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**WITH RESPECT TO BENEFITS**

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# The Elasticity of the Unemployment Rate with Respect to Benefits\*

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## Abstract

If the Mortensen and Pissarides model with efficient bargaining is calibrated to replicate the fluctuations of unemployment over the business cycle, it implies a far too strong rise of the unemployment rate when unemployment benefits rise. This paper explores an alternative, right-to-manage bargaining scheme. This also generates the right degree of fluctuations of unemployment but at the same time implies a reasonable elasticity of unemployment with respect to benefits.

*JEL Classification System:* E24,E32,J64,I38

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# 1 Introduction

By how much does the unemployment rate fall when unemployment insurance benefits are reduced? Calibrations of the textbook search and matching model that generate reasonably strong variations of unemployment over the business cycle, e.g. Hagedorn and Manovskii (2007), imply an unrealistically large drop of the unemployment rate when benefits are reduced; see Costain and Reiter (2008) and Mortensen and Nagypal (2007).

An alternative to the widely used efficient bargaining scheme (EB, henceforth), which follows Trigari's (2006) right-to-manage (RTM, henceforth) assumption, can also be calibrated to achieve sufficient unemployment fluctuations; see Christoffel and Kuester (2008).<sup>1</sup> The current paper shows that RTM, once calibrated to match fluctuations of unemployment over the business cycle, in contrast to EB implies a reasonable elasticity of the steady state unemployment rate to a change in benefits.

Section 2 presents the model. Section 3 describes the calibration. Section 4 reproduces Costain and Reiter's (2008) result that with EB one can either produce reasonable unemployment fluctuations or a reasonable semi-elasticity of unemployment with respect to benefits, but not both. In contrast, the section shows, the model with RTM can achieve both. Section 5 concludes.

## 2 The Model

The model is a real-business-cycle version of Trigari (2006) with fixed costs in period profits.

**Families.** There are infinitely many identical families in the economy with unit measure. These consist of a measure  $u_t$  of unemployed and  $n_t = 1 - u_t$  employed members. The family maximizes the sum of unweighted expected utilities of its individual members

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[ \log(c_t) - \kappa^L \int_0^{n_t} \frac{h_{i,t}^{1+\varphi}}{1+\varphi} di \right] \right\} di, \kappa^L > 0, \varphi > 0. \quad (1)$$

$c_t$  is the average consumption level of family members.  $h_{i,t}$  are the hours worked by employed

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<sup>1</sup> Further explanations exist to solve Shimer's (2005) unemployment volatility puzzle; see Mortensen and Nagypal (2007) for an overview.

worker  $i$ . The family pools all income of its members. Its budget constraint is given by

$$c_t + t_t + \kappa v_t = \int_0^{n_t} w_{i,t} h_{i,t} di + u_t b + d_{t-1} r_{t-1} - d_t + \int_0^{n_t} \Psi_{i,t} di. \quad (2)$$

$t_t$  are lump-sum taxes,  $\kappa v_t$  is the cost of posting  $v_t$  vacancies.  $w_{i,t} h_{i,t}$  is the wage earned by family member  $i$ .  $b$  are real unemployment benefits. The family holds  $d_t$  units of a risk-free one-period real bond (inside debt in zero net supply) with gross real return  $r_t$  in  $t + 1$ . It also owns representative shares of all firms. This generates dividend income of  $\int_0^{n_t} \Psi_{i,t} di$ . The family's consumption Euler equation is given by

$$1 = E_t \left\{ \beta \frac{\lambda_{t+1}}{\lambda_t} r_t \right\}, \quad (3)$$

where the marginal utility of consumption is  $\lambda_t = c_t^{-1}$ .

**Firms.** In period  $t$  there is a mass  $n_t$  of operative one-worker firms (firm-worker matches). Firm/match  $i$  can produce  $y_{i,t}$  units of the homogeneous good according to

$$y_{i,t} = z_t h_{i,t}^\alpha, \quad \alpha \in (0, 1), \quad (4)$$

where  $z_t$  is a shock to productivity. Real period profits are

$$\Psi_{i,t} = z_t h_{i,t}^\alpha - w_{i,t} h_{i,t} - \Phi, \quad (5)$$

where  $\Phi \geq 0$  denotes a per-period fixed cost of production.<sup>2</sup>

**Matching firms and workers.** The matching function governs the number of new matches,  $m_t$ , such that

$$m_t = \sigma_m (u_t)^\xi (v_t)^{1-\xi}, \quad \sigma_m > 0, \xi \in (0, 1). \quad (6)$$

Separations occur with a constant, exogenous probability  $\vartheta \in (0, 1)$ , as in Shimer (2005). Employment evolves according to

$$n_t = (1 - \vartheta)n_{t-1} + m_{t-1}.$$

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<sup>2</sup> See Mortensen and Nagypal (2007) for papers that use fixed costs with EB. Christoffel and Kuester (2008) use fixed costs with RTM.

**Match surplus.** The family takes the labor supply decisions for its workers. The marginal gain of the family from having member  $i$  employed is

$$\begin{aligned} \Delta_{i,t} = & w_{i,t}h_{i,t} - b - \kappa^L \frac{(h_{i,t})^{1+\varphi}}{(1+\varphi)\lambda_t} \\ & + E_t \left\{ \beta \frac{\lambda_{t+1}}{\lambda_t} (1 - \vartheta - s_t) \Delta_{i,t+1} \right\}. \end{aligned} \quad (7)$$

The first line reflects the wage income of the worker, forgone unemployment benefits, and the worker's disutility of work. The second line pertains to the continuation value of the match.  $s_t = \frac{m_t}{u_t}$  is the probability that an unemployed worker finds a job.

The market value/surplus,  $J_{i,t}$ , of firm  $i$  is

$$J_{i,t} = \Psi_{i,t} + E_t \left\{ \beta \frac{\lambda_{t+1}}{\lambda_t} (1 - \vartheta) J_{i,t+1} \right\}. \quad (8)$$

**Vacancy posting.** Vacancy posting costs,  $\kappa > 0$ , in equilibrium equal the discounted expected value of a firm,

$$\kappa = q_t E_t \left\{ \beta \frac{\lambda_{t+1}}{\lambda_t} J_{i,t+1} \right\}, \quad (9)$$

where  $q_t = \frac{m_t}{v_t}$  is the probability of finding a worker.

**Bargaining.** Firms and workers split the match surplus by maximizing the Nash-product  $\Delta_t^\eta J_t^{1-\eta}$ . We look at two different bargaining schemes.<sup>3</sup>

1. Efficient bargaining (EB). Firms and workers simultaneously bargain over wages and hours. The first-order condition for hours equates the marginal product of labor to the worker's marginal rate of substitution between leisure and consumption:  $z_t \alpha h_t^{\alpha-1} = \kappa^L \frac{h_t^\varphi}{\lambda_t}$ . The first-order condition for wages results in the surplus sharing rule

$$\eta J_t = (1 - \eta) \Delta_t. \quad (10)$$

The model generates enough cyclical variation of the unemployment rate if profits in steady state are sufficiently low, and thus  $J$  is low, too; see Hagedorn and Manovskii (2007). Equation (10) then implies that the worker's surplus from working,  $\Delta$ , will also be small in steady state. Therefore, a minor change in benefits can shift the outside option of the worker by enough to generate a sizable change in the unemployment rate; see, e.g., Costain and Reiter (2008).

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<sup>3</sup> In equilibrium all firm-worker matches are identical. Index  $i$  is dropped unless needed for clarity.

2. Right-to-manage (RTM). The firm and the worker bargain about the wage rate only. Given this wage, the firm demands hours of work so as to maximize period profits  $\Psi_t$ , given by (5). The firm's first-order-condition for hours equates the marginal product of labor and the wage rate,  $z_t \alpha h_t^{\alpha-1} = w_t$ . The first-order condition for wages yields a "modified" surplus sharing rule:<sup>4</sup>

$$\eta J_t \left[ \frac{\alpha}{\alpha-1} - \frac{1}{\alpha-1} \frac{mrs_t}{w_t} \right] = (1-\eta) \Delta_t. \quad (11)$$

Whenever the worker's marginal rate of substitution,  $mrs_t := \kappa^L \frac{h_t^\varphi}{\lambda_t}$ , exceeds the wage rate in steady state, relative to EB, the value of the worker's surplus,  $\Delta$ , will be greater for any value of the firm,  $J$ .<sup>5</sup> In the calibration, therefore, profits can be low while still allowing for a large surplus of the worker, even for conventional values of the bargaining power,  $\eta$ . Further, as the first-order condition for hours worked suggests, profits in this scheme are not as responsive to changes to the worker's outside option and  $mrs$ . RTM can replicate fluctuations in the labor market once fixed costs are sufficiently positive.

**Government.** The government collects taxes to offset benefit payments,  $t_t = u_t b$ .

**Market clearing.** Goods market clearing requires

$$y_t = \int_0^{n_t} z_t h_{i,t}^\alpha di = n_t z_t h_t^\alpha = c_t + n_t \Phi + \kappa v_t.$$

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<sup>4</sup> Equation (11) can be derived as follows. Under RTM the wage bargaining problem is

$$\max_{w_t} \Delta_t^\eta J_t^{1-\eta} \text{ s.t. } z_t \alpha h_t^{\alpha-1} = w_t.$$

The first-order condition for the wage is

$$\eta J_t \frac{d\Delta_t}{dw_t} = -(1-\eta) \Delta_t \frac{dJ_t}{dw_t}.$$

The worker's surplus can be written as  $\Delta_t = w_t h_t - \frac{\kappa^L}{\lambda_t} \frac{h_t^{1+\varphi}}{1+\varphi} + \text{tiw}$ , where tiw summarizes terms independent of the current period's wage bargaining (constants and the continuation value). Thereby  $\frac{d\Delta_t}{dw_t} = h_t + w_t \frac{dh_t}{dw_t} - \frac{\kappa^L}{\lambda_t} h_t^\varphi \frac{dh_t}{dw_t}$ .

The first-order condition for hours gives  $h_t = \left( \frac{w_t}{z_t \alpha} \right)^{\frac{1}{\alpha-1}}$ , so  $\frac{dh_t}{dw_t} = \frac{1}{\alpha-1} \frac{h_t}{w_t}$ . Thus  $\frac{d\Delta_t}{dw_t} = h_t + \frac{1}{\alpha-1} h_t - \frac{\kappa^L}{\lambda_t} h_t^\varphi \frac{1}{\alpha-1} \frac{h_t}{w_t}$ , or  $\frac{d\Delta_t}{dw_t} = \left[ \frac{\alpha}{\alpha-1} - \frac{1}{\alpha-1} \frac{mrs_t}{w_t} \right] h_t$ . Similarly,  $J_t = z_t h_t^\alpha - w_t h_t + \text{tiw}$ , so  $\frac{dJ_t}{dw_t} = z_t \alpha h_t^{\alpha-1} \frac{dh_t}{dw_t} - h_t - w_t \frac{dh_t}{dw_t}$ . This gives  $\frac{dJ_t}{dw_t} = \frac{z_t \alpha h_t^{\alpha-1}}{w_t} \frac{1}{\alpha-1} h_t - h_t - w_t \frac{1}{\alpha-1} h_t \frac{1}{w_t}$ . By the first-order condition for hours worked the first term's first fraction equals unity. So the first term can be simplified to  $\frac{1}{\alpha-1} h_t$ . Taking all three terms together yields  $\frac{dJ_t}{dw_t} = -h_t$ . Inserting the terms for  $\frac{dJ_t}{dw_t}$  and  $\frac{d\Delta_t}{dw_t}$  into the wage bargaining first-order condition yields (11).

<sup>5</sup> The condition  $mrs > w$  also holds for EB in the calibration below. It is not special to RTM. In fact, under EB, as  $\alpha \rightarrow 1$ ,  $mrs > w$  is a necessary condition for  $\Psi > 0$  and thus for an equilibrium with  $\kappa > 0$ .

### 3 Calibration

One time period in the model is one month. The model is calibrated to quarterly US data for the period 1984Q1 to 2006Q3. Output is nominal output in the business sector divided by the GDP deflator. The real wage (per hour) is the real compensation (per hour) in the business sector divided by the GDP deflator. Vacancies are the Conference Board’s index of Help-Wanted Advertising. The civilian unemployment rate used is for the age group 16 years old and older.

Table 1 (Calibration) about here.
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Four variants of the model are calibrated: EB and RTM each with and without fixed costs; see Table 1. In the variants with fixed costs,  $\Phi$  is chosen to replicate the standard deviation of unemployment rates.  $\alpha = 0.99$ , so there are only mildly decreasing returns to hours worked per worker. The replacement rate,  $\frac{b}{wh}$ , equals 0.4 following Shimer (2005). The log of  $z_t$  follows an AR(1) process with autocorrelation coefficient  $\rho_z = 0.97$ . The rest of the calibration is standard and explained in the notes to Table 1.

### 4 Results

Table 2 compares the implied second moments (relative to output) of the four model variants to their counterparts in the data. Without fixed costs, neither EB nor RTM is able to replicate the fluctuations in the labor market; see the cells formed by the third and fourth lines and second and fourth columns of Table 2. The third and fifth columns in turn show that both models are able to replicate these fluctuations once fixed costs are introduced. The introduction of fixed costs squeezes steady state profits of firms and thereby increases percentage fluctuations in profits. Profit fluctuations in turn translate into higher variability of vacancy postings and stronger unemployment fluctuations.

Overall, as far as business cycle fluctuations are concerned, the RTM and the EB models with fixed costs behave very similarly. In particular, both replicate the correlation between unemployment, vacancies, and output equally well (see rows labeled “Beveridge curve”), and both generate a similar degree of fluctuations of wages.

Table 2 (Results) about here.

Turning to the key result of this paper, the final row of Table 2, titled  $\frac{d \log(u)}{d \frac{b}{wh}}$ , reports the semi-elasticity of unemployment with respect to benefits in the data and the four model variants. The final column replicates the result in Costain and Reiter (2008): the model with EB, when calibrated to match the fluctuations of vacancies and unemployment, fails to produce a meaningful semi-elasticity of unemployment with respect to changes in the replacement rate. Given a semi-elasticity of 12.28, the steady state unemployment rate would fall from 6% to 5.3% if the replacement rate,  $\frac{b}{wh}$ , were reduced by 1 percentage point, i.e., from 40% in our calibration to 39%. The reason for this strong effect is the following. To reproduce unemployment fluctuations, the firm's surplus in steady state,  $J$ , needs to be small under both EB and RTM. Yet for EB, as the first-order condition (10) shows, a small surplus for the firm also means a small surplus for the worker ( $\Delta$ ). Since the worker is close to indifferent between working and not working, a small change in benefits implies a strong change in work incentives for a given wage, and thus a strong change in unemployment. The same is not true under RTM. First-order condition (11) implies that the worker's surplus can be big even if the firm's surplus is not.<sup>6</sup> This implies a considerably more moderate reaction of work incentives to changes in benefits. If the RTM model is calibrated to match the fluctuations of unemployment over the business cycle, it produces a semi-elasticity of unemployment with respect to benefits equal to 1.68. This is close to the estimates in the literature summarized by Costain and Reiter (2008) and implies that the unemployment rate would fall from 6% to 5.9% in the long run if the replacement rate,  $\frac{b}{wh}$ , were to fall from 40% to 39%.

## 5 Conclusions

This paper argued that a model with right-to-manage bargaining, i.e., a model in which the firm and the worker bargain only about hourly wages, and in which the firm retains the decision on the intensive (hours worked) margin, can be calibrated to quantitatively generate both realistic unemployment fluctuations over the business cycle and a realistic long-run response of unemployment levels to a change in the replacement rate, while the same is not true for a textbook model with

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<sup>6</sup> In our calibration the worker's surplus is 27 times as large as the firm's surplus; see Table 1.



efficient bargaining.

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Table 1: Calibration

	RTM		EB	
	$\frac{\Phi}{zh^\alpha} = 0$	$\frac{\Phi}{zh^\alpha} = .0096$	$\frac{\Phi}{zh^\alpha} = 0$	$\frac{\Phi}{zh^\alpha} = 0.4070$
Preferences				
$\beta$	0.998	.	.	.
$\varphi$	2	.	.	.
$\kappa^L$	28.90	36.22	28.89	48.08
Bargaining and production				
$\alpha$	0.99	.	.	.
$\xi$	0.5	.	.	.
$\vartheta$	0.03	.	.	.
$\eta$	0.50	.	.	.
$\frac{b}{wh}$	0.40	.	.	.
$\sigma_m$	0.394	.	.	.
$\kappa$	0.110	.0046	0.182	0.0174
Implied surplus of the firm and of the worker's family				
$J$	0.332	0.014	0.552	0.053
$\Delta$	0.555	0.378	0.552	0.053
Correlation of the technology shock				
$\rho_z$	0.97	.	.	.

*Notes:* Calibration (RTM with/without fixed costs, EB with/without fixed costs).  $\kappa^L$  is adjusted to imply  $h = \frac{1}{3}$ . Total production is  $y = 1$ .  $\kappa$  is adjusted to match  $u = 6\%$ .  $\Phi$  (when not equal to zero) is set to match the relative standard deviation of unemployment and output in the data. Monthly model. Entry  $\cdot$  means: same value as to the left.

Table 2: Comparison of the data and model properties

	data	RTM		EB	
	hp(1,600)	$\frac{\Phi}{zh^\alpha} = 0$	$\frac{\Phi}{zh^\alpha} = .0096$	$\frac{\Phi}{zh^\alpha} = 0$	$\frac{\Phi}{zh^\alpha} = 0.4070$
Standard deviations					
$\frac{\sigma(w)}{\sigma(y)}$	0.86	0.96	1.31	0.94	1.20
$\frac{\sigma(wh)}{\sigma(y)}$	0.82	0.94	0.58	0.92	0.93
$\frac{\sigma(u)}{\sigma(y)}$	6.77	0.90	6.77	1.10	6.77
$\frac{\sigma(v)}{\sigma(y)}$	8.23	1.08	8.10	1.31	8.12
Beveridge curve (contemporaneous correlation)					
$\rho_{u,y}$	-0.84	-0.96	-0.98	-0.97	-0.99
$\rho_{v,y}$	0.86	0.98	0.97	0.98	0.96
Semi-elasticity of unemployment rate w.r.t. replacement rate					
$\frac{d \log(u)}{d \frac{b}{wh}}$	2*	0.018	1.68	1.84	12.28

*Notes:* The data span is 1984Q1 to 2006Q3. Figures for model variables refer to quarterly aggregates of these. \*: estimate reported by Costain and Reiter (2008).