New Keynesian Dynamics in a Low Interest Rate Environment.*

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Abstract

Recent research has found that the dynamics of the New Keynesian model are very different when the nominal interest rate is zero. Improvements in technology and reductions in the labor tax rate lower economic activity and the size of the government purchase output multiplier can be as large as four. We consider the empirical relevance of these findings using Japanese data. Japan is interesting because it experienced a protracted period of zero nominal interest rates. A prototypical New Keynesian model calibrated to Japan and solved using nonlinear methods exhibits orthodox dynamics with a government purchase multiplier that is less than one.

Keywords: Government purchases; zero nominal interest rates; monetary policy
JEL codes: E3, E5, E6

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1 Introduction

Recent research has found that the dynamics of the New Keynesian model are quite different when the nominal interest rate is zero. A reduction in the labor tax or an improvement in technology can lower output and the size of the government purchase multiplier can be much larger than one. To understand why the dynamics are so different consider the case of a positive, transitory shock to technology. If monetary policy does not respond to this shock output may fall.[1] In the presence of costly price adjustment of goods the arrival of a positive technology shock today has a depressing effect on economic activity. Firms experience temporarily high markups and profits. But, households realize that prices will be lower tomorrow and choose to defer their consumption and investment activities.

One situation where monetary policy cannot respond to technology, or any other shocks for that matter, is when the nominal interest rate is constrained by its lower bound of zero. Braun and Waki (2006) find in this situation that output falls in response to a persistent but transitory improvement in technology using a New Keynesian model calibrated to Japanese data. Eggertsson (2009) illustrates that a reduction in the labor tax has a depressing effect on hours and output. Christiano, Eichenbaum and Rebelo (2009) find that the size of the government purchases multiplier is *much* larger than one.

The objective of this paper is to investigate whether these properties of the New Keynesian model are empirically relevant. Japan is an attractive laboratory for considering the empirical relevance of these previous findings because the policy interest rate was zero between 1999 and 2005. We consider the following questions:

1. Would a lower labor tax have depressed economic activity in Japan?

2. Did the slowdown in the growth rate of TFP that Japan experienced during this period raise output?

3. How big was the government purchase output multiplier?

The answer to the first two questions is no. When we calibrate a prototypical New Keynesian model to Japanese data and subject it to a set of shocks that reproduce Japan's experience, the model has orthodox properties. A lower labor tax and a positive technology shock both increase output when the nominal interest rate is zero.

Why does our model produce orthodox results when the nominal interest rate is zero? The single most important reason is that our solution method explicitly recognizes the resource costs of price adjustment. A common strategy for solving this type of model is to log-linearize the dynamic equations around a steady state with a stable price level. Since the steady state costs of price adjustment are zero, the resource costs of price adjustment disappear from the linearized resource constraint. Explicitly modeling the resource costs of

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[1]See e.g. Basu and Kimball (2003) for an example of a New Keynesian model where this happens.

price adjustment has important effects on the response of hours and output to a labor tax cut and an improvement in technology. We illustrate these points analytically and numerically.

The size of the shocks also matters for our results in the following sense. If we omit the resource costs of price adjustment and posit large enough shocks that lead households to expect a long episode of zero nominal interest rates our model also produces unorthodox results. Labor tax cuts and improvements in technology lower output. These results go away though, when the resource costs of price adjustment are explicitly modeled.

For the *baseline* specification of our economy, the answer to the third question is *less than one*. The value of the government purchase output multiplier ranges from 0.65 to 0.87 depending on the year when government purchases are increased and also how the experiment is defined.

The size of the government purchase multiplier also depends on the treatment of the resource costs of price adjustment and the size of the shock. Resource costs of price adjustment are not explicitly recognized in NIPA accounts data. However, they are recognized in the resource constraint of our model. This yields a distinction between production and output.[2] The government purchase production multiplier is always less than one in our simulations. However, the output multiplier can still exceed one. An improvement in government purchases increases the price level and this lowers the resource costs of price adjustment leaving more of the final good available for consumption and investment.

The size of the shock affects the expected duration of the period of zero nominal interest rates and this also affects the size of the government purchase output multiplier. In our baseline specification households expect the nominal interest rate to be zero for two years and positive afterwards. This assumption is consistent with yield curve data from Japan. However, when shocks are larger and households expect that the nominal interest rate will be zero for 5 years, the size of the government purchase output multiplier increases to 1.31. Most of this boost is due to lower resource costs of price adjustment.[3]

Japan's episode with zero nominal interest rates was a period in which output growth was low but it was also a period of tranquility with low variability in output, consumption and inflation. We use this evidence to argue that Japanese data is most consistent with a government purchase output multiplier that is less than one. Our baseline specification reproduces the empirical evidence of tranquility. However, the model has counterfactual predictions for the data when we posit larger shocks that induce households to expect zero nominal interest rates for a period of five or more years. These specifications imply that the period of zero nominal interest rates should have been a period of macroeconomic instability with high volatility in output and inflation.

We conclude that Japan's experience with zero interest rates is most consistent with a

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[2]Output is defined as the sum of government purchases, investment and consumption. It differs from gross production due to the resource costs of price adjustment.

[3]As noted above the government purchase *production* multiplier is less than one.

New Keynesian model that has the following properties:

1. A lower labor tax rate increases output.

2. An improvement in neutral technology increases output.

3. The government purchases multiplier less than one.

Our results are closely related to research by Braun and Waki (2010). They analyze the role of the solution method for the size of the government purchase multiplier in a New Keynesian model with two alternative forms of costly price adjustment: Rotemberg and Calvo. They find that log-linear solution methods are inaccurate in the presence of preference discount rate shocks that are 3% or higher and induce an upward bias in the government purchase output multiplier that is sometimes very large. However, they find that the government purchase output multiplier is still greater than one for the type of shocks and experiments that they consider.

The remainder of the paper is organized as follows. Section 2 develops a prototypical New Keynesian economy. Section 3 describes the calibration and solution method. Results are reported in Section 4 and Section 5 concludes.

2 Economy

We consider a prototypical New Keynesian economy. The economy is populated by a representative household, a representative final good producer, a continuum of intermediate good producing monopolists that face quadratic costs of adjusting prices, a government and a central bank. We discuss the problems of these agents in turn.

Households

The representative household chooses sequences of consumption $\{c_t\}_{t=0}^{\infty}$ and leisure$\{1 - h_t\}_{t=0}^{\infty}$ to maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t \prod_{j=0}^{t} d_j \left\{ \frac{(c_t^{\nu}(1 - h_t)^{1-\nu})^{1-\sigma}}{1 - \sigma} \right\} \tag{1}$$

where c_t is consumption of the composite good and h_t is hours worked expressed as a fraction of a time endowment of one. β denotes the discount factor, ν is the preference weight a household attaches to consumption and σ determines risk aversion. Finally, d_t is a shock to the subjective discount rate with the law of motion

$$\ln(d_t) = \rho_d \ln(d_{t-1}) + \epsilon_{d,t} \tag{2}$$

where ϵ_t is an I.I.D, homoscedastic, mean zero Gaussian random variable. The household's period t budget constraint is given by

$$(1 + \tau_{c,t})c_t + x_t + \frac{B_t}{P_t} = (1 + R_{t-1})\frac{B_{t-1}}{P_t} +$$

$$\int_0^1 \frac{\Pi_t(i)}{P_t} di + T_t + (1 - \tau_{t,K})r_t k_{t-1} + (1 - \tau_{t,W})w_t h_t + \tau_{t,K}\delta k_{t-1} \tag{3}$$

where P_t is the price level, w_t is the wage rate and r_t is the real interest rate. B_t is the household's holdings of nominal debt at the end of period t, k_{t-1} is the level of capital chosen in period $t-1$ and x_t is investment. Households hold equal shares in each intermediate goods firm so that $\Pi_t(i)$ is per capita nominal profits from intermediate firm indexed i. Households pay taxes $\tau_{c,t}$, $\tau_{t,k}$ and $\tau_{t,w}$ on consumption, capital income and labour income, and receive lump-sum transfers of size T_t from the government. Ponzi schemes are ruled out by limiting attention to solutions that satisfy the standard transversality condition for bonds and capital. Capital is subject to adjustment costs and is accumulated according to

$$k_t = (1 - \delta)k_{t-1} + x_t - \frac{\phi}{2}\left(\frac{x_t}{k_{t-1}} - \mu_k + 1 - \delta\right)^2 k_{t-1} \tag{4}$$

where μ_k is the growth rate of capital in the balanced growth path and δ is the depreciation rate. Let $\lambda_{c,t}$ and $\lambda_{k,t}$ be the Lagrangian multipliers on the household's budget constraint (3) and on the law of motion for capital (4), respectively. The optimal choices of the representative household satisfy

$$\frac{\nu(c_t^\nu(1 - h_t)^{1-\nu})^{1-\sigma}}{c_t} = \lambda_{c,t}(1 + \tau_{c,t}) \tag{5}$$

$$\frac{(1 - \nu)(c_t^\nu(1 - h_t)^{1-\nu})^{1-\sigma}}{1 - h_t} = \lambda_{c,t}(1 - \tau_{w,t})w_t \tag{6}$$

$$\lambda_{c,t} = \lambda_{k,t}\left[1 - \phi\left(\frac{x_t}{k_{t-1}} - \mu_k + 1 - \delta\right)\right] \tag{7}$$

$$0 = \beta E_t d_{t+1}\lambda_{c,t+1}\left[(1 - \tau_{k,t+1})r_{t+1} + \tau_{k,t+1}\delta\right] - \lambda_{k,t}$$

$$+ \beta E_t d_{t+1}\lambda_{k,t+1}\left[1 - \delta + \phi\left(\frac{x_{t+1}}{k_t} - \mu_k + 1 - \delta\right)\frac{x_{t+1}}{k_t} - \frac{\phi}{2}\left(\frac{x_{t+1}}{k_t} - \mu_k + 1 - \delta\right)^2\right] \tag{8}$$

$$-\lambda_{c,t}/P_t + \beta E_t d_{t+1}\lambda_{c,t+1}(1 + R_t)/P_{t+1} = 0 \tag{9}$$

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Final Good Firm

Perfectly competitive final good firms use a continuum of intermediate goods $i \in 0, 1$ to produce a single final good that can be used for consumption and investment. The final good is produced using the following production technology

$$y_t = \left(\int_0^1 y_t(i)^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}} \tag{10}$$

The profit maximizing input demands of the final good firm are

$$y_t(i)^d = \left(\frac{p_t(i)}{P_t} \right)^{-\theta} y_t \tag{11}$$

where $p_t(i)$ denotes the price of the good produced by firm i. The price index P_t is defined as

$$P_t = \left(\int_0^1 p_t(i)^{1-\theta} di \right)^{1/(1-\theta)} \tag{12}$$

Intermediate Goods Firms

There is a continuum of monopolistically competitive firms each producing one differentiated, intermediate good according to the technology

$$y_t(i) = k_{t-1}(i)^\alpha (A_t h_t(i))^{1-\alpha} \tag{13}$$

We assume that there are permanent shocks, $\psi_{A,t}$ and transitory shocks, $\epsilon_{A,t}$ to technology. Both $\psi_{A,t}$ and $\epsilon_{A,t}$ are I.I.D, homoscedastic, mean zero Gaussian random variables. Technology evolves according to

$$A_t = Z_{A,t} e^{v_{A,t}} \tag{14}$$

$$v_{A,t} = \rho_A v_{A,t-1} + \epsilon_{A,t} \tag{15}$$

$$Z_{A,t}/Z_{A,t-1} = \mu_{A,t} \tag{16}$$

$$\ln \mu_{A,t} = \ln \mu_A + \psi_{A,t} \tag{17}$$

Each intermediate firm solves a dynamic profit maximization problem that can be broken down into two parts: The choice of the cost minimizing level of inputs and the choice of the optimal sequence of prices of output. There are two inputs: Labor and capital. Cost minimization implies

$$r_t = \alpha \chi_t k_{t-1}(i)^{\alpha-1} (A_t h_t(i))^{1-\alpha} \tag{18}$$

$$w_t = (1-\alpha) \chi_t A_t^{(1-\alpha)} k_{t-1}(i)^\alpha h_t(i)^{-\alpha} \tag{19}$$

where $\chi_t = \frac{r_t^\alpha w_t^{1-\alpha}}{\alpha^\alpha (1-\alpha)^{1-\alpha} A_t^{1-\alpha}}$ is real marginal cost.

Price rigidity is introduced using a convex cost of price adjustment as in Rotemberg (1996). Define gross inflation $1 + \pi_t(i)$ as $p_t(i)/p_{t-1}(i)$. Given the optimal choice of labor and capital, a typical intermediate goods producer chooses a sequence of prices $p_t(i)$ to maximize

$$\sum_{t=0}^{\infty} \beta^t \prod_{j=0}^{t} d_j \lambda_{c,t} \left[p_t(i) y_t(i) - P_t s \chi_t y_t(i) - \frac{\gamma}{2} P_t (\pi_t(i) - \pi)^2 y_t \right] / P_t \tag{20}$$

subject to the input demands (11). We assume a subsidy $s = \theta/(\theta - 1)$ is in place that corrects the static inefficiency due to monopolistic competition. This subsidy isolates the dynamic distortion caused by the variation in the markup which is the distortion that monetary policy corrects in a New Keynesian model. Introducing a subsidy is also very convenient because it also allows us to nest a real business cycle model as a special case by setting the adjustment costs on prices to zero.

The first order condition for the firms' price setting problem reads

$$\beta E_t \frac{d_{t+1} \lambda_{c,t+1} y_{t+1}}{\lambda_{c,t} y_t} \gamma (\pi_{t+1} - \pi)(1 + \pi_{t+1}) = -\left[1 - \theta + \theta s \chi_t - \gamma (\pi_t - \pi)(1 + \pi_t) \right] \tag{21}$$

Monetary Policy

Interest rate targeting rules have been found to be good empirical specifications of monetary policy in e.g. Taylor (1993) and we refer to monetary policy rules of this form as Taylor rules. The particular Taylor rule considered here is

$$R_t = \max \left[(1 + R) \left(\frac{1 + \pi_t}{1 + \pi} \right)^{\rho_\pi} \left(\frac{1 + R_{t-1}}{1 + R} \right)^{\rho_P} e^{u_{M,t}} - 1, 0 \right] \tag{22}$$

where $u_{M,t}$ is an I.I.D, homoscedastic, mean zero, Gaussian random variable. One special feature of this rule is that it output does not appear. We will consider a sample period during which Japan experienced a long and persistent departure from its trend growth rate. It is not clear how to define the target level of output in this type of situation.[4]

Fiscal Policy

The fiscal authority finances its expenditures by collecting distortionary taxes and lump-sum transfers and by issuing nominal bonds. Fiscal policies satisfy the period budget constraint

$$g_t + (1 + R_{t-1}) \frac{B_{t-1}}{P_t} + S_t = \frac{B_t}{P_t} - T_t + \tau_{w,t} w_t h_t + \tau_{c,t} c_t + \tau_{k,t} k_{t-1} (r_t - \delta) \tag{23}$$

where S_t is a subsidy to intermediate monopolists.[5] Defining $b_t \equiv \frac{B_t}{P_t}$, we can rewrite the government budget constraint as

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[4]One possibility is to introduce the growth rate of output in the Taylor rule. Results for that specification are discussed briefly in Section 4.5 below.

[5]We limit attention to equilibria where the path of the government debt satisfies a transversality condition.

$$g_t + (1 + R_{t-1})b_{t-1}\frac{1}{1 + \pi_t} = b_t - T_t + \tau_{w,t}w_t h_t + \tau_{c,t}c_t + \tau_{k,t}k_{t-1}(r_t - \delta) \tag{24}$$

The tax rates on capital, consumption and labor and government purchases have the following laws of motion

$$\tau_{c,t} = (1 - \rho_c)\tau_c + \rho_c\tau_{c,t-1} + \epsilon_{c,t} \tag{25}$$

$$\tau_{k,t} = (1 - \rho_k)\tau_k + \rho_k\tau_{k,t-1} + \epsilon_{k,t} \tag{26}$$

$$\tau_{w,t} = (1 - \rho_w)\tau_w + \rho_w\tau_{w,t-1} + \epsilon_{w,t} \tag{27}$$

$$log\left(\frac{g_t}{gnp_t}\right) = (1 - \rho_g)log\left(\frac{g}{gnp}\right) + \rho_g log\left(\frac{g_{t-1}}{gnp_{t-1}}\right) + \epsilon_{w,t} \tag{28}$$

where the shocks to each variable are I.I.D, homoscedastic, mean zero Gaussian random variables. Lump-sum transfers are assumed to adjust to satisfy the government budget constraint.

To close the model, the aggregate resource constraint is given by

$$g_t + c_t + x_t = y_t(1 - \frac{\gamma}{2}(\pi_t - \pi)^2) \tag{29}$$

We will solve the model using nonlinear methods that recognize the resource costs of price adjustment. One common approach is to log-linearize the equilibrium conditions around a steady state with zero inflation. Under this assumption, the resource costs of price adjustment do not appear in the log-linearized resource constraint. We will see below that how these adjustment costs are treated has a very important impact on the results.

Equilibrium

The notion of equilibrium considered here is an imperfectly competitive general equilibrium in which the markets for the final good, intermediate goods, labor, capital and government debt clear in each period. The model developed above admits a symmetric monopolistic competitive equilibrium. We start by defining a perfect foresight equilibrium.

Definition A *perfect foresight symmetric monopolistic competitive equilibrium* consists of a sequence of allocations $\{c_t, h_t, x_t, k_t, \lambda_{c,t}, \lambda_{k,t}, y_t\}_{t=0}^{\infty}$, a set of policies $\{R_t\}_{t=0}^{\infty}$, a sequence of prices $\{r_t, w_t, \chi_t, \pi_t\}_{t=0}^{\infty}$ and a finite set of integers I_B that satisfies the

- Households' optimality conditions

- Firms' optimality conditions

- Monetary policy rule:

    - $\forall$ t $\notin I_B$ the zero constraint on interest rates is not binding and the Central Bank follows the Taylor rule

- ∀ t ∈ I_B the zero constraint on interest rates is binding and the Central Bank sets $R_t = 0$

- Aggregate resource constraint and market clearing

given initial conditions (P_{-1}, R_{-1}, k_0), and sequences of shocks to the rules for $\{A_t, d_t, \tau_{k,t}, \tau_{c,t}, \tau_{w,t}, g_t\}_{t=0}^{\infty}$.[6]

Two points are worth mentioning. First, the definition of equilibrium is sequential. Second, the definition of equilibrium includes a statement of specific intervals where the zero lower bond on the nominal rate is binding.

3 Solution Method and Calibration

3.1 Solution Method

Our choice of solution method is motivated by three considerations. First, we choose a nonlinear solution method because recent research by Braun and Waki (2010) has found that log-linear solutions produce significant upward biases in the size of the government purchases multiplier when large shocks to the preference discount drive the nominal interest rate to zero.

We will report results below that show that nonlinearities are also important for other properties of the model such as the sign of the output response to a labour tax or a transitory technology shock.

The second motivation for our choice relates to finding the interval when the nominal interest rate is zero. Braun and Waki (2006) consider the problem of computing an equilibrium for an economy similar to ours in a perfect foresight setting. They limit attention to equilibria of the form where the interest rate is zero for only one finite and contiguous number of periods. Even with this restriction they find that there can be multiple equilibria and they impose two further equilibrium selection devices. First, they impose the restriction that the nominal interest rate in the model hits zero in a specific year that is dictated by Japanese data. Second, they select the equilibrium where the nominal interest rate is zero for the shortest interval of time. We use the same strategies for selecting an equilibrium here.

Third, we want to relax the perfect foresight assumption maintained in e.g. Braun and Waki (2006, 2010) and allow for uncertainty.

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[6]Because we assume that the government adjusts lump-sum transfers such that its budget constraint is satisfied, we omit the government budget constraint from the equilibrium conditions and we omit government bonds and transfers from the list of variables determined in equilibrium.

These three considerations led us to use a variant on a technique called extended shooting.[7] Starting from the initial period, agents solve the set of nonlinear equations that describe their respective decision rules forward for 100 periods. We assume that our economy is at its steady state in period 101. In these future periods, shocks are set to 0. We then move time forward by one period. Agents experience a new set of shocks and have a new set of initial conditions. They once again solve forward for 100 periods. This is repeated for each year from 1988 to 2007.

Because our solution method is sequential, we can limit the problem of dealing with the zero bound constraint to a small set of periods. Prior to 1999, households assign zero probability to the constraint binding in equilibrium. In the periods where households anticipate or experience a binding constraint we solve the model by hand using guess and verify methods to find the interval where the nominal interest rate is zero.

The principal limitation of this approach is that expectations about the future in any given period are degenerate. However, this solution strategy also highlights a hard problem. We know from previous research that the dynamics of the New Keynesian model can change in first order ways when the nominal interest rate is zero. Thus any solution method that allows for non-degenerate expectations formulation will probably require that agents form state dependent probabilities over the interval that the nominal interest rate constraint binds.

3.2   Calibration of Parameters

We calibrate the parameters of our economy by matching model variables to calibration targets in Japanese data between 1981 and 2007. Recently, Bayesian estimation has become popular for parameterizing models like ours. But Bayesian estimation relies on a linearized system. Here the focus is specifically on nonlinear dynamics and estimation techniques that rely on linear time-series methods don't work in the presence of occasionally binding zero interest rate constraints. One could in principal estimate the model parameters using an earlier sample period when the nominal interest rate is positive. However, previous research by e.g. Chen, Imrohoroglu and Imrohoruglu (2006) and Braun, Ikeda and Joines (2009) show that Japan was undergoing large transitional adjustments between 1960 and 1990. This was the period of Japan's growth miracle and it is difficult to derive a stationary representation in the presence of large one off transitional dynamics induced by e.g. a low capital stock.

Table 1 reports the model parameterization. Most of the parameters are computed using averages from Japanese data over the sample period 1981-2007. The data used for calibrating the model are updated versions of the data employed by Hayashi and Prescott (2002).[8] The capital share parameter α is calibrated to match capital's share of income. The depreciation

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[7]Heer and Maussner (2008) provide an excellent description of the extended shooting algorithm

[8]We wish to thank Nao Sudou of the Bank of Japan for providing us with the data.

rate δ reproduces average depreciation in Japanese data. The steady state nominal rate is the average of the Japanese overnight call-rate. The coefficients for the laws of motion of the taxes on, consumption, labor and capital are estimated using Japanese data on average tax rates for these three variables. The parameters for the law of motion of government's share of output are estimated in the same manner.

The preference discount factor β is set to 0.995, a rather high level for a model in which the length of a period is one year. This choice implies that the inflation rate associate with a steady state nominal interest rate of 2.9% is zero. Conditional on the rest of the parameterization, a lower value of β would imply that the steady state inflation rate associated with a nominal interest rate of 2.9% is negative. We set the curvature parameter in preferences to 2. The weight on leisure in the utility function, ν, is calibrated to match the average labor input between 2000 and 2007. We choose this period because prior to 2000, labor input exhibits a significant downward trend. The resulting value of ν is 0.27.

Other parameters are set in a more informal way. The parameter controlling the size of adjustment costs on investment is set to 4 which is a bit larger than the value of 2 used by Braun and Waki (2006).

The average markup is set to 15% as in Braun and Waki (2006). It then follows that the value of the subsidy is 1.15. We assume that technology, A_t, advances at a rate of 2% per annum.

The coefficient on inflation, ρ_π, and the lagged nominal rate in the Taylor rule, ρ_R, are set to 1.7 and 0.4 respectively. The adjustment cost parameter γ is set to 80.[9] These parameter choices imply that the nominal rate increases on impact by 0.4% in response to a 1% shock to monetary policy. This response is a bit lower than the response of 0.6% estimated by Sugo and Ueda (2006) for the Japanese economy.

Finally, we start simulating our economy from 1987 and set the initial capital stock in our economy to the same value as its counterpart in Japanese data in 1987.

3.3   Calibration of Shocks

We use Japanese data to derive sequences of innovations to technology, government purchases, and capital and labour taxes. However, we calibrate the innovations to the preference discount factor, consumption tax and monetary policy to reproduce particular targets. We next describe how this was done.

We started out by simulating our economy using the parameterization described above setting the shocks on the consumption tax, the preference discount rate and monetary policy to zero in all periods. That specification preformed reasonably well in terms of its implica-

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[9]When log-linearized, introducing nominal rigidities via Rotemberg price adjustment costs produces a New Keynesian Phillips curve identical to the one obtained from a Calvo model of nominal rigidities. The Calvo parameter of price stickiness associated with our parameterization is 0.75.

tions for most real variables. However, the model did not produce a large secular decline in labor input after 1987. Between 1987 and 1991, there were some important institutional changes in labor market arrangements in Japan. The number of national holidays were increased and the length of the work week was reduced. However, labor input continues to decline throughout the 1990s. Miyazawa (2010) shows that the secular decline in labor input during the 1990s can partially be attributed to a change in the composition of jobs from full-time to part-time work. We do not explicitly model these factors here and instead treat them as altering the labor wedge as in e.g. Kobayashi and Inaba (2006).

We next introduced shocks to $\tau_{c,t}$ in the years 1987 to 1991 and set them to reproduce movements in Japanese labor input during this sub-sample.[10] The results from this parameterization are reported in Figure 1.

Inspection of Figure 1 indicates that the model does a reasonable job of reproducing some of the basic secular movements in the real side of the Japanese economy. It captures the capital deepening that occurred between 1990 and 2007. The model also captures the decline in output and consumption relative to their trends during the 1990s. However, it does not reproduce the secular decline in labor input after 1991. In addition, the decline in the nominal interest rate and inflation rate is counterfactually small during the 1990s. The model, most importantly, does not predict a period of zero nominal interest rates.

Shocks to the preference discount rate play an important role in getting the interest rate to fall to zero in the work of Eggertsson and Woodford (2003), Taehun, Teranishi and Watanabe (2005) and Christiano Eichenbaum and Rebelo (2009). We follow their approach and introduce shocks to the preference discount factor. Introducing shocks to d_t gets the nominal interest rate to hit its lower bound of zero but these shocks also result in a deterioration in the fit for output and labour input. To counteract the stimulative effect that shocks to the preference discount rate have on these variables, we introduced simultaneous variations in the labor wedge by shocking $\tau_{c,t}$. With some experimentation we found that using a fixed factor of 5 works well.

This calibration scheme resulted in the following sequence of shocks for d_t: For 1993 to 1995, 2%, 1%, and 1%, respectively and for 1999, 2%. This final shock makes the zero lower bound bind in 1999. The value of the log discount rate in 1999 implied by the above shocks is 0.044.

Preference discount rate shocks produce counterfactually low inflation in the second half of the 1990s, too. To counteract the deflationary pressure due to these shocks, we introduced negative monetary policy shocks in the late 1990s. In our economy, a negative shock to monetary policy lowers the nominal interest rate and increases the inflation rate. In other research Sugo and Ueda (2006) have found that negative monetary policy shocks

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[10]Formally, $\tau_{c,t}$ also affects the intertemporal first order condition. However, in our experience this effect is quantitatively very small.

are important for understanding the Japanese economy during this period. The shocks to monetary policy are -0.5% in the years 1993, 1996 and 1998 and -1% in 1997.

These shocks bring the nominal interest rate and the inflation rate down in the 1990s and in particular get the nominal interest rate to hit its lower bound of zero in 1999. However, once the nominal interest rate is zero we were left with a question of how to choose these shocks during the period of zero nominal interest rates. In our baseline specification, we assume that in each period between 1999 to 2005 households expect that the nominal interest rate will be zero for two years. This assumption is based on evidence reported in Ichiue and Ueno (2007). They find using an affine model of the yield curve that the maximum expected duration of zero nominal interest rates during this period was 2.3 years. In 2006, the nominal rate is zero in the current period but agents expect positive nominal interest rates for 2007 and beyond. The Bank of Japan ended its zero nominal interest rate policy in March of 2006 and raised the call rate to 0.25 later in 2006.

The sequence of shocks to d_t, that produces these expectations forecasts gradually declines from 0.3% in 2000 to -0.25% in 2006. We adjust $\tau_{c,t}$ to offset the implications of these shocks for labor input and output in the way described above.[11]

The results from the baseline simulation are reported in Figure 2. A comparison of Figure 2 with Figure 1 has the following noteworthy features. The shocks we have added after 1991 achieve the desired goal of bringing inflation and the nominal rate down during the second half of the 1990s. Moreover, the level of inflation during the period of zero nominal interest rates is about of the same level as we observe in Japanese data.

Relative to Figure 1, there is some deterioration in the fit of the model for real allocations. The baseline economy understates consumption and overstates the extent of capital deepening. The reason for these changes in the fit of the model for real variables is the preference shock. On the one hand, a d_t shock brings the nominal rate down but it also stimulates current labor input and output. We compensate for these effects using a shock to $\tau_{c,t}$. This improves the fit for these variables but also induces households to consume less and save more.

Overall, the baseline model does a good enough job of capturing the main features of Japan's experience between 1990 and 2007 to warrant using it as a laboratory for conducting counterfactuals.

The number of periods that agents expect the nominal interest rate to be zero can have a big effect on the properties of the model. To illustrate this point we will also report results for two other specifications. In the *persistent expectations* specification, the sequence of preference shocks hitting the economy between 1999 and 2007 is set so that agents expect zero nominal rates for 5 years in each year between 1999 to 2003. After 2003, agents expect

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[11]If we do not counteract a shock to the preference discount rate in e.g. 1999, agents expect positive nominal rates in 2000 instead of 2001.

that the nominal rate will become positive sooner. The nominal rate becomes positive in 2007. To implement this scenario, a shock to the preference discount rate of size 3% hits the economy in 1999. From 2000 to 2007, the size of preference discount rate shocks ranges between 0.6% and -0.6%.

In the *large preference shock* specification, we assume that the preference discount shock arriving in 1999 is equal to 3.5% and that the shock to τ_c is 0.175. These shocks lead agents to expect that nominal rates will be zero in each year between 1999 and 2005. After 1999, no other shocks to d_t or $\tau_{c,t}$ arrive. In this specification the equilibrium value of the nominal interest rate becomes positive in 2005.

4   New Keynesian Dynamics in a Low Interest Rate Environment

4.1   Dynamic Responses of the Baseline Economy

We now turn to analyze the dynamics of the model. Previous research by Braun and Waki (2006), Christiano, Eichenbaum and Rebelo (2009) and Eggertsson (2009) finds that the dynamic properties of the New Keynesian model are quite different when the nominal interest rate is zero. We now turn to investigate the quantitative relevance of these results for the Japanese economy during its episode with zero nominal interest rates.

Before discussing the results we wish to emphasize that *production* which we denote by y is distinct from *output* in our model. Measured output does not include the resource costs of price adjustment. We will denote output in our model by *GNP*.[12] GNP is defined as:

$$GNP_t \equiv c_t + g_t + x_t = y_t(1 - \frac{\gamma}{2}(\pi_t - \pi)^2)$$

The distinction between production and GNP plays an important role in the subsequent analysis. Any shock that increases the difference between current and steady state inflation also raises the resource costs of price adjustment. This, in turn, increases the gap between production and GNP.

Table 2 reports impulse responses for the baseline specification. The first row shows the year in which the shocks are perturbed. The second row reports the number of years that agents expect the nominal interest rate to be zero. The third row reports the resource costs of price adjustment as a percent of GNP. The remaining rows report impact responses of GNP, production, and the markup to various shocks.

Consider the responses of GNP and production. Results are reported for permanent and transitory shocks to technology, shocks to the labor tax rate and shocks to government

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[12]We use data constructed by Hayashi and Prescott (2002) to calibrate our model. The notion of gross product in that data set is GNP. That is why we use GNP instead of GDP to denote output. We will hereafter use output and GNP interchangeably.

purchases. In all instances, the sign of the shock is positive. For the first three variables, the responses are the percentage change in GNP to a 1% impulse in the variable that is shocked.[13] For the shock to government purchases the results are expressed as government purchase multipliers which are defined as: $\frac{\Delta GNP}{\Delta G}$ and $\frac{\Delta Y}{\Delta G}$. The first column of results is for shocks that arrive in 1995, which is representative of years in which the current nominal interest rate is positive and expected nominal interest rates are positive in all future years. We also report impulse responses for shocks that arrive in 1999 and 2004. Responses reported for the years 1999 and 2004 differ in that in 1999 none of the shocks affects the equilibrium number of periods that the expected nominal interest rate is zero. In all cases agents continue to expect the nominal interest to be zero for two periods and then positive thereafter. In 2004, in contrast, some of the shocks reduce the expected number of periods of zero nominal interest rates by one period.

Table 2 indicates that the dynamic responses of GNP in our New Keynesian economy are very similar when the nominal interest rate is positive and when it is zero. The signs of the responses are in good accord with what standard economic theory predicts: GNP increases in response to a positive technology shock of either type and falls in response to an increase in the labor tax.

The magnitudes of the government purchase multipliers are also consistent with standard economic theory. The government purchase GNP multiplier is less than one in all periods. In 1995 it is 0.65. It increases to 0.87 in 1999 but never rises above 1 in any period. The government purchase production multiplier is also less than one in all periods. One robust distinction between the two government purchase multipliers is that the production multiplier is always smaller than the GNP multiplier.

Note next that the markup response to each of the shocks is larger when the nominal rate is zero. In the case of a shock to government purchases, for instance, the size of the markup response in both 1999 and 2004 is about three times larger than in 1995. The size of the mark up response increases when the nominal interest rate is zero for the other shocks as well.

Finally, a comparison of the responses in 1999 with those in 2004 shows that the responses to a government purchase or labor tax shock are smaller in 2004. The responses of GNP, production and the markup are all more moderate in 2004. This result is due to the fact that each of these two shocks shortens agents expectations about the number of periods that the interest rate will be zero.

The most significant difference between the period with zero interest rates and the prior period is the markup response. What is the economic mechanism responsible for the approximately threefold increase in markup volatility? It is known from previous work by e.g. Khan, King and Wolman (2002) that optimal government policy in a model with imperfectly

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[13]See Table 3 for the response of production to shocks in these three variables.

competitive intermediate goods markets is to smooth the dynamic response of the markup to shocks. Schmitt-Grohe and Uribe (2007) find that a monetary policy that stabilizes the price level is an effective way to achieve this objective. In the New Keynesian model prices have a close link to the value of the markup via the New Keynesian Philips curve and stabilizing prices acts to limit the size of the response of the markup to shocks to government purchases and other exogenous variables. In practice, a simple Taylor (1993) rule with a large inflation elasticity also works very well. Once the nominal interest rate is zero though, the Taylor rule is no longer operative and monetary policy ceases to stabilize the response of the markup to shocks. This is the mechanism triggering the larger markup responses in Table 2. However, what is noteworthy about Table 2, is that the level of markup variability prior to 1999 is very small. Increasing its variability by a factor of three only has small quantitative effects on the dynamic response of the economy to shocks.

On the one hand, the results reported in Table 2 for the period of zero interest rates are reassuring. They imply that we don't have to change the way we think about the world when the nominal interest rate is constrained by its lower bound of zero. On the other hand, our results are quite different from findings reported elsewhere in the literature. The value of the government purchase multiplier reported in Table 2 is low and in particular, less than one. Christiano, Eichenbaum and Rebelo (2009), in contrast, find that the government purchases multiplier is much larger than one when the nominal interest rates is zero. In addition, the sign of the output response to either type of technology shock is positive in Table 2. Braun and Waki (2006) and Christiano, Eichenbaum and Rebelo (2009) find that it is negative when the nominal interest rate is zero. Finally, we find that output (and hours) fall when the labor tax is increased during the period of zero nominal interest rates. Eggertsson (2009) finds that hours increase in this situation. We now turn to discuss the reasons for these differences.

4.2   Why Our Results are Different

There are three principal reasons why our results are different from results reported in the previous literature: the solution method, expectations about the duration of the period of zero nominal interest rates, and the size of the shock.[14]

The solution method is the single most important factor underlying our results. Most of the previous literature uses log-linear approximations when solving the model. The only nonlinearity that is explicitly recognized comes from imposing the restriction that the nominal interest rate is non-negative in the Taylor rule.[15] In addition, the approximation is typically centered around a zero inflation steady state. Consequently, in these frameworks

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[14]These factors are highly interdependent and their effects cannot be neatly taken apart.

[15]Adam and Billi (2007) use nonlinear methods to solve the model but again the only source of nonlinearity is in the Taylor rule.

there is no distinction between production and GNP because the resource costs of price adjustment are zero. This common assumption is not innocuous. Recognizing the resource cost of price adjustment in the aggregate resource constraint can have both a qualitatively and quantitatively important effect on the responses of the economy to a variety of shocks. We first illustrate this point analytically using a much simpler model and then numerically by reporting some further simulations of our model.

Analytical Analysis

To facilitate discussion with Eggertsson (2009) we consider a reduction in the labor tax rate in a much simpler New Keynesian economy with no physical capital, a convenient functional form of preferences, and Rotemberg (1996) price adjustment costs. [16] The nominal interest rate is brought to zero using the preference shock considered in Eggertsson (2009) and Christiano, Eichenbaum and Rebelo (2009). To keep the analysis as simple as possible we start by assuming that the nominal interest rate is zero in only the first period and that agents expect positive interest rates from period two on.

Figure 3 illustrates the responses of inflation and hours to a tax shock that arrives in period one. The graphs on the left abstract from the resource costs of price adjustment ($\kappa = 0$) while the graphs on the right account for these costs ($\kappa > 0$). The graphs on the top show the aggregate supply (AS) curve in the hours-inflation space and the graphs on the bottom show the aggregate demand (AD) curve in the hours-preference shock space. The explicit AD and AS equations are reported in the Appendix.

Consider first the response of hours and inflation to a tax shock when the resource costs of price adjustment are ignored ($\kappa = 0$). As in Eggertsson (2009), the AD curve is unaffected by a change in the labor tax and equilibrium hours do not change. The tax cut, however, decreases marginal cost and this acts to increase deflationary pressure.

Consider next the responses to a tax cut when the resource costs of price adjustment are recognized ($\kappa > 0$). Now the AD curve also depends on the inflation rate. Thus, both hours and inflation change in response to a tax cut. When $\kappa > 0$, a labor tax cut lowers the inflation rate and this increases the resource costs of price adjustment. A higher κ shifts the AD curve to the right. With the AD curve shifting right hours unambiguously increase. In the new equilibrium (B), hours are higher and inflation is lower.

Expectations are also important. The previous results are premised on the assumption that the nominal interest rate becomes positive in period two with probability one. If agents instead assign positive probability to the nominal interest rate being zero in period two then the results change in the following way. When $\kappa = 0$ expectations about future inflation shift the AD schedule left and hours fall as in Eggertsson (2009). When $\kappa > 0$ though it continues to be the case that the resource costs of price adjustment increase in response to

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[16]The complete details of this economy are provided in the Appendix.

a tax cut and this acts to increase hours. The overall response to hours will depend on the relative magnitude of these two effects which is governed by the expected duration of the low real interest rate episode.

A similar line of reasoning applies for other shocks. For instance, Braun and Waki (2006) and Christiano, Eichenbaum and Rebelo (2009) find that a positive neutral technology shock that arrives in period one can lower production. In their models, κ is zero. However, if $\kappa > 0$ the overall decline in hours is mitigated and production can rise.

Whether hours and production increase or fall is an empirical question that requires an empirically relevant model.

We now turn to illustrate that in our model of the Japanese economy, recognizing the resource costs of price adjustment in the resource constraint has important consequences for the sign and magnitude of the response of hours, production and GNP to a range of shocks.

Numerical Analysis

Table 3 reports impulse responses for alternative specifications of our economy that vary according to the size of the shocks and the treatment of the costs of price adjustment in the resource constraint. We assume that the shocks arrive in 1999 which is the year that the nominal interest rate first hits zero. In the discussion that follows it will be important to keep in mind an important distinction between our model and the simpler analytic example considered above. In our model the number of periods that households expect the nominal interest rate to be is zero is endogenous. In particular, larger shocks increase the number of periods that households expect the nominal interest rate to be zero.

The first two columns of results in Table 3 report impulse responses for the baseline specification. In 1999 the nominal rate falls to zero in response to a positive 2% shock to d_t and an increase in $\tau_{c,t}$ of size 0.1. For purposes of comparison, the first column repeats the baseline responses reported in Table 2. The second column reports responses for the baseline specification when the costs of price adjustment are omitted from the resource constraint. For this configuration of shocks, the resource costs of price adjustment are small (0.60%) and abstracting from them does not have important effects on the properties of the model. The implied resource costs of price adjustment are 0.66% and the impulse responses have a similar size in either column.

Notice that production falls on impact to an increase in the labor tax in either column. It falls by 0.59% in the baseline specification and by 0.55% in the specification that omits the resource costs of price adjustment.

Impact responses to shocks in each type of technology or government purchases are also quite similar in the first two columns of results.

Perhaps the most important distinction between the first two columns of results is in the response of the markup. The markup response is larger in absolute value for each shock

when the resource costs of price adjustment are omitted.

Results reported in columns 3 and 4 are for the persistent expectations specification. For this specification there is a 3% shock to d_t and a simultaneous 15% shock to $\tau_{c,t}$ in 1999. Consider the results reported in column 3. The larger preference shock in 1999 has two effects: First, it lowers the shadow price of the nominal interest rate in 1999 and secondly, it increases the number of periods agents expect nominal rates to be zero from two years to five years. This combination of shocks induces a much larger response in the markup as compared to the baseline specification. Notice also that the resource costs are much larger here. The government purchase GNP multiplier increases to 1.33 which is about 50% larger than its value in the baseline specification. However, the government purchase multiplier for production is only moderately larger and still less than one.[17] Finally, the GNP responses to a positive transitory technology shock or a lower labor tax are also smaller for the persistent expectations specification as compared to the baseline.

Column 4 reports results for the persistent expectations specification with the resource costs of price adjustment omitted from the resource constraint. For shocks of this size we can start to observe some larger differences between the two specifications of the resource constraint. Relative to column 3, a positive, transitory shock to technology or a negative shock to the labor income tax increases GNP by less. The government purchase GNP multiplier increases to 1.45. Most significantly, the markup response is much larger for all types of shocks when the price adjustment costs are omitted from the resource constraint.

Finally, consider the results reported in columns 5 and 6 under the heading large preference shock. In this specification $d_t = 0.035$ and $\tau_{c,t} = 0.175$ in 1999. When the resource costs of price adjustment are reflected in the budget constraint we find that the government purchase GNP multiplier is 1.50. The government purchase production multiplier though is still less than one. GNP continues to respond in a conventional way to impulses to either form of technology shock or the labor tax shock. And the production responses are close in magnitude to results reported in columns 1 and 3.

The picture changes dramatically though when the resource costs of price adjustment are omitted from the resource constraint. The response of the markup to any form of shock is now many orders of magnitude larger than the baseline.

Results reported in the final column of Table 3 indicate that an increase in the labor tax rate now stimulates economic activity. GNP increases and since the capital stock is predetermined on impact, hours also rise. From this we see that two factors are needed to get the type of anomalous responses to a labor tax cut that are described in Eggertsson (2009). First, the shock has to be very large and second one also needs to abstract from the

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[17]Expectations play an important role. Although not reported in Table 3, the value of the government purchase GNP multiplier is also of about the same magnitude in 2000, a year in which there are no shocks to d_t.

resource costs of price adjustment.

The sign of the response of GNP to a transitory improvement in technology is anomalous too. As in Braun and Waki (2006) a positive, transitory shock to technology lowers production. Finally, the government purchase GNP multiplier is large (1.75).

To provide some intuition for the differences between the responses of production and GNP, consider the response of GNP to a government purchases shock.[18] Totally differentiating the resource constraint with respect to a change in government purchases allows us to rewrite the impact response of GNP as

$$\frac{dgnp}{dg} = (1 - \Psi)\frac{dy}{dg} - y\frac{d\Psi}{dg} \tag{30}$$

where Ψ denotes the resource costs of price adjustment and where we have suppressed time subscripts for ease of exposition. Equation (30) decomposes the GNP response to a transitory technology shock into two terms.[19] The first term consists of the response of production to a government purchases shock weighted by one minus the resource costs of price adjustment, $(1 - \Psi)$. The second term is the response of the price adjustment costs to a government purchases shock weighted by production, y.

Table 4 reports this decomposition for alternative specifications of the model. Observe that the first term is almost constant across the different specifications. As in Table 3, the response of production to a change in government purchases is always less than one and falls with the size of the preference shock. The second term is largest in absolute value for the large preference shock specification. A shock to government purchases puts upward pressure on prices and thereby reduces the resource costs of price adjustment. With lower costs of price adjustment, more of production is available for consumption and investment which implies in turn a larger GNP response. From this decomposition one can see that the key factor that is responsible for a large government purchase GNP multiplier is the reduction in the resource costs of price adjustment.

Braun and Waki (2010) use a specification and perform experiments similar in nature to the ones considered in Christiano, Eichenbaum and Rebelo (2009). They find that the multiplier decreases from 4 for the log-linearized solution to about 2.5 when an exact nonlinear solution is computed using Rotemberg (1996) costly price adjustment and the size of the preference discount rate shock is 4%.[20] These two papers compute impulse responses under the assumption that the initial condition is the steady state and that the preference discount rate shock drives the nominal interest rate to zero.[21] If we apply the same procedures to our

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[18]See also the discussion in Braun and Waki (2010).

[19]Strictly speaking, the GNP response to a transitory technology shock, $\Delta gnp/\Delta g$ has a third second order term too. However, for small changes the infinitesimals in (30) are a valid approximation.

[20]Christiano, Eichenbaum and Rebelo (2009) consider Calvo price adjustment. But, the linearized system for Calvo and Rotemberg price adjustment costs is identical when the parameters for price adjustment are chosen appropriately.

[21]Both Christiano, Eichenbaum and Rebelo (2009) and Braun and Waki (2010) consider a 5 % shock to

model a 7 % shock to the discount rate produces a government purchase GNP multiplier of 1.8 when the resource costs of price adjustment are ignored. The remaining gap between our multiplier of 1.8 and the multiplier of 2.5 reported in Braun and Waki (2010) is due to differences in the particular experiment that is considered and the parameterization of the model. Braun and Waki (2010) set the government purchases to a high level for a fixed number of periods. In contrast, we assume that government purchases follow an AR(1). This assumption acts to lower the size of the government purchase multiplier.

The form of the Taylor rule also plays a role. We assume the that the central bank acts to smooth the value of the policy rate over time and introduce serial correlation in the Taylor rule to capture this effect. Braun and Waki (2010) and Christiano, Eichenbaum and Rebelo (2009) don't allow for serial correlation in their Taylor rules. Introducing serial correlation to the Taylor rule increases the size of the shock that is required to make the zero lower bound bind.

4.3 Policy Counterfactuals

In the introduction we asked the following three questions about Japan's episode with zero nominal interest rates:

1. Would a lower labor tax have depressed economic activity in Japan?

2. Did the slowdown in the growth rate of TFP that Japan experienced during this period raise GNP?

3. How big was the government purchase GNP multiplier?

The results reported in Table 2 and 3 provide one set of answers to these questions. The results reported in those tables suggest that the answer to the first and second questions is no. The answer to the third question varies depending on the specification of expectations. A limitation of the results reported in those tables is that they correspond to small changes in the exogenous variable. We now consider the following specific counterfactual simulations that are meant to illustrate how alternative sequences of empirically relevant shocks would have altered economic outcomes in Japan between 1999 and 2004. Table 5 reports results for four counterfactuals for the baseline and persistent expectations specifications. The first counterfactual asks what would have happened had a 3% shock to the government purchases rule arrived in each year from 1999 to 2004. For the baseline specification this sequence of shocks to government purchases adds 0.4 % per year to GNP growth. The corresponding average value of the government purchase GNP multiplier is 0.74. For the specification with persistent expectations GNP growth increases by 0.7 % per year and the average GNP multiplier for the years considered is 1.05.

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the discount factor.

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Christiano, Eichenbaum and Rebelo (2009) report that the size of the government purchase multiplier is larger when government purchases are less persistent and Iwamura, Kudo and Watanabe (2006) provide empirical evidence that the persistence of government purchases fell during the zero interest rate period in Japan. It thus makes sense to consider how the results would change if the government purchases process was less persistent. In order to explore the role of persistence in the government purchase process we set the serial correlation coefficient in the government purchase process to zero and repeated the same counterfactual. Under this alternative assumption the size of the government purchase GNP multiplier increases moderately. The average government purchase GNP multiplier is 1.04 in the baseline specification and 1.09 in the persistent expectations specification.

In the second counterfactual we increase the growth rate of technology by 1% per year between 1999 and 2004 using permanent shocks to technology. This results in higher GNP growth of about 1% per year for both specifications.

In the third counterfactual we consider what would have happened if there had been a sequence of positive transitory shocks to technology of equal size that averaged 0.8% per year. These shocks also increase GNP growth for either specification.

Finally, we ask how GNP in the Japanese economy might have changed if the labor tax rate had been lowered on average by 0.8% per year using equally sized shocks. From Table 5 we can see that this counterfactual increases GNP growth by about 1/2 of one percent per year in both specifications.

The results based on these counterfactuals are consistent with the results based on the impulse response functions. The New Keynesian model exhibits orthodox predictions for the response of production and GNP to shocks to the labor tax rate and technology and the average value of the government purchase GNP multiplier ranges between 0.7 and 1.1.

4.4   Implications of the Model for Volatility

Results reported in the previous sections indicate that the GNP government purchase multiplier is larger than one if preference shocks are sufficiently large. Large preference shocks have implications for the volatility of GNP and other aggregate variables. We now turn to document how the implications of the model for volatility statistics vary with the size of the preference shocks and how these properties of the model line up with Japanese data.

Table 6 reports relative volatility statistics for Japanese data and alternative specifications of the model. For each variable we report the standard deviation from 1988 to 1998 relative to the same variable's standard deviation between 1999 and 2007. A relative volatility statistic of less than one means that the respective variable was less volatile during the period of zero nominal interest rates.

The first row of Table 6 reports relative volatility statistics for Japanese data.[22] Observe

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[22]Because we are only concerned with qualitative implications of the relative volatilities, we use a simple

that the period of zero nominal interest rates was a period of tranquility. The relative volatilities of GNP, consumption, real marginal cost and inflation are all well less than one. Technological growth variability is low in the zero interest rate period, too, providing at least a partial explanation for the relative tranquility of the Japanese economy. Labor input is the only variable that exhibits higher volatility during the 1998-2007 sample period.

Notice next that the baseline specification successfully reproduces the low relative volatility observed in Japanese data. For purposes of comparison we also report results for specifications with lower adjustment costs on prices where we set $\gamma = 10$ and a version of the model with flexible prices.[23] All three of these specifications use the same sequence of shocks. Notice that all three specifications successfully reproduce the basic pattern of low relative volatility found in Japanese data.

Consider in turn the specification with persistent expectations. Recall that this specification calibrates the preference shocks to induce expectations of zero nominal interest rates of 5 years in each period between 1999 and 2004 and produces a government purchase GNP multiplier of 1.35 in 1999. According to this specification the period of zero nominal interest rates should have been a period of relatively large volatility. This, however, is not what actually happened in Japan.

In the final row we report relative volatility for the large preference shock specification. This simulation assumes that there is one large shock in 1999 and that the shocks to d_t and $\tau_{c,t}$ are zero in other periods after 1999. The large preference shock specification also predicts that Japan's episode with zero nominal interest rates should have been a period of large economic volatility.[24]

Overall, the results reported in Table 6 suggest that it may be difficult to reconcile the large shock specifications that produce government purchase multipliers greater than one with Japan's episode with zero nominal interest rates.

4.5   Robustness

In this section, we briefly describe the robustness of our conclusions to the choice of the parameterization and the choice of the preference shock processes.

One of the key parameters in our calibration is the magnitude of the price adjustment cost parameter. Table 6 compares volatilities generated by an economy with a more moderate

labor share measure of real marginal costs that accounts for employees in self-employed firms. Following Muto (2009), we calculate real marginal costs as $\frac{1}{\alpha}$Compensation of employees/(National income - households' operating surplus). We would like to thank Ichiro Muto from the Bank of Japan for his helpful comments on the measurement of real marginal costs.

[23]For the flexible price model we do not impose the zero bound constraint

[24]We have also calculated the volatility of the large preference shock specification when the costs of adjusting prices are omitted from the resource constraint. The relative volatility of endogenous variables is even higher in this scenario and wildly at odds with Japan's experience.

adjustment cost parameter of 10 with volatilities generated by our baseline economy when hit by the same sequences of shocks.[25] While they predict similar volatilities for GNP, consumption and labor input, the model with lower costs of price adjustment creates a higher volatility of real marginal costs and inflation volatility that is closer to what is observed in Japanese data.

The dynamic responses for the economy with a low value for the parameter governing the costs of price adjustment, are similar to results reported in Table 2 for the baseline specification. Under the baseline assumption that agents expect zero nominal rates for two years in each year from 1999 to 2005, we find a government purchase GNP multiplier in 1999 equal to 0.79, which is close to the value of 0.86 for the baseline economy. The responses of GNP to other shocks are also very similar to the baseline responses.

The qualitative nature of our results is also robust to the parameterization of the Taylor rule. We have obtained qualitatively similar results using Taylor rules that set the coefficient on the lagged value of the nominal rate to zero, use a different coefficient on inflation or include output growth.

Turning to the choice of the shock processes, we wish to first mention that our assumption that technology follows a unit root process does have an impact on some of our results. Under our current assumption that shocks to technology are permanent agents best guess of tomorrow's state of technology is today's state of technology plus drift. The past is of no help in forming expectations about the future. Technology shocks play a big role in the dynamics of the model and under the assumption of a unit root process in technology agents never expect the zero lower bound to bind in advance of 1999. If instead technological progress is deterministic and shocks to technology are serially correlated agents start to predict zero nominal interest rates several years before the nominal interest rate falls to zero and this acts to change the dynamics of the model before the nominal interest rate is zero. The dynamics start to change as soon as agents expect zero nominal rates in the future. This finding is significant in the sense that it is not necessary for the nominal interest rate to be zero in order for the dynamics of the model to start to shift. All that is necessary is that agents expect the nominal interest rate to be zero at some point in the future.

Finally, we have also conducted simulations in which we kept the tax rate on the consumption constant.[26] This leads to a deterioration in the fit of the model for GNP and labor input. However, the magnitudes of the GNP impulse responses and the GNP multiplier are very close to those reported for our baseline specification.

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[25]A value of $\gamma = 10$ corresponds to a probability of not being able to adjust prices of 0.45 in the Calvo model.

[26]Under this assumption a 3% shock to the discount factor is needed to induce a binding zero nominal 1999 that agents expect to last for two years.

5 Concluding Remarks

In this paper we have conducted a quantitative investigation aimed at assessing the dynamics of the New Keynesian model in a low interest rate environment.

We produced a baseline specification that does a reasonable job of reproducing some basic stylized facts from the Japanese economy between 1990 and 2007. An investigation of the dynamic properties of that specification implies that the response of GNP to a range of shocks is consistent with standard theory. Moreover, the size of the government purchase GNP multiplier is less than one.

We also produced specifications of the model that have orthodox predictions for the response of GNP to labor tax and technology shocks but generate a government purchase multiplier greater than one. These specifications require large shocks to preferences and this property is inconsistent with Japanese data. Japan's episode with zero interest rates was a relatively tranquil period with low variability of output, consumption, inflation and real marginal cost but these specifications imply that the period of zero nominal interest rates is one of relatively large economic volatility.

We have considered this question in an environment where agents receive new news about the state of the economy in each period and use this information to update their forecasts about future economic activity. A limitation of our analysis though is that those forecasts are degenerate and assign all probability to a single sequence of future outcomes. A valuable line of future inquiry would be to consider how to relax this restriction.

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Table 1: Model Parametrization

Symbol	Value	Description
α	0.362	Capital share
δ	0.085	Depreciation rate
ϕ	4	Adjustment costs on capital
β	0.995	Discount factor
ν	0.27	Preference consumption share
σ	2	Preference curvature
γ	80	Adjustment costs on prices
$\theta/(\theta-1)$	1.15	Steady state gross markup
R	0.029	Steady state nominal rate
ρ_R	0.4	Elasticity of the nominal rate with respect to the lagged nominal rate
ρ_π	1.7	Elasticity of the nominal rate with respect to inflation
μ_A	1.02	Steady state growth rate of technology
G/Y	0.19	Steady state government share
τ_w	0.27	Steady state labor income tax
τ_k	0.41	Steady state capital tax
τ_c	0.05	Steady state consumption tax
ρ_A	0.92	Autocorrelation coefficient of transient technology shocks
ρ_G	0.89	Autocorrelation coefficient of government spending
ρ_w	0.9	Autocorrelation coefficient of labor income tax
ρ_k	0.9	Autocorrelation coefficient of capital income tax
ρ_c	0.9	Autocorrelation coefficient of consumption tax

Table 2
Baseline Specification

Year	1995	1999	2004
Years expected nominal rate is zero	none	1999-2000	2004-2005
Resource costs of price adjustment**	0.22	0.60	0.54
Impact response of GNP to a positive shock in:			
Neutral technology (transitory)	0.64	0.57	0.59
Neutral technology (permanent)	0.62	0.68	0.68*
Labor tax	-0.62	-0.56	-0.60*
Government purchases	0.65	0.87	0.82*
Impact response of production to a positive shock in:			
Government purchases	0.62	0.78	0.75*
Impact response of the markup to a positive shock in:			
Neutral technology (transitory)	0.06	0.21	0.19
Neutral technology (permanent)	-0.03	-0.14	-0.13
Labor tax	-0.06	-0.23	-0.15
Government purchases	-0.20	-0.64	-0.47

*For this shock the zero bound constraint applies only in 2004
**Resource costs of price adjustment are reported in percentage terms of GNP

Table 3
Impulse responses to shocks that arrive in 1999 for alternative specifications of the model

	Baseline	Baseline without price adjustment costs in resource constraint	Persistent Expectations	Persistent Expectations without price adjustment costs in the resource constraint	Large preference shock	Large preference shock without price adjustment costs in the resource constraint
	1999-2000	1999-2000	1999-2004	1999-2004	1999-2006	1999-2006
Years expected nominal rate is zero						
Resource costs of price adjustment*	0.60	0.66	2.49	3.86	5.24	10.43
Impact response of GNP to a positive shock in:						
Neutral technology (transitory)	0.57	0.56	0.29	0.12	0.16	-0.39
Neutral technology (permanent)	0.68	0.68	0.88	0.95	1.00	1.16
Labor tax	-0.56	-0.55	-0.28	-0.18	-0.14	0.09
Government purchases	0.87	0.87	1.33	1.45	1.50	1.75
Impact response of production to a positive shock in:						
Neutral technology (transitory)	0.60	0.56	0.52	0.12	0.58	-0.39
Neutral technology (permanent)	0.66	0.68	0.72	0.95	0.71	1.16
Labor tax	-0.59	-0.55	-0.50	-0.18	-0.71	0.09
Government purchases	0.78	0.87	0.87	1.45	0.74	1.75
Impact response of the markup to a positive shock in:						
Neutral technology (transitory)	0.21	0.26	0.58	1.32	0.64	2.69
Neutral technology (permanent)	-0.14	-0.17	-0.42	-0.80	-0.49	-1.34
Labor tax	-0.23	-0.29	-0.63	-1.22	-0.56	-2.02
Government purchases	-0.64	-0.76	-1.35	-2.42	-1.41	3.57

*Resource costs of price adjustment are reported in percentage terms of GNP.

For the specifications without price adjustment costs in the resource constraint, implied resource costs are reported.

Table 4

Decomposition of the GNP response to a government purchases shock

	gnp = (1- Ψ) production		
	d gnp/d g = (1- Ψ) d y/d g - y d Ψ/ d g		
Specification			
Baseline	0.87 = (1 - 0.006) x 0.78 - 1.01 x (- 0.09)		
Persistent expectations	1.33 = (1 - 0.024) x 0.87 - 0.99 x (- 0.46)		
Large preference shock	1.50 = (1 - 0.050) x 0.74 - 0.98 x (- 0.75)		

Table 5
Policy counterfactuals*

	Baseline specification		Persistent expectations specification	
	Average annual percentage difference in GNP between 1999 and 2004	Average annual percentage difference in the variable shocked between 1999 and 2004	Average annual percentage difference in GNP between 1999 and 2004	Average annual percentage difference in the variable shocked between 1999 and 2004
Counterfactual:				
A positive 3% iid shock to government purchases process in each year from 1999 to 2004	0.8%	4%**	0.9%	4%**
A positive, permanent 1% shock to technology in each year from 1999 to 2004	0.8%	1.0%	0.9%	1.0%
A positive, transitory 1% shock to technology in each year from 1999 to 2004	0.5%	0.8%	0.4%	0.8%
A negative 1% shock to labor taxes in each year from 1999 to 2004	0.6%	-0.8%	0.5%	-0.8%

*Numbers are reported as percentage changes relative to the specification without the shocks

**Change in government purchases when a shock hits the g/y rule

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Table 6
Relative standard deviations for Japanese data and alternative specifications of the model*

	GNP	Consumption	Labor Input	Real Marginal Cost	Consumption deflator	CPI	Preference discount shock	Consumption tax shock
Japanese Data	0.539	0.264	1.23	0.668	0.274	0.283	-	-
Specification								
Flexible Price	0.33	0.71	0.45	0	0.16	0.16	0.19	0.09
Moderate Price adjustment cost	0.43	0.76	0.56	0.76	0.26	0.26	0.19	0.09
Baseline	0.50	0.79	0.58	0.19	0.07	0.07	0.19	0.09
Persistent Expectations	1.17	1.43	1.23	1.49	0.63	0.63	0.4	0.18
Large Preference Shock	1.95	1.84	1.83	2.74	1.28	1.28	0.67	0.30

* All statistics are calculated as the standard deviation of the variable from 1999 to 2005 relative to its standard deviation from 1988 to 1998

All variables except marginal cost are expressed in terms of log growth rates
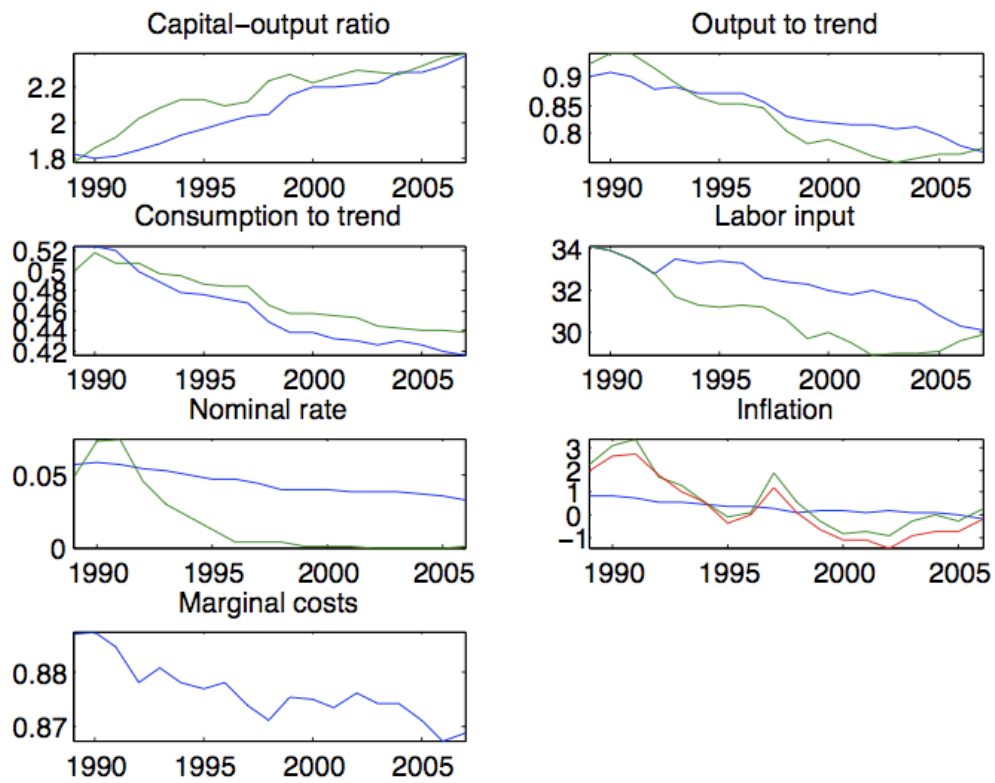
33

Figure 1: Baseline economy if neither preference nor monetary policy shocks arrive after 1991
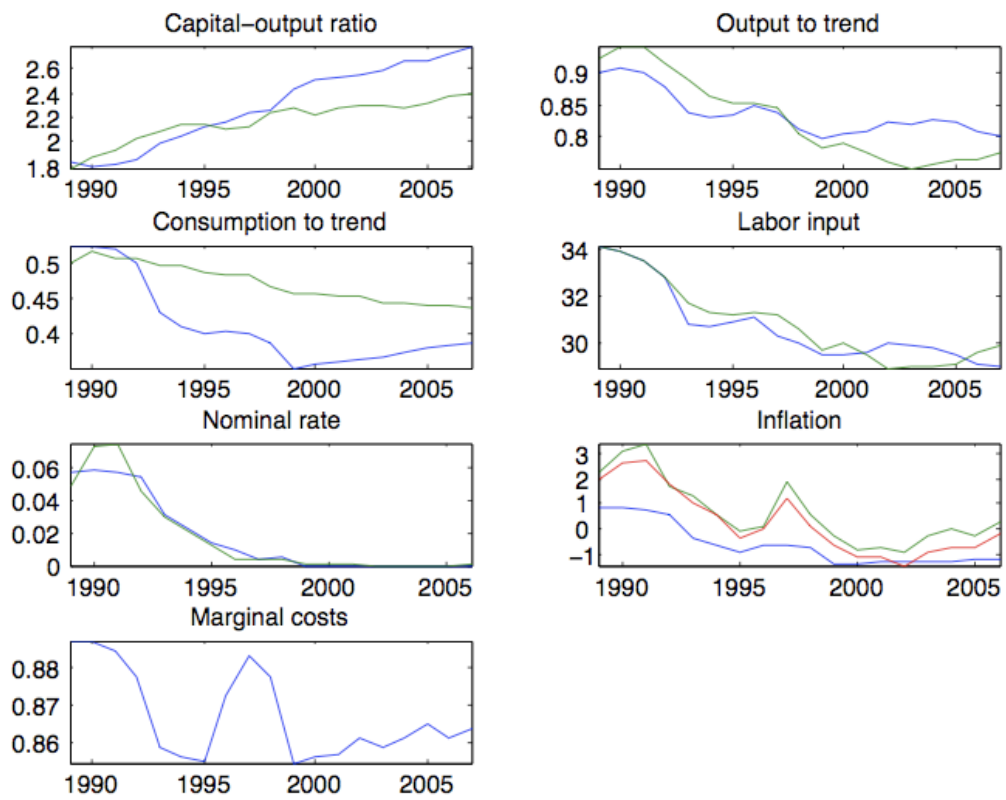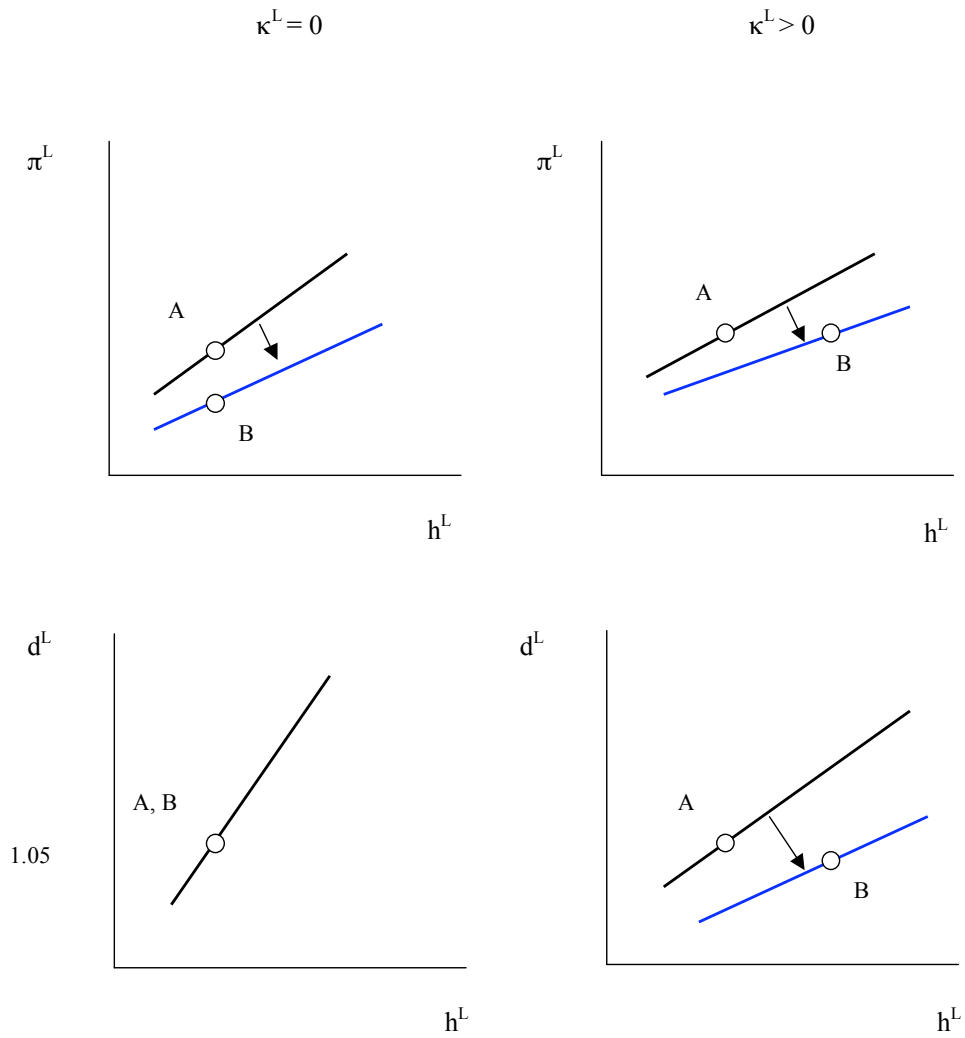
Figure 2: Baseline economy

Figure 3: A tax cut in the analytically tractable New Keynesian economy. The nominal rate is zero in the current period but positive from the next period onwards with probability one. The graphs on the left abstract from the resource costs of price adjustment ($\kappa = 0$) while the graphs on the right account for these costs ($\kappa > 0$). The graphs on the top show the aggregate supply (AS) curve and the graphs on the bottom show the aggregate demand (AD) curve.