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Diagnosing Labor Market Search Models: A Multiple-Shock Approach

by Kenneth Beauchemin and Murat Tasci

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We construct a multiple-shock version of the Mortensen-Pissarides labor market search model to investigate the basic model's well-known tendency to underpredict the volatility of key labor market variables. Data on U.S. job nding and job separation probabilities are used to help estimate the parameters of a three-dimensional shock process comprising labor productivity, job separation, and matching or "allocative" effciency. Although our multiple-shock model generates some more volatility, it has counterfactual implications for the cyclicality of unemployment and vacancies. Our second exercise forces the model to be the data-generating process to uncover the necessary realizations of all three shocks. We show that the Mortensen-Pissarides labor market search model requires significantly procyclical and volatile matching efficiency and job separations to simultaneously account for high procyclical variations in job nding probabilities as well as relatively small net employment changes in the data. Hence, the model is more fundamentally flawed than its inability to amplify shocks would suggest. We also show that variation in job separations accounts for most of the employment fluctuations, suggesting that endogenous separations could be the key feature of an improved model. This leads us to conclude that the model lacks mechanisms to generate procyclical matching efficiency and labor force reallocation. As for the latter, we conjecture that nontrivial labor force participation and job-to-job transitions are promising avenues of research.

Key words: Labor Market Search; Mismatch; Business Cycles; Unemployment; Job Vacancies.

JEL code: E24; E32; J64

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1 Introduction

There is now a fairly rich literature using the Mortensen-Pissarides labor market search model to understand business cycle movements in the labor markets.[1] Shimer (2005a) has recently criticized this model, arguing that it requires implausibly large shocks to labor productivity to generate substantial variation in the key variables of unemployment, vacancies, and the vacancy to unemployment ratio.[2] We explore whether other reasonable sources of exogenous variation, including job separation and matching efficiency shocks, can satisfactorily resolve this puzzle. In particular, we identify the realizations of a multiple-shock process required for the model to fit the data perfectly. Our results are striking. The perfect-fit experiment strongly indicates that the standard labor market search model is more fundamentally flawed than its inability to amplify shocks would suggest.

Our multiple shock approach allows for exogenous shocks to the rate of job separation, labor productivity, and matching efficiency that are mutually correlated over the business cycle. In keeping with the most basic Mortensen-Pissarides model, the rate of job separation is exogenous and simply gives the fraction of employed persons that will separate from their jobs, for whatever reason, during a particular period. The shock to the matching function captures the efficiency with which existing labor market institutions pair searching workers with available jobs. We call this the "allocative efficiency" shock as in Andolfatto (1996). We use data on monthly separation and job finding probabilities as well as unemployment and job vacancies to estimate the process that governs these shocks. The estimation strategy provides us with empirically plausible variations in labor market transition probabilities of the average U.S. worker.

Although realistic variation in the transition probabilities substantially increases the volatil-

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[1]For a textbook treatment of this class of models see Pissarides (2000). Broadly speaking, we can identify two separate but closely related strands in this literature. The first group, including works by Andolfatto (1996), den Haan, Ramey, and Watson (2000) and Merz (1995, 1999), incorporates labor market search into otherwise standard real business cycle environments to improve upon their cyclical implications for labor market variables such as employment. A second group of papers, such as Cole and Rogerson (1999) and Mortensen and Pissarides (1994), focuses on the implications of the standard labor market search model in relation to the empirical evidence on job creation and job destruction provided by Davis and Haltiwanger (1992) and Davis, Haltiwanger, and Schuh (1996).

[2]Earlier studies either failed to address the magnitude of the exogenous forcing process (Mortensen and Pissarides (1994), Cole and Rogerson (1999)) or implied a counterfactually positive relationship between unemployment and vacancies (Andolfatto (1996), Merz (1995)). Merz (1995) provides two versions of the model, one with constant and one with variable search effort. To be precise, her version with variable search effort gives this counterfactual finding.

ity of key variables in the model, it does so at an enormous descriptive cost. The simulated cyclical behavior of unemployment and vacancies is entirely counterfactual, displaying procyclical unemployment and countercyclical job vacancies. We conduct a "perfect-fit" experiment to better understand the counterfactual finding. That is, we posit the model as the actual data-generating process and then infer the shocks that would be required to match the data perfectly. Our findings are startling. To be consistent with the observed fluctuations in unemployment and job vacancies, the multiple-shock model requires volatile *procyclical* job separations to reconcile sharply procyclical variation in job finding with relatively small net employment changes.

These counterfactual findings from the perfect-fit experiment are due to two reasons. First, the substantial fluctuations observed in the vacancy-unemployment ratio, or "labor market tightness," require the multiple-shock model to have significantly procyclical and volatile allocative efficiency shocks. Given the cyclical behavior of allocative efficiency, however, observed net employment changes require significantly procyclical and substantially volatile job separations. Of course, procyclical job separation and destruction runs contrary to the existing empirical evidence on employment flows (Blanchard and Diamond (1990), Davis and Haltiwanger (1992), Davis, Haltiwanger, and Schuh (1996), and Shimer (2005a)). Therefore, the defects in the labor market search model are even more fundamental than Shimer (2005a) argues. Furthermore, these flaws are robust to different calibrations.

We conclude that the basic model lacks sufficiently strong mechanisms to reallocate workers over the course of the business cycle. Our results point to potentially productive extensions of the basic model. If procyclical allocational efficiency is thought inadequate, *a priori*, one then searches for mechanisms that underlie this behavior, and modifies the model accordingly. The results also indicate that any such modification must be accompanied by a theoretical expansion of the pool of searching workers and the incorporation of job-to-job transitions.

This paper relates to various other studies. Our investigation into the mechanics of the standard labor market search model echoes Shimer's (2005a) diagnostic exploration of Mortensen-Pissarides framework. We argue that the model, even when it has substantial degrees of freedom with multiple shocks and empirically plausible transition probabilities, has counterfactual implications. Once we have emphasized this point with the experiment that requires our model to be the actual data-generating process, we then study the unique realization of shocks

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that are required for a perfect fit. Although our objective for this experiment is diagnosis rather than measurement, it is similar to the accounting exercises employed in Chari, Kehoe, and McGrattan (2007) and Ingram, Kocherlakota, and Savin (1994). Our model is identical to Merz (1995) except that we abstract from the capital stock. Finally, we discuss our findings and several avenues for future research in conjunction with the literature that tries to resolve the puzzle presented in Shimer (2005a).

The remainder of the paper is organized as follows. Section 2 outlines our version of the Mortensen-Pissarides model. In Section 3, we briefly describe the data and its basic statistical properties. Section 4 discusses our calibration and presents the simulation results. Section 5 analyzes the simulation results and presents our perfect-fit experiment. We also interpret our findings in the context of recent literature. Section 6 discusses some robustness issues. We briefly outline our conclusions and set a direction for future research in Section 7.

2 The Model

The economy is inhabited by a continuum of infinitely-lived worker/households distributed uniformly along the unit interval; there is also a continuum of firms. At the beginning of each period, a worker is considered either employed or unemployed. The measure of employed workers is denoted N_t; the measure of unemployed workers is the complement, $U_t \equiv 1 - N_t$. The representative household has preferences over state-contingent consumption and employment given by

$$\mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t U\left(C_t, N_t\right), \qquad 0 < \beta < 1, \tag{1}$$

where β is the subjective discount factor. Following Merz (1995), the period utility function is separable in consumption and employment, with

$$U\left(C_t, N_t\right) = \log C_t - \frac{N_t^{1+\frac{1}{\gamma}}}{1+\frac{1}{\gamma}}, \qquad \gamma > 0,$$

where γ defines the wage elasticity of labor supply at a constant marginal utility of wealth (the "Frisch elasticity" of labor supply).

Both workers and firms must undergo a costly search process before jobs are created and

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output is produced. At the beginning of each period, each unemployed worker searches for a job, expending ϕ consumption units in the process. Aggregate search costs incurred in period t therefore equal $\phi\left(1-N_t\right)$ consumption units. Firms create job vacancies, but only by expending κ units of output per vacancy per period, generating aggregate "recruiting" costs equal to κV_t. Here, as in the traditional Mortensen-Pissarides framework, all jobs must be posted as vacancies before they can be filled. Once a job is filled, it produces output equal to Z_t, generating aggregate output

$$Y_t = Z_t N_t, \tag{2}$$

where $Z_t > 0$ is the exogenously determined productivity of labor.

The matching function captures the labor market search frictions. The typical formulation determines the number of job matches formed in a given period, $M\left(V_t, U_t\right)$, as an increasing function M of job vacancies, V_t, and the number of job seekers, U_t, where M exhibits constant returns to scale. Like an aggregate production function, the matching function is a useful reduced form that summarizes a host of complex technological constraints. At its core lies the notion of labor market mismatch, "an empirical concept that measures the degree of heterogeneity in the labor market across a number of dimensions, usually restricted to skills, industrial sector, and location" (Petrongolo and Pissarides, 2001). By this definition, a higher degree of mismatch frustrates the search process, thereby generating more unemployment in the steady state. In this paper we consider the possibility that the efficiency of matching varies systematically over the business cycle, either due to changes in labor market composition, or changes more technological in nature.[3]

Hence, to allow for fluctuations in mismatch, we generalize the matching function to include a multiplicative shock term, χ_t, such that the number of matches formed in period t is given by

$$M_t = \chi_t M\left(V_t, U_t\right) = \chi_t V_t^{\alpha}\left(1 - N_t\right)^{1-\alpha}, \tag{3}$$

where $0 < \alpha < 1$, and χ_t is the period-t realization of an unobserved shock process. Increases in χ_t raise the number of matches formed, given the numbers of searching workers and available positions. From a searching worker's perspective, an increase in χ_t raises the probability of being

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[3]Empirical work by Bowlus (1995) offers evidence that mismatch has cyclical charateristics.

matched with a vacant position; from the perspective of a single firm, it improves the chances of filling a vacancy. Consequently, fluctuations in χ_t signify improvements or deteriorations in the allocative efficiency of the labor market.

As job matches form, others are dissolved. We assume that a fraction of existing matches, σ_t, dissolve each period as the outcome of an exogenous stochastic process. The period-t change in aggregate employment, i.e., the net employment flow, is hence defined as the difference between a period's gross employment inflow and gross employment outflow:

$$N_{t+1} - N_t = M_t - \sigma_t N_t. \tag{4}$$

Note that the unobserved shocks directly impact each stream of workers: the flow into employment by the allocative efficiency term, χ_t, and the outflow by the separation rate, σ_t.

The state of the economy in a given period (N_t, e_t) consists of the beginning-of-period employment level, N_t, and values of the unobserved and exogenous state vector, $e_t = (Z_t, \chi_t, \sigma_t)$. We make the standard Markovian assumption that allows agents to form expectations of future-period quantities using only current-state knowledge. Given that state, the socially efficient allocation of employment, vacancies, and consumption, $\{N_{t+1}, V_t, C_t\}$, solves the following recursively-defined social planner's problem:

$$\upsilon\left(N_t, e_t\right) = \max_{N_{t+1}, V_t, C_t} \left\{ U\left(C_t, N_t\right) + \beta \mathrm{E}_t \upsilon\left(N_{t+1}, e_{t+1}\right) \right\} \tag{5}$$

subject to

$$C_t + \phi\left(1 - N_t\right) + \kappa V_t \leq Z_t N_t \tag{6}$$

$$N_{t+1} = \left(1 - \sigma_t\right) N_t + \chi_t M\left(V_t, 1 - N_t\right), \tag{7}$$

where $\upsilon\left(N_t, e_t\right)$ is the future-discounted social value of employment level N_t and the exogenous state e_t. Equation (6) represents the period-t resource constraint, prohibiting the sum of current expenditures on consumption, job search, and vacancy creation to exceed current output, and equation (7) describes the trajectory of employment (4) with the matching function (3) determining the current-period flow into employment. Finally, we assume that a VAR(1) process governs the exogenous state e_t:

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$$e_{t+1} = Ae_t + \varepsilon_{t+1}, \ \mathrm{E}(\varepsilon\varepsilon') = \Omega. \tag{8}$$

The autoregressive process for exogenous shocks plays a key role in this paper. We will devise a way to estimate this joint process below.

The corresponding first-order and envelope conditions imply an Euler equation describing an intertemporally efficient vacancy-posting scheme for the economy. Suppressing arguments and letting primes denote one-period-ahead quantities, we write

$$U_C \frac{\kappa}{\chi M_V} = \beta \mathrm{E}_t U_C' \left\{ Z' + \phi + \frac{U_N'}{U_C'} + \frac{\kappa}{\chi' M_V'} \left[(1 - \sigma') - \chi' M_U' \right] \right\} \tag{9}$$

equating the loss in welfare due to vacancy creation with its expected future social benefit. In equation (9), $\frac{1}{\chi M_V} = \alpha^{-1} \frac{V}{\chi M}$ gives the average duration of vacancies multiplied by the elasticity of vacancies in matching, $\alpha = \frac{V M_V}{M}$. The left-hand side of (9), therefore, represents the utility loss associated with a marginal increase in vacancies. The expected gain of the marginal vacancy, given by the right-hand side of (9), derives from multiple sources. The expression $Z' + \phi + \frac{U_N'}{U_C'}$ gives the one-period-ahead net social benefit of an additional match formed in the current period. The term Z' equals the output flowing from the match; ϕ represents the (constant) search costs foregone by the worker in the match. The final term in the sum, $\frac{U_N'}{U_C'}$, represents the consumption value of the leisure foregone by the newly matched worker. In the basic Mortensen-Pissarides setup this quantity is a constant, whereas we allow it to vary over the business cycle.

The final term in braces represents the net future social benefit arising from the expected persistence of a job match. Given that any single current-period match survives with probability $1 - \sigma'$, future social welfare will increase simply by reducing expected future recruiting costs by the quantity $\frac{\kappa(1-\sigma')}{\chi' M_V'}$. The second term in this sum, $-\chi' M_U'$, represents the future reduction in the future job-finding rate, $\frac{\chi M}{U}$, due to the current depletion of the unemployment stock; the expected recruiting cost in future consumption units equals $\frac{\kappa M_U'}{M_V'}$.

Equations (6)–(9) characterize the socially optimal allocation of employment, vacancies, and consumption given a joint distribution for the exogenous forcing variables or shocks: Z_t, χ_t,

and σ_t. The traditional Mortensen-Pissarides approach determines these quantities in a market equilibrium, with the real wage determined as a Nash bargain between firms and households. The socially optimal allocation is supported by a similar market allocation mechanism provided that: 1) asset markets are rich enough for households to diversify away employment risk, and 2) the relative bargaining power between households and firms is such that the positive and negative search externalities net out to zero.[4] Although we do not take a position on the precise nature of the allocation mechanism, we maintain that existing market and institutional arrangements direct the realized allocation sufficiently close to the social optimum to establish equations (6)–(9) as a useful instrument of measure.

3   The Data

Before proceeding to the estimation of shocks, we briefly review the salient facts regarding the observed aggregate U.S. labor market measures. Because the model does not require a labor market participation decision for worker/households, we must choose whether to express our employment and unemployment variables, N_t and $U_t \equiv 1 - N_t$, relative to the labor force or the age 16-and-over population. Although there are valid arguments in favor of both normalizations, we find that the choice does not affect our results, so we choose the labor force (employment plus unemployment) as our reference population.[5]

In the absence of a long time series on actual job vacancies, we follow standard practice and construct vacancies from the Conference Board's help-wanted advertising index. The resulting vacancy series, V_t, is also expressed per member of the labor force[6]. Also, since our model abstracts from the capital accumulation decision, we must choose between aggregate output and aggregate consumption – a choice that reflects our desire to preserve a consistent and

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[4]Hosios (1990) determines the conditions under which the Pareto-optimum is supported as a decentralized market equilibrium in a static environment; Merz (1995) and Andolfatto (1996) do the same in dynamic general equilibrium settings. The market equilibrium in the current work closely follows those of Merz and Andolfatto.

[5]Specifically, we use the unemployment rate (unemployed persons per member of the labor force) constructed as a quarterly average of the seasonally adjusted monthly series from the Current Population Survey (CPS) of the Bureau of Labor Statistics (BLS). The civilian labor force measure is also provided by BLS as part of the CPS. Both series can be downloaded from the CPS home page *http://www.bls.gov/cps*.

[6]We construct a vacancy series by multiplying two seasonally adjusted monthly series – the ratio of help-wanted advertising to unemployed workers compiled by the Conference Board (downloaded as the variable LHELX from the DRI Basic database), and the unemployment rate U (defined above) – and averaging the monthly values to obtain the quarterly series. The commonly reported help-wanted advertising index is a scalar transformation of this series.

well-understood labor productivity measure and one that can be more readily compared to those in other studies. Since the aggregate labor input, N_t, produces all goods and services, including private investment goods and those purchased by the government, real GDP provides the appropriate output measure. Therefore, consumption, C_t, is proxied by real GDP per member of the labor force[7]. We divide this series by the seasonally adjusted civilian labor force (averaged from monthly to quarterly), appropriately scaled, to express the variable in year-2000 chained dollars per person. Time series data on U_t, V_t and C_t are constructed at the quarterly frequency and run from 1951:1 to 2003:4.

We use real output per person in the nonfarm business sector as our productivity measure. This particular series is chosen to ensure comparability with the recent literature. It is also a natural way to think about productivity in the standard labor market search model[8]. Although not strictly consistent with our model's output definition, we show later on that the productivity series implied by the model lines up nearly perfectly with the BLS measure.

We also use U.S. labor market transition probabilities for our estimation process. These probabilities were constructed by Shimer (2005a), but our discussion follows that of Shimer (2005b). In accordance with Shimer (2005b), unemployment, u, and employment, e, form the relevant labor market states, with the job finding probability governing the rate at which a worker switches from unemployment to employment, and the separation probability determining the rate at which a worker switches from employment to unemployment.

Shimer's (2005b) definitions of job finding and separation probabilities are as follows:

$$f_t = 1 - \frac{U_{t+1} - U_{t+1}^s}{U_t} \tag{10}$$

$$s_t = \frac{U_{t+1}^s}{E_t(1 - f_t/2)}, \tag{11}$$

where in a given month t, U_t is the number of unemployed, U_t^s is the number of workers unemployed less than one month, and E_t is the number of workers employed. These definitions

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[7]Real GDP (billions of chained 2000 dollars, seasonally adjusted annual rate) was downloaded from the Federal Reserve Bank of St. Louis FRED II database at *http://research.stlouisfed.org/fred2/series/GDPC.1*

[8]This series is part of the BLS's Major Sector Productivity and Costs program and is downloaded from the Federal Reserve Bank of St. Louis FRED II database at *http://research.stlouisfed.org/fred2/series/GDPC*. It is normalized to 100 for 1992.

correct for the time aggregation bias in the job separation probability by allowing for the possibility of short unemployment spells within a given month.

To aggregate monthly transition probabilities, we account for all possible histories of employment states within a quarter. The temporal aggregation of labor market transition probabilities might imply different cyclical features at various frequencies. The idea behind this argument is simple. An unemployed worker at the beginning of a quarter can switch between employment and unemployment before being counted as employed at the end of the quarter. This aggregation implies four possible histories. Therefore, the quarterly job finding probability of an average worker will not only reflect the cyclical features of monthly job finding probabilities, but also of monthly separation probabilities. Since we are interested in the cyclical properties of labor market variables, it is vital for us to be precise in aggregating Shimer's monthly transition probabilities. We use the following aggregation:

$$
\begin{aligned}
F_t \;=\;& \left(1 - f_{(3*(t-1)+1)}\right) * \left(1 - f_{(3*(t-1)+2)}\right) * f_{(3*(t-1)+3)} \;+ \qquad (12) \\
& \left(1 - f_{(3*(t-1)+1)}\right) * f_{(3*(t-1)+2)} * \left(1 - s_{(3*(t-1)+3)}\right) \;+ \\
& f_{(3*(t-1)+1)} * s_{(3*(t-1)+2)} * f_{(3*(t-1)+3)} \;+ \\
& f_{(3*(t-1)+1)} * \left(1 - s_{(3*(t-1)+2)}\right) * \left(1 - s_{(3*(t-1)+3)}\right), \quad \forall t.
\end{aligned}
$$

$$
\begin{aligned}
S_t \;=\;& \left(1 - s_{(3*(t-1)+1)}\right) * \left(1 - s_{(3*(t-1)+2)}\right) * s_{(3*(t-1)+3)} \;+ \qquad (13) \\
& \left(1 - s_{(3*(t-1)+1)}\right) * s_{(3*(t-1)+2)} * \left(1 - f_{(3*(t-1)+3)}\right) \;+ \\
& s_{(3*(t-1)+1)} * f_{(3*(t-1)+2)} * s_{(3*(t-1)+3)} \;+ \\
& s_{(3*(t-1)+1)} * \left(1 - f_{(3*(t-1)+2)}\right) * \left(1 - f_{(3*(t-1)+3)}\right), \quad \forall t.
\end{aligned}
$$

Note the four possible histories implicit in (12) and (13). A simple averaging of monthly probabilities ignores these different experiences.

We summarize the key business cycle features of the data in Table 1. For transition prob-

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abilities, we report both averages of f's and s's and F's and S's to facilitate comparison with Shimer (2005a). To describe the business-cycle variation in these quantities, we follow Shimer (2005a) and remove the low-frequency trend in all variables implied by the Hodrick-Prescott filter, using a smoothing parameter of 10^5. We apply this procedure to remove movements in the aggregates induced by institutional and technological changes associated with job matching, so that they are not spuriously assigned to the matching function instability that arises from cyclical movements in labor market mismatch. Key business cycle features of the U.S. data are summarized in Table 1.

Table 1: U.S. DATA (Quarterly, 1951Q1-2003Q4)

	u	v	v/u	$u{\to}e$	$u{\to}e^*$	$e{\to}u$	$e{\to}u^*$	z^{**}
Standard Dev.	0.190	0.202	0.381	0.117	0.059	0.075	0.118	0.020
Autocorrelation	0.938	0.947	0.946	0.910	0.916	0.731	0.870	0.889
Cross Correlations								
u		-0.894	-0.971	-0.949	-0.938	0.712	0.889	-0.417
v			0.974	0.898	0.908	-0.689	-0.852	0.369
v/u				0.948	0.948	-0.718	-0.893	0.402
$u{\to}e$					0.990	-0.578	-0.841	0.406
$u{\to}e^*$						-0.590	-0.840	0.414
$e{\to}u$							0.910	-0.518
$e{\to}u^*$								-0.546

* Quarterly transition probabilities aggregated as in (12) and (13).

** Official BLS labor productivity measure for the nonfarm business sector.

From Table 1, we observe that employment, vacancies, and the vacancy-unemployment ratio are all strongly procyclical and persistent, while employment is strongly countercyclical and persistent. These data also affirm the Beveridge curve with a strong contemporaneous correlation between vacancies and unemployment of -0.894. Note that unemployment and vacancies are nearly 10 times more volatile than labor productivity, and the volatility of the vacancy-unemployment ratio (market tightness) is extreme, with a standard deviation of 38 percent around its trend. Although these facts are mutually consistent with the qualitative

predictions of the standard model, Table 1 also points out the model's shortcomings as put forth by Shimer (2005a). We will contrast these facts with the standard model's implications in the following section.

Table 1 also highlights the importance of choosing the appropriate temporal aggregation scheme. Although a complete rendering of the monthly transition histories compared to simple arithmetic averaging results in two strongly correlated series (the correlation between u→e and u→e* is 0.99 and *e→u and e→u** is 0.91), the relative variation changes significantly. With aggregations constructed according to (12) and (13), quarterly separations are more volatile than quarterly job findings. It is crucial, however, to keep in mind that the relative variation reversal is a by-product of temporal aggregation. It does not imply that higher-frequency fluctuations in unemployment are dominated by separations.

4   Results

In this section, we explore the cyclical properties of the search model with two sets of simulation results. First, we subject the model to solely a labor productivity shock, holding allocative efficiency and job destruction constant. This experiment provides a direct comparison to the standard labor market search model as in Shimer (2005a). The second simulation incorporates the allocative efficiency and job destruction shocks, with all three governed by a VAR(1) process. Before presenting our results, we briefly describe the calibration of our model.

4.1   Calibration

With a large empirical literature to draw upon and stationary labor market variables at hand, we combine micro-evidence with long-run data averages to calibrate the steady state values of the exogenous shocks and the technology/preference parameters. We begin by setting the steady state values of the labor market variables, N_t, V_t, and U_t, equal to the corresponding first moments of the data: $N = 0.943$, $V = 0.048$, and $U = 0.057$. Given these values, we observe that the steady-state version of the equation-of-motion for employment (7),

$$\sigma N = \chi V^\alpha U^{1-\alpha}, \tag{14}$$

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sharply restricts the steady state values of the shocks, χ_t and σ_t, and the matching technology parameter, α. We set α equal to 0.28, which is the value used by Shimer (2005a).[9] The steady state rate of job separation is chosen to be 6.9 percent of total employment per quarter, or $\sigma = 0.069$, which is the implied quarterly average of job separation probabilities previously discussed. Under these settings, the steady state employment condition (14) subsequently pins down the steady state allocative efficiency level: $\chi = 1.056$. These parameters imply an average vacancy duration, $(M/V)^{-1}$, of 0.85 quarters or about 76 days, which is in the neighborhood of the value, although a bit higher, than that reported by van Ours and Ridder (1992) using data from the Dutch economy. The implied unemployment duration is 0.98 quarters, or about 12.7 weeks, which is consistent with U.S. data.

Without loss of generality, we normalize the steady state of inferred aggregate output to one, $ZN = 1$, yielding steady-state labor productivity $Z = 1/N = 1.06$. Under this assumption, the steady state resource constraint becomes

$$C + \phi U + \kappa V = 1.$$

Note that in the absence of search and recruiting costs, i.e. $\phi = \kappa = 0$, labor productivity reduces to the traditional definition of the average product of labor. Steady state labor productivity equals C^{-1} in that case. (Recall that we must proxy consumption with aggregate output, or real GDP.) In the presence of search and recruiting costs, our imputed output measure somewhat deviates from measured real GDP, but we anticipate that the magnitude of the difference will be small, with the settings of parameters ϕ and κ largely determining the gap. Unlike the model's other parameters, independent evidence regarding these two parameters is scarce to non-existent. We follow Andolfatto (1996) in assuming that steady state recruiting expenditures are one percent of output, or $\kappa V = 0.01$, implying $\kappa = 0.206$. We assume that steady state search costs for workers are also one percent of aggregate output, $\phi U = 0.01$, yielding $\phi = 0.176$. The steady state value of consumption, C, is therefore 0.98, or 98 percent of output.

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[9]This is at the low end of the estimates surveyed by Petrongolo and Pissarides (2001). In our robustness section we allow for variations in this parameter.

Table 2: Calibrated Parameters		
Parameter	Value	Source
β	0.99	4% interest
α	0.28	Shimer (2005a)
γ	1.25	Merz (1995)
ϕ	0.1762	1% of Output
κ	0.2056	1% of Output
χ^{ss}	1.0561	u^{ss} and v^{ss}
z^{ss}	1.0602	Avg. Output = 1
σ^{ss}	0.0609	Shimer (2005b)

Next, we consider the two preference parameters, β and γ, the subjective discount factor and the Frisch elasticity of the labor supply, respectively. We choose $\beta = 0.99$ to be consistent with a steady-state risk-free real interest rate of 4 percent. We follow Merz's (1995) interpretation of the empirical literature and choose $\gamma = 1.5$ for the Frisch elasticity. Table 2 summarizes our calibration. In section 6, we consider different values of γ and α.

To calibrate the shock process, we first define the data series that comprise the VAR(1) specification. Recall that the job separation probability series is taken from Shimer (2005b), corrected for time aggregation, and that the productivity shock, Z_t, is measured by the real output per person in the nonfarm business sector reported by the BLS. Observe that resource constraint along with the data on V_t, U_t, and C_t also defines the period-t productivity shock as follows:

$$ Z_t = \frac{C_t + \phi\left(1 - N_t\right) + \kappa V_t.}{N_t} \tag{15} $$

Since we have reported cyclical properties of U.S. labor market data relative to the official BLS measure of nonfarm business sector labor productivity for reporting consistency, it is natural to measure Z_t by the BLS series rather than what is implied by (15). However, it is comforting to note that the correlation between this series and the BLS series is nearly perfect – 0.998, specifically.[10]

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[10] The correlation between H-P detrended measures is 0.869.

To measure the allocative efficiency shocks, we use job finding probabilities and the matching function. Recall that total flow into employment in a given period is dictated by the matching function, (3), which we can rewrite in the following way:

$$\chi_t V_t^\alpha \left(1 - N_t\right)^{1-\alpha} = \chi_t V_t^\alpha \left(1 - N_t\right)^{-\alpha} \left(1 - N_t\right) = \Pr(u \to e) * \left(1 - N_t\right). \tag{16}$$

In words, total number of matches is equivalent to the job finding probability multiplied by the number of unemployed workers. This decomposition along with data on V_t, U_t, and $\Pr(u \to e)$ identifies a time series for χ_t. In principle, our identification of χ_t is affected by the matching function parameter α. In section 6 we conduct a robustness check, which shows that our conclusions remain intact under different values of α. [11]

We depict the detrended time series for Z_t, χ_t, and σ_t in Figure 1. Knowing that productivity is strongly procyclical, we infer from the figure that allocative efficiency and job separation are both countercyclical. The contemporaneous correlations of χ_t and σ_t with Z_t are -0.37 and -0.55, respectively. Allocative efficiency and job destruction shocks show more volatility than Z_t. The countercyclicality of χ_t follows from the the fact that market tightness, V_t/U_t, is significantly procyclical and more volatile than our measure of job finding, $\Pr(u \to e)$. To see this, observe that (16) implies $\chi_t = \Pr(u \to e)/[V_t/U_t]^\alpha$. Although the job finding probability is procyclical and volatile, it is not enough so to offset the effects of market tightness.

We also want to see whether the inferred series of χ_t and σ_t imply reasonable fluctuations in unemployment. Figure 2 compares the actual unemployment rate with the unemployment rate implied by the shocks depicted in Figure 1 and inferred using the equation of motion (7). Figure 2 reveals that the shocks we use to estimate the VAR(1) lead to cyclical unemployment dynamics that are virtually identical to those of actual unemployment.

Finally, with these data series, we estimate the coefficients of A and Ω using the usual equation-by-equation OLS procedure. Estimates of A and Ω are as follows:

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[11]One could argue that χ_t is picking up the error term in the estimation of the aggrgate matching function, and hence it should be orthogonal to labor market variables. However, we believe that χ_t is omitted in these estimations and therefore estimates of α could be biased. This is another motivation for checking our conclusions under different values of α.

$$
A = \begin{bmatrix} 0.8961 & 0.1216 & -0.0377 \\ -0.0111 & 0.3075 & 0.2930 \\ -1.0368 & 0.0374 & 0.7574 \end{bmatrix} \quad \Omega = \begin{bmatrix} 0.00007 & -0.00003 & -0.00014 \\ -0.00003 & 0.00056 & 0.00047 \\ -0.00014 & 0.00047 & 0.00318 \end{bmatrix}. \quad (17)
$$

Next we turn to the simulation details of our two main experiments.

4.2 Simulating the Benchmark Economy

Before analyzing the multiple shock search model, we wish to establish the correspondence between our discrete-time, centralized model economy and the continuous-time, decentralized version of the standard model used by Shimer (2005b). To do this, we simulate the model with constant job destruction and allocative efficiency (σ and χ), but allow labor productivity, Z_t, to vary stochastically as the only source of exogenous variation. Specifically, we set σ_t and χ_t to their steady state values for all t (given in Table 2) and we assume that Z_t follows a first-order autoregressive process such that the standard deviation and the first-order autocorrelation match the corresponding moments in the data.

Our general solution algorithm is based on Christiano (2002) and relies on the linearized first-order condition (9). We posit linear decision rules for log deviations of the endogenous variables V_t, N_{t+1}, and C_t around their respective steady states as a function of N_t and $e_t = (Z_t, \chi_t, \sigma_t)$. In the benchmark model, the exogenous state consists of only Z_t.

Table 3 presents sample moments computed from 100 simulations of the model economy where each simulation is 500 periods in length. To facilitate comparison with Table 1, each variable is detrended using H-P filter with smoothing parameter of 10^5. We can summarize this table with three broad findings. First, vacancies and market tightness (v/u) are significantly procyclical, and unemployment is countercyclical. Second, the Beveridge curve relationship is consistent with the benchmark model, as shown by the negative correlation between unemployment and vacancies of -0.846. Finally, variation in the labor market variables is much less than the underlying variation in productivity.

	u	v	v/u	$u{\to}e$	$e{\to}u$	z
Standard Dev.	0.005	0.016	0.020	0.006	0.000	0.020
Autocorrelation	0.900	0.845	0.910	0.910	1.000	0.880
Cross Correlations						
u		−0.846	−0.913	0.085	0.000	−0.890
v			0.989	0.989	0.000	0.996
v/u				1.000	0.000	1.000
$u{\to}e$					0.000	1.000
$e{\to}u$						0.000

Table 3: Simulations of Benchmark Economy

This last observation provides the thrust of Shimer's (2005b) argument that the standard search model lacks the mechanisms that enable it to amplify realistically sized productivity shocks to produce the extent of variation in vacancies, unemployment, and market tightness observed in the data. The third rows of Table 1 and Table 3 confirm this point. Therefore, we conclude that the model with only productivity shocks behaves similarly to the criticized search model, even though we focus on the social planner's problem. In what follows, we refer to this discrepancy between the standard model and the data as the *amplification puzzle*.

4.3 Simulating the Multiple Shock Economy

We now focus on the model where the exogenous state space contains the full set of shocks: (Z_t, χ_t, σ_t). Having introduced two additional shocks to the model, we expect to resolve the amplification puzzle to some extent. To help gauge the contribution of the two additional shocks, we report the moments in a fashion similar to Table 1 and 3. Table 4 presents sample averages of moments from 100 simulations of the model economy, where each simulation is 500 periods in length. Once again, we report the percentage deviations from trend.

Table 4: Simulations of Multiple Shock Economy

	u	v	v/u	$u{\to}e$	$e{\to}u$ (σ)	χ	z
Standard Dev.	0.073	0.076	0.032	0.054	0.109	0.051	0.019
Autocorrelation	0.600	0.514	0.335	0.782	0.842	0.807	0.870
Cross Correlations							
u		0.908	−0.134	0.628	0.758	0.690	−0.548
v			0.291	0.785	0.721	0.782	−0.346
v/u				0.423	−0.023	0.274	0.434
$u{\to}e$					0.764	0.987	−0.254
$e{\to}u$ (σ)						0.816	−0.542
χ							−0.345

The simulation results from the multiple-shock economy are surprising. As expected, we observe substantially more volatility in all key variables, especially V_t and U_t. However, as the cross correlations in Table 4 show, the model correctly implies countercyclical unemployment only at the expense of significantly *countercyclical* behavior for vacancies. Therefore, the additional volatility is accompanied by an incorrect Beveridge curve relationship: The correlation between vacancies and unemployment is 0.908. Note that the incorrect Beveridge curve relationship is due to counterfactual behavior in vacancy creation, not unemployment. To sum up, adding two plausible channels of exogenous fluctuation partly resolves the amplification puzzle, but produces seriously counterfactual cyclical features for some of the key endogenous variables.

5   Discussion: Accounting for Imperfection

It is no surprise that the multiple-shock approach produces better results in terms of the volatility of endogenous variables. Beyond that, our results provide more questions than answers. What are the reasons behind conterfactual implications? To gain some traction on this question, we conduct an experiment. In what follows, we compute the innovations of the shock process that would be obtained if the actual data-generating process were indeed the

multiple-shock model.[12]   We then analyze the characteristics of the realized shocks to form economically meaningful conjectures for the counterfactual behavior implied by the simulation in the preceding section.   The exercise is a diagnostic one–one that leads us to potentially productive avenues for future research.

5.1   Perfect-Fit Experiment

Because we use a linearization-based algorithm to solve for the economy's decision rules, solving the exogenous shock series that generates a perfect fit of the multiple-shock model requires only a straightforward inversion of the log-linearized model.

The solution procedure generates log deviations of endogenous variables around the steady state as a function of N_t and $e_t = (Z_t, \chi_t, \sigma_t)$. Dropping the time subscript to denote steady state values and using lower-case letters to represent the corresponding log-deviation from steady state, we define the endogenous variables as follows: $n_t \equiv \ln\left(\frac{N_t}{N}\right)$, $v_t \equiv \ln\left(\frac{V_t}{V}\right)$, and $c_t \equiv \ln\left(\frac{C_t}{C}\right)$. The log-deviations of exogenous variables are similarly defined: $\tilde{z}_t \equiv \ln\left(\frac{Z_t}{Z}\right)$, $\widetilde{\chi}_t \equiv \ln\left(\frac{\chi_t}{\chi}\right)$, and $\widetilde{\sigma}_t \equiv \ln\left(\frac{\sigma_t}{\sigma}\right)$. A similar transformation can be applied to the VAR(1) shock process:

$$\widetilde{e}_{t+1} = A\widetilde{e}_t + \tilde{\varepsilon}_{t+1} \tag{18}$$

where $\widetilde{e}_t = (\tilde{z}_t, \widetilde{\chi}_t, \widetilde{\sigma}_t)'$, A is a 3×3 matrix of constants, and $\tilde{\varepsilon}_t$ is trivariate normal with $\mathrm{E}\tilde{\varepsilon}_t = 0$ and $\mathrm{E}[\varepsilon_t \varepsilon_t'] = \Omega$.

Given values for the parameters comprising the VAR(1) matrix of coefficients A, the decision rules mapping the period t state $(n_t, \widetilde{e}_t)$ into values for the endogenous variables (n_{t+1}, v_t, c_t) are required to be log-linear:

$$\begin{bmatrix} n_{t+1} \\ v_t \\ c_t \end{bmatrix} = \Pi \begin{bmatrix} n_t \\ z_t \\ \widetilde{\chi}_t \\ \widetilde{\sigma}_t \end{bmatrix}, \qquad \Pi = \begin{bmatrix} \pi_{nn} & \pi_{nz} & \pi_{n\widetilde{\chi}} & \pi_{n\widetilde{\sigma}} \\ \pi_{vn} & \pi_{vz} & \pi_{v\widetilde{\chi}} & \pi_{v\widetilde{\sigma}} \\ \pi_{cn} & \pi_{cz} & \pi_{c\widetilde{\chi}} & \pi_{c\widetilde{\sigma}} \end{bmatrix}, \tag{19}$$

where the π parameters comprise expressions of technology and preference parameters. Easy

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[12]This exercise is partly in the spirit of Chari, Kehoe, and McGrattan (2007) and Ingram, Kocherlakota, and Savin (1994).

manipulation segregates the observed variables from the unobserved exogenous variables:

$$
\begin{bmatrix} n_{t+1} - \pi_{nn}n_t \\ v_t - \pi_{vn}n_t \\ c_t - \pi_{cn}n_t \end{bmatrix} = \widehat{\Pi} \begin{bmatrix} z_t \\ \widetilde{\chi}_t \\ \widetilde{\sigma}_t \end{bmatrix}, \qquad \widehat{\Pi} = \begin{bmatrix} \pi_{nz} & \pi_{n\widetilde{\chi}} & \pi_{n\widetilde{\sigma}} \\ \pi_{vz} & \pi_{v\widetilde{\chi}} & \pi_{v\widetilde{\sigma}} \\ \pi_{cz} & \pi_{c\widetilde{\chi}} & \pi_{c\widetilde{\sigma}} \end{bmatrix}. \tag{20}
$$

Given data series for employment, vacancies, and consumption, the left-hand side of this expression is a vector of constants in any given period. Then, the matrix $\widehat{\Pi}$ is easily inverted to yield the period-t realization of the forcing process: $(\tilde{z}_t, \widetilde{\chi}_t, \widetilde{\sigma}_t)$. Our estimates in (17) and the mapping in (20) yield a unique set of realizations for $(\tilde{z}_t, \widetilde{\chi}_t, \widetilde{\sigma}_t)$.

Figure 3 plots the implied time series, and Table 5 summarizes basic statistics about the required shocks for a perfect fit.[13] We see that the standard deviation of allocative efficiency and job destruction are both required to be much larger than that of labor productivity to ensure a perfect fit to the data. More interestingly, the implied job destruction and allocative efficiency shocks are both significantly procyclical. Recall that section 4 led to a different result. We computed the χ_t and σ_t series using the data on job finding and job separation probabilities, and they showed significant countercyclicality over the cycle. However, it is important to note that the computations in section 4 and the exercise here give us two different set of objects. In section 4, we use minimal theory to measure Z_t, χ_t, and σ_t so that we can estimate the VAR(1) that governs these shocks. On the other hand, our perfect-fit experiment in this section generates a set of realizations for Z_t, χ_t, and σ_t that have to satisfy all the first-order conditions, and *also* generate the observed U.S. data. Therefore, these two sets of (Z_t, χ_t, σ_t) are conceptually very different.

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[13] Once again, all variables are log-deviations from their H-P trend, with smoothing parameter, 10^5.

Table 5: Required Shocks for a Perfect Fit			
	Z	χ	σ
Standard Deviation	0.016	0.820	0.745
Autocorrelation	0.894	0.935	0.922
Cross Correlations			
Z		0.596	0.565
χ			0.996

Perhaps a more useful way of diagnosing Mortensen-Pissarides search model is to follow Chari, Kehoe and McGratten (2007) and decompose the fluctuations in employment into the components that are due to different shocks. Specifically, we deduce the variation in employment that can be accounted for by each of the shock variables. To do this we take the sources of variation that were used to generate the perfect-fit, and expose them to the model, one at a time. The remaining shocks are set equal to steady state values in each period.

The results of this exercise is presented are Figure 4. First, consider the model with labor productivity as the only exogenous variable. Observe that the employment response to a labor productivity shock alone is virtually flat at zero, implying that labor productivity shocks have little explanatory power in the labor market search model. This echoes Shimer's critique (2005a). Unlike the Z-only case, both the χ-only and σ-only cases generate significant employment volatility. Note however, that they exhibit divergent cyclical characteristics. The allocative efficiency shock generates procyclical employment pattern, whereas the job destruction alone creates a countercyclical employment pattern. We conclude that allocative efficiency is a productive source of exogenous variation in rectifying the qualitative and quantitative problems of the Mortensen-Pissarides labor market search model.

In a similar vein we can further evaluate the model by sequentially feeding two of the three shocks into the model. The results of this exercise are presented in Figure 5. First consider the experiment which exposes the model to only the χ_t and σ_t that we backed out from the perfect-fit exercise (i.e. exclude Z_t). The implied behavior of employment is virtually indistinguishable from actual employment behavior. This serves as a convincing restatement of the result in the previous experiment in which employment showed no response to a labor productivity shock

alone. The remaining two experiments exclude χ_t and σ_t in turn. The latter one generates the appropriate cyclical employment response. In other words using just labor productivity and allocative efficiency shocks allows us to generate nearly the same behavior as the χ-only shock in the previous experiment. This gives more support for our conclusion that matching efficiency is an important ingredient to labor market dynamics.

This perfect-fit exercise has so far showed us how the shocks must have behaved if we take our model as the true data-generating process. There is obviously nothing surprising with the implied series on labor productivity, Z_t. On the other hand, since we are able to identify χ_t only indirectly when we use restrictions from the model, we can be agnostic about the true nature of the allocative efficiency shock. However, the implied job destruction series poses a significant challenge. It is impossible to reconcile a procyclical job destruction shock with the existing evidence (Blanchard and Diamond (1990), Davis and Haltiwanger (1992), Davis, Haltiwanger, and Schuh (1996) and Shimer (2005a)). Moreover, our decomposition suggests that this component accounts for a significant fraction of the fluctuations in employment alone. Since we cast this experiment as a diagnostic procedure, we need an answer to the following question: What are the properties of the multiple-shock search model that require it to produce a procyclical and volatile job separation rate to account for U.S. employment fluctuations? A successful answer to this puzzle requires a deeper understanding of the mechanics of the standard labor market search model.

5.2   Diagnosing the Search Model

We begin our analysis by identifying the model mechanisms and the cyclical properties of the observed data that produce the results highlighted above. Motivated by the persistent and procyclical movements of labor productivity, Z_t, we first trace out the dynamics generated by the search model in response to a sudden and persistent increase in labor productivity, holding constant allocative efficiency, χ_t, and the rate of job separation, σ_t. In doing so, we make use of the equations (6), (7), and (9).

Consider first the effects of an innovation to labor productivity. By signaling greater future productivity [as captured by the term Z' in the intertemporal efficiency condition (9)], it encourages an immediate spike in vacancies, as firms respond to the higher anticipated produc-

tivity benefits of filled positions. This immediately increases the vacancy-unemployment ratio. In addition, new matches form in the next period, thereby increasing employment and reducing unemployment. These effects are summarized by an increasing vacancy-unemployment ratio.

The productivity innovation also sets in motion forces that oppose the increasing vacancy-unemployment ratio. To see this, one first notes that the resource constraint (6) translates the anticipated increase in future productivity and employment into higher future consumption through a more rapid output flow.[14] The increases in employment and consumption subsequently reduce the representative worker's marginal willingness to substitute nonmarket activities for consumption, i.e., $\frac{U'_N}{U'_C}$ decreases in equation (9). This offsets, to some extent, an individual firm's vacancy-creation motive and the subsequent increase in employment. Furthermore, the draining of the unemployment pool persists and offsets some of the future benefits of currently high productivity by frustrating future hiring efforts through the term $-\frac{\kappa M'_U}{M'_V}$; this term represents the additional future recruiting costs exacted by the depleted stock of searching workers on the right-hand side of (9). Recall that this last quantity (or more precisely, its absolute value) is directly proportional to the vacancy-unemployment ratio–a proxy for the "tightness" of the labor market. The data, as we have seen, display extremely large procyclical variation in this ratio, which casts doubt on the model's ability to produce the required cyclical variation in response to realistically sized shocks to labor productivity.

By allowing both matching efficiency and the job separation rate to vary over the business cycle, the preceding diagnostic procedure responds to this tension by equating the observed vacancy-unemployment ratio with the socially optimal one in each period. The highly variable and procyclical allocative efficiency shock, χ_t, implied by this exercise (Table 5 and Figure 3) effectively increases the expected gains of vacancy creation in response to exogenous increases in labor productivity, thus generating additional vacancies while also increasing the rate at which unemployed workers meet up with them. As a result, the flow of workers from unemployment to employment increases, reducing the unemployment pool. The increase in vacancies, coupled with falling unemployment, gives an additional upward push to the vacancy-unemployment ratio, moving the economy along the Beveridge curve in accordance with the data. Although

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[14]The sum of search and vacancy-creation costs, $\phi(1 - N_t) + \kappa V_t$, is small, and the increase in vacancy-creation costs κV_t counteracts the reduction in search costs, $\phi(1 - N_t)$.

the vacancy-unemployment ratio moves decidedly in the proper direction, it cannot do so with a sizeable increase in net employment, all else constant. However, given that the aggregate employment (or unemployment) data reveal relatively small period-to-period changes, the model requires a much larger employment outflow to restock the unemployment pool depleted by the enhanced matching efficiency. This element could only be provided by the required procyclical rate of job separation, σ_t, (Table 5 and Figure 3). Recall that Figures 4 and 5 indicate that movements in χ_t need an offsetting movement in σ_t to generate empirically consistent employment fluctuations.

In light of Figures 4 and 5 we can also claim that without a movement in χ_t or σ_t the effects of Z_t could end up being negligible. This is consistent with Shimer (2005a). However, our perfect-fit experiment and the discussion above suggest that we cannot expect to have enough variation in vacancies (hence market tightness) to warrant the observed level of variation in the data. This variation is mostly achieved through high-frequency movements in χ_t and an offsetting effect of σ_t to generate enough churning consistent with relatively small movements in employment between two consecutive periods.

5.3 A Resolution: Procyclical Reallocation

At this point we could accept the results of our experiment with a claim that matching efficiency and job separation are indeed both strongly procyclical. To our knowledge, there is no direct evidence on the efficiency of labor market matching over the business cycle. There is some empirical evidence by Bowlus (1995) showing that mismatch has cyclical characteristics, increasing during recessions. We could simply argue that procyclical matching efficiency is a more reasonable outcome than the alternatives. As we have already stated however, sharply procyclical job separation is strongly at odds with existing data. To jointly accept both outcomes would be the equivalent of believing in an extraordinarily unlikely draw from the distribution of shocks. Instead, we look for economic meaning in the results to conjecture a plausible solution to the puzzle, and by doing so, propose potentially productive modifications to the standard labor market search framework. We begin with a brief discussion of the literature regarding the amplification puzzle.

Recent studies have attributed the amplification puzzle to different characteristics of the

standard labor market search model with only a productivity shock. Shimer (2005a) and Hall (2005) suggest that the underlying wage determination mechanism is the reason for the lack of amplification in these models. Hall (2004, 2005), Shimer (2004), and Kennan (2007) build on this presumption and introduce wage rigidity either exogenously or through an endogenous mechanism, such as asymmetric information.

As argued extensively by Mortensen and Nagypal (2007), however, wage rigidity *per se* is not the reason for amplification. For instance, even in a case where the workers' lack of bargaining strength leads to constant wages that are equal to the reservation wage (i.e., the value of leisure), the variability of labor market variables relative to productivity is an order of magnitude smaller than the data (Mortensen and Nagypal (2007)). Moreover, Pissarides (2008) argues that the empirical evidence in favor of wage rigidity over the cycle is not true for newly created matches, which is the important margin for job creation in the canonical labor market search model. Therefore, we conclude that our formulation of the problem as a social planner's problem, thereby ignoring wage determination, is not crucial for understanding the amplification puzzle.

Several recent studies also aim to provide a mechanism that can amplify the effects of business cycles on unemployment and vacancies (Hagedorn and Manovskii (2007), Krause and Lubik (2006), Nagypal (2006), Silva and Toledo (Forthcoming) and Tasci (2007)). Hagedorn and Manovskii (2007) use an unrealistically high value of nonmarket activity to generate amplification, which also implies an excessive unemployment response to a slight increase in unemployment compensation (Costain and Reiter (2008) and Hornstein, Krusell, and Violante (2006)). Silva and Toledo (Forthcoming)'s result depends on a particular constellation of parameter values for separation and hiring and training costs that is hard to quantify empirically.

Introducing a labor force participation decision or the possibility of job-to-job transitions breaks the tight link between job matching and job dissolution in the standard setup by creating an additional pool of workers to draw upon to fill newly created vacancies. Whether these flows are significant is an empirical question. Although the distinction between unemployment and labor force nonparticipation is fairly vague, there is substantial evidence pointing to substantial job-to-job movements. Nagypal (2004) and Shimer (2005b) argue that job-to-job transitions are crucial for cyclical worker reallocation. Exploiting dependent interviewing methods introduced

in the CPS in 1994, Fallick and Fleischman (2004) find that these flows are large: on average 2.6% of employed workers change employers each month. Moreover, job-to-job transitions are procyclical. This particular flow cannot be analyzed by standard search models. Thus, on-the-job search provides a natural research avenue to pursue. Krause and Lubik (2006), Nagypal (2006), and Tasci (2007) are examples of this approach. Recently, Ramey (2008) also argues that endogenous separations accompanied by on-the-job search could improve the fit of the model to a large extent.

The channel through which on-the-job search creates more fluctuations in the standard model's key variables without resorting to counterfactual job separations is straightforward. Consider the case in the preceding section. Following a positive productivity shock, labor demand expands and firms create vacancies. The additional vacancies subsequently imply a higher job finding rate and lower unemployment. If all new matches form through the pool of unemployed workers, an increasingly small pool of unemployed workers dampens the incentive for firms to further create vacancies. This is the mechanism in the model that we emphasized in the previous section, which gives rise to counterfactual job separations to account for unemployment fluctuations in the data. In Nagypal (2006), information frictions generate a bias for firms to hire employed workers, which reduces the counter effect. Alternatively, in Tasci (2007), the underlying match heterogeneity in the form of symmetric incomplete information about the quality of the job-worker match implies that there is a measure of workers employed in relatively low quality matches during expansions. These workers have the incentive to accept better quality matches and provide the additional incentive for firms to post vacancies.

6 Robustness

This section explores the robustness of our results. We check whether our findings vary with the elasticity of the matching function and the elasticity of the labor supply, α and γ, respectively. Recall that we calibrated γ to be 1.25 based on Merz (1995). Since this parameter determines the response of the household labor supply to changes in productivity, it is important to know whether our results are dependent on a particular choice. Similarly, we change the parameter value governing the elasticity of matching and check whether it fundamentally alters

our conclusions. We present the key statistics from our robustness check in Table 6. These statistics include standard deviations of unemployment,vacancies, and vacancy-unemployment ratio and their correlations to the productivity shocks.

Our alternatives for γ are 2 and 0.5. Simulating the multiple-shock economy with these parameter values changes virtually nothing. The model continues to generate more volatility in labor market variables than the benchmark single-shock economy. Moreover, the counterfactual cyclical implications remain in place with countercyclical job vacancies and a positive correlation between unemployment and vacancies. When we repeat our perfect-fit experiment, the model continues to require procyclical job separation and allocative efficiency shocks.[15]

Table 6:	Robustness Checks for Various α and γ.				
	$\alpha = 0.4$	$\alpha = 0.5$	$\alpha = 0.28$	$\gamma = 0.5$	$\gamma = 2$
			$\gamma = 1.25$		
std(u)	0.069	0.061	0.073	0.070	0.0740
std(v)	0.078	0.079	0.076	0.077	0.3075
std(v/u)	0.025	0.028	0.032	0.030	0.032
corr(u,v)	0.947	0.947	0.908	0.921	0.907
corr(u,z)	-0.547	-0.539	-0.548	-0.540	-0.550
corr(v,z)	-0.443	-0.509	-0.346	-0.372	-0.346

Since $\alpha = 0.28$ lies at the lower end of the matching function estimates that Petrongolo and Pissarides (2001) provide, we consider higher values. Increasing the value of α from 0.28 to 0.4 and 0.5 slightly alters our results, generating even less volatility in unemployment and market tightness. This is expected, given that α also determines the share of the match surplus extracted by workers in the decentralized analogue. As the firm's share falls, vacancy-creation becomes less sensitive to the underlying changes in the value of a match. However, the shocks required for a perfect fit continue to exhibit procyclical job separation and allocative efficiency.[16] Hence, we conclude that our results remain in place for reasonably different values of the Frisch elasticity of labor supply and the elasticity of the matching function.

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[15] Results are not shown here but are available upon request.
[16] Results are not shown here but are available upon request.

7 Conclusion

We have extended a basic discrete-time version of the Mortensen-Pissarides model of labor market search to include multiple and mutually correlated sources of exogenous variation. We use our extended model to investigate the model's well-known tendency to underpredict the volatility of key labor market variables. The shock process comprises labor productivity, job separation, and matching efficiency and is partly estimated using data on job finding and separation probabilities for the U.S. economy. Although our model generates more volatility, it has counterfactual implications for the cyclicality of unemployment and vacancies.

We exploit the degrees of freedom facilitated by the multiple-shock structure of our model to uncover the mechanics, or lack thereof, that generate the empirically implausible implications. This leads us to our second exercise, which forces the model to be the data-generating process, allowing us to uncover the necessary realizations of all three shocks. We show that the Mortensen-Pissarides labor market search model requires significantly procyclical and volatile matching efficiency and job separations to simultaneously account for high procyclical variations in the job finding probability as well as the relatively small net employment change in the data. Hence, the standard Mortensen-Pissarides labor market search model is more fundamentally flawed than its inability to amplify shocks would suggest. This leads us to conclude that the model lacks mechanisms to generate procyclical matching efficiency and labor force reallocation. The conclusion points us in the direction of models that allow job-to-job transitions as a productive first step in amending the standard model. We also show that variation in job separations and matching efficiency account for most of the employment fluctuations, suggesting that endogenous separations and cyclical mismatch could be the key feature of an improved model. Our hope is to stimulate further research into the nature of our findings and to generate even richer theoretical structures that will eventually give us a more thorough picture of aggregate labor market fluctuations.

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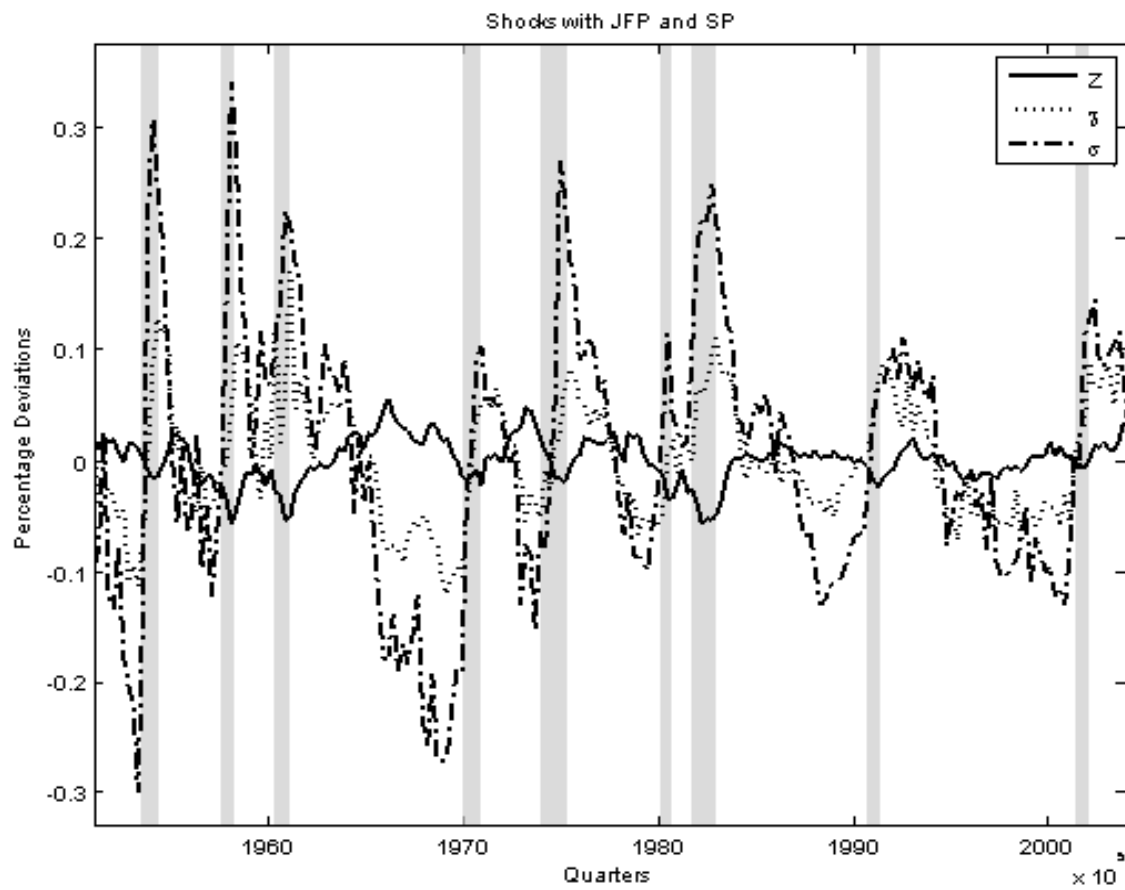
Figure 1: Estimated shocks using job finding (JFP) and separation probabilities (SP). Shaded areas indicate NBER recessions.
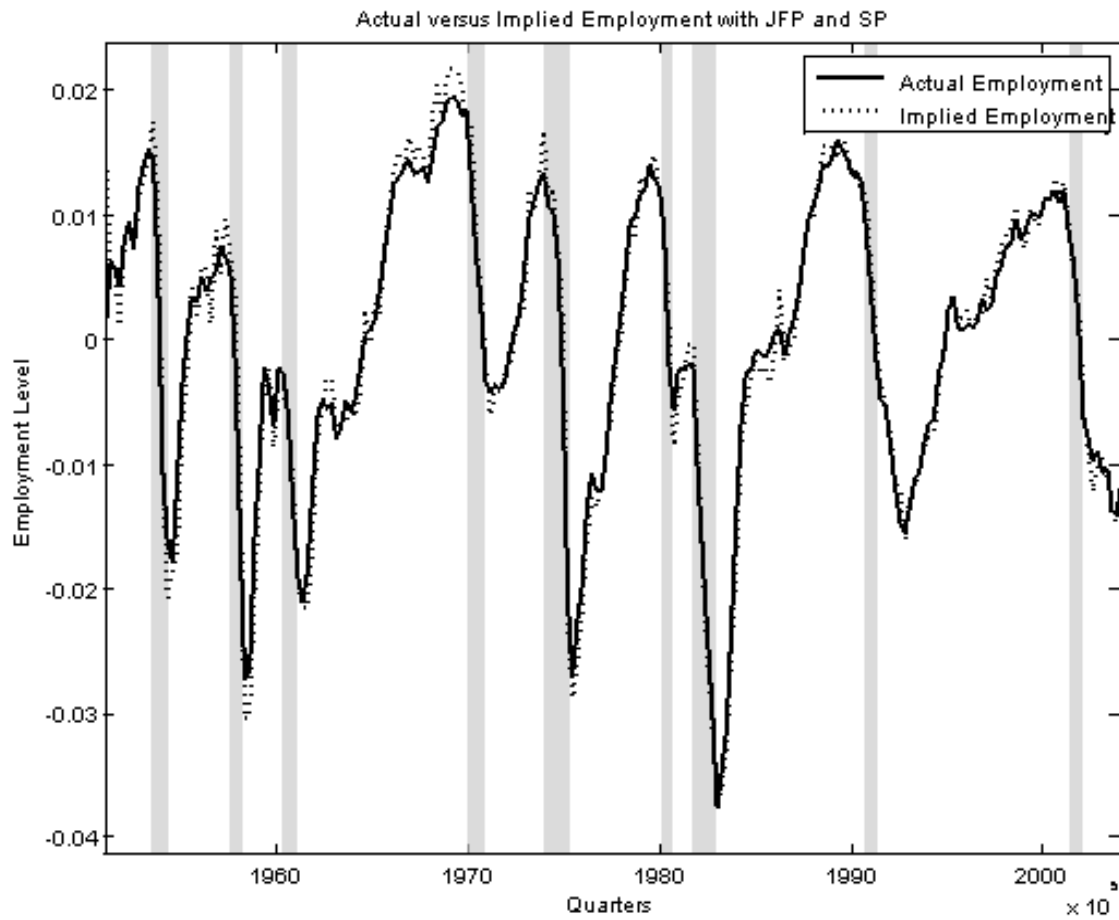
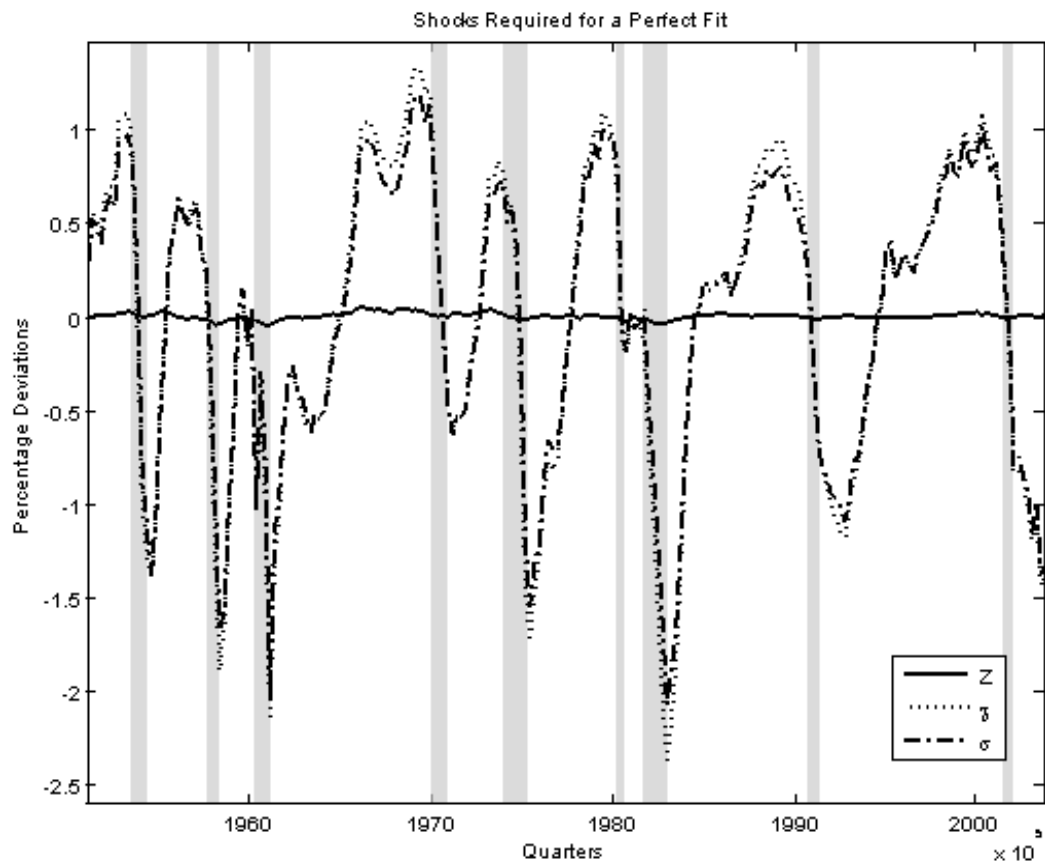Figure 2: Actual versus Implied Unemployment. Shaded areas indicate NBER recessions.

Figure 3: Shocks required for the perfect fit. Shaded areas indicate NBER recessions.
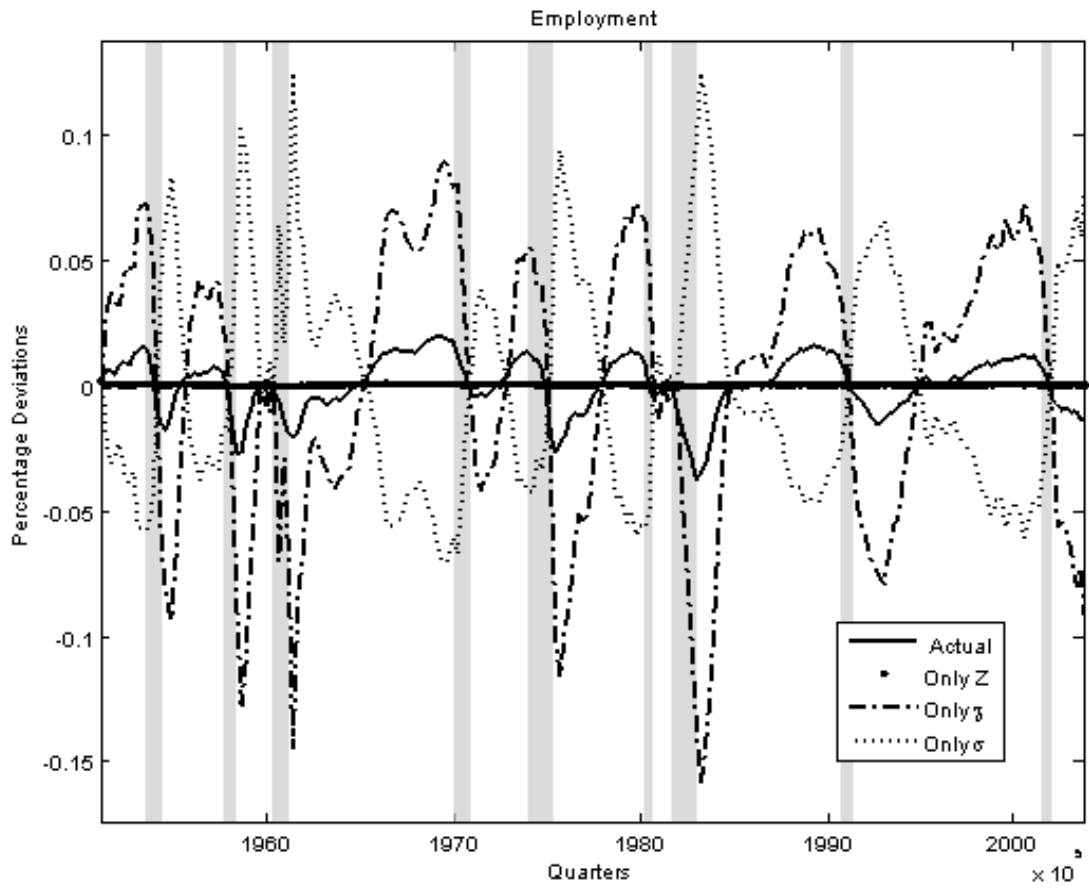
Figure 4: Contribution of each shock to employment fluctuations. Shaded areas indicate NBER recessions.

Figure 5: Contribution of pairs of shocks to employment fluctuations. Shaded areas indicate NBER recessions. The lines for Actual and Without Z coincide in this figure.