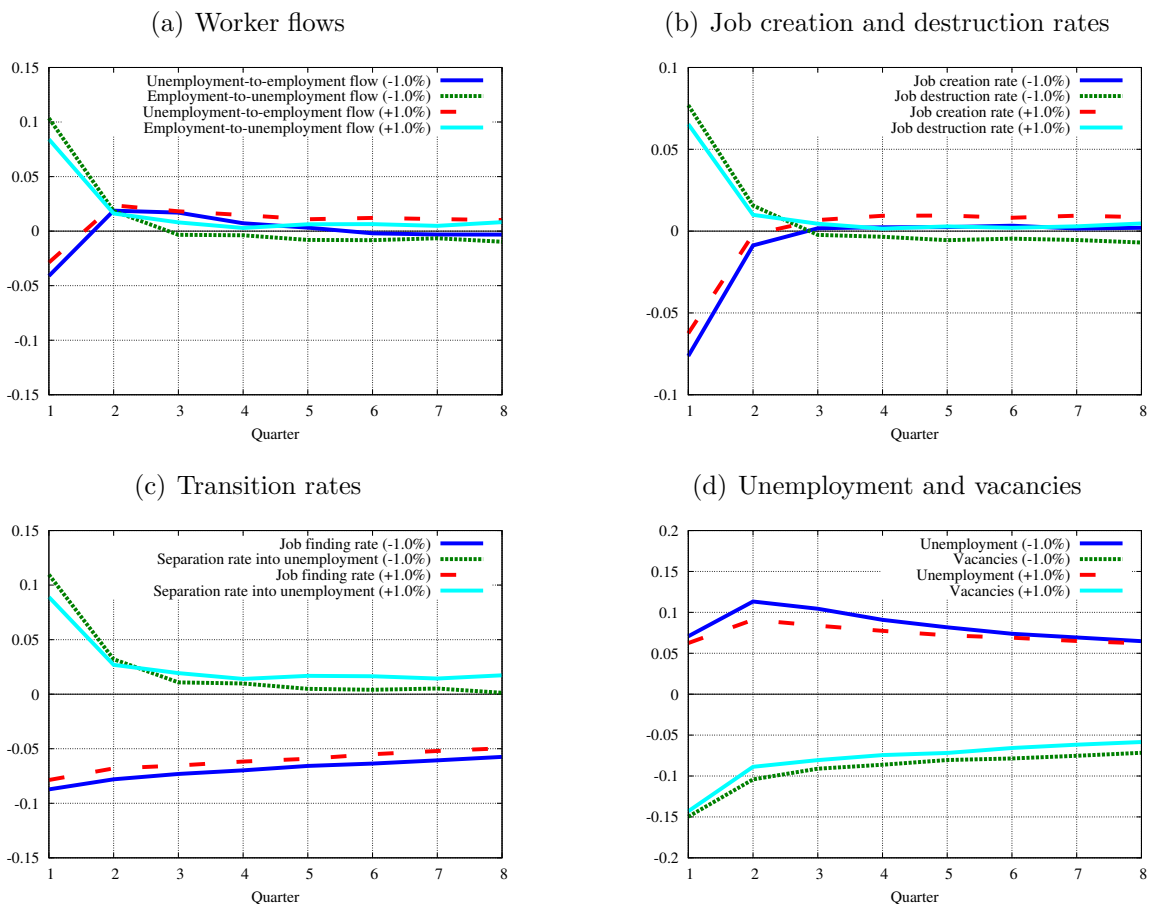|                       | Empirical | Benchmark model | No endog. separation |
|-----------------------|-----------|-----------------|----------------------|
|                       | (a) Standard deviation | | |
| Job destruction rate  | 0.035     | 0.058           | 0.046                |
| Job creation rate     | 0.028     | 0.059           | 0.081                |
| Separation rate       | 0.073     | 0.078           | 0.008                |
| Job finding rate      | 0.086     | 0.084           | 0.088                |
|                       | (b) Relative standard deviation | | |
| Job destruction rate  | 3.838     | 2.963           | 2.432                |
| Job creation rate     | 3.099     | 2.980           | 4.232                |
| Separation rate       | 6.844     | 3.957           | 0.433                |
| Job finding rate      | 7.685     | 4.287           | 4.640                |
|                       | (c) Correlation with output | | |
| Job destruction rate  | -0.398    | -0.257          | -0.313               |
| Job creation rate     | 0.472     | 0.124           | 0.227                |
| Separation rate       | -0.739    | -0.452          | -0.927               |
| Job finding rate      | 0.772     | 0.970           | 0.913                |

is reduced roughly 20%, relative to the benchmark model. While the reduction is substantial, readers may find it somewhat surprising that the model without the endogenous separation decision can generate such non trivial fluctuations in the job destruction rate. However, there are caveats in interpreting this result. First, this number is an upper-bound as mentioned above. In the model, firms are restricted from actively shedding workers. In other words, the result is based on the employment policy that is optimal only when endogenous job destruction imposes infinite costs on the firms. Second, shutting down endogenous separation shifts the balance of relative volatilities toward the job creation side, thereby making the model's quantitative performance worse. Observe that the job creation rate is a lot more volatile than the job destruction rate in the model with exogenous separation only, implying that employment fluctuations are largely accounted for by the job creation rate. This is at odds with the empirical evidence. Furthermore, as can be seen in the last row of each panel, the model generates virtually no variations in the separation rate.[35] This is simply inconsistent with the facts about the separation rate.

## 9.3 Asymmetry and Nonlinearity

The third application looks at the extent of asymmetry and nonlinearity of the model dynamics.[36] In the literature, there is evidence that the U.S. labor market exhibits asymmetry and nonlin-

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[35]The model generates smaller but some variations in the separation rate, which arise due to time aggregation. This bias is emphasized in Shimer (2007), but the result here indicates that the bias is quantitatively very small.

[36]More specifically, asymmetry here means differences in the magnitude of responses to positive and negative shocks; and nonlinearity means a nonlinear relationship between the responses to a small shock and a large shock.

(a) Worker flows



(b) Job creation and destruction rates



(c) Transition rates



(d) Unemployment and vacancies
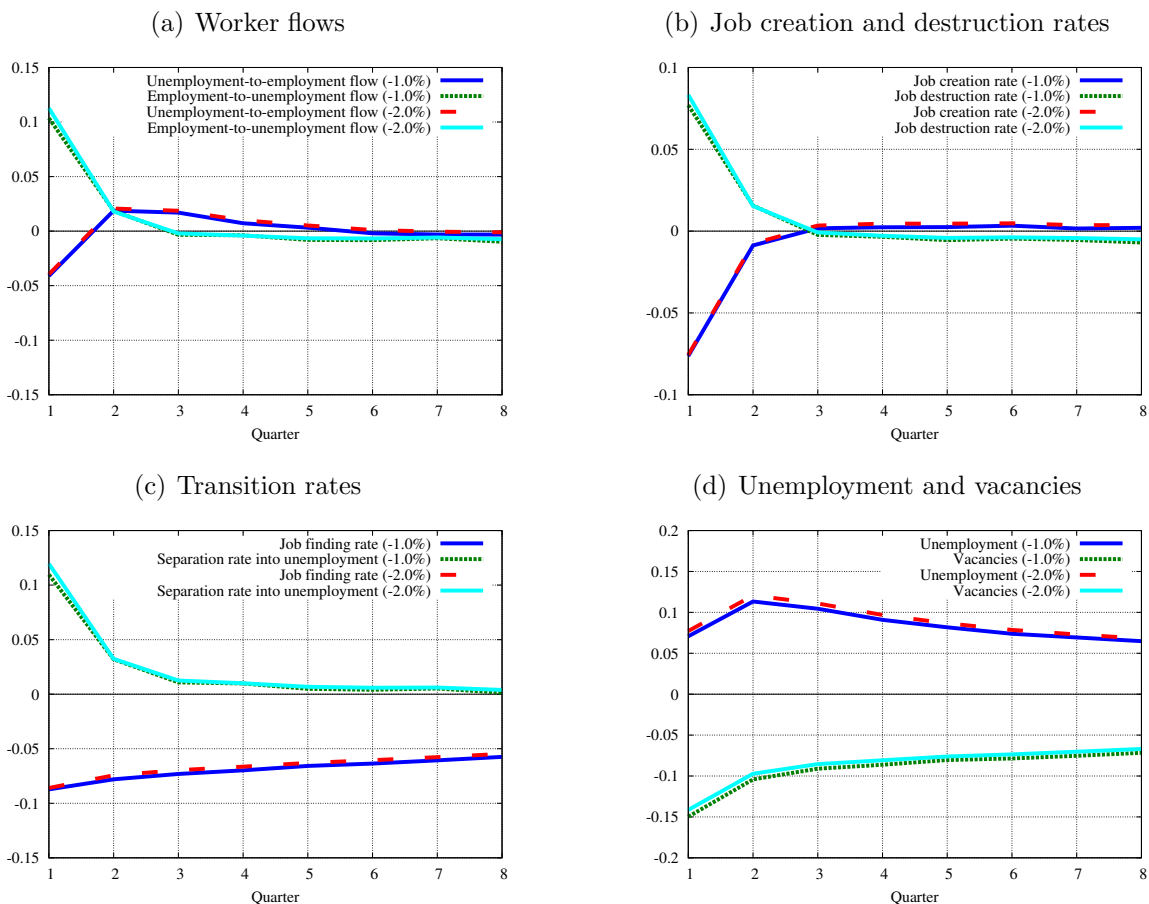


**Figure 6: Asymmetry of impulse responses**

**Notes**: Plotted are responses to $\pm 1\%$ aggregate shocks expressed as deviations from the steady-state levels. Signs of the responses to the positive ($+1\%$) shock are flipped. See notes to Table 5.

earity.[37] Given that our model includes highly nonlinear micro-level features, it is interesting to see how those features translate into asymmetry and nonlinearity of aggregate dynamics.

To examine the asymmetry, we calculate each variable's responses to positive and negative shocks and present the results by flipping signs of the responses to the positive shock (Figure 6). To examine the nonlinearity, we consider the two cases in which the economy is hit by one-% and two-% negative shocks. We then present the results in which the size of responses to the two-% shock is halved (Figure 7).

Figure 6 indicates that the negative shock induces larger responses than the positive shock in our model. The extent of asymmetry is not trivial. For example, an initial response of the separation rate in the face of the negative shock is roughly 20 % larger (in an absolute term) than that in the face of positive shock. Observe also that the negative shock leads to a considerably larger response in unemployment than the positive shock. (The difference is largest in the

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[37]Recent papers that address this issue include McKay and Reis (2008) and Barnichon (2009).

(a) Worker flows

(b) Job creation and destruction rates

(c) Transition rates

(d) Unemployment and vacancies



**Figure 7: Nonlinearity of impulse responses**

**Notes**: Plotted are responses to $-1\%$ and $-2\%$ aggregate shocks expressed as deviations from the steady-state levels. The responses to the $-2\%$ shock are halved, so that they are comparable to those to the $-1\%$ negative shock. See also notes to Table 5.

second quarter.) On the other hand, Figure 7 shows that the model features little nonlinearity; the responses to shocks of different magnitude are related almost linearly.

The results here appear to suggest that the model cannot (at least fully) account for empirical regularities regarding nonlinearity and asymmetry. We suspect that a richer cost structure (as we also discussed elsewhere) may improve the performance of this model along this dimension. Fully exploring this issue is out of the scope of this paper but is certainly an interesting future research topic.

# 10  Conclusion

This paper has quantitatively investigated cyclical properties of the search/matching model with multiple-worker firms. Our main focus was to see whether and how the model can reconcile

differences in the cyclicality of worker flows and job flows. We show that the model is able to replicate countercyclical hires from unemployment and procyclical job creation and that the key to this result is to allow for a large hiring flow that does not go through unemployment, for which procyclicality of the job finding rate dominates its cyclicality.

We identify the lack of propagation as the main drawback of the model. In our model, the effects of the aggregate shock mostly die away in a few quarters. However, labor market adjustments in reality are known to be much more long-lasting. Our model has also faced difficulties in matching the tail of the observed employment growth distribution. We conjecture that both of these problems have to do with the fact that the model features only proportional labor adjustment costs (i.e., the search cost on the hiring side and firing cost on the separation side). Enriching the cost structure, say, by introducing some form of fixed costs, can potentially improve the model's performance along these dimensions. We believe that this is an ambitious yet important future research subject.

# Appendix A    Wage Equation

## A.1    Derivation of the Wage Differential Equation

First, we can write the Bellman equation for the job seeker as follows.

$$W_s(z, m') = b + \beta \int (1 - f(z', m'))W_s(z', m'')dG_z(z'|z)$$
$$+ \beta \int \int f(z', m')W_e(z', x', d', m'', n'')dG_f(x', d', n'')dG_z(z'|z). \quad (28)$$

Using the bargaining solution (7), this can be rewritten as:

$$W_s(z, m') = b + \beta \int W_s(z', m'')dG_z(z'|z)$$
$$+ \beta \int \int f(z', m')\frac{\eta}{1 - \eta}[J(z', x', d', m'', n'') + \tau]dG_f(x', d', n'')dG_z(z'|z). \quad (29)$$

Using (5), it can be re-expressed as:

$$W_s(z, m') = b + \beta \int W_s(z', m'')dG_z(z'|z) + \beta \int \int f(z', m')\frac{\eta}{1 - \eta}[z'x'F'(n'') - w(z', x', d', m'', n'')$$
$$- w_n(z', x', d', m'', n'')n'' + \beta D(z', x', d', m'', n'') + \tau]dG_f(x', d', n'')dG_z(z'|z). \quad (30)$$

Notice that job seekers are only matched with hiring firms ($n'' > (1 - d')n'$). We can use the first-order condition for the hiring firm (17) to obtain the following expression:

$$W_s(z, m') = b + \beta \int W_s(z', m'')dG_z(z'|z)$$
$$+ \beta \int f(z', m')\frac{\eta}{1 - \eta}\left[\frac{\kappa}{q(z', m')} + \tau\right]dG_z(z'|z). \quad (31)$$

Notice that we can eliminate integration with respect to $G_f(x', d', n'')$ since the expression inside the integral is independent of firm types.

Consider next the Bellman equation for the employed worker:

$$W_e(z, x, d, m', n') = w(z, x, d, m', n')$$

$$+ \beta \int \int \int \left( d' + (1 - d')\mathcal{I}_{n'' < (1-d')n'} \frac{(1 - d')n' - n''}{(1 - d')n'} \right) W_s(z', m'')$$

$$+ (1 - d') \left( \mathcal{I}_{n'' > (1-d')n'} + \mathcal{I}_{n'' = (1-d')n'} + \mathcal{I}_{n'' < (1-d')n'} \frac{n''}{(1 - d')n'} \right) W_e(z', x', d', m'', n'')$$

$$dG_d(d'|d)dG_x(x'|x)dG_z(z'|z). \quad (32)$$

Separating out $W_s(z', m'')$, the above equation can be rewritten as:

$$W_e(z, x, d, m', n') = w(z, x, d, m', n') + \beta \int W_s(z', m'')dG_z(z'|z)$$

$$+ \beta \int \int \int (1 - d') \left[ W_e(z', x', d', m'', n'') - W_s(z', m'') \right]$$

$$\left( \mathcal{I}_{n'' > (1-d')n'} + \mathcal{I}_{n'' = (1-d')n'} + \mathcal{I}_{n'' < (1-d')n'} \frac{n''}{(1 - d')n'} \right) dG_d(d'|d)dG_x(x'|x)dG_z(z'|z). \quad (33)$$

Using the bargaining solution (7), we get:

$$W_e(z, x, d, m', n') = w(z, x, d, m', n') + \beta \int W_s(z', m'')dG_z(z'|z)$$

$$+ \beta \frac{\eta}{1 - \eta} \int \int \int (1 - d') \left[ J(z', x', d', m'', n'') + \tau \right]$$

$$\left( \mathcal{I}_{n'' > (1-d')n'} + \mathcal{I}_{n'' = (1-d')n'} + \mathcal{I}_{n'' < (1-d')n'} \frac{n''}{(1 - d')n'} \right) dG_d(d'|d)dG_x(x'|x)dG_z(z'|z). \quad (34)$$

Applying (5) and the first-order conditions (17), we can obtain the following expression:

$$W_e(z, x, d, m', n') = w(z, x, d, m', n') + \beta \int W_s(z', m'')dG_z(z'|z)$$

$$+ \beta \frac{\eta}{1 - \eta} \int \int \int (1-d') \left[ \mathcal{I}_{n'' > (1-d')n'} \left( \frac{\kappa}{q(z', m') + \tau} \right) + \mathcal{I}_{n'' = (1-d')n'} \left[ J(z', x', d', m'', n'') + \tau \right] \right]$$

$$dG_d(d'|d)dG_x(x'|x)dG_z(z'|z). \quad (35)$$

The term associated with firing firms drops out of the equation because the first-order condition implies $J(z', x', d', m'', n'') + \tau = 0$ for firing firms.

Plugging what we obtained above and (5) into the bargaining solution (7), we obtain:

$$w(z, x, d, m', n') - b - \beta \int f(z', m') \frac{\eta}{1 - \eta} \left[ \frac{\kappa}{q(z', m')} + \tau \right] dG_z(z'|z)$$

$$= \frac{\eta}{1 - \eta} \left[ zxF'(n') - w(z, x, d, m', n') - w_n(z, x, d, m', n')n' + \tau - \beta \int (1 - d')\tau dG_d(d'|d) \right]. \quad (36)$$

Solving this equation for $w(z, x, d, m', n')$ gives us the following wage equation:

$$w(z, x, d, m', n') = (1 - \eta)b + \eta \left[ zxF'(n') - w_n(z, x, d, m', n')n' \right.$$
$$\left. +\tau \left( 1 - \beta + \beta \int d' \, dG_d(d'|d) \right) + \beta \int f(z', m') \left( \tau + \frac{\kappa}{q(z', m')} \right) dG_z(z'|z) \right]. \quad (37)$$

## A.2 Solution to the Differential Equation

Assuming $F(n) = n^\alpha$, the above wage equation becomes:

$$w(z, x, d, m', n') = (1 - \eta)b + \eta \left[ zx\alpha n'^{\alpha-1} - w_n(z, x, d, m', n')n' \right.$$
$$\left. +\tau \left( 1 - \beta + \beta \int d' \, dG_d(d'|d) \right) + \beta \int f(z', m') \left( \tau + \frac{\kappa}{q(z', m')} \right) dG_z(z'|z) \right]. \quad (38)$$

To solve the differential equation above, we guess the following wage function (Equation (22)).

$$w(z, x, d, m', n') = (1 - \eta)b + \eta \left[ \frac{zx\alpha n'^{\alpha-1}}{1 - \eta(1 - \alpha)} \right.$$
$$\left. +\tau \left( 1 - \beta + \beta \int d' \, dG_d(d'|d) \right) + \beta \int f(z', m') \left( \tau + \frac{\kappa}{q(z', m')} \right) dG_z(z'|z) \right]. \quad (39)$$

Taking the derivative of the function above, we obtain:

$$w_n(z, x, d, m', n') = \frac{zx\eta\alpha(\alpha - 1)n'^{\alpha-2}}{1 - \eta(1 - \alpha)}. \quad (40)$$

We can compute the right-hand side of Equation (38) to verify that the guess is correct.

$$(1 - \eta)b + \eta \left[ zx\alpha n'^{\alpha-1} - w_n(z, x, d, m', n')n' \right.$$
$$\left. +\tau \left( 1 - \beta + \beta \int d' \, dG_d(d'|d) \right) + \beta \int f(z', m') \left( \tau + \frac{\kappa}{q(z', m')} \right) dG_z(z'|z) \right]$$
$$= (1 - \eta)b + \eta \left[ zx\alpha n'^{\alpha-1} - \frac{zx\eta\alpha(\alpha - 1)n'^{\alpha-1}}{1 - \eta(1 - \alpha)} \right.$$
$$\left. +\tau \left( 1 - \beta + \beta \int d' \, dG_d(d'|d) \right) + \beta \int f(z', m') \left( \tau + \frac{\kappa}{q(z', m')} \right) dG_z(z'|z) \right]$$
$$= (1 - \eta)b + \eta \left[ \frac{zx\alpha n'^{\alpha-1}}{1 - \eta(1 - \alpha)} + \tau \left( 1 - \beta + \beta \int d' \, dG_d(d'|d) \right) \right.$$
$$\left. +\beta \int f(z', m') \left( \tau + \frac{\kappa}{q(z', m')} \right) dG_z(z'|z) \right]$$
$$= w(z, x, d, m', n').$$

# Appendix B   Computational Algorithm

Our numerical solution to the model exploits the *partial information approach* developed by Krusell and Smith (1998).[38] Note that one of the aggregate state variables is $m$, the type distribution of heterogeneous firms. This is an infinite dimensional object, and thus it is impossible to store in a computer. Also, we do not know the function $\Phi_m(z, m)$, which maps the space of the type distribution and the space of aggregate productivity shock into the space of the type distribution. The essence of the Krusell and Smith (1998) approach is to limit the information that agents in the model use to a finite set of statistics that summarize the type distribution and parameterize $\Phi_m(z, m)$ by a known function of the statistics.

We take $N$, the number of employment, as the set of statistics; it is the minimum set of the state variables that could effectively replace $m$. We construct the *approximate equilibrium* by replacing $m$ by $N$ in the model. After solving the approximate equilibrium with $N$, we can evaluate whether $N$ is sufficient to make the approximate equilibrium close to the true equilibrium by adding one more statistic and see how the properties of the model are influenced. If the model properties are intact, we assume that the approximate equilibrium with $N$ is close to the true equilibrium under full information. Accordingly, the functions for aggregate employment in the next period $\Phi_m(z, m)$ and the number of vacancies posted $\Phi_V(z, m)$ are replaced by $\Phi_N(z, N)$ and $\Phi_V(z, N)$, respectively.

We solve the optimal decision of firms for grid points placed on the space of $n$ and $N$. The bounds for $n$ are chosen such that the optimal decision for $n'$ stays within the bounds. The bounds for $N$ are chosen such that the bounds do not bind in simulations.

Logarithms of both aggregate and idiosyncratic shocks are assumed to follow AR(1) processes. The AR(1) process for idiosyncratic productivity is approximated by a finite-state first-order Markov chain using the method proposed by Ada and Cooper (2003). We denote $p_{xx'}$ as the Markov transition probabilities for $\log x$. The AR(1) process for aggregate productivity is also approximated by a finite-state first-order Markov chain.[39] The abscissas for $\log z$ are equally-spaced, and the optimal decision rules are solved at the abscissas. We use piecewise linear approximation to compute the optimal decision off the abscissas. We denote $p_{zz'}$ as the Markov transition probabilities for $\log z$.

We obtain the type distribution of heterogeneous firms via simulation. Let $I$ be the number of firms. Since the total measure of firms is one, each firm carries the weight of $\frac{1}{I}$. The type of firm $i$ in period $t$ is represented by a triplet $(x_{i,t}, d_{i,t}, n_{i,t})$. Employment $N$ and the number of job seekers $S$ in period $t$ can be computed by:

$$N_t = \frac{1}{I} \sum_{i=1}^{I} n_{i,t}, \tag{41}$$

$$S_t = L - N_t, \tag{42}$$

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[38]Ríos-Rull (1999) offers a good summary of the method.

[39]While we appeal to the finite-state approximation in calculating the conditional expectation with respect to aggregate uncertainty (when we solve the firm's problem), we maintain the original AR(1) process in the simulation stage so that the process has a continuous state space.

respectively. We can compute the realized number of vacancies posted in period $t$ as follows:

$$\widetilde{V}_t = \frac{1}{f(z_t, N_t)} \frac{1}{I} \sum_{i=1}^{I} \max(\phi_n(z_t, x_{i,t}, d_{i,t}, N_t, n_{i,t}) - (1 - d_{i,t})n_{i,t}, 0). \tag{43}$$

The detailed solution algorithm takes the following steps.

**Algorithm 1 (Computation Algorithm of the Approximate Equilibrium)**

1. *Parameterize the forecasting functions $\Phi_N(z, N)$ and $\Phi_V(z, N)$. We assume the following functional form with logs of $z$ and $N$.*

$$\log N' = \Phi_N(z, N) = \Phi_{N,0} + \sum_{i=1}^{I_Z} \Phi_{N,Z,i}(\log Z)^i + \sum_{i=1}^{I_N} \Phi_{N,N,i}(\log N)^i, \tag{44}$$

$$\log V = \Phi_V(z, N) = \Phi_{V,0} + \sum_{i=1}^{I_Z} \Phi_{V,Z,i}(\log Z)^i + \sum_{i=1}^{I_N} \Phi_{V,N,i}(\log N)^i, \tag{45}$$

$$\tag{46}$$

*Observe that the functional forms above accommodate higher-order terms of $\log Z$ and $\log N$. In the case of the incomplete-market stochastic growth model studied by Krusell and Smith (1998), it is known that $I_Z = I_N = 1$ is sufficient for obtaining the accurate solution. It is unclear whether the first moment is enough. We thus start from $I_Z = I_N = 1$ and increase $I_Z = I_N$ until the forecasting functions have a sufficiently high prediction power.*

2. *Set an initial guess for the set of coefficients $\{\Phi_{N,0}, \Phi_{N,Z,i}, \Phi_{N,N,i}, \Phi_{V,0}, \Phi_{V,Z,i}, \Phi_{V,N,i}\}$. Denote the initial guess as $\Phi^0$.*

3. *Set a guess of the expected marginal value function $D^0(z, x, d, N', n')$.*

4. *Use the following first-order conditions to obtain the two thresholds $\overline{n}^*(z, x, d, N)$ and $\underline{n}^*(z, x, d, N)$ which characterize the optimal decision rule $n' = \phi_n(z, x, d, N, n)$.*

$$zxF'(\overline{n}^*) - w(z, x, d, N', \overline{n}^*) - w_n(z, x, d, N', \overline{n}^*)\overline{n}^* + \tau + \beta D^0(z, x, d, N', \overline{n}^*) = 0, \tag{47}$$

$$zxF'(\underline{n}^*) - w(z, x, d, N', \underline{n}^*) - w_n(z, x, d, N', \underline{n}^*)\underline{n}^* - \frac{\kappa}{q(z, N)} + \beta D^0(z, x, d, N', \underline{n}^*) = 0, \tag{48}$$

*where $N' = \exp \Phi_N^0(z, N)$ and $V = \exp \Phi_V^0(z, N)$.*

5. *Update $D^0(z, x, d, N', n')$ and obtain $D^1(z, x, d, N', n')$ using the following Bellman operator:*

$$D^1(z, x, d, N', n') = \sum_{z'} \sum_{x'} \sum_{d'} p_{zz'} p_{xx'} p_{dd'} \Pi_n(z', x', d', N', n'), \tag{49}$$

where

$$\Pi_n(z', x', d', N', n') = (1 - d')$$

$$\begin{cases} -\tau & \text{if } \widetilde{n}' > \overline{n}^* \\ z'x'F'(\widetilde{n}') - w(z', x', d', N'', \widetilde{n}') - w_n(z', x', d', N'', \widetilde{n}')\widetilde{n}' + \beta D^0(z', x', d', N'', \widetilde{n}') & \text{if } \widetilde{n}' \in [\underline{n}^*, \overline{n}^*] \\ \frac{\kappa}{q(z', m')} & \text{if } \widetilde{n}' < \underline{n}^*, \end{cases}$$

and $N'' = \exp \Phi_N(z', N')$, $\widetilde{n}' = (1 - d')n'$, $V' = \exp \Phi_V(z', N')$, and $\overline{n}^*$ and $\underline{n}^*$ are characterized by Equations (47) and (48), respectively, for $(z', x', d', N'')$.

6. Compare $D^0(z, x, d, N', n')$ and $D^1(z, x, d, N', n')$. If the chosen norm is smaller than a prespecified tolerance level, stop the iteration and go to the next step. Otherwise, update $D^0(z, x, d, N', n')$ by replacing it with $D^1(z, x, d, N', n')$ and go back to step 4.

7. Simulate the model economy to update $\Phi^0$. First, set the length of simulation $T$. Draw a sequence of $\{z_t\}_{t=1}^{T}$ using a random number generator. Set the initial type distribution of firms $\{(x_{i,1}, d_{i,1}, n_{i,1})\}_{i=1}^{I}$. We use the steady-state type distribution as the initial distribution.

8. Compute $N_t$ and $S_t$ using the period-$t$ type distribution.

9. Start finding a consistent $V_t$. First set $V_t^0 = \exp \Phi_V(z_t, N_t)$.

10. Using $V_t^0$, solve for the optimal decision of firms. Using the optimal decision and the type distribution in period $t$, compute the realized $\widetilde{V}_t^0$.

11. Compare $V_t^0$ and $\widetilde{V}_t^0$. If the chosen norm is smaller than a prespecified tolerance level, take $V_t^0$ as $V_t$ and go to the next step. Otherwise, update $V_t^0$ by taking the weighted average of $V_t^0$ and $\widetilde{V}_t^0$, and go back to step 10.

12. Update the type distribution using the optimal decision rule $\phi_n(z, x, d, N, n)$ under $V_t$.

13. If the simulation reaches the last period, stop and go to the next step. Otherwise, go back to step 8 with the updated distribution and $z$ in the next period.

14. Drop the first $T^0$ observations of the simulated time series of $\{N_t\}_{t=1}^{T}$ and $\{V_t\}_{t=1}^{T}$ to randomize the initial conditions. Run OLS regressions of the form (44) and (46) using the simulated time series. Let $\Phi^1$ be the new coefficients from the regressions.

15. Compare $\Phi^0$ and $\Phi^1$. If the chosen norm is smaller than a predetermined tolerance level, stop and go to the next step. Otherwise, update $\Phi^0$ and go back to step 3.

16. If the coefficients do not converge, or the fit of the regression is not high enough, it is necessary to change the functional forms of $\Phi_N$ and $\Phi_V$ or increase the set of statistics to replace $m$.

17. Once the consistent $\Phi_N$, $\Phi_V$, $w(z, x, d, N', n')$, $D(z, x, d, N', n')$ and $\phi_n(z, x, d, N, n)$ are obtained, run simulations to study cyclical properties of the model.

It turns out that, unlike the case of Krusell and Smith (1998), the parameterized forecasting functions (44) and (46) do not have good forecasting power with $I_Z = I_N = 1$. We thus increased $I_Z = I_N$ and found that raising $I_Z = I_N$ higher than 3 does not improve accuracy of the forecasting functions. In our benchmark model, adjusted $R^2$ for (44) and (46) are 0.9998 and 0.9672, respectively, for $I_Z = I_N = 1$ and 0.9999 and 0.9958, respectively, for $I_Z = I_N = 3$. Note also that adding cross-terms of $\log z$ and $\log N$ in forecasting equations does not significantly affect the accuracy.

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