

Instructions for Applying Statistical Testing to the 2009-2011 ACS 3-Year Data and the 2007-2011 ACS 5-Year Data

This document provides some basic instructions for obtaining the ACS standard errors needed to do statistical tests, as well as performing the statistical testing for multiyear estimates.

In general, ACS estimates are period estimates that describe the average characteristics of the population and housing over a period of data collection. For example, the 2011 ACS 1-year estimates are averages over the period from January 1, 2011 to December 31, 2011 because this is the period of time for which sample data were collected. Similarly, multiyear estimates are averages of the characteristics over several years. For example, the 2009-2011 ACS 3-year estimates are averages over the period from January 1, 2009 to December 31, 2011, and the 2007-2011 ACS 5-year estimates are averages over the period from January 1, 2007 to December 31, 2011. Multiyear estimates cannot be used to say what was going on in any particular year in the period, only what the average value is over the full time period.

For more information regarding multiyear ACS data products see any ACS Multiyear Accuracy document available under Data and Documentation on the ACS website <http://www.census.gov/acs/www/>.

Obtaining Standard Errors

The location and type of ACS data, as well as when it was released, will determine if standard errors are readily available or users have to calculate them. If the estimate of interest is published on American FactFinder (AFF), then AFF should also be the source of the standard errors. Possible sources for the data and where to get standard errors are:

1. ACS data from published tables on American FactFinder

All ACS estimates from tables on AFF include either the 90 percent margin of error or 90 percent confidence bounds. The margin of error is the maximum difference between the estimate and the upper and lower confidence bounds. Most tables on AFF containing single-year or multiyear ACS data display the margin of error.

Use the margin of error to calculate the standard error (dropping the “+/-” from the displayed value first) as:

$$\text{Standard Error} = \text{Margin of Error} / Z$$

where $Z = 1.645$ for 2006 and later years as well as all multiyear estimates and $Z = 1.65$ for 2005 and earlier years.

If confidence bounds are provided instead (as with most ACS data from 2004 and earlier years), calculate the margin of error first before calculating the standard error:

$$\text{Margin of Error} = \max(\text{upper bound} - \text{estimate}, \text{estimate} - \text{lower bound})$$

All published ACS estimates use 1.645 (for 2006 and later years) or 1.65 (for 2005 and previous years) to calculate 90 percent margins of error and confidence bounds. Other surveys may use other values.

2. ACS Public Use Microdata Sample (PUMS) tabulations

Using the methods described in the Accuracy of the PUMS documentation users can calculate standard errors for their tabulations using a design factor method or a replicate weight method. For example, 2009-2011 Accuracy of the PUMS documentation should be used with the 2009-2011 ACS PUMS file to calculate standard errors. This document is available under Data and Documentation on the ACS website <http://www.census.gov/acs/www/>.

NOTE: ACS PUMS design factors *should not* be used to calculate standard errors of full ACS sample estimates, such as those found in data tables on AFF. In addition, Census 2000 design factors *should not* be used to calculate standard errors for *any* ACS estimate.

Obtaining Standard Errors for Derived Estimates

Once users have obtained standard errors for the basic estimates, there may be situations where users create derived estimates, such as percentages or differences that also require standard errors.

All methods in this section are **approximations** and users should be cautious in using them. This is because these methods do not consider the correlation or covariance between the basic estimates. They may be overestimates or underestimates of the derived estimate's standard error, depending on whether the two basic estimates are highly correlated in either the positive or negative direction. As a result, the approximated standard error may not match direct calculations of standard errors or calculations obtained through other methods.

- Sum or Difference of Estimates

$$SE(A + B + \dots) = SE(A - B - \dots) = \sqrt{SE(A)^2 + SE(B)^2 + \dots}$$

As the number of basic estimates involved in the sum or difference increases, the results of this formula become increasingly different from the standard error derived directly from the ACS microdata. Care should be taken to work with the fewest number of basic estimates as possible. If there are estimates involved in the sum that are controlled in the weighting then the approximate standard error can be tremendously different.

- Proportions and Percents

Here we define a proportion as a ratio where the numerator is a subset of the denominator, for example the proportion of persons 25 and over with a high school diploma or higher.

Let:

$$P = \frac{A}{B}$$

$$SE(P) = \frac{1}{B} \sqrt{SE(A)^2 - P^2 \times SE(B)^2}$$

If the value under the square root sign is negative, then instead use:

$$SE(P) = \frac{1}{B} \sqrt{SE(A)^2 + P^2 \times SE(B)^2}$$

If $P = 1$ then use

$$SE(P) = \frac{SE(A)}{B}$$

If $Q = 100\% \times P$ (a percent instead of a proportion), then $SE(Q) = 100\% \times SE(P)$.

- Means and Other Ratios

If the estimate is a ratio but the numerator is not a subset of the denominator, such as persons per household, per capita income, or percent change of an estimate then

$$SE\left(\frac{A}{B}\right) = \frac{1}{B} \sqrt{SE(A)^2 + \left(\frac{A}{B}\right)^2 \times SE(B)^2}$$

- Products

For a product of two estimates - for example if users want to estimate a proportion's numerator by multiplying the proportion by its denominator - the standard error can be approximated as

$$SE(A \times B) = \sqrt{A^2 \times [SE(B)]^2 + B^2 \times [SE(A)]^2}$$

Users may combine these procedures for complicated estimates. For example, if the desired estimate is $P = \frac{A+B+C}{D+E}$, then $SE(A+B+C)$ and $SE(D+E)$ can be estimated first, and then those results used to calculate $SE(P)$.

- Comparing Estimates for Overlapping Periods of Identical Length

Calculating the standard error when comparing two individual estimates for different but overlapping time periods is a special case of the Sum or Difference of Estimates standard error approximation mentioned above. For example, A may represent an estimate of a

characteristic for the period 2008-2010 and B the estimate of the same characteristic for 2009-2011. Data for 2009 and 2010 are included in both estimates, and their contribution is largely subtracted out when differences are calculated.

In this case, it is possible to approximate the sampling correlation between the two estimates to improve upon the previous expression for the standard error of a difference, namely:

$$SE(A - B) = \sqrt{(1 - C)\sqrt{SE(A)^2 + SE(B)^2}}$$

where C is the fraction of overlapping years. For example, the periods 2008-2010 and 2009-2011 overlap by two out of three years, so $C = 2 / 3 = 0.667$.

- Comparing Estimates for Non-Overlapping Periods

The comparison of two individual estimates for different non-overlapping time periods uses the same formula as the Sum or Difference of Estimates standard error approximation mentioned above. For example, A may represent an estimate of a characteristic for the period 2006-2008 and B the estimate of the same characteristic for 2009-2011. In this case, no approximation of the sampling correlation is needed since there is no data used for both estimates. Therefore, the standard error of a difference is simply:

$$SE(A - B) = \sqrt{SE(A)^2 + SE(B)^2}$$

For examples of these formulas, please see any Multiyear Accuracy of the Data document available under Data and Documentation on the ACS website <http://www.census.gov/acs/www/>.

Instructions for Statistical Testing

Once standard errors have been obtained, doing the statistical test to determine significance is not difficult. The determination of statistical significance takes into account the difference between the two estimates as well as the standard errors of both estimates.

For two estimates, A and B, with standard errors SE(A) and SE(B), let

$$Z = \frac{A - B}{\sqrt{[SE(A)]^2 + [SE(B)]^2}}$$

If $Z < -1.645$ or $Z > 1.645$, then the difference between A and B is significant at the 90 percent confidence level. Otherwise, the difference is not significant. This means that there is less than a 10 percent chance that the difference between these two estimates would be as large or larger by random chance alone.

Users may choose to apply a confidence level different from 90 percent to their tests of statistical significance. For example, if $Z < -1.96$ or $Z > 1.96$, then the difference between A and B is significant at the 95 percent confidence level.

This method can be used for any types of estimates: counts, percentages, proportions, means, medians, etc. It can be used for comparing across years, or across surveys. If one of the estimates is a fixed value or comes from a source without sampling error (such as a count from the 2010 Census), use zero for the standard error for that estimate in the above equation for Z.

NOTE: Making comparisons between ACS single-year and multiyear estimates is very difficult, and is not advised.

In addition, using the rule of thumb of overlapping confidence intervals does not constitute a valid significance test and users are discouraged from using that method.