Bridging cyclical DSGE models and the raw data

Fabio Canova *
ICREA-UPF, AMeN and CEPR

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Abstract

I propose a method to estimate cyclical DSGE models using the raw data. The approach links the observables to the model counterparts via a flexible specification which does not require that the cyclical component is solely located at business cycle frequencies and allows the non-cyclical component to take various time series patterns. I show that applying standard data transformation induces distortions in structural estimates and policy conclusions and explain the reasons for their emergence. The proposed approach recovers the features of the cyclical component in selected experimental designs.

JEL classification: E32, C32.

Keywords: DSGE models, Filters, Structural estimation, Business cycles.

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1 Introduction

There has been considerable development in the specification of DSGE models over the last 10 years. The original structure, featuring a single technological disturbance, has been enriched with shocks and frictions and our understanding of the propagation mechanism of important shocks enhanced. Steps forward have also been made in the estimation of these models. While a few years ago it was standard to informally calibrate their structural parameters, now researchers routinely use limited and full information estimation procedures and, perhaps more importantly, this trend is common in academic and policy circles (see, e.g., [29], [12], [31],[21], [28] among others).

Despite recent developments, structural estimation of DSGE models is conceptually and practically difficult. For example, classical estimation is asymptotically justified only when the model is the data generating process (DGP) of the actual data, up to a set of serially uncorrelated measurement errors, and standard validation exercises are meaningless without such an assumption. Identification problems (see [9]) and numerical difficulties appear to be widespread. Finally, the vast majority of the models used in the literature is intended to explain only the cyclical portion of observable fluctuations but macroeconomic data contains many types of fluctuations, some of which can hardly be considered cyclical.

There are a number of reasons for why researchers prefer to work with "cyclical" models. Jointly accounting for cyclical and non-cyclical fluctuations is still an ambitious task and there are very few known theoretical mechanisms able to propagate temporary disturbances for a sufficiently long time (we need e.g. R&D as in [14] or a Schumpeterian creative destruction as in [7]). In addition, since little is known about the features of the non-cyclical component, specification errors may be important. Finally, from the computation and the interpretation point of views, it is convenient to assume that the mechanisms driving cyclical and non-cyclical fluctuations are distinct.

The mismatch between what models are designed to explain and what the data contains creates important headaches to applied investigators. In general, one of two following routes is taken:

• Estimate the model using data transformations which, in theory, are likely to be void of non-cyclical fluctuations, e.g. consider real "great ratios" (as suggested in [13] and [23]) or nominal " great ratios", (as suggested in [33]). As Figure 1 shows, such transformations are unlikely to resolve the mismatch issue because many of these ratios still display important non-cyclical movements.

• Fit the model to filtered data. While popular, such an approach is problematic for at least three reasons. First, while many ways to extract fluctuations with 8-32 quarters average periodicity - the so-called business cycle frequencies - exist in the literature, they all produce contaminated estimates of these fluctuations. For example, a Band Pass (BP) filter, when used with finite stretches of data, only very roughly capture the power of the spectrum at business cycle frequencies and taking growth rates greatly emphasizes the high frequency content of the data.

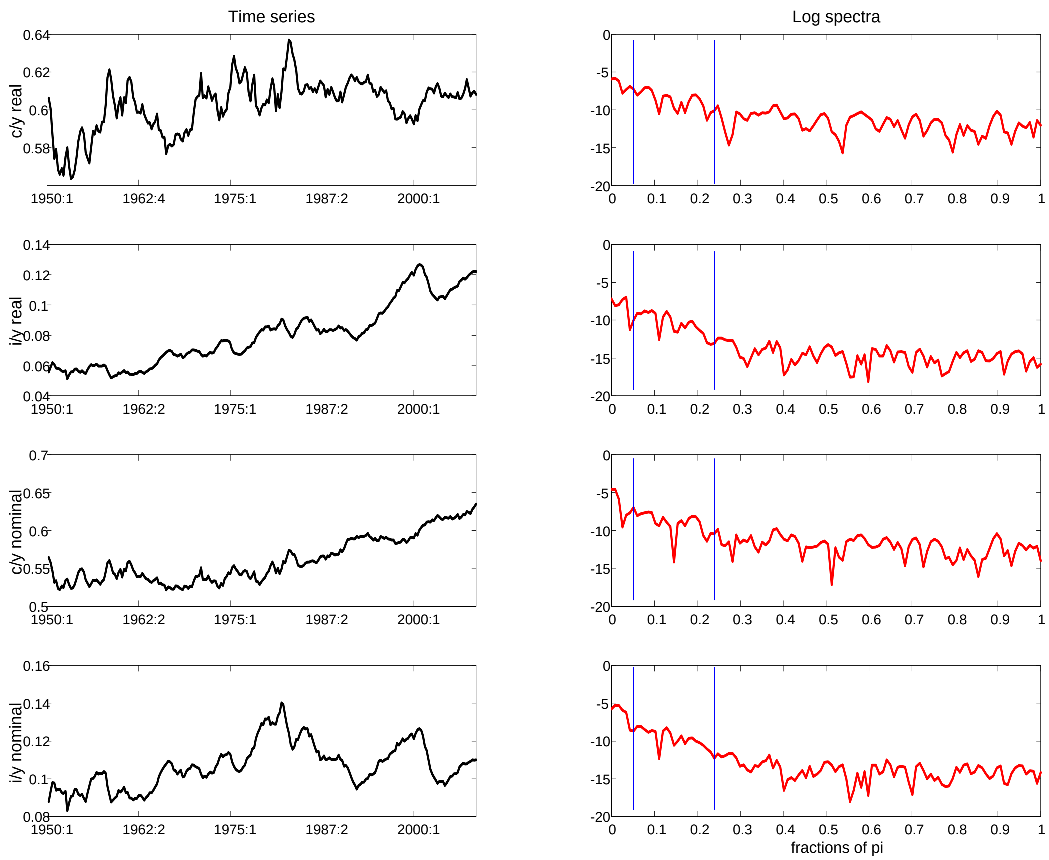

Figure 1: US real and nominal great ratios. Vertical bars in the right column isolate business cycle frequencies.

Second, while it is typical to filter each series separately, there are theoretical reasons to believe that some multivariate consistency condition should be imposed. Along the same lines, it is unclear whether only real variables or all variables should be filtered prior to estimation and there are arguments in favour of both approaches. Many models imply cyclical fluctuations for e.g., inflation and the nominal rate, even when the shocks are non-cyclical; conversely, not all the fluctuations present in nominal variables can be safely considered as cyclical. Third, while usually not appreciated, the cyclical component produced by the majority of the filters can be represented as a symmetric, two-sided moving average of the raw data. Since the timing of information is altered by filtering, dynamic analyses conducted with estimated parameters are difficult to interpret. In sum, rather than resolving the issue, filtering increases the difficulties applied researchers face.

This paper first shows that the data transformation one employs matters for structural parameter estimation and for economic inference in general. Hence, unless one takes a strong but unjustified view of what "cyclical" data the model should explain, one is left wandering how to credibly select among various structural estimates. I then argue that structural estimates obtained with *any* preliminary data transformations should not be trusted for two reasons. On the one hand, the presence of measurement error with low frequencies features in the transformed data distorts the conclusions applied investigators reach. An approach to deal with this type of contamination, which exploits ideas developed in [3], has been recently proposed in [6].

On the other, the cyclical component of a DSGE model has properties which are different from the cyclical component extracted with existing filters, even in large samples. This is because the filters commonly used implicitly assume that the cyclical component has power only at business cycle frequencies. However, a DSGE model generates cyclical and non-cyclical components with power at all frequencies of the spectrum. Hence, at business cycle frequencies, both components may matter and it is not very difficult to build examples where the non-cyclical component dominates. [1] have argued that for Less Developed Countries (LDC) this is an important concern. What I show is that the problem is relevant for structural estimation with the data of any country and, as long as the variance of the disturbances driving the cyclical and the non-cyclical components are roughly of the same magnitude, significant biases may emerge.

I highlight the importance of these two problems using a simple experimental design where data is generated from a DSGE model where the endogenous variables are driven by cyclical and non-cyclical shocks and standard transformations are applied prior to parameter estimation. I interpret the distortions produced using the decomposition of the likelihood function suggested in [19] and show why even "ideal" transformations may lead to incorrect inference.

As an alternative, I propose to estimate the structural parameters of a "cyclical" DSGE model by creating a flexible link between the model and the raw data that allows the cyclical and the non-cyclical components to have power at all the frequencies of the spectrum. Since the specification I use encompasses, as special cases, situations where the non-cyclical component displays deterministic, stochastic or smooth features, the approach does not require researchers to take a stand on the non-cyclical features of the raw data (as is done e.g. in [13] and [18]) and therefore shields the analysis from important specification errors. Also, while for expositional reasons, drivers of the cyclical and the non-cyclical components are assumed to be orthogonal, there are no conceptual difficulties in considering cases where shocks driving the two components are correlated.

I demonstrate that the procedure can effectively capture the cyclical component of the data generating process (DGP) and produces reasonable estimates of the structural parameters when samples of the size currently available in macroeconomics are used. I also show that, when applied to real data, the procedure gives a somewhat different view about important aspects of the model economies relative to standard filtering approaches. In particular, inference about the size of the Phillips curve trade-off and the short run inflation coefficient in the policy rule and the contribution of various shocks to the variance of output and inflation is very different.

To focus attention on the issues of interest, the paper makes a number of simplifying assumptions. In particular, I assume that (i) the estimated model is correctly specified; that is, there are no missing variables or omitted cyclical shocks; (ii) theoretical singularities are absent - the number of shocks is equal to the number of endogenous variables - and (iii) model variables have an exact counterpart in the actual data, i.e. no proxy error is present. While these issues are important in practice and semi-structural methods of the type suggested by [8] produce more robust inference when they are present, I find it useful to keep them separate from the issue of estimating cyclical models with raw data because the problems I highlight occur regardless of whether (i)-(ii) and (iii) are present or not.

The rest of the paper is organized as follows. The next Section presents a simple model and estimates its structural parameters using a number of data transformations. Section 3 shows why estimates obtained in Section 2 should not be trusted. Section 4 presents the methodology to bridge cyclical DSGE models and the raw data, relates it to what is available in the literature and evaluates its properties. Section 5 compares estimates of functions of the parameters in two different models. Section 6 concludes. An appendix contains all the relevant additional material mentioned in the paper.

2 Structural estimation with transformed data

To show how estimates of the parameters a "cyclical" DSGE model depend on the preliminary transformation employed, I consider a standard small scale New-Keynesian model where agents face a labor-leisure choice, there is external habit in consumption, production is stochastic and requires labor, there is an exogenous probability of price adjustments and monetary policy is conducted with a conventional Taylor rule. Details on the structure of this model are in the Appendix. The log-linearized equilibrium conditions are:

$$\lambda_t = \chi_t - \frac{\sigma_c}{1-h}(y_t - hy_{t-1}) \tag{1}$$
$$y_t = z_t + (1-\alpha)n_t \tag{2}$$
$$mc_t = w_t + n_t - y_t \tag{3}$$
$$w_t = -\lambda_t + \sigma_n n_t \tag{4}$$
$$r_t = \rho_r r_{t-1} + (1-\rho_r)(\rho_\pi \pi_t + \rho_y y_t) + \epsilon_t^r \tag{5}$$
$$\lambda_t = E_t(\lambda_{t+1} + r_t - \pi_{t+1}) \tag{6}$$
$$\pi_t = k_p(mc_t + \epsilon_t^\mu) + \beta E_t \pi_{t+1} \tag{7}$$

where $k_p = \frac{(1-\beta\zeta_p)(1-\zeta_p)}{\zeta_p}\frac{1-\alpha}{1-\alpha+\varepsilon\alpha}$, all variables are expressed in deviation from the steady states, λ_t is the Lagrangian on the consumer budget constraint, mc_t are marginal costs, $z_t \rho_z z_{t-1} + \epsilon_t^z$ is a technology disturbance, $\chi_t = \rho_\chi \chi_{t-1} + \epsilon_t^\chi$ a preference disturbance, ϵ_t^r is an iid monetary policy disturbance and ϵ_t^μ an iid markup disturbance. Equation (1) relates the Lagrangian to the marginal utility of consumption, equation (2) is a production function, equation (3) is the definition of marginal costs, equation (4) equates the marginal disutility of leisure to the real wage, equation (5) is the monetary policy rule, equation (6) is the euler equation, equation (7) is a Phillips curve. The structural parameters to be estimated are: σ_c the risk aversion coefficient, σ_n the inverse of the Frisch elasticity, h the coefficient of consumption habit, $1-\alpha$ the share of labor in production, ρ_r the degree of interest rate smoothing, ρ_π and ρ_y the parameters of the policy rule, ζ_p the probability of not changing prices, and ε the elasticity among consumption varieties. The auxiliary parameters are: ρ_χ, ρ_z the autoregressive parameters of preference and technology shocks, and $\sigma_z, \sigma_\chi, \sigma_r, \sigma_\mu$ the standard deviations of the four structural shocks. The discount factor β is kept fixed to 0.99 in the estimation exercises.

I assume that there are four observable variables: output, the real wage, the inflation rate and the nominal interest rate (y_t, w_t, π_t, r_t), and examine a variety of filtering approaches, applied to all or a subset of the variables. In particular, I have considered the following options: (i) independently filter real variables and demean nominal variables (as in [27], [28] or [31]); (ii) independently filter all the variables (as in [21]); (iii) demean all the variables and take ratios for real variables ($\log y_t - \log n_t, \log w_t - \log n_t$, where n_t is hours worked). This last transformation is selected because, if the model is correct, the transformed data must be void of non-cyclical fluctuations no matter what the time series properties of the shocks are. For (i) and (ii), I consider four approaches, which cover the range of filters used in the empirical DSGE literature: linear filtering (LT), Hodrick and Prescott filtering (HP), BP filtering and first difference (FOD) filtering. Since the first three approaches belong to the class of two-sided moving averages and may therefore

Parameter	Distribution	Mean	Standard Deviation
σ_c	$\Gamma(20, 0.1)$	2.00	0.447
σ_n	$\Gamma(20, 0.1)$	2.00	0.447
h	$B(6, 8)$	0.428	0.127
α	$B(3, 8)$	0.277	0.128
ϵ	$N(6, 0.1)$	6.00	0.10
ρ_r	$B(6, 6)$	0.500	0.138
ρ_π	$N(1.5, 0.1)$	1.50	0.10
ρ_y	$N(0.4, 0.1)$	0.40	0.10
ζ_p	$B(6, 6)$	0.500	0.138
ρ_χ	$B(18, 8)$	0.692	0.088
ρ_z	$B(18, 8)$	0.692	0.088
σ_χ	$\Gamma^{-1}(10, 20)$	0.0056	0.0020
σ_z	$\Gamma^{-1}(10, 20)$	0.0056	0.0020
σ_{mp}	$\Gamma^{-1}(10, 20)$	0.0055	0.0020
σ_μ	$\Gamma^{-1}(10, 20)$	0.0056	0.0020
σ_{e_i}	$\Gamma^{-1}(10, 10)$	0.0111	0.0039
σ_{v_i}	$\Gamma^{-1}(10, 100)$	0.0011	0.0003
σ_{u_i}	$\Gamma^{-1}(10, 100)$	0.0011	0.0003

Table 1: Prior distributions

alter the timing of the information of the data, I have also experimented with either a recursive LT filter or a one-sided version of the HP filter. None of the results below are driven by this, nevertheless important, problem.

Estimation is conducted using Bayesian methods. Posterior estimates are obtained with a random walk Metropolis algorithm, where the vector of jumping variables is t-distributed with 5 degrees of freedom, and the variance tuned to have an acceptance rate of roughly 30-35 percent for each transformation considered. One million draws are made for each case-filter combination studied and convergence was checked using CUMSUM graphs. Since convergence to the ergodic distribution is rather slow and draws are highly serially correlated, I keep one every hundred of the last 100,000 draws to compute posterior statistics. The data is from the FRED database and the sample goes from 1980:1 to 2007:2. The priors for the parameters are kept fixed in the exercise, are chosen to be rather loose to let the data talk, and are reported in table 1.

Table 2 contains the median and the standard deviation of the posterior distributions for a subset of the combinations I have tried. Results for other combinations and available in the appendix. Clearly, there are several parameters whose posterior distribution is affected by the preliminary data transformation used (see e.g. the price stickiness parameter ζ_p, the persistence and the volatility of the shocks, and the parameters of the monetary policy rule). Since posterior standard deviations

	LT	HP	FOD	BP	Ratio
	Median (s.d.)	Median (s.d.)	Median (s.d.)	Median (s.d.)	Median (s.d.)
σ_c	2.19 (0.10)	2.25 (0.12)	2.54 (0.16)	2.21 (0.10)	1.69 (0.11)
σ_n	1.79 (0.08)	1.57 (0.10)	1.90 (0.19)	1.78 (0.08)	2.16 (0.10)
h	0.67 (0.01)	0.59 (0.03)	0.44 (0.03)	0.66 (0.02)	0.64 (0.02)
α	0.17 (0.03)	0.12 (0.02)	0.12 (0.03)	0.16 (0.02)	0.13 (0.02)
ϵ	3.90 (0.12)	4.27 (0.14)	2.92 (0.11)	3.72 (0.05)	4.09 (0.12)
ρ_r	0.16 (0.04)	0.52 (0.04)	0.22 (0.06)	0.49 (0.04)	0.22 (0.04)
ρ_π	1.36 (0.08)	1.67 (0.04)	1.74 (0.05)	1.77 (0.08)	1.71 (0.05)
ρ_y	-0.15 (0.02)	0.35 (0.06)	0.13 (0.07)	0.44 (0.05)	-0.02 (0.01)
ζ_p	0.81 (0.01)	0.60 (0.03)	0.33 (0.03)	0.56 (0.03)	0.81 (0.01)
ρ_χ	0.76 (0.02)	0.59 (0.04)	0.29 (0.04)	0.82 (0.03)	0.82 (0.02)
ρ_z	0.96 (0.01)	0.54 (0.05)	0.87 (0.05)	0.46 (0.05)	0.92 (0.01)
σ_χ	0.23 (0.04)	0.37 (0.05)	0.23 (0.04)	0.20 (0.03)	0.95 (0.16)
σ_z	0.12 (0.02)	0.08 (0.01)	0.09 (0.01)	0.09 (0.01)	0.08 (0.01)
σ_r	0.11 (0.01)	0.08 (0.01)	0.12 (0.02)	0.08 (0.01)	0.12 (0.01)
σ_μ	30.54 (1.17)	1.01 (0.40)	0.16 (0.03)	0.63 (0.21)	34.70 (1.04)

Table 2: Posterior estimates. For LT, HP, FOD and BP real variables detrended, nominal demeaned. For Ratio, real variables are in terms of hours and all variables demeaned. Standard deviations in parenthesis. Estimates of σ's and their standard deviations are in percentages. US data, sample 1980:1-2007:2.

are tight, differences across columns are a-posteriori significant. Posterior differences are also economically relevant. For example, the volatility of markup shocks in the LT and Ratio economies is considerably larger than in the other economies and nominal inertia much stronger.

Differences in the location of the posterior of the parameter translate into important differences in the transmission of shocks. This is clear in Figure 2, which reports responses of the four endogenous variables to unitary preference and technology shocks: the magnitude of the impact coefficients and the persistence of the responses vary with the preliminary transformation. Furthermore, at least in the case of technology shocks, the sign of some of the responses is affected. Differences in the responses to monetary and markup disturbances are less dramatic because shocks are iid. Nevertheless, on impact, differences are statistically and economically important.

Important policy recommendations may also depend on the preliminary transformation used. In table 3 I present the median and the standard deviation of the posterior distribution of the Phillips curve trade-off k_p and the short run inflation coefficient in the policy rule $(1 - \rho_r) * \rho_\pi$. The posterior distribution of the trade-off is centered at very low values and is very tight with LT and Ratio estimates but centered at higher values and much more spread out with other data transformations. Similarly, the location and the spread of the posterior distribution of the inflation

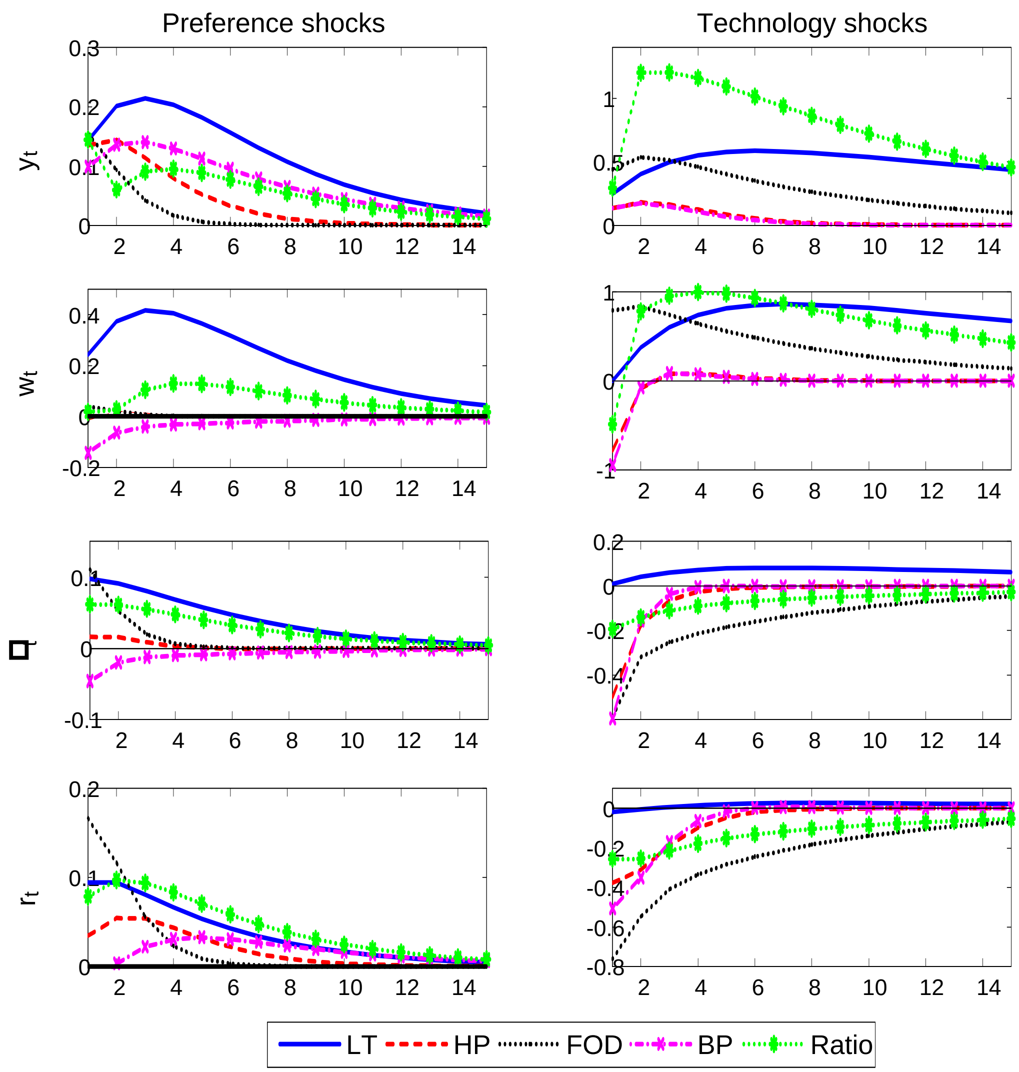

Figure 2: Impulse responses

	LT		HP		FOD		BP		Ratio		Flexible	
	Median	(s.d.)	Median	(s.d.)	Median	(s.d.)	Median	(s.d.)	Median	(s.d.)	Median	(s.d.)
k_p	0.02	(0.001)	0.17	(0.03)	0.95	(0.17)	0.21	(0.03)	0.02	(0.001)	1.20	(0.34)
$(1-\rho_r)*\rho_\pi$	1.16	(0.09)	0.80	(0.06)	1.36	(0.11)	0.89	(0.07)	1.31	(0.07)	1.58	(0.08)

Table 3: Posterior estimates. For LT, HP, FOD and BP real variables detrended, nominal demeaned. For Ratio, real variables are in terms of hours and all variables demeaned. Standard deviations in parenthesis. US data, sample 1980:1-2007:2.

coefficient depends on the preliminary data transformation employed and the posteriors obtained, e.g. with HP and FOD filtered data, display little overlap.

While it is common to sweep these differences under the rug, one should expect them to occur. After all, the growth rate of GDP and the linearly detrended GDP have very different time series properties. These differences would be inconsequential, if applied researchers had a good reason to prefer one preliminary transformation over the other. As argued in [4], it is hard to design criteria to do so. But even if a criteria could be found, none of the columns of table 1 should be trusted ¡for two reasons. First, all the transformation considered only approximately isolate cycles with 8-32 quarters periodicities: the LT transformation leaves both long and short cycles in the filtered data; the HP transformation leaves high frequencies variability unchanged; the FOD transformation emphasizes high frequency fluctuations and reduces the importance of cycles with business cycle periodicity; Great ratios leave important low frequency fluctuations in the transformed data; and even a BP transformation, once truncations due to finite samples are considered, induces significant approximation errors (see e.g. [5], ch.3). Since the "cyclical" data used for estimation contains considerable measurement error, distortions are likely to appear. In addition, since different approaches spread this error across different frequencies, estimates are likely to differ depending on the preliminary transformation used. An estimation approach which can reduce the extent of these measurement errors is suggested in [6].

Second, and perhaps more importantly, all transformations assume that the cyclical and the non-cyclical components are located at different frequencies of the spectrum and that the economic mechanism generating the two is distinct. Such an assumption is crucial, for example, when identifying the frequencies corresponding to fluctuations with 8 to 32 quarters periodicity with the cycle produced by the model. However, the cyclical component produced by a DSGE model has power at frequencies other than those corresponding to 8 to 32 quarters and, viceversa, the non-cyclical component is important at business cycle frequencies. Time series macroeconometricians are generally aware that statistical transformations are unlikely to recover interesting economic objects. For example, [30], [17] and [19] have all emphasized the fallacy of estimating structural models using seasonally adjusted data, precisely for this reason.

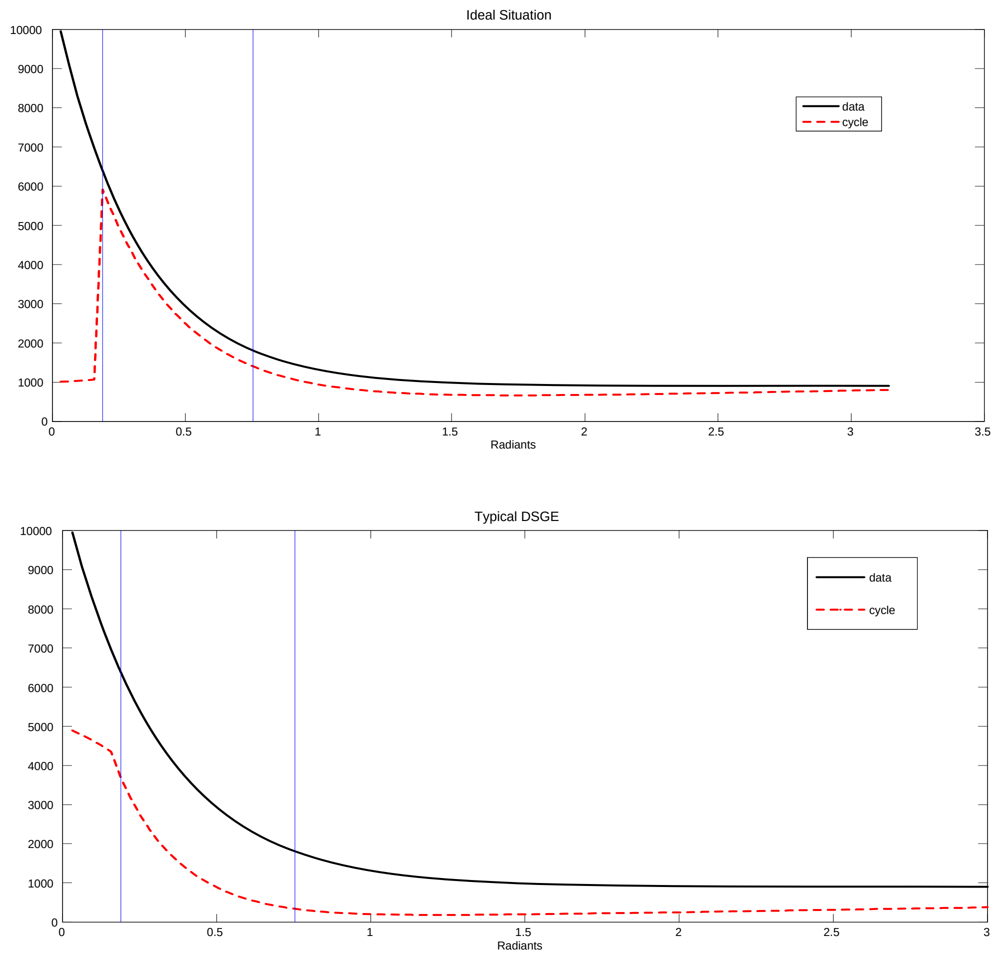

Figure 3: Cyclical and non-cyclical components

Figure 3 illustrates the essence of the problem. On the top panel are plotted the log spectrum of a typical macroeconomic time series and the ideal form of the cyclical component. Here, a statistical filter which attempts to isolate business cycle frequencies (marked by the vertical bars) will produce some measurement errors (due to leakages and compression) but it will not greatly distort the features of the true cyclical component. On the bottom panel, I present instead the more common situation present in DSGE frameworks where the cyclical component has power in the low frequencies of the spectrum and conversely, the non-cyclical component, has power at business cycle frequencies. Here, filtering to extract the power of the spectrum at business cycle frequencies is likely to induce significant distortions. This fundamental mismatch between statistical and economic notions of cyclical fluctuations makes estimation results obtained with preliminary data transformations incredible. The next section quantifies the size of these distortions on estimates of the structural parameters.

3 The problems and their consequences

For illustration, I assume that, in the model of section 2, the preference disturbance has two uncorrelated components: one with unitary autoregressive (AR) coefficient and one with AR coefficient equal to 0.5. This allows me to identify the theoretical non-cyclical component of the four observable variables with the fluctuations generated by the non-stationary component of the preference shock and the theoretical cyclical component with the fluctuations induced by the stationary shocks. Using the parameter values reported in the first column of table 4, I have simulated 1200 data points from the model, discarded 1050 initial observations and passed the experimental data through LT, HP, FOD and BP filters. Figures 4 and 5 show the contribution of the two theoretical components to the log spectrum and to the autocorrelation function of filtered output.

Two features are clear. First, all filters leave considerable power outside the business cycle frequencies (identified by the vertical bars in figure 4). The problem is more evident with LT and FOD but leakages and compressions are present also with HP and BP filters. Second, both theoretical components have power at the business cycle frequencies and, with the chosen parameterization, the theoretical non-cyclical component is as important as the cyclical component. This is very clear in figure 6: the autocorrelation function produced by the two theoretical components is very similar. [1] and [2] have claimed that for LDC countries the trend is the cycle. Figures 5 and 6 show that the problem is relevant for any country. It is only required that the variability of the shocks driving the two components is of similar magnitude.

It is important to stress that the features of figures 5 and 6 do not depend on the assumption that the preference shock has two components: had I assumed that the technology shock has two components similar features would emerge. In other words, what drives the non-cyclical component

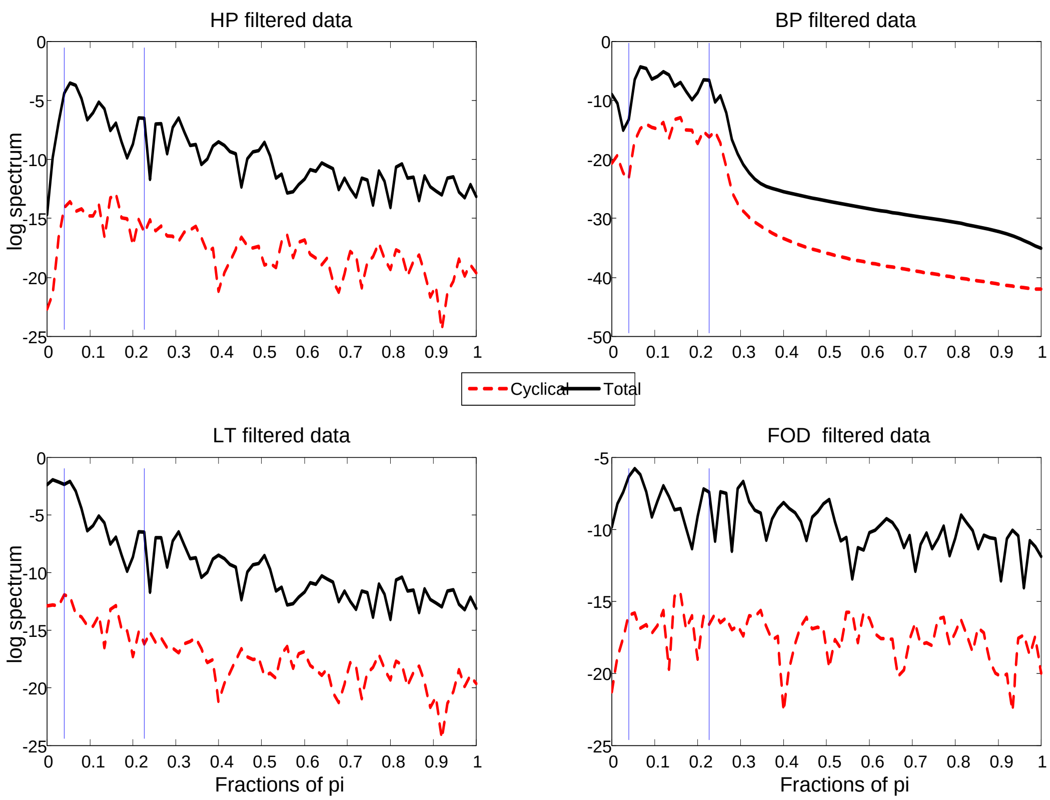

Figure 4: Model based output log-spectra. Vertical bars indicate the frequencies where cycles with 8-32 quarters periodicities are located

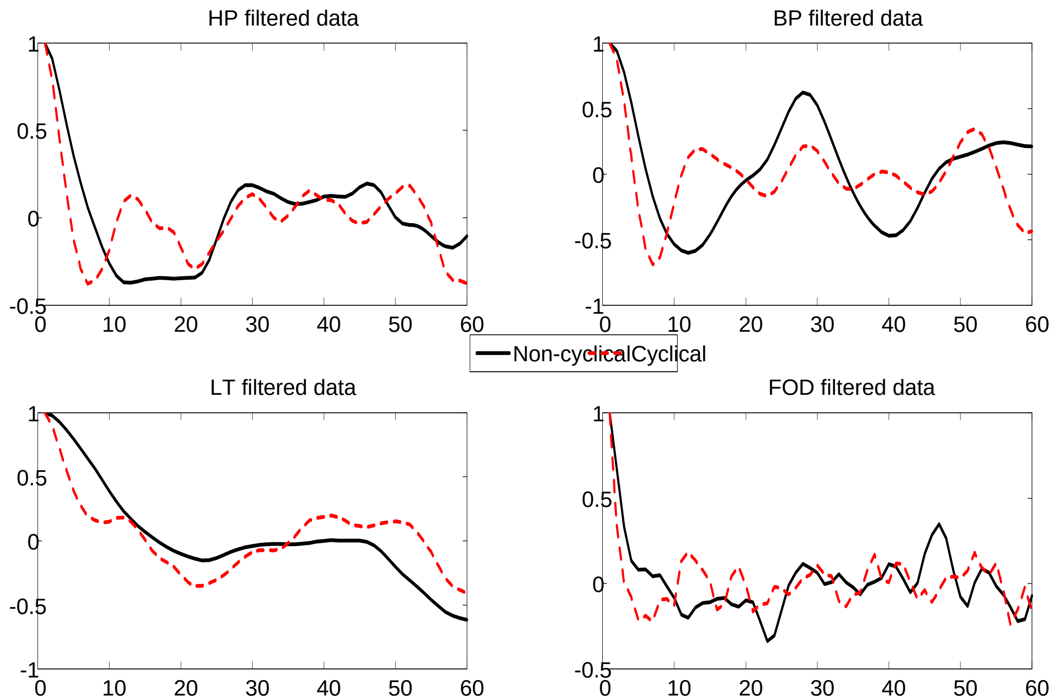

Figure 5: Autocorrelation functions of filtered cyclical and non-cyclical output

is irrelevant for the points I am interested in making here.

To appreciate the distortions that standard data transformation imply, I use LT, HP, BP, and FOD filtered data to estimate the structural parameters of the model in the most ideal situations one could consider. These includes a prior centered at the true parameter vector and initial conditions in the estimation equal to the true parameter vector. Table 4 reports the true parameter values, the posterior median and the posterior standard deviations obtained with each transformation. Clearly, distortions are important. For example, the inverse of the Frisch elasticity σ_n, the lagged interest rate coefficient in the policy rule ρ_r, and the persistence of the preference and the technology shocks ρ_χ, ρ_z are considerably overestimated while the coefficient of relative risk aversion σ_c, and the variance of the preference shocks σ_χ underestimated [1].

To understand why distortions appear, it useful to recall that the posterior distribution of the structural parameters is proportional to the likelihood, given the data, multiplied by the prior. In turn, the log-likelihood can be represented as the sum of three terms $L(\theta|y_t) = [A_1(\theta) + A_2(\theta) + A_3(\theta)|y]$, see [19], where $A_1(\theta) = \frac{1}{\pi}\sum_{\omega_j} \log\det G_\theta(\omega_j)$, $A_2(\theta) = \frac{1}{\pi}\sum_{\omega_j}$ trace $[G_\theta(\omega_j)^{-1}F(\omega_j)]$, $A_3(\theta) = (E(y) - \mu(\theta))G_\theta(\omega_0)^{-1}(E(y) - \mu(\theta))$, $\omega_j = \frac{\pi j}{T}, j = 0, 1, \ldots, T-1$. $G_\theta(\omega_j)$ is the model based spectral density matrix of y_t, $\mu(\theta)$ the model based mean of y_t, $F(\omega_j)$ is the data based spectral density of y_t and $E(y)$ the unconditional mean of y_t. Note that $A_2(\theta)$ and $A_3(\theta)$ are penalty functions: $A_2(\theta)$ sums deviations of the model-based from the data-based spectral density at various frequencies; $A_3(\theta)$, weights deviations of model-based from data-based means, with the spectral density matrix of the model at frequency zero.

Suppose that the actual data is filtered so that frequency zero is eliminated and low frequencies de-emphasized. Then the log-likelihood consists of $A_1(\theta)$ and of $A_2(\theta)^* = \frac{1}{\pi}\sum_{\omega_j}$ trace $[G_\theta(\omega_j)]^{-1}F(\omega_j)^*$, where $F(\omega_j)^* = F(\omega_j)I_{\omega_j}$ and I_{ω_j} is a function describing the effect of the filter at frequency ω_j. Suppose that $I_\omega = I_{[\omega_1,\omega_2]}$, i.e. an indicator function for the business cycle frequencies, as in an ideal BP filter. Then $A_2(\theta)^*$ matters only at business cycle frequencies. Since at this frequencies $[G_\theta(\omega_j)] < F(\omega_j)^*$ (see figure 5), and $A_2(\theta)^*$ and $A_1(\theta)$ enter additively in the log-likelihood function, two types of biases are present in estimates of θ. First, since estimates $\hat{F}(\omega_j)^*$ only approximately capture the features of $F(\omega_j)^*$, $\hat{A}_2(\theta)^*$, the sample version of $A_2(\theta)^*$, has smaller values at business cycle frequencies and a nonzero value at non-business cycle ones. Second, in order to reduce the contribution of the penalty function to the log-likelihood, parameters are adjusted so that $[G_\theta(\omega_j)]$ is close to $\hat{F}(\omega_j)^*$ at those frequencies where $\hat{F}(\omega_j)^*$ is not zero. This is done by allowing fitting errors, (a larger $A_1(\theta)$), at frequencies where $\hat{F}(\omega_j)^*$ is zero - in particular, the low frequencies. Hence, the volatility of the structural shocks will be overestimated (this makes $G_\theta(\omega_j)$ close to $\hat{F}(\omega_j)^*$ at the relevant frequencies), in exchange for misspecifying their persistence.

[1] Distortions in the estimates of α, h and ε occur because these parameters are nearly non-identifiable from the likelihood.

		LT	HP	FOD	BP	Flexible
Parameter	True	Median (s.d.)	Median (s.d.)	Median (s.d.)	Median (s.d.)	Median (s.d.)
σ_c	3.00	1.89 (0.07)	1.89 (0.07)	1.87 (0.07)	2.03 (0.09)	3.26 (0.29)
σ_n	0.70	2.13 (0.08)	2.11 (0.08)	2.15 (0.08)	1.90 (0.08)	0.80 (0.13)
h	0.70	0.58 (0.02)	0.60 (0.02)	0.56 (0.02)	0.69 (0.02)	0.77 (0.04)
α	0.60	0.47 (0.02)	0.46 (0.02)	0.49 (0.02)	0.24 (0.03)	0.41 (0.04)
ϵ	7.00	3.85 (0.13)	3.92 (0.13)	3.46 (0.11)	4.16 (0.13)	6.95 (0.09)
ρ_r	0.20	0.68 (0.03)	0.59 (0.03)	0.43 (0.04)	0.50 (0.03)	0.31 (0.04)
ρ_π	1.20	1.14 (0.04)	1.25 (0.04)	1.25 (0.04)	1.23 (0.04)	1.25 (0.03)
ρ_y	0.05	-0.07 (0.00)	-0.01(0.01)	-0.05 (0.02)	0.23 (0.01)	0.08 (0.10)
ζ_p	0.80	0.81 (0.03)	0.78 (0.03)	0.76 (0.03)	0.89 (0.03)	0.72 (0.02)
ρ_χ	0.50	1.00 (0.03)	1.00 (0.03)	1.00 (0.03)	0.97 (0.03)	0.69 (0.05)
ρ_z	0.80	0.90 (0.03)	0.92 (0.03)	0.91 (0.03)	0.98 (0.03)	0.90 (0.03)
σ_χ	1.12	0.09 (0.01)	0.31 (0.05)	0.61 (0.15)	1.87(0.14)	1.28 (0.03)
σ_z	0.51	0.61 (0.07)	0.30 (0.04)	0.40 (0.05)	0.10 (0.01)	0.69 (0.01)
σ_r	0.10	0.06 (0.01)	0.06 (0.01)	0.06 (0.01)	0.06 (0.01)	0.24 (0.004)
σ_μ	20.60	18.00 (0.74)	18.04 (0.61)	15.89 (0.83)	17.55(0.57)	12.73 (0.04)
σ_χ^{nc}	23.21					

Table 4: Posterior estimates obtained using different filters; σ_χ^{nc} is the standard deviation of the non-cyclical component of the preference shock. Experimental data

Since the volatility and the persistence features of the economy are distorted, agents' decision rules will be altered. Higher perceived volatility, for example, implies distortions in the Frisch elasticity of labor supply and an artificial amplification in the internal features of the model. Inappropriate persistence estimates, on the other hand, imply that perceived substitution and income effects are distorted with the latter typically underestimated. When I_ω is not the indicator function, the derivation of the size and the direction of distortions is more complicated but the same logic applies. Clearly, different I_ω will produce different distortions because income and substitution effects will have different properties.

Since estimates of $F(\omega_j)^*$ are imprecise, even for large T, there are only two situations when estimation biases are small. First, the non-cyclical component has low power at business cycle frequencies - in this case the distortions induced by the penalty function are limited. This occurs when the volatility of the shocks driving the non-cyclical component is considerably smaller than the volatility of the shocks driving the cyclical component. Second, the prior limits the distortions induced by the penalty function. While priors for DSGE parameters are typically tight and this reduces somewhat the ability to trade-off distortions in various portions of the log-likelihood, it is unlikely that biases are wiped out since priors are not designed with such a scope in mind.

While very popular in estimation literature, one could also conceive to fit a filtered version of the model to the filtered data, as it is done e.g. in [22] or [32]. To understand how parameter estimates are affected by this transformation note that, in this case, the log-likelihood is composed of $A_1(\theta)^* = \frac{1}{\pi}\sum_{\omega_j}\log|G_\theta(\omega_j)I_{\omega_j}|$ and $A_2(\theta)$ - since the actual and simulated data are filtered in the same way, the filter does not affect the penalty function. Suppose that $I_\omega = I_{[\omega_1,\omega_2]}$. Then $A_1(\theta)^*$ matters only at business cycle frequencies while the penalty function is present at all frequencies. Therefore, parameter estimates are adjusted so as to reduce the misspecification at all frequencies of the spectrum. Since the penalty function is more important at the low frequencies, parameters are adjusted to make $[G_\theta(\omega_j)]$ close to $\hat{F}(\omega_j)$ at those frequencies. Thus, the log-likelihood is willing to incur large fitting errors at frequencies where $\hat{F}(\omega_j)$ does not differ much from $G_\theta(\omega_j)$ - in particular, the medium and high frequencies. Consequently, the volatility of the shocks will be generally underestimated in exchange for overestimating their persistence and, somewhat paradoxically, this procedure implies that the low frequency components of the data are those that matter most for estimation. Cross frequency distortions imply that agents think they are living in an economy which differs substantially from the true one. For example, since less noise is perceived, agents decision rules will imply a higher degree of predictability of simulated time series, and higher perceived persistence implies that perceived substitution and income effects are distorted with the latter overestimated.

4 The alternative methodology

One solution to the problems I have highlighted is to build a non-cyclical component directly into the DSGE model. I have mentioned in the introduction a few reasons for why researchers may be reluctant to do so. There may also be practical statistical and specification concerns which may not make the approach viable (What time series features should the non-cyclical component have? Should it be deterministic or stochastic? Should it be correlated with the cyclical component or not? What economic mechanism drives its fluctuations? What happens to estimates if the structure of the non-cyclical component is misspecified? What if there are breaks?). Some progress in addressing these latter issues have been reported in [18] and [15].

Rather than augmenting the model with an arbitrary non-cyclical component and conditioning estimation on the chosen specification, making the analysis vulnerable to specification errors, I will use a flexible setup, in the spirit of [20], where the cyclical DSGE structure is unchanged but a link is build from the model to the raw data which permits cyclical and non-cyclical components to jointly appear at all frequencies of the spectrum. Let the (log)-linearized solution of a DSGE model be of the form:

$$x_{2t} = RR(\theta)x_{1t-1} + SS(\theta)\epsilon_t \tag{8}$$

$$x_{1t} = PP(\theta)x_{1t-1} + QQ(\theta)\epsilon_t \tag{9}$$

where $PP(\theta), QQ(\theta), RR(\theta), SS(\theta)$ are functions of the structural parameters $\theta = (\theta_1, \ldots, \theta_k)$, $x_{1t} \equiv (\log \tilde{x}_{1t} - \log \bar{x}_1)$ includes the states and the predetermined variables, $x_{2t} = (\log \tilde{x}_{2t} - \log \bar{x}_2)$ all other endogenous variables, ϵ_t the shocks and $\bar{x}_2, \bar{x}_1$ are the steady states of $\tilde{x}_{2t}$ and $\tilde{x}_{1t}$.

Let $x_t^m(\theta) = W[x_{1t}, x_{2t}]'$, be an $N \times 1$ vector where W is a selection matrix picking out of x_{1t} and x_{2t} those variables which are observable and/or interesting from the point of view of the researcher. Let $x_t^d = \log \tilde{x}_t^d - E(\log \tilde{x}_t^d)$ be the log demeaned vector of observables. I assume that

$$x_t^d = c + x_t^{nc} + x_t^m(\theta) + u_t \tag{10}$$

where $c = \log \bar{x}^m(\theta) - E(\log \tilde{x}_t^d)$, x_t^{nc} is the non-cyclical component, u_t is a iid $(0, \Sigma_u)$ (measurement) noise and x_t^{nc}, x_t^m and u_t are mutually orthogonal. Furthermore, I assume that

$$\begin{aligned} x_t^{nc} &= x_{t-1}^{nc} + \bar{x}_{t-1} + e_t \qquad && e_t \sim \; iid\ (0, \Sigma_e) \\ \bar{x}_t &= \bar{x}_{t-1} + v_t && v_t \sim \; iid\ (0, \Sigma_v) \end{aligned} \tag{11}$$

The specification in (11) is flexible and can account for several time series patterns in x_t^{nc}. For example, if both Σ_e and Σ_v are diagonal, $\Sigma_{v_i} > 0$ and $\Sigma_{e_i} = 0$, $\forall i$, x_t^{nc} is a vector of I(2) processes while if $\Sigma_{v_i} = 0$, and $\Sigma_{e_i} > 0$, $\forall i$, x_t^{nc} is a vector of I(1) processes. Furthermore, if $\Sigma_{v_i} = \Sigma_{e_i} = 0$, $\forall i$, x_t^{nc} is deterministic, while if both $\Sigma_{v_i} > 0$ and $\Sigma_{e_i} > 0$ and $\frac{\Sigma_{v_i}}{\Sigma_{e_i}}$ is large, x_{it}^{nc} is "smooth" and nonlinear, $i = 1, 2, \ldots, N$. Hence, (11) nests, as special cases, the structures which are typically thought to motivate the use of the filters considered in the previous sections.

Given (8)-(9)-(10) and (11), I let the data endogenously select the specification for the non-cyclical component which is more appropriate for each series and this is done jointly with the estimation of the structural parameters θ. In (11) I have assumed that Σ_v^2 and Σ_e^2 are general matrices. However, one can impose further structure by assuming that they are either diagonal (so that the non-cyclical component is series specific) or that they are of reduced rank (so that the non-cyclical component is common across series) and test various specifications using, e.g., marginal likelihood comparisons (see [16]).

There are at least two advantages the suggested specification has. On the one hand, it is not necessary to take a stand on the time series properties of the non-cyclical component and on the choice of filter to tone down its importance. This shields researchers from important specification errors. Second, as I will show below, the cyclical component extracted with this approach is not located at particular frequencies of the spectrum. In fact, by construction, all the components in (10) may have power at each frequency.

In (10) I have assumed, cyclical and non-cyclical fluctuations are driven by independent shocks. While such a setup may appear to be restrictive, it is easy to show that the specification is observationally equivalent to one where the non-cyclical and the cyclical components are correlated. For example, the specification

$$
\begin{aligned}
x_t^d &= x_t^{nc*} + x_t^{m*}(\theta) + u_t & (12)\\
x_t^{nc*} &= x_t^{nc} + \bar{x}_t + y^m(\theta) & (13)\\
x_t^{nc} &= x_{t-1}^{nc} + e_t & (14)\\
\bar{x}_t &= \bar{x}_{t-1} + v_t & (15)\\
x_t^{m*}(\theta) &= x_t^{m\dagger}(\theta) + A(\theta)C(\theta)\bar{x}_t & (16)\\
x_t^{m\dagger}(\theta) &\equiv \tilde{x}_t^m(\theta) - x^m(\theta) - A(\theta)C(\theta)\bar{x}_t = C(\theta)x_t^{m\dagger}(\theta) \equiv C(\theta)(\tilde{x}_t^m(\theta) - x^m(\theta) - A(\theta)\bar{x}_t) & (17)\\
x_t^{m\dagger}(\theta) &= A(\theta)x_{t-1}^{m\dagger}(\theta) + B(\theta)\epsilon_t & (18)
\end{aligned}
$$

which makes y_t^{nc*} and $y_t^{m*}(\theta)$ correlated, is indistinguishable from the point of view of the observed data from the specification I suggest.

4.1 Estimation

Estimation of the hierarchical structure (9)-(8), (11) and (10) can be carried out with both classical and Bayesian methods. In fact, equations (9)-(8), (11) and (10) can be cast into the state space system of the form

$$
\begin{aligned}
s_{t+1} &= Fs_t + G\omega_{t+1} & (19)\\
y_t &= Hs_t & (20)
\end{aligned}
$$

where $F = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & PP & QQ \\ 0 & 0 & 0 & 0 & NN \end{pmatrix}$, $G = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$, $H = \begin{pmatrix} 1 & 0 & 1 & RR & SS \end{pmatrix}$,

$s_{t+1} = \begin{pmatrix} x_t^{nc} & \bar{x}_t & u_t & x_{t-1}^m(\theta) \end{pmatrix}$, $\omega_{t+1} = (e_t, v_t, u_t, \epsilon_{t+1})$ and Σ_ω is block diagonal. Hence, the likelihood of the system can be computed with a modified version of the Kalman filter, which takes into account the possibility of diffuse initial observations, for a given ϑ and maximized using standard tools.

When a Bayesian approach is preferred, one can obtain the non-normalized posterior distribution of ϑ, using standard MCMC tools. For example, estimates presented below are obtained with a Random Walk Metropolis algorithm where, given initial ϑ_{-1}, a Ω, and a prior $g(\vartheta)$, candidate draws are obtained from $\vartheta_* = \vartheta_{-1} + \upsilon$ and υ is distributed $t(0, \kappa * \Omega, 5)$ where κ is a tuning parameter and the draw accepted if the ratio $\chi_* = \frac{\breve{g}(\vartheta_*|y)}{\breve{g}(\vartheta_{-1}|y)}$, where $\breve{g}(\vartheta_i|y) = g(\vartheta_i)\mathcal{L}(y|\vartheta_i)$, $i = *, -1$, and $\mathcal{L}(y|\vartheta_i)$ is the likelihood of ϑ_i, exceeds a uniform random variable. Iterated a large number of times, for κ appropriately chosen, the algorithm ensures that the limiting distribution of ϑ is the target distribution (see e.g. [5], Ch. 9).

4.2 A comparison with existing literature

To the best of my knowledge, the literature has not yet addressed the problems I discuss in this paper. In a seminal work, [13] considers estimating structural parameters when the data is trending and the model has little or nothing to say about the properties of the trends. He builds a useful taxonomy of cases, shows the distortions that incorrect assumptions have on the estimates of the parameters and investigates whether certain estimation methods may downsize the importance of specification errors. I share with Cogley the point of view that economic theory has not much to say about non-cyclical fluctuations. However, rather than distinguishing between trend stationary or difference stationary fluctuations, and attempting to robustify inference, I am concerned with the generic mismatch between statistical and model-based concept of cyclical fluctuations and in designing a procedure which resolves the problem without taking a stand on the time series properties of the non-cyclical component.

[18] extend Cogley's study suggesting how to robustify structural estimation, when the trend specification, arbitrarily built into the model, is potentially misspecified. The suggested approach shares with Gorodnichenko and Ng the idea of jointly estimating structural and auxiliary parameters without specifying the DGP of the data. The two papers differ however in several respects. First, they use minimum distance estimators of the parameters while I use likelihood based estimators. Minimum distance estimators of DSGE parameters are subject to severe identification problems which limits the credibility of the inferential conclusions one draws (see [9]). Second, rather than assuming an arbitrary trend for one of the shocks, I assume that the DSGE model is build to explain only the cyclical component of the data - a much more common assumption in macroeconomics - and link model and observables through a flexible specification. Third, while minimum distance estimators enjoy standard properties only if the data is stationary, my approach works regardless of the time series properties of the raw data.

[1] and [2] have recently pointed out that in emerging markets, variations in trend growth are as important as cyclical fluctuations in explaining the dynamics of macroeconomic variables. While the first paper is primarily interested in characterizing differences between emerging and developing economies and in finding a common mechanism to explain the evidence, the latter is concerned with the misuse of cyclical DSGE models in policy analyses for LDC countries. This paper shows that the problems they highlight are generic and that it is possible to estimate cyclical models without imposing controversial assumptions about the nature of the non-cyclical component.

Finally, [15] investigate the distortions introduced by infrequent switches in trend growth on the estimated parameters of a DSGE model and propose an estimation approach which can deal with these breaks within a standard state space formulation.

Parameter	Small variance			Large variance		
	True	Median	Standard deviation	True	Median	Standard deviation
σ_c	3.00	3.26	(0.40)	3.00	3.26	(0.29)
σ_n	0.70	0.54	(0.14)	0.70	0.80	(0.13)
h	0.70	0.55	(0.04)	0.70	0.77	(0.04)
α	0.60	0.19	(0.03)	0.60	0.41	(0.04)
ϵ	7.00	6.19	(0.07)	7.00	6.95	(0.09)
ρ_r	0.20	0.16	(0.04)	0.24	0.31	(0.04)
ρ_π	1.30	1.30	(0.04)	1.30	1.25	(0.03)
ρ_y	0.05	0.07	(0.03)	0.05	0.08	(0.10)
ζ_p	0.80	0.78	(0.04)	0.80	0.72	(0.02)
ρ_χ	0.50	0.53	(0.02)	0.50	0.69	(0.05)
ρ_z	0.80	0.71	(0.03)	0.80	0.90	(0.03)
σ_χ	1.12	1.29	(0.01)	1.12	1.28	(0.03)
σ_z	0.51	0.72	(0.02)	0.51	0.69	(0.01)
σ_r	0.10	0.21	(0.004)	0.10	0.24	(0.004)
σ_μ	20.60	15.86	(0.06)	20.60	12.73	(0.04)
σ_χ^{nc}	3.21			23.21		

Table 5: Posterior estimates using flexible specification. σ_χ^{nc} is the standard error of the shock to the non-cyclical component of the preference shock. Experimental data.

4.3 The procedure in a controlled experiment

To show the properties of the approach, I simulated data from the model of section 2 assuming again that the preference shock has two components, a nonstationary one and a stationary one. I then estimate the parameters of the DSGE model and those of the flexible non-cyclical part using the suggested specification and a Bayesian approach. The priors for the non-structural parameters are in the lower part of table 1. I use distributions with relatively large standard deviations to allow the likelihood to explore a wide portion of the parameter space without being downweighted by the prior. The median and the standard deviation of the posterior of each parameter are in the last column of table 4 labelled "Flexible". While it is clear that the vector of posterior medians is not exactly on top of the vector of true parameters, many of the distortions that standard procedures produced are reduced or have disappeared - see e.g. estimates of the relative risk aversion coefficient, of the inverse of the Frisch elasticity and of the elasticity of substitution among varieties. Finally, the relative magnitude of the variance of various shocks and their persistence is better estimated. Hence, the estimated and the true economy display decision rules that are roughly similar.

Table 5 shows the location and the spread of the posterior of structural parameters is roughly invariant to the magnitude of the variance of the shock driving the non-cyclical component, while

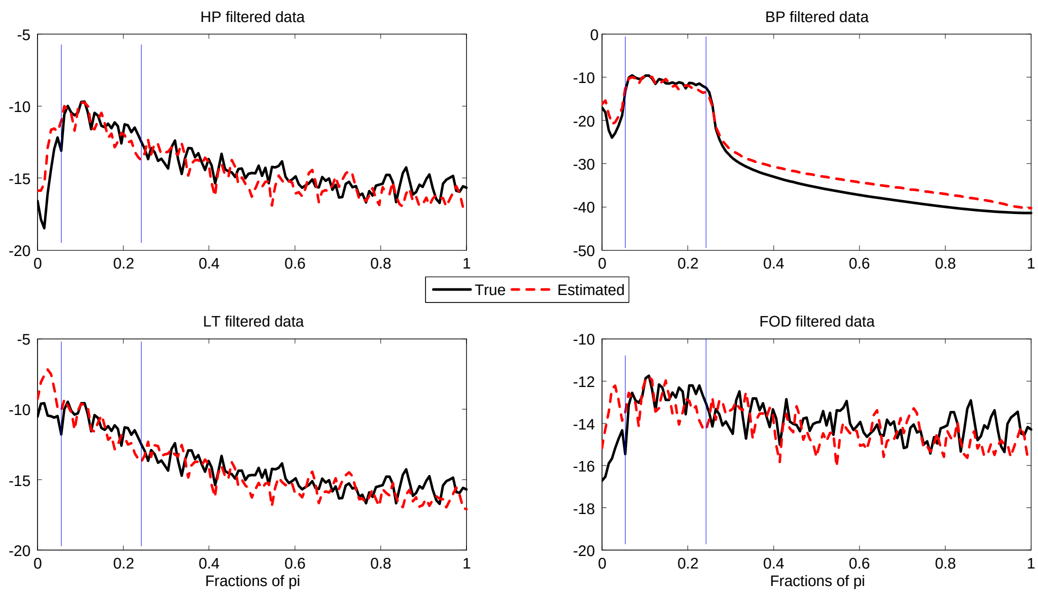

Figure 6: Cyclical output log spectra, true and estimated. Vertical bars indicate the frequencies where cycles with 8-32 quarters periodicities are located

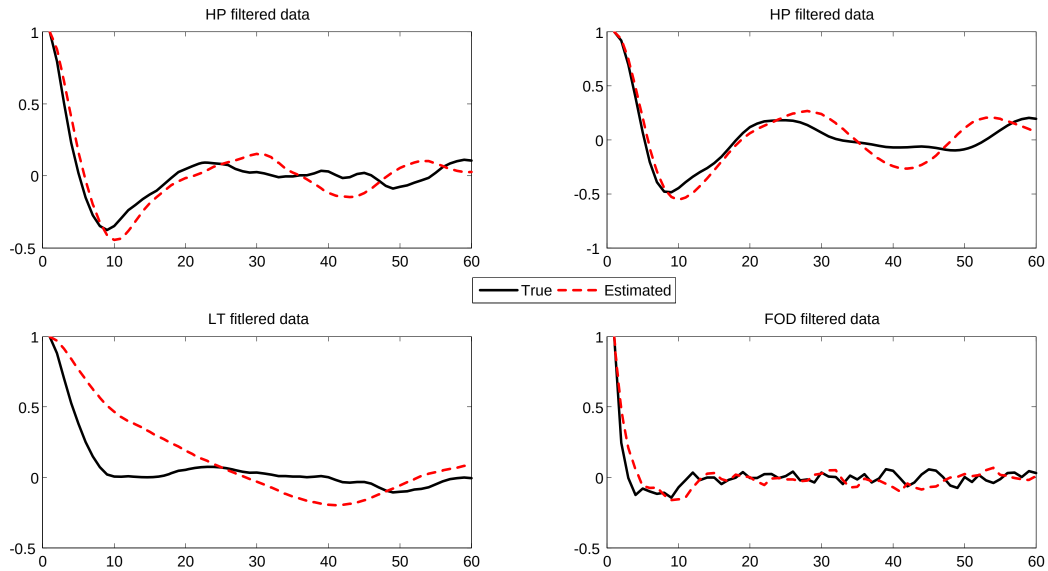

Figure 7: Autocorrelation function of filtered cyclical output, true and estimated.

estimates obtained with standard filtering change considerably with the magnitude of the variance of this shock. Hence, the flexible link I have specified adapts to capture different features of the non-cyclical component of the data.

With the true parameter vector and the estimated median vector obtained with the baseline variance of the non-cyclical shock, I then compute impulse responses to various shocks, which are presented in figure 8, and the model-based cyclical component. I then filter the latter with the same 4 filters used in figures 4 and 5, and plot the log spectrum and the autocorrelation function of cyclical output in figures 6 and 7. Both figures 6 and 7 show that the procedure is successful in recovering the time series features of the true cyclical component and figure 8 indicates that the conditional dynamics in response to each of the shocks are also reasonably matched. As anticipated, figure 6 highlights that both the true and the estimated cyclical components have power at all frequencies of the spectrum.

In sum, the approach I suggest can produce better estimates of the cyclical components. Hence, structural estimates and inference are likely to be less prone to "mismatch" distortions.

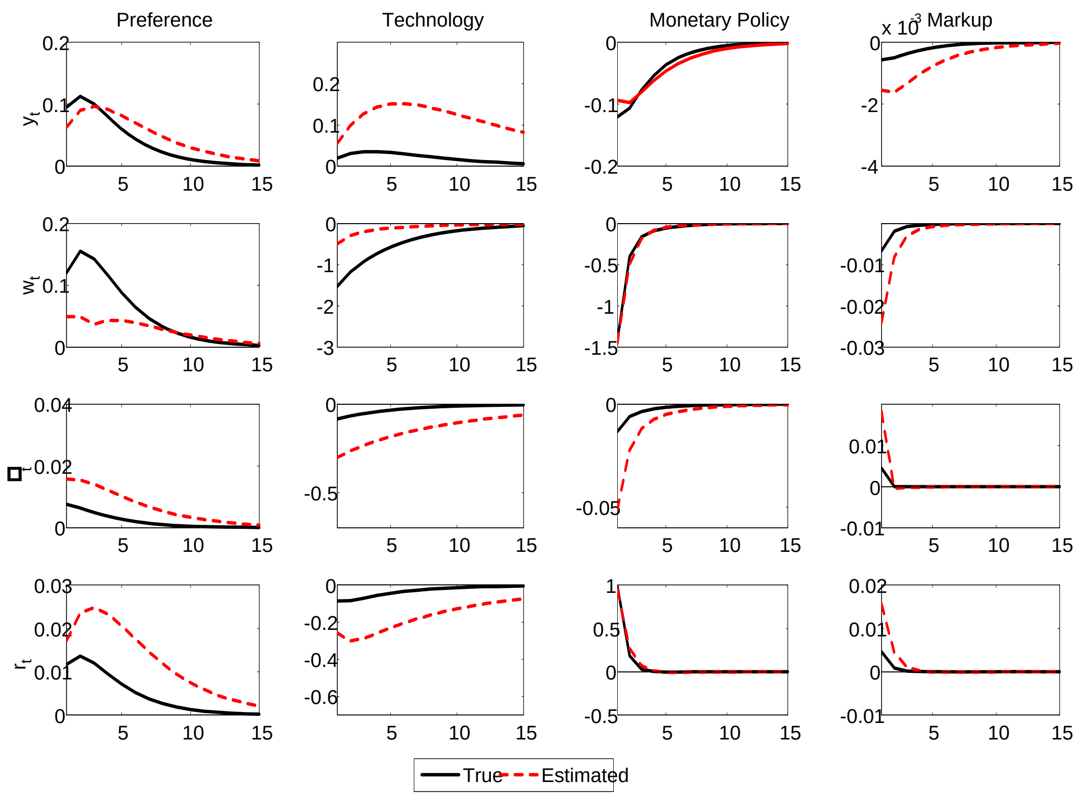

Figure 8: Model based impulse responses, true and estimated.

5 Economic Inference

In this section I show that the proposed approach also produces significantly different interpretations of the economic phenomena. For this reasons, I examine interesting functions of the parameters in the toy model I have been working with so far and in a more standard medium scale DSGE model, which has been widely used in the recent literature.

5.1 The slope of the Phillips curve and policy activism

I have argued that the posterior differences reported in table 2 matter for inference. Table 3 showed that standard approaches give contradictory information both about the location and the spread of the posterior distribution of the Phillips curve trade-off and the short run inflation coefficient in the policy rule in actual data. Since the approach of this paper captures much better the features of the true cyclical component when applied to experimental data, one may be curious as to how the posterior distributions of these functions of the structural parameters look like in our case.

The last column of table 3 shows that the posterior distribution of the Phillips curve trade-off is estimated to be very spread out and there is considerable posterior uncertainty about the location of this distribution. Hence, while it is hard to say by how much a change in the marginal costs will influence inflation, we can safely exclude that the slope coefficients is smaller than 0.5. Comparing across columns, one can see that standard filtering approaches grossly underestimate both the location and the spread of the distribution of this slope coefficient.

The posterior distribution of the short run inflation coefficient in the monetary policy rule is relatively tight, it is centered around 1.6 and displays mild skewness. Contrary to what was obtained with standard transformation, I can exclude with high probability that there is any mass in the region below 1.0. Interestingly, the shape of the posterior resembles the one produced by a FOD filter, but the spread is smaller and the probability that the coefficient is less than 1.2 considerably reduced. Thus, the flexible link I have used seems to prefer for this data set a specification for the non-cyclical component where the variance of v_t is small and the variance of e_t is large.

In a number of closely related papers [24], [25] and [26] Orphanides has stressed the importance of using real time data to assess the stance of monetary policy and argued that distortions may emerge when revised data are used. For example, it may appear that policy looks weak, as far as inflation responses are concerned, or too active, as far as output gap responses are concerned when revised data is used but not with real time data. This could occur because output data is imprecisely measured and revisions are considerable; measures of potential output suffer standard end-of-the-period problems; local trends are difficult to detect and only with hindsight patterns present in the data may be clarified.

Given that the approach may capture the features of the various components of the data better than standard methods, I want to see what the approach tells us about the features of the monetary policy rule during the Great Inflation of the 1970s and the return to norm of the 1980s and 1990s relative to the characterization offered by standard transformations.

In figure 9 I plot the posterior distribution of the policy activism parameter $\frac{\rho_y}{\rho_\pi - 1}$ obtained when the data is linearly detrended or HP filtered prior to estimation and when the alternative approach is employed for the samples running from 1964:1 to 1979:4 and 1984:1 to 2005:4. A few features of the plots deserve some comment. While there is a shift to the left of the posterior of the policy activism parameter in the second sample when HP filtered data is used, the two distributions overlap considerably and the posterior median is the statistically unchanged (equal to -0.23 in the first sample and -0.33 in the second). This left shift of the posterior distribution is absent when the data is linearly detrended prior to estimation and, if anything, the median of the posterior in the second sample increases from -0.38 to 0.12, even though the change is statistically insignificant. Note that with both LT and HP filtered data, the posterior distributions in both samples are relatively tight. This is clearly not the case when the alternative approach is used. In this case, the uncertainty surrounding the median estimate is substantial and although the median of the posterior marginally decreases (from 0.22 to 0.l6) when we move from the first to the second sample, the change is statistically insignificant.

In sum, because standard transformations fails to account for the uncertainty present in the specification of the non-cyclical component, they generally give a much sharper characterization of the properties of the data than otherwise would be. Overall, the evidence for a structural break in the conduct of monetary policy seems to be weak.

5.2 Sources of output and inflation fluctuations

In standard medium scale cyclical DSGE models, like the one employed by [31] and [32], important macroeconomic variables are primarily driven by markup shocks. Since these shocks are an unlikely source of cyclical fluctuations, [10] have argued that misspecification is likely to be present (see [22] for an alternative interpretation). Researchers working with models of this type use filtering devices to fit the model to the data (as in [31]) or arbitrarily build a non-cyclical component in the model (as in [32]), a stochastic one in [22]) and use model-consistent data transformations to estimate the structural parameters. What would the approach of this paper tell us about the sources of cyclical fluctuations in output and inflation relative to standard transformations? To answer this question, I take the same model and the same data set used in [32] but I modify the setup in four ways. First, I do not allow MA terms in price and wage markup disturbances: all shocks have the standard AR(1) structure. Second, the model is solved in deviations from the steady states, rather than from the flexible price equilibrium, which is the most common setup. Third, no rescaling of the shocks is performed. Fourth, the Taylor rule does not include a term concerning output growth,

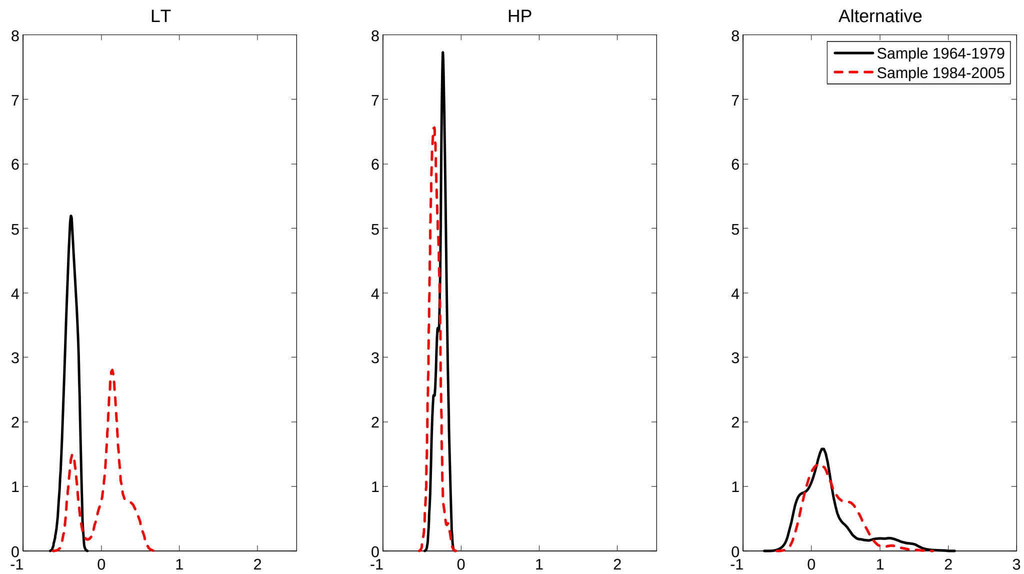

Figure 9: Posterior distributions of policy activism parameter, different samples.

again a more standard approach.

Table 6 suggests some interesting patterns. When either a linear trend is removed from the variables the forecast error variance of output at the five years horizon is indeed primarily driven by price markup shocks, with a considerably smaller contribution of investment specific and preference shocks. For inflation, price markup shocks account for almost 90 percent of the forecast error variability at the five years horizon. When the model is instead fitted to the growth rates of the variables, price markup shocks account for over 98 percent of the variability of both output and inflation at the five years horizon. When the flexible bridge I suggest in this paper is used and the non-cyclical component of real variables is restricted to have the same structure (there are only two variances controlling the non-cyclical component of output, consumption, investment and a measurement error for each equation) the picture is considerably different. Output fluctuation at the five year horizon are driven almost entirely by preference disturbances. On the other hand, inflation fluctuations are jointly accounted for by wage markup, TFP and price markup disturbances. Hence, while it is still true that such a model is less structural than one would like to assume since " black box" disturbances dominate, the role of markup shocks is considerably reduced, at least as far as output is concerned, when one uses the flexible link proposed in this paper.

	LT		FOD		Alternative	
	Output	Inflation	Output	Inflation	Output	Inflation
TFP shocks	0.01	0.04	0.00	0.01	0.01	0.19
Gov. expenditure shocks	0.00	0.00	0.00	0.00	0.00	0.02
Investment shocks	0.08	0.00	0.00	0.00	0.00	0.05
Monetary policy shocks	0.01	0.00	0.00	0.00	0.00	0.01
Price markup shocks	0.75(*)	0.88(*)	0.91(*)	0.90(*)	0.00	0.21
Wage markup shocks	0.00	0.01	0.08	0.08	0.03	0.49(*)
Preference shocks	0.11	0.04	0.00	0.00	0.94(*)	0.00

Table 6: Variance decomposition at the 5 years horizon. Estimates are obtained with the median value of the posterior of the parameters. A (*) indicates that the 68 percent highest credible set is entirely above 0.10. The model and the data set used are the same (as in [32]).

6 Conclusions

I have argued that estimating cyclical DSGE models on either transformed or filtered data is theoretically incorrect and may lead to serious distortions in the estimates of the structural parameters. There are two reasons for this. First, the transformed/ filtered data imperfectly measures fluctuations appearing at frequencies corresponding to cycles of 8-32 quarters. Second, the cyclical component that a model produces is not entirely located at these frequencies and, viceversa, the non-cyclical component may have important power at these frequencies. The consequences of these two specification errors could be important both statistically and economically because income and substitution effects are distorted, the volatilities and persistence of the shocks over or underestimated and the decision rules of the agents altered.

I propose an alternative methodology which allows researchers to estimate cyclical DSGE models using raw data. There are several advantages of the approach. First, there is no need to build non-cyclical components directly into a DSGE model nor to worry about their exact time series features. Second, the procedure eliminates by construction the first source of measurement error and considerably reduces the second because the spectrum of the data is endogenously split into cyclical and non-cyclical parts. Finally, the specification allows us to reinterpret estimated cyclical and non-cyclical components if economic inference requires them to be correlated.

I have shown both the distortions induced by standard filtering approaches and the properties of the alternative methodology using data generated from a simple New Keynesian model and compared the posterior distribution of interesting economic quantities in the actual data. Clearly, the problems I highlight are general and could be very important in a variety of situation. Work in progress indicates, for example, that the role of money in the transmission of cyclical fluctuations, a topic recently addressed with DSGE models by [21], could be severely distorted by the use of preliminary filtering approaches. In fact, the cyclical component generated by, say, a New Keynesian model with or without a role for money is considerably different and, at cyclical frequencies, non-cyclical shocks could potentially matter. I therefore plan to study in the near future a number of interesting economic questions with the methodology I have outlined in this paper.

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Appendix (not intended for publication)

A. The basic DSGE model

The bundle of goods consumed by the representative household is

$$C_t = \left(\int_0^1 C_t(j)^{\frac{\epsilon_t - 1}{\epsilon_t}} dj \right)^{\frac{\epsilon_t}{\epsilon_t - 1}} \tag{21}$$

where $C_t(j)$ is the consumption of the good produced by firm j and ϵ_t the elasticity of substitution between varieties. Maximization of the consumption bundle, given total expenditure, leads to

$$C_t(j) = \left(\frac{P_t(j)}{P_t} \right)^{-\epsilon_t} C_t \tag{22}$$

where $P_t(j)$ is the price of the good produced by firm j. Consequently, the price deflator is $P_t = \left(\int_0^1 P_t(j)^{1-\epsilon_t} dj \right)^{\frac{1}{1-\epsilon_t}}$ and $P_t C_t = [\int_0^1 P_t(j) C_t(j) dj]$.

The representative household chooses sequences for consumption and leisure to maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[X_t \frac{1}{1-\sigma_c} (C_t - hC_{t-1})^{1-\sigma_c} - \frac{1}{1+\sigma_n} N_t^{1+\sigma_n} \right] \tag{23}$$

where X_t is an exogenous utility shifter following an AR(1) in logs:

$$\chi_t = \rho_\chi \chi_{t-1} + \epsilon_t^\chi \tag{24}$$

where $\chi_t = \ln X_t$ and $\epsilon_t^\chi \sim N(0, \sigma_\chi^2)$. The household budget constraint is

$$P_t C_t + b_t B_t = B_{t-1} + W_t N_t \tag{25}$$

where B_t are one-period bonds with price b_t, W_t is nominal wage and N_t is hours worked.

There is a continuum of firms, indexed by $j \in [0, 1]$, each of which produces a differentiated good. The common technology is:

$$Y_t(j) = Z_t N_t(j)^{1-\alpha} \tag{26}$$

where Z_t is an exogenous productivity disturbance following an AR(1) in log,

$$z_t \quad = \rho_z z_{t-1} + \epsilon_t^z \tag{27}$$

where $z_t = \ln Z_t$ and $\epsilon_t^z \sim N(0, \sigma_z^2)$. Each firm resets its price with probability $1 - \zeta_p$ in any t, independently of time elapsed since the last adjustment. Therefore, aggregate price dynamics are

$$\Pi_t^{1-\epsilon_t} = \zeta_p + (1 - \zeta_p)(P_t^* / P_{t-1})^{1-\epsilon_t} \tag{28}$$

A reoptimizing firm chooses the P_t^* that maximizes the current value of discounted profits

$$\max_{P_t^*} \sum_{k=0}^{\infty} \zeta_p^k E_t Q_{t,t+k} \left[P_t^* Y_{t+k|t} - TC_{t+k}(Y_{t+k|t}) \right] \tag{29}$$

subject to the sequence of demand constraints

$$Y_{t+k|t} = \left(\frac{P_t^*}{P_{t+k}} \right)^{-\epsilon_{t+k}} Y_{t+k} \tag{30}$$

$k = 0, 1, 2, ...$ where $Q_{t,t+k} \equiv \beta^k (C_{t+k}/C_t)(P_t/P_{t+k})$, $TC(.)$ is the total cost function, and $Y_{t+k|t}$ denotes output in period $t+k$ for a firm that reset its price at t.

Finally, the monetary authority sets the nominal interest rate according to

$$r_t = \rho_r r_{t-1} + (1 - \rho_r)(\rho_\pi \pi_t + \rho_y gdp_t) + \epsilon_t^{ms} \tag{31}$$

where $\epsilon_t^{ms} \sim N(0, \sigma_{ms}^2)$.

The first order conditions of the optimization problems are:

$$\begin{aligned}
0 &= X_t (C_t - hC_{t-1})^{-\sigma_c} - \lambda_t && (32)\\
0 &= -N_t^{-\sigma_n} - \lambda_t \frac{W_t}{P_t} && (33)\\
1 &= E_t \left[\beta \frac{\lambda_{t+1}}{\lambda_t} \frac{P_{t+1}}{P_t} R_t \right] && (34)\\
0 &= \sum_{k=0}^{\infty} \zeta_p^k E_t Q_{t,t+k} Y_{t+k|t} \left[P_t^* - \mathcal{M}_{t+k} MC_{t+k|t}^n) \right] && (35)
\end{aligned}$$

where λ_t is the Lagrangian multiplier associated with the consumer budget constraint, $R_t \equiv 1 + i_t = 1/b_t$ is the gross nominal rate of return on bonds, $MC^n(.)$ are nominal marginal cost and

$$\mathcal{M}_t = \mu e^{\epsilon_t^\mu} \tag{36}$$

where $\epsilon_t^\mu \sim N(0, \sigma_\mu^2)$ and μ is the steady state markup.

Market clearing requires

$$\begin{aligned}
Y_t(j) &= C_t(j) && (37)\\
N_t &= \int_0^1 N_t(j) dj && (38)
\end{aligned}$$

and letting the aggregate output be $GDP_t \equiv \left(\int_0^1 Y_t(j)^{\frac{\epsilon_t - 1}{\epsilon_t}} dj \right)^{\frac{\epsilon_t}{\epsilon_t - 1}}$ we have $C_t = GDP_t$.

B. The medium scale DSGE model

We only briefly sketch the log-linearized conditions used since the non-linear equations and their transformations are fully described in the appendices of [32] and [22].

(a): The variables of the model

Label	Definition
y_t	: output
c_t	: consumption
i_t	: investment
q_t	: Tobin's q
k_t^s	: capital services
k_t	: capital
z_t	: capacity utilization
r_t	: real rate
μ_t^p	: price markup
π_t	: inflation rate
μ_t^w	: wage markup
N_t	: total hours
w_t	: real wage rate
R_t	: nominal rate

(b): The parameters of the model

Label	Definition
σ_c	elasticity of intertemporal substitution
σ_l	elasticity of labor supply with respect to real wages
h	habit persistence parameter
δ	depreciation rate
$\phi_p - 1$	share of fixed costs in production
χ	steady state elasticity of capital adjustment cost function
ψ	positive function of the elasticity of capital utilization adjustment costs function.
α	share of capital services in production
γ_p	price indexation parameter
ζ_p	price stickiness parameter
ϵ_p	curvature of good market aggregator
γ_w	wage indexation parameter
ζ_w	wage stickiness parameter
ϵ_w	curvature of labor market aggregator

Label	Definition
λ_r	interest smoothing parameter
λ_π	inflation parameter
λ_y	output parameter
gy	government expenditure to output ratio
ky	steady state capital output ratio
$r_* = \beta^{-1}$	steady state rental rate
w_*	steady state real wage rate
N_*/C_*	steady state hours to consumption ratio

(c): The equations of the model (in deviation from steady states)

$y_t = (1 - gy - \delta\ ky)c_t + \delta\ ky\ i_t + r_*\ ky\ z_t + g_t$	(E.1)
$c_t = \frac{h}{1+h} E_t c_{t+1} + \frac{h}{1+h} c_{t-1} - \frac{(\sigma_c - 1) w_* N_*/C_*}{(1+h)\sigma_c}(N_t - E_t N_{t+1}) - \frac{1-h}{(1+h)\sigma_c}(R_t - E_t \pi_{t+1} + e_t^b)$	(E.2)
$i_t = \frac{\beta}{1+\beta} E_t i_{t+1} + \frac{1}{1+\beta} x_{t-1} + \frac{\chi^{-1}}{1+\beta} q_t + e_t^i$	(E.3)
$q_t = \beta(1-\delta) E_t q_{t+1} + (1 - \beta(1-\delta)) E_t r_{t+1} - (R_t - E_t \pi_{t+1} + e_t^b)$	(E.4)
$y_t = \phi_p(\alpha k_t^s + (1-\alpha) N_t + e_t^a)$	(E.5)
$k_t^s = k_{t-1} + z_t$	(E.6)
$z_t = \frac{1-\psi}{\psi} r_t$	(E.7)
$k_{t+1} = (1-\delta)\ k_t + \delta\ i_t + \delta\ (1+\beta)\ \psi\ e_t^i$	(E.8)
$\mu_t^p = \alpha(k_t^s - N_t) + e_t^a - w_t$	(E.9)
$\pi_t = \frac{\beta}{1+\beta\gamma_p} E_t \pi_{t+1} + \frac{\gamma_p}{1+\beta\gamma_p} \pi_{t-1} - T_p \mu_t^p + e_t^p$	(E.10)
$r_t = -(k_t - N_t) + w_t$	(E.11)
$\mu_t^w = w_t - (\sigma_l N_t + (1-h)^{-1}(c_t - h c_{t-1})$	(E.12)
$w_t = \frac{1}{1+\beta} w_{t-1} + \frac{\beta}{1+\beta}(E_t \pi_{t+1} + E_t w_{t+1}) - \frac{1+\beta\gamma_w}{1+\beta} \pi_t + \frac{\gamma_w}{1+\beta} \pi_{t-1} - T_w \mu_t^w + e_t^w$	(E.13)
$R_t = \lambda_r R_{t-1} + (1-\lambda_r)(\lambda_\pi \pi_t + \lambda_y y_t) + e_t^r$	(E.14)

The seven disturbances are: TFP shock (e_t^a); monetary policy shock (e_t^r); investment shock (e_t^i); price markup shock (e_t^p); wage markup shock (e_t^w); risk premium shock (e_t^b); government expenditure shock (e_t^g). The compound parameters in equation (T.11) and (T.13) are defined as: $T_p \equiv \frac{1}{1+\gamma_p} \frac{(1-\beta\zeta_p)(1-\zeta_p)}{((\phi_p - 1)\epsilon_p)\zeta_p}$ and $T_w \equiv \frac{1}{1+\beta} \frac{(1-\beta\zeta_w)(1-\zeta_w)}{((\phi_w - 1)\epsilon_w)\zeta_w}$.

(d): The process for the shocks

$$\boxed{e_t = (e_t^a, e_t^r, e_t^i, e_t^p, e_t^w, e_t^b, e_t^g)}$$
$$\boxed{e_t = \rho e_{t-1} + \eta_t}$$

where both ρ and $\Sigma = E_t \eta_t \eta_t'$ are diagonal.

C. Additional Tables and Figures

	LT	HP	FOD	BP	Ratio
Parameter	Median (s.d.)	Median (s.d.)	Median (s.d.)	Median (s.d.)	Median (s.d.)
σ_c	2.18(0.10)	2.14(0.11)	2.65(0.07)	2.19(0.07)	2.18(0.10)
σ_n	1.80(0.09)	1.82(0.09)	1.52(0.06)	1.80(0.06)	1.83(0.08)
h	0.67(0.01)	0.67(0.01)	0.48(0.01)	0.67(0.01)	0.67(0.01)
α	0.17(0.02)	0.18(0.02)	0.16(0.01)	0.18(0.02)	0.18(0.02)
ϵ	4.53(0.16)	3.73(0.09)	3.91(0.04)	3.84(0.06)	3.71(0.08)
ρ_r	0.17(0.04)	0.19(0.04)	0.23(0.01)	0.06(0.03)	0.18(0.05)
ρ_π	1.37(0.06)	1.38(0.07)	0.81(0.01)	1.79(0.05)	1.36(0.07)
ρ_y	-0.14(0.02)	-0.28(0.05)	-0.01(0.00)	-0.20(0.03)	-0.17(0.02)
ζ_p	0.80(0.01)	0.65(0.02)	0.75(0.01)	0.78(0.02)	0.78(0.01)
ρ_χ	0.76(0.03)	0.52(0.03)	0.99(0.01)	0.83(0.02)	0.71(0.02)
ρ_z	0.96(0.01)	0.92(0.02)	0.99(0.01)	0.93(0.01)	0.95(0.01)
σ_χ	0.23(0.04)	0.19(0.03)	0.29(0.02)	0.11(0.01)	0.23(0.04)
σ_z	0.12(0.02)	0.11(0.01)	0.26(0.03)	0.07(0.01)	0.12(0.02)
σ_{mp}	0.11(0.01)	0.08(0.01)	0.07(0.01)	0.07(0.01)	0.11(0.01)
σ_μ	31.79(0.85)	1.87(0.41)	16.63(0.18)	6.35(1.55)	17.50(2.22)

Table A.1:Posterior estimates, different filters, all variables detrended. Standard deviation in parenthesis. Estimates of σ's and their standard errors in percentages. US data, sample 1980:1-2007:2.

		LT	HP	FOD	BP	Flexible
Parameter	True	Median (s.d.)	Median (s.d.)	Median (s.d.)	Median(s.d.)	Median(s.d.)
σ_c	3.00	2.08 (0.11)	2.08 (0.14)	1.89 (0.14)	2.13 (0.12)	3.26(0.40)
σ_n	0.70	1.72 (0.09)	1.36 (0.07)	1.24 (0.06)	1.58 (0.08)	0.54(0.14)
h	0.70	0.67 (0.02)	0.58 (0.03)	0.36 (0.03)	0.66 (0.02)	0.55(0.04)
α	0.60	0.28 (0.03)	0.15 (0.02)	0.14 (0.02)	0.17 (0.02)	0.19(0.03)
ϵ	7.00	3.19 (0.11)	5.13 (0.19)	3.76 (0.18)	3.80 (0.13)	6.19(0.07)
ρ_r	0.20	0.54 (0.03)	0.77 (0.03)	0.72 (0.04)	0.53 (0.03)	0.16(0.04)
ρ_π	1.20	1.69 (0.08)	1.65 (0.06)	1.65 (0.07)	1.63 (0.10)	1.30(0.04)
ρ_y	0.05	-0.14 (0.04)	0.45 (0.04)	0.63 (0.06)	0.40 (0.04)	0.07(0.03)
ζ_p	0.80	0.85 (0.03)	0.91 (0.03)	0.93 (0.03)	0.90 (0.03)	0.78(0.04)
ρ_χ	0.50	1.00 (0.03)	0.96 (0.03)	0.96 (0.03)	0.95 (0.03)	0.53(0.02)
ρ_z	0.80	0.84 (0.03)	0.96 (0.03)	0.97 (0.03)	0.96 (0.03)	0.71(0.03)
σ_χ	1.12	0.11 (0.02)	0.17 (0.02)	0.21 (0.03)	0.14 (0.02)	1.29(0.01)
σ_z	0.51	0.07 (0.01)	0.09 (0.01)	0.09 (0.01)	0.07 (0.01)	0.72(0.02)
σ_{mp}	0.10	0.05 (0.01)	0.05 (0.01)	0.05 (0.01)	0.05 (0.01)	0.21(0.004)
σ_μ	20.60	6.30 (0.50)	16.75 (0.62)	22.75 (0.83)	14.40 (0.58)	15.86(0.06)
σ_χ^{nc}	3.21					

Table A.2: Posterior estimates, different filters, small variance of the non-cyclical shock.

		LT	HP	FOD	BP	Flexible
Parameter	True	Median (s.d.)	Median (s.d.)	Median (s.d.)	Median(s.d.)	Median(s.d.)
σ_c	3.00	1.90 (0.07)	1.95 (0.09)	1.93 (0.08)	1.96 (0.08)	2.66 (0.19)
σ_n	0.70	2.08 (0.08)	1.96 (0.08)	2.08 (0.10)	1.94 (0.08)	0.56 (0.09)
h	0.70	0.56 (0.02)	0.68 (0.02)	0.57 (0.02)	0.69 (0.02)	0.55 (0.03)
α	0.60	0.49 (0.02)	0.31 (0.02)	0.48 (0.02)	0.28 (0.02)	0.13 (0.03)
ϵ	7.00	3.68 (0.12)	4.00 (0.13)	4.17 (0.13)	4.30 (0.14)	6.15 (0.07)
ρ_r	0.20	0.59 (0.03)	0.74 (0.04)	0.64 (0.04)	0.55 (0.02)	0.28 (0.04)
ρ_π	1.20	1.17 (0.04)	1.47 (0.07)	1.24 (0.04)	1.25 (0.04)	1.60 (0.04)
ρ_y	0.05	-0.05 (0.01)	0.28 (0.02)	0.02 (0.01)	0.20 (0.01)	0.47 (0.03)
ζ_p	0.80	0.80 (0.03)	0.90 (0.03)	0.83 (0.03)	0.89 (0.03)	0.82 (0.02)
ρ_χ	0.50	1.00 (0.03)	0.98 (0.03)	1.00 (0.03)	0.98 (0.03)	0.76 (0.04)
ρ_z	0.80	0.91 (0.03)	0.98 (0.03)	0.95 (0.03)	0.98 (0.03)	0.66 (0.03)
σ_χ	1.12	0.11 (0.02)	1.72 (0.16)	1.02 (0.15)	0.73 (0.08)	0.10 (0.01)
σ_z	0.51	0.17 (0.02)	0.08 (0.01)	0.15 (0.02)	0.08 (0.01)	0.24 (0.04)
σ_{mp}	0.10	0.06 (0.01)	0.06 (0.01)	0.05 (0.01)	0.06 (0.01)	0.10 (0.01)
σ_μ	20.60	13.19 (0.52)	15.97 (0.54)	13.01 (0.50)	13.88 (0.46)	0.14 (0.02)
σ_χ^{nc}	13.21					

Table A.3:Posterior estimates, different filters, medium variance of the non-cyclical shock

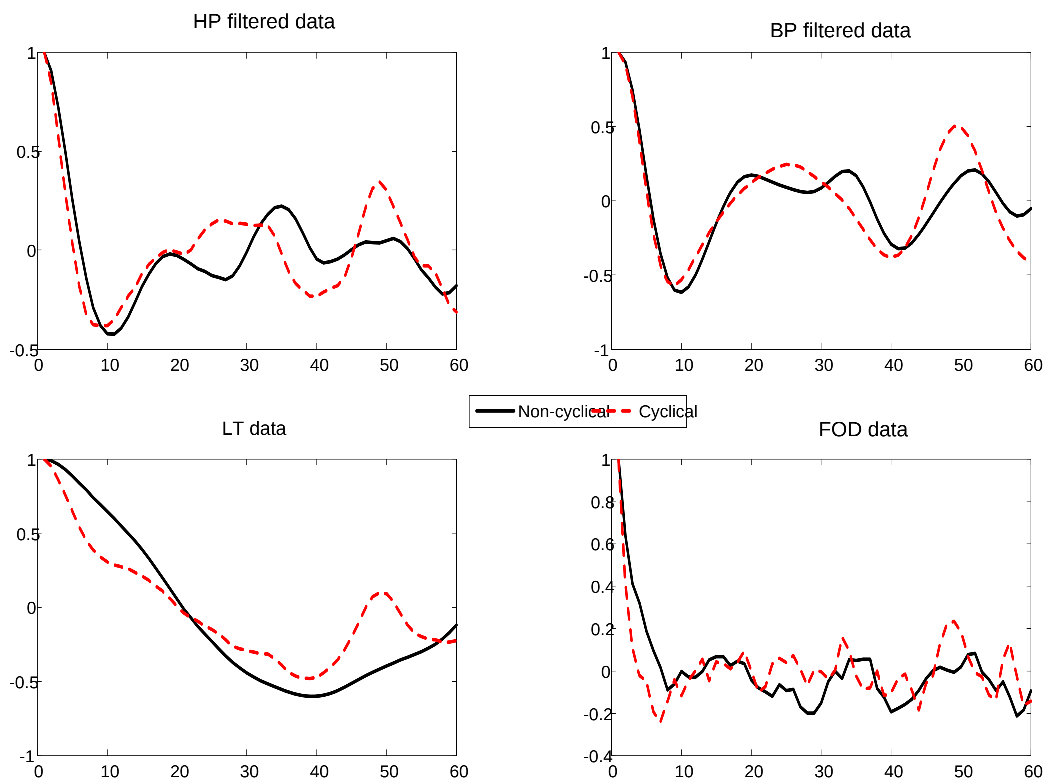

Figure A1: Model based output log-spectra, Technology shocks with two components. Vertical bars indicate the frequencies where cycles with 8-32 quarters periodicities are located

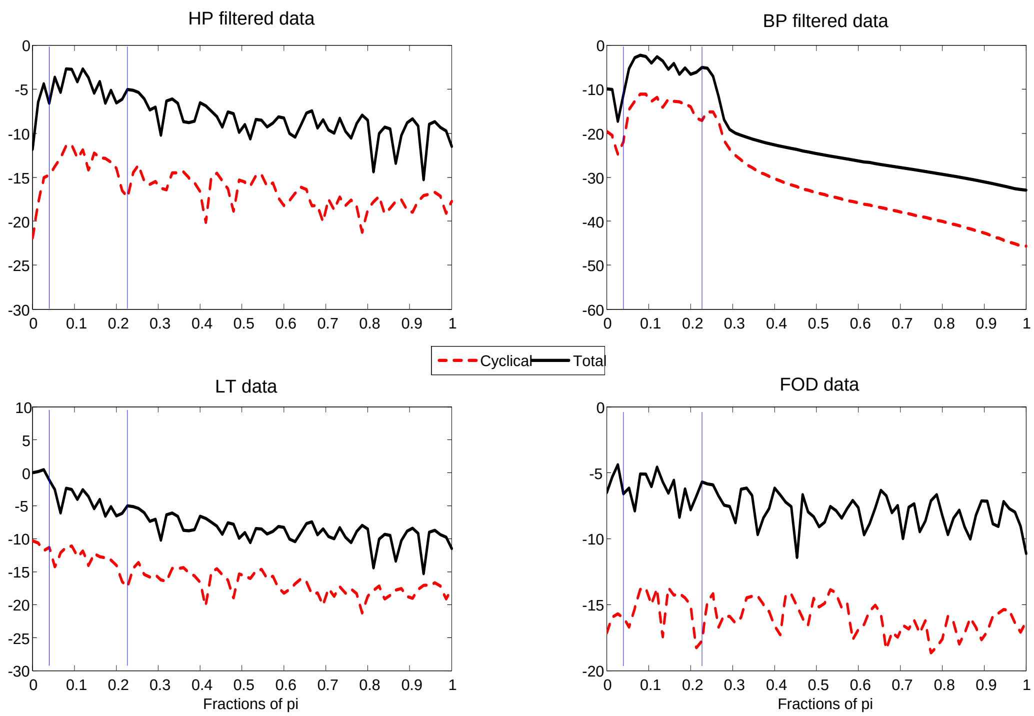

Figure A2: Autocorrelation functions of filtered cyclical and non-cyclical output, Technology shocks with two components.