Are Taste and Technology Parameters Stable? A Test of “Deep” Parameter Stability in Real Business Cycle Models of the U.S. Economy

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Federal Reserve Bank of Boston Working Paper No. 01-5

Original Version: November 2001
This Version: March 2002

Lucas (1976) criticized Keynesian models because of parametric instability. For many economists, the “Lucas critique” provided a reason to replace the Keynesian approach with equilibrium models of the business cycle. One example of the equilibrium approach is the Real Business Cycle (RBC) model. However, RBC models have not been subjected to the same scrutiny of tests of parameter instability. We present a prototypical RBC model and subject it to structural change tests. Our results document evidence of extensive parameter instability. Thus, the “Lucas critique” is applicable and we conclude that the single representative agent class of RBC models has an invalid structural form.

JEL Classification: C32, E1, E13, E32

Keywords: Business Cycles, Real Business Cycle Models, Neoclassical Growth Model, Stochastic Growth Models.

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I. Introduction

As a result of the "Lucas critique" (1976) of parametric instability in Keynesian macroeconomic models, many economists turned to the study of business cycles using equilibrium models of a market economy. Borrowing from the neoclassical growth literature of Solow (1956), Cass (1965), Koopmans (1965), and Brock and Mirman (1972), the "new classical" economists modified the static deterministic equilibrium models that existed prior to Keynes by incorporating dynamic optimizing behavior, stochastic shocks, and forward-looking "rational" expectations. One example of the "new classical" tradition is the Real Business Cycle (RBC) paradigm as presented in the seminal works of Kydland and Prescott (1982) and Long and Plosser (1983). However, RBC models also rely on constant parameter assumptions – the justification is that RBC models contain "deeper" structural parameters than their Keynesian predecessors because the "deep" parameters in RBC models describe the basic concepts of preferences and technology. In this paper, we test the critical assumption that the "deep" parameters describing tastes and technology in prototypical RBC models are constant.

The crux of the "Lucas critique" was that the assumed structural equations of Keynesian models were not really structural, but instead were semi-reduced-form equations. Since the parameters of a reduced-form model are functions of structural parameters, reduced-form equations exhibit parameter instability whenever the more basic structural relationships undergo change. This makes it imperative for economists to uncover the "true" structural form of a model, so that only structural parameters are estimated. Specifically, the problem with the old Keynesian models was that semi-reduced-form parameters were not invariant to changes in government policy. However, it should be emphasized that any economic model, not just Keynesian macro models, may be subject to the more general "Lucas critique" that reduced-form equations exhibit parameter instability, while only the "true" structural equations contain parameters that are invariant over time. Therefore, the assumed structural-form of the "new classical" models also should be subjected to the scrutiny of tests for parameter stability.

However, the "prior" that basic taste and technology parameters are constant is so pervasive that there is very little literature examining the veracity of this assumption, and the literature that does exist assesses parameter stability only within a single Euler-equation; for example, see Oliner, Rudebusch, and Sichel (1996), Nakamura (1996), Saikkonen and Ripatti (2000), and Ghysels and Hall (1990). In part, the paucity of literature questioning the constant parameter assumption is due to the many studies simulating RBC models calibrated with parameter estimates drawn from the literature. However, the use of RBC models for forecasting and policy analysis requires the econometric estimation of "deep" structural parameters from time series data, and these parameter estimates should be tested for stability. There is a small literature econometrically estimating the structural parameters of RBC models before conducting such simulations, but these studies simply assume that "deep" parameters are constant (Altug, 1989; Christiano and Eichenbaum, 1992; Burnside, Eichenbaum, and Rebelo, 1993; McGrattan, 1994; Burnside and Eichenbaum, 1996; McGrattan, Rogerson, and Wright, 1997). This is the first study to analyze the question of parameter stability in RBC models, with the result that we find evidence of extensive parameter instability.

The plan for this paper is as follows. In section II, we present a prototypical RBC model. In section III, we describe our estimation methodology. In section IV, we describe our data. In section V, we discuss the results of our analysis. In section VI, we discuss the very far reaching implications of our results. In section VII, we present our conclusions.

II. A Prototypical Real Business Cycle Model

In this section, we present a prototypical RBC model. The basic RBC model used in the literature was first presented in Kydland and Prescott (1982). However, Prescott (1986) pointed out that the heart of all RBC models is the prototype of a one-sector stochastic neoclassical growth model. The stochastic neoclassical growth model contains the following features: a representative consumer that optimizes discounted utility from a stream of per-capita consumption services; an owner that optimizes the profits of a representative firm producing output with a technology subject to stochastic shocks; and a definition of competitive market equilibrium. RBC models typically add the choice of a stream of leisure services to the consumer's optimization problem in order to analyze the stochastic behavior of a perfectly functioning labor market. While this addition is necessary to study the cyclical behavior of the economy, it is not a necessary element of the prototypical stochastic neoclassical growth model.

In the prototypical model that we present below, all variables are de-trended per-capita variables, where the population is expressed in terms of efficiency units of labor. Thus, output is $\tilde{y} \equiv Y/\tilde{L}$; consumption is $\tilde{c} \equiv C/\tilde{L}$; capital is $\tilde{k} \equiv K/\tilde{L}$; and assets are $\tilde{a} \equiv A/\tilde{L}$. Units of labor are denoted by L and efficiency units of labor by $\tilde{L} \equiv HL$; where $H_t \equiv H_0 e^{\gamma t + \xi_t}$, $\xi_t = \xi_{t-1} + \varepsilon_t$, $\varepsilon_t \sim N(0, \sigma_\varepsilon)$, and γ is the exogenously specified rate of Harrod-neutral technological change.

II.1 The Optimization Problem for a Representative Consumer

In the neoclassical growth model, the representative consumer maximizes discounted utility from a stream of per-capita consumption services: $\tilde{c}_{t+j} \quad \forall j = 0, \ldots, \infty$. We assume the representative consumer's utility function takes the form given in equation (1):

(1) $$U(\tilde{c}_{t+j}) \equiv \frac{\tilde{c}_{t+j}^{1-\theta} - 1}{1-\theta},$$

where: θ is the instantaneous elasticity of intertemporal substitution.

The consumer's problem is to maximize a discounted infinite sum of the utility function in equation (1), subject to a constraint on resources. The consumer's objective function is

(2) $$E_t\left[\sum_{j=0}^{\infty} \beta^{t+j} U(\tilde{c}_{t+j})\right]$$

where: β is the net discount rate, such that $\beta \equiv \dfrac{1}{1 + (\rho - \gamma - \eta)}$,

ρ is the consumer's rate of time-preference, and

η is the rate of growth in the labor force.

The constraint on the consumer's resources, or per-capita assets, $\tilde{a}_{t+j} \quad \forall j = 0, \ldots, \infty$, is:

$$(3)\quad E_t\left[\sum_{j=0}^{\infty}\beta^{t+j}\Delta\tilde{a}_{t+j}\right]=E_t\left[\sum_{j=0}^{\infty}\beta^{t+j}\left(w_{t+j}+\left[r_{t+j}-(\eta+\gamma)\right]\tilde{a}_{t+j}-\tilde{c}_{t+j}\right)\right].$$

II.2 The Optimization Problem for the Representative Firm:

The representative firm solves an optimization problem that is identical in every time period, with no inter-temporal interconnections. The firm maximizes residual profits, which equal real output less factor payments, with the price of output normalized to one. The firm produces output according to a linearly homogenous Cobb-Douglas production function, which takes the intensive form:

$$(4)\quad \tilde{y}_{t+j}=\phi(\tilde{k}_{t+j};\ln v_{t+j})\equiv Ae^{\ln v_{t+j}}\tilde{k}_{t+j}^{\alpha},$$

where: $\ln v_{t+j+1}=\omega\ln v_{t+j}+\ln u_{t+j+1}$, $\ln u_{t+j+1}\sim N(0,\sigma^2_{\ln u})\quad\forall t, j$, and, $A\equiv H_0^{\alpha}$.

The firm's problem is to maximize profits:

$$(5)\quad \underset{\tilde{k}_{t+j}}{Max}\qquad \pi_{t+j}=\phi(\tilde{k}_{t+j};\ln v_{t+j})-\left(r_{t+j}+\delta\right)\tilde{k}_{t+j}-w_{t+j}.$$

II.3 Equilibrium Conditions

If we differentiate equation (5) with respect to $\tilde{k}_{t+j}$ and set it equal to zero, this yields the firm's profit-maximizing condition and the equilibrium rate of return to capital; the marginal product of capital is equal to the real gross rate of return to capital:

(6) $\phi'(\tilde{k}_{t+j};\ln v_{t+j})=\left(r_{t+j}+\delta\right)$ or, equivalently, $\phi'(\tilde{k}_{t+j};\ln v_{t+j})-\delta=r_{t+j}$.

The assumption of perfectly competitive product and factor markets ensures that in long-run equilibrium, maximized profits are zero. Thus, set equation (5) to zero and substitute equation (6) into equation (5) for the real gross rate of return to capital; this yields the equilibrium wage rate:

$$(7)\quad w_{t+j}=\phi(\tilde{k}_{t+j};\ln v_{t+j})-\phi'(\tilde{k}_{t+j};\ln v_{t+j})\tilde{k}_{t+j}.$$

The final equilibrium condition is that the value of financial assets, $\tilde{a}_{t+j}$, must be equal to the value of the physical assets backing them up, $\tilde{k}_{t+j}$. This condition is:

(8) $\tilde{a}_{t+j} = \tilde{k}_{t+j}$.

II.4 The Government Planner's Problem

Substitute the equilibrium conditions represented by equations (6), (7), and (8) into the consumer's resource constraint in equation (3); we obtain the resource constraint for the aggregate economy. We list this aggregate resource constraint in equation (9).

$$(9)\quad E_t\left[\sum_{j=0}^{\infty}\beta^{t+j}\Delta\tilde{k}_{t+j}\right] = E_t\left[\sum_{j=0}^{\infty}\beta^{t+j}\left(\phi(\tilde{k}_{t+j};\ln v_{t+j}) - \tilde{c}_{t+j} - (\eta+\gamma+\delta)\tilde{k}_{t+j}\right)\right].$$

The government planner seeks to optimize the social welfare of consumers, subject to the aggregate resource constraint for the economy. The welfare function for the representative consumer is given by equation (2). Combining equation (2) with the aggregate resource constraint in equation (9) gives us the government planner's problem. Substitute equations (1) and (4) representing the actual functional forms into the generic planner problem given by equations (2) and (9); we obtain the specific problem to be optimized by the planner:

$$(10)\quad \underset{\tilde{c}_{t+j},\tilde{k}_{t+j+1}}{Max}\quad E_t\left[\sum_{j=0}^{\infty}\beta^{t+j}\frac{\tilde{c}_{t+j}^{1-\theta}-1}{1-\theta}\right],$$

$$\text{subject to: } E_t\left[\sum_{j=0}^{\infty}\beta^{t+j}\Delta\tilde{k}_{t+j}\right] = E_t\left[\sum_{j=0}^{\infty}\beta^{t+j}\left(Ae^{\ln v_{t+j}}\tilde{k}_{t+j}^{\alpha} - \tilde{c}_{t+j} - (\eta+\gamma+\delta)\tilde{k}_{t+j}\right)\right].$$

II.5 Approximate Optimal Decision Rules

In general, there is no analytic solution to this problem. However, we follow the approach of King, Plosser, and Rebelo (1988) and take a log-linear, first-order Taylor-series approximation to the first-order necessary conditions of the planner's problem in equation (10). The three first-order conditions of this problem can be reduced to two by substituting out for the Lagrangian multiplier, with the two remaining conditions being used to derive the Taylor-series approximation. These two conditions are the Euler-equation of equation (11) and the single-period resource constraint in equation (12).

(11) $$\frac{\Delta\tilde{c}_t}{\tilde{c}_t} = \frac{1}{\theta[1+(\rho-\gamma-\eta)]} E_t\left[\phi'(\ln\tilde{k}_{t+1};\ln v_{t+1}) - (\rho+\delta)\right] = 0 \text{, and}$$

(12) $$\frac{\Delta\tilde{k}_t}{\tilde{k}_t} = \frac{\phi(\ln\tilde{k}_t;\ln v_t)}{\tilde{k}_t} - \frac{\tilde{c}_t}{\tilde{k}_t} - (\eta+\gamma+\delta) = 0 .$$

The Taylor-series approximation is taken around the logarithm of the non-stochastic steady-state point. This shortcut obtains the same log-linear decision rules as if the planner's problem in equation (10) were a log-linear-quadratic optimization problem. The decision rule for the single endogenous state variable, $\ln\tilde{k}_{t+1}$, is:

(13) $$\ln\tilde{k}_{t+1} = (1-\mu_1)\ln\tilde{\bar{k}} + \mu_1\ln\tilde{k}_t + \vartheta\ln v_t ,$$

where: μ_1, ϑ are non-linear functions of the underlying structural parameter vector, Ψ:

$$\Psi = [\rho, \theta, \eta, H_0, \alpha, \gamma, \delta, \omega, \sigma_{\ln u}, \sigma_\varepsilon] .$$

The law of motion for the single exogenous state variable is given by equation (14):

(14) $$\ln v_{t+1} = \omega\ln v_t + \ln u_{t+1} .$$

III. Estimation Methodology

Our econometric approach is similar to Christiano and Eichenbaum (CE) (1992), who employ Hansen's (1982) generalized method of moments (GMM) estimator. GMM contains a very broad class of consistent estimators, including such commonly used estimators as OLS. The CE (1992) approach equates first-moments of both the data and the model, specifying a sufficient number of these conditions such that the structural parameters are just identified. Our task is similar, except we embed an endogenous break-point finding procedure into the GMM estimator. The first-moment conditions reveal a system of regression equations.

To evaluate parameter instability, we use the Bai (1997) and Bai and Perron (1998) endogenous break-point finding procedure. This procedure employs a Sup-F test – the maximum of a sequence of Wald F-statistics – and estimates both the break date and the value of the parameter change simultaneously. To find a parameter break, compute a Wald F-statistic (using a consistent estimate of the unconstrained residual variance) for every data point in an end-point restricted sample. Compute the maximum of the sequence of Wald F-statistics and record the value of the Sup-F statistic and the date on which it occurs. If the Sup-F statistic does not exceed a stipulated critical value, the procedure stops, since we cannot reject the hypothesis of parameter stability. If the Sup-F statistic exceeds the stipulated critical value, a break-point occurs at this date. When a break-point is found, the full-sample is divided into two sub-samples at the break-date and the procedure is repeated for each of the sub-samples. If additional break-points are found, these divide the full sample into additional sub-samples and each sub-sample is searched until no more break-points are found. The Bai and Perron procedure is valid under very general conditions of serial correlation in the equation's error terms.

Let us define the variables in the first-moment conditions. Per-capita variables are: output, $y \equiv Y / L$; consumption, $c \equiv C / L$; capital, $k \equiv K / L$; gross investment, $i^g \equiv I^g / L$; and net investment, $i^n \equiv I^n / L$. De-trended per-capita variables are as previously defined. The logarithmic first-difference of labor is $dN_t \equiv \ln L_t - \ln L_{t-1}$; the logarithmic first-difference of de-trended per-capita consumption is $d\tilde{c}_t \equiv \ln \tilde{c}_t - \ln \tilde{c}_{t-1}$; the logarithmic first-difference of per-capita output is $dy_t \equiv \ln y_t - \ln y_{t-1}$; and a depreciation rate series is $D_t \equiv \frac{k_t + i_t^g}{k_t} - \frac{k_t + i_t^n}{k_t}$. We denote the ex-ante real interest rate by *r*, and a vector of ones by ι. Dummy variables are used to delineate break-points and are denoted by d_j, for j = 1,…,M, where M is the number of breaks.

The system of regression equations that are estimated is given by equations (15) through equation (23). For each equation, the error term is denoted by e_p, where p is the specific parameter being estimated. For equations (15) through (19) and equations (21) and (23), the first-moment conditions are $E(e_p) = 0 \quad \forall p$, and the orthogonality conditions are $E(\iota' e_p) = 0 \quad \forall p$ and $E(d_j' e_p) = 0 \quad \forall p,\ j = 1, \ldots, M_p$. For equation (20), the first moment condition is $E(e_\theta) = 0$, and the orthogonality conditions are: $E(x_1' e_\theta) = 0$ and $E((d_j x_1)' e_\theta) = 0 \quad \forall\ j = 1, \ldots, M_\theta$. For equation (22), the first moment condition is: $E(\ln u) = 0$ and the orthogonality conditions are $E(\ln v_{t-1}' \ln u) = 0$ and $E((d_j \ln v_{t-1})' \ln u) = 0 \quad \forall\ j = 1, \ldots, M_\omega$.

The growth rate of the labor force, η, is estimated according to:

$$(15) \quad dN = \eta_0 \iota + \sum_{j=1}^{M_\eta} \partial\eta_j d_j^\eta + e_\eta .$$

The depreciation rate, δ, is estimated according to:

$$(16) \quad D = \delta_0 \iota + \sum_{j=1}^{M_\delta} \partial\delta_j d_j^\delta + e_\delta .$$

The Harrod-neutral technological growth rate, γ, is estimated according to:

$$(17) \quad dy = \gamma_0 \iota + \sum_{j=1}^{M_\gamma} \partial\gamma_j d_j^\gamma + e_\gamma .$$

The capital share, α, is estimated according to a logarithmic version of equation (6), obtained by multiplying both sides of equation (6) by the capital-output ratio, k / y . The estimating equation is:

$$(18) \quad \frac{(r + \delta_0 + \sum_{j=1}^{M_\delta} \partial\delta_j d_j^\delta) k}{y} = \alpha_0 \iota + \sum_{j=1}^{M_\alpha} \partial\alpha_j d_j^\alpha + e_\alpha .$$

The consumer's rate of time preference, ρ, is estimated from a modified version of the Euler-equation in equation (11). Substitute the right-hand side of equation (6) into the right-hand side of equation (11) for the marginal product of capital. This yields the first-moment condition, $E(r - \rho) = 0$, from which the implied estimating equation is:

$$(19) \quad r = \rho_0 \iota + \sum_{j=1}^{M_\rho} \partial\rho_j d_j^\rho + e_\rho .$$

The consumer's instantaneous elasticity of intertemporal substitution, θ, is also estimated from the modified version of the Euler-equation in equation (11). Define:

$$x_1 \equiv \frac{e_\rho}{\left[1+\left(\rho_0 + \sum_{j=1}^{M_\rho} \partial\rho_j d_j^\rho - \gamma_0 - \sum_{j=1}^{M_\gamma} \partial\gamma_j d_j^\gamma - \eta_0 - \sum_{j=1}^{M_\eta} \partial\eta_j d_j^\eta\right)\right]}$$, where e_ρ is the residual in equation (19).

Then, the estimating equation is:

(20) $$d\tilde{c} = \frac{1}{\theta} x_1 + \sum_{j=1}^{M_\theta} \frac{1}{\partial\theta_j} d_j^\theta x_1 + e_\theta .$$

Both the technology index, H_0, and the productivity shock, ln *v*, are estimated directly from equation (4), by taking the natural logarithm and then rearranging slightly to yield:

(21) $$\ln \tilde{y} - \left(\alpha_0 + \sum_{j=1}^{M_\alpha} \partial\alpha_j d_j^\alpha\right) \ln \tilde{k} = \ln H_0 \iota + \sum_{j=1}^{M_H} \partial \ln H_j d_j^H + \ln v .$$

The persistence in the productivity shock, ω, and the innovation of the shock, ln *u*, are estimated from the residual of equation (21) as follows:

(22) $$\ln v_t = \omega_0 \ln v_{t-1} + \sum_{j=1}^{M_\omega} \partial\omega_j d_j^\omega \ln v_{t-1} + \ln u_t .$$

Using the residual from equation (22), the variance of the innovation to the productivity shock is estimated from:

(23) $$\ln u^2 = \sigma_0^2 \iota + \sum_{j=1}^{M_\sigma} \partial\sigma_j^2 d_j^\sigma + e_\sigma .$$

IV. Data

The data used in our analysis are drawn from the National Income and Product Accounts (NIPA) of the United States as constructed by the U.S. Bureau of Economic Analysis (BEA), employment statistics collected by the U.S. Bureau of Labor Statistics (BLS), and interest rates collected by the Board of Governors of the Federal Reserve System (FRS). The data are quarterly and span the time period from 1948:1 to 2000:4. All NIPA data were drawn from the June 2001 vintage.

Output, *Y*, is defined as consumption expenditures, *C*, plus gross investment expenditures, I^g. Consumption expenditures include nondurable consumption expenditures plus expenditures on consumer services plus imputed rental expenditures for the consumer durable goods stock used in home production – this imputed rental value of the stock represents the services flow from the durable goods stock. Gross investment includes expenditures on producer fixed assets (structures, equipment, and software), plus expenditures on residential fixed assets (structures), plus consumer purchases of durable goods (residential equipment). Following CE (1992), the capital stock, K, was chosen to match the gross investment series, given an initial value taken from the 1946 value of the BEA's fixed-capital stock series and data on depreciation (consumption of fixed capital) taken from the national accounts. We found the national account measure of depreciation to be problematic because of a stochastic trend with a strong positive drift in the implicit depreciation rate; by definition, the strong positive drift means that the depreciation rate is not constant over time. Because of the positive drift, we treat the depreciation rate as a step function. We discuss this problem in Appendix A.

All variables are adjusted for inflation, using the implicit price deflators in the national accounts. Per-capita variables are created by dividing all real variables (output, consumption, gross investment, net investment, and capital stock) by a full-time equivalent (FTE) measure of employment. The basic employment series used is private, non-farm industry employment collected by the BLS from its establishment survey. To compute the FTE measure, we weight establishment employment by an index that measures how fully the labor force was utilized relative to full-time hours. This index was created by dividing average hours by normal full-time hours; normal full-time hours were measured by the low-frequency (trend) component of full-time hours. We compute de-trended variables from the per-capita data by eliminating the low-frequency component with the Christiano and Fitzgerald (CF) (2001) band-pass filter; by definition this procedure identifies and estimates stochastic trends.

To compute the ex-ante real rate of return series, several series of nominal interest rates were used, along with inflation rates drawn from NIPA. Interest rate data were drawn from FRS data bases. We used the 10-year T-Bond rate, the 10-year BAA-rated corporate bond rate, S&P 500 dividend yields, and the S&P 500 stock price index. We computed a quarterly stock return series by adding dividend yields to the three-month percentage change in the stock price index (computed at an annual rate). We then computed two different risk premiums – a corporate bond risk premium (10-year corporate bond rate less 10-year T-Bond rate), and a corporate stock risk premium (the full-sample average of stock returns in excess of the 10-year corporate bond yield).

These risk premiums are added to the ex-ante real interest rate series created from the 10-year T-Bond series. Using information on the composition of assets from the FRS's flow-of-funds accounts, we compute a weighted real rate of return from the 10-year real corporate bond rate and the real stock return rate. The weighted return is adjusted for taxes using the average personal income tax rate found in the NIPA. To obtain the ex-ante long-run real interest rate, we subtract an estimate of expected inflation from the nominal rate on the 10-year T-Bond. To compute expected 10-year inflation, we used two independent variables to predict the quarterly inflation rate, and then extract the low-frequency component of the quarterly prediction. This low-frequency component is used as an estimate of the expected long-run inflation rate (see Appendix B for further details).

V. Results

Table 1 lists structural parameter estimates under the assumption of parameter constancy, while Table 2 lists unit-root tests for the error terms used in the first-moment conditions. All parameters are statistically significant except θ, the instantaneous elasticity of intertemporal substitution. By construction, the dependent variable in the Euler-equation (20) is stationary while, as Table 2 shows, the single independent variable in the Euler-equation (20) is non-stationary. A simple linear regression of the Euler-equation shows a coefficient very close to zero on the independent variable. Since θ is the inverse of this coefficient, θ is correspondingly very large, but with large standard errors this coefficient is not statistically different from zero.

The demonstration that the regressor in the Euler-equation is non-stationary is the first signal of possible instability in the structural parameters. Table 2 shows that five of the eight error terms in the first moment conditions are non-stationary – a signal that points to possible structural parameter instability. The remaining three moment conditions, represented by equations (20), (21), and (22) have stationary error terms by construction. To examine possible signs of parameter instability in these three conditions, we present three multivariate unit-root tests in Table 3. In a stochastic growth model, balanced growth implies that the variables lnc, lny, lnk, should be cointegrated with one common stochastic trend, and one common deterministic trend. These common trends should be used to de-trend all three variables for use in their respective moment conditions. The multivariate unit root tests in Table 3 show that the balanced growth assumption can be rejected – if we were to de-trend these variables with a single common trend, the corresponding first moment conditions for equations (20), (21), and (22) would be non-stationary.

With evidence of non-stationarity in the first-moment conditions, we proceed to a formal test of structural parameter instability using the Bai (1997) and Bai and Perron (1998) test. Table 4 lists the results from the Bai-Perron Break Point Tests, and Table 5 lists the parameter estimates. The left-hand side of Tables 4 and 5 lists results for the three preference parameters: the gross discount rate, ρ; the instantaneous elasticity of intertemporal substitution, θ; and the growth rate of the labor force, η. The right-hand side of Tables 4 and 5 lists the results for the four technology parameters: the capital share of output, α; the exogenous rate of technological change, γ; the Hicks-neutral (and Harrod-neutral) technology index, H_0; and the depreciation rate, δ. The bottom of Tables 4 and 5 lists results for the two parameters associated with the stationary stochastic productivity shock: the degree of persistence, ω; and the variance of the innovations to the productivity shock, σ^2.

For the growth rate of the labor force, η, we list three different sets of results, each of which depends upon how the business cycle frequencies are filtered out of the data with the CF band-pass filter. The business cycle frequencies are typically stated as oscillations between 1.5 years and 8 years. The first set of results for the growth rate of the labor force filters out these frequencies from dN. However, the low-frequency component exhibits an additional oscillation with frequencies of 10 to 11 years. The remaining two sets of results extend the upper bound of the business cycle frequencies to 10 and 11 years, respectively.

For the persistence parameter, we list two sets of results. Both the productivity shock and the innovations to the productivity shock exhibited a non-constant variance characterized by an ARCH (2,0) stochastic process. The set of results for the persistence parameter is based on the original heteroscedastic productivity shocks and on the productivity shocks standardized by the ARCH conditional standard deviations. For the variance of the innovations to the productivity shock, we list the results of an ARCH Lagrange multiplier test in Table 4 and the coefficients of the ARCH (2,0) process in Table 5.

The break-point tests presented in Table 4 suggest that the basic stochastic neoclassical growth model exhibits extensive parameter instability. All four technology parameters exhibit instability with multiple parameter breaks. Two of the three preference parameters also exhibit instability with multiple parameter breaks. The only parameter for which we cannot reject the hypothesis of parameter constancy is the instantaneous elasticity of intertemporal substitution. However, the Euler-equation from which this parameter is estimated has little explanatory power and the elasticity is not statistically different from either zero or one because of large standard errors – a result found by Hall (1988) and others. The results for the estimated rate of time preference are similar in spirit to the real-rate breaks found by Garcia and Perron (1996).

The two stochastic parameters also exhibit parameter instability. The persistence parameter exhibits one break in early 1975, with very high persistence (0.9) in the period prior to 1975, but post-1975 it exhibits a lower degree of persistence (0.75). Note that standardizing the productivity shocks with the conditional standard deviations reduces to some degree the persistence in the productivity shocks – (0.74) prior to 1978 and (0.59) post-1978. The statistically significant value of the ARCH LM statistic shows that the innovations to the productivity shock exhibit a non-constant degree of volatility over time.[1]

VI. Implications

Our results suggest some very far reaching implications. With respect to households, we found that the rate of time preference is unstable over time. This suggests that, in the aggregate, the preferences

[1] The ARCH (2,0) LM test is χ^2 distributed with 2 degrees of freedom. The 5% critical value is 5.99.

of consumers are dynamically inconsistent; in other words, tastes change over time. The two most likely causes of this result are: (1) a changing demographic structure coupled with heterogeneity in consumer preferences; or (2) preferences that are not additively separable intertemporally. A third reason – that preferences do change over time – would have to be modeled as a set of overlapping generations of consumers with heterogeneous preferences.

With heterogeneous consumer preferences, exact aggregation to a representative consumer requires that the intertemporal utility function be homothetic and that the distribution of wealth be fixed and independent of future expected interest rates and prices.[2] Since most empirical studies of consumer demand routinely reject homothetic preferences and the distribution of wealth is neither fixed over time nor independent of future interest rates and prices, heterogeneity in consumer preferences implies that the representative consumer specification must be abandoned.[3] Furthermore, since homothetic preferences are weakly separable and additive separability is stronger than weak separability, the routine rejection of homothetic preferences implies that additively separable preferences also must be abandoned.[4] Instead, economists will need to construct heterogeneous consumer models with non-separable preferences.[5] Additionally, if a changing demographic structure results in aggregate preferences being dynamically inconsistent, economists will need to incorporate migration and fertility choice into the model. Thus, we conclude that the prototypical aggregate neoclassical growth model has an invalid structural form that is inconsistent with microeconomic consumer behavior.

With preferences that are not intertemporally separable, the equilibrium real interest rate is no longer tied down by a constant rate of time preference; instead equilibrium real interest rates are determined by the multiplicity of slopes contained on the hyperplane that simultaneously supports both the aggregate indifference surface and the aggregate product transformation surface. Since our parameter stability tests demonstrate that aggregate technological frontiers change over time, non-separable

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[2] See Muellbauer (1975,1976) and Lewbel (1989) for the conditions of exact aggregation.
[3] Kirman (1992) makes the same point, but without the econometric results to back up his contention.
[4] Browning (1991) rejects additively separable intertemporal preferences using aggregate consumption data for the U.K.

preferences explains why the mean real interest rate exhibits discrete jumps over different time periods. But, if intertemporal preferences are non-separable, the preceding paragraph suggests that exact aggregation conditions are unlikely to be met so that the representative consumer also will have to be abandoned. Furthermore, in an uncertain world, the likelihood of changing real interest rates over time implies that a representative consumer would have to form expectations about the future path of equilibrium real rates of return; since government tax policy directly affects those rates of return, the specific critique that Lucas applied to the Keynesian model would pertain (i.e., consumer behavior is not invariant to government policy changes). This strengthens our conclusion that the aggregate neoclassical growth model has an invalid structural form that is inconsistent with microeconomic consumer behavior.

With respect to producers, we found that all of the parameters of the aggregate Cobb-Douglas production function were unstable (the capital income share of output, the Harrod-neutral technology index, and the exogenous growth rate of technological change). Additionally, we found that because of a continually changing mix of capital equipment the depreciation rate on the aggregate capital stock also was unstable. The two most likely causes for such extensive technological parameter instability are: (1) an incorrectly specified functional form; (2) a changing final product mix coupled with heterogeneity in producer technologies across products and industries. A third reason – that production sets have maximal frontiers that change over time – would have to be modeled as an overlapping (over time) set of heterogeneous producers. Finally, the only cause of parameter instability in the depreciation rate is that capital inputs are heterogeneous, which imposes a severe restriction for combining inputs into an aggregate capital stock.

The Cobb-Douglas production function is very restrictive such that technological parameter instability could simply be the result of a misspecified functional form. We perform one final test to determine if technological parameter instability is the result of a misspecified functional form. We estimate a translog specification of the capital share equation, with the Cobb-Douglas being a special case

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[5] Studies estimating consumer demands from aggregate data generally reject homothetic preferences. For examples see Christensen, Jorgensen, and Lau (1975), Berndt, Darrough, and Diewert (1977), and Christensen and Manser (1975,1977).

of the translog. The results of this test are presented in Table 6. The test for the Cobb-Douglas special case is that the coefficients on all price variables and trend terms in the respective cost share equations are jointly zero. This hypothesis is rejected, such that the production technology is misspecified if it is represented as a Cobb-Douglas functional form. But, does parametric instability result from this misspecification? The coefficient differences between the constant terms for each regime in the cost share equations are listed in the top half of Table 6. If we examine the translog cost function estimates, under the assumption that the relative price ratio is endogenous, we notice that estimating the translog specification reduces the magnitude of the coefficient differences in the capital share mean, but does not eliminate the parametric instability nor does it eliminate the pattern of the instability found in the Cobb-Douglas specification.[6] The number of regimes for the capital share mean remains at seven and the number of regimes in the exogenous growth rate of Harrod-neutral technology remains at three; in other words both parameters continue to exhibit evidence of instability.[7] Consequently, the only remaining explanation for this instability is heterogeneity in technologies.

With heterogeneous technologies, aggregation to a representative firm employing an aggregate production technology requires that the production function be identical for each individual producer – in other words a homogeneous technology – a condition that Jorgensen (1995) reported was rejected at the two-digit SIC industry level. Thus, it is invalid to use an aggregate production function. When relative product prices and relative factor prices change both the factor mix and the product mix will change. This would manifest itself as parameter instability in the invalidly specified aggregate production function. This further bolsters our conclusion that the aggregate neoclassical growth model has an invalid structural form that is inconsistent with microeconomic producer behavior.[8]

[6] However, under an exogenous relative price assumption both the magnitude and the pattern of the capital share mean coefficient differences is altered by estimating a translog specification, although parametric instability is not eliminated. Because the relative price ratio contains level shifts, the pattern of the level shifts in the capital share is altered by a similar pattern in the shifts in the relative price ratio. However, at an aggregate level, the relative price ratio is extremely unlikely to be exogenous to a single aggregate producer, which implies that the level shifts in relative prices must be endogenously determined.

[7] Because the translog specification includes time trends for factor-augmenting technical change, we include an additional dummy variable for the period 1974:4 to 1979:4 to allow the constant term of the trend to vary along with the slope of the trend.

[8] Long and Plosser (1983) construct a multi-sector RBC model, but also maintain an invalid single representative consumer.

Finally, parameter instability in the aggregate depreciation rate concerns the ability to combine heterogeneous capital inputs into an aggregate capital stock. This harkens back to a debate started by Joan Robinson in the 1950's called the Cambridge Controversy.[9] Our results demonstrate that we can not simply sum the dollar values of heterogeneous capital inputs into an aggregate capital stock. The only other method of functional aggregation requires weak separability in the production function between the aggregate labor input, and all of the different capital inputs. Furthermore, technological change must be capital augmenting (a negative coefficient on the time trend in a translog specification of the capital cost share equation), a condition that does not hold according to Table 6, which demonstrates that technical change is labor augmenting. Therefore, the depreciation rate instability that is caused by a changing capital mix invalidates the premise that heterogeneous capital inputs can be combined into an aggregate capital stock; this result should reopen the Cambridge Controversy debate.

VII. Conclusion

In this paper we subjected a prototypical real business cycle model to the scrutiny of parameter stability tests. For six of the seven structural parameters describing preferences and technology we found evidence of extensive parameter instability. We have argued that heterogeneity in both consumer preferences and producer technologies is the most likely cause of the extensive instability found in the key preference and technology parameters of the aggregate stochastic neoclassical growth model. The implication of such heterogeneity is that economists will be forced to construct and analyze multi-sector, multi-consumer dynamic general equilibrium models at a microeconomic level, and then aggregate, ex-post, across industries, products, and consumers in order to derive aggregate, macroeconomic implications. Therefore, we conclude that the evidence that we found of structural parameter instability in the stochastic neoclassical growth model calls into question the RBC paradigm with the same force that Lucas's critique challenged the Keynesian approach.

[9] See Harcourt (1969).

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Table 1
Structural Parameter Estimates
Under Constant Parameter Assumptions

Parameter	Coefficient	T-statistic
ρ	0.068615	24.9535
	(0.002750)	
θ	45.64085	0.870769
	(52.41446)	
η	0.019813	16.5067
	(0.001200)	
α	0.3074	102.8331
	(0.002989	
γ	0.017418	11.61965
	(0.001499)	
lnH0	7.00484	4490.184
	(0.001560)	
H0	1,101.95	
δ	0.058286	149.5931
	(0.000390)	
ω	0.829887	17.82554
(actual residuals)	(0.046556)	
ω	0.657127	17.37485
(standardized residuals)	(0.037821)	
σ	0.006581	

Table 2
Unit-Root Tests on First Moment Conditions

Moment Condition	ADF-t-Test	5% Critical
ρ	-1.873460	-2.875500
α	-1.939591	-2.875500
γ	-1.921780	-2.875500
η	-1.980263	-2.875500
δ	-1.212197	-2.875500

Table 3
Multivariate Unit-Root Tests

Variables	Balanced Growth				Different Deterministic Linear Trends		
	Number of Unit Roots	Trace Statistic	5% Critical Value		Number of Unit Roots	Trace Statistic	5% Critical Value
lny , lnk	2	10.670930	15.410000		2	27.600380	25.320000
					1	8.258940	12.250000
lnc , lny , lnk	3	57.925870	29.680000		3	73.694710	42.440000
	2	9.342859	15.410000		2	24.977300	25.320000
lnc , lny	2	6.839998	15.410000		2	11.795200	25.320000

Table 4

Bai - Perron Break Point Tests

Preference Parameters

Parameter	Regimes	Break Date	Sup-F Statistic
ρ	1948:2 - 1956:3		
	1956:4 - 1979:4	1956:4	167.4227
	1980:1 - 1980:4	1980:1	2035.9321
	1981:1 - 1985:4	1981:1	43.4712
	1986:1 - 1995:2	1986:1	236.3489
	1995:3 - 2000:4	1995:3	44.0138
θ	1948:2 - 2000:4	None	NA
η	1948:3 - 1954:4		
	1955:1 - 1961:1	1955:1	31.4036
filtering out	1961:2 - 1969:1	1961:2	29.1551
1.5 to 8 year	1969:2 - 1974:1	1969:2	14.2505
oscillations	1974:2 - 1978:4	1974:2	12.4529
	1979:1 - 1983:3	1979:1	10.3906
	1983:4 - 1989:1	1983:4	27.6677
	1989:2 - 1994:1	1989:2	18.2181
	1994:2 - 2000:4	1994:2	14.4830
η	1948:3 - 1954:4		
	1955:1 - 1962:1	1955:1	28.2695
filtering out	1962:2 - 1967:4	1962:2	32.0224
1.5 to 10 year	1968:1 - 1995:2	1968:1	20.4920
oscillations	1995:3 - 2000:4	1995:3	11.9045
η	1948:3 - 1952:1		
filtering out	1952:2 - 1961:1	1952:2	40.1990
1.5 to 11 year	1961:2 - 2000:4	1961:2	23.3478
oscillations			

Technology Parameters

Parameter	Regimes	Break Date	Sup-F Statistic
α	1948:2 - 1950:1		
	1950:2 - 1963:2	1950:2	32.2085
	1963:3 - 1979:4	1963:3	125.5876
	1980:1 - 1980:4	1980:1	1924.1658
	1981:1 - 1985:2	1981:1	39.5261
	1985:3 - 1997:2	1985:3	90.8792
	1997:3 - 2000:4	1997:3	20.2719
γ	1948:3 - 1974:3		
filtering out	1974:4 - 1980:2	1974:4	10.8268
1.5 to 8 year	1980:3 - 2000:4	1980:3	10.1873
oscillations			
H0	1948:2 - 1950:1		
	1950:2 - 1963:2	1950:2	1199.2049
	1963:3 - 1974:3	1963:3	3973.8803
	1974:4 - 1979:4	1974:4	10486.9592
	1980:1 - 1980:4	1980:1	75633.0594
	1981:1 - 1985:2	1981:1	1833.3039
	1985:3 - 1997:2	1985:3	3492.5724
	1997:3 - 2000:4	1997:3	719.5347
δ	1948:2 - 1962:4		
	1963:1 - 1979:4	1963:1	278.5818
	1980:1 - 1995:4	1980:1	1012.0476
	1996:1 - 2000:4	1996:1	31.8408

Stochastic Parameters

Parameter	Regimes	Break Date	Sup-F Statistic	Parameter	Regimes	Break Date	Sup-F Statistic
ω	1948:3 - 1974:4			Var(lnu)	ARCH(2,0) LM		38.5407
original	1975:1 - 2000:4	1975:1	280.6360				
residuals							
ω	1948:3 - 1978:2						
standardized	1978:3 - 2000:4	1978:3	185.1584				
residuals							

Critical Values: 5% critical value for Sup-F Statistic is 9.84 (see Andrews,1993)
5% critical value for ARCH LM Test is 5.99.

Table 5
Parameter Estimates

Preference Parameters

Parameter	Regimes	Coefficient Differences	Parameter Estimates
ρ	1948:2 - 1956:3		0.046244 (0.000827)
	1956:4 - 1979:4	0.013100 (0.001422)	0.059344
	1980:1 - 1980:4	0.039636 (0.001357)	0.085880
	1981:1 - 1985:4	0.058131 (0.002536)	0.104375
	1986:1 - 1995:2	0.039636 (0.001357)	0.085880
	1995:3 - 2000:4	0.030626 (0.001680)	0.076870
θ	1948:2 - 2000:4		1.374414 (1.107635)
η	1948:3 - 1954:4		0.021258 (0.001945)
	1955:1 - 1961:1	-0.020333 (0.002544)	0.000925
filtering out 1.5 to 8 year oscillations	1961:2 - 1969:1	0.009014 (0.003075)	0.030272
	1969:2 - 1974:1	-0.010040 (0.002855)	0.011218
	1974:2 - 1978:4	0.009782 (0.003059)	0.031040
	1979:1 - 1983:3	-0.010782 (0.003267)	0.010476
	1983:4 - 1989:1	0.010557 (0.003075)	0.031815
	1989:2 - 1994:1	-0.014412 (0.003269)	0.006846
	1994:2 - 2000:4	0.007118 (0.003378)	0.028376

Technology Parameters

Parameter	Regimes	Coefficient Differences	Parameter Estimates
α	1948:2 - 1950:1		0.313372 (0.002624)
	1950:2 - 1963:2	-0.029806 (0.003900)	0.283566
	1963:3 - 1979:4	-0.053457 (0.004674)	0.259915
	1980:1 - 1980:4	0.040060 (0.004089)	0.353432
	1981:1 - 1985:2	0.078128 (0.008014)	0.391500
	1985:3 - 1997:2	0.040060 (0.004089)	0.353432
	1997:3 - 2000:4	0.021961 (0.004940)	0.335333
γ filtering out 1.5 to 8 year oscillations	1948:3 - 1974:3		0.023482
	1974:4 - 1980:2	-0.022218 (0.003917)	0.001264
	1980:3 - 2000:4	-0.007295 (0.002550)	0.016187
lnH0	1948:2 - 1950:1		6.895266 (0.007947)
	1950:2 - 1963:2	0.334025 (0.007821)	7.229291
	1963:3 - 1974:3	0.605239 (0.008785)	7.500505
	1974:4 - 1979:4	1.045988 (0.010063)	7.941254
	1980:1 - 1980:4	-0.239587 (0.063182)	6.655679
	1981:1 - 1985:2	-0.841470 (0.008543)	6.053796
	1985:3 - 1997:2	-0.405870 (0.008172)	6.489396
	1997:3 - 2000:4	-0.198671 (0.008628)	6.696595

Standard errors are in parentheses and are calculated as Newey-West heteroscedasticity and autocorrelation consistent standard errors with 4 lags.

Table 5 (continued)
Parameter Estimates

Preference Parameters				Technology Parameters			
Parameter	Regimes	Coefficient Differences	Parameter Estimates	Parameter	Regimes	Coefficient Differences	Parameter Estimates
η	1948:3 - 1954:4		0.019684	δ	1948:2 - 1962:4		0.054883
			(0.001223)				-0.000142
	1955:1 - 1962:1	-0.016976	0.002708		1963:1 - 1979:4	0.002997	0.057880
		(0.002529)				(0.000174)	
filtering out	1962:2 - 1967:4	0.014605	0.034289		1980:1 - 1995:4	0.005773	0.060656
1.5 to 10 year		(0.002301)				(0.000186)	
oscillations	1968:1 - 1995:2		0.019684		1996:1 - 2000:4	0.007232	0.062115
			(0.001223)			(0.000241)	
	1995:3 - 2000:4	0.010054	0.029738				
		(0.001894)					
η	1948:3 - 1952:1		0.029860				
filtering out			(0.002788)				
1.5 to 11 year	1952:2 - 1961:1	-0.025239	0.004621				
oscillations		(0.003301)					
	1961:2 - 2000:4	-0.007321	0.022539				
		(0.002943)					

Stochastic Parameters							
Parameter	Regimes	Coefficient Differences	Parameter Estimates	Parameter	Variables	Parameter Estimates	t-Statistic
ω	1948:3 - 1974:4		0.908213	Var (lnu)			
original			(0.058348)		constant	0.0000191	5.863639
residuals	1975:1 - 2000:4	-0.151096	0.757117			(0.00000325)	
		(0.077132)			Var(lnu)L1	0.111373	1.641741
ω	1948:3 - 1978:2		0.743592			(0.067838)	
standardized			(0.035208)		Var(lnu)L2	0.472447	3.615898
residuals	1978:3 - 2000:4	-0.155452	0.588140			(0.130658)	
		(0.078502)					

Standard errors are in parentheses and are calculated as Newey-West heteroscedasticity and autocorrelation consistent standard errors with 4 lags.

Table 6
Parameter Estimates for Functional Form Test

	Cobb-Douglas Specification			Translog Specification (Assumes relative prices are endogenous: IV Estimation)			Translog Specification (Assumes relative prices are exogenous)		
	Estimated Capital Share Equation		**Implied Labor Share Equation**	**Estimated Capital Share Equation**		**Implied Labor Share Equation**	**Estimated Capital Share Equation**		**Implied Labor Share Equation**
Constants	*Coefficients (standard-errors)*	*t-statistics*	*Coefficients*	*Coefficients (standard-errors)*	*t-statistics*	*Coefficients*	*Coefficients (standard-errors)*	*t-statistics*	*Coefficients*
1948:2 - 1950:1	0.313372	119.107564	0.686628	0.310454	356.433984	0.689546	0.329601	193.768959	0.670399
	(0.002631)			(0.000871)			(0.001701)		
1950:2 - 1963:2	-0.029806	-7.623018	0.029806	-0.022032	-11.010495	0.022032	-0.019165	-10.827684	0.019165
	(0.003910)			(0.002001)			(0.001770)		
1963:3 - 1974:3	-0.054475	-12.448583	0.054475	-0.022032	-11.010495	0.022032	-0.027061	-15.349404	0.027061
	(0.004376)			(0.002001)			(0.001763)		
1974:4 - 1979:4	-0.051276	-5.191455	0.051276	-0.014210	-10.463918	0.014210	-0.019165	-10.827684	0.019165
	(0.009877)			(0.001358)			(0.001770)		
1980:1 - 1980:4	0.040060	9.773115	-0.040060	0.014146	4.718479	-0.014146	-0.019165	-10.827684	0.019165
	(0.004099)			(0.002998)			(0.001770)		
1981:1 - 1985:2	0.078128	9.724670	-0.078128	0.063734	12.256538	-0.063734	-0.019165	-10.827684	0.019165
	(0.008034)			(0.005200)			(0.001770)		
1985:3 - 1997:2	0.040060	9.773115	-0.040060	0.014146	4.718479	-0.014146	-0.027061	-15.349404	0.027061
	(0.004099)			(0.002998)			(0.001763)		
1997:3 - 2000:4	0.021961	4.433878	-0.021961	-0.008615	-6.003484	0.008615	-0.027061	-15.349404	0.027061
	(0.004953)			(0.001435)			(0.001763)		
Other Variables									
ln(r+d)	NA	NA	NA	0.184688	7.976850	-0.184688	0.216690	88.121187	-0.216690
				(0.023153)			(0.002459)		
lnw	NA	NA	NA	-0.184688	-7.976850	0.184688	-0.216690	-88.121187	0.216690
				(0.023153)			(0.002459)		
trend	NA	NA	NA	0.002292	13.325581	-0.002292	0.002848	10.432234	-0.002848
(1948:2 - 1974:3)				(0.000172)			(0.000273)		
trend	NA	NA	NA	0.000752	1.740741	-0.000752	0.002732	3.206573	-0.002732
(1974:4 - 1979:4)				(0.000432)			(0.000852)		
trend	NA	NA	NA	0.004112	17.133333	-0.004112	0.003632	11.947368	-0.003632
(1980:1 - 2000:4)				(0.000240)			(0.000304)		
LR Test for CD									
LR-statistic					333.9414			687.7176	
5% Critical Value					9.4877			9.4877	

Reported standard-errors are heteroscedasticity and autocorrelation consistent.

The instrumental variable for relative prices was constructed from three exogenous and one predetermined variable. The three exogenous variables are (1) the capital tax rate; (2) real government purchases deflated by private employment; (3) real gross exports deflated by private employment. The predetermined variable is the capital-labor ratio lagged one period. To construct the instrument we regressed the log of the relative price ratio (real rate of return / wage rate) on these four weakly exogenous variables, controlling for level shifts and a trend in the relative price ratio. The instrument is the predicted values from the four weakly exogenous variables and the trend in relative prices, which assumes the estimated level shifts are part of the endogenous errors.

Appendix A
Capital Stock Construction and Trends in Depreciation Rates

The capital stock series can be constructed iteratively from equation (A1.1):

(A1.1) $K_t = (1 - D_t)K_{t-1} + I_t^g$.

The necessary ingredients for calculating the capital stock series are a series on gross investment, $I_t^g \quad \forall t$; a series on historical rates of depreciation, $D_t \quad \forall t$; and an initial capital stock, K_0. Chart A1.1 illustrates historical rates of depreciation using BEA data. Panel A of Chart A1.1 illustrates the overall depreciation rate of the capital stock (blue line), which includes private fixed assets and the stock of consumer durables, and the depreciation rate on private fixed assets from NIPA accounts (red line). Panel B of Chart A1.1 illustrates the rate of depreciation on consumer durables, drawn from the BEA's annual data on the consumer durables stock. Note that the overall rate of depreciation and the rate of depreciation on private fixed assets exhibit a strong positive drift. Table A1.1 lists the results from linear trend regressions, which provide evidence of a statistically significant trend in both series. Table A1.1 also shows that a statistically significant drift exists in the depreciation rate for consumer durables. All three depreciation rate series are non-stationary. Table A1.2 lists unit root tests on all three series.

The statistically significant trend in the rate of depreciation means that we can reject the hypothesis of a constant depreciation rate parameter. However, constructing a capital stock series is problematic, since the GMM estimator does not allow for non-stationarity, nor does it allow for deterministic trends in the data. Therefore, we investigate further to find the cause of the trend in depreciation rates. Panel A of Chart A1.1 shows that the overall depreciation rate has a steeper trend than that for fixed assets. This suggests that one problem may be changes in the mix of the capital stock. This hypothesis turns out to be correct. In 1947, consumer durables were 4 percent of the overall capital stock, while in 2000, consumer durables composed 14 percent of the overall capital stock. The share of consumer durables in the capital stock has steadily increased throughout the postwar period. Because consumer durables have a much higher rate of depreciation than private fixed assets (see Chart A1.1), the substitution away from private fixed assets towards consumer durables explains the steeper trend in the overall depreciation rate.

However, the shift toward household capital is not part of the real business cycle model or of the neoclassical growth model. Thus, we construct the overall capital stock to have a constant share (at the full-sample average) of private fixed assets and consumer durables. This does not eliminate the trend in the overall depreciation rate, however, since the depreciation rate on private fixed assets has a strong positive drift, according to Chart A1.1. A change in the mix of private fixed assets turns out to be the cause of the positive drift in this depreciation rate. The mix of capital between structures and equipment and software has continually changed during the postwar period. Chart A1.2 provides a comprehensive illustration of changes in the capital mix. The share of capital in equipment has steadily increased from 12 percent in 1946, to 18 percent in 1960, to 21 percent in 1999. Panel A in Chart A1.2 illustrates the share of equipment in the total capital stock. The substitution away from structures toward equipment is the primary reason for the positive drift in depreciation rates prior to 1960, because equipment has a much higher rate of depreciation than structures.

Shifts in the mix of equipment also explain the drift in fixed-asset depreciation rates after 1960. Panel B in Chart A1.2 illustrates changes in the mix of equipment. The primary change here is an increase in the share of equipment in office and information processing equipment (the red line), which has a high depreciation rate and a corresponding decrease in the share of equipment in transportation equipment (the blue-green line), which has a low depreciation rate. Panel C in Chart A1.2 illustrates changes in the mix of office and information processing equipment, while Panel D illustrates changes in the mix of information processing equipment. The bulk of the change in the mix of office equipment has been a large increase in computers and software. It is this shift toward information processing (computers and software) that is responsible for the positive drift in depreciation rates after 1960. Thus, changes in the mix of the capital stock are responsible for the trend in the depreciation rate.

Chart A1.2 also provides information with which to date major shifts in the capital mix. The postwar boom in industrial equipment investment altered the mix of capital between structures and equipment between 1945 and 1960. The mainframe computer and software revolution caused a gradual shift towards information processing equipment and software between 1960 and 1980. The desktop computer and software revolution caused another shift toward information processing equipment and software between 1980 and 1995, and the Internet software revolution will stimulate a further shift, but in our data this regime occurs between 1996 and 2000.

This is how we construct the depreciation rate series used to construct the capital stock. We weight the asset depreciation rates in Table 3 of Fraumeni (1997) by the corresponding fixed-asset shares. The resulting trended fixed-asset share weighted depreciation rates are illustrated in Panel A of Chart A1.3. We create a depreciation rate regime from 1947 to 1962, using the 1962 weighted depreciation rate in Panel A. We create a second depreciation rate regime from 1963 to 1979, using the 1979 weighted depreciation rate in Panel A. We create a third depreciation rate regime from 1980 to 1995, using the 1995 weighted depreciation rate in Panel A. The fourth depreciation rate regime runs from 1996 to 2000, using the 1999 weighted depreciation rate in Panel A. In this way, we treat the trended share-weighted depreciation rates as a step function – this is illustrated in Panel B of Chart A1.3. To these fixed depreciation rate regimes we add the high-frequency noise of the fixed-asset depreciation rates from the NIPA data displayed on the left-hand panel in Chart A1.1. We assume that the depreciation rate for the consumer durables stock is fixed at the full-sample average of 0.2120. To this depreciation rate we also add the high-frequency noise from the NIPA data. Using the full-sample average shares of fixed-assets and consumer durable stocks, we weight the two sets of depreciation rates. These are illustrated in Panels C through E. The series of depreciation rates that we use in constructing the capital stock is illustrated in Panel E. Although these regimes were constructed, the share-weighted depreciation rate series from which these regimes were constructed does contain a positive trend that is due to shifts in the capital mix. The constructed regimes are an attempt to match the underlying shifts in the capital mix that caused the positive drift in the underlying depreciation rate series.

Chart A1.1
Depreciation Rates in NIPA (1947-2000)

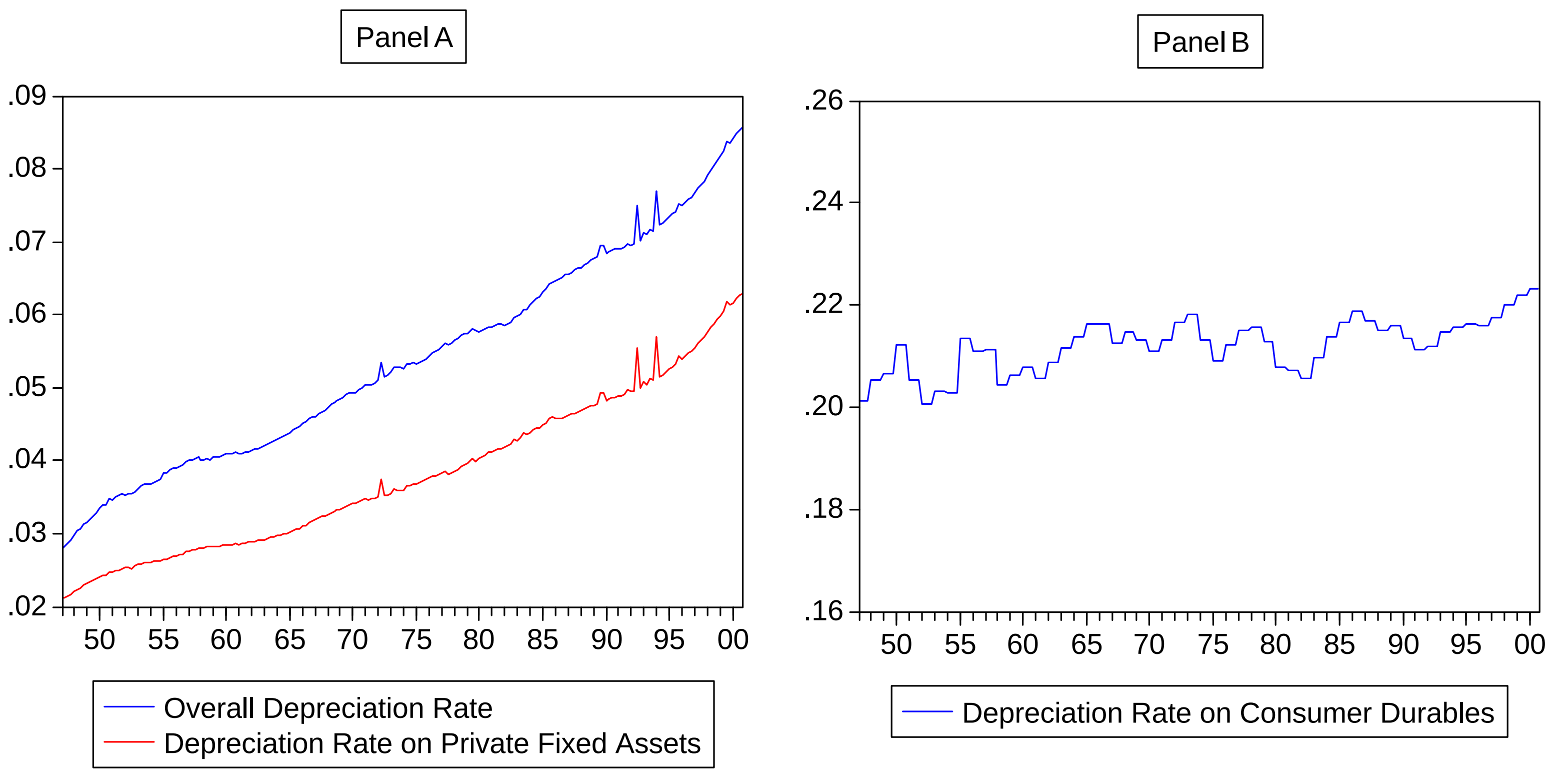

Chart A1.2
Private Fixed Asset Composition

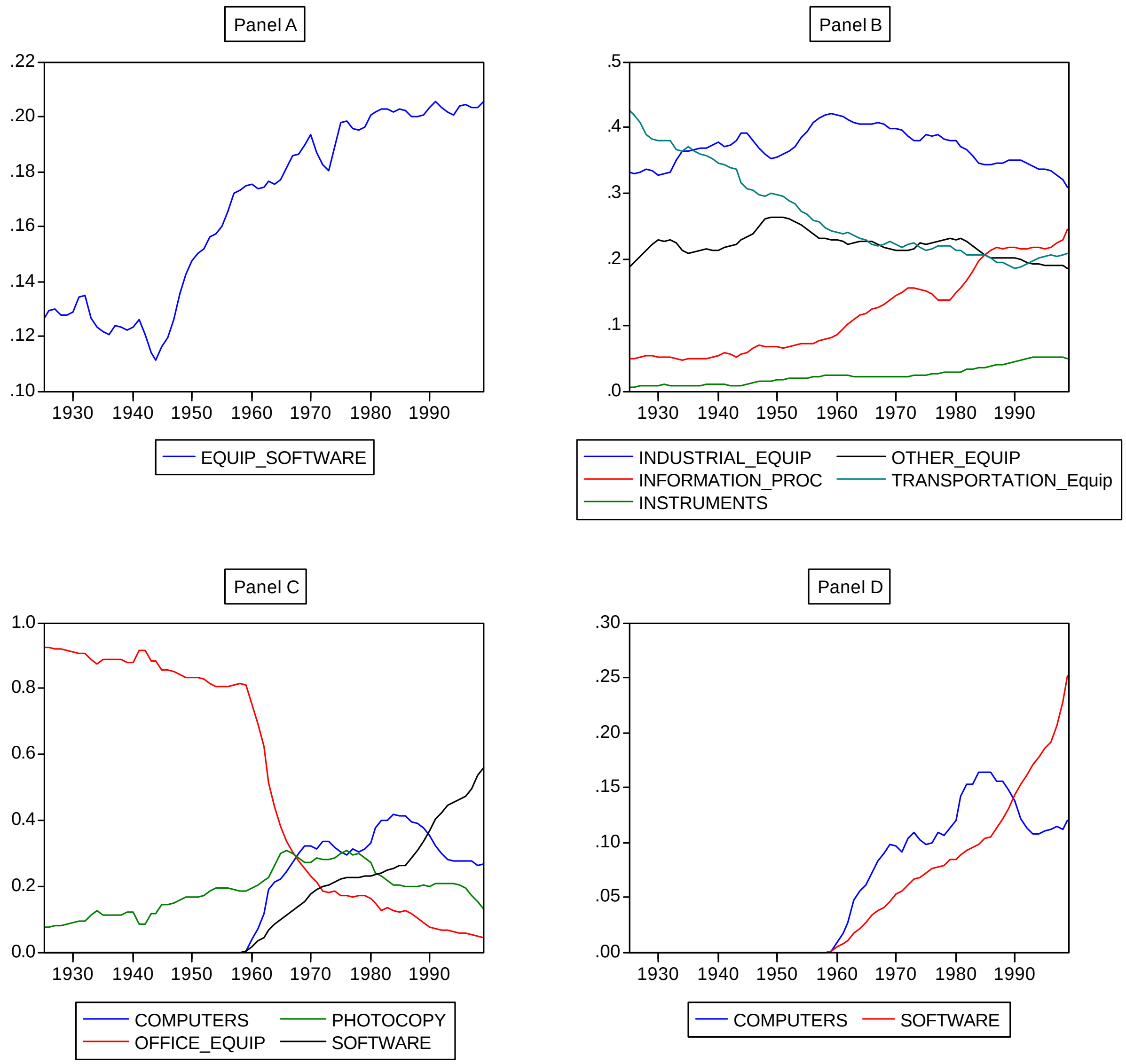

Chart A1.3
Depreciation Rate Construction

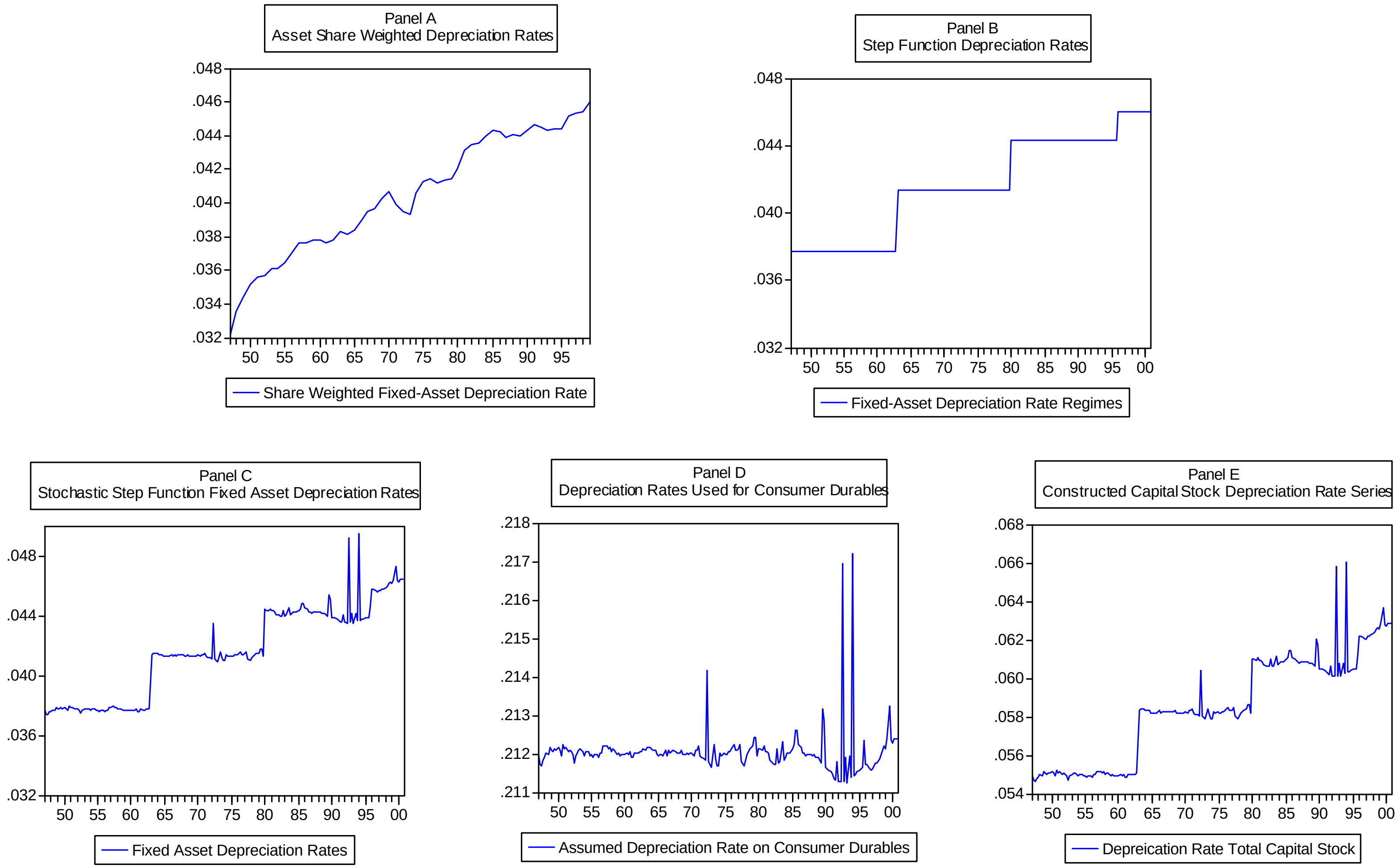

Table A1.1
Depreciation Rate Trend Regressions

Variable	Coefficient	T-Stats
Overall Depreciation Rate:		
Constant	0.028584	51.71772
	(0.000553)	
Trend	0.000235	42.23108
	(0.00000556)	
Depreciation Rate Fixed Assets:		
Constant	0.019556	32.92758
	(0.000171)	
Trend	0.000171	29.40361
	(0.00000583)	
Depreciation Rate Durables:		
Constant	0.205869	205.8406
	(0.001000)	
Trend	0.0000574	7.494817
	(0.00000766)	
Share Weighted Depreciation Rate: (Fixed-Assets)		
Constant	0.034530	109.9779
	(0.000228)	
Trend	0.000228	20.66832
	(0.00000110)	

Table A1.2
Unit-Root Tests on Depreciation Rates

Variable	ADF-t-Test	5% Critical
Overall Depreciation Rate	1.408104	-3.432000
Fixed-Asset Depreciation Rate	2.325543	-3.432400
Durables Depreciation Rate	-2.303800	2.875000
	-3.255273	-3.431800
Share Weighted Depreciation Rate	-3.803541**	-3.498700
** Significant at 5% level		

Appendix B
Calculation of Expected Inflation

Two theories suggest explanations for the long-run inflation rate: the Fisher effect, and the quantity theory of money. The Fisher effect suggests that expected inflation can be inferred from financial markets because of an additive effect on interest rates. The quantity theory says that inflation is always and everywhere a monetary phenomenon, implying that long-run inflation is the result of money growth that is excessive in relation to the long-run potential growth rate of the economy. Tests of the Fisher effect have been conducted by Mishkin (1995) with mixed results. Bullard (1994) has performed tests of the quantity theory, also with mixed results. In this paper we take an agnostic view of these two theories and use a combination of the two in constructing an expected inflation rate variable.

Table A2.1 presents the results of unit root tests on quarterly inflation rates, 3-month T-Bill rates, and the inflation rate that would be predicted by the quantity theory, $\pi*$.[2] Table A2.1 shows that all three variables are characterized by unit-root processes. Since the three variables contain unit roots, a long-run prediction may be extracted from the variables if they share a common trend; thus, we conduct a series of cointegration tests. Table A2.2 presents results from the Johansen cointegration tests. These tests show that a pairwise-cointegration test fails for the Fisher effect, a pairwise-cointegration test is successful for the quantity theory, and that all three variables contain one cointegrating vector and two common trends. Although the quantity theory cointegration test is successful, we remain agnostic regarding these theories.

Our strategy for constructing a long-run inflation expectation is to estimate the single cointegrating vector using the Stock and Watson (1993) DOLS procedure, compute the predicted quarterly inflation rate from the normalized cointegrating vector, and then extract the low-frequency component of this prediction with the HP filter. The estimated cointegrating vector coefficients are presented in Table A2.3. The low-frequency component tracks the actual 10-year inflation rate well, and it is used as our estimate of expected long-run inflation in computing the ex-ante real interest rate.

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[2] $\pi* = d \ln M + d \ln V - d \ln y$

Table A2.1
Unit-Root Tests

Variable	ADF-t-Test	5% Critical
π	-2.012881	-2.875500
π^*	-2.605128	-2.875500
3-month T-Bill rate	-2.005955	-2.875500

Table A2.2
Multivariate Unit-Root Tests

Variables	Johansen Cointegration Test Under No Linear Trend Assumption		
	Number of Unit Roots	Trace Statistic	5% Critical Value
π , π^*	2	46.235400	19.960000
(5 lags by AIC)	1	2.652600	9.240000
π , T-Bill Rate	2	14.303880	19.960000
(3 lags by SC)			
π , π^*, T-Bill Rate	3	66.896900	34.910000
(7 lags by AIC)	2	14.258300	19.960000

Table A2.3
Cointegrating Vector Coefficient Estimates

Dependent vbl: 3 month Inflation Rate		
Variables	Coefficients	t-Statistic
Constant	-0.319139	-1.150291
	(0.277442)	
π^*	0.607167	7.011606
	(0.086595)	
T-Bill Rate	0.309292	5.848615
	(0.052883)	