# Evaluating Interest Rate Rules
# in an Estimated DSGE Model[*]

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## Abstract

The empirical DSGE literature pays surprisingly little attention to the behavior of the monetary authority. Alternative policy rule specifications abound, but their relative merit is rarely discussed. We contribute to filling this gap by comparing the fit of a large set of interest rate rules (55 in total), which we estimate within a simple New Keynesian model. We find that specifications in which monetary policy responds to inflation and to deviations of output from its efficient level—the one that would prevail in the absence of distortions— have the worst fit within the set we consider. Policies that respond to measures of the output gap based on statistical filters perform better, but the best fitting rules are those that also track the evolution of the model-consistent efficient real interest rate.

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# 1　Introduction

Most central banks have a dual mandate: stabilize inflation and real activity. This dual mandate is explicit and symmetric in the United States, where the Federal Reserve Act instructs the Federal Reserve to "...promote effectively the goals of maximum employment," and "stable prices." But even in inflation targeting countries, whose formal mandates tend to focus on inflation, the implementation of monetary policy usually involves balancing this objective with the stabilization of a real criterion. "In practice, inflation targeting is never 'strict' inflation targeting but always 'flexible' inflation targeting...", according to Svensson (2007) .

And yet, while the interpretation of the price stability mandate has become increasingly transparent and uniform around the world, the real stability objective remains vague everywhere. This lack of clarity reflects in part the absence of a consensus in the academic literature and among policymakers. Economists agree that inflation should be low and stable, but they do not share an operational definition of a real target for monetary policy. In applied contexts, full employment, or potential, output has been traditionally defined as a smooth trend for GDP, and it is often measured through some filtering or de-trending procedure.[1] From a more theoretical perspective, the New Keynesian literature suggests that output should be stabilized around its counterfactual efficient level, the one that would be observed in the absence of distortions (Woodford, 2003).[2] The problem is that these two notions of potential output—one purely statistical, the other one based on theory—can differ significantly, since the latter incorporates the efficient response of the economy to shocks, and hence might be far from smooth.[3]

The absence of a standard definition of the real objective of monetary policy is not only relevant in normative contexts. For example, it is evident in the empirical dynamic stochastic general equilibrium (DSGE) literature. In the last few years, DSGE models have incorporated

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[1] Orphanides and Van Norden (2002) review several of these statistical procedures. Growth accounting represents another popular approach to the measurement of potential output as a smooth trend (see CBO, 2001).

[2] A third approach to the measurement of potential output, which is intermediate between the two described above, involves positing a statistical relationship between inflation and the output gap (a Phillips curve). This relationship then forms the basis for a multivariate Kalman filter to extract potential output (see for example Kuttner, 1994, and Laubach and Williams, 2003). Mishkin (2007) provides an excellent survey of various statistical and model-based methods for the estimation of potential output and discusses their policy implications.

[3] For example, in an estimated DSGE model with several frictions, Edge, Kiley and Laforte (2007) find that the time series of efficient output does not resemble much the more traditional potential output derived within FRB/US. Justiniano and Primiceri (2009) reach opposite conclusions in their simpler model comparing efficient output with an HP trend.

ever more detailed and realistic descriptions of private sector behavior and of the monetary transmission mechanism, following the seminal work of Christiano, Eichenbaum and Evans (2005) and Smets and Wouters (2007). When it comes to modeling the monetary authority, though, most studies simply posit an interest rate feedback rule broadly inspired by Taylor (1993), usually with no discussion of its details and of potential alternatives. As a result, we have witnessed a proliferation of estimated policy rules, especially with respect to the specification of the real variables the central bank reacts to, but with very little guidance on their positive or normative merit.[4]

This paper attempts to impose some order on this wilderness by comparing the fit of a large set of interest rate rules within an estimated, small scale DSGE model of the U.S. economy. Most of the rules we consider have previously appeared in the literature. Others, including the best fitting ones, have not.

Our analysis proceeds in three steps. First, we show how to integrate statistical measures of the output gap (in particular those obtained through filtering) into a general equilibrium model. The idea, which we adapt from Christiano and Fitzgerald (2003), is to use the DSGE model as a forecasting (and backcasting) device to construct a two-sided version of the filter, in which the model's forecasts substitute the realized forward values of the variable of interest (here GDP). This filter produces a real-time measure of a "statistical" output gap, which at the same time is one of the endogenous variables in the DSGE model.

In the second step of the analysis, we assemble a catalog of interest rate rules of the general form

$$i_t = \rho i_{t-1} + (1 - \rho) \left[ \phi_{ct} + \phi_\pi \left( \pi_t - \pi_t^* \right) + \phi_x x_t \right] \tag{1}$$

where $i_t$ is the Federal Funds Rate, $\phi_{ct}$ is a potentially time-varying intercept, $\pi_t - \pi_t^*$ is the deviation of inflation from a target value, which can also be time-varying, and $x_t$ is the output gap. As a baseline, we adopt a simple specification with constant intercept and inflation target, and with the efficient output gap as the measure of economic slack.

We then consider a few alternative classes of policies, each with several variants, for a total of 55 estimated rules. For example, one alternative class of policies replaces the efficient output gap with a statistical one, with variants corresponding to different types of filters.

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[4]For a recent normative analysis of alternative simple interest rate rules within a calibrated DSGE model see Schmitt-Grohe and Uribe (2007). Svensson (2003) recommends modeling central banks as optimizing agents that maximize an objective function, as it is customary for the private sector, rather than as automatons committed to an interest rate feedback rule. The optimal targeting rule obtained in this framework, however, still depends on the arguments of the loss function policymakers are assumed to minimize. See Adolfson, Laséen, Lindé and Svensson (2008) for a state-of-the-art implementation of this approach within a DSGE model for Sweden.

This set of rules is designed to be close to those estimated in empirical analyses of monetary policy behavior based on partial information estimation methods, which tend to measure slack as the deviation of GDP from a smooth trend (e.g. Clarida, Galí and Gertler, 2000; Judd and Rudebusch, 1998; English, Nelson and Sack, 2003 and the survey by Orphanides, 2003). Another class of policies we examine allows the intercept $\phi_{ct}$ to move over time. In particular, we study specifications in which the monetary authority tracks the evolution of an "equilibrium" real interest rate, the real rate that would maintain the economy at potential. These policy rules echo Wicksell's suggestion that a "natural" rate of return determined by real factors represents a useful target for monetary policy (Woodford, 2003), an idea familiar to Fed policymakers at least since the early 1990s (e.g. Greenspan, 1993). However, to our knowledge, this paper is the first to estimate interest rate rules consistent with this idea.[5]

Finally, in the third step of the analysis, we embed each of the candidate interest rate rules within a DSGE framework with given tastes and technology. We estimate the resulting set of models with Bayesian methods, and compare their fit using marginal data densities.[6] The objective of this exercise is not necessarily to pick *the* best fitting rule, and discard all others, but rather to identify a class of policies that offer the best promise to account for the behavior of the data and, perhaps more importantly, weed out those whose fit is clearly inferior.

We can summarize the main results as follows. First, and to our surprise, the baseline rule ranks $47^{th}$ in terms of fit, out of the 55 rules we have estimated. Moreover, the evidence against this specification is very strong, according to our model evaluation criterion (Kass and Raftery, 1995). Second, the fit of the model improves significantly when we resort to a statistical filter to measure slack in the policy rule. In this context, the quarterly HP filter performs particularly well.

Third, the fit improves further when we let the intercept of the policy rule track the efficient rate of interest, the one that would prevail in the economy with no distortions. In fact, this measure of the equilibrium interest rate is a better proxy for the real economic developments to which monetary policy seems to respond, than any of the several measures of the output gap we have experimented with, though both sources of information are useful in helping fit the data. This is the main result of the paper, which sets it apart from the large

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[5] Trehan and Wu (2007) discuss the biases in the reduced-form estimation of policy rules with a constant intercept, when in fact the central bank responds to a time-varying equilibrium real rate. However, they do not estimate this response.

[6] An and Schorfheide (2007) provide a comprehensive survey of the application of Bayesian methods to the estimation and comparison of DSGE models. Lubik and Schorfheide (2007) use these methods to estimate the response of monetary policy to exchange rate movements in several small open economies.

literature on the estimation of Taylor rules with partial information techniques. It takes a complete general equilibrium model, in fact, to compute equilibrium measures of the interest rate of the kind analyzed here.

Fourth, policy rules with a slowly evolving inflation target perform best, since this target captures some of the low frequency variation in inflation and the nominal interest rate that is evident even in our relatively short sample (1987Q3 to 2009Q3). However, this improvement in fit comes at the cost of introducing one more exogenous process into the model, even if one with a clear economic interpretation. Therefore, we take the empirical success of this specification as an indication that more research is needed to understand the low frequency movements in nominal variables, rather than a reflection of the actual behavior of the Federal Reserve.

The rest of the paper proceeds as follows. The next section presents our model of private sector behavior, together with the baseline interest rate rule. Section 3 discusses the econometric methodology and the estimation results for the baseline model. Section 4 introduces the alternative classes of policy rules we consider and compares their empirical performance. Section 5 concludes.

## 2    A Simple Model of the Monetary Transmission Mechanism

We augment the purely forward-looking textbook New Keynesian framework (Woodford, 2003) with two sources of inertia, to improve its ability to fit the data. On the demand side, we include habits in consumption in the utility specification. On the supply side, we allow for partial indexation to past inflation of the subset of prices that are not reoptimized in each period.

The resulting model is smaller than the workhorse empirical DSGE model of Smets and Wouters (2007). It abstracts from capital accumulation and the attending frictions—such as endogenous utilization and investment adjustment costs—and from non-competitive features in the labor market—such as monopolistic competition and sticky wages. This modeling choice allows us to estimate and compare the fit of as many interest rate rules as we like—55 in the current version, and a multiple of this number if we consider various revisions—without having to worry about computational constraints. This is an important consideration for our exercise, given the very large number of policy specifications found in the literature, many of which we have not (yet) considered.

The remainder of this section presents the linearized equilibrium conditions of the model, which constitute the basis for estimation. Appendix *A* contains details of the model's mi-

crofoundations, including the mapping of the tastes and technology parameters into those of the approximate log-linear equations.

## 2.1 Private Sector

An Euler equation summarizes the demand side of the model

$$\lambda_t = E_t\lambda_{t+1} + (i_t - E_t\pi_{t+1}) - E_t\gamma_{t+1} - E_t\delta_{t+1},$$

where $\lambda_t$ is the marginal utility of real income, $i_t$ is the (continuously compounded) nominal interest rate and $\pi_t$ is inflation, while $\gamma_t$ and $\delta_t$ are the (exogenous) growth rate of total factor productivity and a shock to consumers' impatience, both distributed as stationary AR(1) processes. All variables are expressed as log deviations from their balanced growth paths. The intertemporal elasticity of substitution is restricted to unity, because we assume logarithmic utility.

Manipulating this Euler equation, we can obtain the gap representation

$$\tilde{x}_t = E_t\tilde{x}_{t+1} - \varphi_\gamma^{-1}\left(i_t - E_t\pi_{t+1} - r_t^e\right). \tag{2}$$

Here, $r_t^e$ is the efficient real interest rate and the measure of real activity $\tilde{x}_t$,

$$\tilde{x}_t \equiv \left(x_t^e - \eta_\gamma x_{t-1}^e\right) - \beta\eta_\gamma E_t\left(x_{t+1}^e - \eta_\gamma x_t^e\right),$$

is a distributed lag of the efficient output gap $x_t^e \equiv y_t - y_t^e$, where $y_t$ is output and $y_t^e$ is its efficient counterpart. The lead-lag structure in the definition of $\tilde{x}_t$ reflects the presence of internal habits in consumption, to a degree indexed by the parameter $\eta_\gamma$.

The efficient output $y_t^e$ is an important construct in what follows. It represents the level of aggregate output that would prevail in equilibrium if prices were, and always had been, flexible and there were no markup shocks. Efficient output evolves according to the difference equation

$$\omega y_t^e + \varphi_\gamma\left(y_t^e - \eta_\gamma y_{t-1}^e\right) + \beta\varphi_\gamma\eta_\gamma\left(E_t y_{t+1}^e - \eta_\gamma y_t^e\right) = \varphi_\gamma\eta_\gamma\left(\beta E_t\gamma_{t+1} - \gamma_t\right) + \frac{\beta\eta_\gamma}{1 - \beta\eta_\gamma}E_t\delta_{t+1}, \tag{3}$$

from which we observe that $y_t^e$ is a linear combination of the past and future expected values of the productivity and intertemporal taste shocks alone. This observation implies that the counterfactual environment in which prices are flexible is a parallel universe, which evolves independently from the outcomes observed in the actual economy. In this parallel universe,

the intertemporal Euler equation implies

$$r_t^e = E_t \gamma_{t+1} + E_t \delta_{t+1} - \omega \left( E_t y_{t+1}^e - y_t^e \right),$$

where we used the production function and the intratemporal efficiency condition (i.e. marginal rate of substitution equal to marginal product of labor) to map the efficient marginal utility of consumption $\lambda_t^e$ into output $y_t^e$.

Turning now to the supply side of the model, the optimal pricing decisions of firms produce a Phillips curve of the form

$$\tilde{\pi}_t = \xi \left( \omega x_t^e + \varphi \tilde{x}_t \right) + \beta E_t \tilde{\pi}_{t+1} + u_t, \tag{4}$$

where

$$\tilde{\pi}_t \equiv \pi_t - \zeta \pi_{t-1}$$

depends on the degree of indexation to past inflation, parametrized by $\zeta$, and $u_t$ is an AR(1) cost-push shock, generated by exogenous fluctuations in desired markups. These fluctuations are the only source of a tradeoff between inflation and real activity in this model.

Without markup shocks, the efficient level of aggregate production can be achieved together with price stability (i.e. $\pi_t = 0$), as we can see by substituting $u_t = 0$ and $y_t = y_t^e$, or $x_t^e = 0$, $\forall t$ in equation (4). This is the first best outcome in this economy, since no price needs to change when aggregate inflation is zero, thus eliminating price dispersion across monopolistic producers and the distortions in the allocation of resources associated with it (Woodford, 2003). When markup shocks are present, on the contrary, the efficient allocation is no longer feasible, because the efficient level of *aggregate* output could only be achieved by allowing cost-push shocks to pass-through to inflation entirely, as we can see by solving equation (4) forward with $y_t = y_t^e$ $\forall t$

$$\pi_t = \zeta \pi_{t-1} + \sum_{s=0}^{\infty} \beta^s E_t u_{t+s}.$$

The resulting fluctuations in inflation would then produce an inefficient dispersion of prices and production levels across varieties. At the other extreme of the policy spectrum, perfect inflation stabilization would require cost-push shocks to show-through entirely in deviations of output from its efficient level. Optimal policy, therefore, will distribute the impact of these shocks between output and inflation, as to balance the objectives of price stability and efficient aggregate production.

One implication of this trade-off is that an ex-ante real interest rate, $i_t - E_t \pi_{t+1}$, set to

perfectly shadow the efficient rate of return $r_t^e$, would not be optimal, although the Euler equation (2) implies that such a policy would close the output gap every period and thus achieve the efficient level of aggregate production. This is the main reason for including some feedback from inflation and the output gap even in the interest rate rules that include $r_t^e$ in their intercept, as we do below.[7]

## 2.2   Monetary Policy: Baseline Specification

In the baseline policy specification, the central bank sets the nominal interest rate in response to the current inflation rate and efficient output gap, with a certain degree of inertia

$$i_t = \rho i_{t-1} + (1 - \rho) \left( \phi_\pi \pi_t + \phi_x x_t^e \right) + \varepsilon_t^i. \tag{5}$$

Expression (5) represents a natural starting point for our comparative analysis, since it brings the basic ingredients of the empirical literature on interest rate rules into the context of our DSGE framework. Inflation and real activity are standard arguments of monetary policy rules at least since Taylor (1993), while interest rate inertia typically improves their ability to fit the data, as shown for example by Clarida, Galí and Gertler (2000). We choose the efficient output gap as the baseline policy measure of real economic developments for internal consistency with the rest of our theoretical apparatus. In our model, in fact, this gap is both the fundamental driver of inflation, as shown in equation (4), as well as the measure of slack that is relevant for welfare analysis (e.g. Woodford, 2003).

## 3   Inference

We estimate the model laid out in the previous section, and all the variants discussed below, with Bayesian methods, as surveyed for example by An and Schorfheide (2007). Bayesian estimation combines prior information on the parameters with the likelihood function of the model, to form a posterior density function. We construct the likelihood using the Kalman filter based on the state space representation of the rational expectations solution of each model under consideration, assuming a likelihood of zero for the parameter values that imply

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[7]Another reason is that a policy rule of the form $i_t = r_t^e + E_t \pi_{t+1}$ would not deliver the efficient output uniquely, since it does not satisfy the Taylor principle (e.g. Clarida, Galí and Gertler, 1999).

indeterminacy. The observation equations are

$$\begin{aligned}
\Delta \log GDP_t &= \gamma + y_t - y_{t-1} + \gamma_t \\
\Delta \log PCE_t &= \pi^* + \pi_t \\
FFR_t &= r + \pi^* + i_t,
\end{aligned}$$

where $\Delta$ is the first difference operator, $GDP_t$ is real GDP, $PCE_t$ is the core PCE deflator ex food and energy, and $FFR_t$ is the average effective Federal Funds rate, all sampled at a quarterly frequency. The constants in these equations represent the average growth rate of productivity, $\gamma$, the long run inflation target, $\pi^*$, and the average real interest rate, $r$. The sample period runs from 1987:Q3 to 2009:Q3, although the main results are not affected by truncating the sample at 2008:Q4, when the Federal Funds rate first hit the zero bound.

The left panel of Table 1 reports our choice of priors, which are maintained across all the model specifications we consider. On the demand side, we calibrate the discount factor as $\beta = 0.99$, and impose a loose prior between zero and one on the habit coefficient $\eta$, only slightly favoring higher values. These two parameters, together with the average balanced growth rate $\gamma$, determine the slope of the Euler equation (2), $\varphi_\gamma^{-1} \equiv \left(1 - \eta_\gamma\right)\left(1 - \beta\eta_\gamma\right)$, where $\eta_\gamma \equiv \eta e^{-\gamma}$.

On the supply side, the prior on the indexation parameter $\zeta$ is centered around 0.6, but is quite dispersed over the unit interval. The slope of the Phillips curve is a convolution of deep parameters, $\xi \equiv \frac{(1-\alpha)(1-\alpha\beta)}{\alpha(1+\omega\theta)}$, where $\alpha$ is the fraction of firms that do not change their price in any given period, $\theta$ is the elasticity of demand faced by each monopolistic producer and $\omega$ is the inverse Frisch elasticity of labor supply. Only the slope $\xi$ can be identified from our observables, so this is the parameter on which we formulate our prior, which is a Gamma distribution with mean 0.1. This is a somewhat higher value than those typically recovered in partial information estimates of the New Keynesian Phillips curve (e.g. Galí and Gertler, 1999, Sbordone, 2002), but it is consistent with the low degree of price stickiness found in microeconomic studies such as Bils and Klenow (2004), given reasonable values for $\omega$ and $\theta$.[8]

Turning now to the interest rate rule, the prior on the smoothing parameter $\rho$ is a Beta centered at 0.7, with a 90% probability interval wide enough to encompass most existing estimates. The priors for the feedback coefficients on inflation and real activity are normally distributed with means 1.5 and 0.5 respectively, as in the original Taylor (1993) rule.

The autocorrelations of the exogenous shocks, the $\rho$'s in the table, have Beta priors with

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[8]For example, with $\omega = 1$ and $\theta = 8$, which corresponds to a desired markup of 14%, $\xi = 0.1$ implies $\alpha = 0.4$, or an expected duration of prices of about five months.

mean 0.5, while the standard deviations, denoted by $\sigma$, have Inverse Gamma priors also centered at 0.5.

We obtain the posterior mode and inverse Hessian by minimizing the negative of the log posterior density function and use Markov Chain Monte Carlo methods, more specifically a Random Walk Metropolis algorithm, to build a representative sample of the parameters' joint posterior distribution. We monitor the convergence of the chains of draws in each step using a variety of tests. Finally, upon convergence, we combine the chains in the last step, after discarding the initial 25% of the draws in each chain, to form a full sample of the posterior distribution, which represents the source of our inference information.[9]

To evaluate the fit of different policy rules, we compare the marginal data densities, or posterior probabilities, of the DSGE models in which they are embedded, using Geweke's (1999) modified harmonic mean estimator. In particular, we compute twice the log of the Bayes factor of each alternative model against the baseline. This is the measure of relative fit recommended by Kass and Raftery (1995), since it is on the same scale as a Likelihood Ratio Statistic.[10] This procedure results in an overall ranking of the interest rate rules under consideration, as well as in a measure of their individual fit against a common benchmark, and thus implicitly against each other.

## 3.1 Estimation Results in the Baseline Model

The right panel of Table 1 reports selected moments of the marginal posterior distributions of the parameters under the baseline interest rate rule. Although the data are quite informative on most parameters, and many of the posterior estimates fall within reasonable ranges, close inspection of the results also reveals some anomalies with this specification. To better visualize these anomalies, Figure 1 graphs the prior and posterior marginal distributions for the group of problematic parameters.

First, note that the posterior estimate of the slope of the Phillips curve, $\xi$, is minuscule, with a mean of 0.002, two orders of magnitude smaller than the prior mean and at the extreme lower edge of the available estimates in the DSGE literature (see for example the survey by Schorfheide, 2008). This posterior estimate implies that there is no discernible trade-off between inflation and real activity, and that inflation is close to an exogenous process driven

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[9]Detailed convergence and inference analysis for each specification discussed in the paper is available upon request.

[10]The Bayes factor of model 1 against model 2 is the ratio of their marginal likelihoods. Kass and Raftery (1995) suggest that values of $2 \log BF$ above 10 can be considered very strong evidence in favor of model 1. Values between 6 and 10 represent strong evidence, between 2 and 6 postive evidence, while values below 2 are "not worth more than a bare mention." We refer to this statistic as the KR criterion.

by movements in desired markups. As a consequence, there is little hope of distinguishing between dynamic inflation indexation and persistent markup shocks, as drivers of the observed inflation persistence. This lack of identification is reflected in the bimodal marginal posterior distributions of the parameters $\zeta$ and $\rho_u$, which are generated by MCMC draws with high $\zeta$ and low $\rho_u$, or vice versa, and that correspond to local peaks of the joint posterior density of similar heights. Finally, the last two panels of Figure 1 show that the estimated parameters of the interest rate rule imply a strong reaction of policy to the output gap, and an extremely weak reaction to inflation, with about half of the posterior draws for $\phi_\pi$ below one. These values are puzzling, in light of the large literature that has argued that a forceful reaction to inflation has been one of the hallmarks of U.S. monetary policy since the mid-eighties.

The anomalous features of the posterior distribution highlighted above, to the extent that they conflict with the prior information, reduce the baseline model's marginal data density and contribute to its extremely poor overall fit. For now, we are not in a position to quantify the extent of this empirical failure, since we have not introduced any alternative model yet, but we can say that the baseline specification ranks 47th in terms marginal likelihood, among the 55 evaluated in this version of the paper.

## 4   Evaluating Alternative Interest Rate Rules

Many aspects of our baseline model could be problematic. In the rest of the paper, we focus on one potential source of these problems, which in our judgement has been largely, and surprisingly, overlooked in the DSGE literature: the specification of the interest rate rule. As we will see, relatively minor adjustments to the policy rule compared to the baseline specification can improve the fit of our simple DSGE model dramatically, at the same time contributing to solve some of the anomalous estimates and identification problems highlighted in Figure 1.[11]

### 4.1   Statistical Output Gaps

The measure of economic slack that we chose to include in the baseline interest rate rule is the deviation of GDP from its efficient level. This choice is fairly common in DSGE work (e.g. Smets and Wouters, 2007), although far from universal. However, it has the drawback of making the resulting policy rule impossible to compare with those estimated in the vast

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[11] We do not address directly here the extent to which different policy rules aid or hinder the identification of the model's parameters, although this is an issue that would deserve further scrutiny. For a recent study of identification in DSGEs, see Canova and Sala (2009), who find that identification is often problematic in this class of models.

literature that employs partial information econometric techniques, since the construction of the counterfactual efficient output requires a general equilibrium model. Moreover, the efficient output gap might be considered an implausible choice as a summary statistic for policymakers' views on the level of resource utilization, precisely because it depends on the details of the model used to construct it.

To bridge the gap between our general equilibrium framework and the work based on single equation methods, and perhaps the real world, we begin our catalog of alternative policy rules with specifications in which the output gap is measured through statistical filters. In particular, we focus on the Hodrick and Prescott (HP) filter as a tool to construct smooth versions of potential output, given its popularity in applied macroeconomics.[12]

One difficulty in making the HP filter operational within a DSGE model is that its ideal representation is a two-sided, infinite moving average, whose standard approximation to finite samples requires different coefficients on the observations at the beginning, in the middle, and at the end of the sample. Such a pattern of coefficients is difficult to replicate within a dynamic system of rational expectation equations with a parsimonious state space. To circumvent this problem, we adapt the methodology proposed by Christiano and Fitzgerald (2003) for the approximation of ideal band pass filters. Christiano and Fitzgerald (2003) suggest to use forecasts (and backcasts) from an auxiliary time-series model—in their case a simple unit root process—to extend the sample in the past and in the future. In our implementation of their idea, the auxiliary model that generates the dummy observations is the linearized DSGE itself.

This approach is particularly convenient for our purposes, because it produces a very parsimonious recursive expression for the DSGE-HP gap

$$\left[1 + \lambda(1 - L)^2 (1 - F)^2\right] x_t^{HP} = \lambda(1 - L)^2 (1 - F)^2 y_t, \tag{6}$$

where the operators $L$ and $F$ are defined by $Ly_t = y_{t-1}$ and $Fy_t = E_t y_{t+1}$, and the smoothing parameter $\lambda$ is set at the typical quarterly value of 1600. This expression can thus be added to the system of rational expectations equations that defines the equilibrium of the model. Of course, the time series for the output gap obtained through this procedure will not be the same as the one produced by the finite sample approximation usually employed in applied work. However, it has a very similar flavor, as we will see shortly. More details on the derivation of equation 6 and on its interpretation, together with some background on linear filtering, can be found in the Appendix.

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[12] See Orphanides and van Norden (2002) for a comprehensive survey of the use of statistical filters as measures of the output gap, and of their pitfalls.

When we estimate the model by replacing the efficient output gap with $x_t^{HP}$ in the interest rate rule, the marginal likelihood increases by about 10 log-points, or 21.6 points on the KR criterion. This improvement represents very strong evidence in favor of the latter specification. For some insight into this result, Figure 2 reports prior and posterior marginal distributions for the same parameters we highlighted as anomalous in the baseline model. Although the slope of the Phillips curve remains extremely low, its mean is now twice as large as before (0.004 vs 0.002). Moreover, the posterior points to a small indexation coefficient ($\zeta$), and to relatively little persistence in the cost-push shock ($\rho_u$), with no immediate evidence of identification problems. Finally, the feedback coefficients on inflation and the output gap in the Taylor rule are closer to more typical values, although $\phi_\pi$ remains on the low side.

Another interesting posterior object in this model is the distribution of the time-series for the DSGE-HP output gap, which is depicted in Figure 3, together with the standard finite sample approximation of the HP filter, denoted by Data-HP, and the output gap computed using the measure of potential output produced by the Congressional Budget Office (CBO). The two HP approximations co-move fairly closely, although far from perfectly. In particular, the dips in the DSGE-based approximation around the NBER recessions, which are shaded in grey, are more pronounced than in the standard HP. In fact, the DSGE-HP conveys a view of the timing and extent of expansions and recessions over our sample period very similar to that of the CBO output gap (at least in two of the three recessions experienced over the sample period). Overall, this evidence supports our use of the DSGE-HP filter as an effective de-trending tool, which produces a measure of capacity utilization similar to those often used in single-equation estimates of the Taylor rule.

Given the promising empirical performance of the quarterly HP output gap as an argument of the model's interest rate rule, we explored several alternative filter formulations. In particular, we consider HP filters in which the smoothing parameter $\lambda$ is either estimated, with a very diffused prior centered at somewhat higher values than 1600, or calibrated to a "high" value of $\lambda = 160000$. The motivation for both these specifications is to test the data's appetite for a smoother trend than in the baseline HP, closer to those obtained through the production function approach, for example by the CBO (2001). In addition, we evaluate models with simpler, one-sided filters, such as the exponential filter[13]

$$\left[ 1 + \tilde{\lambda}(1 - L) \right] x_t^{Exp} = \tilde{\lambda} \left( 1 - L \right) y_t,$$

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[13]In one version of the exponential filter we set the smoothing parameter to $\tilde{\lambda} = 61.5$, to match the gain of the HP filter at frequency $\omega = 2\pi/32$, which corresponds to an eight year cycle (King and Rebelo, 1993). We also consider a version where $\tilde{\lambda}$ is estimated, with a prior centered at the same value as above.

the four quarter moving average of GDP growth, $(y_t - y_{t-4})/4$ and its simple quarterly growth rate, $\Delta y_t$.

The impact on the model's fit of using these alternative de-trending methods to measure the output gap in the interest rate rule are summarized in Panel I of Table 2. This table reports the log marginal likelihood of all the model specifications we have estimated, together with twice the log Bayes factor for each model against the baseline, the KR statistic. Moreover, in the first column of the table, we report the ranking of each model, from the best to worst fitting. Column two reports a shortcut with which we sometimes refer to the rules in the text, while column three describes each rule in mathematical notation, focusing on its long-run arguments (ignoring interest rate smoothing). For example, the baseline model, whose long-run arguments are $\phi_\pi \pi_t + \phi_x x_t^e$, has a $\log ML$ of $-379$, which makes it number 47 in terms of fit out of the 55 rules we estimated in this paper.

From the table, we see that the DSGE-HP filter with $\lambda = 1600$ produces the best fit, among the models with a statistical output gap. The evidence in favor of this specification (model HP in the table) against the baseline is very strong, as we already pointed out. The model in which the HP smoothing parameter is estimated ($\mathrm{HP}\hat{\lambda}$) does only slightly worse. This is because the posterior distribution of this parameter has a median of about 1100 (and the posterior distribution concentrates around this level), which produces a gap almost identical to $\lambda = 1600$. The performance of all the other filters, on the contrary, is clearly inferior, although most of them fit better than the baseline specification.

Finally, to round up our exploration of the role of the output gap in the policy rule, Table 2 reports results for two more specifications. The first one is the one adopted by Smets and Wouters (2007) (model SW), which also includes a term in the growth rate of the efficient gap. The second one is a "control", in which the output gap is excluded altogether, and the federal funds rate only responds to inflation (NoGap). Smets and Wouter's (2007) rule performs significantly better than the baseline, which probably explains why they included the somewhat unusual rate of change term in the first place. In fact, its fit is very close to that of the HP rule, although it is still in the lower half of the overall ranking.

On the contrary, the restriction $\phi_x = 0$ is strongly rejected by the data, leading to a significant deterioration in fit even with respect to the baseline. This result confirms that the identification of a good indicator of real economic developments is a crucial factor in the search for a parsimonious, but reasonably accurate, description of the behavior of the policy rate. Our results so far suggest that common measures of de-trended output, such as those obtained through the HP filter, are more likely to represent such an indicator than the flexible-price gaps consistent with the structure of the DSGE model. In the next section,

we move the search for this indicator further, by exploring the properties of an alternative flexible-price construct implied by our general equilibrium model: the efficient real interest rate.

## 4.2   Tracking the Efficient Real Interest Rate

The idea that an "equilibrium" interest rate (EIR) might represent a useful reference point for monetary policy was familiar to Federal Reserve policymakers well before Woodford (2003) revitalized its Wicksellian roots. For example, in his Humphrey Hawkins testimony to Congress in May 1993, Chairman Alan Greenspan stated that: "In assessing real rates, the central issue is their relationship to an equilibrium interest rate, specifically, the real rate level that, if maintained, would keep the economy at its production potential over time. Rates persisting above that level, history tell us, tend to be associated with slack, disinflation, and economic stagnation—below that level with eventual resource bottlenecks and rising inflation, which ultimately engenders economic contraction. Maintaining the real rate around its equilibrium level should have a stabilizing effect on the economy, directing production toward its long-term potential" (Greenspan, 1993).[14]

In this section, we investigate the extent to which Chairman Greenspan's reasoning had a measurable impact on the evolution of the observed nominal interest rate over our sample. To measure the EIR within our DSGE model, we follow the Chairman's description and compute the counterfactual "real rate level that, if maintained, would keep the economy at its production potential over time." When "potential" output is defined as the efficient aggregate level of production, $y_t^e$, the EIR is the efficient rate of return $r_t^e$. This is our preferred measure of the EIR, since it is grounded in the microeconomic structure of the DSGE model. However, we also consider the equilibrium real rates that correspond to the potential outputs implied by the HP and exponential filters.[15] We then embed these measures of the EIR, which we generically denote by $r_t^*$, in a class of policy rules of the form

$$i_t = \rho i_{t-1} + (1-\rho)\left[r_t^* + \phi_\pi \pi_t + \phi_x x_t\right] + \varepsilon_t^i, \tag{7}$$

where we consider several permutations in the definitions of both $r_t^*$ and $x_t$.

The first rule in this class that we consider uses the DSGE's efficient equilibrium as its

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[14] Quantitative measures of the EIR are a regular input in the monetary policy debate at the Federal Reserve, as demonstrated by the fact that a chart with a range of estimates of the EIR is included in most published Bluebooks at least since May 2001 (see http://www.federalreserve.gov/monetarypolicy/fomc_historical.htm).

[15] We report here only results for the filters with the quarterly smoothing parameters ($\lambda = 1600$ for the HP and $\tilde{\lambda} = 61.5$ for the exponential), although we also experimented with the other approaches to the choice of these values described in the previous section.

notion of potential, so that $r_t^* \equiv r_t^e$ and $x_t \equiv x_t^e$. This choice of arguments for the policy rule improves the model's marginal likelihood by approximately 20 log-points with respect to the baseline specification and by 10 log-points with respect to the best fitting rule among those discussed in the previous section. These differences represent very strong evidence in favor of policy rules that allow a gradual adjustment of the nominal interest rate to movements in the efficient real rate. To our knowledge, this paper is the first to document this evidence, although policymakers have been discussing the equilibrium real rate as a potentially useful indicator for monetary policy for a long time, as witnessed by Chairman Greenspan's remarks above (see also Amato, 2005). The empirical success of interest rate rules that track the efficient rate of interest is also interesting, because these rules have desirable stabilization properties, as shown for instance by Galí and Gertler (2007). However, we are not aware of a systematic study of their normative performance.

Panel II of Table 2 shows that the Re specification, the one that is probably most appealing on theoretical grounds, is also preferred by the data over the others in the same class, although in some cases only slightly. For example, the differences in fit with some of the specifications in which the output gap is measured through statistical filters, rather than in deviation from the DSGE's efficient output, are minor.

On the other hand, the deterioration in fit is more significant when we restrict the feedback coefficient on the output gap, $\phi_x$, to zero, as in model ReNoGap. This result suggests that $r_t^e$ is not a sufficient statistic for the real developments in the economy that drive the movements in the federal funds rate. However, there is strong evidence in favor of $r_t^e$ as a more useful real indicator for monetary policy than the DSGE-HP output gap, as we can see by comparing model ReNoGap to model HP. Finally, alternative approaches to the measurement of the EIR, in which potential output is measured through a statistical filter, and the equilibrium real rate is one consistent with that notion of potential, do not fare nearly as well (models RexpExp and RhpHP).

The reason for the success of specifications that include $r_t^e$ among the arguments of the interest rate rule can be further appreciated from Figure 4, where we plot the posterior distribution of $r_t^e$ implied by the model. As we can see, the estimated $r_t^e$ is a good business cycle indicator over our sample. It drops sharply during recessions and rises over booms. However, $r_t^e$ conveys somewhat different information than the HP output gap, which is also reported in Figure 4. For example, $r_t^e$ peaks earlier than the HP output gap before the recessions of 1990 and of 2007, although the peaks coincide in the 2001 recession. Moreover, the efficient real rate is fairly stable above its mean in the mid-nineties, while the HP output gap turns negative in 1995. These inferred movements in $r_t^e$ mirror those in the effective

federal funds rate quite closely, helping to explain the empirical success of the Re policy specification.

The close comovement between the effective federal funds rate and the estimates of $r_t^e$, which is depicted in Figure 5, raises the concern that the observations on the nominal interest rate might be "explaining" the estimates of $r_t^e$, and not vice versa. This is not the case, however, as demonstrated by the fact that we obtain almost identical estimates of the time path of $r_t^e$ in the baseline model, in which the efficient real rate is not included in the policy rule. The main difference between the two estimates is that the posterior distribution is tighter when $r_t^e$ enters the interest rate rule, as shown in Figure 6. This enhanced precision of the estimates suggests that, indeed, the nominal interest rate carries useful information on $r_t^e$ in specification Re, as we would expect, but that this information does not distort the inference on its average time-path.

Some intuition for the robustness of the estimates of $r_t^e$ across models can be gleaned from the expression for the efficient rate of interest derived in section 2, which we report here for convenience

$$r_t^e = E_t \gamma_{t+1} + E_t \delta_{t+1} - \omega \left( E_t y_{t+1}^e - y_t^e \right).$$

If the log-deviations of efficient output from the balanced growth path were a martingale (i.e. $E_t y_{t+1}^e = y_t^e$), this expression would imply that the efficient real interest rate is the sum of the forecastable movements in the growth rate of productivity $\gamma_t$ and in the intertemporal taste shock $\delta_t$. In our estimated models, the deviations from the condition $E_t y_{t+1}^e = y_t^e$ are "small", as are the forecastable movements in $\gamma_t$. The taste shock $\delta_t$, on the contrary, is persistent, and its innovations are sizable, so that its forecastable movements tend to be the main driving force of the movements in $r_t^e$.[16] Moreover, the cyclical behavior of these forecastable movements in $\delta_t$ is precisely and robustly pinned down in our estimates, with little variation across specifications. As a result, the inference on the evolution of the efficient real rate over time is remarkably consistent across all the models we consider.

## 4.3 A Time-Varying Inflation Target

In this section, we further enlarge the set of policy rules subject to our evaluation, by introducing a feature that is fairly common in the recent empirical DSGE literature: a time-varying

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[16]The important role of the intertemporal shock $\delta_t$ in reconciling this class of DSGE models with the data is a manifestation of the well-known deficiencies of standard Euler equations in pricing returns, as first documented by Hansen and Singleton (1982) and more recently re-emphasized in a DSGE context by Primiceri, Schaumburg and Tambalotti (2006).

inflation target (TVIT). This addition creates a new class of feedback rules, of the form

$$i_t = \rho i_{t-1} + (1 - \rho) \left[ r_t^* + \pi_t^* + \phi_\pi \left( \pi_t - \pi_t^* \right) + \phi_x x_t \right] + \varepsilon_t^i, \tag{8}$$

where $\pi_t^*$ is an exogenous AR(1) process that represents persistent deviations of the inflation target from its long-run value $\pi^*$.[17] The motivation for considering this feature in the policy rule is that it helps to capture the low-frequency movements in inflation and the nominal interest rate that are evident even in our relatively short sample. In particular, inflation hovered around 4% in the late 1980s, until the recession of the early 1990s contributed to reduce it to its more recent range around 2%. This process of so-called opportunistic disinflation took until the middle of the decade to complete. One simple way of capturing the central bank's willingness to delay the achievement of its ultimate inflation objective until the "next" recession, which is at the heart of the opportunistic approach to disinflation, is to allow smooth time-variation in its short-run inflation target, as in specification (8).

When we allow for this type of time-variation in the best rule so far, rule Re, the fit improves by another 15 points on Kass and Raftery's (1995) likelihood ratio scale. This improvement constitutes very strong evidence in favor of the inclusion of a time-varying inflation objective in the policy rule. With respect to the baseline, the marginal likelihood of specification RePistar is 27 log-points higher. Moreover, specification RePistar, in which the EIR is measured by the efficient real rate and the output gap by the deviation of output from its efficient level, is the best-fitting one among those with a time-varying inflation objective, as shown in Panels III and IV of Table 2.

Two more results from the table are worth emphasizing. First, the role of $r_t^e$ remains crucial even in the specifications that include a TVIT. In fact, rule RePistar improves the model fit by about 17 KR points with respect to the best-fitting rule with a TVIT, but without the equilibrium real rate among its arguments (rule PistarExp in Panel IV). The improvement in fit is even larger (30 KR points) when rule RePistar is compared to a simple baseline specification with a TVIT (rule Pistar), which is a more relevant comparison if we want to isolate the marginal contribution of the EIR in the policy rule. Interestingly, this improvement in performance is comparable to the one obtained when the EIR is included in the equivalent specifications with a constant inflation target, i.e. when comparing the baseline rule to rule Re. This result suggests that the efficient real rate and a smoothly evolving inflation target enhance the empirical performance of the model through fairly independent channels and should thus be complementary features in policy specifications with good empirical properties.

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[17] The autocorrelation coefficient of $\pi_t^*$ has a Beta prior tightly distributed around a mean of 0.95.

Second, our ability to draw sharp conclusions on the most appropriate measure of the output gap is complicated by the presence of a TVIT. For example, the deterioration in fit when the output gap is measured through various statistical filters, rather than in deviation from the efficient level of output, or even excluded from the policy rule altogether, as in rule RePistarNoGap, is negligible. This latter result, in particular, might suggest that the efficient interest rate and a TVIT are all that is needed to account for the movements in the federal funds rate, and that measures of output slack are redundant. However, this conclusion is probably unwarranted, since there are fairly clear signs of weak identification of the output gap coefficient $\phi_x$, especially in specification RePistar. This identification problem should not be too surprising, since the interest rate rules with a TVIT include at least three latent variables: the inflation target itself, the i.i.d. monetary policy shock and potential output.[18] Drawing sharp inferences on the contributions of these three factors to the movements in the interest rate, therefore, is bound to be problematic, even though the structure of the model imposes restrictions on the behavior of potential output. In fact, this consideration suggests that similar problems are likely to persist even in richer models—at least as long as the inflation target is treated as an exogenous variable. Given the promising empirical performance of this class of policy rules, these identification issues probably deserve further scrutiny.

## 4.4   Summary of Main Results

So far, we have surveyed the empirical performance of about 40 different interest rate rules, while trying to develop some leads on the sources of their successes and failures. This exercise brought four main themes to our attention. First, the simplest and most natural extension of the original Taylor (1993) rule to our DSGE framework, which we adopted as our baseline policy specification, fits the data extremely poorly, compared to most of the alternative specifications we have considered. Second, this poor performance can be improved significantly if the model-implied efficient output gap is substituted by an HP filter as the measure of economic slack in the policy rule. Simpler, one-sided filters also perform better than the efficient output gap, although worse than the HP filter. Third, further significant improvements in fit can be achieved by allowing the policy rate to respond to movements in the efficient real interest rate implied by the DSGE model. Documenting the empirical success of policy rules with this feature is the main contribution of this paper, given the normative appeal of

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[18]The equilibrium real rate is a fourth latent variable in some specifications, but this does not appear to worsen the identification challenge, since $r_t^e$ is restricted to enter the intercept of the policy rule (i.e. to have a coefficient of one) and its evolution is pinned down fairly precisely by the demand side of the model.

these rules and the frequent discussion of the potential uses of measures of the equilibrium real rate in the policy debate. Fourth, feedback rules in which the inflation target evolves smoothly over-time perform best. However, tracking the efficient real interest rate remains an important feature even in this class of rules, suggesting that both these extensions to the baseline specification should be standard in applied DSGE modelling.

In the next, and last, section, we investigate the extent to which these main themes survive variations in the arguments of the policy rule, which have often appeared in the literature. We plan to explore the themes' robustness to a broader set of modifications to the framework we have adopted here in future work. In particular, we are currently working on comparing a set of exemplary rules in a medium-scale model of the U.S. business cycle, similar to that popularized by Smets and Wouters (2007).

## 4.5   Robustness, and the Best Rule

In this section, we conduct a series of robustness exercises that involve relatively small variations in the policy rule, but that result in specifications commonly found in the literature. We subject to these experiments only the best-fitting rules within each class, to avoid an exponential proliferation of estimated models. In this process, we also discover the best-fitting rule among those we have estimated.

The first variation we consider replaces the contemporaneous values of inflation and the output gap in the interest rate rule with their rational expectations forecasts, as in Clarida, Galí and Gertler (2000), for instance. The resulting policy rule specifications, and their fit, are reported in Panel V of table 2. We emphasize two findings. First, the forward-looking rules maintain the relative ranking of the broader classes of policy specifications emphasized above. For example, rules that include $r_t^e$ and/or $\pi_t^*$ fit better than rules without these factors and the evidence in their favor is still very strong. Second, the forward-looking specification with $r_t^e$ and $\pi_t^*$ is preferred to its contemporaneous counterpart.

However, this result does not survive when the measure of inflation we include in the feedback rule is a four quarter moving average, rather than its quarterly value, as in Panel VI of table 2. Once again, the improvements in fit obtained by including $r_t^e$ and $\pi_t^*$ in the policy rule are very similar to those documented before. The log marginal likelihood of rule RePistarPi4Q, which includes both features, is 6 points higher than that of rule RePi4Q, in which the inflation target is constant, and about 23 points higher than that of rule Pi4Q, which has the same structure as the baseline. In fact, rule RePistarPi4Q is the best-fitting rule among the 55 analyzed in this paper.

Several features of this rule are worth emphasizing. First, the improvement in fit it

achieves over the baseline, 67 points on KR's scale, is remarkable. The evidence in favor of this rule against the equivalent version with quarterly inflation is also very strong, although of course much less decisive. Second, the best rule is a sensible blend of theoretical and practical considerations. For example, most policymakers would agree that a four quarter moving average of inflation is a more reliable guide to inflationary pressures than a quarterly measure. On the other hand, they might object to the proxies for real economic developments included in this rule, the efficient rate of return and output gap. But at least, these measures have the virtue of being linked directly to the objectives that monetary policy should pursue according to the DSGE model, and are thus appealing on theoretical grounds. Finally, the posterior estimates of the model that embeds the best rule are all reasonable and do not point to any obvious identification or other specification problem. This is true for the parameters, as well as for the latent variables that enter the policy rule, whose posterior distributions we report in Figures 7 and 8.

Among the parameters, the slope of the Phillips curve, $\xi$, has a posterior mode of 0.05, and a mean of 0.07, very close to the typical values in the literature. Both the indexation parameter ($\zeta$) and the autocorrelation of the cost-push shock ($\rho_u$) are distributed around low values, although both display a fairly long tail. This is because the observed persistence in inflation is well captured by the slow-moving inflation target, whose estimated autocorrelation ($\rho_{\pi^*}$) has a mode of 0.99. Finally, the coefficients on inflation ($\phi_\pi$) and the output gap ($\phi_x$) in the Taylor rule have modes (and means) of 1.7 and 0.6, respectively, both in line with most empirical estimates for this period, although the data do not appear very informative on the latter coefficient, as we already pointed out.

Turning now to the latent variables in Figure 8, we see that the posterior median of $\pi_t^*$ captures well the step-down in inflation in the first few years of the sample, although the posterior uncertainty on the level of this target is very large. Of course, the estimates continue to fluctuate even in the second half of the sample. In fact, they dip around 2003 and in the more recent period, at the same time as observed inflation was falling. These movements remind us that time-variation in the inflation target is a useful statistical device, but it is not a substitute for a more structural analysis of the low-frequency movements in inflation. The second panel of the figure depicts the posterior distribution of the efficient real interest rate, which is very similar to the one reported in Figure 5. This similarity confirms the robustness of the inference on $r_t^e$ across different models. Finally, the third panel of the figure reports the posterior estimate of the efficient output gap. Although the uncertainty on the level of $x_t^e$ is large, its evolution over time is broadly consistent with the business cycle as identified by the NBER, whose recessions are shaded in the picture.

# 5    Conclusions

The existing positive DSGE literature focuses an overwhelming share of its attention on specifying the behavior of the private sector, while treating that of the central bank as an afterthought. This state of affairs is not too surprising, since reducing the real world complexity of the private sector to fit into a macroeconomic model offers a vast menu of modeling choices. In comparison, capturing the broad contours of the behavior of monetary policy is certainly much easier and less controversial. Yet, paying virtually no attention to this step in the specification of a general equilibrium model seems suboptimal, for at least two reasons. First, in the current vintage of monetary DSGE models, the systematic response of policy to economic developments can have large effects on the equilibrium, as demonstrated by the vast body of normative work in the field (see Woodford, 2010, for a survey). Second, one of the main objectives of these models is to offer a quantitative tool to study the consequences of different approaches to the conduct of monetary policy. This study is complicated by the lack of systematic guidance on the extent to which different plausible policy rules, once embedded into a general equilibrium apparatus, enhance or detract from its ability to account for the historical relations between the macroeconomic variables of interest.

This paper attempted to provide some of that guidance, by estimating a large set of interest rate rules (55 in the current draft) in the context of a simple DSGE model, and comparing their empirical fit. We can summarize what we learned from this exercise as follows. First, the improvements in fit that can be achieved by a careful choice of the arguments of the monetary policy rule, with respect to the specifications more often used in the literature, are very strong. Second, a robust feature of the best fitting rules is that they include a previously unexplored factor among their arguments, namely the efficient real interest rate, the rate of return that would prevail in equilibrium if the economy were perfectly competitive. Third, this feature remains true in the now canonical medium-scale estimated DSGE model of Christiano, Eichenbaum and Evans (2005) and Smets and Wouters (2007).

Of course, our results do not represent a definitive guide to "good" interest rate rules, for at least two related reasons. First, they depend on the exact model specifications we adopted. More work on the results' robustness across different models would therefore be desirable. Second, model comparison through marginal data densities and Bayes factors applied to DSGE models is subject to some pitfalls, highlighted for example by Del Negro and Schorfheide (2010). However, we hope to have at least contributed to narrowing significantly the set of rules researcher will entertain as empirically plausible in the future. Going forward, we expect to devote some of our research to further scrutinize the role of the efficient real interest rate $r_t^e$ as a useful explanatory factor for the movements in nominal interest rates.

In particular, we would like to understand better the origins of this combination of shocks, which in our current simple model is largely a reflection of the empirical shortcomings of the intertemporal Euler equation, as captured by the intertemporal presence shock $\delta_t$. Moreover, it would be interesting to explore more realistic assumptions on the information available to policy makers when making their decisions, focusing in particular on the obvious fact that, unlike in our model, the efficient real interest rate is not observable in practice.

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# A    The Model

This appendix presents the microfoundations of the model.

## A.1    Households

A continuum of households of measure one populates the economy. All households, indexed by $j \in (0,1)$, discount the future at rate $\beta \in (0,1)$ and have the same instantaneous utility function, additively separable over consumption and labor, so that their objective is

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t e^{\delta_t} \left[ \log(C_t^j - \eta C_{t-1}^j) - \frac{(h_t^j)^{1+\omega}}{1+\omega} \right] \right\}.$$

The aggregate preference shock $\delta_t$ shifts the intertemporal allocation of consumption without affecting the intratemporal margin between labor and leisure.[19] We assume that $\delta_t$ follows a stationary process with mean zero of the form

$$\delta_t = \rho_\delta \delta_{t-1} + \varepsilon_t^\delta.$$

The consumption index $C_t^j$ is a constant elasticity of substitution aggregator over differentiated goods indexed by $i \in (0,1)$

$$C_t^j \equiv \left[ \int_0^1 c_t^j (i)^{\frac{\theta-1}{\theta}} \, di \right]^{\frac{\theta}{\theta-1}}. \tag{9}$$

Households supply their specialized labor input for the production of a specific final good. As a consequence of labor market segmentation, the wage $w_t^j$ differs across households. However, household $j$ can fully insure against idiosyncratic wage risk by buying at time $t$ state-contingent securities $D_{t+1}^j$ at price $Q_{t,t+1}$. Besides labor income, households earn after-tax $\Gamma_t^j$ from ownership of the firm. The flow budget constraint for household $j$ is

$$\int_0^1 p_t (i) c_t^j (i) \, di + E_t(Q_{t,t+1} D_{t+1}^j) = w_t^j h_t^j + D_t^j + \Gamma_t^j,$$

where $p_t (i)$ is the dollar price of the $i^{th}$ good variety.

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[19] We could have also introduced a purely intratemporal shock affecting labor supply decisions only. However, in our empirical implementation of the model, hours and wages are not included among the observables. Therefore, such a shock would only affect the flexible price level of output, making it indistinguishable from a technology shock.

## A.2 Firms

Firm $i$ produces the differentiated consumption good $y_t(i)$ with a linear production function in labor

$$y_t(i) = A_t h_t(i). \tag{10}$$

We assume that productivity grows at rate $\gamma_t \equiv \Delta \log A_t$ and that growth rate shocks display some persistence

$$\gamma_t = (1 - \rho_\gamma) \gamma + \rho_\gamma \gamma_{t-1} + \varepsilon_t^\gamma. \tag{11}$$

Firms take wages as given and sell their products in monopolistically competitive goods markets, setting prices in a staggered fashion, as in Calvo (1983). Every period, independently of previous adjustments, each firm faces a probability $(1 - \alpha)$ of optimally choosing its price. The $\alpha$ firms that do not fully optimize in a given period adjust their price according to the indexation scheme

$$p_t(i) = p_{t-1}(i) \left( \frac{P_{t-1}}{P_{t-2}} \right)^\zeta e^{(1-\zeta)\pi^*},$$

where $P_t$ is the aggregate price level consistent with the consumption aggregator (9) and we allow for partial indexation to the long run central bank's inflation target $\pi^*$. In the event of a price change at time $t$, firm $i$ chooses $p_t(i)$ to maximize the present discounted value of profits net of sales taxes $\tau_t$

$$E_t \left\{ \sum_{s=t}^{\infty} \alpha^{T-t} Q_{t,s} \left[ (1 - \tau_s) p_t(i) \left( \frac{P_{s-1}}{P_{t-1}} \right)^\zeta e^{(1-\zeta)\pi^*(s-t)} y_{t,s}(i) - w_s(i) h_s(i) \right] \right\}, \tag{12}$$

subject to its production function (10) and the demand for its own good conditional on no further price change after period $t$

$$y_{t,s}(i) = \left[ \frac{p_t(i)}{P_s} \right]^{-\theta} Y_s, \tag{13}$$

where $Y_t$ is an index of aggregate demand of the same form as (9).

## A.3 Monetary Policy

The central bank sets the net nominal interest rate $i_t$ with a certain degree of inertia in response to departures of aggregate demand and inflation from their respective objectives.

The non-linear formulation of the baseline interest rate rule is

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\rho} \left[\left(\frac{P_t}{P_{t-1}e^{\pi^*}}\right)^{\phi_\pi} \left(\frac{Y_t}{Y_t^e}\right)^{\phi_x}\right]^{1-\rho} e^{\varepsilon_t^i}, \tag{14}$$

where the gross nominal interest rate is defined as

$$R_t \equiv \frac{1}{E_t Q_{t,t+1}}$$

and its average can be decomposed via the Fisher equation as $R = e^{r+\pi^*}$, which defines the steady state net real interest rate $r$. The continuously compounded nominal interest rate in the text is defined as $i_t \equiv \log R_t$.

# B    Statistical Filters in DSGE Models

This appendix illustrates how to embed a linear filter into a dynamic rational expectation model. We begin with a brief general description of linear filtering problems. We then focus on the application to the Hodrick and Prescott (HP) filter (Hodrick and Prescott, 1997).

## B.1    Linear Filters

The objective of "filtering" is to decompose the stochastic process $x_t$ into two orthogonal components

$$x_t = y_t + \tilde{x}_t,$$

where the process $y_t$ has power only in some frequency interval $\{(a, b) \cup (-a, -b)\} \in (-\pi, \pi)$. Then, we can represent $y_t$ as

$$y_t = B(L)x_t,$$

where $B(L)$ – the *ideal* band-pass filter – is of the form

$$B(L) = \sum_{j=-\infty}^{\infty} B_j L^j.$$

Therefore, implementation of the ideal filter requires an infinite dataset. We can think about approximating the ideal filter as a projection problem. Given a sample $x = [x_1, ..., x_T]$,

the estimate of $y = [y_1, ..., y_T]$ is $\hat{y} = P[y|x]$, which is of the form

$$\hat{y}_t = \sum_{=-f}^{p} \hat{B}_j^{p,f} x_{t-j},$$

where $f = T - t$ and $p = t - 1$. The main problem of this estimates is that the $B$ coefficients require knowledge of $f_x(\omega)$, the spectral density of $x$.

Christiano and Fitzgerald (2003) show that, for most macro variables, the coefficients obtained by assuming that $x$ is a random walk work quite well. One approach to the calculation of these coefficients is then to "expand" the available sample with the least squares optimal guesses of the missing data at the beginning and end of the sample. For the random walk, these data are just $x_1$ and $x_T$. Our proposal is to adopt the same philosophy (i.e. to expand the available dataset) in the context of our framework, using the rational expectations forecasts of the missing data obtained from the model.[20]

## B.2   Application to the HP Filter

In this section, we discuss the application of our methodology to the HP filter. We focus on the HP filter because of its wide use in macroeconomics as a flexible device (through the choice of $\lambda$) to draw a smooth trend through the data. The HP filter provides a typical example of a "traditional" smooth measure of potential output and of the associated output gap. Its added advantage in out context is that the expression for the ideal filter is a relatively simple function of lag polynomials. The result is a parsimonious (i.e. two leads and lags) recursive representation, that requires only a modest expansion of the model's state space.

The ideal HP filter is of the form (e.g. Baxter and King, 1999)

$$
\begin{aligned}
HP^g &= \frac{\lambda(1-L)^2(1-F)^2}{1 + \lambda(1-L)^2(1-F)^2} \\
HP^t &= \frac{1}{1 + \lambda(1-L)^2(1-F)^2}
\end{aligned}
$$

where $HP^g$ denotes the filter whose application results in the "gap", while $HP^t$ denotes the filter whose application produces the trend.[21] Practical application of these filters requires an

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[20] Watson (2007) proposes a similar procedure using unrestricted ARIMA processes as forecasting tools. Julliard at al. (2006) is the only example we could find of an application to DSGEs models. The main objective of all these papers is to improve the end-of-sample performance of the filters they consider.

[21] King and Rebelo (1993) originally derived these expressions as the solution of a "smoothing" problem. However, they also showed that this filter, with $\lambda = 1600$, approximates very well a high pass filter with cutoff frequency $\pi/16$ or 32 quarters.

approximation, since they embed a two-sided, infinite moving average of the data.[22] However, application of Christiano and Fitzgerald's (2003) insight to a rational expectations context allows us to use the ideal filter directly, where the approximation relies on the substitution of the infinite leads and lags implicit in $HP(L)$ with rational expectation forecasts. In particular, given observations on $\log GDP_t = y_t$, we define the HP gap with parameter $\lambda$ as

$$\left[1 + \lambda(1-L)^2 (1-F)^2\right] x_t^{HP(\lambda)} = \lambda(1-L)^2 (1-F)^2 y_t,$$

where now the forward and backward operators are defined by

$$Ly_t = y_{t-1}$$
$$Fy_t = E_t y_{t+1}$$

as it is standard in rational expectations models (e.g. Blanchard and Fischer, 1989).

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[22] Details on this approximation can be found, for example, in Baxter and King (1999).

# C  Tables

| Parameter | Prior | | | | Posterior | | | |
|:---:|:---:|:---:|:---:|:---:|:---:|:---:|:---:|:---:|
| | Distribution | 5% | Median | 95% | Mode | 5% | Median | 95% |
| $\omega$ | $G(1,0.2)$ | 0.70 | 0.99 | 1.35 | 0.96 | 0.67 | 0.96 | 1.32 |
| $\xi$ | $G(0.1,0.05)$ | 0.03 | 0.09 | 0.19 | 0.001 | 0.000 | 0.002 | 0.004 |
| $\eta$ | $B(0.6,0.2)$ | 0.25 | 0.61 | 0.90 | 0.56 | 0.46 | 0.59 | 0.70 |
| $\zeta$ | $B(0.6,0.2)$ | 0.25 | 0.61 | 0.90 | 0.69 | 0.10 | 0.53 | 0.80 |
| $\rho$ | $B(0.7,0.15)$ | 0.43 | 0.72 | 0.92 | 0.70 | 0.62 | 0.72 | 0.80 |
| $\phi_\pi$ | $N(1.5,0.25)$ | 1.09 | 1.50 | 1.91 | 0.89 | 0.66 | 1.03 | 1.49 |
| $4\phi_x$ | $N(0.5,0.2)$ | 0.17 | 0.50 | 0.83 | 1.19 | 0.97 | 1.21 | 1.45 |
| $400\pi^*$ | $N(2,1)$ | 0.36 | 2.00 | 3.64 | 2.36 | 1.89 | 2.38 | 2.85 |
| $400r^a$ | $N(2,1)$ | 0.36 | 2.00 | 3.64 | 1.90 | 0.83 | 1.90 | 2.95 |
| $400\gamma^a$ | $N(3,0.35)$ | 2.42 | 3.00 | 3.58 | 2.94 | 2.48 | 2.94 | 3.40 |
| $\rho_\delta$ | $B(0.5,0.2)$ | 0.17 | 0.50 | 0.83 | 0.92 | 0.87 | 0.92 | 0.95 |
| $\rho_\gamma$ | $B(0.5,0.2)$ | 0.17 | 0.50 | 0.83 | 0.56 | 0.29 | 0.53 | 0.72 |
| $\rho_u$ | $B(0.5,0.2)$ | 0.17 | 0.50 | 0.83 | 0.11 | 0.06 | 0.32 | 0.71 |
| $\sigma_\delta$ | $IG1(0.5,2)$ | 0.17 | 0.34 | 1.24 | 1.23 | 0.95 | 1.37 | 2.04 |
| $\sigma_\gamma$ | $IG1(0.5,2)$ | 0.17 | 0.34 | 1.24 | 2.05 | 1.34 | 2.13 | 3.02 |
| $\sigma_u$ | $IG1(0.5,2)$ | 0.17 | 0.34 | 1.24 | 0.53 | 0.19 | 0.43 | 0.60 |
| $\sigma_i$ | $IG1(0.5,2)$ | 0.17 | 0.34 | 1.24 | 0.27 | 0.23 | 0.30 | 0.39 |

Table 1: Prior and posterior marginal distributions for the paramaters in the baseline model. G stands for Gamma, B stands for Beta, N stands for Normal and IG1 stands for Inverse Gamma 1, with mean and standard deviation in parenthesis

| Rank | Name | Policy Rule | logML | 2(logBF) |
|------|------|-------------|-------|----------|
| 47 | Baseline | $\phi_\pi \pi_t + \phi_x x_t^e$ | -379.0 | 0.0 |
| | | *Panel I: Alternative Output Gaps* | | |
| 36 | HP | $\phi_\pi \pi_t + \phi_x x_t^{HP}$ | -368.2 | 21.6 |
| 38 | SW | $\phi_\pi \pi_t + \phi_x x_t^e + \frac{\phi_{\Delta x}}{1-\rho} \Delta x_t^e$ | -369.0 | 20.1 |
| 39 | HP$\hat{\lambda}$ | $\phi_\pi \pi_t + \phi_x x_t^{HP(\hat{\lambda})}$ | -369.8 | 18.4 |
| 42 | Growth | $\phi_\pi \pi_t + \phi_{\Delta y} \Delta y_t$ | -374.8 | 8.3 |
| 43 | Exp$\hat{\lambda}$ | $\phi_\pi \pi_t + \phi_x x_t^{Exp(\hat{\lambda})}$ | -375.1 | 7.9 |
| 44 | Growth4Q | $\phi_\pi \pi_t + \frac{\phi_{\Delta y}}{4}\left(y_t - y_{t-4}\right)$ | -375.5 | 7.1 |
| 46 | Exp | $\phi_\pi \pi_t + \phi_x x_t^{Exp}$ | -378.6 | 0.7 |
| 48 | HP$\lambda^H$ | $\phi_\pi \pi_t + \phi_x x_t^{HP(\lambda^H)}$ | -380.8 | -3.5 |
| 52 | NoGap | $\phi_\pi \pi_t$ | -393.2 | -28.3 |
| | | *Panel II: Equilibrium Real Rate* | | |
| 16 | Re | $r_t^e + \phi_\pi \pi_t + \phi_x x_t^e$ | -359.3 | 39.4 |
| 17 | ReHP$\lambda^H$ | $r_t^e + \phi_\pi \pi_t + \phi_x x_t^{HP(\lambda^H)}$ | -360.0 | 38.0 |
| 20 | ReExp | $r_t^e + \phi_\pi \pi_t + \phi_x x_t^{Exp}$ | -360.4 | 37.2 |
| 22 | ReExp$\hat{\lambda}$ | $r_t^e + \phi_\pi \pi_t + \phi_x x_t^{Exp(\hat{\lambda})}$ | -361.3 | 35.5 |
| 23 | ReHP$\hat{\lambda}$ | $r_t^e + \phi_\pi \pi_t + \phi_x x_t^{HP(\hat{\lambda})}$ | -361.4 | 35.1 |
| 27 | ReNoGap | $r_t^e + \phi_\pi \pi_t$ | -364.6 | 28.9 |
| 29 | ReHP | $r_t^e + \phi_\pi \pi_t + \phi_x x_t^{HP}$ | -366.1 | 25.7 |
| 30 | RexpExp | $r_t^{Exp} + \phi_\pi \pi_t + \phi_x x_t^{Exp}$ | -366.3 | 25.4 |
| 32 | ReGrowth4Q | $r_t^e + \phi_\pi \pi_t + \frac{\phi_{\Delta y}}{4}\left(y_t - y_{t-4}\right)$ | -367.5 | 22.9 |
| 33 | ReGrowth | $r_t^e + \phi_\pi \pi_t + \phi_{\Delta y}\Delta y_t$ | -367.7 | 22.6 |
| 55 | RhpHP | $r_t^{HP} + \phi_\pi \pi_t + \phi_x x_t^{HP}$ | -397.3 | -36.6 |

Table 2: Ranking of alternative policy rules. First column shows the overall ranking, the second column the designation of the policy rule, the third column the long run component of the policy rule equation (excluding the smoothing component), the fourth column the log marginal likelihood and the fifth column the log of the Bayes factor. The table shows six panels, corresponding to different groups of policy rules.

| Rank | Name | Policy Rule | logML | 2(logBF) |
|------|------|-------------|-------|----------|
| | | Panel III: Time-Varying Inflation Target and Equilibrium Real Rate | | |
| 5 | RePistar | $r_t^e + \pi_t^* + \phi_\pi(\pi_t - \pi_t^*) + \phi_x x_t^e$ | -351.8 | 54.4 |
| 6 | RePistarNoGap | $r_t^e + \pi_t^* + \phi_\pi(\pi_t - \pi_t^*)$ | -353.6 | 50.7 |
| 7 | RePistarHP$\hat\lambda$ | $r_t^e + \pi_t^* + \phi_\pi(\pi_t - \pi_t^*) + \phi_x x_t^{HP(\hat\lambda)}$ | -355.0 | 48.0 |
| 8 | RePistarGrowth4Q | $r_t^e + \pi_t^* + \phi_\pi(\pi_t - \pi_t^*) + \frac{\phi_{\Delta y}}{4}\left(y_t - y_{t-4}\right)$ | -355.7 | 46.6 |
| 9 | RePistarExp$\hat\lambda$ | $r_t^e + \pi_t^* + \phi_\pi(\pi_t - \pi_t^*) + \phi_x x_t^{Exp(\hat\lambda)}$ | -355.8 | 46.4 |
| 10 | RePistarHP | $r_t^e + \pi_t^* + \phi_\pi(\pi_t - \pi_t^*) + \phi_x x_t^{HP}$ | -356.2 | 45.6 |
| 11 | RePistarGrowth | $r_t^e + \pi_t^* + \phi_\pi(\pi_t - \pi_t^*) + \phi_{\Delta y}\Delta y_t$ | -356.4 | 45.1 |
| 12 | RePistarHP$\lambda^H$ | $r_t^e + \pi_t^* + \phi_\pi(\pi_t - \pi_t^*) + \phi_x x_t^{HP(\lambda^H)}$ | -356.6 | 44.8 |
| 13 | RePistarExp | $r_t^e + \pi_t^* + \phi_\pi(\pi_t - \pi_t^*) + \phi_x x_t^{Exp}$ | -356.7 | 44.6 |
| 18 | RexpPistarExp | $r_t^{Exp} + \pi_t^* + \phi_\pi(\pi_t - \pi_t^*) + \phi_x x_t^{Exp}$ | -360.2 | 37.7 |
| 51 | RhpPistarHP | $r_t^{HP} + \pi_t^* + \phi_\pi(\pi_t - \pi_t^*) + \phi_x x_t^{HP}$ | -388.8 | -19.6 |
| | | Panel IV: Time-Varying Inflation Target | | |
| 19 | PistarExp | $\pi_t^* + \phi_\pi(\pi_t - \pi_t^*) + \phi_x x_t^{Exp}$ | -360.2 | 37.5 |
| 21 | PistarExp$\hat\lambda$ | $\pi_t^* + \phi_\pi(\pi_t - \pi_t^*) + \phi_x x_t^{Exp(\hat\lambda)}$ | -360.6 | 36.9 |
| 24 | PistarHP | $\pi_t^* + \phi_\pi(\pi_t - \pi_t^*) + \phi_x x_t^{HP}$ | -361.6 | 34.9 |
| 25 | PistarHP$\hat\lambda$ | $\pi_t^* + \phi_\pi(\pi_t - \pi_t^*) + \phi_x x_t^{HP(\hat\lambda)}$ | -362.0 | 34.0 |
| 26 | PistarHP$\lambda^H$ | $\pi_t^* + \phi_\pi(\pi_t - \pi_t^*) + \phi_x x_t^{HP(\lambda^H)}$ | -362.0 | 34.0 |
| 28 | PistarGrowth4Q | $\pi_t^* + \phi_\pi(\pi_t - \pi_t^*) + \frac{\phi_{\Delta y}}{4}\left(y_t - y_{t-4}\right)$ | -365.0 | 28.0 |
| 31 | Pistar | $\pi_t^* + \phi_\pi(\pi_t - \pi_t^*) + \phi_x x_t^e$ | -366.8 | 24.4 |
| 34 | PistarGrowth | $\pi_t^* + \phi_\pi(\pi_t - \pi_t^*) + \phi_{\Delta y}\Delta y_t$ | -367.8 | 22.3 |
| 40 | PistarNoGap | $\pi_t^* + \phi_\pi(\pi_t - \pi_t^*)$ | -371.4 | 15.1 |

Table 2: (Continued) Ranking of alternative policy rules. First column shows the overall ranking, the second column the designation of the policy rule, the third column the long run component of the policy rule equation (excluding the smoothing component), the fourth column the log marginal likelihood and the fifth column the log of the Bayes factor. The table shows six panels, corresponding to different groups of policy rules.

| Rank | Name | Policy Rule | logML | 2(logBF) |
|---|---|---|---|---|
| | | Panel V: Forward-Looking Rules | | |
| 3 | RePistarEPiEx | $r_t^e+\pi_t^*+\phi_\pi E_t(\pi_{t+1}-\pi_{t+1}^*)+\phi_x E_t x_{t+1}^e$ | -346.6 | 64.9 |
| 15 | ReEPiEx | $r_t^e+\phi_\pi E_t\pi_{t+1}+\phi_x E_t x_{t+1}^e$ | -358.6 | 40.9 |
| 50 | EPiExHP | $\phi_\pi E_t\pi_{t+1}+\phi_x E_t x_{t+1}^{HP}$ | -388.1 | -18.3 |
| 53 | EPiEx | $\phi_\pi E_t\pi_{t+1}+\phi_x E_t x_{t+1}^e$ | -394.7 | -31.5 |
| 54 | EPi | $\phi_\pi E_t\pi_{t+1}+\phi_x x_t^e$ | -394.9 | -31.7 |
| | | Panel VI: 4Qtr Inflation | | |
| 1 | RePistarPi4Q | $r_t^e+\pi_t^*+\phi_\pi(\pi_t^{4Q}-\pi_t^*)+\phi_x x_t^e$ | -345.4 | 67.2 |
| 2 | RePistarEPi4QEx | $r_t^e+\pi_t^*+\phi_\pi E_t(\pi_{t+4}^{4Q}-\pi_t^*)+\phi_x E_t x_{t+1}^e$ | -345.6 | 66.9 |
| 4 | RePi4Q | $r_t^e+\phi_\pi\pi_t^{4Q}+\phi_x x_t^e$ | -351.4 | 55.1 |
| 14 | RePi4QEx | $r_t^e+\phi_\pi E_t\pi_{t+4}^{4Q}+\phi_x E_t x_{t+1}^e$ | -357.7 | 42.7 |
| 35 | Pi4QHP | $\phi_\pi\pi_t^{4Q}+\phi_x x_t^{HP}$ | -368.0 | 22.0 |
| 37 | Pi4Q | $\phi_\pi\pi_t^{4Q}+\phi_x x_t^e$ | -368.6 | 20.8 |
| 41 | EPi4QExHP | $\phi_\pi E_t\pi_{t+4}^{4Q}+\phi_x E_t x_{t+1}^{HP}$ | -373.1 | 11.7 |
| 45 | Pi4QGrowth4Q | $\phi_\pi\pi_t^{4Q}+\frac{\phi_{\Delta y}}{4}\Delta^4 y_t$ | -377.8 | 2.4 |
| 49 | Pi4QEx | $\phi_\pi E_t\pi_{t+4}^{4Q}+\phi_x E_t x_{t+1}^e$ | -385.1 | -12.1 |

Table 2: (Continued) Ranking of alternative policy rules. First column shows the overall ranking, the second column the designation of the policy rule, the third column the long run component of the policy rule equation (excluding the smoothing component), the fourth column the log marginal likelihood and the fifth column the log of the Bayes factor. The table shows six panels, corresponding to different groups of policy rules.
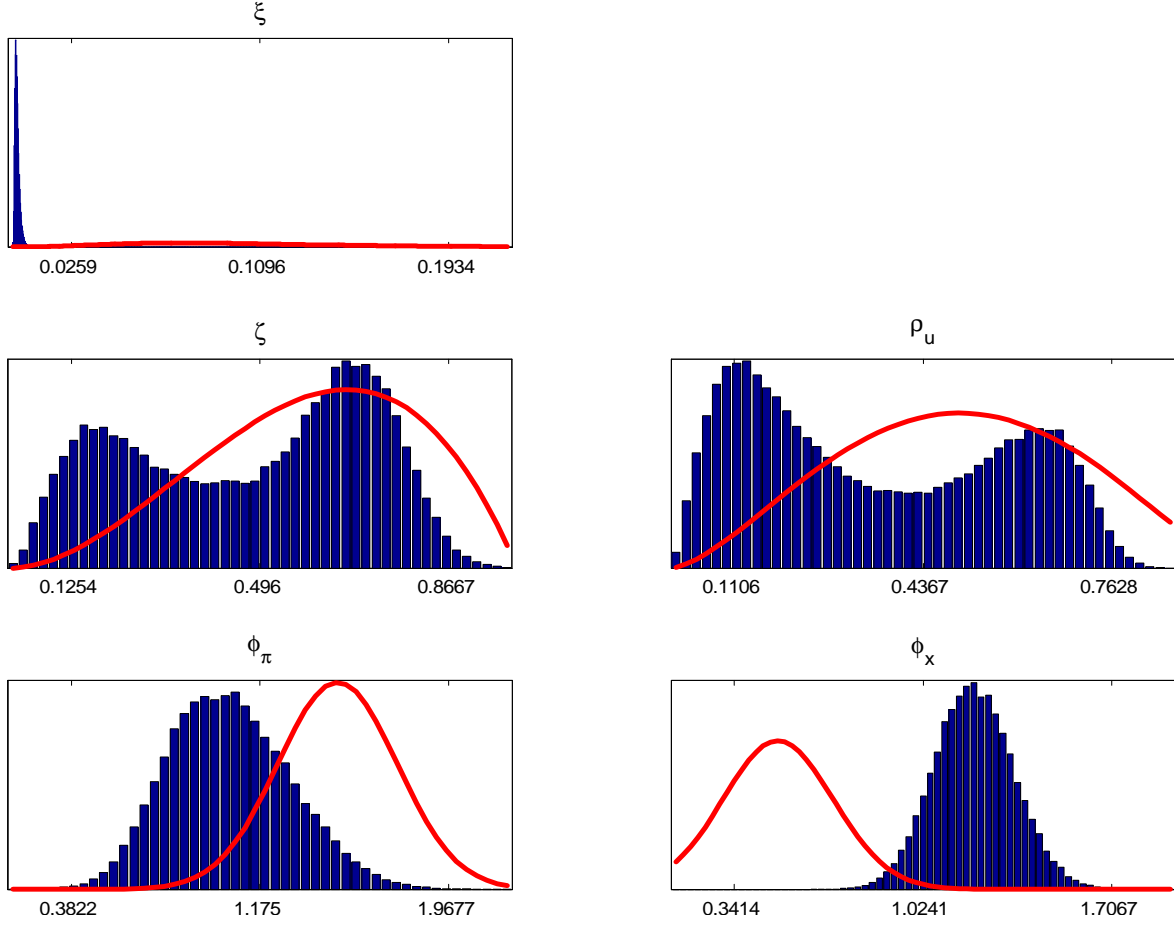
# D    Figures



Figure 1: Prior and posterior distributions for $\xi$, $\zeta$, $\rho_u$, $\phi_\pi$, and $\phi_x$ under the baseline specification of interest rate rule: $i_t = \rho i_{t-1} + (1 - \rho)(\phi_\pi \pi_t + \phi_x x_t^e) + \varepsilon_t^i$. For each parameter, the solid red line represents the prior while the blue histogram is the posterior.
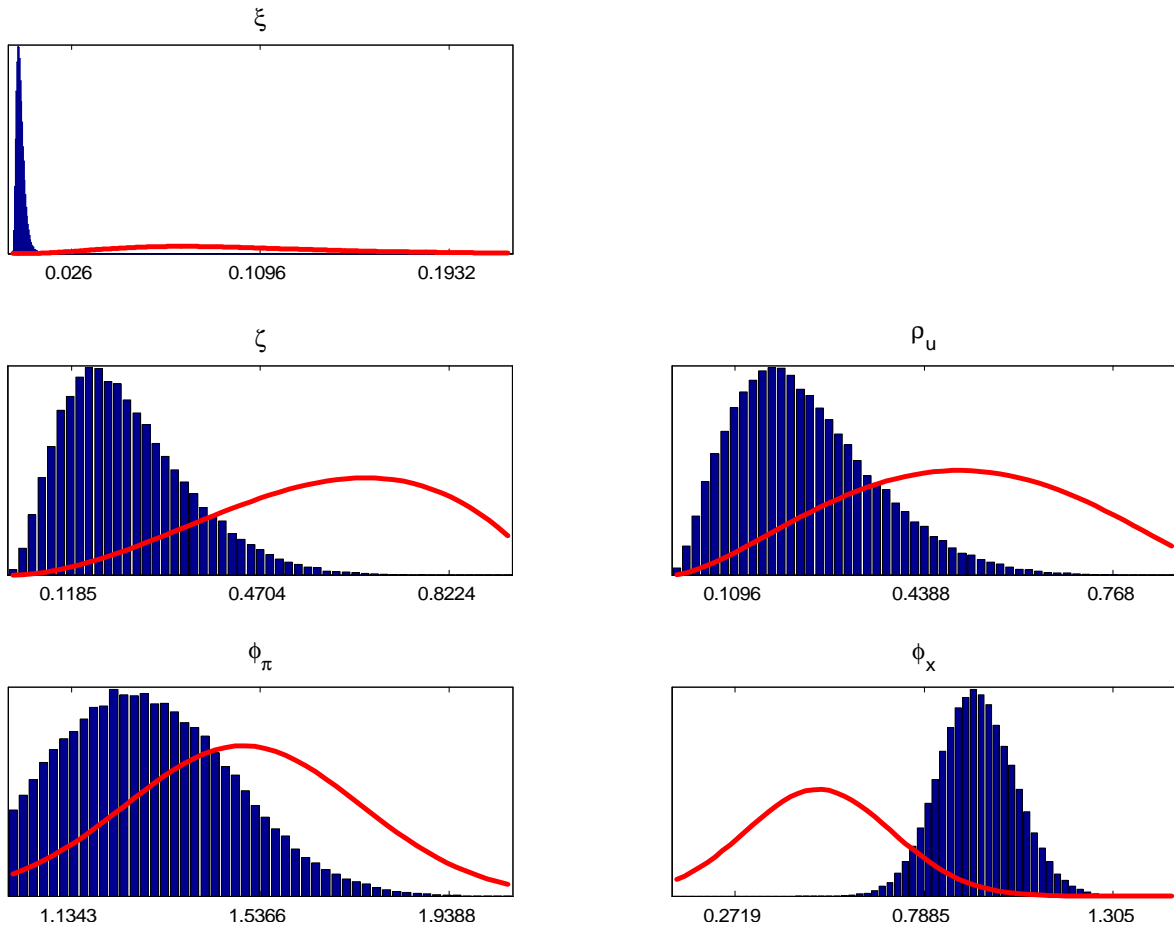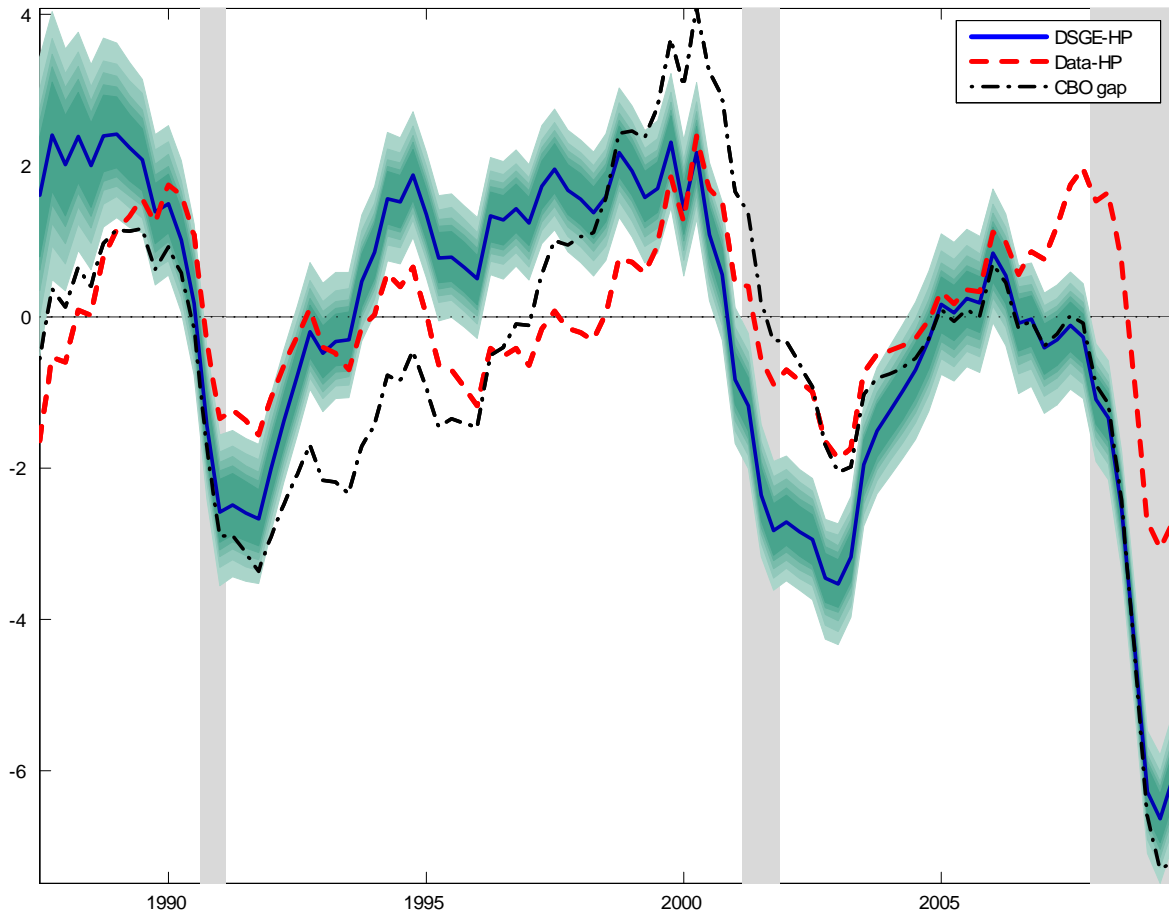
Figure 2: Prior and posterior distributions for $\xi$, $\zeta$, $\rho_u$, $\phi_\pi$, and $\phi_x$ under the HP specification of interest rate rule: $i_t = \rho i_{t-1} + (1 - \rho)\left(\phi_\pi \pi_t + \phi_x x_t^{HP}\right) + \varepsilon_t^i$. For each parameter, the solid red line represents the prior while the blue histogram is the posterior.

Figure 3: Evolution of the model HP output gap (DSGE-HP) and empirical HP output gap (Data-HP), all in percentage points. The blue continuous line and the shaded area around it are the posterior median estimate of the model-based HP-filtered output gap $x_t^{HP}$ and the 90% uncertainty bands when the interest rate rule is $i_t = \rho i_{t-1} + (1 - \rho)\left(\phi_\pi \pi_t + \phi_x x_t^{HP}\right) + \varepsilon_t^i$. The red dashed line is the cyclical component which results from applying the HP filter on the real GDP data used in the estimation. The black dash-dotted line is the output gap produced by the CBO.
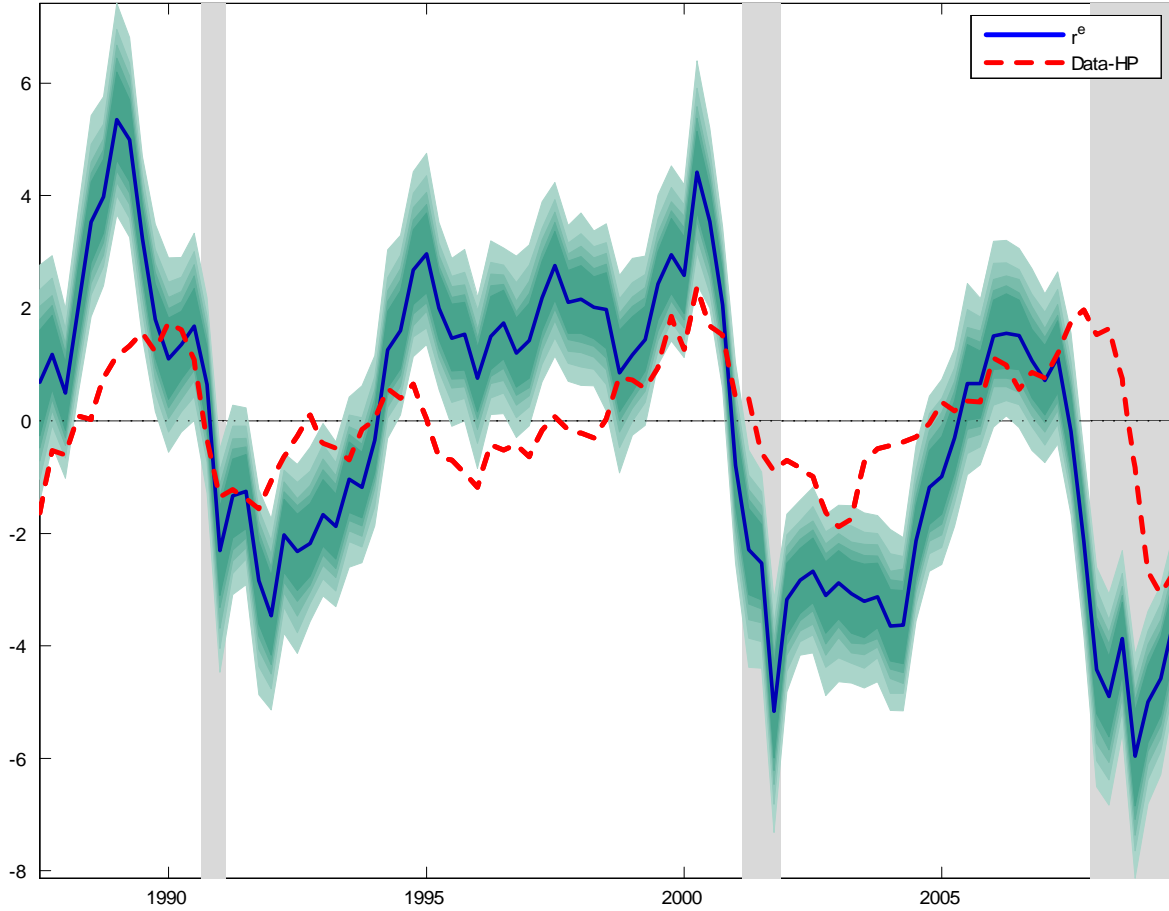
Figure 4: Evolution of model efficient annualized real interest rate ($r^e$) and empirical HP output gap (Data-HP), both in percentage points. The blue continuous line and the shaded area around it are the posterior median estimate of the model efficient real interest rate $r_t^e$ and the 90% uncertainty bands when the interest rate rule is $i_t = \rho i_{t-1} + (1-\rho)\left(r_t^e + \phi_\pi \pi_t + \phi_x x_t^e\right) + \varepsilon_t^i$. The red dashed line is the cyclical component which results from applying the HP filter on the real GDP data used in the estimation.
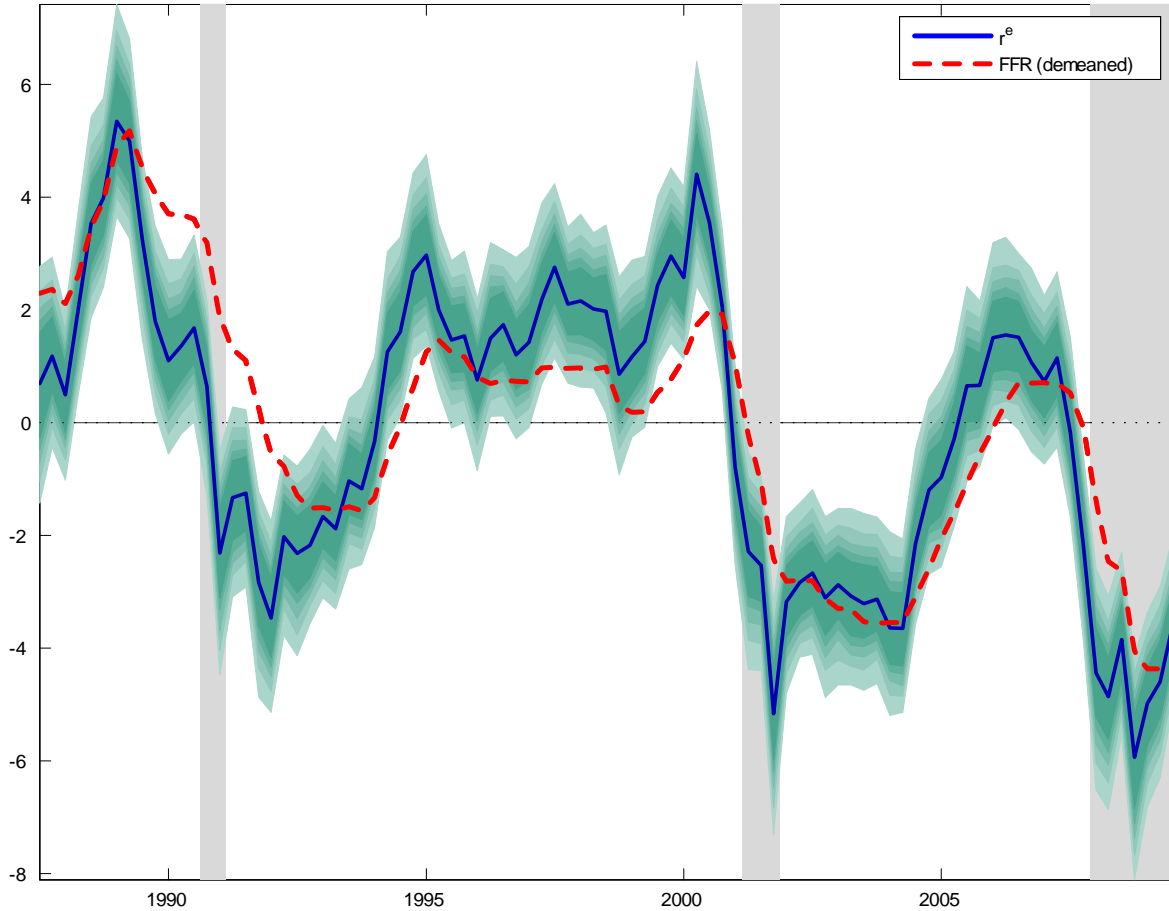
Figure 5: Evolution of model efficient real interest rate $(r^e)$ and demeaned Federal Funds Rate (FFR demeaned), both annualized and in percentage points. The blue continuous line and the shaded area around it are the posterior median estimate of the model efficient real interest rate $r^e$ and the 90% uncertainty bands when the interest rate rule is $i_t = \rho i_{t-1} + (1-\rho)\left(r_t^e + \phi_\pi \pi_t + \phi_x x_t^e\right) + \varepsilon_t^i$. The red dashed line is the demeaned FFR (sample mean equal to 4.5%).
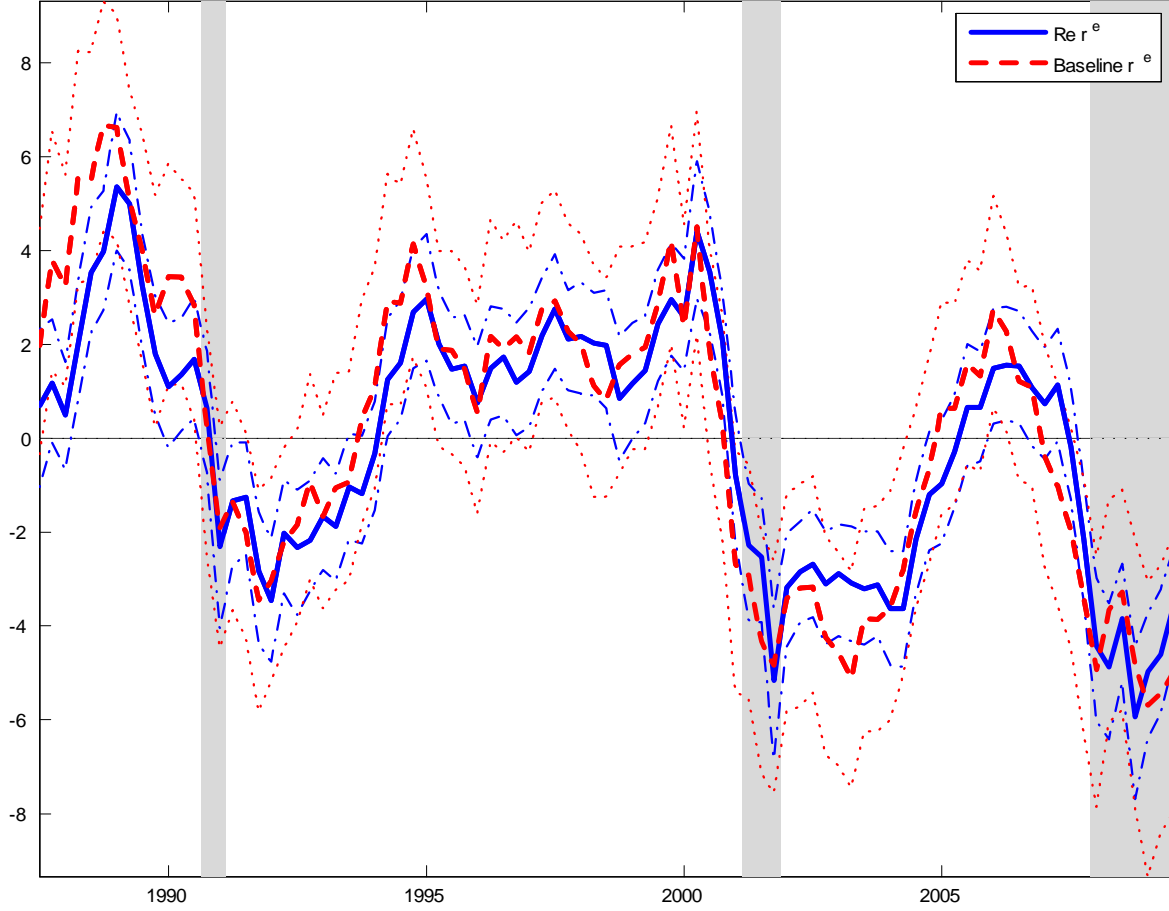
Figure 6: Evolution of efficient real interest rate $(r^e)$ in baseline model (Baseline) versus model with time-varying intercept (Re), both annualized and in percentage points. The blue continuous line and the dashed blue lines around it are the posterior median estimate of the model efficient real interest rate $r^e_t$ and the 90% uncertainty bands when the interest rate rule is $i_t = \rho i_{t-1} + (1 - \rho) \left( r^e_t + \phi_\pi \pi_t + \phi_x x^e_t \right) + \varepsilon^i_t$. The red continuous line and the dotted red lines around it are the posterior median estimate of the model efficient real interest rate $r^e_t$ and the 90% uncertainty bands around it when the interest rate rule is $i_t = \rho i_{t-1} + (1 - \rho) \left( \phi_\pi \pi_t + \phi_x x^e_t \right) + \varepsilon^i_t$.
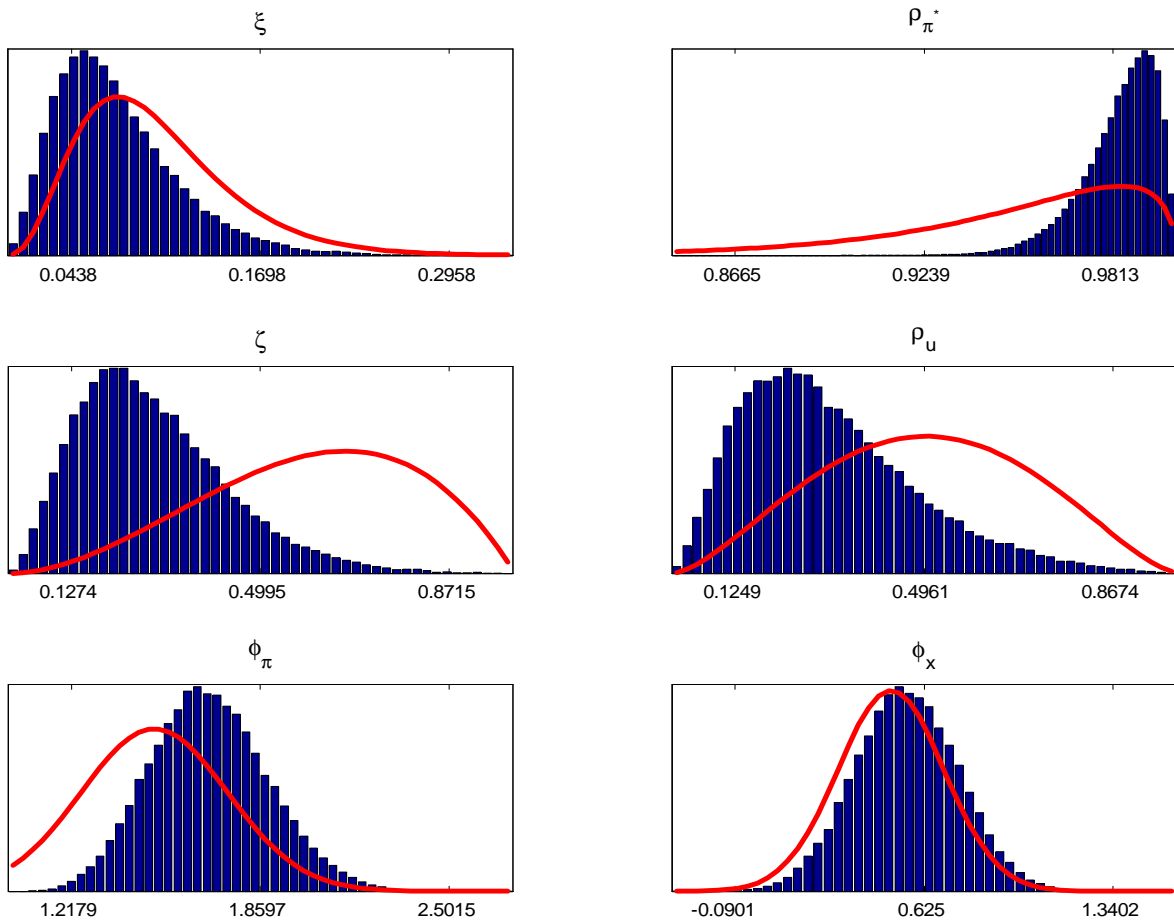
Figure 7: Prior and posterior distributions for $\xi$, $\rho_\pi^*$, $\zeta$, $\rho_u$, $\phi_\pi$, and $\phi_x$ under the RePistarPi4Q specification of the interest rate rule: $i_t = \rho i_{t-1} + (1-\rho)\left(r_t^e + \pi_t^* + \phi_\pi\left(\pi_t^{4Q} - \pi_t^*\right) + \phi_x x_t^e\right) + \varepsilon_t^i$. For each parameter, the solid red line represents the prior while the blue histogram is the posterior.
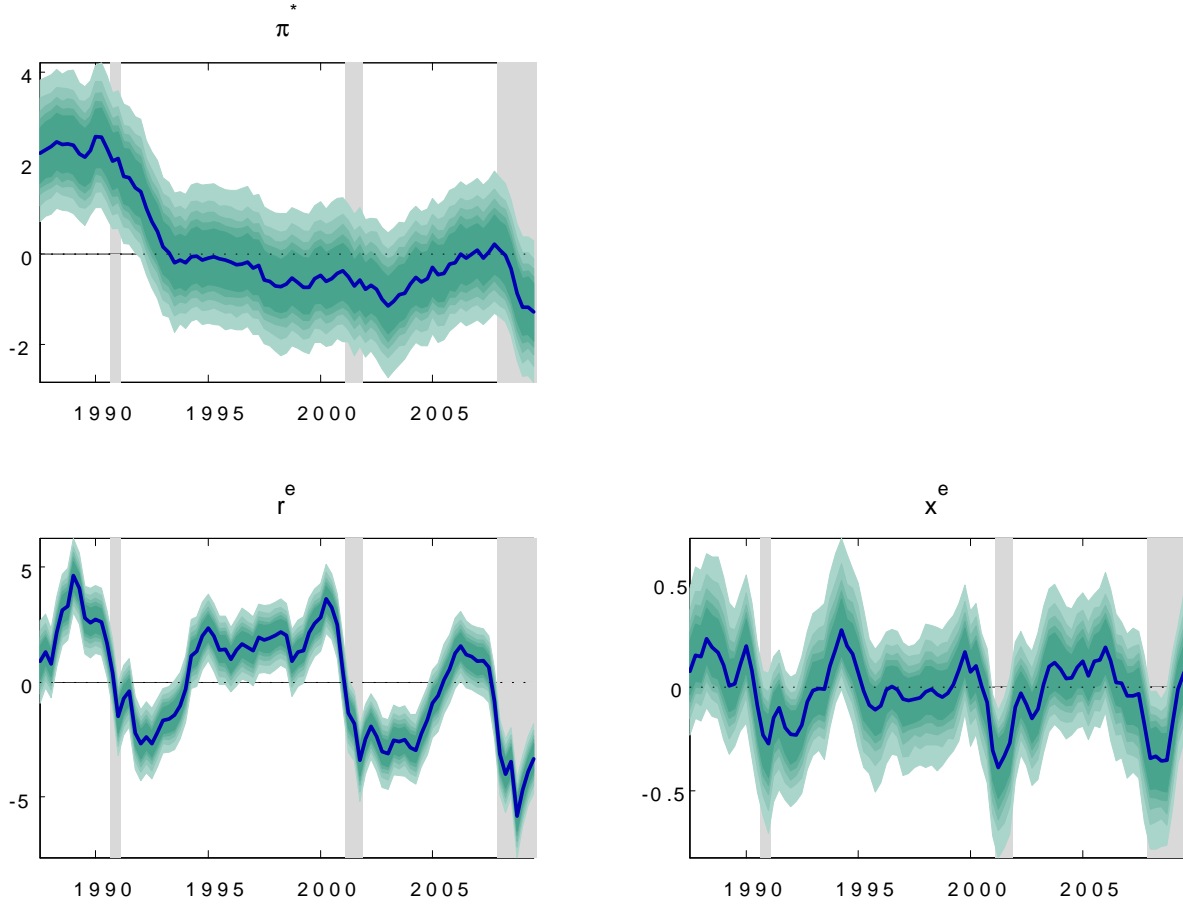
Figure 8: Evolution of inflation target $(\pi_t^*)$, efficient real interest rate $(r^e)$, and efficient output gap $(x_t^e)$ in the RePistarPi4Q specification of the interest rate rule: $i_t = \rho i_{t-1} + (1-\rho)\left(r_t^e + \pi_t^* + \phi_\pi\left(\pi_t^{4Q} - \pi_t^*\right) + \phi_x x_t^e\right) + \varepsilon_t^i$. Inflation and interest rate are annualized and all are shown in percentage points. The blue continuous lines and the shaded areas around them are the posterior median estimates and the 90% uncertainty bands.