Is the U.S. Economy Characterized by Endogenous Growth?: A Time-Series Test of Two Stochastic Growth Models

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Abstract

In this paper, I conduct a structural change test that casts doubt on the validity of exogenous growth assumptions. Cross-sectional empirical support for non-stochastic convergence in the neoclassical growth model is the reason that the literature rejects endogenous growth. But, in a stochastic world, both neoclassical and endogenous growth models exhibit disequilibrium adjustment dynamics, thus convergence is not sufficient to reject endogenous growth. After testing for cointegration in regional per-capita incomes, I extract a single common trend to control for non-stationarity in regressions including both linear and stochastic trends. Structural change tests demonstrate that the data contain segmented linear trends, which is inconsistent with an exogenous growth assumption, but is consistent with endogenous growth.

(JEL C10,C22, O30,O40, O41, R11) Keywords: Endogenous Growth Models, Neoclassical Growth Models, Time-Series Analysis, Structural Change, Cointegration.

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I. Introduction

Solow (1956, 1957,1988,1994) developed the neoclassical growth model to show that long-run steady-state growth was both feasible and dynamically stable. In the neoclassical model, disequilibrium adjustment is a temporary phenomena. Once disequilibrium dynamics are played out, stable long-run growth is driven by technological change, deemed by Solow to be any factor causing the production function to shift. Focusing on dynamic stability, Solow *assumed* that technological change was exogenously determined outside of the model. This assumption was maintained in subsequent extensions (Cass (1965), Koopmans (1965), Brock & Mirman (1972)).

However, maintaining the exogenous growth assumption hampered growth theory because of the inability to explain long-run growth. Recently, a series of authors (Lucas (1988), Romer (1986, 1990), Jones & Mannueli (1990), Grossman & Helpman (1991), Rebelo (1991), Aghion & Howitt (1992), Jones (1995)) have remedied this shortcoming by endogenizing the technological growth rate. It is unfortunate for this "new" growth theory that recent empirical tests have failed to confirm endogenous growth. In contrast to the existing empirical literature, I conduct structural change tests that call into question the validity of the exogenous growth assumption which has been traditionally maintained in the neoclassical growth model.

Most of the empirical work, such as Barro (1991), Barro and Sala-i-Martin (1991,1992, 1995), Mankiew, Romer, and Weil (1992), employs cross-sectional data to test the disequilibrium dynamics of the neoclassical growth model in contrast to the no-convergence implication of non-stochastic versions of the AK endogenous growth model (Rebelo (1991)). However, it should be emphasized that convergence is *consistent* with the endogenized technological growth rate model of Romer (1990) as well as with a stochastic version of the AK endogenous growth model. Therefore, tests of convergence do not constitute a sufficient rejection of endogenous growth.

In comparison to the convergence literature, there are few time-series tests of endogenous growth (Kremer (1993), Jones (1995), Evans (1997), Lau and Sin (1997a, 1997b), Karras (1999), and Kocherlakota and Yi (1996, 1997), Yip (1999)). These studies assess whether certain variables – investment, private capital, tax rates, public capital, or public spending – have lagged permanent effects on productivity growth rates, with most rejecting endogenous growth. But, this conclusion is subject to the caveats of both endogenous variable bias and omitted variable bias. In fact, Kocherlakota and Yi (1997), demonstrate that omitted variable bias is a serious problem.

Ultimately, how well the exogenous growth assumption serves the theory depends upon the question of parameter constancy. If the technological growth rate rarely changes, then it is unnecessary to complicate growth models by adding an endogenous process that determines this growth rate. On the other hand, if technological growth rates change with greater frequency, then the simplicity provided by an exogeneity assumption is less attractive. In this case, an endogenous process must be added not only to explain how the growth rate is determined, but also to explain how the growth rate changes over time.

The growth rate expressions in the neoclassical, the AK endogenous growth, and the endogenized growth rate models are composed of deep parameters that describe preferences, technology, and policy – one deep parameter is exogenously specified in the neoclassical model and an algebraic expression combines the deep parameters in the two endogenous growth models. Suppose that these deep parameters are constant for all time. In a stochastic world, the three growth models are observationally equivalent. Thus, an endogenous process may determine the growth rate, but because of parameter constancy, this growth rate can be exogenously specified.

Now suppose that the deep parameters change over time. In the two endogenous growth models, altering the values of the deeper parameters changes the growth rate. In the neoclassical model, there can be no explanation for changes in the single growth rate parameter without endogenizing it. In other words, the justification for maintaining exogenous growth must be an underlying assumption of parameter constancy. It is this distinction that is exploited in using structural change econometric techniques to answer the question of whether productivity growth rates should be modeled as endogenously determined or can be exogenously specified.

Quarterly time series data is employed in this study – per-capita incomes for the eight census regions of the United States, spanning the time period from 1948 through 1998. The empirical results are threefold. First, per-capita incomes are non-stationary. Under the hypothesis of convergence across regions a single trend would be common to all regions. Thus, I test the hypothesis of a single common trend. After taking into account parameter instability in the cointegrating vectors, the test is consistent with a single common stochastic trend, which I interpret as a common growth shock. After extracting the common trend, the growth shock is used to control for non-stationarity in regressions that also include a linear trend – called a structural time-series. Third, I test for parameter instability in the structural time-series, and find that per-capita incomes contain segmented linear trends – evidence demonstrating that growth rates change frequently over time. This casts doubt on the validity of the exogenous growth rate assumption and suggests that technological growth rates require an endogenous specification. The question of which particular endogenous specification is left to future research.

The plan for the remainder of this paper is as follows. In section two, I present the time paths of productivity for the three classes of stochastic growth models. In section three, I briefly present the empirical model. Statistical results are presented in section four, and the conclusions in section five. The optimization problems and the data are discussed in separate appendices.

II. A Tale of Three Stochastic Growth Models

2.1 The Time Path of Productivity

Given constant parameters over time, the time paths of labor productivity in the neoclassical growth model, the endogenized growth rate model, and the AK growth model are observationally equivalent. The stochastic time path of observed labor productivity, or equivalently per-capita income, is given by the following four equations (in appendix 2, we derive these laws of motion for the three stochastic growth models, employing a representative, optimizing consumer that resides in a small open economy with imperfect capital mobility):

$$(1)\ \ln y_t = \ln \bar{y}_t + \mu_t$$

$$(2)\ \mu_t \equiv (\ln y_t - \ln \bar{y}_t) = \sum_{i=1}^{p} a_i \mu_{t-i} + \sum_{j=0}^{q} b_j \nu_{t-j} : \quad \text{an ARMA(p,q) process}$$

$$(3)\ \ln \bar{y}_t = \ln \bar{y}_0 + \gamma t + \xi_t$$

$$(4)\ \xi_t = \xi_{t-1} + \varepsilon_t : \ \text{a non-stationary growth shock.}$$

where: $y_t \equiv \dfrac{Y_t}{L_t}$: denotes labor productivity observed in period t..

$\bar{y}_t$: denotes the steady-state labor productivity in period t.

Y_t: denotes the level of income observed in period t..

L_t: denotes the labor input observed in period t.

ε, ν : are gaussian stochastic disturbances with variances of $\sigma_\varepsilon^2, \sigma_\nu^2$.

The time path of labor productivity is presented in equation (1). Growth in observed labor productivity, is driven by long-run growth in the steady-state. Equation (1) also shows that short-run movements in observed productivity levels are stochastic – equation (2) gives the general law of motion for this stationary process – which also describes the *stochastic* convergence dynamics for the three growth models. Our derivation in appendix 2 demonstrates that the AK growth model exhibits convergence dynamics in a stochastic setting, which is a very different result than that obtained from the non-stochastic model. Because the data are stochastic by their very nature, tests based upon the presence of convergence dynamics are not sufficient to reject endogenous growth.

The law of motion for the steady-state is presented in equation (3). As this equation shows, the time path of the steady-state is also stochastic, with equation (4) giving the dynamics of the steady-state shock. Because numerous authors find the data to be non-stationary, we present this steady-state growth shock as a difference stationary process. The productivity growth rate – γ , the linear trend slope in equation (3) – represents the upward drift of this non-stationary process. The growth rate is specified exogenously in the neoclassical growth model, and determined endogenously in both the endogenized growth rate and AK growth models. The determination of γ for these two different classes of growth models is given by:

The neoclassical growth model

(5) $\gamma = \bar{\gamma}$: is an exogenously specified constant.

The endogenous growth models

(6) $$\gamma = \frac{\beta}{\theta(1-\overline{m})}\left[B - \left[(1-\overline{m})(\eta+\delta+\zeta) + \overline{m}(r+\delta)\right]\right]$$

(a) The AK endogenous growth model

$B = A$

where: A: is the technology index in the AK model.

(b) The endogenized technological growth rate model

$$B = [\alpha\tilde{A}]^{\frac{\phi}{\phi+(1-\alpha)}}[(1-\phi)\Phi]^{\frac{(1-\alpha)}{\phi+(1-\alpha)}}\left[\frac{\phi(1-\alpha)}{\alpha(1-\phi)}\right]^{\frac{\phi(1-\alpha)}{\phi+(1-\alpha)}},$$

where:

α : is capital's share of income from the output sector.
$\tilde{A}$: is an output scaling factor in the output technology.
ϕ :is capital's share of the income from knowledge accumulation.
Φ : is a technological index in the knowledge accumulation technology.

The remaining parameters contained in equation (6) are:

β : the consumer's discount rate, with $\beta = \frac{1}{1+\zeta}$ where: $\zeta = (\rho - \eta)$.
ρ : the consumer's rate of time preference.
η : the exogenously specified growth rate of the labor force.
θ : the consumer's elasticity of intertemporal substitution.
δ : the rate of depreciation.
$\overline{m}$: is an exogenously specified debt/wealth ratio. It is institutionally determined because of capital market imperfections limiting the mobility of capital.
r : the real world interest rate.

2.2 The Productivity Growth Rate and Structural Change

Equations (5) and (6) suggest an important difference in the nature and composition of the drift terms in the respective random walk representations of each growth model. The technological growth rate in the neoclassical model – equation (5) – is an exogenously specified constant. It should be emphasized that *if* the "deep" parameters in equations (5) and (6) remain constant for all time, then, for all practical purposes, equation (6) is identical to equation (5) – a growth rate that is constant for all time can be specified exogenously – and the three growth models are observationally equivalent. In this case, the only effect of an endogenous growth model is to explain how this constant growth rate is determined.

Furthermore, while there is no reason that an exogenously specified growth rate cannot vary over time, any such structural change cannot be explained without endogenizing the growth rate using an endogenous growth model. But, an endogenous process adds an extra (and unnecessary) layer of analysis when the technological growth rate is constant for all time. Thus, the reason for suppressing this endogeneity and specifying the growth rate exogenously must be an assumption that structural changes occur rarely, if they occur at all. However, if the growth rate changes frequently, then an endogenous process must be used to explain this variation, which results in the two similar expressions specified in equation (6).

We endogenize the growth rate of equation (5), γ, via the addition of a constant-returns-to-scale knowledge accumulation technology – a generalized version of the Lucas (1988) endogenous growth model. This growth model incorporates the second specification listed for the parameter, B, into the expression in equation (6), which contains the production technology parameters of both the output and knowledge accumulation sectors. In contrast, the AK growth model endogenizes the growth rate, γ, through the elimination of diminishing returns to capital. The expression for the growth rate of the AK growth model is also given by equation (6), but it incorporates the first specification listed for the parameter B – the technology index, A, of the output sector in the model. Equation (6) also contains "deep" parameters common to both endogenous growth models.

However, unlike the growth rate, γ, in the neoclassical model, we argue that these “deeper” parameters *can* change over time for a variety of reasons, and that this will induce corresponding changes in the productivity growth rate. Therefore, regardless of which approach is used to endogenize the growth rate, γ, it should be apparent that evidence of structural changes in γ implies endogenous growth. It is this distinction that is exploited in testing for endogenous growth. Lau (1997) first suggested this conjecture, which is essentially correct.

The growth rate expressions for the two types of endogenous growth models contain many deep parameters emanating from the production technology as well as from preferences.

Let's discuss how these parameters could change. The technological parameters contain: the technology indexes, $A, \tilde{A}, \Phi$, which may themselves be functions of other endogenous variables and parameters (such as energy, materials, and public capital); the capital share parameters, α, ϕ; and the depreciation rate, δ, which can be affected by technological improvements in capital as well as in the substitution among different forms of capital that result from tax changes. In addition, endogenous changes in the industrial composition of aggregate output will induce parameter shifts in all of the aggregate technology parameters.

The preference parameters are the intertemporal substitution elasticity, θ; the rate of time preference, ρ; and the labor force growth rate, η. In an aggregate model, these preference parameters can all be influenced by disaggregated changes in population demographics. In an open-economy model, the growth rate expressions contain two additional parameters: $\overline{m}$, which represents the institutional constraint on the debt/wealth ratio that stems from capital market imperfections; and, r, which represents the national (or international) interest rate. Thus, institutional and other external changes to world credit markets lead to the possibility of additional regime shifts.

In order to discriminate between the neoclassical and the endogenous growth models, I conduct statistical tests for structural changes in a deterministic linear trend, while at the same time modeling the stochastic growth process as a random walk.

III. The Empirical Model

The empirical model is presented in equations (7) through (9) below, which are multivariate extensions of equations (1), (3) and (4), respectively (detail is suppressed on the convergence process, μ_t^i, because the parameters of the ARMA processes are not estimated).

These equations encompass a structural time-series system:

$$(7) \quad \ln y_t^i = \ln \bar{y}_t^i + \mu_t^i \qquad \forall \quad i = 1, \ldots, n.$$

$$(8) \quad \ln \bar{y}_t^i = a^i(t) + b^i(t)t + c^i(t)\xi_t \qquad \forall \quad i = 1, \ldots, n.$$

$$(9) \quad \xi_t = \xi_{t-1} + \varepsilon_t \equiv \sum_{k=1}^{t} \varepsilon_k$$

where:

$$a^i(t) = a_0^i + \Delta a_j^i d_j \qquad d_j = \begin{cases} 1 & if \quad T_j + 1 \le t \le T_{j+1} \\ 0 & otherwise \end{cases} \qquad \text{for: } j = 1, \ldots, M_1$$

$$b^i(t) \equiv \gamma^i(t) = b_0^i + \Delta b_j^i d_j \qquad d_j = \begin{cases} 1 & if \quad T_j + 1 \le t \le T_{j+1} \\ 0 & otherwise \end{cases} \qquad \text{for: } j = 1, \ldots, M_2$$

$$c^i(t) = c_0^i + \Delta c_j^i d_j \qquad d_j = \begin{cases} 1 & if \quad T_j + 1 \le t \le T_{j+1} \\ 0 & otherwise \end{cases} \qquad \text{for: } j = 1, \ldots, M_3.$$

M_1, M_2, M_3: represent the number of breaks in a, b, c, respectively.

$T_0 = 0, \quad T_{M+1} = T$.

μ_t^i: a stationary stochastic "convergence" process.

ε_t: a common growth shock.

However, equations (7) through (9) also include four additional wrinkles. First, along with the multivariate nature of the steady-state in equation (8), the coefficients of the linear trend are allowed to vary discretely over time – this wrinkle allows structural changes to be estimated in the deterministic part of the growth process. Second, regional scaling coefficients on the "common" non-stationary stochastic trend, ξ_t, are included. Third, these scaling coefficients are allowed to vary discretely over time. These two wrinkles enable the stochastic trends to vary across regions, but in a systematic way.

Fourth, using cointegration techniques, we separately estimate and extract the "common" non-stationary stochastic trend represented in equation (9). The reason for using cointegration techniques is that if we were to estimate equations (7) through (9) as a univariate system for each region, we would have an identification problem caused by the difficulty in separating out the influence of two stochastic shocks with only one data series. Employing a dynamic factor analysis (i.e., with cointegration techniques) using many productivity indicators across regions avoids this problem, and enables the identification and estimation of the stochastic trend (i.e., the growth shock) separately from the stochastic convergence process.

IV. The Empirical Test for Endogenous Growth

4.1 Unit-Root Tests

The statistical tests employ quarterly per-capita personal income for each of the eight census regions, spanning the time period from 1948:1 to 1998:3. In appendix 3, I provide a more complete description of this data set. The first test is for unit roots, using Dickey-Fuller (DF) and Augmented Dickey-Fuller (ADF) tests. The test results are presented in Table 1. The results for per-capita income levels indicate that the unit-root null cannot be rejected, while the results for the first differences indicate that the unit-root null is strongly rejected. The conclusion is that regional per-capita incomes are generated from a stochastic process that is integrated of order one (i.e., is non-stationary).

The test for endogenous growth entails a search for regime shifts in the deterministic part of the growth process. As discussed in section 2, multiple changes in the trend growth rate, γ, are consistent with endogenous growth. However, since the data are non-stationary there is an identification problem in implementing this test in a univariate framework – the problem occurs because it is difficult to identify two stochastic shocks (a stationary productivity shock and a non-stationary growth shock) with a single data series.[2] In a univariate context, this problem can be solved only with prior knowledge of at least one of the stochastic error variances, or with their ratio. However, in a multivariate context, the identification problem can be solved if one of the shocks is common to all regions. In this case, there are multiple indicators with which to conduct a dynamic factor analysis to extract the common factor, or the common shock.

4.2 Dynamic Factor Analysis

Dynamic factor analysis and Stock's (1988) concept of a common stochastic trend (i.e., cointegration) are synonymous. Therefore, I test for cointegration and, if possible, attempt to extract a single common trend. Barro and Sala-i-Martin's (1992) assumption of unconditional convergence across states directly implies the hypothesis of a single common trend. Table 2 summarizes the results from a Johansen cointegration test. While the data are cointegrated, the results show that a large number of unit-roots exist across regions. However, structural changes in growth model parameters – the basis for our endogenous growth test – suggests that the cointegrating vectors would contain structural breaks, contaminating the Johansen test results.

To examine the possibility of parameter instability, the Stock and Watson (1993) DOLS procedure is used to estimate the seven cointegrating vectors, with the West Coast series as the single driving process. Because the DOLS estimator is linear, the Bai (1997) and Bai and Perron (1998) procedure is used to endogenously search for multiple breaks (one at a time) in each of the estimated cointegrating vectors. This procedure is described in appendix 4. Not surprisingly,

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2 There are a number of trend break tests for univariate series. However, these tests have been designed mainly to show that trend breaks may be the cause of a unit root. Thus, this type of test is an adjunct to the unit root tests. Second, the reason that we do not use these additional unit root tests is that they have been designed to search for one break, rather than the multiple breaks that we expect to find. Third, while both trend stationary data and non-stationary data are consistent with the stochastic neoclassical and endogenized growth rate models, the stochastic AK model is consistent only with non-stationary data. Thus, incorporating all three growth models under one hypothesis test for endogenous growth requires non-stationary data – and a solution to the identification problem.

I find multiple breaks in all three parameters of the cointegrating vectors – intercepts, linear trend slopes, and cointegrating coefficients. Table A4.1 in appendix 4 presents the break-dates that the procedure found, along with the associated Sup-F statistics for each of the cointegrating vectors.

Next, DF and ADF tests are performed on the cointegrating residuals as an alternative test for cointegration in the presence of structural breaks, the results of which are presented in Table 3. These results show that the non-stationary null-hypothesis is rejected for all seven equilibrium relationships. Therefore, I conclude that, after adjusting for regime shifts, the data contain a single common stochastic trend. However, since each of the estimated cointegrating vectors contains a statistically significant trend term, I also conclude that unconditional convergence does not occur.[3] Thus, the common trend represents a common growth shock.

A dynamic factor analysis is performed to extract the common growth shock. I follow Johansen (1995) in forming the common trend via a linear combination of the integrated residuals obtained from a Johansen vector error-correction mechanism (VECM), which is estimated using data that are adjusted for the regime shifts. The resulting drift-less series is the common growth shock, ξ_t, in equation (9). Chart 1 plots a an upward drifting version of this growth shock – the drift is an average estimated across regions with panel data methods. The graph suggests that along with the drift, the growth shock is characterized by two different cyclical movements – the business cycle, and a longer-run technological cycle. These two stochastic movements appear to account for the non-stationarity in per-capita incomes.

4.3 The Test for Endogenous Growth

With the identification problem solved, I proceed to estimate the structural time-series

3 We perform a formal test of unconditional convergence in another paper.

system of equations (7) through (9). If equation (8) is substituted into equation (7) for $\bar{y}_t^i$, then there is a nine equation system: eight regional versions of the expanded equation (7) and the common growth shock of equation (9), which was estimated using a dynamic factor analysis. Employing the original data series as dependent variables, a constant, linear trend, the drift-less common growth shock, and two leads and lags of the first differences of the growth shock are included as the regressors in each of the eight DOLS estimations. The Bai and Perron (1998) procedure is used to endogenously search for multiple breaks (one at a time) in three coefficients (constant, trend growth rate, and the scaling coefficients on the growth shock) in each of these eight regressions. Table 4 lists the regimes and the Sup-F statistics found by the procedure, while Table 5 lists the coefficient estimates for the growth rates in each regime.

There are many regime shifts in the trend growth rate. But, some changes are of relatively short duration, suggesting that these may be picking up cyclical movement. To explore this possibility, I compare regime dates to NBER business cycle dates, graph the regional growth shocks along with the original series in Chart 2, and graph the segmented trends along with the original series in Chart 3. After examining these comparisons, it appears that only four trend breaks (highlighted in yellow in Table 4) may be picking up cyclical movement. These four are incorporated into the regional growth shocks depicted in Chart 2, and eliminated from Chart 3 and Table 5. Chart 2 demonstrates that the growth shocks pick up most of the cyclical movement.[4] Similarly, Chart 3 shows that while some trend breaks originate during a business cycle movement, these breaks are capturing long-lived changes in growth rates. According to

4 The plotted growth shocks are trended, with the trend growth rate being the average growth rate over the complete time span for each region. The regional growth shock is the common growth shock scaled by its estimated regional coefficient from equation (7), and incorporates the regime shifts in these regional scaling coefficients along with the regional level shifts, that is the regime shifts in the regional intercepts.

Chart 3, the remaining short-duration trend breaks represent a gradual ratcheting upward, or downward in the long-run trend growth rate.

Tables 4 and 5 along with Chart 3 demonstrate that there is strong evidence that regional growth rates change frequently over time.[5] This is consistent with frequent changes in the "deep" parameters of a regional specific endogenous growth model, inducing changes in the corresponding productivity growth rate. Therefore, I conclude that the process of regional growth must emanate from an endogenous growth process, although we cannot determine which particular endogenous process (i.e., an AK growth model, or an endogenized growth rate model), nor which particular deep parameter(s) caused these changes. We leave this for future work. However, this study does provide empirical evidence consistent with endogenous growth.

V. Conclusions

The main contribution of this paper is to provide empirical evidence supporting endogenous growth. This stands in contrast to most of the literature, both cross-sectional and time-series, that finds against endogenous growth. The cross-sectional literature reaches this conclusion on the basis of empirical evidence of convergence – which seems to confirm this implication of the neoclassical model as opposed to the no-convergence implication of a non-stochastic AK endogenous growth model. But, convergence is also consistent with the endogenized growth rate model of Romer (1990) as well as with a stochastic AK endogenous growth model, so that convergence, in and of itself, is not sufficient to reject endogenous growth.

The time-series literature reaches its conclusion on the basis that there is limited empirical evidence of certain determinants having lagged permanent effects on productivity

5 The break-point dates may be sensitive to changes in a common trend that is not unique. However, this does not invalidate my conclusions because while the break dates may vary with changes to the common trend, the ability to find multiple regime shifts

growth rates. However, the conclusions of this literature are subject to both omitted variable bias and endogenous variable bias. In the only time-series study that shows support for endogenous growth, Kocherlakota and Yi (1997) demonstrated that omitted variable bias is indeed a serious problem in these studies. However, the government policy (as well as investment) variables that predominate on the right-hand-side in these studies are endogenous variables (see Yip (1999) for one study that tests the exogeneity assumption). Therefore, this vein of the empirical literature also suffers from the equally serious problem of endogenous variable bias, such that their conclusions should be highly suspect.

In comparison, the structural change test employed in this paper provides powerful evidence that productivity growth rates change frequently over time. This evidence is sufficient to reject the parameter constancy assumption that is the basis for exogenously specifying the technological growth rate in the neoclassical model. Such frequent changes cannot be explained unless the productivity growth rate is endogenized with an endogenous growth model. Thus, the results in this paper provide evidence consistent with the hypothesis that regional economic growth in the U.S. emanates from an endogenous process.

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in the linear trends will not change.

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Table 1 [a,b]
Unit Root Tests of Regional Per-Capita Incomes

Region	Levels			First Differences	
	DF t-test	ADF t-test		DF t-test	ADF t-test
New England	1.04	0.14		-9.65**	-5.51**
Middle Atlantic	0.73	0.10		-11.61**	-6.33**
Great Lakes	0.55	-0.05		-10.63**	-6.07**
Plains	0.68	0.11		-12.06**	-6.75**
Southeast	-0.78	-1.02		-10.33**	-5.65**
Southwest	-0.30	-0.58		-12.49**	-6.28**
Mountains	1.08	0.78		-14.12**	-6.76**
West Coast	0.40	-0.38		-10.42**	-5.87**

a. Regional per-capita incomes are quarterly series running from 1948:1 to 1998:3.
Critical Value for the DF and ADF t-test of levels at a 5% significance level: 1.96.
Critical Value for the DF and ADF t-test of first differences at a 5% significance level: -2.88.

b. ADF test based on a regression containing 2 lags of first differences

** Denotes significance at the 5% level.

Table 2

Eight Census Region Johansen Cointegration Tests

Cointegrating Equations		Contain a Constant				Contain a Linear Trend			
VARS		Contain a Drift				Contain a Drift			
Number of Trends	Number of Trends	Trace	5% C.V.	Max Eigenvalue	5% C.V.	Trace	5% C.V.	Max Eigenvalue	5% C.V.
0									
>=1	1 vs 0	0.056	3.76	0.056	3.76	2.930	12.25	2.930	12.25
>=2	2 vs 1	9.380	15.41	9.324	14.07	14.680	25.32	11.750	18.96
>=3	3 vs 2	21.336	29.68	11.956	20.97	28.098	42.44	13.418	25.54
>=4	4 vs 3	35.352	47.21	13.996	27.07	52.466	62.99	24.368	31.46
>=5	**5 vs 4**	**63.598**	**68.52**	**28.266**	**33.46**	**83.958**	**87.31**	**31.492**	**37.52**
>=6	**6 vs 5**	**95.512****	**94.15**	**31.914**	**39.37**	**123.33****	**114.90**	**39.372**	**43.97**
>=7	7 vs 6	146.776**	124.24	51.264**	45.28	178.858**	146.76	55.528**	49.42
>=8	8 vs 7	216.606**	156.00	69.83**	51.42	252.89**	182.82	74.032**	55.50

** Significant at the 5% level.

Table 3 [a,b]

Alternative Cointegration Test:

Unit Root Test of Cointegrating Residuals

(After adjustment for parameter instability in cointegrating equations)

Region	Residual				C.E. Adjusted with Leads and Lags		
	DF t-test		ADF t-test		DF t-test		ADF t-test
New England	-7.430**		-6.038**		-7.821**		-6.042**
Middle Atlantic	-8.244**		-7.551**		-8.681**		-7.654**
Great Lakes	-7.606**		-6.619**		-7.656**		-7.062**
Plains	-6.896**		-7.452**		-7.010**		-7.307**
Southeast	-7.159**		-6.786**		-7.948**		-6.978**
Southwest	-7.609**		-7.253**		-8.216**		-6.861**
Mountains	-6.311**		-5.274**		-6.290**		-5.255**

a. Sample runs from 1949:4 to 1998:1.
Cointegrating Residual from Stock and Watson DOLS estimator using the West Coast region as the non-stationary series.
Critical Value for the DF and ADF t-test at a 5% significance level: 2.88.

b. ADF test based on a regression containing 2 lags of first differences

** Denotes significance at the 5% level.

Table 4 (a)
Structural Time-Series Equation -- Regime Change Tests

Region	Coefficient	Regime Dates: Beginning Date		Regime Dates: Ending Date	Sup-F Statistics
New England					
	Constant				
		1949:4		1953:3	
		1953:4		1973:4	16.9784
		1974:1		1985:4	641.1672
		1986:1		1986:4	101.3854
		1987:1		1990:1	21.1025
		1990:2		1998:1	94.5061
	Trend				
		1949:4		1955:2	
		1955:3		1982:4	38.7780
		1983:1		1985:4	117.9593
		1986:1		1989:2	1,247.3549
		1989:3		1991:1	23.8490
		1991:2		1998:1	37.4178
	Growth shock				
		1949:4		1961:2	
		1961:3		1963:2	42.4051
		1963:3		1972:4	19.8266
		1973:1		1998:1	26.5019
Middle Atlantic					
	Constant				
		1949:4		1974:2	
		1974:3		1986:4	1,065.4879
		1987:1		1998:1	291.7386
	Trend				
		1949:4		1951:3	
		1951:4		1955:3	50.6973
		1955:4		1973:4	33.6032
		1974:1		1976:3	29.4929
		1976:4		1983:2	164.6536
		1983:3		1987:3	30.2907
		1987:4		1998:1	22.4695
Great Lakes					
	Constant				
		1949:4		1972:3	
		1972:4		1980:1	27.2375
		1980:2		1993:1	454.8053
		1993:2		1998:1	154.9867
	Trend				
		1949:4		1961:4	
		1962:1		1970:4	50.6116
		1971:1		1978:2	246.3882
		1978:3		1998:1	113.1922

a. The 5% Critical Value is 15.2. See Bruce Hansen, "Tests for Parameter Instability in Regressions with I(1) Processes," Journal of Business and Economic Statistics, July 1992

Table 4 (Continued)
Structural Time-Series Equation -- Regime Change Tests

Region	Coefficient	Regime Dates: Beginning Date	Regime Dates: Ending Date	Sup-F Statistics
Great Plains				
	Constant			
		1949:4	1953:2	
		1953:3	1954:3	78.3484
		1954:4	1969:2	59.0218
		1969:3	1972:3	424.1866
		1972:4	1975:4	37.7396
		1976:1	1988:2	60.4358
		1988:3	1990:3	176.6094
		1990:4	1998:1	32.6311
	Trend			
		1949:4	1961:4	
		1962:1	1964:4	27.0468
		1965:1	1993:3	140.7396
		1993:4	1998:1	41.9053
	Growth shock			
		1949:4	1956:3	
		1956:4	1958:4	18.2346
		1959:1	1998:1	17.8547
Southeast				
	Constant			
		1949:4	1957:3	
		1957:4	1959:2	19.9026
		1959:3	1968:1	33.3376
		1968:2	1970:4	2,296.5020
		1971:1	1975:4	168.8359
		1976:1	1998:1	25.4800
	Trend			
		1949:4	1953:3	
		1953:4	1955:2	23.2708
		1955:3	1975:4	34.9683
		1976:1	1977:3	57.9971
		1977:4	1990:4	327.2663
		1991:1	1995:1	583.4392
		1995:2	1998:1	41.4929
	Growth shock			
		1949:4	1978:3	
		1978:4	1998:1	26.9264
Southwest				
	Constant			
		1949:4	1970:1	
		1970:2	1980:3	45.6678
		1980:4	1986:2	92.4129
		1986:3	1990:4	1,409.4865
		1991:1	1998:1	24.5535
	Trend			
		1949:4	1966:3	
		1966:4	1968:1	73.2759
		1968:2	1983:4	638.4706
		1984:1	1985:3	80.2725
		1985:4	1993:3	20.3197
		1993:4	1998:1	30.1693

a. The 5% Critical Value is 15.2. See Bruce Hansen, "Tests for Parameter Instability in Regressions with I(1) Processes," Journal of Business and Economic Statistics, July 1992

Table 4 (Continued)
Structural Time-Series Equation -- Regime Change Tests

Region	Coefficient	Regime Dates: Beginning Date	Regime Dates: Ending Date	Sup-F Statistics
Mountain				
	Constant			
		1949:4	1953:2	
		1953:3	1986:2	257.9041
		1986:3	1990:3	135.9453
		1990:4	1998:1	28.8684
	Trend			
		1949:4	1969:2	
		1969:3	1994:1	341.2107
		1994:2	1998:1	49.6590
West Coast				
	Constant			
		1949:4	1969:4	
		1970:1	1991:1	22.1223
		1991:2	1998:1	24.4311
	Trend			
		1949:4	1993:1	
		1993:2	1998:1	29.6081
	Growth shock			
		1949:4	1953:3	
		1953:4	1998:1	24.1592

a. The 5% Critical Value is 15.2. See Bruce Hansen, "Tests for Parameter Instability in Regressions with I(1) Processes," Journal of Business and Economic Statistics, July 1992

Table 5 (a)
Structural Time-Series Equations -- Growth Rate Coefficients

Region	Regime Dates		Coefficients		
Region	Beginning Date	Ending Date	Annual Growth Rate	Quarterly Growth Coeff	Coeff Differences
New England					
	1949:4	1955:2	1.54%	0.003820	
				(13.95541)	
	1955:3	1985:4	2.19%	0.005439	0.001619
					(6.813603)
	1986:1	1989:2	3.87%	0.009533	0.005713
					(11.92012)
	1989:3	1991:1	3.76%	0.009265	0.005445
					(11.6451)
	1991:2	1998:1	3.64%	0.008990	0.005170
					(12.11349)
Middle Atlantic					
	1949:4	1955:4	1.89%	0.004701	
				(15.41910)	
	1955:4	1973:4	2.31%	0.005736	0.001035
					(3.864387)
	1974:1	1976:3	2.13%	0.005289	0.000588
					(2.077411)
	1976:4	1987:3	1.96%	0.004866	0.000165
					(0.584857)
	1987:4	1998:1	2.11%	0.005236	0.000535
					(1.860474)
Great Lakes					
	1949:4	1961:4	1.43%	0.003547	
				(18.83315)	
	1962:1	1970:4	1.68%	0.004167	0.000620
					(5.308937)
	1971:1	1978:2	1.86%	0.004610	0.001063
					(8.444407)
	1978:3	1998:1	1.72%	0.004271	0.000724
					(4.269166)

a. Statistics in parentheses are t-statistics, which are computed using Newey-West Heteroscedasticity and Autocorrelation adjusted standard errors.

Table 5 (Continued)
Structural Time-Series Equations -- Growth Rate Coefficients

Region	Regime Dates		Coefficients (a)		
	Beginning Date	Ending Date	Annual Growth Rate	Quarterly Growth Coeff	Coeff Differences
Great Plains					
	1949:4	1961:4	1.64%	0.004078	
				(17.26690)	
	1962:1	1964:4	1.86%	0.004622	0.000544
					(5.115076)
	1965:1	1993:3	1.98%	0.004924	0.000846
					(5.790380)
	1993:4	1998:1	2.06%	0.005107	0.001029
					(6.761185)
Southeast					
	1949:4	1955:2	3.49%	0.008617	
				(31.49769)	
	1955:3	1975:4	3.27%	0.008077	-0.000540
					(-3.427646)
	1976:1	1977:3	2.64%	0.006536	-0.002081
					(-6.179580)
	1977:4	1990:4	2.55%	0.006323	-0.002294
					(-7.398140)
	1991:1	1995:1	2.46%	0.006088	-0.002529
					(-8.268224)
	1995:2	1998:1	2.42%	0.005986	-0.002631
					(-8.681164)
Southwest					
	1949:4	1966:3	1.78%	0.004422	
				(35.159000)	
	1966:4	1968:1	2.03%	0.005034	0.000612
					(5.487312)
	1968:2	1983:4	2.18%	0.005399	0.000977
					(9.057914)
	1984:1	1985:3	2.11%	0.005233	0.000811
					(6.978622)
	1985:4	1993:3	2.02%	0.005005	0.000583
					(4.903233)
	1993:4	1998:1	2.09%	0.005185	0.000763
					(6.481533)

a. Statistics in parentheses are t-statistics, which are computed using Newey-West Heteroscedasticity and Autocorrelation adjusted standard errors.

Table 5 (Continued)
Structural Time-Series Equations -- Growth Rate Coefficients

Region	Regime Dates: Beginning Date	Regime Dates: Ending Date	Coefficients (a): Annual Growth Rate	Coefficients (a): Quarterly Growth Coeff	Coefficients (a): Coeff Differences
Mountain					
	1949:4	1969:2	2.00%	0.004961	
				(33.885034)	
	1969:3	1994:1	2.17%	0.005371	0.000410
					(4.325418)
	1994:2	1998:1	2.26%	0.005599	0.000638
					(6.200793)
West Coast					
	1949:4	1993:1	1.84%	0.004562	
				(87.34277)	
	1993:2	1998:1	1.88%	0.004666	0.000104
					(4.223410)

a. Statistics in parentheses are t-statistics, which are computed using Newey-West Heteroscedasticity and Autocorrelation adjusted standard errors.

Chart 1
Common Stochastic Trend (with a common drift)

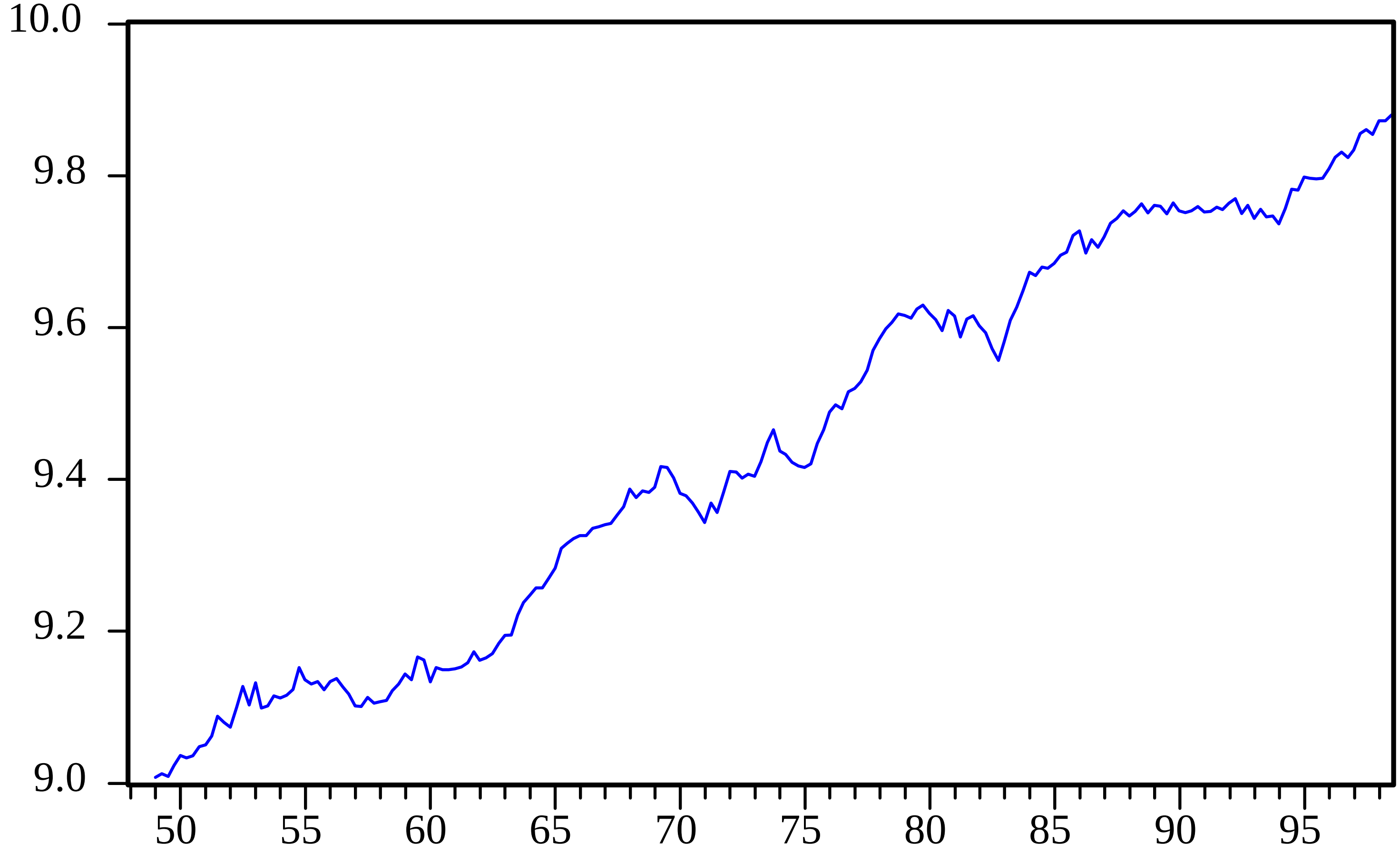


Chart 2
Per-Capita Income Levels and Trended Growth Shocks
(including level shifts)

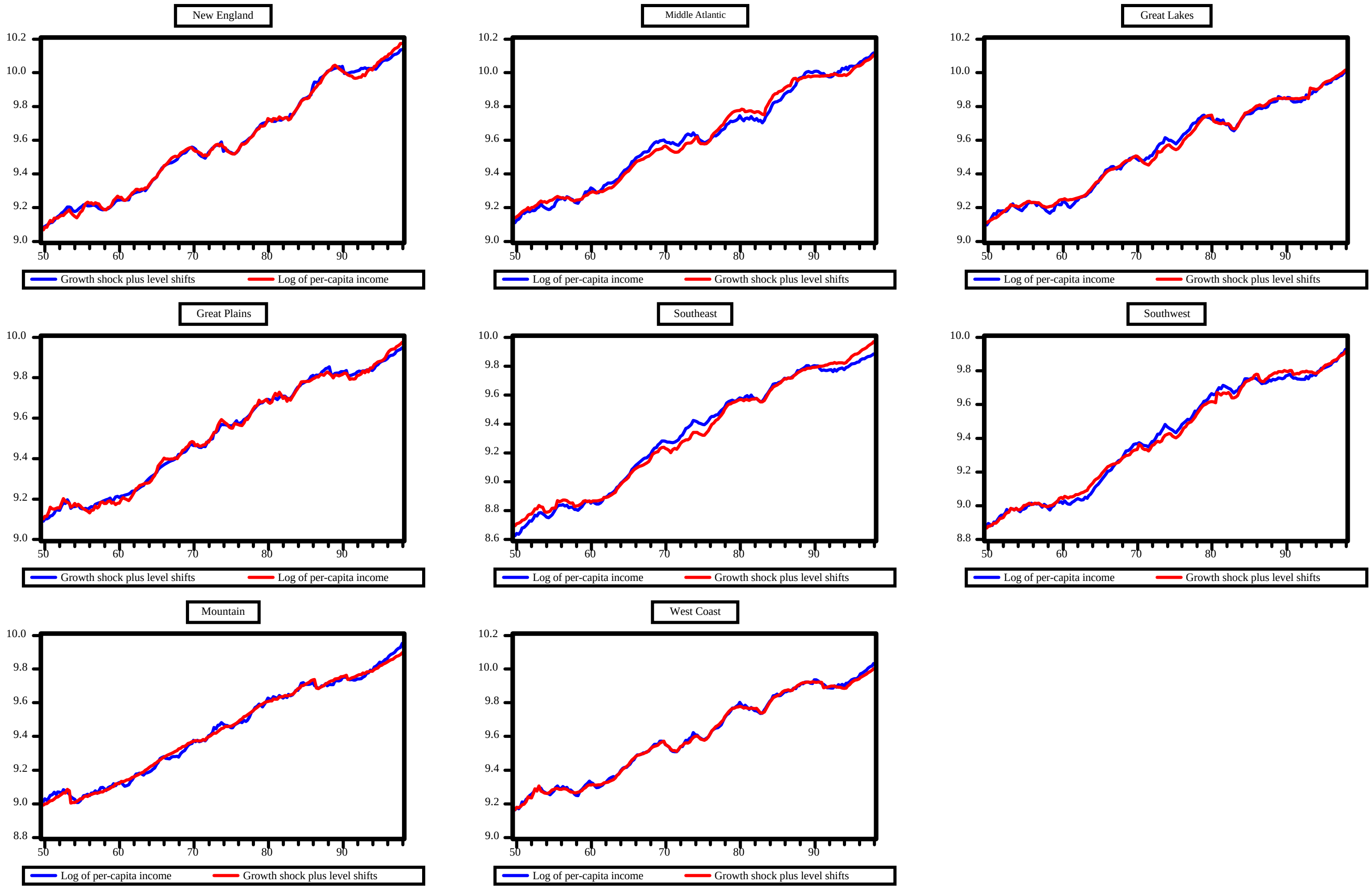
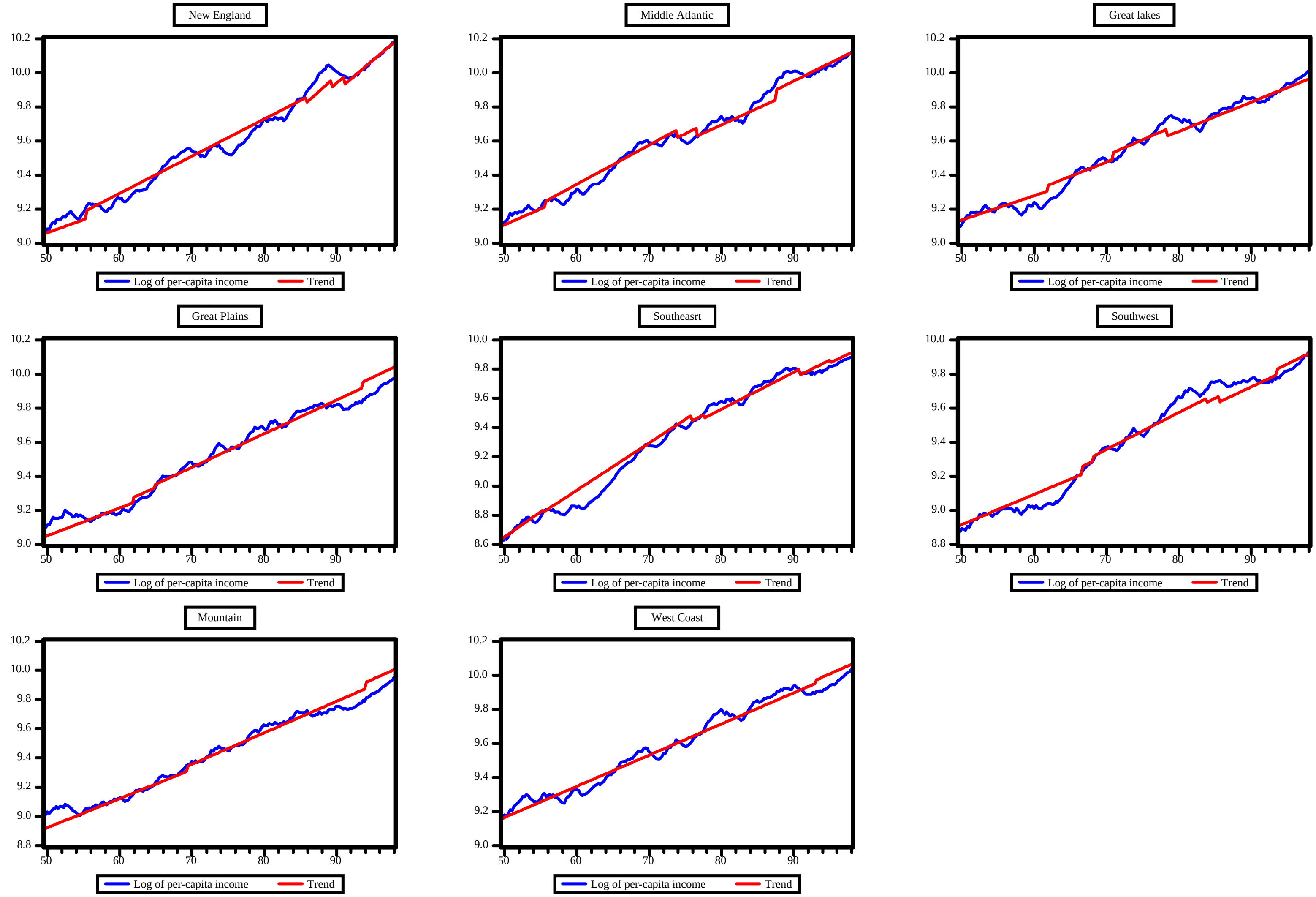


Chart 3
Per-capita income levels and Segmented Trends



Appendix 1
Variable Glossary

L : denotes labor input. $\eta \equiv \frac{\Delta L}{L}$: denotes the labor force growth rate.

Y: denotes aggregate output. $y \equiv \frac{Y}{L}$: is observed labor productivity.

K: denotes aggregate capital stock. $k \equiv \frac{K}{L}$: is the observed capital-labor ratio.

C denotes aggregate consumption. $c \equiv \frac{C}{L}$: is observed per-capita consumption.

d : denotes per-capita external debt: $d = \overline{m}k$

$\overline{m}$: denotes a constant debt-capital ratio. δ : is the depreciation rate.

v : is a productivity shock. r : denotes the real world interest rate.

β : denotes the discount rate: $\beta = \frac{1}{1+\zeta}$, and $\zeta = \rho - \eta$.

θ : the intertemporal rate of substitution: $\theta = -\frac{U''(c)c}{U'(c)}$.

ρ : denotes the rate of time preference.

Differences Across Growth Models

The Neoclassical Growth Model

$\tilde{L} = hL$: is efficiency labor. $\beta = \frac{1}{1+\zeta}$, and $\zeta = \rho - \gamma - \eta$.

h : is a Harrod-neutral technology index: $\gamma \equiv \frac{\Delta h}{h}$: denotes its exogenous growth rate.

$\tilde{y} \equiv \frac{Y}{\tilde{L}}$: is labor productivity: $\tilde{y} = \phi(\tilde{k}) \equiv Ae^{\ln v}\tilde{k}^{\alpha}$.

$\tilde{k} \equiv \frac{K}{\tilde{L}}$: is the capital-labor ratio. $\tilde{c} \equiv \frac{C}{\tilde{L}}$: is per-capita consumption.

$\tilde{d}$: per-capita debt (in efficiency units): $\tilde{d} = \overline{m}\tilde{k}$.

The Endogenized Growth Model

H : denotes efficiency labor (knowledge): $H = hL$.

$h \equiv \frac{H}{L}$: denotes per-capita knowledge: $\Delta h = g(k_h, h_h) \equiv \Phi\left((1-u)k\right)^{\phi}\left((1-w)h\right)^{(1-\phi)}$.

$y \equiv \frac{Y}{L}$: is observed labor productivity: $y = f(k_y, h_y) \equiv Ae^{\ln v}(uk)^{\alpha}(wh)^{(1-\alpha)}$.

u: share of capital used in output sector. w: share of knowledge used in output sector.

$d_t = \overline{m}(k_t + h_t)$. $\overline{m}$: denotes a constant debt-wealth ratio.

δ_k, δ_h: are the depreciation rates on capital and knowledge.

The AK Endogenous Growth Model

$y \equiv \frac{Y}{L}$: is observed labor productivity: $y = \phi(k) \equiv Ae^{\ln v}k$.

Appendix 2
Derivation of the law of motion for productivity

The Neoclassical Model

The problem of the representative consumer is to:

$$\underset{\tilde{c}}{Max} \quad \sum_{t=0}^{\infty} \beta^t U(\tilde{c}_t)$$

$$s.t. \quad \Delta\tilde{k}_t = \frac{1}{(1-\overline{m})}\left[\phi(\tilde{k}_t) - \tilde{c}_t - \left[(1-\overline{m})(\eta+\gamma+\delta) + \overline{m}(r+\delta)\right]\tilde{k}_t\right]$$

To solve this problem, we set up a Lagrangian and take derivatives with respect to the control variable, $\tilde{c}_t$, the state variable, $\tilde{k}_{t+1}$, and the co-state variable, λ_t. These derivatives form the first-order necessary conditions, which can be reduced to the following two equations:

$$\text{(A2.1)} \quad \frac{\Delta\tilde{k}}{\tilde{k}} = \frac{1}{(1-\overline{m})}\left[\frac{\phi(\tilde{k})}{\tilde{k}} - \frac{\tilde{c}}{\tilde{k}} - \left[(1-\overline{m})(\eta+\gamma+\delta) + \overline{m}(r+\delta)\right]\right] = 0$$

$$\text{(A2.2)} \quad \frac{\Delta\tilde{c}}{\tilde{c}} = \frac{\beta}{(1-\overline{m})\theta}\left[\phi'(\tilde{k}) - \left[(1-\overline{m})(\eta+\gamma+\delta+\zeta) + \overline{m}(r+\delta)\right]\right] = 0.$$

Assuming perfect certainty, the expression inside the brackets of equation (A2.2) is solved for the steady-state, $\overline{\overline{y}}, \overline{y}_t$, which is given by:

(A2.3) $\bar{y}_t \equiv \tilde{\bar{y}} h_t$

where: $\tilde{\bar{y}} = A^{\frac{1}{(1-\alpha)}} \left[\frac{\alpha}{[(1-\bar{m})(\eta+\gamma+\delta+\zeta)+\bar{m}(r+\delta)]} \right]^{\frac{\alpha}{(1-\alpha)}}$

$h_t = h_0 e^{\gamma t + \xi_t}$.

Equations (A2.1) and (A2.2) also imply the following two conditions in the steady-state:

(A2.4) $Ae^{-(1-\alpha)\ln \tilde{k} + \ln v} - e^{\ln \tilde{c} - \ln \tilde{k}} = [(1-\bar{m})(\eta+\gamma+\delta)+\bar{m}(r+\delta)]$

(A2.5) $\alpha A e^{-(1-\alpha)\ln \tilde{k} + \ln v} = [(1-\bar{m})(\eta+\gamma+\delta+\zeta)+\bar{m}(r+\delta)]$.

We take a first-order Taylor series expansion of equations (A2.1) and (A2.2), evaluated at the steady-state conditions in equations (A2.4) and (A2.5). The resulting simultaneous system of log-linear, stochastic difference equations are solved for the choice rule of $\tilde{k}_{t+1}$. The production function is rearranged and substituted into this rule, giving the time path for productivity:

(A2.6) $(\ln y_{t+1} - \ln \bar{y}_{t+1}) = \vartheta_1(\ln y_t - \ln \bar{y}_t) + \frac{[(1+\alpha\psi_2)\vartheta_2 - \omega]}{\vartheta_2 - \omega} \ln v_{t+1} + (\alpha\psi_1 - \vartheta_1)\ln v_t$.

where: ψ_1, ψ_2 : are complicated functions of all the parameters in the model.

ϑ_1 : is the stable eigenvalue of the equation system and corresponds to the speed of convergence in the deterministic beta-convergence model.

ϑ_2 : is the unstable eigenvalue of the equation system.

ω: is an AR(1) coefficient governing the expectation of future shocks: v_{t+j} .

Adding $\ln \bar{y}_{t+1}$ to both sides of equation (A2.6) yields equations (1) and (2) in the text:

(A2.7) $\ln y_{t+1} = \ln \bar{y}_{t+1} + \mu_{t+1}$

(A2.8) $\mu_{t+1} \equiv (\ln y_{t+1} - \ln \bar{y}_{t+1}) = \vartheta_1 \mu_t + \frac{[(1+\alpha\psi_2)\vartheta_2 - \omega]}{\vartheta_2 - \omega} \ln v_{t+1} + (\alpha\psi_1 - \vartheta_1)\ln v_t$.

Equation (A2.8) is a restricted form of equation (2) in the text, where $p = q = 1$, $a_1 = \vartheta_1$, $b_0 = \frac{[(1+\alpha\psi_2)\vartheta_2 - \omega]}{\vartheta_2 - \omega}$, $b_1 = (\alpha\psi_1 - \vartheta_1)$. The steady-state time path is obtained by taking the natural logarithm of equation (A2.3), with $\ln \bar{y}_0 = \ln(h_0 \bar{\tilde{y}})$. Finally, the restriction of equation (2) to an ARMA(1,1) should not be taken too seriously – the length of this lag specification actually depends on the empirical validity of the log-linear approximation.

<u>The Endogenized Growth Rate Model</u>

The problem of the representative consumer is to:

$$\underset{c}{Max} \quad \sum_{t=0}^{\infty} \beta^t U(c_t)$$

$$s.t. \quad \Delta k_t = \frac{1}{(1-\bar{m})}\left[f(u_t k_t, w_t h_t) - c_t - \left[(1-\bar{m})(\eta + \delta_k) + \bar{m}(r+\delta_k)\right]k_t \right]$$

$$\Delta h_t = \frac{1}{(1-\bar{m})}\left[g((1-u_t)k_t, (1-w_t)h_t) - \left[(1-\bar{m})(\eta + \delta_h) + \bar{m}(r+\delta_h)\right]h_t \right]$$

To solve this problem, we set up a Lagrangian and take derivatives with respect to the three control variables, c_t, u_t, w_t; two state variables, k_{t+1}, h_{t+1}; and two co-state variables, λ_t, Λ_t. These derivatives form the first-order necessary conditions, which are reduced to the following five equations:

(A2.9) $$\frac{f_k}{f_h} = \frac{g_k}{g_h}$$

(A2.10) $$f_k - \delta_k = g_h - \delta_h$$

(A2.11) $$\frac{\Delta c}{c} = \frac{\beta}{\theta(1-\bar{m})}\left[f_k - \left[(1-\bar{m})(\eta + \delta_k + \zeta) + \bar{m}(r+\delta_k)\right]\right]$$

(A2.12) $$\frac{\Delta h}{h} = \frac{1}{(1-\bar{m})}\left[\frac{g((1-u)k,(1-w)h)}{h} - \left[(1-\bar{m})(\eta + \delta_h) + \bar{m}(r+\delta_h)\right]\right]$$

(A2.13) $$\frac{\Delta k}{k} = \frac{1}{(1-\bar{m})}\left[\frac{f(uk, wh)}{k} - \frac{c}{k} - \left[(1-\bar{m})(\eta + \delta_k) + \bar{m}(r+\delta_k)\right]\right].$$

<u>Steady-State Solution</u>

If we assume perfect certainty, a closed-form solution can be obtained only if $\delta_k = \delta_h$, otherwise, the five equations must be solved numerically. Assuming $\delta_k = \delta_h$, equations (A2.9) and (A2.10) are solved for $\tilde{k}_y \equiv \frac{uk}{wh}$, which is substituted into equation (A2.11) to obtain $\gamma \equiv \frac{\Delta y}{y} = \frac{\Delta c}{c} = \frac{\Delta k}{k}$. This expression is set equal to the LHS of equation (A2.12) and solved for $w, (1-w)$. The values of w and $\tilde{k}_y$ are substituted into the production function to obtain $\bar{\tilde{y}}, \bar{y}_t$:

$$\text{(A2.14)}\quad \gamma = \frac{\beta}{\theta(1-\overline{m})}\left[B - \left[(1-\overline{m})(\eta+\delta+\zeta) + \overline{m}(r+\delta)\right]\right]$$

$$\text{where: } B = [\alpha A]^{\frac{\phi}{\phi+(1-\alpha)}}[(1-\phi)\Phi]^{\frac{(1-\alpha)}{\phi+(1-\alpha)}}\left[\frac{\phi(1-\alpha)}{\alpha(1-\phi)}\right]^{\frac{\phi(1-\alpha)}{\phi+(1-\alpha)}}.$$

$$\text{(A2.15)}\quad \bar{y}_t = h_t \bar{\tilde{y}}$$

$$\text{where: } \bar{\tilde{y}} = AM\left[\frac{\alpha A}{(1-\phi)\Phi}\right]^{\frac{\alpha}{\phi+(1-\alpha)}}\left[\frac{\alpha(1-\phi)}{\phi(1-\alpha)}\right]^{\frac{\alpha\phi}{\phi+(1-\alpha)}}$$

$$h_t = h_0 e^{\gamma t+\xi_t}$$

$$M = \frac{D}{\theta}\left[\theta D^{-1} - \beta\left[B-(1-\overline{m})\zeta\right] - (\theta-\beta)\left[(1-\overline{m})(\eta+\delta)+\overline{m}(r+\delta)\right]\right]$$

$$D = \Phi^{\frac{(\alpha-1)}{\phi+(1-\alpha)}}\left[\frac{(1-\phi)}{\alpha A}\right]^{\frac{\phi}{\phi+(1-\alpha)}}\left[\frac{\alpha(1-\phi)}{\phi(1-\alpha)}\right]^{\frac{\phi(1-\alpha)}{\phi+(1-\alpha)}}.$$

Time Path Solution

The solution for the time path of productivity is identical to the neoclassical model, except that it is derived from an output production function that is a reduced form of the two-sector problem, with γ given by equation (A2.14) and $\bar{\tilde{y}}$ given by equation (A2.15). The expression for the reduced-form production function is:

$$\text{(A2.16)}\quad \tilde{y}_t = e^{\ln v}\tilde{A}\tilde{k}^{\alpha}$$

$$\text{where: } \tilde{A} = AMR^{\frac{\alpha\phi}{\phi+(1-\alpha)}}Q^{\frac{\alpha[(\alpha-1)-\phi]}{\phi+(1-\alpha)}}$$

$$R = \left[\frac{\alpha(1-\phi)}{\phi(1-\alpha)} \right]$$

$$Q = \left[(1-M)R^{\frac{\alpha-1}{\phi+(1-\alpha)}} + MR^{\frac{\phi}{\phi+(1-\alpha)}} \right].$$

The AK Model

The problem of the representative consumer is to:

$$\underset{c}{Max} \quad \sum_{t=0}^{\infty} \beta^t U(c_t)$$

$$s.t. \quad \Delta k_t = \frac{1}{(1-\overline{m})} \left[\phi(k_t) - c_t - \left[(1-\overline{m})(\eta+\delta) + \overline{m}(r+\delta) \right] k_t \right]$$

To solve this problem, we set up a Lagrangian and take derivatives with respect to the control variable, c_t, the state variable, k_{t+1}, and the co-state variable, λ_t. These derivatives form the first-order necessary conditions, which are reduced to the following two equations:

$$\text{(A2.17)} \quad \frac{\Delta k}{k} = \frac{1}{(1-\overline{m})} \left[\frac{\phi(k)}{k} - \frac{c}{k} - \left[(1-\overline{m})(\eta+\delta) + \overline{m}(r+\delta) \right] \right]$$

$$\text{(A2.18)} \quad \frac{\Delta c}{c} = \frac{\beta}{(1-\overline{m})\theta} \left[\phi'(k) - \left[(1-\overline{m})(\eta+\delta+\zeta) + \overline{m}(r+\delta) \right] \right].$$

Assuming perfect certainty, we substitute expressions derived from the linear production function for $\frac{\phi(k)}{k}$ and $\phi'(k)$ into equations (A2.17) and (A2.18) and then ensure that equation (A2.17) satisfies the transversality condition. Thus, we obtain the steady-state growth rate:

$$\text{(A2.19)} \quad \gamma \equiv \frac{\Delta y}{y} = \frac{\Delta k}{k} = \frac{\Delta c}{c} = \frac{\beta}{(1-\overline{m})\theta} \left[A - \left[(1-\overline{m})(\eta+\delta+\zeta) + \overline{m}(r+\delta) \right] \right].$$

Adding the productivity shock to equation (A2.19), we take a first-order Taylor series expansion of these equations, evaluated at the steady-state given by equation (A2.19). The first equation of the resulting system is the choice rule for capital, k_{t+1}. The production function is rearranged and substituted into this choice rule, providing the time path of productivity:

$$\text{(A2.20)}\quad \ln y_{t+1} = \gamma + \ln y_t + \vartheta_{t+1}$$

$$\text{where: } \vartheta_{t+1} = \frac{1}{(1-\overline{m})\theta}\left[(\beta A + (1-\overline{m})\theta)\ln v_{t+1} - (1-\overline{m})\theta \ln v_t\right].$$

Equation (A2.20) is a random walk. A random walk can be decomposed into the sum of a stochastic trend and a stationary stochastic process. Employing this decomposition for equation (A2.20), we obtain equations (1) through (4) in the text:

$$\text{(A2.21)}\quad \ln y_t = \ln \overline{y}_t + u_t$$

$$\text{(A2.22)}\quad \mu_t \equiv (\ln y_t - \ln \overline{y}_t) = \sum_{i=1}^{p} a_i \mu_{t-i} + \sum_{j=1}^{q+1} b_j \nu_{t-j} \text{ : an ARMA(p, q) process}$$

$$\text{(A2.23)}\quad \ln \overline{y}_t = \ln \overline{y}_0 + \gamma t + \xi_t \text{ : a non-stationary stochastic trend with drift}$$

$$\text{(A2.24)}\quad \xi_t = \xi_{t-1} + \varepsilon_t \text{ : a non-stationary growth shock.}$$

Appendix 3
The Data

I employ quarterly data on per-capita personal income for the eight census regions of the United States, spanning the time period from 1948:1 to 1998:3. The growth models presented in the paper are designed to analyze productivity. However, since there is no labor-leisure choice, nor any labor participation choice in these models, productivity is equivalent to per-capita income. Ideally, I would like to use regional product as the income measure, but the gross state product data released by the U.S. Bureau of Economic Analysis have too short a time span for the analysis conducted in the paper. Thus, we use the less than ideal measure of personal income.

Two data series make up per-capita personal income: personal income and population. The Bureau of Economic Analysis publishes a quarterly state personal income series. The most recent release spanned the time period from 1969:1 to 1998:3. To go back further in time, I compiled additional data from the publication: *State Personal Income: 1929-1982*. I collected quarterly state personal income data from this publication that spanned the time period from 1948:1 to 1968:4. I spliced these data series onto the prior data series, providing a consistent data series spanning the entire time period. We then subtracted a transfer payment series from personal income, to obtain personal income net of transfers, which is the nominal personal income series used in the analysis. Because regional pricing data are very poor, nominal personal income was deflated with the annual GDP price deflator as published in *The Economic Report of the President*.

The population data comes from the Bureau of the Census and are the annual population estimates that the Bureau releases every year. This data series is annual, so I linearly interpolated between every pair of years to generate a quarterly data series. Both the inflation- adjusted personal income series and the quarterly population estimates were then aggregrated from the state to the regional level for each of the eight census regions for every point in time. The per-capita income series is then calculated as inflation-adjusted personal income divided by the population estimates.

Preliminary analysis of the first differences of this data set revealed two major problems requiring adjustment: the presence of outliers, combined with heteroscedasticity in the data. We employed an iterative procedure developed by Chen and Liu (1993) to simultaneously treat both problems. Because serial correlation estimates of ARCH/GARCH processes are biased in the presence of outliers, the Chen and Liu procedure endogenously searches for outliers in combination with providing an estimate of the ARCH 1 parameters in the outlier-free process. Once the procedure achieves convergence, we use these maximum likelihood ARCH estimates to obtain a one-step-ahead forecast of the uncontaminated conditional variances, and then add back the outliers to form a conditional variance series that we use to weight the first differences of per-capita incomes. The adjusted first differences are then integrated to form a series of adjusted per-capita income levels. These adjusted series are used in all subsequent analysis.

Appendix 4
The Bai and Perron Procedure to Search for Structural Breaks in Linear Equations

The Bai and Perron procedure is a Sup-F test – the maximum of a sequence of Wald F-statistics – determining both the break date and the break-point estimator simultaneously. To find the first break, the Wald F-statistic (using a consistent estimate of the unconstrained residual variance) is computed for every data point in the series. The date that achieves the maximum of the sequence of F-statistics determines the date of the candidate break-point. If the Sup-F statistic exceeds the stipulated critical value, then a break in the relationship occurs at this date.[6] With this break-point, the procedure is repeated, searching through each of the sub-samples for additional breaks. If additional breaks are found, then these break-points divide the data into additional sub-samples, each of which is searched for breaks until no additional breaks-points are found. This procedure is valid under the conditions of serial correlation in the errors, which is why we choose not to estimate the ARMA convergence processes (see Bai (1997), or Bai and Perron (1998) for additional information).

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6 The critical value employed in our tests is 15.2, which was computed by Bruce Hansen for a single break. See Bruce Hansen, "Tests for Parameter Instability in Regressions with I(1) Processes," **Journal of Business and Economic Statistics**, July 1992.

Table A4.1 (a)
Cointegrating Equations -- Parameter Instability Tests

Region	Coefficient	Regime Dates		Break Dates	Sup-F Statistics
		Beginning Date	Ending Date		
New England	Constant				
		1949:1	1965:1		
		1965:2	1973:3	1965:2	44.6836
		1973:4	1981:2	1973:4	574.2981
		1981:3	1989:4	1981:3	70.3851
		1990:1	1998:1	1990:1	55.7037
	Trend				
		1949:1	1954:3		
		1954:4	1981:2	1954:4	24.3962
		1981:3	1985:3	1981:3	67.6707
		1985:4	1998:1	1985:4	1,253.8619
	Cointegrating Coeff				
		1949:1	1987:1		
		1987:2	1998:1	1987:2	33.5372
Middle Atlantic	Constant				
		1949:1	1964:3		
		1964:4	1974:2	1964:4	290.4326
		1974:3	1985:1	1974:3	1,908.6797
		1985:2	1987:2	1985:2	173.2195
		1987:3	1998:1	1987:3	601.6554
	Trend				
		1949:1	1973:4		
		1974:1	1975:5	1974:1	89.8863
		1976:1	1978:4	1976:1	19.3385
		1979:1	1995:1	1979:1	75.5246
		1995:2	1998:1	1995:2	22.3836
	Cointegrating Coeff				
		1949:1	1951:3		
		1951:4	1973:1	1951:4	74.7290
		1973:2	1984:1	1973:2	20.8689
		1984:2	1998:1	1984:2	30.4848
Great Lakes	Constant				
		1949:1	1957:3		
		1957:4	1972:3	1957:4	307.7112
		1972:4	1979:2	1972:4	77.8547
		1979:3	1981:3	1979:3	77.5635
		1981:4	1998:1	1981:4	405.8063
	Trend				
		1949:1	1963:1		
		1963:2	1964:4	1963:2	28.6102
		1965:1	1992:3	1965:1	706.8869
		1992:4	1998:1	1992:4	38.5790
	Cointegrating Coeff				
		1949:1	1970:4		
		1971:1	1998:1	1971:1	30.1466

(a.) The 5% Critical Value is 15.2. See Bruce Hansen, "Tests for Parameter Instability in Regressions with I(1) Processes," Journal of Business and Economic Statistics, July 1992

Table A4.1 (Continued)
Cointegrating Equations – Parameter Instability Tests

Region	Coefficient	Regime Dates Beginning Date	Regime Dates Ending Date	Break Dates	Sup-F Statistics
Great Plains	Constant				
		1949:1	1954:3		
		1954:4	1969:2	1954:4	51.5788
		1969:3	1975:4	1969:3	340.6436
		1976:1	1988:2	1976:1	27.5003
		1988:3	1998:1	1988:3	125.5702
	Trend				
		1949:1	1956:3		
		1956:4	1961:4	1956:4	20.6572
		1962:1	1964:3	1962:1	17.6449
		1964:4	1972:4	1964:4	116.3199
		1973:1	1994:1	1973:1	20.3841
		1994:2	1998:1	1994:2	22.6396
Southeast	Constant				
		1949:1	1959:2		
		1959:3	1968:1	1959:3	28.9956
		1968:2	1969:2	1968:2	2,098.0996
		1969:3	1984:1	1969:3	41.1398
		1984:2	1987:2	1984:2	55.3695
		1987:3	1998:1	1987:3	23.6003
	Trend				
		1949:1	1954:4		
		1955:1	1990:3	1955:1	46.7077
		1990:4	1998:1	1990:4	20.6622
	Cointegrating Coeff				
		1949:1	1968:1		
		1968:2	1971:2	1968:2	559.6296
		1971:3	1972:2	1971:3	347.6117
		1972:3	1998:1	1972:3	43.0173
Southwest	Constant				
		1949:1	1980:2		
		1980:3	1998:1	1980:3	67.2659
	Trend				
		1949:1	1959:2		
		1959:3	1965:3	1959:3	37.6630
		1965:4	1968:2	1965:4	52.0963
		1968:3	1982:4	1968:3	599.4966
		1983:1	1998:1	1983:1	68.1631
	Cointegrating Coeff				
		1949:1	1969:4		
		1970:1	1980:4	1970:1	42.5454
		1981:1	1986:2	1981:1	15.7148
		1986:3	1998:1	1986:3	1,349.7491

a. The 5% Critical Value is 15.2. See Bruce Hansen, "Tests for Parameter Instability in Regressions with I(1) Processes," Journal of Business and Economic Statistics, July 1992

Table A4.1 (Continued)
Cointegrating Equations – Parameter Instability Tests

Region	Coefficient	Regime Dates		Break Dates	Sup-F Statistics
		Beginning Date	Ending Date		
Mountain	Constant				
		1949:1	1953:1		
		1953:2	1986:1	1953:2	299.2389
		1986:2	1987:4	1986:2	151.7546
		1988:1	1998:1	1988:1	20.6920
	Trend				
		1949:1	1969:2		
		1969:3	1994:1	1969:3	551.9840
		1994:2	1998:1	1994:2	61.9057
	Cointegrating Coeff				
		1949:1	1953:1		
		1953:2	1998:1	1953:2	22.3251

a. The 5% Critical Value is 15.2. See Bruce Hansen, "Tests for Parameter Instability in Regressions with I(1) Processes," Journal of Business and Economic Statistics, July 1992