A Quantile Regression Analysis of the Cross Section of Stock Market Returns

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JEL Classifications: G12; C14; C21.

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1. Introduction

Classic approaches to modelling returns and testing the Capital Asset Pricing Model (CAPM) of Sharpe (1964), Lintner (1965), Mossin (1966) and Black (1972) include the two-pass method of Fama and MacBeth (1973), the generalized method of moments (GMM) approach of Harvey (1989), and MacKinlay and Richardson (1991) and the seemingly unrelated regression (SUR) approach of Gibbons (1982) and Gibbons, Ross, and Shanken (1989). All of these methods and their embellishments effectively model returns and test the CAPM at the mean of the conditional distribution of returns; that is, they model and test assets that behave as the mean would predict that they behave.

It is in the second, cross-sectional pass of the Fama and MacBeth (1973) framework that most researchers have tried to improve upon the traditional method. The innovation of the present paper is to use the quantile estimator developed by Koenker and Bassett (1978), and popularized, in part, by Buchinsky (1998), in the second stage of the Fama and MacBeth (FM) regressions. By so doing, it is possible to model the relationship between returns and beta not just for those firms that behave according to the mean of the conditional distribution, but also for firms that overperform and underperform relative to the mean. In this sense, not only are we evaluating the CAPM relationship at different points in the conditional distribution, we are also exploring why conditional mean approaches have yielded ambiguous results regarding the impact of beta on returns.[2] Quantile regression methodology provides a way of understanding and testing how the relationship between returns and other conditioning variables or risk factors changes across the distribution of conditional returns, it is these changes that are our primary focus here. We demonstrate that the marginal effects differ greatly, both across firm size and conditional quantile. In so doing, we add new evidence to the debates on the relationships between beta and returns, size and returns, and financial paper and returns.

A fortunate by-product of using this approach is that we can address some of the statistical problems that have plagued the literature. The standard problem faced by the two-pass method is errors-in-variables (EIV).[3] Starting with Fama and MacBeth (1973), studies in the two-pass tradition try to solve the EIV problem by grouping the firms into portfolios[4]

[2] If the coefficient on beta is of opposite sign at opposite ends of the distribution of conditional returns, then at some point in the distribution the coefficient will pass through zero. In fact, we show that this is the case in the empirical example considered here, and that the zero point is around the center of the distribution of conditional returns, underscoring the fact that looking at just the conditional mean can sweep a lot of interesting economic relationships under the carpet. In this case, it can lead us to conclude, as much of the literature has concluded, that beta is insignificant, whereas it is statistically significant for under- and over-performing firms.

[3] This approach performs time-series estimation of beta in the first step, and then uses these estimates in the second stage cross-sectional regression of returns on the time-series estimates of beta along with any other variables thought to be correlated with the cross section of returns.

[4] The rationale for this procedure is that if the errors in the betas for the different firms are not perfectly

per Blume's (1970) suggestion.[5] Shanken (1992) derives an asymptotically valid correction to the precision of the Fama and MacBeth (FM) estimates of the price of beta risk due to the EIV problem.[6] Both the approaches of Gibbons (1982) and Harvey (1989) were developed in large part to ameliorate the EIV problem by simultaneously exploiting the time-series and cross-sectional dimensions of the data. In practice, however, most applications using the GMM and maximum likelihood-based methodologies also use the grouping procedures popularized by Fama and MacBeth (1973). Like ordinary least squares (OLS), the quantile regression estimator is inconsistent under EIV. Though the statistical properties of the quantile estimator are not the focus of the present paper, we motivate its use by presenting the results of a small Monte Carlo.[7] We demonstrate that the effect of EIV on quantile regression and OLS is similar - estimates are invariably biased towards zero. Shrinkage of the estimates in this way will have the effect of masking any interquantile variation, implying that attenuation will force us to be more conservative in our analysis than strictly necessary. Since we are comparing results across quantiles, it is important, however, that any estimator bias be uniform across the distribution. We show that in small samples the bias is not uniform, but it becomes so as the sample size increases, suggesting that we could benefit from pooling the cross sections and using panel data techniques. We use these results to justify our divergence from the literature's practice of using portfolio grouping which dramatically reduces the available number of observations. Any aggregation bias is also avoided when using individual firm data.

Another statistical problem faced by these types of studies is that cross-sectional regressions with stock returns as the dependent variable are susceptible to heteroskedastic and correlated errors. Fama and MacBeth (1973) suggest using a simple average of rolling betas and associated t-statistics estimated from data prior to each cross-sectional regression to address these issues.[8] Ferson and Harvey (1999) improve the Fama and MacBeth (1973) approach by developing an efficient weighting scheme. Jagannathan and Wang (1998) develop the asymptotic distribution of the FM estimators without assuming conditional homoskedasticity and show that when this assumption is violated, the FM standard errors

correlated, the betas of the portfolios could provide more precise estimates of true beta than the betas for individual securities due to noise cancellation.

[5] Litzenberger and Ramaswamy (1979) recommend an errors-in-the-variables regression model as a substitute for the grouping procedures typically used in this literature.

[6] Jagannathan and Wang (1996) follow Shanken (1992) and develop a formula for correcting the EIV bias in a GMM framework.

[7] A result of the experiment, of interest to the statistical literature on quantile regression is that, in small samples, the median regression estimator (a special case of quantile regression) has less attenuation bias than OLS. This result demands further attention.

[8] Shanken (1992) discusses the properties of this two-pass method using OLS and generalized least squares (GLS), and shows that the GLS version is asymptotically equivalent to the Gauss-Newton maximum-likelihood approach of Gibbons (1982). Hence, the linearization required to use the Gauss-Newton estimator leaves the SUR approach of Gibbons (1982) vulnerable to the EIV criticism.

do not necessarily overstate the precision of the estimates. Ahn and Gadarowski (1999) develop a minimum distance estimation framework for which the two-pass estimator is robust to conditional heteroskedasticity and/or autocorrelation in asset returns and compare their estimator to generalized least squares (GLS) and maximum likelihood (ML). While the quantile regression estimator is not robust to heteroskedasticity per se it can be used to help elucidate the nature of any heteroskedasticity that may be present, indeed the quantile regression estimator has been employed in the service of a test for heteroskedasticity that is robust to departures from Gaussianity (Koenker and Bassett (1982), Buchinsky (1998)). The existence of such a test may lead one to the conclusion that any differences found in regressions at different quantiles are merely due to heteroskedasticity. As Buchinsky (1998) asserts, however, "...potentially different solutions at distinct quantiles may be interpreted as differences in the response of the dependent variable to changes in the regressors at various points in the conditional distribution of the dependent variable...", implying that it is possible to interpret changing coefficients across the distribution as the result of systematic differences in firm behavior. Another heteroskedasticity related concern is obtaining estimates of the variance covariance matrix that are robust under nonstandard error distributions. Following Buchinsky (1998), we choose the design matrix bootstrap estimator which is robust to dependencies between the regressors and the regression errors.

Most studies implicitly assume that the error distribution is Gaussian.[9] Since quantile estimators may be more efficient than OLS when the distribution is nonnormal (Buchinsky (1998)), they may be more appropriate than LS-based methods in the context of stock market returns for which the normality assumption may not be appropriate.

Quantile methods can be useful for thinking about omitted variables bias too. One interpretation of the quantile is that it identifies points in the conditional distribution where omitted variables are favorably and/or unfavorably influencing returns. For example, if a firm falls in the bottom tail of the conditional distribution, this could imply that the combination of factors that were omitted from the model meant that the firm performed worse than predicted by the factors that were included in the model. One may think of these omitted factors as representing idiosyncratic shocks, or as the receipt of bad news during the sample period by firms located in the lower quantiles. By using quantile regression, we are therefore exploring whether the market prices firms' underlying characteristics consistently given different degrees of good versus bad news[10].

[9] The use of portfolio grouping techniques ensures that sample sizes are moderate at best, implying that asymptotic approximations may be poor.

[10] A nice interpretation of the quantile would be that it indicates firms with "good" versus "bad" managers. We find, however, that there is very little intertemporal persistence in the location of firms in the conditional distribution, implying that firms tend not to consistently under- or overperform in the context used here. Too, idiosyncratic shocks likely influence idiosyncratic volatility which Campbell, Lettau, Malkiel, and Xu (2001) and Goyal and Santa-Clara (2002) find to be a major component of the volatility of individual stock returns. For these reasons, we prefer the interpretation provided here.

The traditional CAPM is built on the theory that idiosyncratic risk is irrelevant when pricing assets; in this way, the construction of interquantile tests, or tests of whether, say, the 10% and 90% quantile regression coefficients on beta significantly differ, can be loosely interpreted as a test of CAPM across the entire conditional distribution. If the coefficients on beta are different, then the market price of beta risk is different for firms that under- or overperform, or for firms that receive bad versus good news. If the coefficient on beta differs across the conditional distribution, we cannot claim that beta conditionally explains the entire cross section of returns in a linear fashion.[11] Note that using methods that estimate these relationships at the conditional mean, i.e. LS-based methods, implicitly assumes a constant price of beta risk across firms, as CAPM theory would predict. Our main focus, however, will be in analyzing how the relationship between risk factors and returns changes across the conditional distribution. In this context, we will also ask the standard question of whether the market price of beta risk is statistically significant, but we will focus on the question for particular points of the conditional distribution, as opposed to looking collectively at the entire distribution, or just at the mean.

A host of firm or portfolio-specific and marketwide conditional variables and risk factors have been put forward in the literature to test CAPM. These include firm size (Banz 1981, Berk 1995 and Kothari, Shanken, and Sloan 1995), bid-ask spread (Amihud and Mendelson 1986, 1989), leverage (Bhandari 1988), book to market value (Chan, Hamao, and Lakonishok 1991) and marketwide phenomena (Ferson and Harvey 1999) among other variables. Although sometimes marketwide factors are aggregated into a single variable (Ferson and Harvey (1999)), in this paper they are disaggregated in order to better understand both what may cause CAPM to fail at different points of the conditional distribution of returns and the relationship between returns and these variables over the entire distribution. Here, the empirical CAPM includes both marketwide and firm-specific conditional variables.

Results from the interquantile tests indicate that the coefficients in the cross-sectional regression are indeed significantly different from each other at the 10% and 90% quantiles. Beta is a strongly significant cross-sectional explanatory variable for firms that underperform and overperform vis-à-vis the mean, but is insignificant for firms with average performance. This illustrates that one should not summarize an entire distribution of conditional returns by just one point, what Koenker (2000) calls "throwing the mountains into the sea", since we lose much of the informational content of the distribution. Economic relationships that are insignificant at the mean may be highly significant over other parts of the conditional distribution of the dependent variable. This may also explain why the literature has achieved such mixed results on the relationship between beta and returns.

[11] Technically, the CAPM statement is a statement about the conditional mean of the cross-section of returns, and not a statement about how the relationship should behave at other parts of the conditional distribution of returns, which is why we claim that the test of interquantile variation can be "loosely" interpreted as a test of the CAPM.

The interquantile relationship between average returns and beta differs markedly for large and small firms, with large under- (over-) performing firms having a significantly positive (negative) coefficient on beta and small under- (over-) performing firms having a significantly negative (positive) coefficient on beta. Large firms have smaller coefficients on beta (in absolute value) than small firms too. The market price of beta risk crosses the zero axis around the center of the distribution, corroborating some of the existing evidence on the (ir)relevance of beta. Many of the marketwide variables considered here are statistically significant at least over some quantiles; as with the firm-specific variables, the coefficients tend to take on both negative and positive values, depending on the quantile of interest.

Some of the differences between our results and those found in the literature may arise because we consider daily, firm-level data instead of monthly or quarterly portfolio-level data. We assert that since the quantile estimates change so dramatically across the distribution, it is unlikely that mere data differences could be solely responsible. Too, at the center of the conditional distribution of returns, we do obtain results similar to the existing literature. The quantile regression methodology enables an entirely fresh perspective on the standard types of relationships studied in this literature. Indeed, we uncover several results that are at odds with past empirical studies that focused exclusively on the mean.

The paper is organized as follows. In Section 2, the relevant theory of quantile regression is presented and testing issues are discussed. Section 3 details the empirical example that forms the main focus of the paper. The small sample bias properties of the quantile and least absolute deviation regression estimators in the presence of EIV are examined in Section 4. This section also includes a discussion of why daily individual firm data are employed in this study instead of the standard monthly portfolio data. Section 5 discusses the empirical findings and the consequences of this research for the theoretical and empirical CAPM literature, while the final section concludes the discussion.

2. Quantile Regression

Quantile regression, developed by Koenker and Bassett (1978), is an extension of the classical least squares estimation of the conditional mean to a collection of models for different conditional quantile functions. As the median (quantile) regression estimator minimizes the symmetrically weighted sum of absolute errors (where the weight is equal to 0.5) to estimate the conditional median (quantile) function, other conditional quantile functions are estimated by minimizing an asymmetrically weighted sum of absolute errors, where the weights are functions of the quantile of interest.[12] Thus, quantile regression is robust to the presence of outliers. This technique has been used widely in the past decade in many areas of applied econometrics; applications include investigations of wage structure

[12] Koenker and Hallock's (2001) survey on quantile regression provides an excellent discussion of the intuition behind this class of estimators.

(Buchinsky and Leslie 1997), earnings mobility (Eide and Showalter 1999; Buchinsky and Hunt 1996), and educational attainment (Eide and Showalter 1998). Financial applications include Engle and Manganelli (1999) and Morillo (2000) to the problems of Value at Risk and option pricing respectively, but to date the application of Koenker and Bassett's (1978) method to the cross section of stock market returns has not been considered.

The general quantile regression model, as described by Buchinsky (1998), is

$$y_i = x_i' \beta_\theta + u_{\theta i},$$

or, alternatively,

$$\theta = \int_{-\infty}^{x_i' \beta_\theta} f_y(s|x_i) ds,$$

where β_θ is an unknown $k \times 1$ vector of regression parameters associated with the θ^{th} percentile, x_i is a $k \times 1$ vector of independent variables, y_i is the dependent variable of interest and $u_{\theta i}$ is an unknown error term. The θ^{th} conditional quantile of y given x is $Quant_\theta(y_i|x_i) = x_i' \beta_\theta$. Its estimate is given by $x_i' \widehat{\beta}_\theta$. As θ increases continuously, the conditional distribution of y given x is traced out. Although many of the empirical quantile regression papers assume that the errors are independently and identically distributed (i.i.d.), the only necessary assumption concerning $u_{\theta i}$ is

$$Quant_\theta(u_{\theta i}|x_i) = 0,$$

that is, the conditional θ^{th} quantile of the error term is equal to zero. Thus, the quantile regression method involves allowing the marginal effects to change for firms at different points in the conditional distribution by estimating β_θ using several different values of θ, $\theta \in (0, 1)$. It is in this way that quantile regression allows for parameter heterogeneity across different types of assets.

Thus, the quantile regression estimator can be found as the solution to the following minimization problem:

$$\widehat{\beta}_\theta = \arg\min_\beta \left(\sum_{i:y_i > x_i'\beta} \theta \left| y_i - x_i'\beta \right| + \sum_{i:y_i < x_i'\beta} (1-\theta) \left| y_i - x_i'\beta \right| \right),$$

i.e., by minimizing a weighted sum of the absolute errors, where the weights are symmetric for the median regression case ($\theta = 0.5$) and asymmetric otherwise. This minimization can be formulated either as a linear programming or as a GMM problem. The former implies that the method is computationally straightforward while the latter implies that

$$\sqrt{n} \left(\widehat{\beta}_\theta - \beta_\theta \right) \xrightarrow{d} N(0, \Omega_\theta),$$

so tests can be constructed using critical values from the normal distribution with asymptotic justification. A plot of $\widehat{\beta}_\theta$ against θ is called the quantile plot. If this indicates significant variation, it implies that the effect of the x_i variable changes as the conditional performance of the stock improves. Note that all data observations are used to construct each quantile regression estimate; there is no partitioning of data performed on the dependent variable as this would incur sample selection bias. Several estimators are available for Ω_θ; the most commonly used in practice, and the one most favored by Buchinsky (1995) because it is more efficient in small samples and is robust to dependence between the regressors and the regression errors, is based on the design matrix bootstrap. This method involves computing

$$\widehat{\Omega}_\theta^{BS} = \frac{n}{B} \sum_{j=1}^{B} \left(\widehat{\beta}_{\theta j}^{BS} - \widehat{\beta}_\theta\right) \left(\widehat{\beta}_{\theta j}^{BS} - \widehat{\beta}_\theta\right)'$$

where $\widehat{\beta}_{\theta j}^{BS}$ is the quantile regression estimator based on the j^{th} bootstrap sample, $j = 1, \cdots, B$. The bootstrap samples, say $\left(y_i^{BS}, x_i'^{BS}\right)$, are obtained by sampling with replacement from the original sample, (y_i, x_i'). This procedure can be implemented using the software program Stata.

In order to construct joint tests, including tests of interquantile restrictions across m quantiles $(\theta_1, \cdots, \theta_m)$, we can define the hypotheses of interest as

$$\begin{aligned} H_0 &: R\beta^* = r \\ & \quad vs. \\ H_0 &: R\beta^* \neq r \end{aligned}$$

where $\beta^* = \left(\beta_{1,\theta_1}, \cdots, \beta_{k,\theta_1}, \cdots, \beta_{1,\theta_m}, \cdots, \beta_{k,\theta_m}\right)'$. Here, R is a $q \times km$ matrix and r is a $q \times 1$ vector that characterizes the restrictions of interest, where q is the number of restrictions imposed under the null. We can use the F-statistic, described by Buchinsky (1998),

$$F = \frac{\left(R\widehat{\beta}^* - r\right)' \left(R\widehat{\Sigma}R'\right)^{-1} \left(R\widehat{\beta}^* - r\right)}{q},$$

which is asymptotically $F(q, n-k-1)$ under H_0. In this case, $\widehat{\Sigma}$ is the estimated variance-covariance matrix for $\widehat{\beta}^*$ which, again, can be obtained via application of the design matrix bootstrap. We use this bootstrap to compute standard errors and variance-covariance matrices throughout the paper. In every case we use 100 bootstrap samples due to the large sample sizes involved.

3. Methodology and Data

The CAPM states that the expected excess return from asset i at time period t, is directly proportional to the beta estimated using information available at time t. Mathematically,

$$E_t(R_{i,t+1}) = \gamma_{1,t+1}\beta_{i,\tau}$$

where $\tau < t+1$ denotes that the beta-risk is determined over moving samples, $\beta_{i,\tau}$ is the beta-risk obtained from a time series regression

$$R_{i,\tau} = \alpha_i + \beta_{i,\tau}R_{m,\tau} + u_{i,\tau}$$

and $R_{i,\tau}$ and $R_{m,\tau}$ are the excess return on the asset and the market portfolio, respectively. The standard FM approach involves OLS regression of the average return from several portfolios of assets on values of beta and a vector of marketwide (and/or firm-specific) factors, that is,

$$R_{i,t+1} = \gamma_{0,t+1} + \gamma_{1,t+1}\hat{\beta}_{i,\tau} + \gamma_{2,t+1}' Z_{i,t} + u_{i,t+1}.$$

The CAPM is then tested by investigating whether these $\hat{\beta}_{i,\tau}$ and $Z_{i,t}$ are able to explain variations in the conditional mean of the cross section of stock market returns. More recent extensions include, among others, additional risk factors whose coefficients are estimated along with beta in a time series regression and then included in the cross-sectional regression as above.

For our purposes, we are most interested in whether or not $\gamma_{1,t+1}$, the market price of beta risk, is significant over any portion of the conditional distribution, since many studies have found beta to be a weak or even insignificant explanatory variable in such conditional mean cross-sectional regressions. We also explore whether the coefficient changes significantly across quantiles. If this is so, it would imply that the market prices beta risk differently for under- and overperforming stocks. In turn this would imply that the theoretical prediction of a single market price of beta risk for all stocks, a hallmark of capital asset pricing theory, would be violated for the empirical example considered in this paper.

Here, our additional variables include both firm-specific and marketwide variables. For the marketwide variables, which vary intertemporally but are constant across firms, we include just the estimated coefficients from the multivariate time series regression of individual returns on the lagged variables in the cross-sectional regression. In effect, this treats these lagged predictors or conditional variables as additional factors. More precisely, there is little qualitative difference between treating these lagged variables as conditioning or instrumental variables (i.e., including the estimated coefficient as a multiplicative of the lagged exogenous variable) or as risk factors (i.e., including just the estimated coefficient) in the cross-sectional regression since the marketwide (lagged exogenous) variables have the same value for all firms at a particular point in time.

We analyze the cross section of returns using quantile regression methods applying two distinct methods of pooling. In the first method, 36 different cross sections of 1,093 firms are studied in the two-pass framework. In order to construct these cross sections, we run, for each firm, a standard time series regression of 100 observations of excess returns on the independent variables - hence we obtain simple OLS estimates of beta and the coefficients on the marketwide variables. The BIDASK variable is calculated as a simple average of the bid-ask spread over these 100 observations. The average return over the subsequent 25 days is then computed; this constitutes one cross section. We do this for 36 different periods in time by rolling forward in 25-day steps. Thus, the dependent variable is constructed in such a way that it is nonoverlapping over time; this is important to avoid possible intertemporal correlation across cross-sections. We then compute quantile regression estimates (and OLS estimates for comparison) for each cross section and combine the results using the efficient weighting scheme recommended by Ferson and Harvey (1999).

The second method involves pooling the 36 cross sections together and performing quantile regressions of the returns on all the risk factors mentioned above. In this case we use time dummies to control for any time-specific fixed effects that may be present in the data.[13] The value of pooling, motivated by Koenker and Machado (1999), is to dramatically improve the precision of the estimates by increasing the available degrees of freedom.[14]

The firm-specific data used in this paper for the individual returns are drawn from the Center for Research in Security Prices (CRSP) database and consist of 1,093 firms observed across 1028 consecutive trading days, ending on December 30, 1994. The associated market capitalization and bid and ask prices for these firms are also extracted from CRSP, along with the value-weighted CRSP (VWCRSP) return index series. Marketwide data are obtained from the Federal Reserve Bank, Board of Governors' web site. A list of the variables used for this part of the analysis is contained in Appendix 1. The returns data for the market and individual firms are filtered for day of the week and month of the year effects using the dummy variables described in the appendix. They are then transformed into excess returns by subtracting away the proxy for the risk-free rate, labelled "Finpaper." In all cases, the returns data are expressed as daily returns; they are not annualized. The money market yield data are annualized.

A short summary of the independent variables follows:

BETA - A simple estimate of $\beta_{i,\tau}$ for each firm obtained by regressing the individual

[13] Estimating fixed individual effects in this framework defeats the purpose of quantile regression in the sense that the distinction between underperforming and overperforming firms will be lost when different intercept terms are allowed for each firm.

[14] We investigate whether estimated coefficients vary over time by considering four separate panels consisting of nine cross sections each. We find that the results are qualitatively similar to those presented in the paper, hence they are not reported. In other words, the results are robust to different levels of pooling. Naturally they are available on request.

firm's daily excess return on the market excess return, that is,

$$R_{i\tau} = \alpha_i + \beta_{i,\tau} R_{m\tau} + u_{i\tau}$$

using 100 time series observations. This variable is a measure of reward for holding systematic risk associated with asset i.

LOGSIZE - The natural logarithm of the market capitalization of the firm at the mid-point in the data (the same for all 36 cross sections).[15]

BIDASK - The average bid-ask spread for the firm across 100 observations. The bid-ask spread is calculated as

$$BIDASK_{it} = \frac{Askhi_{it} - Bidlo_{it}}{\frac{1}{2}\left(Askhi_{it} + Bidlo_{it}\right)}.$$

This variable is a measure of the liquidity of asset i. The larger is the bid-ask spread, the lower is the liquidity of the stock and hence the more the investor must be compensated for holding it. Theoretically we expect a positive relationship between BIDASK and expected return.

RF - The risk factors, or coefficients from the time-series regression of excess return on the following (lagged) marketwide variables simultaneously: finp (financial paper - proxy for the risk-free rate), junk (yield on Moody's Baa-rated corporate bonds less the yield on Aaa-rated corporate bonds), slfh (Ten-year constant maturity T-bond yield less the 3-month constant maturity T-bill rate), term (spread between a ten-year and a one-year T-bond yield), mnt6 (six-month Treasury constant maturity yield less the one-month yield, finpaper) and mnt3 (three-month yield less the one-month yield). These are similar to the variables used to test the CAPM by Fama and French (1989, 1992), Campbell (1987), Harvey (1989), Breen, Glosten, and Jagannathan (1989), and Ferson and Harvey (1991, 1999) among others.

The model we estimate is

$$RETURN_i = \beta_0 + \beta_1 BETA_i + \beta_2 LOGSIZE_i + \beta_3 BIDASK_i + RF_i \beta_4 + u_i,$$

where β_4 is a 6×1 vector and RF_i is an $n \times 6$ matrix containing the risk factors.

For the combined cross-section method (i.e. the first method of pooling) we consider Ferson and Harvey's (1999) efficient-weighted FM estimator (labelled EOLS), efficient-weighted least squares (EWLS), the various efficient-weighted quantile regressions as well as efficient-weighted OLS, similar to EOLS, but where the weights and standard errors are computed based on White's (1980) heteroskedasticity robust variance-covariance matrix

[15] Berk (1995) shows that size-related regularities should be apparent in an economy and why size can explain that part of the cross section of returns which remains unexplained due to model misspecification. He maintains that size-related measures should be included in cross-sectional tests to detect model misspecifications.

(labelled EOLS-WC). This method is less efficient than EOLS but may be the preferred method if heteroskedasticity is suspected. The estimators used in the fixed time effects panels (i.e. the second method of pooling) are OLS, OLS with White (1980) heteroskedasticity robust standard errors, weighted least squares and quantile regression considering several different values for θ. Although the pseudo R^2 (standard R^2 for LS procedures) are presented, they are not directly comparable across estimators or quantiles. Finally, we disaggregate into large and small firms, or for firms with midsample market-capitalization values above and below the mean market capitalization value, and compute the various estimates for the disaggreagted data.[16]

4. Quantile Regression in the Presence of Errors-in-Variables

The attenuation problem has been well-documented in the finance literature (see Fama and MacBeth 1973 and Jagannathan and Wang 1996). Simply put, because the BETA variable consists of estimates from an OLS time series regression, it suffers from an EIV problem that causes estimates of the effect of BETA on average return to be biased towards zero. The standard method of dealing with this is to aggregate the individual firms into portfolios. It is argued that portfolio betas can be estimated more precisely than betas for individual firms, thus reducing, but not eliminating, the resultant bias.

There are several problems associated with this approach. Firstly, aggregation can reduce the informational content inherent in the individual returns data. Further, the construction of large portfolios may induce an aggregation bias. Since individual firms are themselves merely portfolios of individual assets, and since the parameters will be estimated more precisely using a larger data set, the optimal solution to the attenuation problem involves a trade-off between reducing bias (via aggregation) and improved efficiency (via disaggregation). Under standard conditions, using OLS regression, the main concern is bias, so typically only about 20-25 portfolios (Kothari, Shanken, and Sloan 1995 and Ferson and Harvey 1999) are considered.

In our application, using so few cross-sectional (portfolio) observations would prevent identification of the quantile regression estimates across a wide range of quantiles (see Buchinsky (1998)). Those estimates that could be computed, particularly in the tails of the conditional distribution, would be measured very imprecisely and subsequent significance tests would lack power. Further, by using quantile regression, we seek to delineate the behavior of bad news versus good news firms (i.e. underperforming and overperforming firms). If we aggregate the firms into large portfolios, this will have the effect of diversifying away the effect of the idiosyncratic shocks or the effect of the omitted variables for a

[16] A comparison of LS-based versus quantile regression errors, or evaluation of model mispricing of assets, would yield an additional way of seeing which regression approach is better suited to the problem of modelling asset returns.

particular firm and will dilute the impact of the quantile regression findings.[17] Since we are more interested in the relative values of the parameters across the quantiles, the absolute amount of bias is of secondary concern; in this section we study how the bias of the quantile regression estimator changes across θ.

The properties of the quantile regression estimator in the presence of errors-in-variables have not been considered in the statistics or econometrics literature. If quantile methods can be shown to help deal with bias caused by EIV, these methods may gain wider acceptance with those who analyze aggregated data since portfolio formation reduces, but does not eliminate, the EIV problem. Here, a Monte Carlo simulation is employed to examine the small sample properties. The asymptotic properties are unknown and beyond the scope of this paper but, like OLS, the quantile regression estimator is inconsistent when applied in the presence of omitted variables.

The design of the experiment is as follows. A design matrix is taken from one of the cross sections considered in the empirical example. Then, a series of average returns is generated according to the equation

$$RETURN_i = \beta_0 + \beta_1 BETA_i + \gamma Z_i + u_i, u_i \sim IN(0, \sigma_u^2)$$

where Z_i is a matrix containing the other variables. The parameters used are extracted from OLS estimates based on the actual returns data. β_1, the parameter of interest here, is allowed to take the values 0.05 and 0.1. A new series for $BETA_i$ that incorporates measurement error is then constructed according to

$$BETA_i^* = BETA_i + \varepsilon_i, \varepsilon_i \sim IN(0, \sigma_\varepsilon^2).$$

In this case, determining a realistic value for σ_ε is difficult so it is allowed to vary from 0 (no errors-in-variables), through 0.2 to 0.4. The sample size takes the values of 100 and 1,093 (the sample used for the individual cross sections in this paper). We use 5,000 Monte Carlo replications throughout. The primary aim here is to assess the bias of the quantile regression estimator across θ and to compare it to the OLS estimator. Under the assumptions of the classical linear regression model (namely independence of the errors and regressors), the true quantile slope parameters are the same for all quantiles and the quantile regression estimator is consistent (Buchinsky (1998)). We must use this method to generate data with known quantile properties because of the difficulties associated with generating data with different slopes at different points in the distribution, this property is discussed by Buchinsky (1995). Including $\sigma_\varepsilon^2 = 0$ allows determination of the small sample bias where the asymptotic bias is zero - it thus represents a control experiment against

[17] Again, the reader should note that we use terms such as "overperforming" and "underperforming" firms in a specific sense. We prefer to think of location in a quantile as corresponding to the magnitude and sign of the idiosyncratic shock to a firm at a particular time. More precisely, it can be viewed as the effect of omitted variables on the pricing of risk.

which the results for the case of EIV can be compared. The Monte Carlo output is included in Table 1.

The results indicate that for every experiment considered, quantile regression estimates of β_1 are larger, on average, for low values of θ than for high values of θ. Where σ_ε is zero and the classical assumptions hold, there is a small sample bias evident in the quantile regression estimator that is positive where $\theta < 0.5$ and negative where $\theta > 0.5$. For the special case of median regression ($\theta = 0.5$), the quantile regression estimator is unbiased (up to a small Monte Carlo error) as expected. The bias is generally smaller for $n = 1,093$ at all quantiles when compared to $n = 100$, demonstrating the asymptotic convergence properties. This small sample bias deserves greater attention, which is beyond the scope of this paper.

When EIV are induced, the shape of the function relating bias to θ is roughly equivalent to that observed for $\sigma_\varepsilon = 0$, that is, the average coefficient is higher for $\theta = 0.1$ than for $\theta = 0.9$. In the case of EIV, however, the median regression estimator is actually less biased than OLS and thus may be the preferred estimator when dealing with the EIV problem. When $\theta = 0.5$, the improved bias of the median regression estimator means that it suffers less squared error loss than OLS, despite being less efficient. This result provides strong evidence that, at least for the present problem, median regression is a better estimator than OLS.[18]

When the EIV problem is rendered more extreme by considering $\sigma_\varepsilon = 0.4$, the bias of the quantile regression estimator is less than for OLS even when $\theta > 0.5$. For $n = 100$, $\beta_1 = 0.1$, the bias of the OLS estimator exceeds that of the quantile estimator for all θ under consideration. The same experiments were run for the case where $\beta_1 = -0.05$ and -0.1 and the results were found to be qualitatively the same but the direction of the attenuation bias was still towards zero. The effect of the bias means that the quantile plot (the plot of $\widehat{\beta}_\theta$ against θ) will generally be flatter than it is in reality,[19] i.e. patterns will appear to be less extreme in the sample than they are in the population.

There is evidence that the plot of the bias of the quantile regression estimator against θ 'flattens out' as n is increased to 1,093. This finding was confirmed by a small experiment involving pooled cross sections, suggesting that pooling is favored relative to methods based on combining individual cross sections. Since we use pooling in this paper, interquantile variation solely due to bias differences across θ can be ruled out.

5. Results and Discussion

The results of the empirical analysis are contained in Tables 2 through 8 and Figures 1 through 12. The general conclusion that can be drawn is that there exist wide disparities

[18] This point demands further consideration, including a treatment of the relevant asymptotic theory.

[19] A useful way to think of this is to consider a case where the coefficient changes sign, say from negative to positive. The negative estimate is attenuated and so is the positive estimate.

		$\beta_1 = 0.05$						$\beta_1 = 0.1$					
		$n = 100$			$n = 1093$			$n = 100$			$n = 1093$		
σ_ε	θ	Coef	S.D.	RMS	Coef	S.D.	RMS	Coef	S.D.	RMS	Coef	S.D.	RMS
0.0	0.1	0.061	0.079	0.080	0.056	0.027	0.028	0.111	0.079	0.080	0.106	0.027	0.028
0.0	0.3	0.056	0.060	0.060	0.052	0.020	0.020	0.106	0.060	0.060	0.102	0.020	0.020
0.0	0.5	0.050	0.049	0.049	0.050	0.017	0.017	0.100	0.049	0.049	0.100	0.017	0.017
0.0	0.7	0.045	0.061	0.062	0.044	0.020	0.021	0.095	0.061	0.062	0.094	0.020	0.021
0.0	0.9	0.041	0.081	0.081	0.045	0.027	0.028	0.091	0.081	0.081	0.095	0.027	0.028
0.0	LS	0.051	0.058	0.058	0.050	0.015	0.015	0.101	0.058	0.058	0.100	0.015	0.015
0.2	0.1	0.051	0.073	0.073	0.045	0.025	0.025	0.092	0.072	0.073	0.086	0.024	0.028
0.2	0.3	0.046	0.055	0.055	0.043	0.018	0.019	0.088	0.055	0.056	0.083	0.018	0.025
0.2	0.5	0.044	0.046	0.046	0.042	0.016	0.017	0.087	0.047	0.048	0.084	0.017	0.023
0.2	0.7	0.037	0.055	0.056	0.036	0.018	0.023	0.079	0.056	0.059	0.076	0.018	0.030
0.2	0.9	0.034	0.075	0.077	0.036	0.025	0.028	0.075	0.074	0.078	0.077	0.025	0.034
0.2	LS	0.039	0.050	0.051	0.040	0.014	0.017	0.077	0.050	0.055	0.079	0.014	0.025
0.4	0.1	0.033	0.059	0.061	0.029	0.019	0.029	0.060	0.059	0.071	0.054	0.020	0.050
0.4	0.3	0.031	0.045	0.048	0.027	0.014	0.027	0.060	0.046	0.061	0.053	0.014	0.049
0.4	0.5	0.033	0.039	0.043	0.029	0.015	0.026	0.064	0.042	0.055	0.058	0.020	0.047
0.4	0.7	0.026	0.045	0.051	0.022	0.014	0.031	0.053	0.046	0.066	0.047	0.015	0.055
0.4	0.9	0.022	0.061	0.068	0.023	0.019	0.033	0.050	0.060	0.078	0.050	0.020	0.054
0.4	LS	0.022	0.038	0.047	0.024	0.011	0.028	0.044	0.038	0.068	0.048	0.011	0.053

Table 1: **Small Sample Performance of the Quantile Regression Estimator in the Presence of Errors in Variables.** The results of the Monte Carlo experiment used to assess the properties of the quantile regression estimator in the presence of errors-in-variables. Coef. is the average coefficient value, S.D. is the standard deviation of the estimates, RMS is root mean square loss, σ_ε represents the amount of error in the estimate of β_1, n is sample size, θ is the quantile and LS stands for the least squares estimator.

Figure 1: BETA
Quantile Plot of Coefficient Against the Quantile. Pooled Quantile Regression with Fixed Time Effects

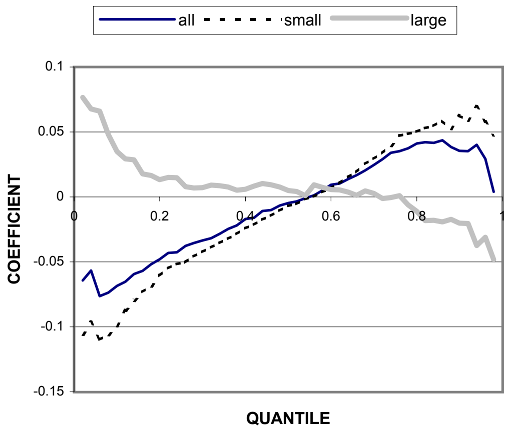


Figure 2: BETA
Quantile Plot of T-statistic Against the Quantile. Pooled Quantile Regression with Fixed Time Effects

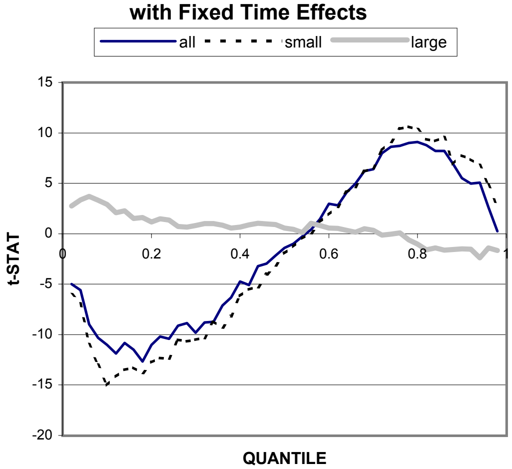


Figure 3: BIDASK
Quantile Plot of Coefficient Against the Quantile. Pooled Quantile Regression with Fixed Time Effects

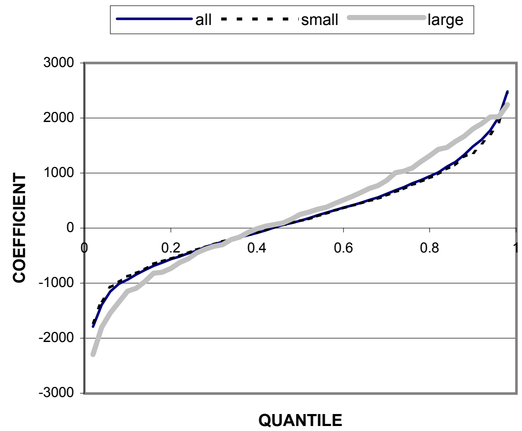


Figure 4: BIDASK
Quantile Plot of T-statistic Against the Quantile. Pooled Quantile Regression with Fixed Time Effects

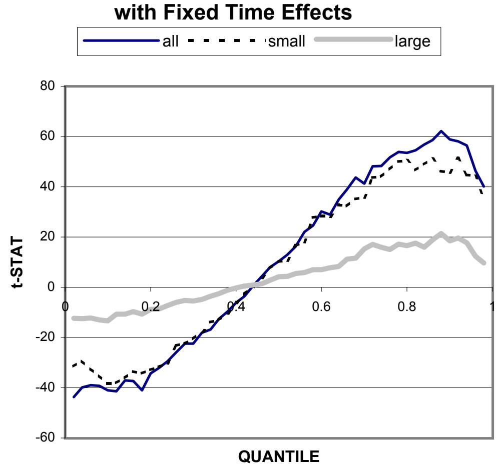


Figure 5: LOGSIZE
Quantile Plot of Coefficient Against the Quantile. Pooled Quantile Regression with Fixed Time Effects

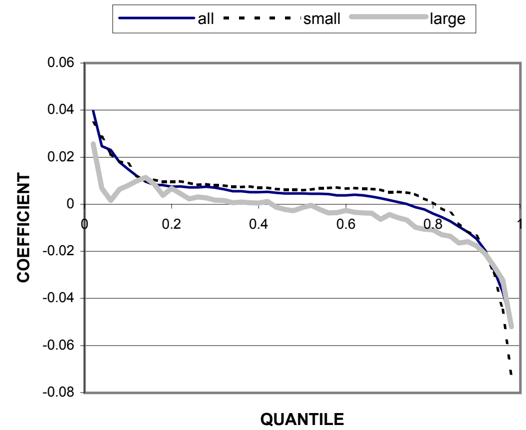
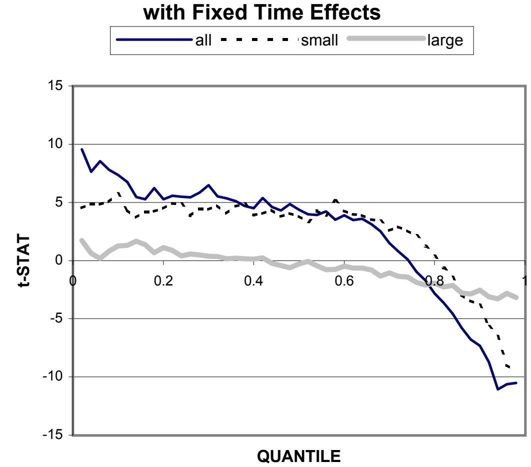


Figure 6: LOGSIZE
Quantile Plot of T-statistic Against the Quantile. Pooled Quantile Regression with Fixed Time Effects



in behavior between underperforming and overperforming stocks, or firms that may be receiving negative or positive idiosyncratic shocks, and that such behavior differs for large as opposed to small firms. The quantile regression technique therefore provides considerable insight that cannot be obtained by using standard regression techniques.

Although focusing this results section on the two-pass efficient-weighted quantile regression results for the 36 individual cross sections would provide greater conceptual comparability with the existing literature, we choose to focus our discussion on the results for the pooled, fixed time effects quantile regressions for the following reasons: 1) The interquantile F-test of equality of coefficients is easily interpretable and need not be averaged over 36 cross sections; 2) Greater sample size and degrees of freedom improve the precision of estimates; 3) Monte Carlo evidence suggesting that larger samples display more evenly-distributed attenuation bias across quantiles than observed for smaller samples; and 4) Both approaches yield qualitatively similar results.

Reported p-values are constructed using the design matrix bootstrap approach and hence are robust to serial correlation, heteroskedasticity and any general dependence between the regressors and the regression errors. Throughout this section, we primarily discuss the pooled results but note any discrepancies between the results from the two levels of pooling.

CAPM Performance Across Quantiles. Table 2 presents the results for interquantile tests of whether or not the parameters are equal across quantiles. The table compares parameter values for $\theta = 0.1$ and 0.5, for $\theta = 0.5$ and 0.9, and for $\theta = 0.1$ and 0.9. Interquantile tests are performed individually for each risk factor, for groups of factors such as firm-specific or marketwide factors and for all factors jointly.

All of the individual firm-specific tests resoundingly reject the null hypothesis that the coefficients are the same across the quantiles. The only exceptions are for size and beta for large firms where the p-values comparing the 10^{th} and 50^{th} percentiles are 0.2834 and 0.0838 respectively. This result may in part be due to the data containing fewer large firms than small firms. The joint tests for all firm-specific factors find a significant difference between the coefficients across quantiles for both large and small firms. The marketwide factors generally do not significantly differ across the quantiles when considered individually, but jointly, the coefficients on these factors vary significantly with θ. Also, joint tests of the null that the coefficients on all firm-specific and marketwide factors are equal to zero are also strongly rejected across all quantiles and firm sizes. Similar estimation results obtain for the 36 efficiently-weighted pooled cross sections too.

This body of evidence implies that the particular version of conditional multifactor CAPM tested here fails, since the market price of beta risk changes across quantiles (and is different for firms of different sizes). Furthermore, these results imply that there are locations in the conditional distribution of returns where the market price of beta risk is strongly significant, even without the two extra Fama and French factors HML and SMB, or

	ALL	ddof = 39303		SMALL	ddof = 30123		BIG	ddof = 9135	
	10-50	50-90	10-90	10-50	50-90	10-90	10-50	50-90	10-90
beta	65.88	19.69	84.62	120.47	46.44	171.98	2.99	2.97	6.62
ndof=1	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0838	0.0850	0.0101
bidask	600.48	656.79	1281.33	967.32	445.58	1273.29	142.93	187.77	444.75
ndof=1	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
logsize	30.87	127.75	148.06	12.25	30.37	62.51	1.15	6.81	6.33
ndof=1	0.0000	0.0000	0.0000	0.0005	0.0000	0.0000	0.2834	0.0091	0.0119
finp	0.43	0.83	0.07	1.35	0.60	0.03	0.90	1.66	2.67
ndof=1	0.5107	0.3623	0.7889	0.2446	0.4370	0.8532	0.3423	0.1980	0.1025
junk	0.15	0.02	0.14	0.01	0.49	0.39	0.00	3.91	2.26
ndof=1	0.6989	0.8800	0.7079	0.9186	0.4846	0.5330	0.9574	0.0481	0.1327
mnt3	0.21	0.10	0.00	0.54	0.00	0.20	0.09	0.89	0.23
ndof=1	0.6454	0.7512	0.9616	0.4610	0.9490	0.6546	0.7668	0.3442	0.6315
mnt6	0.95	0.37	1.54	1.33	0.25	1.38	0.27	1.16	1.53
ndof=1	0.3297	0.5441	0.2142	0.2489	0.6149	0.2398	0.6036	0.2825	0.2159
slfh	0.00	0.29	0.21	0.05	0.26	0.32	0.36	0.23	0.07
ndof=1	0.9968	0.5877	0.6482	0.8217	0.6108	0.5701	0.5507	0.6324	0.7945
term	5.75	0.02	2.90	2.26	0.00	0.89	2.23	0.82	3.17
ndof=1	0.0165	0.8805	0.0888	0.1331	0.9679	0.3460	0.1353	0.3656	0.0751
FS-MW	110.50	135.44	256.08	161.29	84.40	226.54	24.30	31.24	79.12
ndof=9	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
FS	323.97	386.70	661.17	471.61	218.17	588.88	64.67	89.00	233.73
ndof=3	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
MW	3.57	0.42	2.89	2.88	0.40	1.23	2.39	1.26	3.79
ndof=6	0.0015	0.8644	0.0081	0.0083	0.8780	0.2887	0.0264	0.2701	0.0009

Table 2: **Individual and Joint Interquantile Tests Using Pooled Quantile Regressions.** Individual and joint interquantile tests based on pooled quantile regressions with time-specific fixed effects. We include F-statistics and p-values. The p-values are based on the F-distribution with the stated number of numerator (ndof) and denominator (ddof) degrees of freedom. FS-MW is a joint test of whether firm-specific and marketwide factors are constant across the stated quantiles; FS tests just the firm-specific factors; and MW tests just the marketwide factors. ALL represents the case where all firms are included in the data set; SMALL includes only firms with lower than average market capitalization; and BIG includes only larger than average firms. $10-50$ indicates that the test is for determining whether or not the coefficients differ across the 10th and 50th quantiles; $50-90$ indicates the test is for the 50th and 90th quantiles; and $10-90$ indicates the test is for determining whether the coefficients differ across the 10th and 90th quantiles.

other factors that have typically helped to resuscitate the CAPM. Since it is the case that we can reject the hypothesis that the coefficients are the same across quantiles, we find that none of the empirical or theoretical results previously uncovered in the literature (positive, negative or zero market prices of risk) are supported unambiguously. By construction, we show that the results obtained using LS-based methods are but one perspective on the more complex set of patterns revealed by quantile regression.

Firm-Specific Conditional Variables. Theory says that returns should increase with beta, so we would expect a positive and significant coefficient on the market price of beta risk across all quantiles. Furthermore, the expected return for an individual firm should share the same market price of risk as all firms in equilibrium. Above, we showed that the market price of risk is not the same across all firms. Here, by inspection of the coefficients and t-statistics from the pooled quantile regression in Figures 1 and 2 and in Tables 3 through 5, we reveal that beta is not a significant explanator of returns across the entire conditional distribution. In general, an "S" pattern is observed, where beta has a negative coefficient for firms that underperform the mean (or have a negative idiosyncratic shock or bad news) and a positive coefficient for firms that overperform the mean. In particular, beta is significant and positive for the top 30% of firms, implying that the theoretical CAPM relationship applies conditionally, but only for firms that out-perform the mean. Firms in the lower quantiles have unobserved characteristics that are not conducive to high returns. The market appears to price these assets in such a way that there is a significant negative return to beta risk. Moving across the quantiles, the quality of the unobserved factors improves and the effect of additional beta risk becomes positive. An interesting result here is the negative slope that occurs in the quantile plot in the upper 25% of the distribution. This implies that in this range, as the unobserved factors improve, firms experience a lower return to increased beta risk.

The opposing results for large and small firms are striking. As the unobserved factors improve, the return to beta risk for large firms decreases, suggesting that the market places a positive price on beta risk for large firms that receive negative idiosyncratic shocks or are underperforming and a negative price on this risk for overperforming or good-news large firms. On the contrary, for small firms that are underperforming, the price of beta risk is negative while for overperforming small firms, the market price of beta risk is positive. Small firms are dominating the overall pattern in the quantile plot, as the majority that are included in our sample are small. Not surprisingly, around the center of the distribution, the coefficients lie on either side of zero without statistical significance, underscoring the ambiguous results typically found in the literature where the mean is the only point considered.[20]

[20] The differences between the results for large and small firms may be due to the fact that the excess market return, and hence beta risk, is largely constituted by large firms. Perhaps if the market return were

These results suggest that the conditional or multi-factor CAPM studied here holds only outside the center of the conditional distribution of returns. To date, empirical results have generally found that the CAPM predicted a positive (though often insignificant and occasionally negative and insignificant) relationship between cross-sectional returns and beta. It may also be the case that the results found in the literature to date are dominated by large firms and overperforming small firms, or, perhaps, because these types of firms are over-represented in the samples used. As is evident from Tables 6 through 8, the results for the efficient-weighted (à la Ferson and Harvey 1999) 36 cross sections are similar to the pooled results reported here.

A long debate has ensued as to whether size should be significantly related to returns and, if it is, then what sign should its coefficient take. Some, like Banz (1981), demonstrate empirically that smaller firms have higher risk-adjusted returns, on average, than larger firms and that this is evidence against the CAPM, i.e. it represents an anomaly. Merton (1987), in contrast, relaxes some of the informational assumptions of the CAPM and shows theoretically that the expected return on the asset should be an increasing function of size. Finally, Berk (1995) also argues that size-related stylized facts are not anomalies, but instead maintains that size and returns should be inversely related if the covariance of cash flows and returns is non-negative.

As demonstrated in Figures 3 and 4 and in Tables 3 through 5, quantile methods provide some support for Merton's argument, in that the coefficient on size in these pooled quantile regressions is largely positive overall, except for the top 10% of overperforming firms. The result that small firms have a significant positive coefficient on size for all but the overperforming firms is a strong result from a statistical point of view and one that rekindles the debate on what the pricing of size represents. For large firms, the coefficients are negative in the second half of the conditional distribution, but are significant only for the top 10% of performers. So, the inverse relationship between size and returns documented in the literature occurs in our empirical example only for firms that perform significantly better than average (very good-news firms) and for large firms.

If beta were to fully explain the cross section of returns as per the traditional (unconditional single-factor) CAPM, size, along with other factors and conditioning variables, should be insignificant. We find that it is significant, though, indicating that size is at least proxying for a factor that should be priced but is not included in the model.[21] In

constructed with the same proportion of large and small firms as our sample, the results would not be so striking. A recent working paper by Chen and Bassett (2002) demonstrates a related point in a different context. They show that large-cap portfolios can have positive SMB cefficients in the Fama-French 3-factor model, giving the illusion of being small simply because the first factor, the market excess return, already includes components related to size. If the market factor is already 90% big, then a portfolio that is less than 90% big would have a positive SMB.

[21] Berk (1995) suggests using the significance of size as a barometer of how well the model prices risk.

	$\theta = 10$	$\theta = 30$	$\theta = 50$	$\theta = 70$	$\theta = 90$	OLS	OLS(WC)	WLS
beta	-0.0684	-0.0334	-0.0047	0.0247	0.0355	-0.0078	-0.0078	-0.0088
z-stat	-7.97	-6.56	-0.92	4.14	3.95	-1.88	-1.26	-2.10
bidask	-937.38	-293.85	134.13	623.87	1480.25	261.18	261.18	263.06
z-stat	-22.47	-10.71	5.16	22.30	29.57	16.08	8.03	16.60
logsize	0.0149	0.0070	0.0046	0.0018	-0.0147	0.0025	0.0025	0.0027
z-stat	7.91	5.00	4.95	1.38	-6.78	1.93	1.84	2.01
finp	-0.0088	-0.0082	-0.0185	-0.0113	-0.0022	-0.0091	-0.0091	-0.0099
z-stat	-0.57	-0.71	-1.69	-1.15	-0.12	-1.13	-0.70	-1.24
junk	-0.0003	0.0119	0.0062	0.0034	0.0090	0.0147	0.0147	0.0137
z-stat	-0.01	1.00	0.50	0.27	0.43	1.69	1.08	1.58
mnt3	-0.0022	-0.0151	0.0066	0.0108	-0.0007	0.0047	0.0047	0.0084
z-stat	-0.12	-1.14	0.54	0.79	-0.03	0.47	0.28	0.83
mnt6	0.0066	0.0050	-0.0128	-0.0290	-0.0249	-0.0030	-0.0030	-0.0065
z-stat	0.34	0.34	-0.95	-1.84	-1.04	-0.30	-0.18	-0.66
slfh	0.0176	0.0152	0.0176	0.0021	0.0057	0.0023	0.0023	0.0031
z-stat	1.03	1.14	1.41	0.19	0.24	0.25	0.16	0.34
term	0.0045	-0.0344	-0.0352	-0.0231	-0.0384	-0.0199	-0.0199	-0.0186
z-stat	0.26	-2.61	-3.05	-1.76	-1.59	-2.15	-1.39	-2.02
Pseudo R^2	0.0996	0.0612	0.0509	0.0753	0.1515	0.0931	0.0931	0.0917
All	129.36	133.71	168.08	257.08	189.49	91.73	81.56	90.23
ndof=44	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
FS	267.49	81.00	15.62	186.90	498.46	93.30	23.64	98.69
ndof=3	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
MW	0.52	2.34	4.23	3.97	2.04	2.12	0.78	1.97
ndof=6	0.7903	0.0290	0.0003	0.0006	0.0566	0.0481	0.5872	0.0664
FTE	82.69	136.55	151.73	214.99	64.81	104.23	96.83	102.38
ndof=35	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Table 3: **Pooled Quantile, OLS, and WLS Regressions for All Firms.** Pooled quantile, OLS and WLS regressions including time-specific fixed effects. The top panel presents coefficients and z-statistics. The bottom panel gives joint (F) test statistics and p-values. The p-values are calculated using the stated number of numerator degrees of freedom (ndof), with denominator degrees of freedom (ddof) = 39303. ALL tests whether all slope parameters are jointly significant, FS tests the firm-specific factors, MW tests the markewide factors, FTE tests the fixed-time effects. θ denotes quantile for which relationship is estimated. All firms included in the regressions.

	$\theta = 10$	$\theta = 30$	$\theta = 50$	$\theta = 70$	$\theta = 90$	OLS	OLS(WC)	WLS
beta	-0.0993	-0.0424	-0.0068	0.0293	0.0625	-0.0115	-0.0115	-0.0119
z-stat	-9.10	-7.02	-1.21	4.32	5.73	-2.38	-1.62	-2.45
bidask	-878.86	-297.86	130.89	594.70	1373.59	260.56	260.56	262.68
z-stat	-22.33	-10.48	5.27	17.09	28.42	14.63	7.72	14.98
logsize	0.0171	0.0080	0.0060	0.0052	-0.0136	0.0036	0.0036	0.0035
z-stat	5.34	4.27	3.37	2.40	-3.30	1.65	1.55	1.62
finp	0.0014	-0.0039	-0.0185	-0.0091	-0.0032	-0.0094	-0.0094	-0.0101
z-stat	0.08	-0.33	-1.45	-0.76	-0.16	-1.02	-0.64	-1.11
junk	0.0058	0.0106	0.0077	0.0035	0.0224	0.0138	0.0138	0.0128
z-stat	0.32	0.80	0.55	0.24	1.06	1.39	0.90	1.28
mnt3	0.0038	-0.0058	0.0194	0.0145	0.0177	0.0129	0.0129	0.0151
z-stat	0.17	-0.36	1.54	0.91	0.55	1.12	0.69	1.31
mnt6	-0.0031	-0.0081	-0.0276	-0.0331	-0.0402	-0.0118	-0.0118	-0.0137
z-stat	-0.13	-0.47	-1.78	-2.07	-1.32	-1.04	-0.63	-1.21
slfh	0.0099	0.0012	0.0057	-0.0050	-0.0053	-0.0038	-0.0038	-0.0012
z-stat	0.53	0.08	0.44	-0.31	-0.23	-0.36	-0.23	-0.11
term	0.0053	-0.0186	-0.0208	-0.0170	-0.0218	-0.0118	-0.0118	-0.0124
z-stat	0.30	-1.35	-1.79	-1.06	-0.86	-1.12	-0.74	-1.18
Pseudo R^2	0.1028	0.0626	0.0492	0.0760	0.1535	0.0925	0.0925	0.0913
All	116.05	113.03	129.94	108.31	158.56	69.76	62.24	68.81
ndof=44	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
FS	268.79	132.73	12.62	152.86	331.90	75.14	20.86	78.86
ndof=3	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
MW	0.51	1.06	3.12	2.93	0.84	1.31	0.48	1.32
ndof=6	0.7983	0.3813	0.0047	0.0074	0.5383	0.2465	0.8210	0.2418
FTE	51.77	120.15	116.46	82.22	59.31	79.60	74.74	78.27
ndof=35	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Table 4: **Pooled Quantile, OLS, and WLS Regressions for Small Firms.** Pooled quantile, OLS and WLS regressions including time-specific fixed effects. The top panel presents coefficients and *z*-statistics. The bottom panel gives joint (F) test statistics and *p*-values. The *p*-values are calculated using the stated number of numerator degrees of freedom (ndof), with denominator degrees of freedom (ddof) = 30123. ALL tests whether all slope parameters are jointly significant, FS tests the firm-specific factors, MW tests the marketwide factors, FTE tests the fixed-time effects. θ denotes quantile for which relationship is estimated. Only firms withsmaller than average market capitalization included in the regressions.

	$\theta = 10$	$\theta = 30$	$\theta = 50$	$\theta = 70$	$\theta = 90$	OLS	OLS(WC)	WLS
beta	0.0349	0.0070	0.0048	0.0028	-0.0201	0.0078	0.0078	0.0079
z-stat	1.97	0.55	0.46	0.22	-1.59	0.92	0.80	0.94
bidask	-1147.78	-334.22	252.73	868.17	1800.69	305.29	305.29	305.83
z-stat	-9.23	-3.91	2.97	9.19	17.20	5.14	4.07	5.16
logsize	0.0079	0.0017	-0.0013	-0.0044	-0.0176	-0.0036	-0.0036	-0.0037
z-stat	1.01	0.35	-0.28	-0.93	-2.67	-0.83	-0.85	-0.83
finp	-0.0584	-0.0288	-0.0273	-0.0025	0.0152	-0.0134	-0.0134	-0.0143
z-stat	-1.71	-1.22	-1.16	-0.12	0.52	-0.82	-0.71	-0.88
junk	0.0289	-0.0070	0.0309	0.0010	-0.0400	0.0097	0.0097	0.0096
z-stat	0.76	-0.27	1.24	0.04	-1.14	0.56	0.46	0.55
mnt3	-0.0456	-0.0752	-0.0567	-0.0400	-0.0240	-0.0441	-0.0441	-0.0429
z-stat	-1.43	-2.58	-2.85	-1.42	-0.87	-2.13	-1.90	-2.08
mnt6	0.0638	0.0750	0.0430	0.0255	0.0050	0.0557	0.0557	0.0556
z-stat	1.73	2.34	1.79	0.89	0.21	2.74	2.29	2.74
slfh	0.0676	0.0774	0.0434	0.0473	0.0566	0.0477	0.0477	0.0467
z-stat	1.81	3.43	1.92	1.55	1.83	2.67	2.19	2.61
term	-0.0201	-0.0741	-0.0836	-0.0943	-0.1143	-0.0727	-0.0727	-0.0714
z-stat	-0.47	-2.66	-3.24	-3.20	-3.18	-3.75	-3.22	-3.69
Pseudo R^2	0.0927	0.0704	0.0680	0.0843	0.1429	0.1236	0.1236	0.1240
All	28.90	42.29	70.75	57.92	41.71	29.28	26.80	29.39
ndof=44	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
FS	35.74	6.18	4.66	42.95	157.09	15.61	10.36	15.78
ndof=3	0.0000	0.0003	0.0030	0.0000	0.0000	0.0000	0.0000	0.0000
MW	2.28	4.00	4.05	3.44	3.53	4.34	3.16	4.27
ndof=6	0.0338	0.0005	0.0005	0.0021	0.0017	0.0002	0.0042	0.0003
FTE	16.45	41.59	68.25	29.36	16.95	31.48	28.42	31.62
ndof=35	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Table 5: **Pooled Quantile, OLS, and WLS Regressions for Large Firms.** Pooled quantile, OLS and WLS regressions including time-specific fixed effects. The top panel presents coefficients and *z*-statistics. The bottom panel gives joint (F) test statistics and *p*-values. The *p*-values are calculated using the stated number of numerator degrees of freedom (ndof), with denominator degrees of freedom (ddof) = 9135. ALL tests whether all slope parameters are jointly significant, FS tests the firm-specific factors, MW tests the marketwide factors, FTE tests the fixed-time effects. θ denotes quantile for which relationship is estimated. Only firms with larger than average market capitalization included in the regressions.

	$\theta = 10$	$\theta = 30$	$\theta = 50$	$\theta = 70$	$\theta = 90$	EOLS	EOLS(WC)	EWLS
beta	-0.0508	-0.0273	-0.0060	0.0208	0.0414	-0.0042	-0.0052	-0.0051
z-stat	-5.97	-4.61	-1.10	3.50	4.34	-0.95	-0.86	-1.16
bidask	-764.42	-301.56	92.11	523.33	1213.37	210.31	187.19	209.94
z-stat	-20.05	-10.97	3.63	17.90	26.42	12.81	6.46	13.14
logsize	0.0133	0.0040	0.0036	0.0026	-0.0122	0.0034	0.0038	0.0038
z-stat	7.18	3.28	3.26	2.17	-5.97	2.63	2.93	2.90
finp	-0.0385	-0.0064	-0.0278	0.0078	0.0142	-0.0066	-0.0039	-0.0107
z-stat	-1.53	-0.32	-1.48	0.38	0.50	-0.47	-0.19	-0.77
junk	-0.0117	-0.0046	-0.0205	0.0065	0.0098	0.0178	0.0068	0.0211
z-stat	-0.45	-0.26	-1.21	0.34	0.35	1.36	0.36	1.61
mnt3	0.0602	0.0551	0.0466	0.0256	0.0377	0.0457	0.0396	0.0505
z-stat	1.89	2.53	1.98	0.95	1.00	2.57	1.51	2.85
mnt6	0.0091	-0.0161	-0.0276	-0.0665	-0.0734	-0.0447	-0.0311	-0.0492
z-stat	0.23	-0.66	-1.10	-2.26	-1.77	-2.36	-1.10	-2.60
slfh	-0.0401	-0.0014	0.0133	0.0047	0.0457	0.0020	0.0034	-0.0015
z-stat	-0.98	-0.05	0.51	0.16	1.06	0.10	0.11	-0.07
term	0.0726	-0.0309	-0.0153	-0.0064	-0.0004	-0.0129	-0.0103	-0.0148
z-stat	2.07	-1.22	-0.62	-0.23	-0.01	-0.72	-0.40	-0.83

Table 6: **Efficient-Weighted Quantile, OLS, OLS with White (1980) Correction, and WLS Regression Results for All Firms.** Efficiently combined cross-section quantile, OLS, OLS using weights based on White (1980) heteroskedasticity corrected variance-covariance matrix and WLS regressions. We present coefficients and z-statistics. θ denotes quantile for which relationship is estimated. All firms included in the regressions.

other words, size contributes to the evidence against an unconditional single-factor CAPM for nearly all points in the conditional distribution and, using Berk's (1995) interpretation, against the particular conditional CAPM we consider. Note that for most of the quantile plot, the coefficient on size for small firms lies above the coefficient for large firms, mirroring Banz's (1981) empirical results. Finally, these results are slightly different for the efficiently weighted 36 cross sections as illustrated in Tables 6 through 8: Unexpectedly, for large firms, the size effect is more significant in these efficient-weighted cross-sectional results than in the pooled quantile regression results.

Merton (1987) also suggests that returns are a decreasing function of the public availability of information on returns. Amihud and Mendelson (1989) link this with bid-ask spread and find a positive relationship between returns and spread in a conditional CAPM. Figures 5 and 6 and Tables 3 through 5 illustrate that when we revisit the relationship using quantile methods, a positive and significant relationship is uncovered only for the

	$\theta = 10$	$\theta = 30$	$\theta = 50$	$\theta = 70$	$\theta = 90$	EOLS	EOLS(WC)	EWLS
beta	-0.0793	-0.0394	-0.0026	0.0196	0.0526	-0.0084	-0.0103	-0.0087
z-stat	-7.66	-5.76	-0.42	2.77	4.64	-1.66	-1.50	-1.71
bidask	-788.48	-299.30	95.01	499.05	1159.32	211.07	190.79	210.46
z-stat	-19.99	-11.09	3.63	16.21	24.06	11.73	6.39	11.91
logsize	0.0136	0.0050	0.0053	0.0055	-0.0088	0.0056	0.0056	0.0057
z-stat	4.05	2.75	3.12	2.86	-2.51	2.68	2.59	2.72
finp	-0.0059	0.0043	-0.0210	-0.0038	0.0322	-0.0052	-0.0008	-0.0101
z-stat	-0.19	0.18	-0.95	-0.17	0.90	-0.32	-0.04	-0.63
junk	-0.0164	0.0078	-0.0353	0.0066	0.0274	0.0172	0.0036	0.0210
z-stat	-0.57	0.39	-1.78	0.30	0.85	1.15	0.17	1.41
mnt3	0.0589	0.0636	0.0713	0.0361	0.0510	0.0483	0.0426	0.0524
z-stat	1.53	2.35	2.61	1.15	1.10	2.38	1.46	2.58
mnt6	0.0212	-0.0465	-0.0502	-0.0742	-0.0729	-0.0548	-0.0416	-0.0572
z-stat	0.50	-1.59	-1.76	-2.20	-1.53	-2.54	-1.33	-2.65
slfh	-0.0582	-0.0352	0.0061	-0.0189	0.0077	-0.0093	-0.0116	-0.0097
z-stat	-1.27	-1.11	0.21	-0.58	0.15	-0.41	-0.35	-0.43
term	0.0693	-0.0127	-0.0128	0.0212	0.0274	-0.0056	0.0020	-0.0096
z-stat	1.74	-0.43	-0.46	0.71	0.64	-0.27	0.07	-0.47

Table 7: **Efficient-Weighted Quantile, OLS, OLS with White (1980) Correction, and WLS Regression Results for Small Firms.** Efficiently combined cross-section quantile, OLS, OLS using weights based on White (1980) heteroskedasticity corrected variance-covariance matrix and WLS regressions. We present coefficients and z-statistics. θ denotes quantile for which relationship is estimated. Only firms with smaller than average market capitalization included in the regressions.

second half of the distribution, or for firms that perform at or better than average. For bad-news firms, below the middle quantiles, the coefficient on spread is actually negative, for both large and small firms. For these firms, increased illiquidity has a negative effect on subsequent return. The coefficient on spread does increase, though, as θ increases - this is a highly significant result. Bid-ask spread is another firm-specific variable which uncovers the failings of traditional CAPM. There does not appear to be a large difference in the pattern of coefficients for large and small firms across quantiles, but big firms do have a larger positive coefficient on spread for firms that overperform and an even more negative coefficient for firms that underperform relative to the mean. The most notable difference between the pooled and cross-sectional results (c.f. Tables 6-8) for this factor is that at the median, for large firms, without pooling the 36 cross sections, the positive coefficient is not significant at the 5% level.

	$\theta = 10$	$\theta = 30$	$\theta = 50$	$\theta = 70$	$\theta = 90$	EOLS	EOLS(WC)	EWLS
beta	0.0276	0.0166	0.0036	0.0000	-0.0322	0.0133	0.0113	0.0136
z-stat	1.58	1.33	0.30	0.00	-1.95	1.48	1.13	1.52
bidask	-1073.31	-478.56	159.44	820.54	1641.88	218.55	202.89	219.84
z-stat	-9.38	-5.25	1.81	9.36	13.97	3.57	2.85	3.59
logsize	0.0078	0.0028	-0.0068	-0.0094	-0.0192	-0.0049	-0.0034	-0.0050
z-stat	1.18	0.56	-1.49	-1.95	-2.96	-1.16	-0.84	-1.16
finp	-0.0374	-0.0822	-0.0559	0.0207	0.0426	-0.0256	-0.0193	-0.0270
z-stat	-0.71	-1.88	-1.39	0.50	0.85	-0.89	-0.60	-0.94
junk	0.0421	0.0590	0.0610	0.0002	-0.0136	0.0325	0.0441	0.0328
z-stat	0.78	1.49	1.67	0.01	-0.29	1.21	1.46	1.22
mnt3	0.0930	0.0037	-0.0279	-0.0294	-0.0764	-0.0143	-0.0067	-0.0125
z-stat	1.37	0.08	-0.57	-0.61	-1.22	-0.40	-0.18	-0.35
mnt6	-0.0875	0.0871	0.0520	0.0316	0.0607	0.0456	0.0325	0.0450
z-stat	-0.96	1.50	0.98	0.60	0.90	1.14	0.73	1.12
slfh	-0.0185	0.0817	0.0894	0.1137	0.1265	0.0909	0.0929	0.0874
z-stat	-0.24	1.31	1.61	1.90	1.70	2.14	1.87	2.06
term	-0.0185	-0.1391	-0.0952	-0.0999	-0.1012	-0.0795	-0.0843	-0.0767
z-stat	-0.26	-2.66	-1.73	-1.62	-1.48	-2.04	-1.93	-1.97

Table 8: **Efficient-Weighted Quantile, OLS, OLS with White (1980) Correction, and WLS Regression Results for Large Firms.** Efficiently combined cross-section quantile, OLS, OLS using weights based on White (1980) heteroskedasticity corrected variance-covariance matrix and WLS regressions. We present coefficients and z-statistics. θ denotes quantile for which relationship is estimated. Only firms with larger than average market capitalization included in the regressions.

Overall, we see evidence that the results from the LS-based mean and the median estimators appear to be some summary or average measure of the multifaceted patterns revealed by quantile regression. In general, the 10% and 90% quantile-estimated coefficients tend to be much larger in absolute value and more significant than the median or mean (LS-based) estimated coefficients. In addition, these two extreme quantile estimates flip signs, meaning that extremely overperforming and underperforming firms have relationships between excess returns and firm-specific variables or factors that are qualitatively opposite in nature. The Monte Carlo simulation reported in Section 3 illustrates that for such cases where the sign flips across quantiles, the actual quantile plot is likely to be steeper than the estimated quantile plot. Finally, in the center of the distribution where the market price of beta risk is insignificant, both spread and size are significant.

Marketwide Factors. Numerous authors, including Breen, Glosten and Jagannathan (1989), Ferson and Harvey (1991) and Campbell (1987) follow the Fama and Schwert (1977) tradition and find a negative relationship between returns and financial paper (finp). It is clear from the quantile plot of the t-statistics (Figure 7) and from Tables 3 through 5 that this pattern is significant for large firms only in the bottom 10% of the conditional distribution for the empirical example considered here.[22] Small firms tend to exhibit a negative partial correlation between these variables for the top 70% of conditional returns, but the coefficients are largely insignificant. Overall, the small firms appear to dominate the pattern of the quantile plot since our sample largely consists of small firms. This negative relationship is significant at the median for all firms at the 10% level of significance.

In Figure 8, the quantile plot for the t-statistic on the coefficient on the three-month yield spread factor (mnt3), a measure of the short-maturity term structure, shows very little substantiating evidence for the Harvey (1989) and Ferson and Harvey (1991) predictions of a positive relationship between the three-month yield spread and returns. Mostly, the results indicate that there tends to be a positive but largely insignificant relationship for small firms (with some statistical significance around the center of the distribution) and a negative relationship for large firms which is significant at or just below the median (at the 30% and 50% quantiles). This is corroborated by the results in Tables 3 through 5. In contrast, Figure 9 demonstrates that for the six-month yield spread (mnt6), another measure of the term structure used by Campbell (1987), the coefficient is positive almost everywhere for large firms, and is generally significant for the first 50% of the conditional distribution. For small firms, the coefficient is largely insignificant (except for the 50% and 70% quantiles) and negative.

The default or junk bond premium, or the spread between Moody's Aaa and Baa yields (junk), is used as a measure of business conditions by Fama and French (1989) and as a measure of changes in the risk of corporate default by Ferson and Harvey (1991). Both of these studies find empirical support for a positive relationship between returns and the junk premium. Fama and French (1989) argue that the default premium captures time variation in expected stock returns as a result of changes in business conditions that endure longer than business cycles. They find that the default premium moves in opposition to business conditions, and argue that returns move in a similar way using a consumption-smoothing argument. They also note that some consumption-smoothing-based arguments yield the opposite prediction. From Figure 10 and Tables 3 through 5 it is apparent that their positive prediction holds true across most of the quantile plot for both large and small firms. However, in this empirical example, this is not a significant result. In fact, the only (slightly) significant results obtain for small and large firms in the 50^{th} percentile for the

[22] The extensive literature on the Fisher hypothesis in the context of stock returns demonstrates that such results can be sensitive to data frequency, so the results here may differ from those found in the literature due to our choice of data frequency.

Figure 7: FINP
Quantile Plot of T-statistic Against the Quantile. Pooled Quantile Regression with Fixed Time Effects

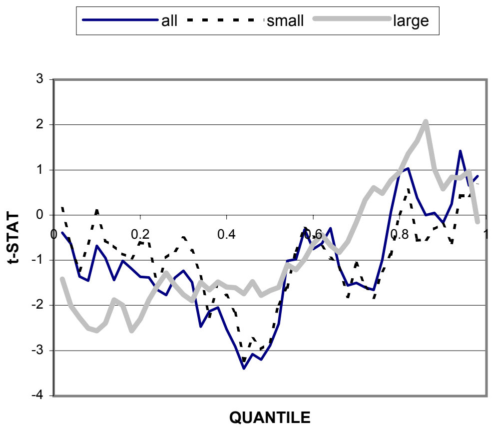


Figure 8: MNT3
Quantile Plot of T-statistic Against the Quantile. Pooled Quantile Regression with Fixed Time Effects

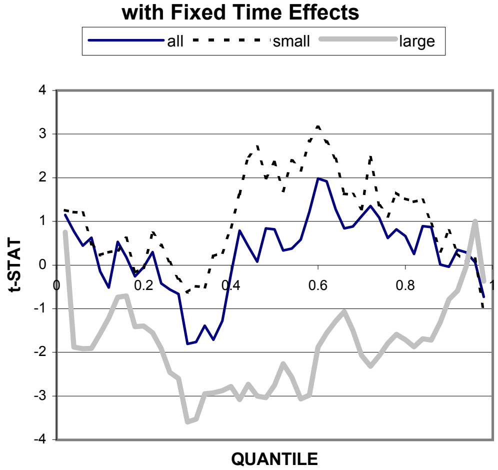


Figure 9: MNT6
Quantile Plot of T-statistic Against the Quantile. Pooled Quantile Regression with Fixed Time Effects

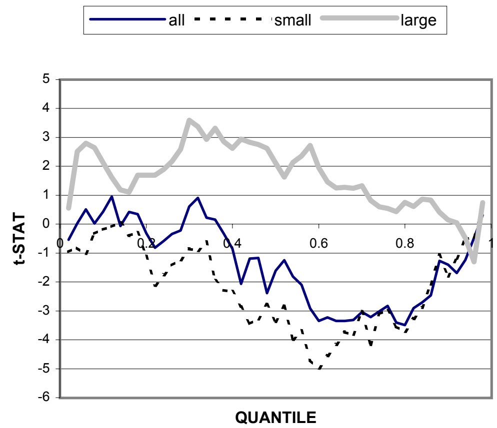


Figure 10: JUNK
Quantile Plot of T-statistic Against the Quantile. Pooled Quantile Regression with Fixed Time Effects

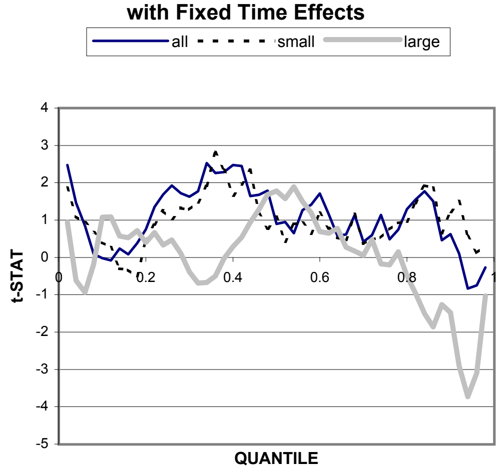


Figure 11: TERM
Quantile Plot of T-statistic Against the Quantile. Pooled Quantile Regression with Fixed Time Effects

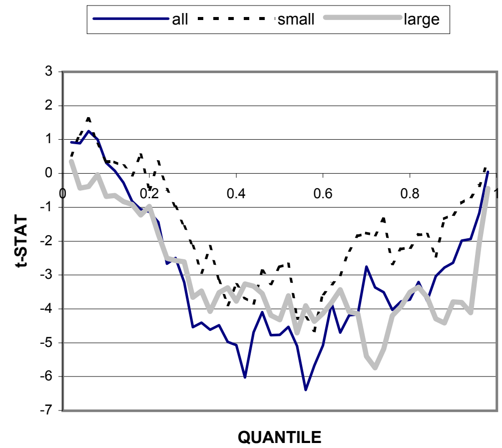
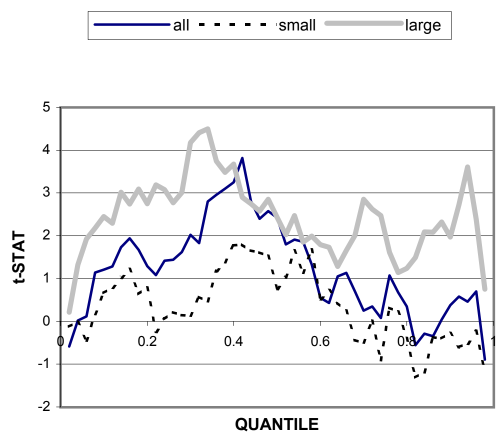


Figure 12: SLFH
Quantile Plot of T-statistic Against the Quantile. Pooled Quantile Regression with Fixed Time Effects



efficient-weighted 36 cross sections, with a negative coefficient on this factor for small firms and a positive coefficient for large firms.

Fama and French (1989) similarly argue that the term spread (term), which can be measured as the difference between long-term and short-term bond yields, also varies inversely with business conditions, again implying a positive relationship between expected returns and the term premium. Figure 11, the quantile plot for the t-statistic of the coefficient on the term spread factor, demonstrates that the price of this source of risk is largely negative across the conditional distribution and is largely significant for the central 40% of the distribution. Both large and small firms display negative coefficients on this risk factor, but these coefficients are significant only for large firms. In comparing Tables 3 through 5 with Tables 6 through 8, the most striking difference between the results of these pooled regressions and the efficient-weighted 36 cross sectional regressions is that in the latter, a significant and positive relationship occurs for small firms in the bottom 10% of performers.

A measure of the changing slope of the Treasury yield curve, the difference between the yield of a ten-year Treasury bond and a three-month Treasury bill (slfh), is also used as an economic risk variable by Ferson and Harvey (1991). Although they find that the relationship between the price of this type of risk and returns is significant and positive, we find that the coefficient on this variable is largely insignificant across the entire conditional distribution for small firms and for all firms overall. It is positive for large firms and underperforming small firms and is significant and positive only for large firms, with larger coefficients and t-statistics for underperforming firms.

In summary, many of the marketwide variables considered here are statistically significant, at least over some range of the conditional distribution of returns, and, as with the firm-specific variables, the coefficients tend to take on both negative and positive values, depending on the quantile. In other words, the quantile regression approach supports qualitatively different theoretical predictions simultaneously. In fact, some of the previously discovered empirical relationships between marketwide conditioning variables and returns are further substantiated when quantile methods are used as the second pass estimator, but these results generally do not hold, even weakly, over the entire distribution. Again, the existing literature exclusively analyzes these relationships at the center of the conditional distribution, so it is not surprising that more intricate patterns are revealed when these relationships are analyzed for other parts of the distribution. What is apparent is that, taken together, many of these marketwide conditioning variables are significantly related to the cross section of returns over some part of the quantile plot and jointly contribute more evidence against unconditional single-factor CAPM.

The fact that we find joint significance but less evidence of individual significance may indicate the presence of multicollinearity. A cursory glance at the correlation matrix contained in Table 9 indicates that the marketwide factors are all positively correlated with each other and the magnitudes are larger for the marketwide factors than for the firm-specific

	beta	bidask	logsize	finp	junk	mnt3	mnt6	slfh	term
beta	1								
bidask	0.225	1							
logsize	0.257	-0.271	1						
finp	0.048	0.158	-0.022	1					
junk	-0.023	0.078	-0.015	0.562	1				
mnt3	0.038	0.105	-0.006	0.401	0.382	1			
mnt6	0.028	0.098	0.002	0.408	0.473	0.838	1		
slfh	0.019	0.100	-0.002	0.561	0.583	0.379	0.406	1	
term	0.049	0.130	-0.012	0.631	0.520	0.380	0.391	0.760	1

Table 9: **Correlation Matrix for the Independent Factors.** Correlation matrix for the independent variables used in the analysis. See Section 3 for variable descriptions. Coefficients are based on the 36 pooled cross sections.

factors.[23]

Finally, since joint interquantile tests support the notion that the coefficients on these risk factors vary significantly across the distribution, the version of conditional multi-factor CAPM considered here is also rejected since this implies the market price of risk for these factors differs across quantiles and hence firms, whereas theory posits that the market price of any factor should be the same for all firms. Whether the rejection is due to inherent heterogeneity across firms or heteroskedasticity unrelated to such economically-determined heterogeneity is an open question.[24]

Comparison of Quantile Methods with LS-based Approaches. In Tables 3 through 5 we compare the results of using quantile regression methods to the results of using LS-based methods (OLS, White-corrected OLS (OLS-WC), and WLS) for the pooled 36 cross sections. Similarly, in Tables 6 through 8, the efficient-weighted quantile regression method is compared to the efficient-weighted LS-based methods (EOLS, White-corrected EOLS (EOLS-WC), and EWLS) for the 36 different cross sections. Focusing the discussion on the firm-specific factors will ensure a brief and clear discussion.

Generally, the expected results are manifested and hold for both the 36 efficient-weighted cross sections as well as for the pooled regressions. The White-corrected OLS and EOLS-WC have smaller t-statistics than OLS and EOLS respectively, and WLS and EWLS estimates are more significant and often larger in size than those from the OLS (EOLS) and OLS-WC (EOLS-WC) estimators. The median regression estimate is typically smaller and

[23] The highest correlations are between the mnt3 and mnt6 factors (0.838), between the term and the slfh factors (0.760) and between the term and finp factors (0.631).

[24] Of course, the classical CAPM test would likely reject our model as well, since the market price of risk is insignificant around the center of the conditional distribution of returns.

less significant than those from the mean regression estimators, which is notable since the median regression estimator is most robust to EIV bias, outliers, and nonnormality.

6. Conclusion

Using quantile methods in the second pass of the FM procedure uncovers a potential reason why so many researchers have found conflicting or inconclusive evidence regarding the importance of beta in explaining the cross section of returns: around the mean of the distribution, beta 'flops around' either side of zero and is insignificant; for both overperforming and underperforming firms, however, beta is strongly significant though with contrasting signs. Previous studies concentrate their efforts on the central tendency of conditional returns, and thus overlook those regions of the distribution where beta has bite.

By using quantile methods, some additional light is shed on other controversies such as the relevance of firm size and the negative relationship between returns and financial paper. Here we discover that size is significant and positive for firms in the bottom 50% of conditional returns (that is, for firms who receive negative idiosyncratic shocks or bad news of varying magnitude), and significant and negative for the top 25% of overperforming (or good news) firms. This provides new empirical support for Merton's (1987) prediction that size and returns are positively related and is in contrast to many of the empirical findings reported in the literature. In addition, overperforming firms have a positive and sometimes significant association between returns and financial paper, and underperforming firms tend to have a negative and sometimes significant relationship in the empirical example considered here.

In this paper we establish that the quantile regression method is a statistically viable and appropriate way of analyzing the cross section of returns. One of the implications of the failure of the interquantile tests to find parameter homogeneity across quantiles is that the multifactor conditional CAPM considered here is rejected, since the prices of risk factors, including beta risk, systematically change across the conditional distribution of returns. A more profound implication is that we open the door for further theoretical exploration of *why* such risk factors should be priced differently, depending on firm size and whether the firm received good, bad or indifferent news during the sample period.

References

Ahn, Seung C. and Christopher Gadarowski. (1999) Two-Pass Cross-Sectional Regression of Factor Pricing Models: Minimum Distance Approach, Working Paper, Department of Economics, Arizona State University.

Amihud, Yakov and Haim Mendelson. (1986) Asset Pricing and the Bid-Ask Spread, *Journal of Financial Economics*, vol. 15(2), pp. 223-49.

Amihud, Yakov and Haim Mendelson. (1989) The Effects of Beta, Bid-Ask Spread, Residual Risk and Size on Stock Returns, *Journal of Finance*, vol. 44(2), pp. 479-486.

Banz, Rolf W. (1981) The Relationship between Return and Market Value of Common Stocks, *Journal of Financial Economics*, vol. 9(1), pp. 3-18.

Berk, Jonathan B. (1995) A Critique of Size-Related Anomalies, *Review of Financial Studies*, vol. 8(2), pp.275-286.

Bhandari, Laxmi Chand. (1988) Debt/Equity Ratio and Expected Common Stock Returns: Empirical Evidence, *Journal of Finance*, vol. 43(2), pp. 507-28.

Black, Fischer. (1972) Capital Market Equilibrium with Restrictive Borrowing, *Journal of Business*, vol. 45(3), pp. 444-55.

Blume, Marshall. E. (1970) Portfolio Theory: A Step toward Its Practical Application, *Journal of Business*, vol. 43(2), pp. 152-73.

Breen, William, Glosten Lawrence R., and Ravi Jagannathan. (1989) Economic Significance of Predictable Variations in Stock Index Returns, *Journal of Finance*, vol. 44(5), pp. 1177-1189.

Buchinsky, Moshe. (1995) Estimating the Asymptotic Covariance Matrix for Quantile Regression Models: A Monte Carlo Study, *Journal of Econometrics*, vol. 68(2), pp. 303-38.

Buchinsky, Moshe and Jennifer Hunt. (1996) Wage Mobility in the United States, National Bureau of Economic Research Working Paper No. 5455.

Buchinsky, Moshe and Phillip Leslie. (1997) Educational Attainment and the Changing US Wage Structure: Some Dynamic Implications, Brown University, Department of Economics Working Paper.

Buchinsky, Moshe. (1998) Recent Advances in Quantile Regression Models: A Practical Guideline for Empirical Research, *Journal of Human Resources*, vol. 33(1), pp. 88-126.

Campbell, John Y. (1987) Stock Returns and the Term Structure, *Journal of Financial Economics*, vol. 18(2), pp. 373-99.

Campbell, John Y., Lettau, Martin, Malkiel, Burton G., and Yexiao Xu. (2001) Have Individual Stocks Become More Volatile? An Empirical Exploration of Idiosyncratic Risk, *Journal of Finance*, vol. 56(1), pp. 1-43.

Chan, Louis K. C., Hamao, Yasushi and Joseph Lakonishok. (1991) Fundamentals and Stock Returns in Japan, *Journal of Finance*, vol. 46(5), pp. 1739-64.

Chen, Hsiu-lang and Gilbert Bassett. (2002) Attribution in the Fama-French 3-Factor Model, Working Paper, Department of Finance, University of Illinois at Chicago.

Eide, Eric and Mark H. Showalter. (1998) The Effect of School Quality on Student Performance: A Quantile Regression Approach, *Economic Letters*, vol. 58(3), pp. 345-50.

Eide, Eric and Mark H. Showalter. (1999) Factors Affecting the Transmission of Earnings Across Generations: A Quantile Regression Approach, *Journal of Human Resources*, vol. 34(2), pp. 253-67.

Engle, Robert F. and Simone Manganelli. (1999) CAViaR: Conditional Value at Risk by Quantile Regression, National Bureau of Economic Research Working Paper No. 7341.

Engsted, Tom. (1998) Evaluating the Consumption-Capital Asset Pricing Model Using Hansen-Jagannathan Bounds: Evidence from the UK, *International Journal of Finance and Economics*, vol. 3(4), pp. 291-302.

Fama, Eugene F. and Kenneth R. French. (1989) Business Conditions and Expected Returns on Stocks and Bonds, *Journal of Financial Economics*, vol. 25(1), pp. 23-49.

Fama, Eugene F. and Kenneth R. French. (1992) The Cross-Section of Expected Stock Returns, *Journal of Finance*, vol. 47(2), pp. 427-65.

Fama, Eugene F. and James D. MacBeth. (1973) Risk, Return, and Equilibrium: Empirical Tests, *Journal of Political Economy*, vol. 81(3), pp. 607-36.

Fama, Eugene F. and William Schwert. (1977) Asset Returns and Inflation, *Journal of Financial Economics*, vol. 5(2), pp. 115-46.

Ferson, Wayne E. and Campbell R. Harvey. (1991) The Variation of Economic Risk Premiums, *Journal of Political Economy*, vol. 99(2), pp. 385-415.

Ferson, Wayne E. and Campbell R. Harvey. (1999) Conditioning Variables and the Cross Section of Stock Returns, *Journal of Finance*, vol. 54(4), pp. 1325-60.

Gibbons, Michael R. (1982) Multivariate Tests of Financial Models: A New Approach, *Journal of Financial Economics*, vol. 10(1), pp. 3-27.

Gibbons, Michael R., Ross, Stephen. A., and Jay Shanken. (1989) A Test of the Efficiency of A Given Portfolio, *Econometrica*, vol. 57(5), 1121-52.

Goyal, Amit, and Pedro Santa-Clara. (2002) Idiosyncratic Risk Matters!, Working Paper, Anderson Graduate School of Management, University of California at Los Angeles.

Hansen, Lars Peter and Ravi Jagannathan. (1991) Implications of Security Market Data for Models of Dynamic Economies, *Journal of Political Economy*, vol. 99(2), pp. 225-62.

Harvey, Campbell. R. (1989) Time-Varying Conditional Covariances in Tests of Asset Pricing Models, *Journal of Financial Economics*, vol. 24(2), pp. 289-317.

Jagannathan, Ravi, and Zhenyu Wang. (1996) The Conditional CAPM and the Cross-Section of Expected Returns, *Journal of Finance*, vol. 51(1), pp. 3-53.

Jagannathan, Ravi and Zhenyu Wang. (1998) An Asymptotic Theory for Estimating Beta-Pricing Models Using Cross-Sectional Regression, *Journal of Finance*, vol. 53(4), pp. 1285-1309.

Koenker, Roger W. (2000) Galton, Edgeworth, Frisch, and Prospects for Quantile Regression in Econometrics, *Journal of Econometrics*, vol. 95(2), pp. 347-74.

Koenker, Roger W. and Gilbert Basset Jr. (1978) Regression Quantiles, *Econometrica*, vol. 46(1), pp. 33-50.

Koenker, Roger W. and Gilbert Basset Jr. (1982) Robust Tests for Heteroscedasticity Based on Regression Quantiles, *Econometrica*, vol. 50(1), pp. 43-61.

Koenker, Roger W. and Kevin F. Hallock. (2001) Quantile Regression, *Journal of Economic Perspectives*, vol. 15(4), pp. 143-56.

Koenker, Roger W. and Jose A. F. Machado. (1999) Goodness of Fit and Related Inference Processes for Quantile Regression, *Journal of the American Statistical Association*, vol. 94(448), pp. 1296-1310.

Kothari, S. P., Shanken, Jay and Richard G. Sloan. (1995) Another Look at the Cross-Section of Expected Stock Returns, *Journal of Finance*, vol. 50(1), pp. 185-224.

Lintner, John. (1965) The Valuation of Risk Asset and the Selection of Risky Investments in Stock Portfolios and Capital Budgets, *Review of Economics and Statistics*, vol. 47(1), pp. 13-37.

Litzenberger, Robert. H., and Krishna Ramaswamy. (1979) The Effect of Personal Taxes and Dividends on Capital Asset Prices: Theory and Empirical Evidence, *Journal of Financial Economics*, vol. 7(2), pp. 163-96.

Merton, Robert C. (1987) A Simple Model of Capital Market Equilibrium with Incomplete Information, *Journal of Finance*, vol. 42(3), pp. 483-510.

MacKinlay, A. Craig and Matthew P. Richardson. (1991) Using Generalized Method of Moments to Test Mean-Variance Efficiency, *Journal of Finance*, vol. 46(2), pp. 511-27.

Morillo, Daniel S. (2000) Monte Carlo American Option Pricing with Nonparametric Regression, in: *Essays in Nonparametric Econometrics*, Dissertation, University of Illinois.

Mossin, Jan. (1966) Equilibrium in a Capital Asset Market, *Econometrica*, vol. 34(4), pp. 768-83.

Shanken, Jay. (1992) On the Estimation of Beta Pricing Models, *Review of Financial Studies*, vol. 5(1), pp. 1-33.

Sharpe, William. (1964) Capital Asset Prices: A Theory of Market Equilibrium Under Conditions of Risk, *Journal of Finance*, vol. 19(3), pp. 425-42.

White, Halbert. (1980) A Heteroskedasticity-Consistent Covariance Matrix Estimator and a Direct Test for Heteroskedasticity, *Econometrica*, vol. 48(4), pp. 817-38.

Appendix 1
Data Descriptions

Firm-Specific Data (Source is Center for Research in Security Prices - CRSP)

Bidlo: The lowest sale or closing bid on the day.

Askhi: The highest sale or closing ask on the day.

Price: The closing price or the negative bid/ask average on the day.

Return: The change in the total value of an investment in a common stock over some period of time per dollar of initial investment. It is the holding period return for a sale on that day. It is based on a purchase on the most recent time previous to the current day when the security had a valid price (usually the day before).

Marketwide Data (Sources are CRSP and the Federal Reserve's Board of Governors' web page)

Market: The return on the value-weighted market portfolio, constructed by CRSP.

finp: Financial paper. The proxy for the risk-free rate, defined as directly placed, unsecured, short-term negotiable promissory notes. The yield is the unweighted average of offering rates reported each business day to the Federal Reserve Bank of New York.

junk: The yield on Moody's Baa-rated corporate bonds less the yield on Aaa-rated corporate bonds. Aaa and Baa types of yield are defined as average yield to maturity on selected long-term bonds.

slfh: Ten-year constant maturity T-bond yield less the three-month constant maturity T-bill rate.

term: The spread between a ten-year and a one-year T-bond yield, ten-year yield minus one-year yield. Constant maturity yields on Treasury securities at constant, fixed maturity are constructed by the Treasury Department, based on the most actively traded marketable Treasury securities. Yields on these issues are based on composite quotes reported by U.S. government securities dealers to the Federal Reserve Bank of New York. To obtain the constant maturity yields, personnel at Treasury construct a yield curve each business day and yield values are then read from the curve at fixed maturities.

mnt6: Six-month Treasury constant maturity yield less the one month yield (finp). Yields on Treasury securities at constant, fixed maturity are constructed by the Treasury Department, based on the most actively traded marketable Treasury securities. Yields on these issues are based on composite quotes reported by U.S. government securities dealers to the Federal Reserve Bank of New York.

mnt3: Same as above, three-month yield less the one-month yield.

Dummy Variables for Calendar and Trading Effects

Dummy variables: DV1-4 are dummies for Tuesday through Friday; DV5 is the square root of the gap between trading days; DV6-14 are dummies for months March through November; and DV15-22 are dummies for weeks of the month for December and January.