An Appendix to

A Tale of Tails: An Empirical Analysis of Loss Distribution Models for Estimating Operational Risk Capital

Kabir Dutta and Jason Perry

The public release of *A Tale of Tails* generated many emails to the authors with questions about the research results not included in the paper. This appendix contains tables that summarize the unpublished results relevant to those emails, and is intended to enhance the reader's understanding of the research.

These tables are similar to tables included in the paper. They differ in the exposure indicators used to scale the data (gross income and total assets) and include some business lines and event types originally excluded. As in the paper, the designs of the tables are such that they anonymize the participant institutions.

The authors produced these results while they were employees of the Federal Reserve Bank of Boston.

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Table 1: Power Law Capital Estimates at the Business LineLevel as a Percentage of Total Assets

	Business Line 6: Age	ncy Services						
Panel A: ξ Values by Institution								
	0.86	0.90						
	1.00	1.17						
	1.02	1.02						
	-	0.80						
	-	0.85						
	-	0.93						
Panel B: Summary Statistics of Capital Estimates as a Percentage of Assets								
25th	9.84	0.30						
Med	15.71	3.43						
75th	21.17	13.23						
	Business Line 9	: Other						
	Panel A: ξ Values by	/ Institution						
	-	0.90						
	-	1.41						
	-	1.98						
Panel B: S	Summary Statistics of Percentage of J	Capital Estimates as a Assets						
25th	-	79.53						

75th	-	10484.09
Med	-	158.60
25th	-	79.53

Panel A presents ξ for each business line, computed using the Hill estimator, with 5 and 10 percent of the data in the tail. Panel B presents summary statistics for capital estimates at the 99.9% level for each institution as a percentage of total assets. These capital estimates were not simulated, but instead calculated using the power law approximation method described in the paper. No goodness-of-fit tests are presented in this table.

5% Threshold	10% Threshold								
Event Type 2: Exter	nal Fraud								
Panel A: ξ Values by Institution									
0.71	0.76								
0.74	0.74								
0.80	0.73								
0.95	1.00								
Summary Statistics of	Capital Estimates as a								
Percentage of A	Assets								
0.10	0.10								
0.11	0.11								
0.15	0.19								
3: Employment Practi	ices & Workplace Safety								
Panel A: ξ Values by	Institution								
0.73	0.80								
0.86	0.97								
0.89	0.96								
Summary Statistics of	Capital Estimates as a								
Percentage of A	Assets								
0.20	0.39								
0.30	0.58								
0.37	0.89								
pe 4: Clients, Products	& Business Practices								
Panel A: ξ Values by	Institution								
0.78	0.81								
1.56	1.21								
1.62	1.62								
-	2.98								
Summary Statistics of	Capital Estimates as a								
Percentage of A	Assets								
734.25	27.11								
1468-13	5305 10								
1400.15	5505.10								
	5% Threshold Event Type 2: Exter Panel A: ξ Values by 0.71 0.74 0.80 0.95 Summary Statistics of Percentage of A 0.10 0.11 0.15 3: Employment Practi Panel A: ξ Values by 0.73 0.86 0.89 Summary Statistics of Percentage of A 0.20 0.30 0.37 pe 4: Clients, Products Panel A: ξ Values by 0.78 1.56 1.62 - Summary Statistics of Percentage of A								

 Table 2: Power Law Capital Estimates at the Event Type

 Level as a Percentage of Total Assets

Event Type 7: Execution, Del Management	livery & Process								
Panel A: ξ Values by Institution									
0.80	0.90								
0.83	0.99								
0.86	1.00								
0.89	0.98								
0.90	0.83								
1.09	0.93								
1.09	1.08								
Panel B: Summary Statistics of Capital Estimates as a									

Percentage of Assets

25th	2.10	7.59	
Med	4.53	8.87	
75th	6.06	15.62	

Panel A presents ξ for each event type, computed using the Hill estimator, with 5 and 10 percent of the data in the tail. Panel B presents summary statistics for capital estimates at the 99.9% level for each institution as a percentage of total assets. These capital estimates were not simulated, but instead calculated using the power law approximation method described in the paper. No goodness-of-fit tests are presented in this table.

		Std. error		Std. error		Std. error		Std. error		Std. error
	g	for g	h/η_0	for h/η ₀	η_1	for η_1	η_2	for η_2	η_3	for η_3
				Panel	A: Busin	ess Line Leve	el			
2	1 6554	0.0007	0 4107	0 0004	_	-	_	-	-	-
-	1 9613	0.0004	0.2095	0.0006	_	_	-	_	-	_
	2 6413	0.0009	0.2102	0.0010	_	_	_	-	-	-
3	1.5473	0.0004	0.3250	0.0002	_	_	_	_	-	_
•	1.5611	0.0005	0.5420	0.0004	-	-	-	_	-	-
	1.7750	0.0006	0.4326	0.0004	-	-	-	_	-	-
	1.7941	0.0001	0.2663	0.0009	-0.1199	0.0004	0.0224	0.0001	-0.0010	0.0000
	1.8615	0.0010	0.1684	0.0007	_	_	-	_	_	_
	1.9410	0.0010	-0.0403	0.0006	-	-	-	-	-	-
4	1.8568	0.0020	0.4831	0.0012	_	-	-	-	_	-
	2.0952	0.0006	0.3298	0.0005	-	-	-	-	-	-
	2.2063	0.0012	-0.1076	0.0004	-	-	-	-	-	-
5	1.2487	0.0004	0.2690	0.0004	-	-	-	-	-	-
	1.5550	0.0005	0.0615	0.0005	-	-	-	-	-	-
	1.6361	0.0006	-0.0173	0.0005	-	-	-	-	-	-
	1.6523	0.0022	0.5079	0.0012	-	-	-	-	-	-
6	1.7197	0.0007	0.4113	0.0005	-	_	-	-	-	-
	1.7604	0.0009	0.0978	0.0005	-	-	-	-	-	-
	1.7665	0.0011	0.2331	0.0006	-	-	-	-	-	-
	1.8576	0.0004	0.0868	0.0002	-	-	-	-	-	-
	1.8861	0.0005	0.2663	0.0004	-	-	-	-	-	-
7	1.3902	0.0011	0.1108	0.0015	-	-	-	-	-	-
	1.7967	0.0018	0.0331	0.0011	-	-	-	-	-	-
	1.8526	0.0011	-0.0518	0.0006	-	-	-	-	-	-
	1.9809	0.0014	0.0340	0.0007	-	-	-	-	-	-
8	1.4874	0.0004	-0.0825	0.0002	-	-	-	-	-	-
	1.5582	0.0003	0.1530	0.0002	-	-	-	-	-	-
	1.5896	0.0011	-0.0637	0.0009	-	-	-	-	-	-
9	1.3932	0.0006	0.1254	0.0004	-	-	-	-	-	-
	1.4855	0.0006	0.0480	0.0004	-	-	-	-	-	-
	2.2056	0.0010	0.2418	0.0010	-	-	-	-	-	-

Table 3: Standard Errors for g-and-h Parameter Estimates at the Business Line and Event Type Level

TABLE CONTINUED ON NEXT PAGE

		Std. error		Std. error		Std. error		Std. error		Std. error
	g	for g	h/η_0	for h/η_0	η_1	for η_1	η_2	for η_2	η_3	for η_3
				Pano	el B: Even	t Type Leve	1			
1	1.5363	0.0010	0.3349	0.0005	-	-	-	-	-	-
	1.8990	0.0006	0.2250	0.0005	-	-	-	-	-	-
2	1.4264	0.0002	0.2471	0.0003	-	-	-	-	-	-
	1.5319	0.0009	0.1418	0.0008	-	-	-	-	-	-
	1.5446	0.0004	0.1341	0.0003	-	-	-	-	-	-
	1.6084	0.0006	0.2445	0.0007	-	-	-	-	-	-
	1.7683	0.0003	-0.1583	0.0016	0.0192	0.0008	0.0012	0.0001	-0.0001	0.0000
3	1.1909	0.0004	0.1943	0.0003	-	-	-	-	-	-
	1.4207	0.0006	0.2031	0.0010	-	-	-	-	-	-
	1.4672	0.0004	0.3239	0.0002	-	-	-	-	-	-
4	1.5876	0.0003	0.5658	0.0006	-	-	-	-	-	-
	1.6956	0.0005	0.0026	0.0002	-	-	-	-	-	-
	2.1618	0.0005	0.2905	0.0003	-	-	-	-	-	-
	2.4195	0.0017	0.2066	0.0021	-	-	-	-	-	-
6	1.9810	0.0016	0.1702	0.0010	-	-	-	-	-	-
7	0.9910	0.0005	0.9800	0.0004	-	-	-	-	-	-
	0.9920	0.0003	0.9909	0.0002	-	-	-	-	-	-
	0.9942	0.0003	0.9933	0.0002	-	-	-	-	-	-
	0.9970	0.0006	0.9910	0.0003	-	-	-	-	-	-
	0.9989	0.0003	0.9972	0.0003	-	-	-	-	-	-
	0.9992	0.0005	0.9900	0.0003	-	-	-	-	-	-
	1.6600	0.0002	-0.0314	0.0017	0.1149	0.0009	-0.0068	0.0001	0.0000	0.0000
8	1.9534	0.0014	0.4146	0.0006	-	_	-	_	-	_

This table presents the g and h parameters and the bootstrap standard error estimates for the g-and-h distribution. Panel A shows the enterprise level, Panel B shows the business line Level, and Panel C shows the event type level. Standard errors were estimated using 50,000 samples bootstrapped from the original data for each institution. Some institutions were fitted using only one h parameter. For the other institutions, four h parameters were used and $h(Z^2) = \eta_0 + \eta_1 * Z^2 + \eta_2 * Z^4 + \eta_3 * Z^6$.

	Reasonable							Generally Yielded Unreasonable Capital					
	Resu	lts	Rar	ely Fit the	Data		-	Estimate	S	-			
						EVT	EVT		Log-	Truncated			
	g-and-h	Emp	Exp	Gamma	Weibull	5%	10%	GPD	logistic	Lognormal			
				Business	rading &	Sales							
Panel	A: Sumn	nary St	tats of C	Capital Est	timates as	a Percent	tage of Tot	tal Assets	for All M	odels			
# Modeled	3	6	6	6	6	3	3	6	6	5			
# that Fit	3	6	1	1	2	2	2	6	6	5			
Mean	0.27	0.05	0.02	0.02	0.01	12.75	2.32	225.72	2.85	5.83			
Med	0.26	0.01	0.01	0.01	0.01	2.46	1.28	7.57	0.62	0.19			
SD	0.09	0.07	0.02	0.03	0.01	19.34	2.65	532.98	3.91	12.77			
25th	0.22	0.00	0.00	0.00	0.00	1.60	0.82	0.11	0.17	0.00			
75th	0.31	0.10	0.03	0.03	0.01	18.77	3.31	22.98	6.00	0.26			
Pane	l B: Capi	tal Est	imates a	as a Perce	ntage of T	Total Asset	ts for Mod	els that Fi	it (Frequ	ency)			
0-1.5%	3	6	1	1	2	0	1	3	4	4			
1.5-3%	0	0	0	0	0	1	0	0	0	0			
3-20%	0	0	0	0	0	0	1	1	2	0			
20-100%	0	0	0	0	0	1	0	1	0	1			
100-500%	0	0	0	0	0	0	0	0	0	0			
500+%	0	0	0	0	0	0	0	1	0	0			
				Business	s Line 3: I	Retail Ban	king						
Panel	A: Sum	nary St	tats of C	Capital Est	timates as	a Percent	tage of Tot	tal Assets	for All M	lodels			
# Modeled	6	6	6	6	6	4	4	6	6	6			
# that Fit	6	6	0	0	0	4	4	3	4	5			
Mean	0.30	0.04	0.02	0.02	0.01	41.51	4.32	1.14	0.96	1.40			
Med	0.11	0.05	0.02	0.02	0.01	13.76	0.67	0.63	0.79	0.36			
SD	0.42	0.03	0.01	0.01	0.01	65.14	7.71	0.99	0.48	2.73			
25th	0.07	0.02	0.01	0.01	0.01	2.98	0.20	0.53	0.75	0.12			
75th	0.29	0.06	0.03	0.03	0.02	52.29	4.79	1.42	0.94	0.59			
Pane	l B: Capi	tal Est	imates a	as a Perce	ntage of T	Total Asset	ts for Mod	els that Fi	it (Frequ	ency)			
0-1.5%	6	6	0	0	0	1	3	1	3	4			
1.5-3%	0	0	0	0	0	0	0	2	1	0			
3-20%	0	0	0	0	0	1	1	0	0	1			
20-100%	0	0	0	0	0	1	0	0	0	0			
100-500%	0	0	0	0	0	1	0	0	0	0			
500+%	0	0	0	0	0	0	0	0	0	0			
						,	TABLE C	ONTINU	ED ON N	EXT PAGE			

Table 4: Capital Estimates at the Business Line Level as a Percentage of Total Assets

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	Reason	nable			Generally Yielded Unreasonable Capital							
	Resu	ılts	Rarely Fit the Data				-	Estimate	s	-		
						EVT	EVT		Log-	Truncated		
	g-and-h	Emp	Exp	Gamma	Weibull	5%	10%	GPD	logistic	Lognormal		
			В	usiness Li	nmercial H	Banking						
Panel	A: Sumr	nary St	tats of C	Capital Est	a Percent	tage of Tot	al Assets	for All M	odels			
# Modeled	3	5	5	5	5	-	-	5	5	5		
# that Fit	3	5	0	0	1	-	-	5	4	5		
Mean	0.16	0.01	0.00	0.00	0.00	-	-	9.09	0.64	0.22		
Med	0.20	0.01	0.00	0.00	0.00	-	-	5.50	0.73	0.11		
SD	0.15	0.01	0.00	0.00	0.00	-	-	13.54	0.54	0.29		
25th	0.10	0.00	0.00	0.00	0.00	-	-	0.31	0.14	0.07		
75th	0.24	0.01	0.00	0.00	0.00	-	-	6.83	0.94	0.18		
Pane	Panel B: Capital Estimates as a Percentage of Total Assets for Models that Fit (Frequency)											
0-1.5%	3	5	0	0	1	-	-	2	4	5		
1.5-3%	0	0	0	0	0	-	-	0	0	0		
3-20%	0	0	0	0	0	-	-	2	0	0		
20-100%	0	0	0	0	0	-	-	1	0	0		
100-500%	0	0	0	0	0	-	-	0	0	0		
500+%	0	0	0	0	0	-	-	0	0	0		
				Business	Line 6: A	gency Ser	vices					
Panel	A: Sumr	nary St	tats of C	Capital Est	timates as	a Percent	tage of Tot	al Assets	for All M	odels		
# Modeled	5	6	6	6	6	4	4	6	6	6		
# that Fit	5	6	0	1	1	4	4	6	6	6		
Mean	0.59	0.04	0.02	0.02	0.02	1.03	0.17	2.85	2.00	0.35		
Med	0.14	0.02	0.01	0.01	0.01	0.90	0.12	2.92	1.60	0.20		
SD	0.97	0.05	0.02	0.02	0.02	1.00	0.19	2.57	1.98	0.39		
25th	0.04	0.00	0.00	0.00	0.00	0.44	0.05	0.52	0.28	0.04		
75th	0.48	0.05	0.02	0.03	0.02	1.50	0.24	5.06	3.55	0.67		
Pane	l B: Capi	ital Est	imates a	ns a Perce	ntage of T	Total Asset	ts for Mod	els that F	it (Freque	ency)		
0-1.5%	4	6	0	1	1	3	4	2	3	6		
1.5-3%	1	0	0	0	0	1	0	1	1	0		
3-20%	0	0	0	0	0	0	0	3	2	0		
20-100%	0	0	0	0	0	0	0	0	0	0		
100-500%	0	0	0	0	0	0	0	0	0	0		
500+%	0	0	0	0	0	0	0	0	0	0		

This table presents a summary of the 99.9% capital estimates for each Basel business line as a percentage of total assets. The capital estimates were simulated from one million trials. Panel A presents the first, second and third quartiles, which were calculated across each model. These statistics include both capital estimates that statistically fit and do not fit. Panel B presents a frequency distribution of banks whose capital estimates fit according to one or more of the following goodness-of-fit tests: Kolmogorov-Smirnoff, Chi-Square, and Anderson-Darling. The fit for the g-and-h distribution was only tested using Q-Q plots. No goodness-of-fit tests were performed for the empirical distribution. By construction, the empirical distribution would fit the data. All empirical estimates are included in these counts. The total number of business lines modeled and the number that fit are also presented in Panel A.

	Reasor	nable										
	Resu	lts	R	arely Fit th	ne Data	Gener	ally Yielded	Unreasona	ble Capital E	Estimates		
						EVT	EVT		Log-	Truncated		
	g-and-h	Emp	Exp	Gamma	Weibull	5%	10%	GPD	logistic	Lognormal		
				Bus	iness Line :	5: Payment & S	Settlement			<u> </u>		
	Panel	A: Sun	ımarv	Stats of Ca	apital Estim	ates as a Perce	entage of To	tal Assets f	or All Models	8		
# Modeled	4	5	5	5	5	3	3	5	5	5		
# that Fit	4	5	0	0	0	3	3	5	5	5		
25th	0.00	0.00	0.00	0 00	0.00	0.09	0.01	0.02	0.05	0.00		
Med	0.00	0.00	0.00	0.00	0.00	0.17	0.01	0.02	0.08	0.00		
75th	0.01	0.00	0.00	0.00	0.00	0.19	0.01	0.12	0.22	0.02		
Panel B: Capital Estimates as a Percentage of Total Assets for Models that Fit (Frequency)												
0-50%	4	5	_	_		3	3	5	5	5		
50-100%	-	-	_	_	_	-	-	-	-	-		
100-200%	_	_	_	_	-	_	_	_	_	_		
200-1000%	-	-	_	_	-	_	_	-	_	_		
1000%+	-	_	_	-	-	-	-	-	-	-		
100070				B	usiness Lin	e 7: Asset Man	agement					
Panel A: Summary Stats of Capital Estimates as a Percentage of Total Assets for All Models												
# Modeled	4	7	5	5	5	0	0	5	5	5		
# that Fit	4	7	0	0	2	0	0	5	5	1		
75th	0.01	0 00	0.00	0.00	0.00	-	-	1 33	0.56	0.01		
Med	0.01	0.00	0.00	0.00	0.00	_	_	1.55	1.08	0.01		
75th	0.01	0.01	0.01	0.01	0.01	_	_	3.62	1.00	0.03		
/otil	Panel	B: Ca	pital E	stimates as	a Percenta	ge of Total Ass	sets for Mod	lels that Fit	(Frequency)	0.00		
0_50%	4	7	_	_	2	-	_	1	5			
50_100%	-	/		_	-			-	5	-		
100_200%		_		_	_			- 1	_	_		
200-1000%	_	_	_	_	-	_	_	-	_	_		
1000%+	-	_	_	-	-	-	-	-	_	_		
				I	Business Lir	ne 8: Retail Bro	okerage					
	Panel	A: Sun	ımarv	Stats of Ca	apital Estim	ates as a Perce	entage of To	tal Assets f	or All Models	8		
# Modeled	3	5	4	4	4	2	2	4	4	4		
# that Fit	3	5	1	1	1	2	2	3	3	3		
25th	0.01	0.00	0.00	0.00	0.00	0.27	0.09	0.01	0.18	0.00		
Med	0.02	0.00	0.00	0.00	0.00	0.47	0.10	0.06	0.35	0.01		
75th	0.02	0.01	0.01	0.01	0.01	0.68	0.10	0.16	1.01	0.01		
	Panel	B: Ca	pital E	stimates as	a Percenta	ge of Total Ass	sets for Mod	lels that Fit	(Frequency)			
0-50%	3	5	1	1	1	2	2	3	3	3		
50-100%	-	-	-	_	-	-	-	-	-	-		
100-200%	-	-	-	-	-	-	-	-	-	-		
200-1000%	-	-	-	-	-	-	-	-	-	-		
1000%+	-	-		-	-		-	-	-			
									NEDULED O	NUMBER DACE		

Table 5: Capital Estimates at the Business Line Level as a Percentage of Total Assets	
(Additional Business Lines Not Originally Included in Paper)	

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					Busin	ess Line 9: Othe	r				
Panel A: Summary Stats of Capital Estimates as a Percentage of Total Assets for All Models											
# Modeled	3	7	0	0	0	3	3	5	5	0	
# that Fit	3	7	0	0	0	0	0	3	4	0	
25th	0.01	0.04	-	-	-	1648.21	1.33	0.10	0.26	-	
Med	0.01	0.11	-	-	-	3295.69	2.19	0.56	0.70	-	
75th	0.10	1.34	-	-	-	67851981.14	816.36	251.26	3.97	-	
	Pane	l B: Caj	pital Es	stimates a	s a Percenta	age of Total Ass	ets for Mo	dels that Fit	(Frequency)		
0-50%	3	7	-	-	-	-	-	1	3	-	
50-100%	-	-	-	-	-	-	-	-	-	-	
100-200%	-	-	-	-	-	-	-	-	-	-	
200-1000%	-	-	-	-	-	-	-	1	-	-	
1000%+	-	-	-	-	-	-	-	1	1	-	

This table presents a summary of the 99.9% capital estimates for each Basel business line as a percentage of total assets. The capital estimates were simulated from one million trials. Panel A presents the first, second and third quartiles, which were calculated across each model. These statistics include both capital estimates that statistically fit and do not fit. Panel B presents a frequency distribution of banks whose capital estimates fit according to one or more of the following goodness-of-fit tests: Kolmogorov-Smirnoff, Chi-Square, and Anderson-Darling. The fit for the g-and-h distribution was only tested using Q-Q plots. No goodness-of-fit tests were performed for the empirical distribution. By construction, the empirical distribution would fit the data. All empirical estimates are included in these counts. The total number of business lines modeled and the number that fit are also presented in Panel A.

	Reasonable									
	Resu	ults	Rai	ely Fit th	e Data	Gener	ally Yiel	ded Unreasonabl	e Capital	Estimates
						EVT	EVT		Log-	Truncated
	g-and-h	Emp	Exp	Gamma	Weibull	5%	10%	GPD	logistic	Lognormal
				Business	Line 5: Pa	yment &	Settlem	ent		
Panel A: Su	nmary S	stats of	Capita	l Estimat	es as a Pei	centage	of Busin	ess Line Gross In	come for	All Models
# Modeled	4	5	5	5	5	3	3	5	5	5
# that Fit	4	5	0	0	0	3	3	5	5	5
25th	1.28	0.93	0.53	0.60	0.57	12.65	3.30	10.44	28.09	1.81
Med	1.81	1.12	0.60	0.65	0.60	14.50	4.80	11.57	35.63	3.17
75th	16.52	2.67	0.99	1.04	0.70	64.65	5.54	54.38	38.61	8.58
Panel B: Ca	pital Est	timates	as a Po	ercentage	of Busine	ss Line G	Fross Inc	ome for Models t	that Fit (F	Frequency)
0-50%	3	5	-	-	-	2	3	3	4	5
50-100%	1	-	-	-	-	-	-	2	1	-
100-200%	-	-	-	-	-	1	-	-	-	-
200-1000%	-	-	-	-	-	-	-	-	-	-
1000%+	-	-	-	-	-	-	-	-	-	-
				Busines	s Line 7: A	Asset Ma	nagemei	nt		
Panel A: Su	nmary S	tats of	Capita	l Estimat	es as a Pei	centage	of Busin	ess Line Gross In	come for	All Models
# Modeled	4	7	5	5	5	0	0	5	5	5
# that Fit	4	7	0	0	2	0	0	5	5	4
25th	2.46	1.10	0.79	0.94	0.95	-	-	178.84	145.22	6.92
Med	4.03	1.61	1.17	1.39	0.96	-	-	275.18	156.90	11.39
75th	6.71	3.14	1.17	1.50	1.42	-	-	1064.43	362.04	18.95
Panel B: Ca	pital Est	timates	as a Po	ercentage	of Busine	ss Line G	Fross Inc	come for Models (that Fit (F	Frequency)
0-50%	4	6	-	-	2	-	-	1	1	4
50-100%	-	-	-	-	-	-	-	-	-	-
100-200%	-	1	-	-	-	-	-	1	2	-
200-1000%	-	-	-	-	-	-	-	1	1	-
1000%+	-	-	-	-	-	-	-	2	1	-
				Busine	ess Line 8:	Retail B	rokerage	2		
Panel A: Su	nmary S	stats of	Capita	l Estimat	es as a Pei	centage	of Busin	ess Line Gross In	come for	All Models
# Modeled	3	5	4	4	4	2	2	4	4	4
# that Fit	3	5	1	1	1	2	2	3	3	3
25th	1.15	1.54	1.17	1.24	1.13	24.83	10.51	6.80	85.09	1.96
Med	1.24	1.76	1.38	1.44	1.27	36.45	15.32	15.01	130.02	2.04
75th	2.68	1.91	1.49	1.57	1.44	48.08	20.14	22.05	159.82	2.31
Panel B: Ca	pital Est	imates	as a Po	ercentage	of Busine	ss Line G	ross Inc	come for Models t	that Fit (F	requency)
0-50%	3	5	1	1	1	1	2	3	1	3
50-100%	-	-	-	-	-	1	-	-	-	-
100-200%	-	-	-	-	-	-	-	-	2	-
200-1000%	-	-	-	-	-	-	-	-	-	-
1000%+	-	-	-	-	-	-	-	-	-	-

 Table 6: Capital Estimates at the Business Line Level as a Percentage of BL Gross Income
 (Additional Business Lines Not Originally Included in Paper)

This table presents a summary of the 99.9% capital estimates for each Basel business line as a percentage of business line gross income. The capital estimates were simulated from one million trials. Panel A presents the first, second and third quartiles, which were calculated across each model. These statistics include both capital estimates that statistically fit and do not fit. Panel B presents a frequency distribution of banks whose capital estimates fit according to one or more of the following goodness-of-fit tests: Kolmogorov-Smirnoff, Chi-Square, and Anderson-Darling. The fit for the g-and-h distribution was only tested using Q-Q plots. No goodness-of-fit tests were performed for the empirical distribution. By construction, the empirical distribution would fit the data. All empirical estimates are included in these counts. The total number of business lines modeled and the number that fit are also presented in Panel A.

	Reasor	nable				Gene	erally Yie	lded Un	reasonabl	e Capital
	Resu	lts	Rar	ely Fit tl	he Data			Estima	ites	
						EVT	EVT		Log-	Truncated
	g-and-h	Emp	Exp	Gamma	Weibull	5%	10%	GPD	logistic	Lognormal
				Event	Гуре 1: Іп	ternal Fra	aud			
Panel A: S	Summary	v Statis	tics of	Capital	Estimates	as a Perc	entage of	' Total A	ssets for A	All Models
# Modeled	2	7	4	4	4	-	-	4	4	4
# that Fit	2	7	0	2	2	-	-	4	4	4
25th	0.022	0.001	0.001	0.001	0.003	-	-	0.146	0.623	0.014
Med	0.028	0.004	0.001	0.001	0.004	-	-	0.379	0.886	0.019
75th	0.035	0.005	0.002	0.003	0.004	-	-	0.619	1.900	0.117
Panel E	B: Capital	l Estim	ates as	a Perce	entage of T	Fotal Asse	ts for Mo	dels tha	t Fit (Freq	[uency]
0-1.5%	2	7	-	2	2	-	-	4	3	4
1.5-3%	-	-	-	-	-	-	-	-	-	-
3-20%	-	-	-	-	-	-	-	-	1	-
20-100%	-	-	-	-	-	-	-	-	-	-
100%+	-	-	-	-	-	-	-	-	-	-

 Table 7: Capital Estimates at the Event Type Level as a Percentage of Total Assets

 (Additional Event Type Not Originally Included in Paper)

This table presents a summary of the 99.9% capital estimates for each Basel event type as a percentage of total assets. The capital estimates were simulated from one million trials. Panel A presents the first, second and third quartiles, which were calculated across each model. These statistics include both capital estimates that statistically fit and do not fit. Panel B presents a frequency distribution of banks whose capital estimates fit according to one or more of the following goodness-of-fit tests: Kolmogorov-Smirnoff, Chi-Square, and Anderson-Darling. The fit for the g-and-h distribution was only tested using Q-Q plots. No goodness-of-fit tests were performed for the empirical distribution. By construction, the empirical distribution would fit the data. All empirical estimates are included in these counts. The total number of event types modeled and the number that fit are also presented in Panel A.

Table 8: Summary Statistics for Allocation of g-and-h Capital Estimates at the Business Line Level

	BL1	BL2	BL3	BL4	BL5	BL6	BL7	BL8	BL9
Mean	2.07%	12.91%	20.88%	12.88%	2.13%	32.98%	6.70%	1.14%	21.22%
SD	3.01%	14.83%	26.23%	15.55%	2.23%	38.10%	7.01%	1.21%	29.89%
25	0.03%	3.23%	3.23%	0.53%	0.20%	3.57%	0.06%	0.10%	1.38%
Med	0.06%	10.58%	10.04%	5.17%	0.91%	14.79%	5.27%	1.11%	4.54%
75	2.19%	12.32%	23.66%	26.00%	3.78%	65.82%	12.87%	1.18%	33.40%

This table gives the allocation of the 99.9% capital estimates across Basel event types (as a percentage of the total capital) for the g-and-h distribution. For the g-and-h distribution, event types with 100 or more observations were estimated. Some event types were combined in order to estimate using g-and-h. In these cases, the total capital for the combined event types was then reallocated among those event types using the empirical model results. For all other event types that could not be estimated due to limited data, estimates from the empirical model were substituted.

Table 9: Summary Statistics for Allocation of g-and-h Capital Estimates at the Event Type Level

	10/01		7774			777		770
	ETI	ET2	ET3	ET4	ET5	ET6	ET7	ET8
Mean	1.11%	1.43%	8.72%	40.03%	9.70%	2.91%	34.44%	3.91%
SD	1.49%	1.45%	20.52%	44.43%	24.35%	5.51%	40.64%	2.58%
25	0.32%	0.82%	0.52%	3.92%	0.07%	0.34%	4.98%	2.64%
Med	0.62%	0.99%	0.97%	10.14%	0.44%	0.70%	17.94%	4.06%
75	1.02%	1.21%	1.83%	85.13%	1.18%	1.79%	58.06%	5.25%

This table gives the allocation of the 99.9% capital estimates across Basel event types (as a percentage of the total capital) for the g-and-h distribution. For the g-and-h distribution, event types with 100 or more observations were estimated. Some event types were combined in order to estimate using g-and-h. In these cases, the total capital for the combined event types was then reallocated among those event types using the empirical model results. For all other event types that could not be estimated due to limited data, estimates from the empirical model were substituted.

Table 10: Summary Statistics for Allocation of g-and-h Capital Estimates
at the Business Line Level as a Percentage of Total Assets

					_	-			
	BL1	BL2	BL3	BL4	BL5	BL6	BL7	BL8	BL9
Mean	0.01%	0.07%	0.19%	0.06%	0.02%	0.20%	0.02%	0.01%	0.21%
SD	0.01%	0.07%	0.30%	0.06%	0.02%	0.28%	0.03%	0.01%	0.31%
25	0.00%	0.02%	0.03%	0.01%	0.00%	0.02%	0.00%	0.00%	0.00%
Med	0.00%	0.07%	0.07%	0.04%	0.01%	0.08%	0.02%	0.01%	0.04%
75	0.01%	0.12%	0.12%	0.11%	0.04%	0.22%	0.05%	0.01%	0.29%

This table gives the allocation of the 99.9% capital estimates across Basel business lines (as a percentage of total assets) for the g-and-h distribution. For the g-and-h distribution, business lines with 100 or more observations were estimated. Some business lines were combined in order to estimate using g-and-h. In these cases, the total capital for the combined business lines was then reallocated among those business lines using the empirical model results. For all other business lines that could not be estimated due to limited data, estimates from the empirical model were substituted.

Table 11: Summary Statistics for Allocation of g-and-h Capital Estimates at the Event Type Level as a Percentage of Total Assets

			$r_{P} = r_{P}$					
	ET1	ET2	ET3	ET4	ET5	ET6	ET7	ET8
Mean	0.01%	0.01%	0.04%	0.30%	0.11%	0.03%	0.20%	0.04%
SD	0.01%	0.01%	0.07%	0.39%	0.27%	0.06%	0.30%	0.03%
25	0.00%	0.01%	0.00%	0.02%	0.00%	0.00%	0.04%	0.02%
Med	0.00%	0.01%	0.01%	0.06%	0.00%	0.00%	0.10%	0.03%
75	0.01%	0.01%	0.02%	0.49%	0.01%	0.01%	0.18%	0.05%

This table gives the allocation of the 99.9% capital estimates across Basel event types (as a percentage of total assets) for the g-and-h distribution. For the g-and-h distribution, event types with 100 or more observations were estimated. Some event types were combined in order to estimate using g-and-h. In these cases, the total capital for the combined event types was then reallocated among those event types using the empirical model results. For all other event types that could not be estimated due to limited data, estimates from the empirical model were substituted.

		EVT	EVT					Log-	
	Emp	5%	10%	Exp	Gamma	g-and-h	GPD	logistic	Weib
Α	0.05%	0.04%	0.03%	0.01%	0.01%	0.41%	-6.44%	-0.59%	0.01%
В	0.05%	-324.43%	-0.37%	0.01%	0.01%	0.10%	0.22%	0.26%	0.01%
С	0.24%	-36.65%	-629.08%	0.02%	0.05%	0.25%	-80661.37%	-29.44%	0.04%
D	0.10%	-0.10%	-0.97%	0.01%	0.02%	0.49%	32.61%	-15.15%	0.01%
Е	0.07%	-7.32%	-296.59%	0.01%	0.01%	0.05%	-120.04%	-1.02%	0.01%
F	0.07%	0.00%	0.00%	0.02%	0.03%	0.49%	-7.19%	-1.31%	0.03%
G	0.00%	0.00%	0.00%	0.00%	0.00%	0.02%	-5.07%	-0.56%	0.00%
Mean	0.08%	-52.64%	-132.43%	0.01%	0.02%	0.26%	-11538.18%	-6.83%	0.02%
Std Dev	0.07%	120.59%	245.27%	0.01%	0.02%	0.21%	30480.50%	11.35%	0.01%
25th	0.05%	-21.98%	-148.78%	0.01%	0.01%	0.08%	-63.62%	-8.23%	0.01%
Med	0.07%	-0.10%	-0.37%	0.01%	0.01%	0.25%	-6.44%	-1.02%	0.01%
75th	0.09%	0.00%	0.00%	0.01%	0.02%	0.45%	-2.43%	-0.58%	0.02%

 Table 12: Difference Between Aggregate Capital Estimates (as a Percentage of Enterprise Assets) Under Comonotonic and Independence Structures at the Business Line Level

This table shows the difference between the 99.9% capital estimates calculated under two dependence assumptions: comonotonicity (simple addition) and independence (zero correlation), summed from the Basel business line level to the enterprise level and given as a percentage of the total assets. The mean, median, standard deviation, and interquartile statistics were calculated across all institutions. If certain business lines were not large enough to estimate under the given distribution, those business lines were left off of these calculations.

 Table 13: Difference Between Aggregate Capital Estimates (as a Percentage of Enterprise Assets) Under Comonotonic and

 Independence Structures at the Event Type Level

		EVT	EVT					Log-	
	Emp	5%	10%	Exp	Gamma	g-and-h	GPD	logistic	Weib
Α	0.56%	0.00%	0.00%	0.01%	0.02%	0.01%	-4.56%	-0.31%	0.02%
В	0.05%	-82.26%	-2.40%	0.01%	0.01%	0.10%	-0.56%	0.06%	0.01%
С	0.09%	1.13%	-52.17%	0.00%	0.01%	0.07%	-18.06%	-0.56%	0.00%
D	0.04%	0.05%	0.94%	0.01%	0.01%	0.11%	-850.49%	-8.60%	0.01%
Е	0.03%	-0.39%	-34.48%	0.00%	0.01%	0.11%	-0.44%	0.14%	0.01%
F	0.08%	0.00%	0.00%	0.02%	0.03%	0.10%	-215.42%	-6.53%	0.04%
G	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	-1.36%	-0.80%	0.00%
Mean	0.12%	-11.64%	-12.59%	0.01%	0.01%	0.07%	-155.84%	-2.37%	0.01%
Std Dev	0.20%	31.14%	21.63%	0.01%	0.01%	0.05%	316.25%	3.61%	0.01%
25th	0.03%	-0.20%	-18.44%	0.00%	0.01%	0.04%	-116.74%	-3.66%	0.01%
Med	0.05%	0.00%	0.00%	0.01%	0.01%	0.10%	-4.56%	-0.56%	0.01%
75th	0.08%	0.02%	0.00%	0.01%	0.01%	0.11%	-0.96%	-0.13%	0.01%

This table shows the difference between the 99.9% capital estimates calculated under two dependence assumptions: comonotonicity (simple addition) and independence (zero correlation), summed from the Basel event type level to the enterprise level and given as a percentage of the independence estimates. The mean, median, standard deviation, and interquartile statistics were calculated across all institutions. If certain event types were not large enough to estimate under the given distribution, those event types were left off of these calculations.

	Largest	Second	Third	Fourth	Fifth	Median
	Loss	Largest	Largest	Largest	Largest	Loss
Mean	12.8	20.5	47.4	61.5	111.7	155,510.6
Std Dev	12.2	15.1	30.0	32.1	76.7	204,049.2
25th	2.2	9.4	24.2	46.7	66.0	31,233.5
Med	8.1	17.1	49.3	57.4	91.6	50,157.5
75th	21.7	28.1	66.4	80.6	154.3	194,616.3

Table 14: g-and-h Loss Multiplier at the Enterprise Level

This table presents sample statistics for the g-and-h distribution loss multipliers, where the loss multiplier is defined as the 99.9% g-and-h capital estimate divided by the given loss. These statistics are calculated across all institutions for the median and five largest losses.

	Largest	Second	Third	Fourth	Fifth	Median
	Loss	Largest	Largest	Largest	Largest	Loss
		Business L	ine 2: Tradi	ing & Sales		
Mean	24.7	57.1	102.3	141.2	244.3	97,441.6
Std Dev	29.4	56.9	118.9	138.8	109.7	53,720.3
25th	7.7	25.0	34.5	61.0	181.3	78,628.1
Med	7.9	37.3	52.0	65.1	192.4	121,336.8
75th	33.2	79.3	145.1	183.3	281.4	128,202.6
	•	Business I	Line 3: Reta	il Banking		
Mean	49.8	119.3	188.4	218.2	258.4	105,407.4
Std Dev	49.8	129.4	218.6	252.7	308.0	191,654.6
25th	7.7	29.4	39.3	45.6	48.8	7,405.0
Med	40.6	60.9	84.4	105.0	121.1	20,548.4
75th	79.4	196.3	306.0	333.8	375.9	77,960.4
	ł	Business Line	e 4: Comme	rcial Bankin	g	
Mean	67.7	321.5	355.0	419.3	715.5	67,036.7
Std Dev	81.5	423.9	428.2	505.8	998.7	74,976.9
25th	21.8	80.0	113.7	134.4	142.5	26,264.7
Med	39.2	149.0	206.8	244.9	253.5	51,453.4
75th	99.4	476.7	522.2	617.1	1,057.5	100,017.1
	В	usiness Line	5: Payment	t & Settleme	nt	
Mean	153.5	185.9	276.5	316.6	343.5	7,654.4
Std Dev	298.4	341.7	496.2	555.3	582.9	12,488.6
25th	3.7	11.4	23.4	28.9	32.6	906.4
Med	5.0	19.6	32.5	45.1	62.9	1,736.1
75th	154.8	194.1	285.5	332.8	373.8	8,484.1
		Business L	ine 6: Agen	cy Services		
Mean	45.7	55.3	80.3	124.9	166.2	14,809.1
Std Dev	27.4	30.8	50.2	68.1	84.3	13,145.6
25th	26.4	38.1	38.2	102.2	110.6	5,406.2
Med	34.6	45.2	81.5	121.4	216.6	10,205.4
75th	73.4	81.4	127.6	147.6	225.1	20,396.3
		Business Li	ne 7: Asset N	Management		
Mean	16.0	30.8	37.0	54.3	72.5	1,311.6
Std Dev	15.0	9.0	9.5	22.1	31.8	752.0
25th	4.6	25.7	33.5	41.9	59.5	752.9
Med	15.7	28.7	37.5	58.5	81.0	1,176.8
75th	27.2	33.8	41.0	70.9	94.0	1,735.4
		Business Li	ine 8: Retail	Brokerage		
Mean	14.9	19.8	25.1	39.3	45.7	3,014.1
Std Dev	16.8	15.1	22.0	21.1	20.6	3,918.8
25th	5.5	11.1	12.4	32.7	39.7	769.0
Med	9.1	11.5	13.9	50.5	57.6	1,254.5
75th	21.5	24.4	32.2	51.5	57.6	4,379.4
		Busin	ess Line 9:	Other		
Mean	0.4	1.9	8.1	35.6	73.8	31,501.4
Std Dev	0.2	1.3	5.8	32.3	84.6	51,313.1
25th	0.3	1.2	5.3	18.1	26.0	1,876.7
Med	0.4	1.8	8.6	29.2	43.7	2,277.8
75th	0.5	2.5	11.1	49.9	106.5	46,514.3

 Table 15: g-and-h Loss Multiplier at the Business Line Level

This table presents sample statistics for the g-and-h distribution loss multipliers, where the loss multiplier is defined as the 99.9% g-and-h capital estimate divided by the given loss. These statistics are calculated across all institutions for the median and five largest losses.

	Largest	Second	Third	Fourth	Fifth	Median		
	Loss	Largest	Largest	Largest	Largest	Loss		
Event Type 2: External Fraud								
Mean	7.3	21.6	54.1	71.4	83.9	4,572.6		
Std Dev	6.9	14.1	33.9	36.5	44.7	3,106.3		
25th	2.2	14.0	24.5	50.4	54.5	3,110.6		
Med	4.4	18.8	50.2	56.5	73.0	3,807.6		
75th	13.4	24.4	78.8	103.3	119.2	6,552.9		
Event Type 3: Employment Practices & Workplace Safety								
Mean	18.8	37.9	52.4	87.8	156.3	167,607.7		
Std Dev	27.0	51.4	50.9	41.7	97.2	182,269.3		
25th	2.8	13.7	24.3	61.4	96.0	12,761.2		
Med	4.1	18.3	24.7	85.5	148.9	118,821.0		
75th	18.9	20.3	67.3	101.6	153.1	274,325.5		
Event Type 4: Clients, Products & Business Practices								
Mean	10.2	18.1	35.0	53.0	126.0	206,165.6		
Std Dev	8.7	3.1	21.6	26.5	131.0	185,442.2		
25th	2.8	17.1	24.1	36.6	55.7	89,814.2		
Med	9.6	19.2	24.5	51.1	81.1	196,573.3		
75th	17.0	20.1	35.4	67.4	151.3	312,924.7		
	Event Type 7: Execution, Delivery & Process Management							
Mean	14.5	20.9	39.7	61.7	68.1	12,715.2		
Std Dev	7.4	7.7	21.7	34.9	39.4	4,889.0		
25th	9.2	15.8	24.1	31.8	35.4	10,404.0		
Med	12.1	18.5	38.0	74.7	80.2	13,862.7		
75th	19.0	28.4	50.6	83.3	87.5	15,228.2		

Table 16: g-and-h Loss Multiplier at the Event Type Level

This table presents sample statistics for the g-and-h distribution loss multipliers, where the loss multiplier is defined as the 99.9% g-and-h capital estimate divided by the given loss. These statistics are calculated across all institutions for the median and five largest losses.

	b	
Business Line 2: Trading & Sales	0.0043	
Business Line 3: Retail Banking	0.0019	
Business Line 4: Commercial Banking	0.0039	
Business Line 5: Payment & Settlement	0.0023	
Business Line 6: Agency Services	0.0028	
Business Line 7: Asset Management	0.0044	
Business Line 8: Retail Brokerage	0.0039	
Event Type 1: Internal Fraud	0.0031	
Event Type 2: External Fraud	0.0011	
Event Type 3: Employment Practices & Workplace Safety	0.0036	
Event Type 4: Clients, Products & Business Practices	0.0033	
Event Type 6: Business Disruption & System Failures		
Event Type 7: Execution, Delivery & Process Management		
Overall	0.0024	

Table 17: Average b Parameterby Business Line, Event Type and Overall