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Heterogeneous Beliefs and Inflation Dynamics: A General Equilibrium Approach

Fabià Gumbau-Brisa

Abstract:
This paper looks at the implications of heterogeneous beliefs for inflation dynamics. Following a monetary policy shock, inflation peaks after output, is inertial, and can be characterized by a Hybrid Phillips Curve. It presents a novel channel through which systematic monetary policy can affect the degree of inflation persistence. It does so by altering the effective extent of strategic complementarities in pricing, and hence the role of higher-order expectations in the equilibrium. In particular, stronger inflation targeting reduces the impact of uncertainty on the economy and therefore the degree of inertia. It is possible to calibrate at around 25 percent the fraction of relevant information processed every period by the private sector. The imperfect common knowledge framework does not require any exogenous shocks to create heterogeneity. Despite the fact that prices can be adjusted at no cost in every period, there are nominal rigidities, and monetary policy has real effects.

JEL Classifications: D5, D8, E3, E5

Fabià Gumbau-Brisa is an Economist at the Federal Reserve Bank of Boston. His email address is fabia@bos.frb.org.

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1 Motivation

This paper presents a channel through which heterogeneous beliefs shape how monetary policy impacts the macroeconomy. In particular, it determines the importance of endogenous nominal rigidities in an imperfect common knowledge (ICK) environment with learning. Moreover, this rational expectations model displays realistic dynamics in several dimensions: inflation inertia, a Hybrid Phillips Curve, and inflation's lagging the cycle after a monetary policy shock.

The Calvo-pricing mechanism *assumed* in New Keynesian (NK) models implies that inflation must lead the cycle.[1] In particular, it generates a forward-looking Phillips Curve, in which inflation responds to current and forecastable future output gaps.[2] Contrary to this prediction, evidence from estimated VAR models indicates that inflation has a maximal response to a monetary policy shock several quarters after output does.[3] The NK paradigm requires prices to be rigid for unrealistically long periods of time, as reported by Chari et al. (2000). Additionally, empirical evidence seems to point to the need to motivate a backward-looking component in the Phillips Curve relationship in order to generate sufficient inflation inertia.[4]

The complications that these empirical regularities imply for standard NK models have spawned a vast amount of empirical and theoretical research. There have been several attempts to sidestep these problems, by amending and expanding the NK framework.[5] Others have focused on alternative modeling strategies for nominal rigidities.[6] This paper adopts the latter strategy, avoiding the Calvo-pricing assumption.

The model presented here consists of a monetary DSGE model with rational expectations and ICK. It is able to generate substantial inflation inertia even though nominal contracts last only one period and can be adjusted without cost. Price rigidities are endogenous, in contrast with the main building block in NK models. The model contrasts also with the State Dependent Pricing literature, as here price-setters do not need to optimally choose the time for a price change.[7] In particular, there is no marginal benefit from not re-optimizing prices in any given period.

Because prices can be adjusted every period, expected economic conditions do not need to affect pricing today. This makes it possible for the inflation peak to take place quarters after that of output. Here, systematic monetary policy can change the degree of inflation inertia. It does so by affecting the way price-setters base their decisions on higher-order expectations about the price level. In particular, policies that are more aggressive against inflation reduce the impact of ICK on the economy and hence the intensity of inflation persistence. Monetary policy is able to take the economy closer to the perfect information equilibrium.

This paper shows that it is possible to endogenize and sustain ICK in a general equilibrium

[1] See Galí (2003), among others.

[2] The usual reference is Calvo (1983), but the criticism here applies to other models such as Rotemberg (1982a, 1982b). These are frameworks that motivate the forward-looking Phillips Curve.

[3] See Altig et al. (2004) or Christiano, Eichenbaum and Evans (2005).

[4] For instance, Fuhrer and Moore (1995), Fuhrer (1997, 2005) and Mankiw (2001).

[5] Erceg and Levin (2003), Calvo, Celasun and Kumhof (2002), Galí, López-Salido and Vallés (2004), Blanchard and Galí (2005), among others.

[6] Woodford (2002), Adam (2005), Mankiw and Reis (2002), and Gumbau-Brisa (2003), among others.

[7] Dotsey, King and Wolman (1999), Dotsey and King (2005), and the references therein.

rational expectations model, without exogenous idiosyncratic shocks.[8] The producer/consumer households face cognitive constraints that result in heterogeneity of beliefs (ICK).[9] The presence of a persistent policy shock provides incentives to accumulate any relevant information over time. Namely, it leads to endogenous learning. It is possible to calibrate the percentage of relevant information processed every period by the private sector. This measure appears to be in the neighborhood of 25 percent.

Inflation inertia arises endogenously because optimal pricing decisions are strategic complements. Price-setting requires learning about the behavior of aggregate inflation. History matters because the underlying shock is persistent, implying that learning can provide more accurate perceptions of the economy. Therefore, households accumulate information about this shock over time, leveraging the extent of inflation inertia. When households have a lower capacity to process information, beliefs are more heterogeneous. In this case, they rely less on their noisier observations when updating their beliefs, adjusting prices more slowly. Hence, the degree of inflation inertia depends on how much learning takes place in equilibrium. Taylor (1999a) suggests that persistence in the conduct of monetary policy might be the source of inflation inertia. In the model discussed here, this is a necessary but not sufficient condition to generate inertial behavior.

In order to understand why monetary policy affects the extent of inflation inertia, consider a Taylor rule that targets inflation more aggressively. This policy coordinates expectations around the known inflation target. In the equilibrium, the level of strategic interdependence in pricing is reduced. Learning about others' decisions is now less important, and price adjustment takes place more quickly, resulting in less inflation inertia. Hence the extent to which households need to engage in learning is endogenous and dependent on the systematic part of monetary policy.

Summarizing these key intuitions, inflation persistence results from the interaction of two phenomena. In the first place, households' limited capacity to process information leads to heterogeneous beliefs about the persistent shock. Second, monetary policy has some control over how that heterogeneity impacts the equilibrium level of inflation inertia. It does so by altering the effective degree of strategic complementarity in pricing, and hence the importance of learning. This indicates that Taylor rules are not only an answer to the central bank's time-inconsistency problem, but also a mechanism that influences the impact of uncertainty on the equilibrium.

After a monetary policy shock takes place, the relationship between inflation and output takes the form of a Hybrid Phillips Curve (HPC), as it includes both past and future inflation. This relationship can also be expressed as a Backward-Looking Expectations-Augmented (BLEA) Phillips Curve. In both formulations, some of the coefficients of the Phillips Curve are not constant over time. Some of the coefficients in both relationships change as price-setters learn from their environment.

It is important to note that both the Hybrid and the BLEA Phillips Curves are obtained within a rational expectations framework. It is not necessary to assume adaptive expectations (or, more gener-

[8] The exogeneity of those idiosyncratic shocks has long been the traditional way to sustain the existence of heterogeneous beliefs. Examples abound in the literature on global games, or more generally in any models with ICK. See, for instance, Carlsson and van Damme (1993), Morris and Shin (1998, 2000), Hellwig (2002), and Angeletos and Calvet (2005, 2006), among others.

[9] Similar motivations are used in the models by Sims (2003), Woodford (2002), and Adam (2005). Formally, the model here adds a general equilibrium structure in which the producer/consumer can observe both prices and quantities relevant for her optimization problem.

ally, bounded rationality), in order to make current inflation depend on its own past values/forecasts. The endogenous learning process discussed here shows that a purely rational framework is sufficient.

2 Structure of the Private Sector

Time is discrete and the economy is populated by yeoman producer households indexed by $h \in H \equiv [0,1]$. A household's utility is given by

$$U\left(\{C_t(h), L_t(h)\}_{t=1}^{\infty}\right) = \sum_{t=1}^{\infty} \beta^{t-1} \left[u\left(C_t(h)\right) - \frac{1}{\nu}\left[L_t(h)\right]^{\nu}\right].$$

The consumption index $C_t(h)$ is defined below, $L_t(h)$ is the household's labor supply at t, and the rest of the parameters are constrained as usual: $\beta \in (0,1)$, and $\nu \in (0,+\infty)$. Each household h produces the differentiated good h. The set of commodities produced in this closed economy is A.

The notation $x_t(h)$ indicates a variable that is a choice for household h at time t. In contrast, the household-specific $x_{h,t}$ is not a choice variable.

The consumption index for household $\widehat{h}$ is given by

$$C_t\left(\widehat{h}\right) = \left(\int_A \left[C_t\left(\widehat{h}, h\right)\right]^{\frac{\theta-1}{\theta}} dh\right)^{\frac{\theta}{\theta-1}},$$

where $C_t\left(\widehat{h}, h\right)$ denotes household $\widehat{h}$'s consumption of the differentiated good h. Parameter $\theta > 1$ gives the elasticity of substitution between goods. With $P_t(h)$ denoting the price of good h, the aggregate price level is given by

$$P_t = \left(\int_A \left[P_t(h)\right]^{1-\theta} dh\right)^{\frac{1}{1-\theta}}.$$

For simplicity, production of good h is given by

$$Y_t(h) = L_t(h).$$

Aggregate output and labor supply are related by the following expressions

$$\begin{aligned} Y_t &= \left(\int_H \left[Y_t(h)\right]^{\frac{\theta-1}{\theta}} dh\right)^{\frac{\theta}{\theta-1}} \\ L_t &= \left(\int_H \left[L_t(h)\right]^{\frac{\theta-1}{\theta}} dh\right)^{\frac{\theta}{\theta-1}} \\ Y_t &= L_t. \end{aligned} \tag{1}$$

Demand for good h is given by

$$Y_t^d(h) = \left(\frac{P_t(h)}{\widetilde{P}_{h,t}}\right)^{-\theta} Y_t, \tag{2}$$

where $\widetilde{P}_{h,t}$ is the idiosyncratic average price level of the competitors of good h at time t, which may or may not coincide with P_t. I also assume that household h faces an idiosyncratic price $P_{h,t}$ for its consumption basket, which needs not equal P_t. The following equalities must hold

$$P_t = \int_H \widetilde{P}_{h,t} dh = \int_H P_{h,t} dh. \tag{3}$$

Asset markets are incomplete. Nominally denominated riskless discount bonds, are the only asset traded in the economy. The nominal face value of the bonds carried from period t to period $t+1$ is given by $B_t(h)$. Their return is given by $i_{h,t}$, where the subindex h indicates that this might vary across households.

The budget constraint is

$$C_t(h) \leq \frac{B_{t-1}(h)}{P_{h,t}} - \frac{B_t(h)}{(1+i_{h,t}) P_{h,t}} + \frac{P_t(h)}{P_{h,t}} Y_t(h).$$

Using (1) and (2), this constraint becomes

$$C_t(h) \leq \frac{B_{t-1}(h)}{P_{h,t}} - \frac{B_t(h)}{(1+i_{h,t}) P_{h,t}} + \frac{P_t(h)}{P_{h,t}} \left(\frac{P_t(h)}{\widetilde{P}_{h,t}}\right)^{-\theta} Y_t.$$

Assuming that the utility of consumption is CRRA, and given $\rho \in (0, +\infty)$, the problem of household h can be written as:

$$\begin{aligned}
&\textit{Choose } \{P_t(h),\ B_t(h)\}_{t=1}^{\infty} && \textit{to maximize:} \\
U\left(\{P_t(h), C_t(h)\}_{t=1}^{\infty}\right) &= \sum_{t=1}^{\infty} \beta^{t-1} \left[\frac{C_t(h)^{1-\rho} - 1}{1-\rho} - \frac{1}{\nu} L_t(h)^{\nu}\right] \\
s.t.\ C_t(h) &\leq \frac{B_{t-1}(h)}{P_{h,t}} - \frac{B_t(h)}{(1+i_{h,t}) P_{h,t}} + \frac{P_t(h)}{P_{h,t}} L_t(h) \\
L_t(h) &= L_t \left(\frac{P_t(h)}{\widetilde{P}_{h,t}}\right)^{-\theta} \\
\lim_{T\to\infty} & \frac{b_T(h)}{\Pi_{t=0}^{T}(1+i_{h,t})} \geq 0 \\
B_0(h) &= 0.
\end{aligned} \tag{4}$$

3 The Imperfect Common Knowledge Economy

Each household faces an idiosyncratic information-processing constraint that limits its ability to fully perceive relevant macroeconomic variables. Given this constraint, households form expectations and make decisions rationally. Because the information-processing constraints are specific to each household, beliefs are heterogeneous. In the relevant case analyzed, there is no common knowledge about macro aggregates. It is important to note that there is no bounded rationality; although agents may be "ignorant," they are immersed in a rational expectations model.[10]

The fraction of information processed by households per unit of time, denoted by $k^* \in (0,1)$, is constant and endogenous. How this parameter is determined is discussed in detail in Section 4.

The structure provided below allows us to sidestep the introduction of the exogenous idiosyncratic shocks commonly used to sustain heterogeneity of beliefs.

Assuming that households can process a fraction of the information about the economy is conceptually inconsistent with allowing them to observe an infinite number of consumption goods (and prices). The most sensible modeling assumption is to impose the same cognitive limitation in perceiving the consumption space. Hence, consumption baskets are idiosyncratic and limited by each household's capacity to process information.[11] As a result, they do not observe the economy-wide price level, but rather an idiosyncratic one. The following definition formalizes this idea.

Definition 1: *The* **commodity space for household** h **at time** t *is denoted by* $A_{h,t}$*, and determined as follows:*

1- *Consider the probability space* $\mathbb{A} \equiv (A, A_\sigma, M(.))$*, where* A_σ *is the* σ*-algebra of* A*, and the probability measure* $M(.)$ *corresponds to a uniform distribution.*

2- *Given* $k^* \in (0,1)$*, define for each household* h *the set:* $\Theta_A(\mathbb{A}, k^*, h) \equiv \left\{ \widetilde{A} \in A_\sigma \mid M\left(\widetilde{A}\right) = k^* \text{ and } h \in \widetilde{A} \right\}$.

3- *Then the* **commodity space for household** $h \in H$ **at time** $t \geq 1$ *is characterized as:*

$$\begin{aligned} A_{h,t} &\in \Theta_A(\mathbb{A}, k^*, h) \\ .\bigcup_{h \in H} A_{h,t} &= A \text{ at each } t. \end{aligned} \tag{5}$$

The endogenous consumption basket of household h, is therefore:

$$C_t(h) = \left(\int_{A_{h,t}} \left[C_t(h,i) \right]^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}}.$$

Result (5) plus the uniform distribution assumed in the definition of the probability space imply that:[12]

$$\int_H P_{h,t} dh = P_t \text{ at each } t. \tag{6}$$

[10] Alternatively, they can be said to suffer "rational inattention," as in Sims (2003).

[11] Rationality implies that households know that there are other goods in the economy. They are just unable to gather information about them.

[12] If the household-specific consumption space does not change from one period to another, it is possible to learn over time the relationship between $P_{h,t}$ and P_t. This would lead to an increasingly accurate assessment of P_t.

Additionally, given the limited capacity to observe the consumption space, it is assumed that only a fraction k^* of all goods is effectively competing against any given good h. This set of competitors is also assumed to vary over time for any given h, and does not need to equal $A_{h,t}$. Since the information set of household h includes only goods in $A_{h,t}$, the price level of the competitors, denoted by $\widetilde{P}_{h,t}$, is not directly observed. Hence the following definition:

Definition 2: *The* **price level of the competition of good** $h \in H$ **at time** t *is denoted by* $\widetilde{P}_{h,t}$ *and defined as follows:*

1- Consider the probability space $G \equiv (G, G_\sigma, N(.))$*, where* G_σ *is the* σ*-algebra of* G*, and the distribution function* $N(.)$ *corresponds to a uniform distribution.*

2- Given $k^* \in (0,1)$, *define:*

$$\Pi_G(\mathbb{G}, k^*, h) \equiv \left\{ \widetilde{G} \in G_\sigma \mid N\left(\widetilde{G}\right) = k^* \right\}.$$

3- Then the set of goods competing with good h *for each* $t \geq 1$ *is given by:*

$$G_{h,t} \in \Pi_G(\mathbb{G}, k^*, h, P).$$

4- Let $N_{h,t}(.)$ *denote the conditional distribution over subset* $G_{h,t}$*. Then the* **price level of the competition of good** h **at time** t *is given by*

$$\widetilde{P}_{h,t} = \int_{G_{h,t}} x \, dN_{h,t}(x).$$

It is assumed that by the law of large numbers, for each $t \geq 1$

$$\int_H \widetilde{P}_{h,t} = P_t. \tag{7}$$

Finally, asset markets are segmented. The following definition provides the necessary structure:

Definition 3: *The* **asset market segments at time** t *are given by the partition* $\mathcal{I}_t(\iota)$ *of* H*. For market segment* $S_t \in \mathcal{I}_t(\iota)$, *the measure of households participating in* S_t *is given by the uniform distribution* $\overline{M}(S_t) = \iota$, with $\iota \in (0,1)$.

The parameter ι is assumed to be small, in the sense discussed in Subsection 4.1. It is important to note that the partition $\mathcal{I}_t(\iota)$ is time-dependent.

In the ICK model presented below, households can form estimates of P_t over time given the sequence of realizations of $P_{h,t}$, which amounts to facing idiosyncratic consumption-basket inflation rates $\pi_{h,t}$ that may differ from their beliefs on π_t.[13] They also face both heterogeneous nominal and

[13]The assumption that the idiosyncratic consumption space changes each period can be relaxed at the expense of more analytical complexity. If there were a systematic presence of a set of goods in the households' consumption space, it would be easier to figure out an approximate relationship between the idiosyncratic price level and the aggregate one. According to the discussion that follows, this would increase the speed of learning and diminish the need to conjecture what others are doing. Hence the lack of common knowledge would have a smaller impact on the economy's dynamics.

real rates of interest. Hence in a given period, the intertemporal consumption allocation decisions are also heterogeneous. For instance, holding everything else constant, household h increases its consumption if it believes that it faces a low $P_{h,t}$ relative to the conjectured aggregate price level. This also has an impact on price-setting, as the household's desired labor supply is likely to be affected (depending on the extent of complementarity/substitutability between leisure and consumption).

3.1 Private Sector Information Structure: Perceptions and Beliefs

In contrast with the previous literature on models with imperfect common knowledge, this rational expectations framework generates endogenously the observation noise that generates and sustains heterogeneity of beliefs.

As a simplification, the household optimization problem is completely symmetric. The two restrictions specific to each household are its budget constraint *and* its information-processing constraint. This last cognitive constraint is identical (by assumption) across households *but is independent.*

Once the economy is hit by an aggregate shock, the information-processing constraints introduce observation noise that is idiosyncratic. Household conjectures about the realization of the shocks are *almost surely* heterogeneous. The resulting endogenous imperfect common knowledge structure is sustained without deviating from the rational expectations framework. The necessary elements are definitions 1 and 2, plus Assumption 1.

A signal ${}_F s_{h,t}$ indicates the observation of variable F_t by household h. The following vector summarizes the signals perceived by the household:

$$s_{h,t} \equiv \left({}_\pi s_{h,t},\ {}_y s_{h,t},\ {}_i s_{h,t}\right).$$

The information set of household h at time t is defined as[14]

$$I_t\left(h\right) \equiv \left\{I_{t-1}\left(h\right),\ s_{h,t},\ i_{h,t},\ P_{h,t},\ B_{t-1}\left(h\right),\ \pi_{t-1}\left(h\right)\right\}.$$

The timing of decisions and of information flows is given in Figure 1.

It is necessary here to introduce further notation. The expectation operator E_t^h indicates an expectation conditional on the information set $I_t\left(h\right)$. Expression $F_{s/t}^{h,(j)}$ is the expectation held at time t by household h of the average expectation of the average expectation... (j times) of the *aggregate* stochastic variable F_s. The superscript $j \geq 0$ indicates the order of the expectation. The average expectation across the economy is given by $F_{s/t}^{(j)} \equiv \int_H F_{s/t}^{h,(j)} dh$. Given an expectation of order j, the expectation of order $j+1$ is given by $F_{s/t}^{h,(j+1)} = E_t^h \left\{\int_H F_{s/t}^{h,(j)} dh\right\}$.

By convention, $F_{s/t}^h \equiv F_{s/t}^{h,(1)}$ and $F_s \equiv F_{s/t}^{h,(0)}$. The argument (h) still indicates household

[14]Information sets are denoted by $I_t\left(h\right)$ instead of $I_{h,t}$ because they also contain household h's choice variables. By definition, households cannot forget.

h's choice variables, past and present, or a set containing them.[15,16] Finally, define the operator $E_t\left\{F_{s/t}^{(j)}\right\} \equiv \int_H F_{s/t}^{h,(j+1)} dh = F_{s/t}^{(j+1)}$.

It is assumed throughout the paper that at $t = 0$ the economy is at the deterministic steady state, which is common knowledge. Functional arguments are not specified when confusion is not possible.

4 Learning in The Private Sector

All relevant stochastic variables are normally distributed. The constraint adopted to model the household's information-processing constraint is based on Sims (2003). Its particular functional form and a brief discussion of its statistical basis is given in Appendix A.1. This constraint places bounds on how much a rational decision-maker can reduce per period the uncertainty of her beliefs. It establishes that each decision maker can process a maximum of $K \in (0, +\infty)$ bits of information per unit of time. In controlling flows of information, it provides the foundation to model learning.

The following proposition introduces the main parameter of the learning process used throughout the paper, and is proven in Appendix A.2.

Proposition 1 *Let k^* denote the fraction of all information that each household can absorb per period. Then, there is a one-to-one relationship between K and k^* given by*

$$k^* = 1 - e^{-K \cdot 2\ln(2)}.$$

Figure 2 presents this functional relationship. For relatively low values of K the fraction of relevant information absorbed is already rather large ($K = 1$ already implies $k^* = 0.75$).

By assumption, the value k^* is the same for all households and constant over time. Nonetheless, the noise in the observed signals is independent across households and over time, due to the idiosyncrasy of the constraints.

Given an imperfectly observed normally distributed variable F_t, household h's optimal first-order conjecture is

$$F_{t/t}^{h} = (1 - k^*) F_{t/t-1}^{h} + k^* \cdot_F s_{h,t}. \tag{8}$$

This last expression is a Kalman filter with a constant gain equal to the fraction of information processed per unit of time (k^*). Given symmetry of the problem, equation (8) governs learning from every variable in $s_{h,t}$, for all $h \in H$. Therefore, Proposition 1 provides an intuitive interpretation for the Kalman gain involved in the learning process.

[15] For instance, expressions such as $F_{s/t}^{h,(j)}(h)$ denote expectations of order j held by household h at time t, about the choice of $F_s(h)$.

[16] With common knowledge, $F_{s/t}^{h,(j)} = F_{s/t}^{h} = F_{s/t}$ for all $h \in H$, $t \geq 1$, $s \geq 1$, and $j \geq 1$.

4.1 Monetary Policy and Interest Rates in the Private Sector

Monetary policy follows a simple Taylor rule that is subject to persistent shocks:

$$i_t = \phi_\pi \pi_t + \phi_y l_t + \zeta_t, \tag{9}$$

where i_t denotes the federal funds rate, π_t is the (net) inflation rate, and l_t is employment, all in deviations from their steady-state levels. It is therefore assumed that the target for $(\pi_t,\ l_t)$ is the steady state. The shock ζ_t is given by:[17]

$$\zeta_t = \delta\zeta_{t-1} + \varepsilon_t \ \text{ with } \delta \in (0,1) \text{ and } \varepsilon_t \sim N\left(0, \sigma_\varepsilon^2\right), \tag{10}$$

where ε_t is the *innovation* to the policy shock. Rudebusch (2002) estimates the same Taylor rule, obtaining an $AR(1)$ policy shock with $\delta = 0.92$ and standard error 0.06.[18] Monetary policy is the only shock in the model, allowing for a much simpler exposition of the basic ideas.[19]

In what follows I analyze the effect of a unique innovation at $t = 1$:

$$\varepsilon_t \neq 0 \iff t = 1, \tag{11}$$

which implies by rationality

$$\varepsilon_{t/t-1} = 0 \text{ for all } t. \tag{12}$$

The pair $\phi \equiv \left(\phi_\pi, \phi_y\right)$ is referred to as the coefficients of the systematic part of monetary policy.

This specification presents some advantages over a Taylor rule with interest-rate smoothing for the purpose of this paper. Adding a lagged interest rate to the right-hand side would introduce additional intertemporal dynamics. This makes it more difficult to separate out the source of inertia. The only variables that persist from one period to the next are asset holdings and the policy shock.

Note that $i_{h,t} = i_{\widetilde{h},t}$ if both households are in the same asset market segment $(h, \widetilde{h} \in S_t)$. It is assumed that

$$\int_H i_{h,t} dh = i_t.$$

The size of the market segments $\iota > 0$ is asssumed to be small enough so that $i_{h,t} \neq i_t$ in any market segment, with probability one. Households perceive the federal funds rate i_t with noise, and

[17] Note that because households understand that nominal rates are set according to (9), the vector of signals $s_{h,t}$ included in $I_t(h)$ can be alternatively defined as

$$s_{h,t} \equiv \left({}_\pi s_{h,t},\ {}_y s_{h,t},\ {}_\zeta s_{h,t}\right).$$

This is because (9) implies that the four signals $\left({}_\pi s_{h,t},\ {}_y s_{h,t},\ {}_\zeta s_{h,t},\ {}_i s_{h,t}\right)$ are linearly dependent.

[18] Rudebusch (2002) obtains the following Taylor rule with serially correlated shocks

$$i_t = \underset{(0.24)}{1.24}\,\pi_t + \underset{(0.10)}{0.33}\,l_t + \zeta_t, \qquad \zeta_t = \underset{(0.06)}{0.92}\,\zeta_{t-1} + \varepsilon_t$$

$$\text{with } \sigma_\varepsilon = 0.36 \quad \text{and} \quad \overline{R}^2 = 0.96$$

(his equation (18), page 1178). A Dickey-Fuller test rejects the hypothesis of a unit root in the policy shock process with a p-value smaller than 0.05.

[19] As in Chari, Kehoe and McGrattan (2000), Christiano, Eichenbaum and Evans (2005), or Woodford (2002), for instance.

their conjectures about it are given by the set $\left\{i_{t/t}^{h}\right\}_{h\in H}$. The average federal funds signal observed by household h is given by ${}_{i}s_{h,t} \equiv i_t +_i \eta_{h,t}$, where ${}_{i}\eta_{h,t} \sim N\left(0,\ {}_{i}\sigma_{\eta}^{2}\right)$ is the observation noise.[20] After aggregation the noise cancels out and the average conjecture is

$$i_{t/t} = (1-k)\, i_{t/t-1} + k^{*} i_t. \tag{13}$$

4.2 Formulation of the Private Sector Problem

The specification of production and demand functions implies

$$L_t(h) = L_t\left(P_t(h)\ /\ \widetilde{P}_{h,t}\right)^{-\theta}.$$

The rate of inflation for commodity h is

$$1+\pi_t(h) = P_t(h)\ /\ P_{t-1}(h),$$

and the economy-wide inflation rate $\pi_t = \int_H \pi_t(h)\, dh$.

The inflation rate of the competitors of commodity h is given by

$$1+\widetilde{\pi}_{h,t} = \widetilde{P}_{h,t}\ /\ \widetilde{P}_{h,t-1}.$$

Next, define the following endogenous state variables[21]

$$\begin{aligned} 1+\tau_{t-1}(h) &= P_{t-1}(h)\ /\ P_{h,t-1}, & (14)\\ 1+\lambda_{t-1}(h) &= P_{t-1}(h)\ /\ \widetilde{P}_{h,t-1}, & (15)\\ b_t(h) &= B_t(h)\ /\ P_{h,t}. \end{aligned}$$

After taking into account the presence of uncertainty, the problem (4) becomes

$$\begin{aligned} &\textit{Choose } \{\pi_t(h),\ b_t(h)\}_{t=1}^{\infty} \textit{ to maximize:} & (16)\\ &\widetilde{U}(.) = \sum_{t=1}^{\infty}\beta^{t-1}E_t^h\left\{\frac{C_t(h)^{1-\rho}-1}{1-\rho} - \frac{1}{\nu}\left[\left((1+\lambda_{t-1}(h))\,\frac{1+\pi_t(h)}{1+\widetilde{\pi}_{h,t}}\right)^{-\theta} L_t\right]^{\nu}\right\}\\ &\qquad s.t.\ C_t(h) \le \frac{b_{t-1}(h)}{1+\pi_{h,t}} - \frac{b_t(h)}{(1+i_{h,t})}\\ &\qquad\quad +\left[1+\lambda_{t-1}(h)\right]^{-\theta}\left(\frac{1+\pi_t(h)}{1+\widetilde{\pi}_{h,t}}\right)^{1-\theta} L_t\left[1+\tau_{t-1}(h)\right]\frac{1+\widetilde{\pi}_{h,t}}{1+\pi_{h,t}} & (17)\\ &\qquad \lim_{T\to\infty}\frac{b_T(h)}{\Pi_{t=0}^{T}(1+i_{h,t})} \ge 0\\ &\qquad b_0(h)=0;\ \lambda_0(h)=0;\ \tau_0(h)=0 \text{ (common knowledge).} \end{aligned}$$

[20] Appendix A.2 discusses how ${}_{i}\sigma_{\eta}^{2}$ is determined.

[21] Defining $\tau_t(h)$ and $\lambda_t(h)$ allows us to express the household optimization problem in terms of inflation rates instead of price levels. It is possible then to define the problem around a steady state.

The solution method has two stages. First, a linear quadratic (LQ) approximation to the economy is performed around its stationary zero-inflation *deterministic* steady state, solving the resulting household optimization problem. This yields the optimal decision functions for the household. The second stage uses these decision functions to solve the general equilibrium problem under imperfect common knowledge. The problem can be broken down into these two stages because the optimal decision function from an LQ-problem displays certainty equivalence.

5 First Stage: Optimal Decision Functions

The LQ approximation is standard and only the main results are presented.[22] Note that in the deterministic steady state $\widetilde{\pi}_{h,t} = \pi_t$ for all h and t, because all prices are observable at no cost. Define then

$$W_t(h) \equiv \left(1, \pi_t, \pi_{h,t}, l_t, i_t, \lambda_{t-1}(h), \tau_{t-1}(h), b_{t-1}(h), \pi_t(h), b_t(h)\right)^T.$$

The approximation to the felicity function, once the constraints are substituted into it, is a quadratic form[23]

$$W_t^T Q W_t$$

and the corresponding deterministic Bellman Equation $V(.)$ is[24]

$$V\left(\zeta_{t-1}, b_{t-1}(h), \lambda_{t-1}(h), \tau_{t-1}(h)\right) = \max_{\{\pi_t(h),\, b_t(h)\}} \left\{ \begin{array}{c} W_t^T Q W_t \\ +\beta V\left(\zeta_t, b_t(h), \lambda_t(h), \tau_t(h)\right) \end{array} \right\}.$$

The only state variables in the model are the exogenous policy shock and asset holdings (other than $\tau_t(h)$ and $\lambda_t(h)$, which are introduced for modelling convenience). Additional states could be introduced, for instance, assuming habit formation or federal funds rate smoothing. The approach adopted here allows for a more direct analysis of the impact of policy shocks on inflation inertia.

In terms of deviations from the deterministic steady state, the resulting optimal decision functions for $(\pi_t(h), b_t(h))$ are

$$\begin{aligned} \pi_t(h) &= a_1\pi_t + a_2\pi_{h,t} + a_3 l_t + a_4 i_t + a_5\lambda_{t-1}(h) + a_6\tau_{t-1}(h) \\ &\quad + a_7 b_{t-1}(h) \qquad (18) \\ b_t(h) &= d_1\pi_t + d_2\pi_{h,t} + d_3 l_t + d_4 i_t + d_5\lambda_{t-1}(h) + d_6\tau_{t-1}(h) \\ &\quad + d_7 b_{t-1}(h). \qquad (19) \end{aligned}$$

Household rationality implies an understanding of the relationship between the Taylor rule (9)

[22] For more details, see, for instance, Kydland and Prescott (1982), Hansen and Prescott (1995), and Díaz-Giménez (1999).

[23] For the details of this method applied to a similar model, see Gumbau-Brisa (2003).

[24] Note that the quadratic felicity function is bounded in the relevant domain and meets the conditions required to apply Theorem 4.8 in Stokey and Lucas (1989). This ensures the existence of $V(.)$.

and both inflation and output:[25]

$$\begin{aligned}\pi_t(h) &= (a_1 + a_4\phi_\pi)\,\pi_t + a_2\pi_{h,t} + \left(a_3 + a_4\phi_y\right) l_t + a_4\zeta_t \\ &\quad + a_5\lambda_{t-1}(h) + a_6\tau_{t-1}(h) + a_7 b_{t-1}(h) \qquad (20)\\ b_t(h) &= (b_1 + b_4\phi_\pi)\,\pi_t + b_2\pi_{h,t} + \left(b_3 + b_4\phi_y\right) l_t + b_4\zeta_t \\ &\quad + b_5\lambda_{t-1}(h) + b_6\tau_{t-1}(h) + b_7 b_{t-1}(h). \qquad (21)\end{aligned}$$

Let $D \equiv \{a, d\}$ where $a \equiv \{a_1, ..., a_7\}$, and $d \equiv \{d_1, ..., d_7\}$. Tables 1 and 2 in Appendix B present the coefficients in D for several parameter value configurations.[26] The parameter values considered are $\rho \in \{0.5, 1, 2\}$, $\theta \in \{4, 6, 8\}$, $\nu \in \{2, 5\}$ for a quarterly model with $\beta = 0.96$. The Taylor rule coefficients are $\phi = (1.5,\ 0.5)$, as proposed in Taylor (1993).[27] In the next sections, the benchmark set of parameters consists of $\rho = 2$, $\theta = 6$ and $\nu = 2$.

The coefficients $(a_4,\ d_4)$ indicate the effects of changes in the federal funds rate on the choice variables. It should be the case that $a_4 < 0$ and $d_4 > 0$. According to Table 1, $a_1 + a_2 = 1$, as in the perfect-information equilibrium each producer changes prices one-to-one with the aggregate price level. Moreover, Table 2 shows that $d_1 + d_2 = 0$, indicating that the desired level of *real* savings does not depend *directly* on the current inflation rate.

6 Second Stage: Imperfect Common Knowledge Problem

The constraint on households' ability to observe economic variables is introduced now. It does not affect the optimal decision functions, given the LQ nature of the problem.[28]

Households understand the process that generates $P_{h,t}$ and $\widetilde{P}_{h,t}$. Hence, for all h and $t \geq 1$:

$$E_t^h\left\{\widetilde{P}_{h,t+j}\right\} = E_t^h\left\{P_{t+j}\right\} \quad \text{for } j = 0, 1, ... \qquad (22)$$

$$E_t^h\left\{P_{h,t+j}\right\} = E_t^h\left\{P_{t+j}\right\} \quad \text{for } j = 1, 2, ... \qquad (23)$$

Note that they can observe $P_{h,t}$ but can only conjecture $\widetilde{P}_{h,t}$ from their sales.

From now on, given (22), $\widetilde{\pi}^h_{h,t/t}$ is substituted for the expectation of the aggregate variable $\pi^h_{t/t}$.

[25] Performing the LQ approximation with respect to ζ_t instead of i_t results in the same coefficients with four exceptions:

$$\begin{aligned}\widetilde{a}_j &= a_j + a_4\phi_\pi \text{ for } j = 1, 3.\\ \widetilde{d}_j &= d_j + d_4\phi_\pi \text{ for } j = 1, 3.\end{aligned}$$

Nonetheless, the approach used here allows us to obtain more analytical results regarding the impact of policy on the equilibrium.

[26] The coefficients in a consist of elasticities and semi-elasticities (a_4 and a_7). The coefficients in d indicate the total effect on the units of real savings of percent changes of the right-hand-side variables. Coefficient d_7 is a simple derivative, providing the unit change in the optimal choice of real savings, given a unit change in current real financial wealth.

[27] Many authors have argued that these coefficients characterize well the conduct of recent U.S. monetary policy. See, for instance, Gertler (1999), Taylor (1999b), Rudebusch (2002), McCallum and Nelson (1999), and the references in the latter.

[28] The solution displays certainty equivalence.

This is done because

$$\pi^h_{t/t} = E^h_t \{\widetilde{\pi}_{h,t}\}.$$

6.1 Forecasts and Higher-Order Expectations of the Shocks

The current average conjecture of the policy shock can be expressed as

$$\zeta_{t/t} = (1 - k^*)\,\delta\zeta_{t-1/t-1} + k^*\zeta_t. \tag{24}$$

In this case, ζ_t is exogenously determined by the stochastic process in (10).

The difference equation shows how information about policy is absorbed over time at a constrained rate (k^*). The *accumulation* of information over time improves the (cross-sectional) average estimates of the policy shock.[29] Nonetheless, individual beliefs are affected every period by an idiosyncratic *iid* observation noise. This maintains heterogeneity in households' conjectures of ζ_t (or equivalently, of ε_1).[30]

Solving the difference equation (24), it is possible to write the expectation of a shock as

$$\zeta_{t-a/t-b} = \zeta_{t-a}\left[1 - (1-k)^{t-b}\right] \text{ for } t > b \geq a \geq 0, \tag{25}$$

given $\zeta_{0/0} = 0$, (11) and (12). For easier interpretation, set $a = 0$ in (25) and express the policy shock in terms of ε_1 :

$$\zeta_{t/t-b} = \varepsilon_1\delta^{t-1} \cdot \left[1 - (1-k^*)^{t-b}\right] \text{ for } t > b \geq 0, \tag{26}$$

This illustrates that the passage of time has two effects on $\zeta_{t/t-b}$. On the one hand, δ^{t-1} represents the diminishing impact of the innovation ε_1 on the shock ζ_t. On the other hand, the term $(1-k^*)^{t-b}$ indicates that accumulation of information has taken place between $t' = 1$ and $t'' = t - b$. As b increases, conjectures are formed farther back in time; hence, households have accumulated less information. In this case, the forecast about the shock at t is more inaccurate (closer to 0).

Given $\{\zeta_t\}_{t=0}^{\infty}$, $\zeta_{0/0} = 0$, (11), and (12), equation (26) for $b = 0$ yields the entire sequence of average conjectures $\left\{\zeta_{t/t}\right\}_{t=1}^{\infty}$.[31]

Repeating $(j-1)$ times the process of taking expectations of (25) with respect to $I_{t-b}(h)$, and integrating over $h \in H$ yields:

$$\zeta^{(j)}_{t-a/t-b} = \zeta_{t-a}\left[1 - (1-k)^{t-b}\right]^j \text{ for } t > b \geq a \geq 0 \text{ and } j = 0, 1, ... \tag{27}$$

Therefore, higher-order expectations are less sensitive to changes in ζ_{t-a}. The higher their order, the smaller the adjustment after a shock. Forming the expectation of order (j) requires processing information (learning) about the average expectation of order $(j-1)$. This introduces additional

[29] There is no household superscript in (24).

[30] The shock can be expressed as $\zeta_t = \delta^{t-1}\varepsilon_1$. See equation (10) and condition (11).

[31] The assumption that $\zeta_{0/0} = 0$ can be relaxed at the expense of lengthier expressions, without relevant offsetting gains.

observation noise, which makes the expectation of order (j) less responsive to shocks than the expectation of order $(j-1)$. This effect, plus the persistence of the underlying shock, constitutes the engine of the macroeconomic system's sluggish adjustment.

6.2 Definition and Characterization of the ICK Equilibrium

The formal definition of equilibrium for this economy is

Definition 4: *A* **Competitive Equilibrium in the ICK Monetary Economy at** t *consists of a plan* $\Lambda(h) = \{\pi_t(h),\ b_t(h)\}_{t=1}^{\infty}$ *for each* $h \in H$, *and sequences of prices*

$$P = \left\{\pi_t,\ i_t,\ \left(\widetilde{\pi}_{h,t},\ \pi_{h,t},\ i_{h,t}\right)_{h\in H}\right\}_{t=1}^{\infty}$$

and of information structures

$$I = \left\{\left(I_t(h)\right)_{h\in H}\right\}_{t=1}^{\infty}$$

such that:

1$-$*Given* P *and* I, *plan* $\Lambda(h)$ *solves the problem* (16) *for all* $h \in H$.

$2 - \int_H b_t(h)\,dh = 0$ *for all* $h \in H$ *and* t.

The economy's equilibrium is Markov: the stochastic process of the shock is AR(1), the other relevant state variables are determined at $t-1$ ($b_{t-1}(h)$, $\lambda_{t-1}(h)$, and $\tau_{t-1}(h)$), and learning takes the form of a Kalman filter. Hence, choices at t depend only on information about the shock available at t, and the state variables inherited from $t-1$.

Under ICK, household h is aware that its observations are *almost surely* different from those of a non-zero-measure set of households. It also knows that all other households are in the same situation, and that everybody knows this fact. It has to take this into account in forming its expectation about π_t, and, correspondingly, choosing $\pi_t(h)$. Hence, in conjecturing π_t, household h has to take into account that the other households take into account that other households take into account... (iterated to infinity) that there are different perceptions of the economy-wide level of inflation. Given the structure of monopolistic competition, this affects price-setting (and hence inflation) in period t.

Households form conjectures about the aggregate inflation level by processing information from their environment, including $\pi_{h,t}$and $\widetilde{\pi}_{h,t}$. Therefore,

$$\pi^h_{t/t} \neq \pi_{h,t} \text{ and } \pi^h_{t/t} \neq \widetilde{\pi}_{h,t} \textit{ almost surely } \text{for all } h \text{ and } t.$$

In period t household h can be saving or lending, depending on the values of $\pi_{h,t}$, $\widetilde{\pi}_{h,t}$, and $\pi^h_{t/t}$, even though it knows that aggregate savings are zero. As discussed above, the relative movements in idiosyncratic prices lead to heterogeneous beliefs, which result in heterogeneous consumption/savings decisions.

6.3 ICK Equilibrium Solution

To simplify notation, define

$$z_t^{h,(j)}(h;\mathbf{n}) \equiv n_1 b_{t-1}^{h,(j-1)}(h) + n_2 b_t^{h,(j-1)}(h) \text{ for } j = 1, 2, ... \tag{28}$$

$$z_t^{h,(0)}(h;\mathbf{n}) \equiv z_t^{h,(1)}(h;\mathbf{n}) \tag{29}$$

$$x_t^{h,(j)}(h;\mathbf{n}) \equiv n_3 \lambda_{t-1}^{h,(j-1)}(h) + n_4 \tau_{t-1}^{h,(j-1)}(h) \text{ for } j = 1, 2, ... \tag{30}$$

$$x_t^{h,(0)}(h;\mathbf{n}) \equiv x_t^{h,(1)}(h;\mathbf{n}) \tag{31}$$

for $\mathbf{n} = (n_1,\ n_2,\ n_3,\ n_4) \in \mathbb{R}^4$.

The following property is proven in Appendix A.3.

Lemma 1 for $t_0 \geq t$

$$z_{t/t_0}^{(j)}(\mathbf{n}) = 0 \text{ for all } \mathbf{n} \in \mathbb{R}^4 \text{ and } j \geq 0.$$

$$x_{t/t_0}^{(j)}(\mathbf{n}) = 0 \text{ for all } \mathbf{n} \in \mathbb{R}^4 \text{ and } j \geq 0.$$

Appendix A.4 shows that higher-order expectations ($j \geq 1$) of inflation are

$$\pi_{t/t}^{h,(j)} = \Gamma_1 \pi_{t/t}^{h,(j+1)} + \Gamma_4 \zeta_{t/t}^{h,(j+1)} + \Gamma \left(x_{t/t}^{h,(j+1)}(h;\mathbf{n}^\pi) + z_{t/t}^{h,(j+1)}(h;\mathbf{n}^\pi) \right) \tag{32}$$

and provides the functional forms for the endogenous parameters $\Gamma_1(D,\phi)$, $\Gamma_4(D,\phi)$, and $\Gamma(D,\phi)$. The parameter Γ_1 captures the effective degree of strategic interdependence in pricing under imperfect common knowledge.[32] In particular $\Gamma_1 > 0$ because there is complementarity in price-setting. For the parameter values considered, $\Gamma_1 < 1$, which, as discussed below, is required to ensure the existence of equilibrium, and $\Gamma_4 < 0$.

Iterated substitution of equation (32) for all $j \geq 1$, yields:

$$\pi_{t/t}^{h} \equiv \sum_{j=1}^{\infty} (\Gamma_1)^{j-1} \left[\Gamma_4 \zeta_{t/t}^{h,(j)} + \Gamma \left(x_{t/t}^{h,(j)}(h;\mathbf{n}^\pi) + z_{t/t}^{h,(j)}(h;\mathbf{n}^\pi) \right) \right]. \tag{33}$$

In order to find a reduced-form expression for $\pi_{t/t}$, it is then necessary to solve (and aggregate) the series

$$\Delta_{t/t}^{h} \equiv \sum_{j=1}^{\infty} (\Gamma_1)^{j-1} \left[\Gamma_4 \zeta_{t/t}^{h,(j)} \right]. \tag{34}$$

Aggregating (33) and using Lemma 1 implies:

$$\pi_{t/t} = \Delta_{t/t}.$$

[32] Strictly speaking, it can be shown that the coefficient measuring the strategic interdependence of household actions is $\widehat{\gamma} = \gamma_1 \cdot \Gamma_1$, with $\gamma_1 \in (0,1)$ for the relevant parameter values. Coefficient γ_1 is given in Appendix A.4. Parameter γ_1 turns out to be of little relevance.

Appendix A.4 shows that aggregate inflation can be expressed as

$$\pi_t = \Gamma_1 \pi_{t/t} + \Gamma_4 \zeta_{t/t}. \tag{35}$$

Consequently, Γ_1 determines how pricing is affected by the higher-order expectations of inflation. The weight placed in (34) on higher-order expectations increases geometrically with Γ_1. In particular, larger values of Γ_1 imply that the weights given to expectations of any order become more similar to one another.

$\Gamma_1 (D, \phi)$ is endogenous and affected by the particular Taylor rule implemented. As shown in Section 7, monetary policy has an effect on the extent of persistence because it affects the degree of household interdependence in decision-making. When monetary policy is more aggressive in stabilizing the variable that creates strategic interdependence, conjectures about others' decisions are less important. Information about what others are doing is less important, since the relevant variable becomes easier to predict. This reduces the extent of inertia, as households respond by placing a greater weight on low-order conjectures, which adjust more quickly.

The average of expectation (34) can be expressed recursively:[33]

$$\Delta_{t/t} = E_t \left\{ \Gamma_4 \zeta_t + \Gamma_1 \int_H \sum_{j=2}^{\infty} (\Gamma_1)^{j-2} \left[\Gamma_4 \zeta_{t/t}^{h,(j-1)} \right] dh \right\} \tag{36}$$

and therefore

$$\Delta_{t/t} = \Gamma_4 \zeta_{t/t} + \Gamma_1 E_t \left\{ \Delta_{t/t} \right\}. \tag{37}$$

Appendix A.5 relies on this last equation to provide the proof for the next proposition.

Proposition 2 For $\Gamma_1 \in (0,1)$ and $k^* \in (0,1)$, a solution for π_{t+j} exists, is unique, and is given by

$$\pi_{t+j} = \zeta_{t+j} \cdot Z_{t+j} \text{ for } j = 0, 1, ...,$$

where

$$Z_{t+j} \equiv \left\{ \begin{array}{c} \Gamma_1 \left[\mu_{1,t+j} \left(1 - (1-k^*)^{t+j-1} \right) + \mu_2 \right] \\ +\Gamma_4 \left(1 - (1-k^*)^{t+j} \right) \end{array} \right\} < 0 \tag{38}$$

$$\mu_{1,t+j} = \frac{(1-k^*)}{1 - \Gamma_1 \cdot \left(1 - (1-k^*)^{t+j} \right)} \left[\frac{\Gamma_4}{(1-k^*\Gamma_1)} \right] < 0 \tag{39}$$

$$\mu_2 = k^* \left[\frac{\Gamma_4}{(1-k^*\Gamma_1)} \right] < 0. \tag{40}$$

The functional shape of μ_1 illustrates the need for $\Gamma_1 < 1$ if an equilibrium has to exist. Equation

[33] An early version of Adam (2005) used a similar handling of higher-order expectations.

(78) in Appendix A.5.4 establishes that[34]

$$\pi_t = \Gamma_1 \left[\mu_{1,t}\zeta_{t/t-1} + \mu_2\zeta_t\right] + \Gamma_4\zeta_{t/t}. \tag{41}$$

It is possible, then, to derive an expression for the forecast of inflation:

$$\pi_{t-a/t-b} = \zeta_{t-a/t-b}\left\{\Gamma_1\left[\mu_{1,t-a} + \mu_2\right] + \Gamma_4\right\} \quad \text{for } t > b \geq a \geq 0, \tag{42}$$

where (25) allows us to provide a closed-form solution for $\zeta_{t-a/t-b}$ as function of ζ_{t-a}.

Given that Z_{t+j} is deterministic for all t and j, equations (27) and (42) indicate that the higher the expectation order of inflation, the milder is its reaction to a policy shock.[35]

The comparative statics with respect to the information-processing parameter yields

$$\left|\frac{\partial \pi_t}{\partial k^*}\right| > 0. \tag{43}$$

Conditional on a policy rule ϕ, and for the same size of the shock, the aggregate inflation responds more on impact to the current shock, as it is easier to learn about it.

Dependence on higher-order expectations reduces the sensitivity of the *inflation expectation* to shocks, as well as the expectation of the shock. This slows down the adjustment of expectations and, given the relationship (35), it also slows down the adjustment of inflation. Hence, the degree of inflation inertia is increased. *Both* strategic complementarities in the nominal side and imperfect common knowledge about nominal values are necessary for this result. Nonetheless, the design of monetary policy plays also an important role, as discussed in the next section.

6.4 Impact of Monetary Policy on Equilibrium Inflation

Definitions (58), (59), and (64) given in Appendix A.4 can be used to determine the following relationships between both Γ_1 and Γ_4 with respect to ϕ :

$$\begin{aligned} \frac{\partial \Gamma_1(\phi)}{\partial \phi_\pi} &< 0 \;;\; \frac{\partial \Gamma_1(\phi)}{\partial \phi_y} > 0 \;;\; \frac{\partial^2 \Gamma_1(\phi)}{\partial \phi_y \partial \phi_\pi} > 0 \\ \frac{\partial \Gamma_4(\phi)}{\partial \phi_\pi} &= 0 \;;\; \frac{\partial \left|\Gamma_4(\phi)\right|}{\partial \phi_y} < 0 \;;\; \frac{\partial^2 \Gamma_4(\phi)}{\partial \phi_y \partial \phi_\pi} = 0. \end{aligned} \tag{44}$$

These signs hold for all the considered parameter values. Figure 3 (Appendix C) graphs Γ_1 for $\phi_\pi \in [1.05, 2.5]$ and $\phi_y \in [0.05, 2.5]$. The coefficient Γ_4 is an increasing and strictly concave negative function of ϕ_y.

The positive coefficient Γ_1 decreases with ϕ_π for given ϕ_y. This indicates that stronger systematic responses to inflation reduce the relevance of higher-order expectations of π_t for any price-setter.

[34] Some algebra shows that as $k^* \to 1$ this solution converges to the equilibrium under perfect information. The limit $k^* \to 0$ implies $\pi_t = 0$, the deterministic steady-state level of inflation.

[35] Woodford (2002) also provides a closed-form solution for the higher-order expectations of inflation in his model with the same property.

Strategic complementarities are weaker, resulting in a narrower channel for ICK to cause inertia.[36] Γ_1 increases with ϕ_y, conditional on ϕ_π. A higher ϕ_y makes the nominal rate more sensitive to movements in employment around its natural rate, affecting the nominal rate and hence inflation. With a more volatile inflation, higher-order expectations play a greater role. This increases the role of strategic complementarities in pricing, as measured by Γ_1.

The negative coefficient Γ_4 is unaffected by ϕ_π, but approaches zero with ϕ_y. A higher value of ϕ_y decreases the impact of a shock on inflation going through Γ_4.

Using numerical simulation, it is possible to show the following two properties of Z_t. They are crucial in understanding the impact of monetary policy on the equilibrium:

$$\frac{\partial \left| Z_t (\phi) \right|}{\partial \phi_\pi} < 0 \quad ; \quad \frac{\partial \left| Z_t (\phi) \right|}{\partial \phi_y} < 0.$$

Hence, $Z_t (\phi) < 0$ will be closer to 0 for higher values of ϕ_π and ϕ_y. First, the systematic part of policy implies a larger response of the nominal rate to inflation (larger ϕ_π) or output (larger ϕ_y), allowing smaller departures of inflation from the target. And second, with a larger ϕ_π, households understand the stronger anti-inflationary goal of policy and rely less on information coming from the behavior of others. In this case, the expectation of inflation stays closer to the target. Higher values of ϕ_y reduce the magnitude of Γ_4, implying a smaller reaction of prices to the expectation of the policy shock (see equation (35)).

7 The Phillips Curve

The Phillips Curve can be obtained starting from the expression for the equilibrium level of output, provided in the following proposition.

Proposition 3 Given $k^* \in (0,1)$ *the reduced form for the equilibrium level of output is*

$$l_t = \zeta_t \cdot \Psi_t \tag{45}$$

$$\Psi_t \equiv -\frac{a_4}{\widetilde{a}_3} \left(\phi_\pi \left(1 - (1-k^*)^t \right) - \frac{a_1}{a_4} (1-k^*)^t \right) \cdot Z_t < 0. \tag{46}$$

The proof is provided in Appendix A.6.

It is possible to express the Phillips Curve in different ways. This results from the presence of a single shock and linearity of the model. At least two Phillips Curve relationships are worth mentioning, given their economic content and role in the literature. The first is a Hybrid Phillips Curve (with both backward and forward-looking terms), and the second a Backward-Looking, Expectations-Augmented (BLEA) Phillips Curve.

The Hybrid Phillips Curve takes the form

$$\pi_t = \Omega_1 \cdot l_t + \Omega_{2,t} \cdot \pi_{t-1} + \Omega_{3,t} \cdot \pi_{t+1}$$

[36] An additional interpretation would be that a high ϕ_π reduces the effect of potential control errors or other shocks in the monetary transmission mechanism.

with $\Omega_1,\ \{\Omega_{i,t}\}_{i=2,3} > 0$ for all t and all relevant parameter values. The functional form of the coefficients is given by (82) – (84) in Appendix A.7, where there is a brief discussion on how to obtain this expression.

The analysis that follows is constrained to $\phi_\pi \in (1,\ 2.5)$ and to $\phi_y \in (0,\ 2.5)$. Coefficient Ω_1 is constant through time, but $\Omega_{2,t}$ and $\Omega_{3,t}$ are not, and all three depend on k^*. Coefficient $\Omega_1 \in (0, 0.12)$, and is strictly increasing with k^*. The dynamic behavior of $\Omega_{2,t}$ and $\Omega_{3,t}$ is given in Figures 4 and 5 as a function of k^*. These two coefficients illustrate how, with the passage of time, new information is less relevant for price-setting. Since households keep learning in every period about the persistent shock, new information has a decreasing value relative to the accumulated stock of information. Conjectures formed exclusively with past information become increasingly precise over time, which explains the increasing value of $\Omega_{2,t}$. Future inflation is related to current inflation because there is common information absorbed at t that is used for price-setting both at t and at $t+1$. The larger k^* is, the more common information is processed, increasing $\Omega_{3,t}$. Nonetheless, the importance of π_{t+1} in the Phillips Curve is extremely small when compared with π_{t-1}. While $\Omega_{2,t}$ approximates the value 0.9 $(= \delta)$ as households accumulate information, coefficient $\Omega_{3,t} \leq 0.07$ for $k^* \in (0, 1)$. Fuhrer (1997) estimates a similar Phillips Curve, obtaining a coefficient for an average of past inflation of 0.8, with 0.9 within the 5 percent confidence interval.

The BLEA Phillips Curve takes the form:

$$\pi_t = \Omega_1 \cdot l_t + \Omega_{2,t} \cdot \pi_{t-1} + \Omega^*_{3,t} \cdot \pi_{t/t-1},$$

where again $\Omega^*_{3,t} > 0$ for all relevant parameter values. The expression for this parameter is given by (85) in Appendix A.7, as a function of $(t,\ k^*,\ a,\ \phi,\ \delta,\ Z_t)$. Figure 6 displays the behavior of $\Omega^*_{3,t}$ over time after a policy shock. The forecast $\pi_{t/t-1}$ is important only for rather low values of k^* and for a short time. This illustrates the high value of new information right after the shock takes place. As information accumulates, $\pi_{t/t-1}$ basically vanishes from the equation.

7.1 Impact of Monetary Policy on Inflation Persistence

Next, I define an index of inflation persistence (or inertia) and analyze its properties.

Definition 4: *The* **Index of Inflation Inertia** *is given by*

$$\Phi\left(.\right) \equiv \frac{Corr\left(\pi_{t+j}, \pi_t\right)}{Corr\left(\zeta_{t+j}, \zeta_t\right)} = Z_{t+j}/Z_t \text{ for } j = 0, 1, 2, ... \tag{47}$$

Figure 7 presents this index as function of both time and k^*. The most important property of this index is given by the following proposition.

Proposition 4

$$\begin{aligned} \Phi\left(.\right) &> 1 \text{ for } j = 1, 2, ... \\ \Phi\left(.\right) &= 1 \text{ for } j = 0. \end{aligned} \tag{48}$$

The proof, and the full expression of the index, are both provided in Appendix A.8.

For $j = 1, 2, ...$the index also has the following relevant properties:

$$\Phi(t) > \Phi(t') \Longleftrightarrow t < t' \text{ for all } t \;\; ; \;\; \lim_{t\to\infty} \Phi(.) = 1 \tag{49}$$

$$\frac{\partial \Phi(.)}{\partial \Gamma_1} > 0 \tag{50}$$

$$\frac{\partial \Phi(.)}{\partial k^*} < 0 \;\; ; \;\; \lim_{k^*\to 1} \Phi(.) = 1 \tag{51}$$

$$\Phi(.) \quad \text{depends only on } (\Gamma_1,\ k^*,\ t). \tag{52}$$

Proposition 4 states that inflation is inertial, more persistent than the underlying shock. Nonetheless, the extent of inertia dies off monotonically over time, while the private sector learns about the policy shock (see (49)). Moreover as $t \to \infty$ it disappears completely; eventually, both the shock and inflation must return to their steady-state levels.

The partial derivative in (50) can be related to monetary policy through (44). Hence (50) indicates that stronger anti-inflationary systematic policy reduces the extent of inertia, while a stronger response to the output gap increases it. The degree of inflation inertia can be understood as a measure of the impact of imperfect common knowledge. Focusing policy on inflation brings the economy closer to the perfect information (or common knowledge) case by making heterogeneity less relevant in the equilibrium. In particular, knowing that policy keeps inflation closer to target diminish the importance of higher-order beliefs in equilibrium. This in turn translates into a lower need for learning about others' actions, and hence in smaller effective strategic complementarities in pricing. The extent to which households engage in learning is therefore endogenous and dependent on monetary policy (ϕ).

The relationship between $\Phi(.)$ and policy is shown in Figure 8, since policy affects this index only through its impact on Γ_1 (see (52)). Stronger strategic complementarities (lower ϕ_π or higher ϕ_y) lead to more inflation inertia.

Expressions (43) and (51) highlight the relationship between inflation persistence and the rigidity in the adjustments of inflation (Fuhrer and Moore (1995)). In particular, Section 6.3 establishes that a low k^* implies a small response *on impact* of inflation to the monetary policy shock. The analysis of $\Phi(.)$ shows that in that same case, there is more inflation persistence (or "stickiness" in the inflation rate).

7.2 Dynamic Behavior of the Economy: Impact of Taylor Rules and ICK

The equations used to determine the equilibrium dynamics of the economy are

$$\begin{aligned}
\zeta_t &= \delta\zeta_{t-1} \text{ for } t = 2, 3, ... \text{ with } \zeta_1 \neq 0, \text{ and } \zeta_0 = 0 \\
\pi_t &= \zeta_t \cdot Z_t \text{ with } Z_t \text{ given in } (38) \\
l_t &= \zeta_t \cdot \Psi_t \text{ with } \Psi_t \text{ given in } (46) \\
i_t &= \phi_\pi \pi_t + \phi_y l_t + \zeta_t.
\end{aligned}$$

Figure 9 displays the response of the economy to a monetary policy innovation in the first quarter, for several cognitive capacities (k^*). The series presented are the price level, inflation, output, and the federal funds rate. The shock at $t = 1$ is the same for all cases in the figure. This shock is set to increase on impact the federal funds rate by 25 basis points for the series with the lowest k^*. The Taylor rule used is the benchmark $\phi^0 = (1.5,\ 0.5)$, for which $\Gamma_1 = 0.73$.

The peak effect on the inflation rate takes place quarters after the onset of the policy shock. More "ignorance" (lower k^*) delays further the maximum impact of the policy shock on inflation. In all cases considered, the output response always peaks before the inflation response does, but never after the second quarter. For the most informationally constrained case plotted, $k^* = 0.15$, the trough of output takes place 2 quarters after the shock ($t = 3$).[37]

After the positive policy innovation, the dynamics of the federal funds rate are dominated mostly by the systematic part of policy. On impact the rate rises, due to the inertial behavior of inflation. After the onset of the shock, the Taylor rule dictates a drop in the federal funds rate, in an attempt to stabilize both output and inflation.

For the benchmark $(\phi_\pi,\ \phi_y) = (1.5,\ 0.5)$, processing capacities of at most 25 percent ($k^* = 0.25$) are required if the maximum impact on inflation has to take place no sooner than the sixth quarter after the onset of the policy shock ($t = 7$).

The following exercise allows us to better assess the impact of ICK in the equilibrium. Figure 10 compares the effects of two Taylor rules that differ only in the weight placed on the inflation rate gap. The two policies are $\phi^L = (2.5, 0.5)$, with implied $\Gamma_1 = 0.55$, and $\phi^H = (1.1, 0.5)$, with $\Gamma_1 = 0.8$. For a given shock and without accounting for the effect that monetary policy has on beliefs, policy ϕ^L should result in the largest employment gaps. The figure shows that this is in fact the other way around once ICK is introduced. Policy ϕ^H allows for more heterogeneity of beliefs, a higher degree of strategic complementarity in pricing, and therefore stronger nominal rigidities. As a result, ϕ^H implies stronger effects on output, as Figure 10 shows for different information processing capacities.

8 Conclusions

The dynamics of the rational expectations, monetary model with heterogeneous beliefs discussed here present certain realistic features, difficult to reproduce within the standard NK framework. After a monetary policy shock, the inflation response peaks after the response of output does, inflation is inertial, and can be characterized by a Hybrid Phillips Curve. Moreover, the model shows that a Taylor rule with a stronger emphasis on inflation targeting reduces the extent of nominal rigidities and inflation inertia. This type of policy approximates the behavior of the economy to that of the perfect-information equilibrium.

Monetary policy and the extent of inflation inertia are linked through the effects of policy on the effective extent of strategic complementarities in pricing. In a setting with imperfect common

[37] For this value of k^*, output increases slightly on impact. This results from the assumed low elasticity of the marginal disutility of labor. On impact, the real rate increases substantially, providing incentives to raise labor income in order to save. In equilibrium, this effect dominates the incentives to transfer resources intertemporally by reducing consumption. Closedness of the economy implies, then, that output rises, although by a very small percentage. By increasing slightly the elasticity of the disutility of labor (ν), it is possible to eliminate this effect.

knowledge (ICK) and monopolistic competition, price-setters need to form conjectures about the aggregate price level. In this way, price-setting depends on the beliefs of what others perceive. The importance of higher-order expectations for pricing reduces the size of nominal adjustments, as those expectations adjust more sluggishly to shocks. When price-setters know that policy is more aggressive in maintaining inflation around the known target, the dependence on others' beliefs is reduced. Therefore, heterogeneity has a smaller impact on the equilibrium dynamics, reducing the extent of inertia and the real effects of monetary shocks. In this way, monetary policy can shift the equilibrium closer to the perfect information case.

The ICK setup used here does not require any exogenous shocks to generate heterogeneity. Therefore, the model presents a departure from the traditional strategy adopted to sustain ICK. There are nominal rigidities and monetary policy has real effects, despite the fact that producers can adjust prices at no cost in every period. This is another key difference from the commonly assumed Calvo-pricing mechanism and from State Dependent Pricing models.

It is possible to perform a rough calibration of the amount of information processed by the private sector. In particular, the degree of absorption of relevant macroeconomic information appears to be around 25 percent.

The analysis is limited to the case in which the only state variables are the exogenous policy shock and asset holdings. The purpose of this is to better assess the impact of policy on inflation inertia. Nonetheless, possible extensions of the model include interest-rate smoothing and habit formation. These can increase the level of inertia displayed by output after a shock.

The convenience of a strong focus on inflation targeting needs to be reassessed by introducing other shocks in the model. In particular, this is so for shocks that cause output and inflation to move in opposite directions (for example, an oil price shock). In such a case, aggressive anti-inflationary policy would lead to larger output gaps, with potentially detrimental welfare effects. A more general shock structure would then allow for the analysis of the optimal monetary policy.

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A Appendices

A.1 Information-Processing Constraints and ICK

Suppose that at each point in time household h chooses to observe a signal ${}_F s_{h,t}$ about the relevant variable $F_t \sim N\left(0, \sigma_F^2\right)$. This is, ${}_F s_{h,t} = F_t + \eta_{h,t}$, where $\eta_{h,t} \sim iid$ is the household-specific observation noise.

Let ${}_F S$ and ${}_F V$ be the variance of household h's estimate of variable F_t before and after observing the signal ${}_F s_{h,t}$, respectively. These variances are constant, as the problem is stationary by assumption. Let K denote the maximum amount of information that the household can process per unit of time, measured in bits of information. The information-processing constraint that affects each household's information set is[38]

$$ln_F S - ln_F V \leq K \cdot 2 \ln(2). \tag{53}$$

A household's utility decreases with the uncertainty it faces (a smaller K), as its value function can be shown to be strictly concave. As long as information has value for the household, this expression holds with equality. Given K and ${}_F S$, the choice is ${}_F V$, the variance of the ex-post estimate of F_t. Hence, ${}_F V$ is set to the minimum value that (53) allows for.

A.2 Proof of Proposition 1

Suppose that F_t is an autocorrelated but normally distributed random variable that can only be observed subject to (53). The optimal filter used by the household to process this information is given by

$$F^h_{t/t} = (1 - k^*) F^h_{t/t-1} + k^* \left(F_t +_F \eta_{h,t}\right),$$

where again ${}_F \eta_{h,t}$ is the idiosyncratic observation noise. The parameter k^* (the *gain*) determines how new observations of F_t are incorporated into today's estimate $F^h_{t/t}$. This updated estimate is also normally distributed (see Shannon (1948) and Adam (2005) for the details).

Let σ_η^2 denote the variance of the observation error, still to be determined. The gain in the

[38] If Y is a random variable, this constraint is the result of the equation

$$Entropy_t^b(Y) - Entropy_t^a(Y) \leq K.$$

The entropy is the measure of uncertainty of a given distribution. The expression $Entropy_t^b(Y)$ denotes the uncertainty about Y before the processing of the new K bits of information absorbed in period t. The variable $Entropy_t^a(h)$ denotes the uncertainty after processing that information.

The entropy of a random variable $Y \sim N\left(0, \sigma_Y^2\right)$ takes the form:

$$Entropy(Y) = \frac{1}{2} log_2 \left(2\pi e \sigma_Y^2\right).$$

Hence, because the model is linear and the shock is normally distributed, the information processing constraint is (53).

See Shannon (1948) or Cover and Thomas (1991) for the definition and properties of the entropy of a random variable. Sims (2003) discusses the use of the concept in economic modeling.

Kalman filter is then

$$k^* =_F S \left(_F S + \sigma_\eta^2 \right)^{-1}. \tag{54}$$

After processing the new observation, the *ex-post* variance is given by

$$_F V =_F S - \frac{_F S^2}{_F S + \sigma_\eta^2}.$$

Since all information is used, the information constraint (53) is binding. Note that setting $_F V$ as a function of $_F S$ and K is equivalent to minimizing the variance of the observation noise subject to (53). Substituting the last equation into the constraint yields:

$$\sigma_\eta^2 = \frac{_F S}{e^{K \cdot 2 \ln(2)} - 1}.$$

It is possible then to compute k^* clearing σ_η^2 from equation (54), obtaining

$$k^* = 1 - e^{-K \cdot 2 \ln(2)}. \tag{55}$$

This implies that the Kalman gain is in fact an exponential distribution with argument K. Distribution functions are themselves uniformly distributed. Therefore, the same percentage of information is accumulated between equidistant values of $k^* \in (0,1)$. This measure then can be interpreted as the percentage of relevant information absorbed by the households.

■

A.3 Proof of Lemma 1

Rationality implies that each household h chooses its savings (borrowing) $b_t(h)$ knowing that the rest of the economy is saving (borrowing) in total $-b_t(h)$. Hence, expectations of order $j \geq 2$ about the economy-wide level of savings will be 0. For $t_0 \geq t$, the fact that $I_{t-1}(h) \in I_t(h)$ for all t means that

$$\begin{aligned} z_{t/t_0}^{h,(j)}(h; \mathbf{n}) &= \begin{cases} n_1 b_{t-1}(h) + n_2 b_t(h) & \text{for } j = 0, 1 \\ 0 & \text{for } j \geq 2 \end{cases} \\ \Rightarrow z_{t/t_0}^{(j)}(\mathbf{n}) &= \int_H z_{t/t_0}^{h,(j)}(h; \mathbf{n})\, dh = 0 \text{ for all } j \geq 0. \end{aligned}$$

Finally, using definitions (14), (15), and expressions in (3), one obtains

$$x_{t/t_0}^{(j)}(h; \mathbf{n}) = 0 \text{ for all } \mathbf{n} \in \mathbb{R}^4 \text{ and } j \geq 0.$$

■

A.4 Computation of Γ, Γ_1 and Γ_4

Using expressions (20) and (21) to characterize the equilibrium behavior of households, one obtains

$$\begin{aligned} \pi_t(h) &= \gamma_1 \pi^h_{t/t} + \gamma_2 \pi_{h,t} + \gamma_4 \zeta^h_{t/t} + \gamma_5 \lambda_{t-1/t}(h) + \gamma_6 \tau_{t-1/t}(h) \\ &\quad + \gamma_7 b_{t-1}(h) + \gamma_8 b_t(h) \end{aligned} \tag{56}$$

$$\begin{aligned} l^h_{t/t} &= -\frac{\widetilde{d}_1}{\widetilde{d}_3}\pi^h_{t/t} - \frac{d_2}{\widetilde{d}_3}\pi_{h,t} - \frac{d_4}{\widetilde{d}_3}\zeta^h_{t/t} - \frac{d_5}{\widetilde{d}_3}\lambda_{t-1}(h) - \frac{d_6}{\widetilde{d}_3}\tau_{t-1}(h) \\ &\quad - \frac{d_7}{\widetilde{d}_3} b_{t-1}(h) - \frac{1}{\widetilde{d}_3} b_t(h), \end{aligned} \tag{57}$$

where the coefficients are given by

$$\gamma_1 \equiv \left((a_1 + a_4\phi_\pi) - (d_1 + d_4\phi_\pi)\frac{a_3 + a_4\phi_y}{d_3 + d_4\phi_y} \right) \tag{58}$$

$$\gamma_n \equiv \left(a_n - d_n \frac{a_3 + a_4\phi_y}{d_3 + d_4\phi_y} \right) \text{ for } n \in \gamma \equiv \{2, 4, .., 7\} \tag{59}$$

$$\begin{aligned} \gamma_8 &\equiv \left(a_3 + a_4\phi_y\right) \;/\; \left(d_3 + d_4\phi_y\right) \\ \widetilde{d}_1 &\equiv d_1 + d_4\phi_\pi \\ \widetilde{d}_3 &\equiv d_3 + d_4\phi_y. \end{aligned}$$

Using definitions $(28) - (31)$, equations (56) and (57) can be expressed as

$$\pi_t(h) = \gamma_1 \pi^h_{t/t} + \gamma_2 \pi_{h,t} + \gamma_4 \zeta^h_{t/t} + x^h_{t/t}(h; \mathbf{n}^\pi) + z^h_{t/t}(h; \mathbf{n}^\pi) \tag{60}$$

$$l^h_{t/t} = -\frac{d_1}{\widetilde{d}_3}\pi^h_{t/t} - \frac{d_2}{\widetilde{d}_3}\pi_{h,t} - \frac{d_4}{\widetilde{d}_3}\zeta^h_{t/t} - x^h_{t/t}\left(h; \mathbf{n}^l\right) - z^h_{t/t}\left(h; \mathbf{n}^l\right) \tag{61}$$

for the following parameter configuration:

$$\mathbf{n}^\pi = (\gamma_i)_{i=5}^8 \text{ and } \mathbf{n}^l = \frac{1}{\widetilde{d}_3}(d_5,\ d_6,\ d_7,\ 1).$$

Averaging equations (60) and (61) over h, and taking expectations with respect to $I_t(h)$ yields for $j = 1, 2, ...$

$$\begin{aligned} \pi^{h,(j)}_{t/t} &= \gamma_1 \pi^{h,(j+1)}_{t/t} + \gamma_2 \pi^{h,(j)}_{t/t} + \gamma_4 \zeta^{h,(j+1)}_{t/t} + x^{h,(j+1)}_{t/t}(h; \mathbf{n}^\pi) \\ &\quad + z^{h,(j+1)}_{t/t}(h; \mathbf{n}^\pi) \end{aligned} \tag{62}$$

$$\begin{aligned} l^{h,(j+1)}_{t/t} &= -\frac{d_1}{\widetilde{d}_3}\pi^{h,(j+1)}_{t/t} - \frac{d_2}{\widetilde{d}_3}\pi^{h,(j)}_{t/t} - \frac{d_4}{\widetilde{d}_3}\zeta^{h,(j+1)}_{t/t} - x^{h,(j+1)}_{t/t}\left(h; \mathbf{n}^l\right) \\ &\quad - z^{h,(j+1)}_{t/t}\left(h; \mathbf{n}^l\right). \end{aligned} \tag{63}$$

It is possible to define then

$$\Gamma \equiv 1/(1-\gamma_2) \quad ; \quad \Gamma_1 \equiv \gamma_1 \cdot \Gamma \quad ; \quad \Gamma_4 \equiv \gamma_4 \cdot \Gamma. \tag{64}$$

The household-specific equations for $j=0$ are still given by (60) and (61).

For $j \geq 1$ it is possible then to write

$$\pi_{t/t}^{h,(j)} = \Gamma_1 \pi_{t/t}^{h,(j+1)} + \Gamma_4 \zeta_{t/t}^{h,(j+1)} + \Gamma \left(x_{t/t}^{h,(j+1)} \left(h; \mathbf{n}^{\pi}\right) + z_{t/t}^{h,(j+1)} \left(h; \mathbf{n}^{\pi}\right) \right).$$

Finally, note that using Lemma 1, equation (60) can be aggregated as

$$\pi_t = \Gamma_1 \pi_{t/t} + \Gamma_4 \zeta_{t/t}.$$

A.5 Proof of Proposition 2

The equilibrium level of inflation is the solution to

$$\begin{aligned} \pi_t &= \Gamma_1 \Delta_{t/t} + \Gamma_4 \zeta_{t/t} \\ \Delta_{t/t} &= \Gamma_1 E_t \left\{ \Delta_{t/t} \right\} + \Gamma_4 \zeta_{t/t} \text{ for all } t. \end{aligned} \tag{65}$$

Equation (25) provides the expression for $\zeta_{t-a/t-b}$, with $t > b \geq a$ as a function of the shock. Therefore, in order to prove Proposition 2, it is only necessary and sufficient to find a solution for $\Delta_{t/t}$. In order to do so, the following Lemma is used.

Lemma 2 For $\Gamma_1 \in (0,1)$ and $k^* \in (0,1)$, a solution for $\Delta_{t/t}$ exists, it is unique, and given by

$$\Delta_{t/t} = \mu_{1,t} \cdot \zeta_{t/t-1} + \mu_2 \cdot \zeta_t \tag{66}$$

$$\mu_{1,t} = \frac{(1-k^*)}{1 - \Gamma_1 \cdot \left(1 - (1-k^*)^t\right)} \left[\frac{\Gamma_4}{(1 - k^* \Gamma_1)} \right] < 0 \tag{67}$$

$$\mu_2 = k^* \left[\frac{\Gamma_4}{(1 - k^* \Gamma_1)} \right] < 0. \tag{68}$$

Its proof is divided in 3 steps given in subsections A.5.1-A.5.3. Subsection A.5.4 uses Lemma 2 to prove Proposition 2.

A.5.1 Proof of Lemma 2, Step 1: Prove Existence and Uniqueness of $\Delta_{t/t}$

Time subscripts are omitted where doing so cannot lead to a confusion.

Consider the probability space $\mathbb{O} = \{\mathbb{R}, \mathbb{R}_{\sigma}, O\}$, where $\mathbb{R}_{\sigma}$ is the $\sigma-$algebra defined over the real line $\mathbb{R}$, and O is a $N\left(0, \sigma^2\right)$ distribution function. Let $O_{I_t(h)}$ denote the conditional distribution with respect to the information set $I_t(h)$.

Let $\mathbb{Z}$ be the set of measurable functions with respect to $\mathbb{R}_{\sigma}$. Then, define the linear mapping $T : \mathbb{Z} \rightarrow \mathbb{Z}$ such that for $f_1 \in \mathbb{Z}$

$$\begin{aligned} f_2 &= T f_1 \\ f_2 &= \int_H \int_{\mathbb{R}} \left(\Gamma_4 \zeta_t + \Gamma_1 f_1 \right) dO_{I_t(h)} dh. \end{aligned}$$

Alternatively, the mapping T can be expressed as $f_2 = \Gamma_4 \zeta_{t/t} + \Gamma_1 E_t \{f_1\}$. I need to show that there exists a unique $\overline{f} \in \mathbb{Z}$, such that

$$\overline{f} = \Gamma_4 \zeta_{t/t} + \Gamma_1 E_t \{\overline{f}\}.$$

Note that it has already been shown in (37) that such a fixed point exists:

$$\begin{aligned} \Delta_{t/t} &= \Gamma_4 \zeta_{t/t} + \Gamma_1 E_t \{\Delta_{t/t}\} \\ \Longrightarrow \Delta_{t/t} &= T\Delta_{t/t}, \quad \text{for } \Gamma_1 \in (0,1). \end{aligned} \tag{69}$$

A tentative solution to the functional equation is found in Appendix A.5.2, and is verified as such in Appendix A.5.3 (it is shown that it solves (69)). Therefore, the conditions for Theorem 9.12 in Stokey and Lucas (1989) are met, and the solution obtained in Appendix A.5.2 is unique.[39]

A.5.2 Proof of Lemma 2, Step 2: Compute Solution for $\Delta_{t/t}$[40]

Since expression (34) is linear in the normally distributed expectations of ζ_t, the optimal filter of the household's signal is a Kalman filter. An educated guess is that the reduced form of $\Delta_{t/t}$ is linear in the shocks and its expectations. By inspecting expression (37), the educated guess for the vector of relevant variables determining $\Delta_{t/t}$ is given by

$$X_t = \begin{bmatrix} \Delta_{t/t} & \zeta_{t/t-1} & \zeta_t \end{bmatrix}^T.$$

Define then

$$X_t = M g_t \;\; where\; M = \begin{bmatrix} \mu_{1,t} & \mu_{2,t} \\ \multicolumn{2}{c}{I_2} \end{bmatrix} \;\; and \;\; g_t = \begin{bmatrix} \zeta_{t/t-1} \\ \zeta_t \end{bmatrix}, \tag{70}$$

which needs to be fulfilled in the equilibrium.

These definitions imply that

$$\Delta_{t/t} = \overline{\xi} X_{t/t} \quad \text{for } \; \overline{\xi} = [\Gamma_1,\ 0,\ \Gamma_4].$$

Therefore, the relevant filter that determines household-level learning is

$$\overline{\xi} X_{t/t} = (1-k^*)\,\overline{\xi} M g_{t/t-1} + k^* \overline{\xi} M g_t. \tag{71}$$

Using the fact that the first row of M is simply:

$$M_{1\bullet} = \begin{bmatrix} \mu_{1,t} & \mu_{2,t} \end{bmatrix},$$

it is easy to compute

[39] Note that here there is no optimization problem to consider.

[40] The solution procedure used in this step of the proof of Lemma 2 extends work in an early version of Adam (2005).

$$\bar{\xi}Mg_t = \Gamma_1\mu_{1,t}\zeta_{t/t-1} + \left(\Gamma_1\mu_{2,t} + \Gamma_4\right)\zeta_t. \tag{72}$$

Note that the conditional expectation $E_{t-1}\left\{\zeta_{t/t-1}\right\}$ is a second order expectation. Using equation (26) for $b = 1$:

$$\begin{aligned} E_{t-1}\left\{\zeta_{t/t-1}\right\} &= E_{t-1}\left\{\zeta_t\left(1-(1-k^*)^{t-1}\right)\right\} \\ &= \zeta_{t/t-1}\left(1-(1-k^*)^{t-1}\right). \end{aligned}$$

This last expression implies

$$\bar{\xi}Mg_{t/t-1} = \left[\Gamma_1\left(\mu_{1,t}\left(1-(1-k^*)^{t-1}\right)+\mu_{2,t}\right)+\Gamma_4\right]\zeta_{t/t-1}. \tag{73}$$

The initial guess (70) must be consistent with (71) plus (72) and (73). Therefore

$$\begin{aligned} \mu_{1,t} &= \{\Gamma_1\mu_1 + (1-k^*)[\Gamma_1\mu_2 + \Gamma_4]\} \\ \mu_{2,t} &= k^*\left[\Gamma_1\mu_{2,t} + \Gamma_4\right]. \end{aligned}$$

Given that $k^* \in (0,1)$, the solution to the system yields (note that $\mu_{2,t}$ is time independent):

$$\mu_{1,t} = \frac{1-k^*}{1-\Gamma_1\cdot\left(1-(1-k^*)^t\right)}\frac{\Gamma_4}{(1-k^*\Gamma_1)} < 0 \tag{74}$$

$$\mu_2 = k^*\frac{\Gamma_4}{(1-k^*\Gamma_1)} < 0. \tag{75}$$

The law of motion for this last variable is then given by

$$\Delta_{t/t} = \mu_{1,t}\cdot\zeta_{t/t-1} + \mu_2\cdot\zeta_t. \tag{76}$$

In order to complete the proof, (76) needs to satisfy the functional equation (69).

A.5.3 Proof of Lemma 2, Step 3: Verify Functional Equation.

The solution to be verified is

$$\Delta_{t/t} = \mu_{1,t}\cdot\zeta_{t/t-1} + \mu_2\cdot\zeta_t,$$

where μ_1 and μ_2 are given in (74) and (75).

It is necessary to check that this solution satisfies the recursive formulation given in (36), namely

$$\Delta_{t/t} = \Gamma_4\zeta_{t/t} + \Gamma_1 E_t\left\{\Delta_{t/t}\right\}.$$

Using this last equation and (76) yields

$$\Delta_{t/t} = \Gamma_4 \zeta_{t/t} + \Gamma_1 E_t \left\{ \mu_{1,t} \cdot \zeta_{t/t-1} + \mu_2 \cdot \zeta_t \right\}. \tag{77}$$

Expression (26) for $b = 1$ implies

$$E_t \left\{ \zeta_{t/t-1} \right\} = \zeta_{t/t-1} \left(1 - (1 - k^*)^t \right),$$

and hence (77) can be rewritten as

$$\Delta_{t/t} = \Gamma_4 \zeta_{t/t} + \Gamma_1 \left[\mu_{1,t} \cdot \left(1 - (1 - k^*)^t \right) \cdot \zeta_{t/t-1} + \mu_2 \cdot \zeta_{t/t} \right].$$

Some algebra shows that

$$\Delta_{t/t} = \frac{\Gamma_4}{1 - k^* \Gamma_1} \zeta_{t/t} + \Gamma_1 \mu_{1,t} \cdot \left(1 - (1 - k^*)^t \right) \cdot \zeta_{t/t-1},$$

which, according to the learning rule used to form the expectation $\zeta_{t/t}$, can be expressed as

$$\Delta_{t/t} = \frac{\Gamma_4}{1 - k^* \Gamma_1} \left[(1 - k^*) \cdot \zeta_{t/t-1} + k^* \cdot \zeta_t \right] + \Gamma_1 \mu_{1,t} \cdot \left(1 - (1 - k^*)^t \right) \cdot \zeta_{t/t-1}.$$

This last equation can be simplified:

$$\Delta_{t/t} = \left[\frac{\Gamma_4 (1 - k^*)}{1 - k^* \Gamma_1} + \Gamma_1 \mu_{1,t} \cdot \left(1 - (1 - k^*)^t \right) \right] \zeta_{t/t-1} + \mu_2 \cdot \zeta_t.$$

Some algebra shows that

$$\left[\frac{\Gamma_4 (1 - k^*)}{1 - k^* \Gamma_1} + \Gamma_1 \mu_{1,t} \cdot \left(1 - (1 - k^*)^t \right) \right] = \mu_{1,t}$$

and therefore

$$\Delta_{t/t} = \mu_{1,t} \zeta_{t/t-1} + \mu_2 \cdot \zeta_t,$$

which means that the conjectured solution for $\Delta_{t/t}$ satisfies the functional equation (69), completing the proof of Lemma 2.

■

A.5.4 Final Part of Proof of Proposition 2: Equilibrium π_t:

It suffices to substitute the solution to Lemma 2, expression (66), into (65), obtaining:

$$\pi_t = \Gamma_1 \left(\mu_{1,t} \zeta_{t/t-1} + \mu_2 \zeta_t \right) + \Gamma_4 \zeta_{t/t}. \tag{78}$$

Using (25) in this last equation yields

$$\pi_t = \zeta_t \cdot \left[\Gamma_1 \ \left(\mu_{1,t} \left(1 - (1 - k^*)^{t-1} \right) + \mu_2 \right) + \Gamma_4 \left(1 - (1 - k^*)^t \right) \right],$$

which proves Proposition 2.

■

A.6 Proof of Proposition 3: The Equilibrium Level of Output

Households form expectations about $l_{t/t}^h$, optimally filtering their perceptions. After aggregation of the households' filtering rules for l_t, and rearranging the terms of the equation:

$$l_t = \frac{1}{k^*}\left[l_{t/t} - (1-k^*)\, l_{t/t-1}\right]. \tag{79}$$

Aggregating equation (20) under an ICK information structure yields:

$$l_{t/t} = -\frac{a_1 + a_4\phi_\pi}{\widetilde{a}_3}\pi_{t/t} - \frac{a_2 - 1}{\widetilde{a}_3}\pi_t - \frac{a_4}{\widetilde{a}_3}\zeta_t. \tag{80}$$

The reduced forms of the equilibrium level and any expectation of l_t must be linear in ζ_t, as the model is itself linear.

Therefore, using Lemma 2 (Appendix A.5),

$$l_{t/t-1} = \frac{a_4\phi_\pi}{\widetilde{a}_3}\pi_{t/t-1} - \frac{a_4}{\widetilde{a}_3}\zeta_{t-1}, \tag{81}$$

where the fact that $a_1 + a_2 = 1$ has been used (see Table 1).

Next, substitute (80) and (81) in (79).

Finally, use the resulting expression with equations (25) and (42) to express l_t as a linear function of ζ_t.

■

A.7 Obtaining the Phillips Curves

The Hybrid Phillips Curve can be found starting out from (80) after imposing ICK.

Next substitute away ζ_t using (25) and (42). Then obtain π_{t+1} as one of the right-hand-side variables in (80) by using Proposition 2.

Taking expectations of equation (45), conditional on information at t, substitute $l_{t/t}$. Next, replace $\zeta_{t/t}$ by a linear function of ζ_t using (25). Finally, rearrange the terms in the resulting expression.

The HPC can be rewritten as a BLEA-PC by exploting the relationship between π_{t+1} and $\pi_{t/t-1}$ provided by equation (42) and Proposition 2 (compare the expressions of $\Omega_{3,t}$ and $\Omega_{3,t}^*$ below).

The coefficients in the two Phillips Curves discussed in the main text are:

$$
\begin{aligned}
\Omega_1 &= \frac{\widetilde{a}_3 k^*}{a_1 - \phi_\pi a_4 k^*} > 0 && (82)\\
\Omega_{2,t} &= \Omega_{1,t} \cdot \delta \cdot \frac{a_1 \left(1 - (1-k^*)^t\right)}{\widetilde{a}_3 k^*} > 0 && (83)\\
\Omega_{3,t} &= \Omega_{1,t} \cdot \frac{a_4}{\widetilde{a}_3 \delta Z_{t+1}} > 0 && (84)\\
\Omega_{3,t}^* &= \Omega_{3,t} \cdot \frac{a_4}{\widetilde{a}_3 Z_t \cdot \left(1 - (1-k^*)^{t-1}\right)} > 0, && (85)
\end{aligned}
$$

where Z_t and Z_{t+1} are defined in (38).

A.8 Proof of Proposition 4: The Index of Inflation Inertia.

The case for $j = 0$ is trivial. For $j = 1, 2, ...$

$$
\begin{aligned}
&\left\{ \frac{\Gamma_1}{1 - k^*\Gamma_1} \cdot \left[\frac{(1-k^*) \cdot \left(1 - (1-k^*)^{t-1+j}\right)}{1 - \Gamma_1 \cdot \left(1 - (1-k^*)^{t+j}\right)} + k^* \right] + \left[1 - (1-k^*)^{t+j}\right] \right\} \\
&> \left\{ \frac{\Gamma_1}{1 - k^*\Gamma_1} \cdot \left[\frac{(1-k^*) \cdot \left(1 - (1-k^*)^{t-1}\right)}{1 - \Gamma_1 \cdot \left(1 - (1-k^*)^{t}\right)} + k^* \right] + \left[1 - (1-k^*)^{t}\right] \right\} > 0,
\end{aligned}
$$

which implies

$$
\begin{aligned}
\Phi(.) &= \frac{Z_{t+j}}{Z_t} && (86)\\
&= \frac{\left\{ \frac{\Gamma_1}{1-k^*\Gamma_1} \cdot \left[\frac{(1-k^*)\cdot\left(1-(1-k^*)^{t-1+j}\right)}{1-\Gamma_1\cdot\left(1-(1-k^*)^{t+j}\right)} + k^* \right] + \left[1 - (1-k^*)^{t+j}\right] \right\}}{\left\{ \frac{\Gamma_1}{1-k^*\Gamma_1} \cdot \left[\frac{(1-k^*)\cdot\left(1-(1-k^*)^{t-1}\right)}{1-\Gamma_1\cdot\left(1-(1-k^*)^{t}\right)} + k^* \right] + \left[1 - (1-k^*)^{t}\right] \right\}} > 1 && (87)
\end{aligned}
$$

for $j = 1, 2, ...$ Note that Γ_4 does not enter the expression of $\Phi(.)$.

Hence,

$$\Phi(.) > 1 \text{ for } j = 1, 2, ...$$

■

B Tables

TABLE 1

	Parameters			Household's Pricing Policy Function						
Row	**ρ**	**θ**	***v***	**a1**	**a2**	**a3**	**a4**	**a5**	**a6**	**a7**
1	0.5	4	2	0.9853	0.0147	0.2320	-0.0683	0.0039	-0.9853	-0.0147
2	0.5	4	5	0.9936	0.0064	0.2436	-0.0240	0.0015	-0.9936	-0.0064
3	0.5	6	2	0.9904	0.0096	0.1584	-0.0468	0.0026	-0.9904	-0.0096
4	0.5	6	5	0.9957	0.0043	0.1638	-0.0162	0.0010	-0.9957	-0.0043
5	0.5	8	2	0.9929	0.0071	0.1203	-0.0355	0.0020	-0.9929	-0.0071
6	0.5	8	5	0.9968	0.0032	0.1234	-0.0122	0.0007	-0.9968	-0.0032
7	1	4	2	1.0359	-0.0359	0.2329	-0.0683	0.0070	-1.0359	0.0359
8	1	4	5	1.0126	-0.0126	0.2440	-0.0240	0.0028	-1.0126	0.0126
9	1	6	2	1.0246	-0.0246	0.1589	-0.0468	0.0047	-1.0246	0.0246
10	1	6	5	1.0085	-0.0085	0.1640	-0.0162	0.0019	-1.0085	0.0085
11	1	8	2	1.0187	-0.0187	0.1206	-0.0355	0.0035	-1.0187	0.0187
12	1	8	5	1.0064	-0.0064	0.1235	-0.0122	0.0014	-1.0064	0.0064
13	2	4	2	1.1131	-0.1131	0.2344	-0.0683	0.0118	-1.1131	0.1131
14	2	4	5	1.0467	-0.0467	0.2446	-0.0240	0.0052	-1.0467	0.0467
15	2	6	2	1.0751	-0.0751	0.1595	-0.0468	0.0077	-1.0751	0.0751
16	2	6	5	1.0310	-0.0310	0.1643	-0.0162	0.0034	-1.0310	0.0310
17	2	8	2	1.0563	-0.0563	0.1209	-0.0355	0.0057	-1.0563	0.0563
18	2	8	5	1.0232	-0.0232	0.1236	-0.0122	0.0026	-1.0232	0.0232

ρ - Coefficient of Relative Risk Aversion.
θ - Elasticity of Demand.
***v* -** Elasticity of Disutility of Labor.

TABLE 2

	Parameters			Household's Savings Policy Function						
Row	ρ	θ	*v*	d1	d2	d3	d4	d5	d6	d7
1	0.5	4	2	-0.5976	0.5976	0.2845	2.0625	0.9246	0.5976	-0.5976
2	0.5	4	5	-0.5249	0.5249	0.2499	1.9241	0.9246	0.5249	-0.5249
3	0.5	6	2	-0.6133	0.6133	0.1946	2.0924	0.9246	0.6133	-0.6133
4	0.5	6	5	-0.5297	0.5297	0.1681	1.9332	0.9246	0.5297	-0.5297
5	0.5	8	2	-0.6215	0.6215	0.1479	2.1080	0.9246	0.6215	-0.6215
6	0.5	8	5	-0.5321	0.5321	0.1267	1.9378	0.9246	0.5321	-0.5321
7	1	4	2	-0.5976	0.5976	0.2845	1.1379	0.9246	0.5976	-0.5976
8	1	4	5	-0.5249	0.5249	0.2499	0.9995	0.9246	0.5249	-0.5249
9	1	6	2	-0.6133	0.6133	0.1946	1.1679	0.9246	0.6133	-0.6133
10	1	6	5	-0.5297	0.5297	0.1681	1.0086	0.9246	0.5297	-0.5297
11	1	8	2	-0.6215	0.6215	0.1479	1.1834	0.9246	0.6215	-0.6215
12	1	8	5	-0.5321	0.5321	0.1267	1.0132	0.9246	0.5321	-0.5321
13	2	4	2	-0.5976	0.5976	0.2845	0.6756	0.9246	0.5976	-0.5976
14	2	4	5	-0.5249	0.5249	0.2499	0.5372	0.9246	0.5249	-0.5249
15	2	6	2	-0.6133	0.6133	0.1946	0.7056	0.9246	0.6133	-0.6133
16	2	6	5	-0.5297	0.5297	0.1681	0.5463	0.9246	0.5297	-0.5297
17	2	8	2	-0.6215	0.6215	0.1479	0.7212	0.9246	0.6215	-0.6215
18	2	8	5	-0.5321	0.5321	0.1267	0.5509	0.9246	0.5321	-0.5321

ρ - Coefficient of Relative Risk Aversion.
θ - Elasticity of Demand.
***v* -** Elasticity of Disutility of Labor.

C Figures

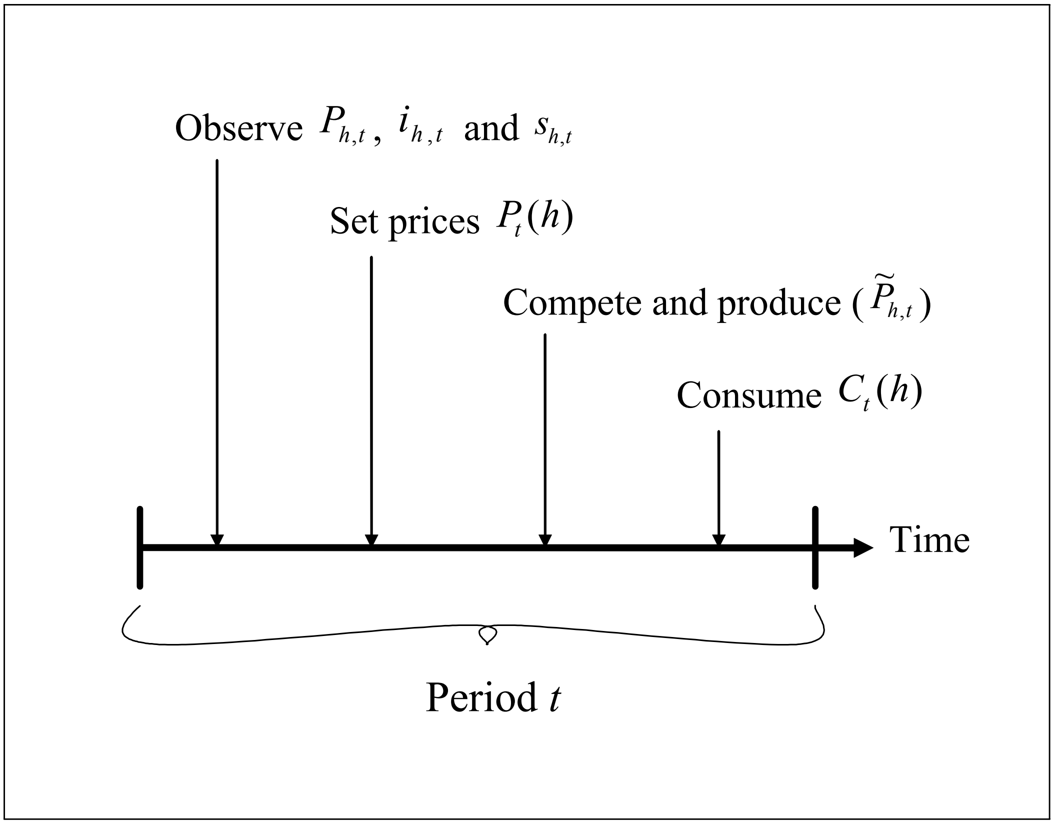


FIGURE 1

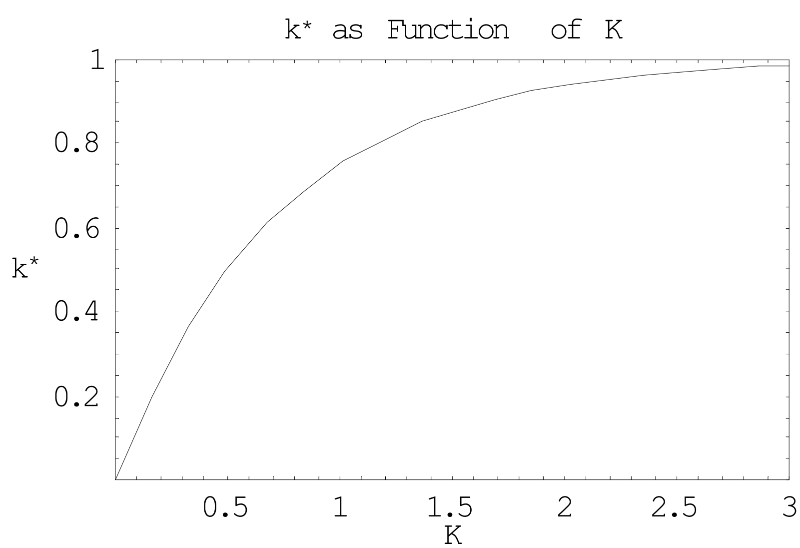


FIGURE 2

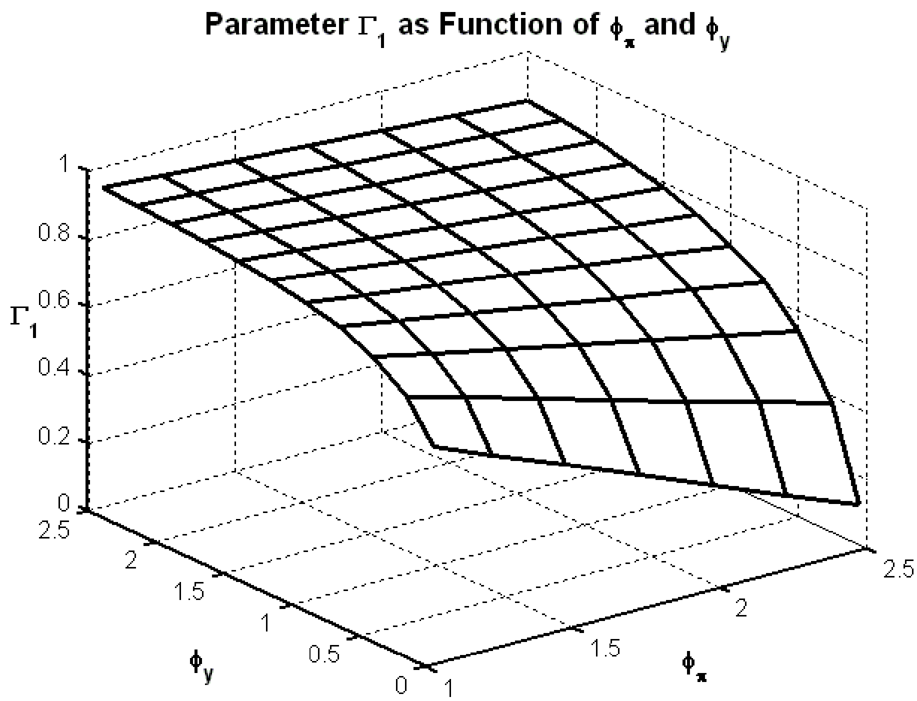


FIGURE 3

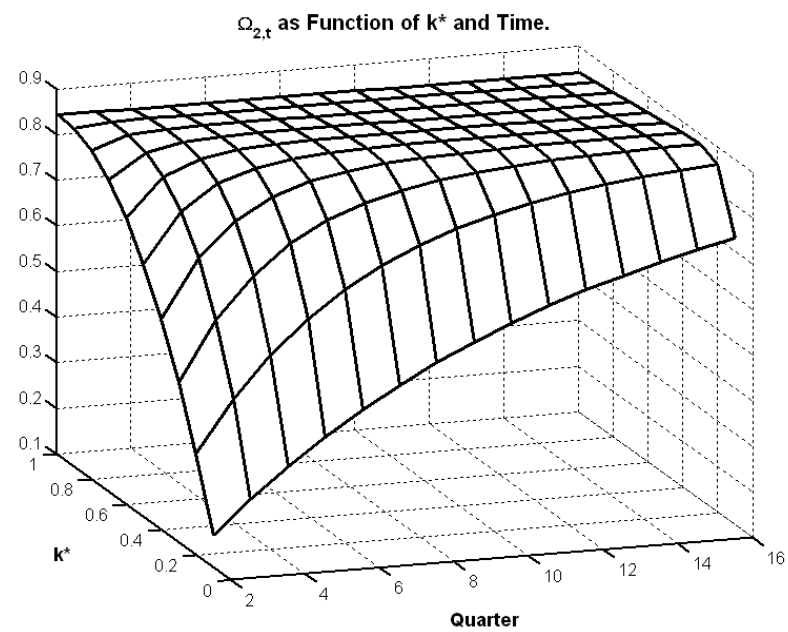


FIGURE 4

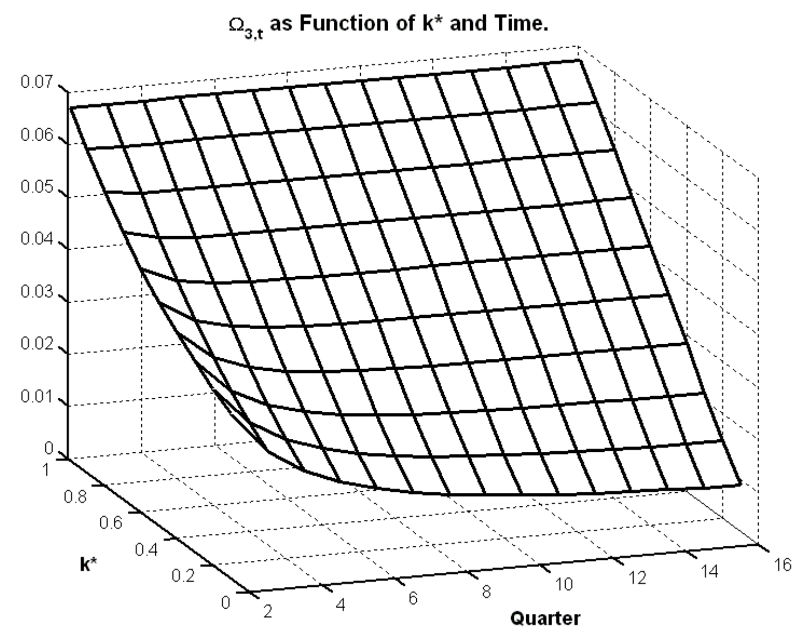


FIGURE 5

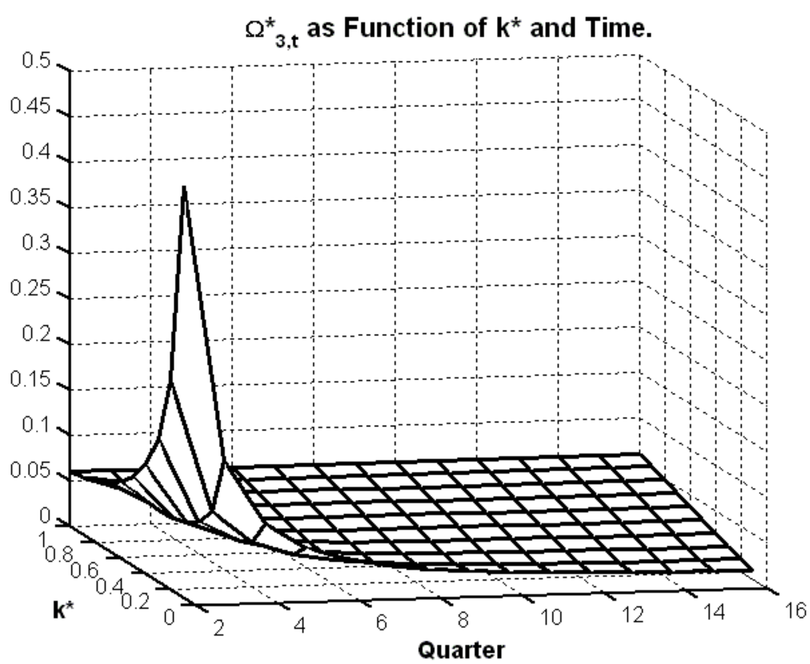


FIGURE 6

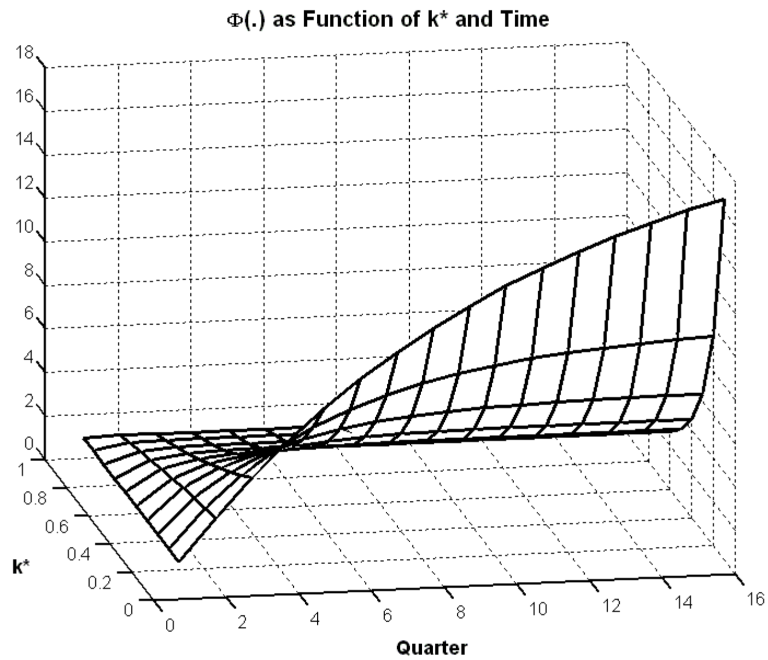


FIGURE 7

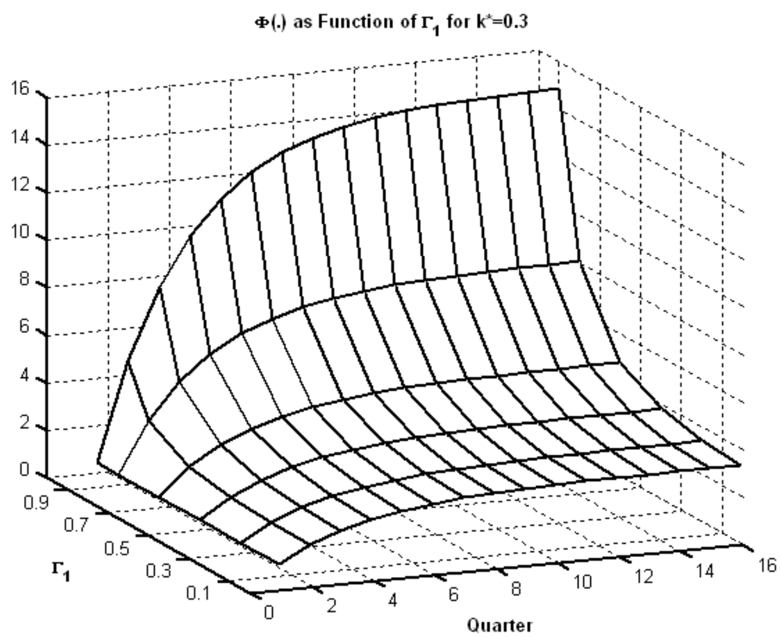


FIGURE 8

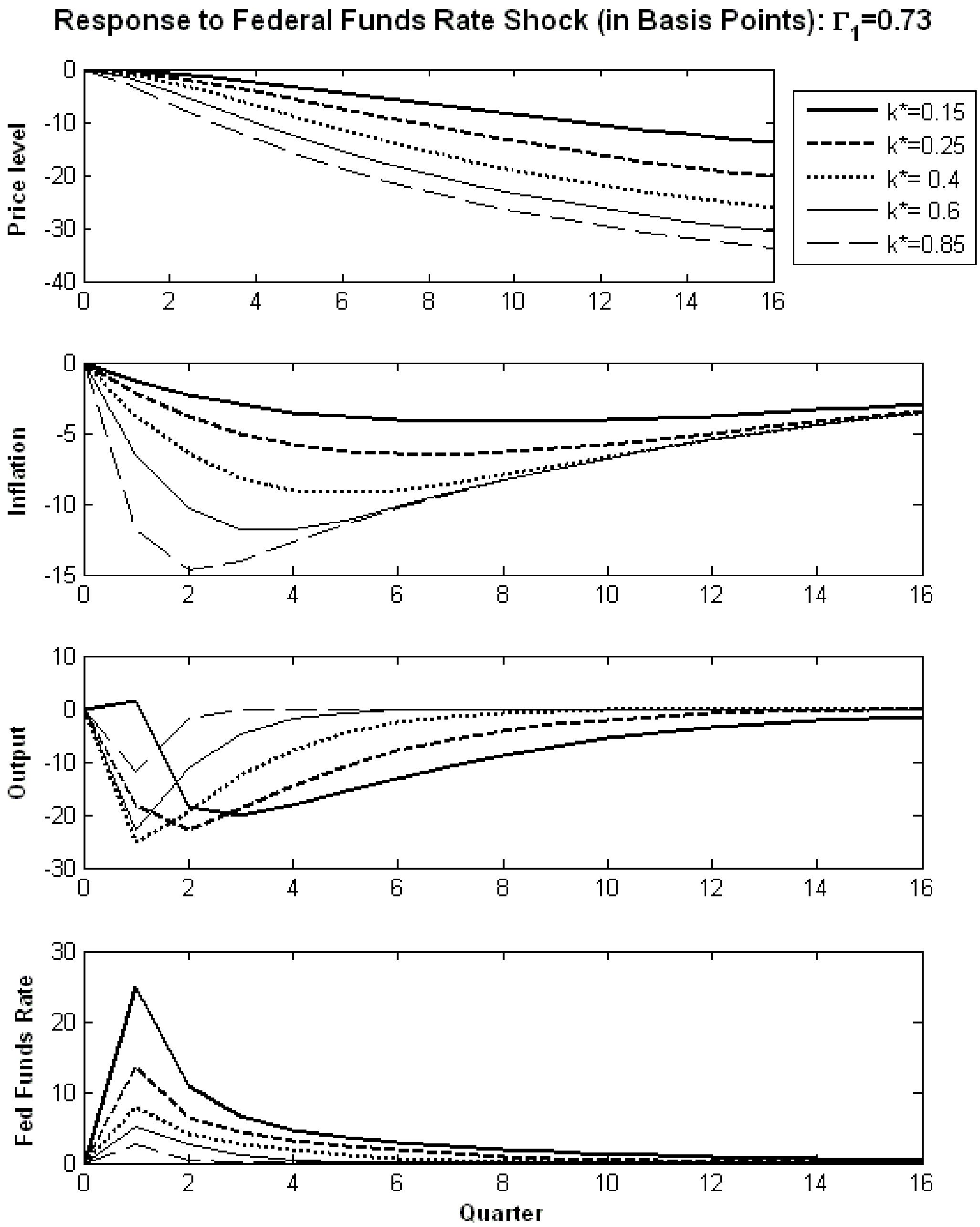
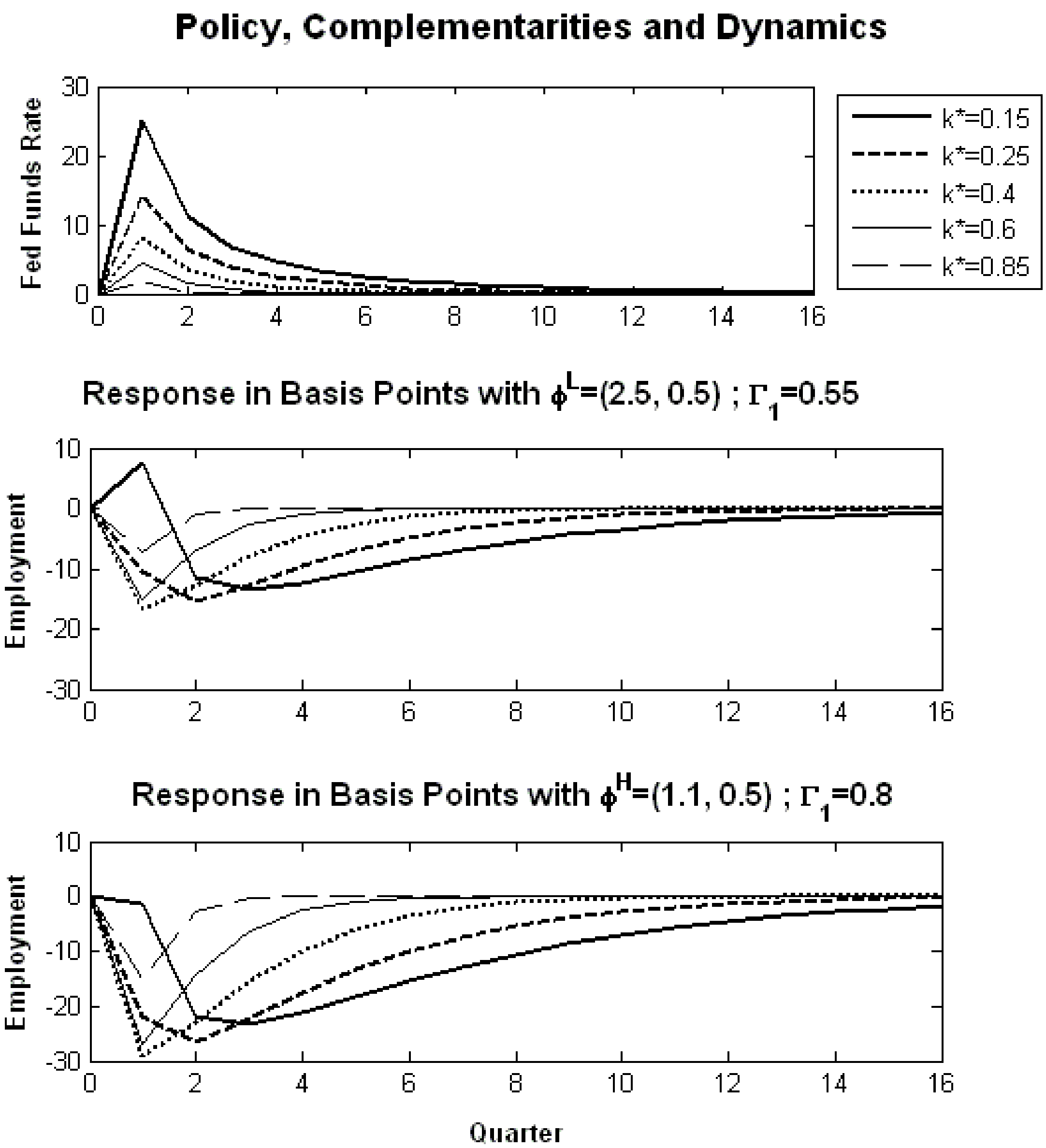

Response to Federal Funds Rate Shock (in Basis Points): Γ_1=0.73
Price level
Inflation
Output
Fed Funds Rate
Quarter
k*=0.15
k*=0.25
k*= 0.4
k*= 0.6
k*=0.85

FIGURE 9



Policy, Complementarities and Dynamics
Fed Funds Rate
k*=0.15
k*=0.25
k*=0.4
k*=0.6
k*=0.85
Response in Basis Points with ϕ^L=(2.5, 0.5) ; Γ_1=0.55
Employment
Response in Basis Points with ϕ^H=(1.1, 0.5) ; Γ_1=0.8
Employment
Quarter

FIGURE 10