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**Positive and Normative Effects of a  
Minimum Wage**

by Guillame Rocheteau and Murat Tasci



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We review the positive and normative effects of a minimum wage in various versions of a search-theoretic model of the labor market.

Key words: Minimum wage, Search, Unemployment, Participation, Working Hours, Job Destruction.

JEL code: J08; J38; J42; E24

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# 1 Introduction

The federal minimum wage was established in 1938 by the Fair Labor Standards Act. Initially set at 25 cents per hour, the minimum wage has been raised periodically to reflect changes in inflation and productivity. On May 24, 2007, Congress approved the first increase in the federal minimum wage since September 1, 1997.

For the 10 years in between, the minimum wage stayed at \$5.15 an hour, but its real value declined steadily from about 40 percent of the average private nonsupervisory wage to a mere 30 percent. Adjusted for inflation, the minimum wage was lower at the beginning of 2007 than at any time since 1955 (see figure 1). Moreover, the fraction of hourly workers who earned no more than the minimum wage dropped from around 15 percent in 1980 to just 2.2 percent in 2006. By the beginning of 2007, the federal minimum was binding in only 21 states. On May 24, Congress passed a bill raising the historically low real federal minimum wage to \$7.25 in three phases over two years.

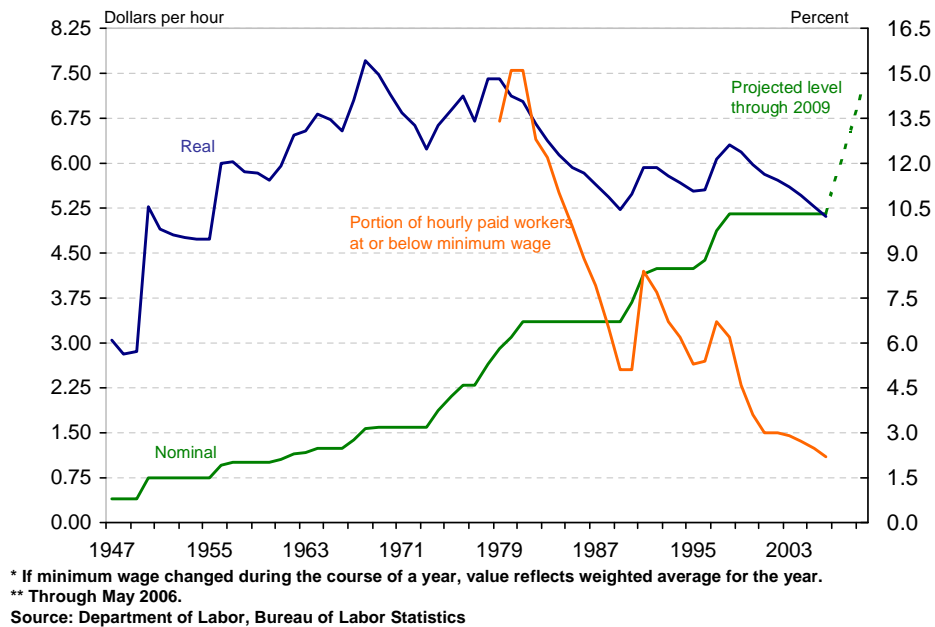


Figure 1: **Federal Minimum Wage and Portion of Workers at or Below Minimum Wage**

When it was established in 1938, Fair Labor Standards Act emphasized using minimum wage policy to reduce poverty. In this Policy Discussion Paper, we want to clarify the debate

about the minimum wage by analyzing how the main economic theories view its effects on the labor market. Broadly speaking, opponents of a minimum wage believe that labor markets are competitive and any wage regulation is therefore bound to reduce employment, especially among low-skilled workers. On the other hand, the wage's proponents believe that labor markets are dominated by some employers, and argue that a minimum wage can exert positive effects on labor market outcomes by reducing employers' excessive market power. Unfortunately, both descriptions are extremely stylized. In this Policy Discussion Paper, we study alternative and more realistic environments and we investigate whether they deliver similar conclusions about the effects of minimum wage. We focus on labor markets featuring search frictions in various different forms as in Pissarides (2000). Our analysis consists of examples with endogenous search effort, labor force participation decision, a decision about hours of work, and endogenous job destruction due to heterogeneity in match productivity. We also calibrate our model economies to match some key U.S. labor market moments and then present the effects of minimum wage through numerical examples.

## 2 Search Effort

We start with a simple version of the labor market search model with endogenous search intensity.

### 2.1 Environment

The environment is similar to chapter 5 in Pissarides (2000). Time is discrete. Agents are risk-neutral and discount future utility according to the factor  $\beta \in (0, 1)$ . There is a unit-measure of workers indexed by  $i$  in  $[0, 1]$  and a large measure of firms which are free to enter the market. Each worker is endowed with one unit of labor and each firm corresponds to a single job. A match composed of one job and one worker produces  $z$  units of output per period. The wage paid by a firm to its worker is  $w \leq z$ .

When unemployed a worker receives an income  $b \leq w$  that can be interpreted as unemployment benefits, or the utility that the worker derives from not working. An unemployed worker must also expend some effort, denoted  $s$ , to find a job. The disutility associated with this

search effort (or intensity) is  $c(s)$  where  $c'(\cdot) > 0$ ,  $c''(\cdot) < 0$ ,  $c(0) = c'(0) = 0$  and  $c'(\infty) = \infty$ . Similarly, a firm with a vacant job must incur a cost  $\gamma > 0$  to advertise its vacancy.

The labor market is subject to search-matching frictions captured by an aggregate matching function that specifies the number of matches formed in each period,

$$M\left(\int_U s_i di, v\right),$$

where  $U \subseteq [0, 1]$  is the set of unemployed workers and  $v$  is the measure of vacancies. The first input of the matching function is the sum of unemployed workers' search efforts while the second input is the measure of vacancies posted by firms. The matching function exhibits constant returns to scale, is strictly concave and increasing with respect to each of its arguments. Furthermore, we impose the following feasibility condition,  $M\left(\int_U s_i di, v\right) \leq \min\left(\int_U di, v\right)$  (i.e., the number of matches cannot be greater than the measure of unemployed workers or the measure of vacancies).

From the aggregate matching function we are able to derive the matching probabilities for an unemployed worker and a vacancy. Denote  $u$  the measure of unemployed workers,  $u = \int_U di$ , and  $\bar{s} = \int_U s_i di / u$  denotes the average search effort of an unemployed workers. We define market tightness as  $\theta = v / \bar{s}u$ . The job finding probability of an unemployed worker searching with intensity  $s$  is  $sp(\theta)$  with

$$p(\theta) = \frac{M(\bar{s}u, v)}{\bar{s}u} = M(1, \theta).$$

Similarly, a vacant job finds an unemployed worker with probability

$$q(\theta) = \frac{M(\bar{s}u, v)}{v} = M\left(\frac{1}{\theta}, 1\right).$$

The elasticity of matching function with respect to unemployment could be defined as  $\eta(\theta) \equiv \frac{-q'(\theta)\theta}{q(\theta)}$ .

Finally, ongoing matches are destroyed exogenously with probability  $\delta$  every period. Firms enter the market as long as they make nonnegative expected profits.

## 2.2 Workers and firms

Let  $W^u$  denote the expected lifetime utility of an unemployed worker, and  $W^e(w)$  the expected lifetime utility of an employed worker who is paid a wage  $w$ . The Bellman equation for the value of being unemployed is<sup>1</sup>

$$W^u = \max_{s \geq 0} \{b - c(s) + \beta [sp(\theta)W^e(w) + (1 - sp(\theta))W^u]\} \quad (1)$$

According to (1) an unemployed worker enjoys an income  $b$  and searches for a job with intensity  $s$ . With probability  $sp(\theta)$  he finds a job and starts the next period as employed, and with the complement probability he remains unemployed. The optimal choice of search intensity solves

$$c'(s) = \beta p(\theta) [W^e(w) - W^u] \quad (2)$$

Since  $c'(\cdot)$  is strictly increasing,  $c'(0) = 0$  and  $c'(\infty) = \infty$ , there is a unique solution to (2). Consequently, all unemployed workers search for a job with the same intensity.

The Bellman equation for the value of being employed is

$$W^e(w) = w + \beta [(1 - \delta)W^e(w) + \delta W^u] \quad (3)$$

An employed worker gets  $w$  and remains employed next period with exogenous probability  $(1 - \delta)$ . If the match dissolves with probability  $\delta$ , she becomes unemployed next period.

Next we turn to firms. Let  $J^u$  be the value of a vacant job and  $J^e(w)$  the value of a filled job when the wage paid to the worker is  $w$ . The Bellman equation for the value of a vacancy is

$$J^u = -\gamma + \beta [q(\theta)J^e(w) + (1 - q(\theta))J^u] \quad (4)$$

According to (4) a firm posting a vacancy incurs an advertising cost  $\gamma$  and the job is filled with probability  $q(\theta)$ . Firms enter into the market as long as they make nonnegative profits.

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<sup>1</sup>We assume that the optimal  $s$  is such that  $sp(\theta) \in [0, 1]$ .

Therefore,  $J^u = 0$  and equation (4) implies

$$J^e(w) = \frac{\gamma}{\beta q(\theta)} \quad (5)$$

According to (5) the value of a filled job must be equal to the expected recruiting cost incurred by the firm to fill a vacancy.

The Bellman equation for the value of a filled job is

$$J^e(w) = z - w + \beta(1 - \delta)J^e(w). \quad (6)$$

According to (6) a filled job generates a profit  $z - w$  per period, and the job survives destruction with probability  $(1 - \delta)$ .

### 2.3 Equilibrium

There are essentially three endogenous variables in the model:  $s$ ,  $u$  and  $\theta$  (therefore  $v$ ). Search effort is determined by (2). Using (3) and (1) the equilibrium condition for workers' search intensity becomes

$$c'(s) = \frac{\beta p(\theta)}{1 - \beta + \beta [sp(\theta) + \delta]} [w - b + c(s)] \quad (7)$$

Differentiating (7), it can be checked that

$$\{[1 - \beta(1 - \delta)] c''(s) + \beta p(\theta) [c''(s)s - c'(s)]\} \frac{ds}{d\theta} = \frac{1 - \beta + \beta\delta}{1 - \beta + \beta [sp(\theta) + \delta]} [w - b + c(s)] \beta p'(\theta)$$

If  $c'''(s) > 0$  then  $ds/d\theta > 0$ . Workers' search effort increases as the market becomes tighter.

To determine the equilibrium market tightness substitute  $J^e(w)$  by its expression given by (5) into (6) to get

$$\frac{\gamma}{\beta q(\theta)} = \frac{z - w}{1 - \beta(1 - \delta)} \quad (8)$$

Notice from (8) that  $\theta$  is determined independently of workers' search intensity.

Finally, the law of motion for unemployment is



$$u_{+1} = u + (1 - u)\delta - sp(\theta)u \quad (9)$$

Hence, unemployment next period,  $u_{+1}$ , is equal to the unemployment in the current period plus the inflow of job destructions,  $(1 - u)\delta$ , minus job creations,  $M(\int_U s_i di, v)$ . At the steady state ( $u_{+1} = u$ ),

$$u = \frac{\delta}{\delta + sp(\theta)} \quad (10)$$

Having described the equilibrium conditions, we can formally state the definition of the equilibrium.

**Definition 1** A steady state equilibrium with exogenous wage is a triple  $(s, \theta, u)$  that satisfies (7), (8) and (10).

Equilibrium has a simple recursive structure. Equation (8) determines  $\theta$ . Knowing  $\theta$ , (7) gives  $s$ . Finally, given  $\theta$  and  $s$ , (10) gives  $u$ .

The following Proposition describes the effects of a change in the wage on the equilibrium outcome.

**Proposition 2** (i) Market tightness decreases with  $w$ . (ii) Search effort is a non-monotonic function of  $w$ . If  $w \in \{b, z\}$  then  $s = 0$ . (iii) Equilibrium unemployment is a non-monotonic function of the wage. If  $w \in \{b, z\}$  then  $u = 1$ .

**Proof.** (i) Direct from (8). (ii) Since  $c(0) = c'(0) = 0$ , it is easy to check that  $s = 0$  solves (7) when  $w = b$ . If  $w = z$  then  $\theta = 0$  from (8) which implies  $s = 0$  from (7). (iii) If  $s = 0$  then  $u = 1$  from (10). ■

Intuitively, if worker gets all the surplus than firms have no incentives to post vacancies, knowing that there are no vacancies, workers will not search at all. Similarly, if workers' outside option is no better than their income while unemployed, they do not search for a job. In both cases unemployment will be maximum.

## 2.4 Endogeneizing the wage

A standard assumption in the literature is to assume that wages are determined according to the generalized Nash bargaining solution where the worker's bargaining power is  $\lambda \in (0, 1)$ . The negotiated wage solves

$$w = \arg \max [W^e(w) - W^u]^\lambda [J^e(w)]^{1-\lambda} \quad (11)$$

From (3) and (6) this can be reformulated as

$$W^e(w) - W^u = \frac{\lambda}{1-\lambda} J^e(w) \quad (12)$$

Substituting  $W^e(w) - W^u$  by its expression given by (12) into (2) and using (5) the optimal search effort satisfies

$$c'(s) = \frac{\lambda}{1-\lambda} \gamma \theta \quad (13)$$

Multiplying both sides of (12) by  $1 - \beta(1 - \delta)$  and using (3) and (6),

$$w = \lambda z + (1 - \lambda)(1 - \beta)W^u \quad (14)$$

So the wage is a weighted-mean of the worker's productivity ( $z$ ) and his permanent income when unemployed ( $(1 - \beta)W^u$ ). From (1), (5) and (12)  $(1 - \beta)W^u$  satisfies

$$(1 - \beta)W^u = b - c(s) + s \frac{\lambda}{1-\lambda} \gamma \theta \quad (15)$$

Thus, the expression for the wage is

$$w = \lambda z + (1 - \lambda)[b - c(s)] + s\lambda\gamma\theta \quad (16)$$

Finally, substitute  $w$  by its expression into (8) and rearrange using (13) to find

$$\frac{\gamma}{(1-\lambda)\beta q(\theta)} = \frac{z - b + c(s) - sc'(s)}{1 - \beta(1 - \delta)} \quad (17)$$

Notice that the right-hand side of (17) is increasing in  $s$ . Therefore, (17) gives a negative relationship between  $\theta$  and  $s$ .

In the presence of minimum wage, the equation for wage becomes

$$w = \max\{\lambda z + (1 - \lambda) [b - c(s)] + s\lambda\gamma\theta, \underline{w}\} \quad (18)$$

**Definition 3** A steady state equilibrium with endogenous wage formation is a list  $(\theta, s, w, u)$  that solves (10), (13), (18) and (17).

The pair  $(\theta, s)$  is uniquely determined by (13) and (17). Then, given  $(\theta, s)$ ,  $w$  is determined by (16) and  $u$  is given by (10).

## 2.5 Welfare

We now ask whether the decentralized equilibrium is optimal. To this end, we consider the problem of a social planner who is subject to the matching frictions captured by  $M(\bar{s}u, v)$  and who maximizes the sum of all agents utility. For simplicity, suppose that the planner is infinitely patient ( $\beta \rightarrow 1$ ) and only cares about the steady state welfare. His problem is

$$\begin{aligned} \max_{u, s, v, \theta} & [(1 - u)z + u [b - c(s)] - \theta su\gamma] \\ \text{s.t.} & u = \frac{\delta}{\delta + sp(\theta)} \end{aligned} \quad (19)$$

where we have used that  $v = \theta su$ .

**Proposition 4** Equilibrium is efficient iff worker's bargaining power,  $\lambda$ , is equal to the elasticity of matching function with respect to unemployment,  $\eta(\theta)$ . Equivalently, the expression for the efficient wage is

$$w = \eta(\theta)z + [1 - \eta(\theta)] [b - c(s)] + s\eta(\theta)\gamma\theta \quad (20)$$

**Proof.** Substituting  $u$  by its expression, the maximization problem in (19) can be simplified to

$$\max_{s, \theta} \left\{ \frac{sp(\theta)z + \delta [b - c(s)] - \theta s\gamma\delta}{\delta + sp(\theta)} \right\}$$

The first-order conditions with respect to  $s$  and  $\theta$  are

$$s : p(\theta)z - \delta c'(s) - \theta\gamma\delta = p(\theta) \frac{sp(\theta)z + \delta [b - c(s)] - \theta s\gamma\delta}{\delta + sp(\theta)} \quad (21)$$

$$\theta : sp'(\theta)z - s\gamma\delta = sp'(\theta) \frac{sp(\theta)z + \delta [b - c(s)] - \theta s\gamma\delta}{\delta + sp(\theta)} \quad (22)$$

Divide (21) by (22) and use the fact that  $\eta(\theta) \equiv \frac{-q'(\theta)\theta}{q(\theta)}$  and  $1 - \eta(\theta) \equiv \frac{\theta p'(\theta)}{p(\theta)}$  to obtain

$$c'(s) = \frac{\eta(\theta)}{1 - \eta(\theta)} \gamma\theta \quad (23)$$

Then, rearrange (21) by using the fact that  $p(\theta) = \theta q(\theta)$  in order to get

$$\frac{\delta\gamma}{[1 - \eta(\theta)]q(\theta)} = z - b + c(s) - sc'(s) \quad (24)$$

The equilibrium conditions (13) and (17) when  $\beta \rightarrow 1$  can be rewritten as

$$c'(s) = \frac{\lambda}{1 - \lambda} \gamma\theta \quad (25)$$

$$\frac{\delta\gamma}{(1 - \lambda)q(\theta)} = z - b + c(s) - sc'(s) \quad (26)$$

The comparison of (23)-(24) and (25)-(26) shows that equilibrium is efficient iff  $\lambda = \eta(\theta)$ .

Substituting  $\lambda = \eta(\theta)$  into (16) the expression for the wage is given by (20). ■

Proposition 4 states that equilibrium is efficient when the worker's bargaining power ( $\lambda$ ) coincides with the elasticity of the matching function ( $\eta$ ). This is the so-called Hosios (1990) condition for efficiency in environments with search frictions. The interpretation for this condition is as follows. Since the matching function exhibits constant returns to scale, it satisfies

$$M(su, v) = M_u su + M_v v,$$

where  $M_u$  and  $M_v$  are the partial derivatives of  $M$  with respect to each of its arguments. The fraction of matches that can be attributed to worker's search effort is then

$$\frac{M_u su}{M} = \eta.$$

According to Mortensen (1982), efficiency requires that workers get the entire surplus of the match in those matches that they are responsible for, that is, a fraction  $\eta$  of the matches. Equivalently, since workers are risk neutral, they should receive a fraction  $\eta$  of all match surpluses, that is  $\lambda = \eta$ . Of course, there are no reasons that  $\lambda$  and  $\eta$  coincide and therefore the equilibrium is in general inefficient.

**Proposition 5** Worker’s search intensity is increasing with worker’s bargaining power whenever  $\eta(\theta) > \lambda$  and it reaches its maximum when  $\eta(\theta) = \lambda$ .

**Proof.** Total differentiate (13) and (17). It can be shown after some calculation that

$$\text{sign} \left( \frac{ds}{d\lambda} \right) = \text{sign} [\eta(\theta) - \lambda]$$

■

According to Proposition 5, an increase in the bargaining power of workers raises their search intensity if the elasticity of the matching function is less than their bargaining power. In other words, if the wage is too low –lower than the level that maximizes social welfare– then a mandatory increase in the wage can raise the search effort of workers and society’s welfare together.

## 2.6 Calibration and numerical exercise

We calibrate our model to match some simple features of U.S. labor markets<sup>2</sup>. We present two different calibrations here; one for exogenously given wage and one for the endogenous wage determined through Nash bargaining, as usual in the literature. We assume a Cobb-Douglas functional form for the matching function:  $M = (\bar{s}u)^\eta v^{1-\eta}$ , implying that  $p(\theta) = \theta^{1-\eta}$  and  $q(\theta) = \theta p(\theta)$ . With this functional form, (8) can be solved closed-form for  $\theta$ :

$$\theta = \left[ \frac{\beta(z - w)}{\gamma(1 - \beta(1 - \delta))} \right]^{1/\eta} \tag{27}$$

A model period is normalized to be a month, implying  $\beta = 0.9967$  to match about 4%

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<sup>2</sup>Our calibration targets aggregate labor market outcomes. A more relevant calibration might target a specific group such as low-skilled young workers.

interest rate. We normalize the match output to be 1. Given this, value of leisure is calibrated to  $b = 0.4$  following Shimer (2005a). The parameter of the matching function,  $\eta$ , comes from Merz (1995). Shimer (2005b) computes average monthly separation probabilities for a worker in the U.S. to be around 4%. This pins down  $\delta$ .

We also need a specific functional form, and an estimate for the cost function  $c(s)$ . We follow Christensen et al. (2005) and assume the following functional form:

$$c(s) = c_0 \frac{s^{1+1/\alpha}}{1 + 1/\alpha}$$

They estimate  $\alpha$  to be 1.18 using Danish labor market data. We then calibrate  $c_0$  such that the steady state unemployment rate is 0.056, which is the long-run average in the U.S. Finally, we calibrate wage to be the midpoint of the feasible set and  $\gamma$  to match average vacancy duration of 1.5 months.

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*Table 1: Benchmark Calibration with Search Effort*

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Parameter	Exogenous Wage	Nash Bargaining	Source
$\beta$	0.9967	0.9967	4% interest rate
$\eta$	0.4	0.4	Merz (1995)
$\delta$	0.0339	0.0339	Shimer (2005b)
$z$	1	1	Normalization
$b$	0.4	0.4	Shimer (2005a)
$w$	0.7	<i>n.a.</i>	$(z - b)/2$
$\gamma$	8.0621	1.7144	Match $q(\theta) = 0.67$
$\alpha$	1.18	1.18	Christensen et.al (2005)
$c_0$	1.645	8.4082	Match $u = 0.056$
$\underline{w}$	<i>n.a.</i>	0.7	$(z - b)/2$
$\lambda$	<i>n.a.</i>	0.4	Hosios (1990)

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Given this benchmark calibration, we want to understand how the wage affects equilibrium unemployment and search effort. Therefore the numerical exercise involves solving the equilibrium for different wages in the feasible set  $[b, z]$ , given all other parameters in Table 1. Figure 2 plots equilibrium unemployment, search effort and welfare for different  $w$ . The level of wage

that minimizes unemployment rate does not necessarily coincide with the wage that maximizes social welfare as defined by equation (19). In our numerical example, for instance, society will be better off by increasing the wage above the level that minimizes unemployment rate. The non-monotonic nature of search effort is also evident in Figure 2.

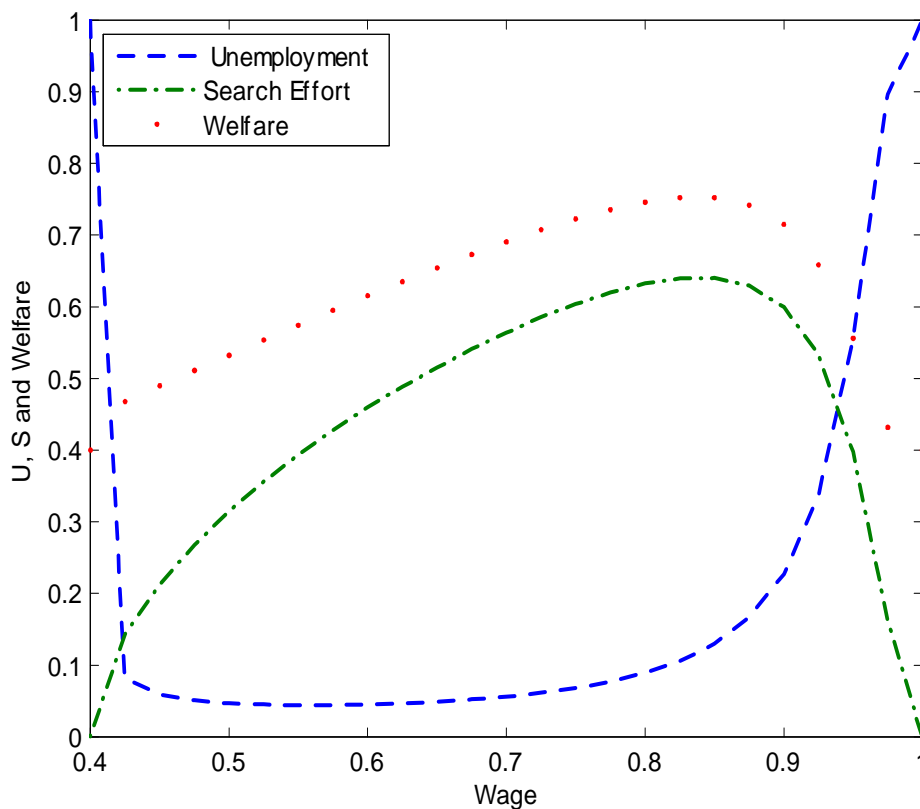


Figure 2: Unemployment, Search Effort and Welfare for Different Wages

The calibration when the wage is endogenously determined by Nash bargaining is only different with respect to four parameters:  $\underline{w}$ ,  $\lambda$ ,  $\gamma$  and  $c_0$ . We set  $\underline{w} = 0.7$ , which guarantees that in the benchmark equilibrium, minimum wage is not binding. The worker's bargaining power,  $\lambda$ , is assumed to satisfy the Hosios condition i.e.,  $\lambda = \eta$ , for the benchmark equilibrium (Hosios, 1990). The remaining two parameters,  $\gamma$  and  $c_0$  are calibrated with the same targets in mind; expected vacancy duration and unemployment rate, respectively. Implied values are,  $\gamma = 1.7144$  and  $c_0 = 8.4082$ .

We compute steady state equilibrium for different values of the bargaining power parameter,

$\lambda$ . The effects of changing the bargaining parameter is presented in Figure 3, which numerically confirms propositions (4) and (5). Steady state welfare is maximized when the Hosios condition is satisfied, which happens when  $\lambda = \eta = 0.4$ . This situation also leads to the highest search effort exerted by workers in this economy. The level of bargaining power that minimizes unemployment is lower than the level of bargaining power that maximizes welfare.

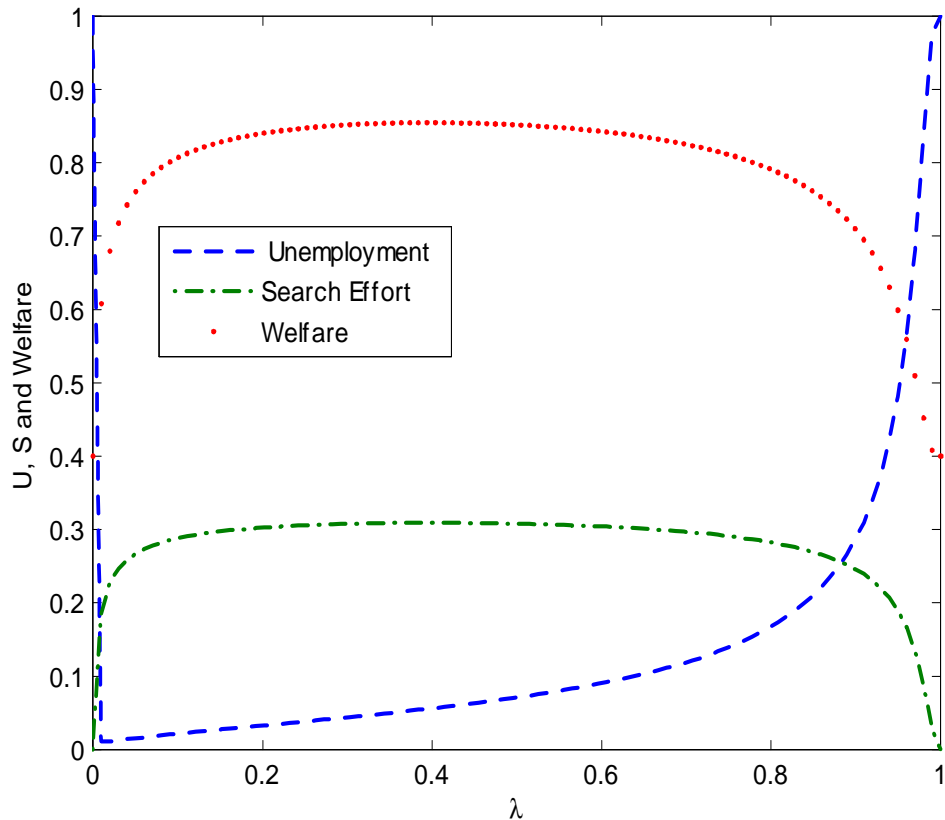


Figure 3: Unemployment, Search Effort and Welfare for Different Bargaining Power

### 3 Participation in the labor force

Up to now we have ignored workers' decisions to participate in the labor force in order to focus on their search behavior when unemployed. To start with, suppose that the population is normalized to 1. We introduce a participation decision by assuming that a worker out of the labor force gets some utility flow  $\kappa$ . This utility stems from non-market activities such



as raising children, doing some cooking and cleaning, and enjoying leisure. We assume that workers differ in terms of their utility at home. The distribution of utilities is  $G(\kappa)$ , where  $G'(\kappa) > 0$ . For simplicity, assume that unemployed workers no longer need to search ( $s = 1$ , and  $c(1) = 0$ ). Equations describing expected utilities for workers and firms remain the same with this last qualification.

The expected lifetime utility of a worker out of the labor force is  $W^o(\kappa)$  that satisfies  $W^o(\kappa) = \frac{\kappa}{1-\beta}$ . A worker will choose to participate in the labor force if  $W^o(\kappa) < W^u$ , or equivalently if  $\kappa < \kappa_u$  where

$$\kappa_u = (1 - \beta)W^u. \quad (28)$$

If the wage is exogenous, then this expression can be simplified further by making use of (1) and (3), i.e.,

$$\kappa_u = \frac{b(1 - \beta(1 - \delta)) + w\beta p(\theta)}{\{1 - \beta[1 - p(\theta) - \delta]\}} \quad (29)$$

The wage is set in accordance with the generalized Nash solution then, from (15),

$$\kappa_u = b + \frac{\lambda}{1 - \lambda} \gamma \theta \quad (30)$$

The equilibrium participation rate is  $L = G(\kappa_u)$ .

The equation of motion for unemployment changes slightly to accommodate for variations in the labor force, i.e.,

$$U_{+1} = U + (L - U)\delta - p(\theta)U \quad (31)$$

At the steady state, (31) implies

$$U = \frac{L\delta}{\delta + p(\theta)} \quad (32)$$

Unemployment rate,  $u = U/L$ , will be the familiar equation.

$$u = \frac{\delta}{\delta + p(\theta)} \quad (33)$$

An equilibrium is then defined as follows.

**Definition 6** A steady state equilibrium with endogenous participation and exogenous wage is a 3-tuple  $(\kappa_u, \theta, u)$  that satisfies (8), (29), and (32).

We consider next the effects of an increase in wage on participation, market tightness and unemployment rate.

**Proposition 7** A higher wage reduces market tightness and it increases unemployment rate. It increases the participation in the labor force provided that  $w < \hat{w}$ , where  $\hat{w}$  satisfies (20).

The proof is similar to the ones in previous section and therefore omitted. See also proof of Proposition 9.

**Definition 8** A steady state equilibrium with endogenous participation and endogenous wage determination is a 4-tuple  $(\kappa_u, w, \theta, u)$  that satisfies (8), (18), (30) and (32).

We have a similar proposition about the effects of an increase in the binding minimum wage on participation, market tightness and unemployment rate.

**Proposition 9** A binding minimum wage reduces market tightness and raises unemployment rate. It can raise participation in the labor force provided that  $\lambda < \eta(\theta)$ .

**Proof.** From (8) and (28) it is easy to check that an increase in  $w$  reduces  $\theta$  and increases  $u$  (provided it is binding). Since we know that an increase in  $\lambda$  generates an increase in  $w$ , we focus in the following on the effect of raising workers' bargaining power. Total differential (17) and (30) to get

$$\begin{aligned}\frac{d\theta}{d\lambda} &= \frac{q(\theta)}{(1-\lambda)q'(\theta)} < 0 \\ \frac{d\kappa_u}{d\lambda} &= \frac{\gamma\theta}{(1-\lambda)^2} \left[ 1 - \frac{\lambda}{\eta(\theta)} \right]\end{aligned}$$

■

### 3.1 Calibration and numerical exercise

We assume that  $G(\kappa)$  is exponentially distributed i.e.,

$$g(\kappa/\mu) = \frac{1}{\mu} e^{-\frac{\kappa}{\mu}}$$

We calibrate  $\mu$  to match the labor force participation rate of 66 percent in the U.S. The cost of posting a vacancy,  $\gamma$  is calibrated to match the long-run average unemployment rate, which implies  $\gamma = 35.164$ . We assume a standard Cobb-Douglas matching function,  $M = u^\eta v^{1-\eta}$ , and calibrate  $\eta$  to be 0.72 following Shimer (2005a). The remaining parameters,  $\beta$ ,  $\delta$ ,  $z$ ,  $b$  and  $w$  follow the same calibration strategy employed in section 2.6. When we extend the model to incorporate endogenous wage determination through Nash bargaining, we set  $\lambda = \eta$  due to Hosios (1990). In addition,  $\gamma$  and  $\mu$  also changes to match unemployment rate and participation rate targets respectively. This calibration is summarized in Table 2.

Parameter	Exogenous Wage	Nash Bargaining	Source
$\beta$	0.9967	0.9967	4% interest rate
$\eta$	0.72	0.72	Shimer (2005a)
$\delta$	0.0339	0.0339	Shimer (2005b)
$z$	1	1	Normalization
$b$	0.4	0.4	Shimer (2005a)
$w$	0.7	<i>n.a.</i>	$(z - b)/2$
$\gamma$	35.164	1.65	Match $u = 0.056$
$\mu$	0.6315	0.88	Match $G(\kappa_u) = 0.67$
$\underline{w}$	<i>n.a.</i>	0.7	$(z - b)/2$
$\lambda$	<i>n.a.</i>	0.72	Hosios (1990)

Our numerical exercise is to evaluate how endogenous variables like unemployment rate, participation rate and welfare, change in response to changes in the level of the minimum wage. We undertake the same exercise for both the model with exogenous wage and the model with endogenous wage determined through Nash bargaining.

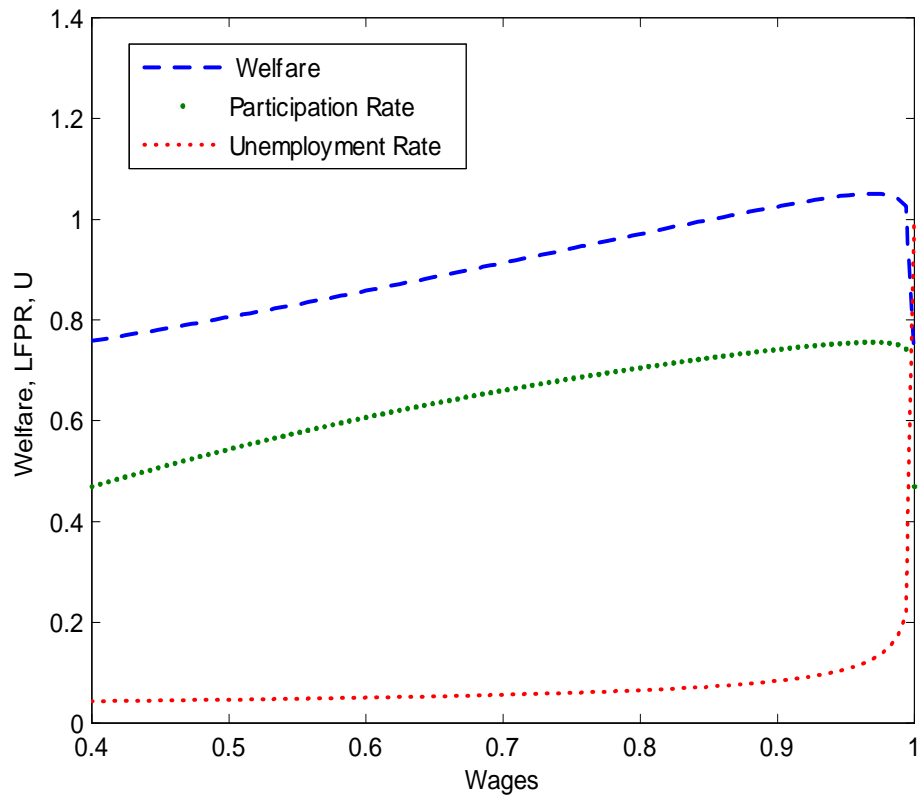


Figure 4: Welfare, Participation and Unemployment Rate for Different Wages

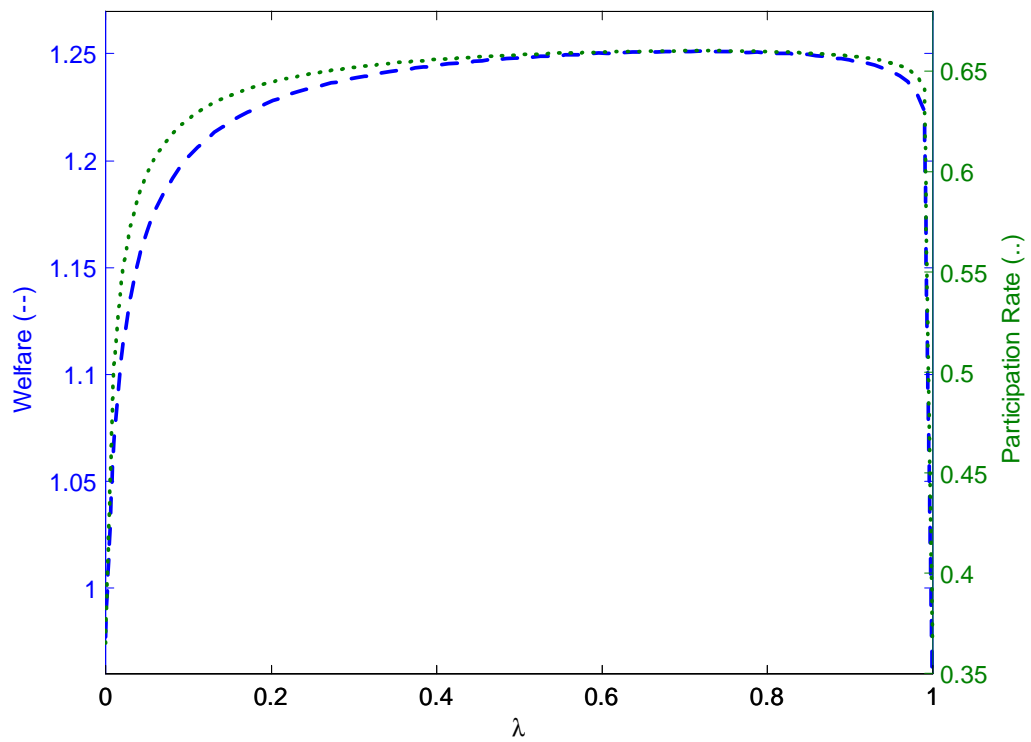


Figure 5: Welfare and LFPR for Different Bargaining Powers

Figures 4 and 5 confirm Propositions (7) and (9). Social welfare can increase as long as wage (or the bargaining power) is lower than the efficient level. The participation rate closely follows social welfare qualitatively, peaking when welfare is maximized.

## 4 Working time

In this section we endogenize the number of working hours and we study the effects of a minimum-wage regulation on employment and unemployment. Suppose that the match output is a function  $z(h)$  of the number of working hours spent by the employee. It satisfies  $z(0) = 0$ ,  $z'(h) > 0$  and  $z''(h) < 0$ . The disutility of work is  $e(h)$  with  $e(0) = 0$ ,  $e'(h) > 0$  and  $e''(h) > 0$ . To simplify the presentation, we will assume in this section that  $b = 0$ .

The Bellman equation for the value of an employed worker is

$$W^e(w, h) = wh - e(h) + \beta [(1 - \delta)W^e(w, h) + \delta W^u], \quad (34)$$

where  $w$  is the hourly wage. Similarly, the value of a filled job satisfies

$$J^e(w, h) = z(h) - wh + \beta [(1 - \delta)J^e(w, h) + \delta J^u] \quad (35)$$

The job creation condition (8) generalizes to

$$\frac{\gamma}{\beta q(\theta)} = \frac{z(h) - wh}{1 - \beta(1 - \delta)} \quad (36)$$

Since we have seen that a minimum wage could be welfare enhancing when employers have a sufficiently high bargaining power, we assume in the following that wages and working time are set unilaterally by firms. Thus, the firm will choose  $(w, h)$  so as to maximize  $J^e(w, h)$  subject to the participation constraint of the worker,  $W^e(w, h) \geq W^u$ , and the minimum wage constraint,

$w \geq \underline{w}$ . From (34) and (35) this problem can be simplified to

$$\max_{w,h} [z(h) - wh] \quad (37)$$

$$\text{s.t. } wh - e(h) \geq (1 - \beta)W^u \quad (38)$$

$$w \geq \underline{w} \quad (39)$$

It is easy to check from (37)-(39) that if the minimum wage constraint is not binding the laissez-faire equilibrium is such that

$$z'(h) = e'(h) \quad (40)$$

$$w = \frac{e(h)}{h} \quad (41)$$

According to (40) the number of hours is set so as to maximize the match surplus. According to (41) the wage is chosen to allow the firm to extract the entire surplus of the match. From the strict convexity of  $e(h)$  we have  $e'(h)h > e(h)$  and therefore

$$w = \frac{e(h)}{h} < e'(h) = z'(h)$$

So the hourly wage is less than the marginal product of an hour.

We define a laissez-faire equilibrium as follows.

**Definition 10** A laissez-faire equilibrium with endogenous working time is a list  $(w, h, \theta, u)$  that satisfies (10), (36), (40) and (41).

Following the same reasoning as before, one can establish that there is a unique equilibrium and it is inefficient. Since firms have all the bargaining power, market tightness is too high and unemployment is too low.

Consider next the case where  $\underline{w} \geq w^* \equiv e(h^*)/h^*$  where  $h^*$  denotes the solution to (40). There are two regimes to consider. The first regime is such that the worker's participation constraint (38) is binding. Then,  $h$  satisfies

$$\underline{w}h = e(h) \quad (42)$$

It is optimal for the firm to choose  $h$  that satisfies (42) if and only if  $z'(h) > \underline{w}$ . In this case, the firm has no incentive to cut hours to increase its profits. Also, (42) holds then  $\underline{w} < e'(h)$  so that the firm cannot raise hours without violating the worker's participation constraint. The condition  $z'(h) > \underline{w}$  implies  $h < \tilde{h}$  where  $\tilde{h} > h^*$  is the unique solution to  $z'(h) = e(h)/h$  and  $\underline{w} \leq \tilde{w} \equiv e(\tilde{h})/\tilde{h}$ .

The second regime is such that the worker's participation constraint (38) does not bind. In this case,

$$z'(h) = \underline{w} \tag{43}$$

The worker's participation constraint does not bind if  $\underline{w}h - e(h) > 0$  which requires  $h \geq \tilde{h}$  and  $\underline{w} \geq \tilde{w}$ .

**Definition 11** An equilibrium with binding minimum wage and endogenous working time is a list  $(h, \theta, u)$  that satisfies (10), (36), (42) if  $\underline{w} \leq \tilde{w}$  and (43) otherwise.

The effects of an increase in the minimum wage are as follows.

**Proposition 12** An increase in the minimum wage reduces market tightness and increases unemployment. If  $\underline{w} \leq \tilde{w}$  the number of working hours increases while if  $\underline{w} > \tilde{w}$  the number of working hours decreases.

**Proof.** From (37)-(39) an increase in  $\underline{w}$  reduces  $z(h) - wh$ . We deduce from (10) and (36) that  $\theta$  falls and  $u$  increases. From (42)  $h$  increases with  $\underline{w}$ . From (43)  $h$  decreases with  $\underline{w}$ . ■

We can also make a statement about the welfare effects of a limited increase in the minimum wage.

**Proposition 13** A binding minimum wage in  $[w^*, \tilde{w}]$  is Pareto-worsening.

**Proof.** If  $\underline{w} \in [w^*, \tilde{w}]$  then (38) is binding and  $W^u = W^e = 0$ . Since  $\theta$  decreases with  $\underline{w}$ ,  $J^e = \gamma/\beta q(\theta)$  is lower and firms are worse-off. ■

#### 4.1 Calibration and numerical exercise

In this section, our calibration requires functional forms for match output and disutility of work. We assume that match output takes the simple form,  $z(h) = \phi h^{1/2}$  with  $\phi > 0$ . The disutility



of work is  $e(h) = ah^2$  where  $a > 0$ .

In our calibration we target unemployment rate as well as average monthly hours of work in the U.S. The average hours of work is approximately 33 in a week in the U.S., which implies a target of 143 for our monthly model. Given these targets in mind,  $\phi$  is calibrated to normalize monthly output to 10. Then, from (40),  $a$  is required to be 0.0001 to match target hours of work in the model. For simplification, we set  $b$  and  $\lambda$  to zero. Finally, we set the value of the minimum wage arbitrarily low such that it is not binding in the benchmark calibration. Calibration for this section is summarized in Table 3.

*Table 3: Benchmark Calibration with Hours of Work*

Parameter	Value	Source
$\beta$	0.9967	4% interest rate
$\eta$	0.72	Shimer (2005a)
$\delta$	0.0339	Shimer (2005b)
$\phi$	0.8362	Normalize $z(h^*) = 10$
$b$	0	Assumption
$\gamma$	879	Match $u = 0.056$
$a$	0.0001	Match $h^* = 143$
$\underline{w}$	0.012	Benchmark, not binding
$\lambda$	0	Assumption

We have shown in Proposition 12 that increasing the level of the minimum wage unambiguously reduces equilibrium market tightness, thereby decreasing the job finding probability and raising the unemployment rate. We confirm this point numerically in Figure 6. In our benchmark, equilibrium hourly wage in the absence of minimum wage,  $w^*$ , implied by (41) is 0.0175. As the hourly minimum wage level increases beyond  $w^*$ , minimum wage becomes binding and affects the unemployment rate and market tightness in the predicted way. However, the implied increase in unemployment could be quantitatively small. For this stylized model, if minimum wage is raised by 40 percent from an initial level that is not binding, say 0.0175, unemployment rate could increase by a mere 2.8 percent, from 5.67 percent to 5.83 percent.

One interpretation for the small effect of a minimum wage increase on unemployment is

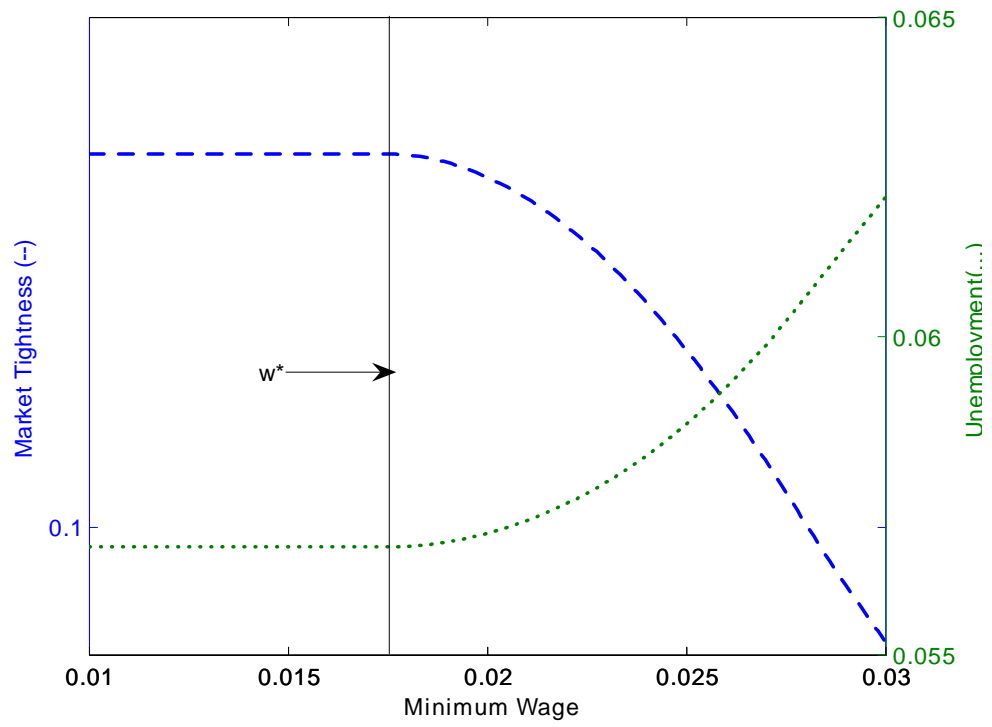


Figure 6: Unemployment and Market Tightness for Different Minimum Wage Levels

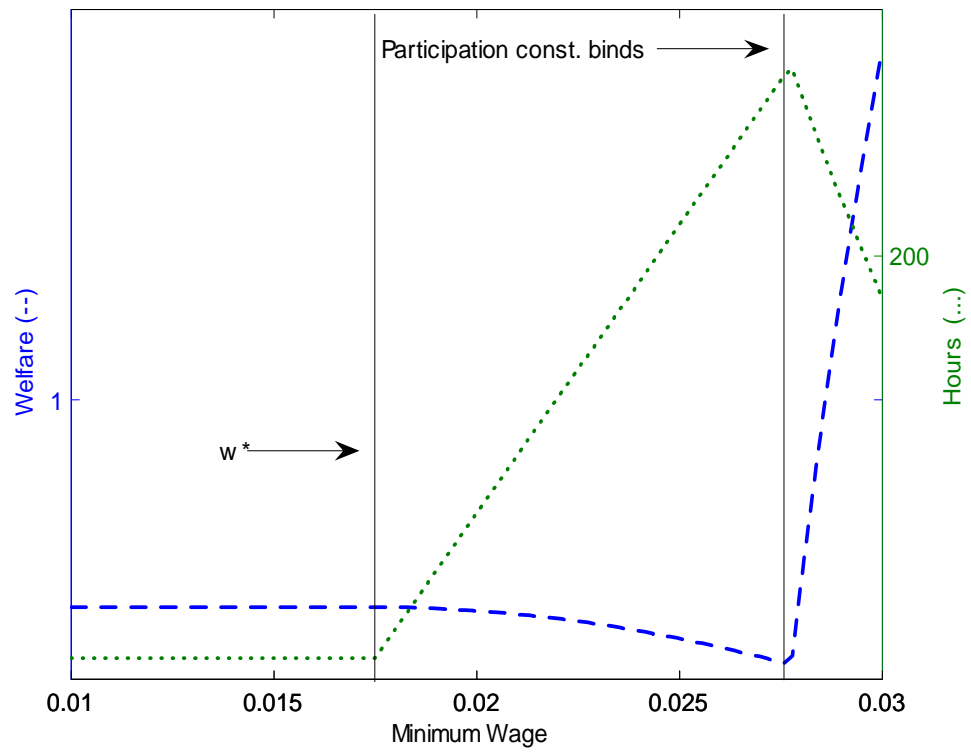


Figure 7: Hours and Welfare for Different Minimum Wage Levels

that firms can easily adjust labor at the intensive margin by increasing hours initially. This is possible in this example, because firms have all the bargaining power ( $\lambda = 0$ ). Therefore, as long as workers are willing to work, firms can partially undo the effects of a minimum wage increase by raising hours. Following the same example of a 40 percent raise in the minimum wage, hours increase by almost 40 percent from 143 to 200. However, as figure 7 shows, such a raise reduces welfare. This result directly follows from Proposition 13. Finally, note that the decline in welfare and the increase in hours are both sustained as long as (38) is binding. However, after the inflection point in figure 7 a higher minimum wage increases can be welfare improving.

## 5 Job destruction

Up to now, we have assumed that the productivity  $z$  of a job is constant and jobs are destroyed according to some exogenous probability  $\delta$ . In order to endogenize the decision by firms to destroy jobs we follow Mortensen and Pissarides (1994) and assume that the productivity of a match changes over time.

The productivity is assumed to be the product of two components:  $z$ , which is an aggregate component common to all jobs, and  $x \in [0, 1]$  which is specific to the firm. Introducing a specific component for the productivity of firms captures the heterogeneity among jobs.

The idiosyncratic component  $x$  takes a new value each time the match receives a signal with probability  $\alpha$ . Consider a firm with current productivity  $xz$  which receives a signal  $\tilde{x}$  where  $\tilde{x}$  is a random draw from  $H(x)$ . Then, the new productivity of the match is  $x'z$  where

$$x' = \min(x, \tilde{x})$$

This assumption guarantees that the productivity of the match declines over time.

A newly-created job starts with the highest productivity, i.e.,  $x = 1$ . After some random period of time, the firm is subject to an idiosyncratic shock, and the productivity of the match starts decreasing. When the productivity reaches a low value, the firm finds worthwhile to destroy the job.

One can interpret the production technology as "putty-clay". The production units embody the most advanced techniques available at the time of their creation. However, a firm cannot change its technology and adopt the leading one once production has started. There is complete irreversibility of initial choices.

## 5.1 Workers, firms and the match surplus

The lifetime expected utility of an employed worker in a match with productivity  $zx$  satisfies

$$W^e(x) = w(x) + \beta \left\{ \alpha \int_0^x [W^e(x')\chi(x') + W^u [1 - \chi(x')]] dH(x') + [1 - \alpha H(x)] W^e(x) \right\}, \quad (44)$$

where  $\chi(x)$  is an indicator function equal to one if the match is maintained and 0 if the match is destroyed (either unilaterally by the worker or the firm or, by mutual agreement). Similarly, the value of a filled job satisfies

$$J^e(x) = zx - w(x) + \beta \left\{ \alpha \int_0^x J^e(x')\chi(x')dH(x') + [1 - \alpha H(x)] J^e(x) \right\} \quad (45)$$

Define the total surplus of a match with productivity  $zx$  as  $S(x) \equiv W^e(x) + J^e(x) - W^u$ . From (44) and (45) the value of a match satisfies the following Bellman equation

$$S(x) = xz - (1 - \beta)W^u + \beta \left\{ \alpha \int_0^x S(x')\chi(x')dH(x') + [1 - \alpha H(x)] S(x) \right\} \quad (46)$$

According to (46) a match generates output  $xz$  minus the opportunity cost for the worker of being employed,  $(1 - \beta)W^u$ . With probability  $\alpha$  an idiosyncratic shock occurs; the new productivity is lower than the current one with probability  $H(x)$ ; the firm and the worker can then decide to maintain the match or destroy it.

The decision to maintain a match is given by

$$\chi(x) = 1 \iff \min [W^e(x) - W^u, J^e(x)] \geq 0.$$

In the absence of a minimum wage, and provided that the worker and the firm can renegotiate

the wage when an idiosyncratic productivity shock occurs,  $W^e(x) - W^u = \lambda S(x)$  and  $J^e(x) = (1 - \lambda)S(x)$ . Therefore, the match is maintained as long as  $S(x) \geq 0$ . In the presence of a minimum wage,  $W^e(x) - W^u \geq \lambda S(x)$  and  $J^e(x) \leq (1 - \lambda)S(x)$ . Therefore, the match is maintained as long as  $J^e(x) \geq 0$  (which does not necessarily coincide with  $S(x) \geq 0$ ). Using a guess-and-verify method, we assume that both  $S(x)$  and  $J^e(x)$  are increasing functions of  $x$  and verify later that this conjecture is correct. As a consequence, there is a threshold  $x_R$  for  $x$  below which a match is destroyed. It satisfies  $J^e(x_R) = 0$  (as well as  $S(x_R) = 0$  in the absence of a minimum wage.)

Using integration by parts, (46) can be rearranged as

$$(1 - \beta)S(x) = xz - (1 - \beta)W^u - \beta\alpha \int_{x_R}^x S'(x')H(x')dx' - \beta\alpha H(x_R)S(x_R) \quad (47)$$

The first term on the right-hand side of (47) is the flow surplus of a match and the last two terms are the capital losses when a new productivity is drawn. We can solve for  $S(x)$  closed-form as follows. First, differentiate (47) with respect to  $x$  to get

$$S'(x) = \frac{z}{1 - \beta [1 - \alpha H(x)]} \quad (48)$$

Thus, (48) confirms our guess that  $S'(x) > 0$ . Second, integrate (48) from  $x_R$  to  $x$  to compute the expression for a match surplus,

$$S(x) = \frac{x_R z - (1 - \beta)W^u}{1 - \beta [1 - \alpha H(x_R)]} + \int_{x_R}^x \frac{z}{1 - \beta [1 - \alpha H(x')]} dx', \quad (49)$$

where the first term is the expression for  $S(x_R)$  derived from (47).

Let us turn to unemployed workers and vacancies. The value of an unemployed worker and a vacant job satisfy

$$(1 - \beta)W^u = b + \beta p(\theta) [W^e(1) - W^u] \quad (50)$$

$$(1 - \beta)J^u = -\gamma + \beta q(\theta) J^e(1) \quad (51)$$

## 5.2 Job creations and destructions

We assume in the following that the minimum wage constraint is not binding at  $x = 1$ . (The case where it does bind for all  $x$  is similar to the model above where  $w = \underline{w}$  and  $\delta = H(\underline{w}/z)$ .) Since wages are determined according to the generalized Nash solution then  $W^e(1) - W^u = \lambda S(1)$  and  $J^e(1) = (1 - \lambda)S(1)$ .

Consider first job creations. Market tightness is determined by the free-entry condition  $J^u = 0$ . From (49) and (51) we obtain

$$S(1) = \frac{\gamma}{\beta q(\theta)(1 - \lambda)} \quad (52)$$

According to (52) the firm's surplus at the beginning of the relationship (i.e., when  $x = 1$ ) must be equal to the average advertising cost incurred by the firm to find a worker.

In order to compute the value of the match at the time when it is created, we need to determine the permanent income of an unemployed worker,  $(1 - \beta)W^u$ . Substituting  $S(1)$  by its expression given by (52) into (50) we obtain

$$(1 - \beta)W^u = b + \frac{\lambda\gamma\theta}{1 - \lambda} \quad (53)$$

According to (53) the value of an unemployed worker increases with  $\theta$ . From (49) and (52) market tightness in equilibrium solves

$$\frac{x_R z - b - \lambda\gamma\theta/(1 - \lambda)}{1 - \beta [1 - \alpha H(x_R)]} + \int_{x_R}^1 \frac{z}{1 - \beta [1 - \alpha H(x')]} dx' = \frac{\gamma}{\beta q(\theta)(1 - \lambda)} \quad (54)$$

Consider next job destructions. If the minimum wage constraint is not binding then  $x_R z = (1 - \beta)W^u$  and the first term on the left-hand side of (54) vanishes. If the minimum wage constraint is binding then  $J^e(x_R) = 0$  which from (45) implies  $z x_R = \underline{w}$ . So,

$$x_R = z^{-1} \max \left( \underline{w}, b + \frac{\lambda\gamma\theta}{1 - \lambda} \right) \quad (55)$$

### 5.3 Wages

We next establish that there is a threshold  $\bar{x}$  below which the minimum wage constraint binds. For the minimum wage to bind, it has to be that  $W^e(x) - W^u \geq \lambda S(x)$  when  $w(x) = \underline{w}$ . Since  $\partial W^e(x)/\partial x = 0$  when  $w(x) = \underline{w}$  we deduce that if the minimum wage binds at  $x = \bar{x}$  then it binds for all  $x < \bar{x}$ .

From (44), for all  $x < \bar{x}$  the surplus of an employed worker satisfies

$$W^e(x) - W^u = \frac{\underline{w} - (1 - \beta)W^u}{1 - \beta [1 - \alpha H(x_R)]}, \quad \forall x < \bar{x} \quad (56)$$

Notice from (56) that the worker's surplus is independent from  $x$ . Using the fact that  $J^e(x_R) = 0$  we deduce that  $W^e(x) - W^u = S(x_R)$  for all  $x < \bar{x}$ . For all  $x < \bar{x}$  the value of an employed worker satisfies

$$\begin{aligned} (1 - \beta) [W^e(x) - W^u] &= w(x) - (1 - \beta)W^u + \beta \left\{ \alpha \int_{\bar{x}}^x [W^e(x') - W^u] dH(x') \right. \\ &\quad \left. + \alpha \int_{x_R}^{\bar{x}} S(x_R) dH(x') - \alpha H(x) [W^e(x) - W^u] \right\} \end{aligned} \quad (57)$$

Using the fact that  $W^e(x) - W^u = \lambda S(x)$  for all  $x > \bar{x}$  we rewrite (57) as follows,

$$(1 - \beta)\lambda S(x) = w(x) - (1 - \beta)W^u + \beta\alpha \left\{ \lambda \int_{\bar{x}}^x S(x') dH(x') + \int_{x_R}^{\bar{x}} S(x_R) dH(x') - H(x)\lambda S(x) \right\} \quad (58)$$

Using (46) and (53) after some calculation we find the following expression for the wage

$$w(x) = \lambda xz + (1 - \lambda)b + \lambda\gamma\theta + \beta\alpha \int_{x_R}^{\bar{x}} [\lambda S(x') - S(x_R)] dH(x') \quad (59)$$

So, if the minimum wage constraint is never binding ( $\bar{x} < x_R$ ) then the expression for the wage is  $w(x) = \lambda xz + (1 - \lambda)(1 - \beta)W^u$ . If the minimum wage constraint binds for some productivity above the reservation productivity then the worker is able to increase his share in the surplus of the match. However, firms anticipate that the minimum wage constraint will be binding for low productivity levels and as a consequence they reduce the wage paid at higher productivity levels. Using integration by parts, and the fact that  $\lambda S(\bar{x}) = S(x_R)$ , the expression for the



wage can be rewritten as

$$w(x) = \lambda x z + \frac{1 - \beta}{1 - \beta [1 - \alpha H(x_R)]} [(1 - \lambda)b + \lambda \gamma \theta] + \frac{(1 - \lambda)\beta \alpha H(x_R)}{1 - \beta [1 - \alpha H(x_R)]} x_R z - \beta \alpha \lambda \int_{x_R}^{\bar{x}} \frac{z}{1 - \beta [1 - \alpha H(x')] } H(x') dx' \quad (60)$$

The threshold  $\bar{x}$  is determined by the condition  $W^e(\bar{x}) - W^u = \lambda S(\bar{x})$ . From (49) and (56),

$$(1 - \lambda) \frac{\underline{w} - (1 - \beta)W^u}{1 - \beta [1 - \alpha H(x_R)]} = \lambda \int_{x_R}^{\bar{x}} \frac{z}{1 - \beta [1 - \alpha H(x')] } dx' \quad (61)$$

## 5.4 Equilibrium

Before we turn to the definition of an equilibrium, we need to characterize the case  $\bar{x} > 1$  when the minimum wage constraint binds for all productivity levels. In this case,

$$W^e(x) - W^u = \frac{\underline{w} - (1 - \beta)W^u}{1 - \beta [1 - \alpha H(x_R)]} \quad (62)$$

From (50) the permanent income of an unemployed worker satisfies

$$(1 - \beta)W^u = \frac{\{1 - \beta [1 - \alpha H(x_R)]\} b + \beta p(\theta) \underline{w}}{1 - \beta [1 - \alpha H(x_R) - p(\theta)]} \quad (63)$$

The value of a filled job at  $x = 1$  is  $J^e(1) = S(1) - W^e(x) - W^u$  which from (49) and (62) gives

$$J^e(1) = \int_{x_R}^1 \frac{z}{1 - \beta [1 - \alpha H(x')] } dx' \quad (64)$$

From the free-entry condition  $J^u = 0$ , (51) and (64) we deduce that market tightness satisfies

$$\frac{\gamma}{\beta q(\theta)} = \int_{x_R}^1 \frac{z}{1 - \beta [1 - \alpha H(x')] } dx' \quad (65)$$

The minimum wage constraint is binding at all productivity levels if  $(1 - \lambda) [W^e(1) - W^u] > \lambda J^e(1)$  which requires

$$\frac{\underline{w} - b - \lambda \theta \gamma / (1 - \lambda)}{1 - \beta [1 - \alpha H(x_R)]} > \frac{\lambda}{1 - \lambda} \int_{\underline{w}/z}^1 \frac{z}{1 - \beta [1 - \alpha H(x')] } dx' \quad (66)$$

Finally, to complete our description of equilibrium we need to specify the distribution of workers' states. The dynamics for unemployment satisfies  $u_{t+1} = u_t + \alpha H(x_R)(1 - u_t) - \theta q(\theta)u_t$ . Therefore, at the steady-state ( $u_{t+1} = u_t$ ) the equilibrium unemployment rate satisfies

$$u = \frac{\alpha H(x_R)}{\alpha H(x_R) + \theta q(\theta)} \quad (67)$$

Denote  $G(x)$  the distribution of employed workers' productivity. At the steady-state,  $[1 - G(x)]\alpha [H(x) - H(x_R)] = G(x)\alpha H(x_R)$  for all  $x \in [x_R, 1)$ . Therefore,

$$G(x) = 1 - \frac{H(x_R)}{H(x)}, \quad \forall x \in [x_R, 1) \quad (68)$$

The fraction of employed workers at the  $x = 1$  satisfies

$$G(1) - G(1^-) = H(x_R) \quad (69)$$

**Definition 14** A steady-state equilibrium is a list  $[x_R, \theta, w(x), u, G(x)]$  that satisfies (54), (55), (60), (67) and (68)-(69).

The model has a simple recursive structure. Equations (54) and (55) can be used to solve for  $x_R$  and  $\theta$ . Then, (60) gives  $w(x)$  and (67) gives  $u$ .

**Proposition 15** Equilibrium exists and is unique. The minimum wage constraint binds if

$$\frac{\gamma}{\beta q(\bar{\theta})(1 - \lambda)} > \int_{\underline{w}/z}^1 \frac{z}{1 - \beta [1 - \alpha H(x')]} dx' \quad (70)$$

where  $\bar{\theta} = (\underline{w}z - b)(1 - \lambda)/\lambda\gamma$ .

**Proof.** (i) *Existence and uniqueness.* Differentiate (54)

$$\frac{d\theta}{dx_R} = -\frac{[x_R z - b - \lambda\gamma\theta/(1 - \lambda)]}{\{1 - \beta [1 - \alpha H(x_R)]\}^2} \beta \alpha h(x_R) \left\{ \frac{-\gamma q'(\theta)}{\beta(1 - \lambda) [q(\theta)]^2} + \frac{\lambda\gamma/(1 - \lambda)}{1 - \beta [1 - \alpha H(x_R)]} \right\}^{-1}$$

In the space  $(x_R, \theta)$  the curve that represents (54) is hump-shaped and it reaches a maximum when it intersects  $x_R z = b + \lambda\gamma\theta/(1 - \lambda)$ . When  $\theta = 0$  the curve representing (54) is located

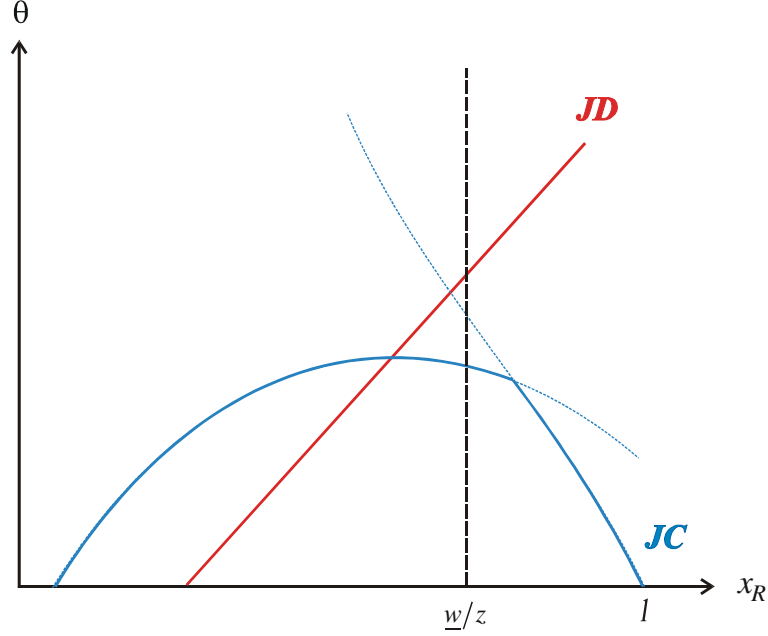


Figure 8: Equilibrium with endogenous job destructions

to the left of the curve representing (55). When  $x_R = 1$  the curve representing (54) is located below the curve representing (55). Thus, (54) and (55) intersect and an equilibrium exists. To establish uniqueness, recall that (54) intersects once with  $x_R z = b + \lambda \gamma \theta / (1 - \lambda)$  at its maximum. Using this observation one can show that (54) and (55) intersect once. (ii) *Binding minimum wage.* The minimum wage is binding if at  $x_R = \underline{w}/z$  the curve representing (54) is located below the curve representing (55). The value of  $\theta$  given by (55) at  $x_R = \underline{w}/z$  is  $\bar{\theta}$ . The solution to (54) at  $x_R = \underline{w}/z$  is smaller than  $\bar{\theta}$  if

$$\frac{x_R z - b - \lambda \gamma \bar{\theta} / (1 - \lambda)}{1 - \beta [1 - \alpha H(x_R)]} + \int_{x_R}^1 \frac{z}{1 - \beta [1 - \alpha H(x')]} dx' < \frac{\gamma}{\beta q(\bar{\theta})(1 - \lambda)}$$

Notice that the first term on the left-hand side of the previous expression is 0 to get (70). ■

Next we turn to the effects of raising the minimum wage on the equilibrium outcome.

**Proposition 16** Assume (70) holds. An increase in the minimum wage reduces  $\theta$  and raises  $x_R$  and  $u$ .

**Proof.** When (70) holds the minimum wage constraint binds and the curve representing (55) intersects the curve representing (54) in its downward-sloping part. An increase in  $\underline{w}$  moves the

curve representing (55) to the right in the space  $(x_R, \theta)$ . Thus,  $x_R$  increases and  $\theta$  falls. From (67) we deduce that  $u$  increases. ■

According to Proposition 16, an increase in the minimum wage reduces job creations, raises job destructions and increases unemployment.

## 5.5 Calibration and numerical exercise

We follow a simple benchmark calibration that targets the average unemployment rate and job destruction in the model to match the U.S. counterparts. For simplicity, we will be silent about the implications of  $H(x)$  on the cross sectional distribution of employment as it relates to wage and tenure distribution. Such a calibration would be beyond the scope of this paper.

First, we assume that  $H(x)$  is normally distributed with mean  $\mu$  and standard deviation  $\sigma$ , appropriately reweighted such that  $x \in [0, 1]$ . Since we assume that all matches start with the highest productivity, we choose a right skewed distribution by setting  $\mu = 1$  and  $\sigma = 0.5$ .

A Cobb-Douglas matching function does not necessarily imply well-defined probabilities for a given  $\theta$ .<sup>3</sup> Therefore, following Hagedorn and Manovskii (2006), we assume a functional form that guarantees this.

$$M(u, v) = \frac{uv}{(u^\eta + v^\eta)^{\frac{1}{\eta}}} \quad (71)$$

We calibrate  $\eta$  to be 0.4, closely following Hagedorn and Manovskii (2006).

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<sup>3</sup>To see this point, consider the job finding probability under Cobb-Douglas specification,  $p(\theta) = \theta^{1-\eta}$ . For  $\theta > 0$ ,  $p(\theta)$  is well-defined if it is restricted to be less than 1. Hence,  $p(\theta) = \min\{1, \theta^{1-\eta}\}$ . Enforcing this restriction throughout the computation of the equilibrium with endogenous job destruction could be very difficult. Matching function in equation 71 does not require such a restriction.

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*Table 4: Calibration with Endogenous Destruction*

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Parameter	Value	Source
$\beta$	0.9967	4% interest rate
$\eta$	0.4	Hagedorn and Manovskii (2006)
$\lambda$	0.4	Hosios (1990)
$\alpha$	0.05	Match $\alpha H(x_R)$
$b$	0.4	Assumption
$\mu$	1	Highest productivity
$\sigma$	0.5	Arbitrary
$\gamma$	0.005	Match $u = 0.056$
$z$	1	Normalization
$\underline{w}$	0	No minimum wage

---

The value of posting a vacancy is once again calibrated to match average U.S. unemployment rate in the post-war period, implying a parameter value of 0.005. The probability of receiving a new productivity signal,  $\alpha$ , is an important determinant of the equilibrium separation probability in the model. We calibrate this parameter to 0.05 to approximately match the average separation probability reported in Shimer (2005b), 0.0339. In the benchmark equilibrium, we do not want to have a binding minimum wage, hence  $\underline{w} = 0$ . Remaining parameters follow the same calibration as in the previous sections and summarized in Table 4.

We investigate numerically how endogenous variables respond to variations in the minimum wage. To this end, we increase minimum wage from 0.8 to 1. The findings are, not surprisingly, in accord with Proposition 16. Figures 9 and 10 show that as the minimum wage increases, unemployment increases and market tightness declines, whereas  $x_R$  increases.

## 6 Conclusion

We have analyzed the effects of a minimum wage in different versions of a search model of the labor market. We showed that a minimum wage can increase social welfare, labor force participation and search effort of workers. We also argue that if firms have other instruments than the wage to maximize profits, they can mitigate the negative effects of the minimum wage.

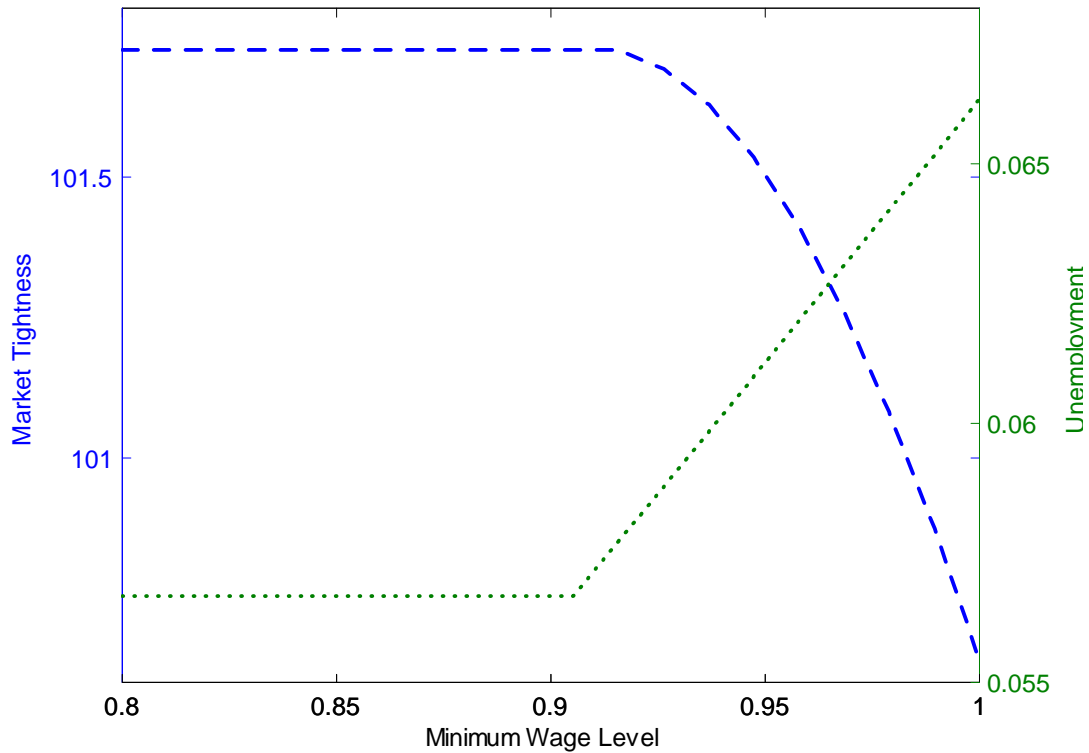


Figure 9: Market Tightness and Unemployment Rate for Different Minimum Wages

The effects of a minimum wage could change depend on the structure of the labor market. In particular, the bargaining power of workers is a crucial determinant. In practice, it is difficult to assess firms’ bargaining power in the labor market, or the extent of search frictions. A 2006 study by Christopher Flinn, which estimates workers’ bargaining power, finds that the market wage exceeds the maximum effort wage. In this case, increasing the minimum wage would have negative consequences for both employment and social welfare. Hence, the question could ultimately be an empirical one.

Many empirical studies have sought to quantify the employment effects of a minimum wage. According to Neumark and Washer’s (2006) survey of this literature, “the preponderance of the evidence points to disemployment effects.” Furthermore, “when researchers focus on the least-skilled groups most likely to be adversely affected by minimum wages, the evidence for disemployment effects seems especially strong.”

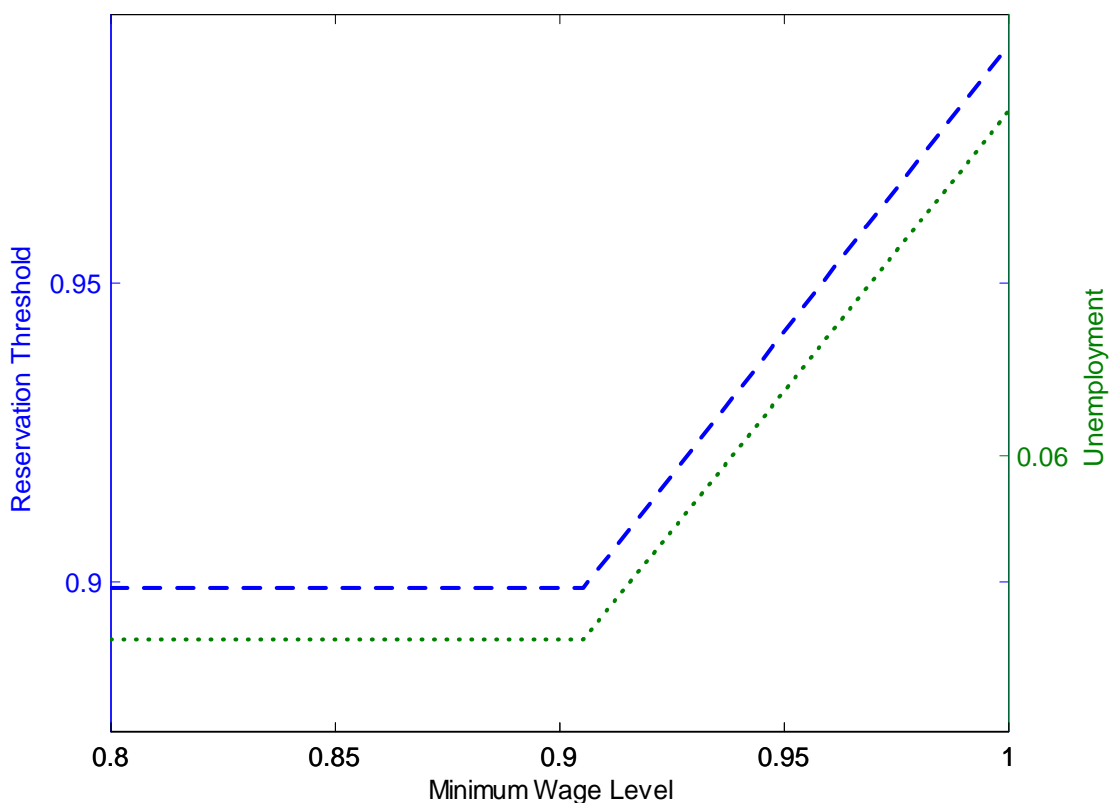


Figure 10: Reservation Threshold and Unemployment Rate for Different Minimum Wages

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