

Threatening to Offshore in a Search Model of the Labor Market*

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Abstract

We develop a two-country labor search model in which a multinational firm engages in production sharing by hiring both domestic and foreign labor to produce a final good. A key innovation to the model is the sequential nature of labor markets which allows the ability of the multinational to shift production overseas to enter into its outside option in domestic wage negotiations. This feature allows us to articulate the threat effect in a very tractable way. Using this framework, we derive a model-based estimate of the effect that the threat of offshoring has on global wages and labor market allocations. In the short run, when firm entry and the capital stock are both impeded from fully adjusting to an increase in globalization, we find that the threat has sizable effects: *ceteris paribus*, domestic wages are lower by as much as 8 percent. In contrast, when entry and the capital stock are free to adjust over the long run, we find that the threat effect is muted considerably. These results highlight the importance of taking into account transition dynamics when evaluating the effects of changes in trade policy.

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1 Introduction

Does the threat of offshoring have an important effect on wages and unemployment? Surveys generally indicate that the public thinks so.¹ A 2004 AP poll reported that nearly 70 percent of Americans think offshoring hurts the US economy. Moreover, anecdotal evidence supports this perception. In September 2010, Sergio Marchionne, CEO of the Italian automaker Fiat, explicitly threatened to pull all production out of Italy and offshore it to lower-cost plants located in Serbia and Poland. In doing so, he obtained major concessions from the Italian unions in labor negotiations.² Clearly, in an environment of increased globalization the ease with which multinational firms can move production plants offshore should strengthen their outside options in wage negotiations.

Yet, standard models of international macroeconomics are ill-suited to address some important channels through which offshoring can impact labor market outcomes. For instance, labor markets in standard models are assumed to be perfectly competitive and wages are determined in spot markets. As a result, fear that a firm may relocate a job abroad doesn't enter into the wage determination process. Yet, as the Fiat example suggests, one channel through which offshoring may have an important impact on wages is via the associated loss in workers' bargaining power and the decline in economic rent that accrues to them. In a recent attempt to quantify this channel, Blinder (2009) estimates that offshorability in the services sector, that is, the characteristics of a job that makes it more likely to be offshored, may lower wages by up to 14 percent for the service jobs most at risk of being moved abroad.

In this paper, we complement this empirical work by analyzing the effect of the threat of offshoring on wages and unemployment in an open economy model in which the labor market is subject to search frictions à la Diamond-Mortenson-Pissarides and in which firms and workers bargain over wages. In the search framework employment relationships generate a surplus that must be divided between a worker and a firm. This surplus, and more specifically the fact that it must be split between the two parties, is essential to modeling the threat of offshoring.³ In our framework, multinational firms need to post vacancies to fill job openings, but can do so either in the domestic or foreign markets. Since firms operate both domestic and foreign plants, offshoring

¹Not surprisingly, this sentiment has worked its way into the political arena. Mankiw and Swagel (2005) called offshoring the single most important, and least understood, economic issue for the 2004 US presidential campaign. Most recently, in late 2010, the Obama administration proposed legislation, the Creating American Jobs and End Offshoring Act, that would impose a direct tax on firms that are engaged in offshoring domestic jobs.

²"Fiat: Marchionne's gamble", Financial Times, Sept. 29, 2010.

³We choose to generate the surplus via search frictions because it is analytically convenient. That said, we could have relied on any number of labor market imperfections—such as models of efficiency or fair wages, or models of union influence. We leave that for future work.

in our model captures an intra-firm production-sharing activity whereby the parent company is able to shift production from the domestic country to its foreign affiliates.

Within this open economy labor search framework, we model the threat of offshoring by introducing two additional innovations. First, we assume that, in order to create a new position, firms must have capital in place and therefore must pay a cost prior to entry. This cost of entry implies that once a firm has entered the market, an unfilled vacancy retains a strictly positive value under free entry. Second, we introduce a sequential matching problem where firms first post vacancies in the domestic market (the day market), but have the outside option of waiting to subsequently fill the vacancies with foreign workers (the night market). Taken together, these two innovations allow us to formalize the threat of offshoring in a tractable way.

Our main result is that the threat of offshoring production can put significant downward pressure on wages in the source country, *even if the existing amount of offshoring is very small*. In our benchmark calibration, offshored production accounts for only one percent of total output. Nevertheless, we show that, *ceteris paribus*, the ability of the multinational to exercise the outside option of offshoring domestic production lowers the domestic wage in the bargaining process by nearly 8 percent. However, this appears to be largely a short-run effect. In the long run, we find that the quantitative magnitude of impact that the threat of offshoring has on domestic wages is muted considerably when firm entry and the capital stock are allowed to freely adjust. This suggests that the threat of offshoring is primarily a short-run phenomena and, as such, it is important to consider the threat effect taking into account transition dynamics.⁴ We will address this in a subsequent draft of the paper.

Our paper adds to a growing literature that builds on early work by Davidson, Martin, and Matusz (1988) by embedding labor market search frictions into open economy models (see, e.g., Helpman and Itskhoki (2010), Helpman, Itskhoki, and Redding (2011, 2010a, 2010b), Boz, Durdu, and Li (2009), Dutt, Mitra, and Ranjan (2009), and Mitra and Ranjan (2010)). Much of this work has concentrated on the impact of labor market frictions on trade flows, although Mitra and Ranjan (2010) explicitly considers offshoring. Our work, like Felbermayr, Prat, and Schmerer (2010), differs in that it focuses instead on wage formation. In particular, what is unique about our work is that by concentrating specifically on the impact of the threat of offshoring on wage negotiation outcomes we are able to provide a model-based answer to a policy-relevant question that has thus far proved largely elusive.⁵ To this end, our model is also related to the earlier work of Borjas and Ramey (1995) who studied the impact of trade on firms' rent, wages, and employment in

⁴The importance of taking into account transition dynamics in open economy models with equilibrium unemployment is a point that is stressed in multiple chapters of Davidson and Matusz (2010).

⁵Davidson, Matusz, and Shevchenko (2008) look at the the influence of offshoring on wages through the firm's outside option, but this analysis is of the partial equilibrium labor market.

a model in which firms and unions bargain over pay and the number of workers employed. Finally, our results complement the perviously mentioned empirical findings of Blinder (2009) who classifies the offshorability of jobs and its impact on wages and employment.

The idea that the value of outside options is important in wage negotiations has recently been challenged by Hall and Milgrom (2008). They argue that threatening to walk away from the negotiating table once a match has been formed is not credible. Instead, the more credible threat is to extend bargaining: job-seekers' best option is to try to hold on for a better deal, while firms should delay negotiations as long as possible. This approach to wage bargaining lowers the influence of outside options on negotiated outcomes and is useful for solving the well known Shimer (2006) puzzle in dynamic labor search models. However, in the case of the firms' ability to move production offshore, the value of offshoring may be so high that the threat of terminating employment becomes credible as demonstrated by Fiat's threat to Italian workers. Moreover, using Swedish data, Lachowska (2010) presents empirical evidence indicating that outside options are important in the wage formation process.

The remainder of this paper is organized as follows. The next section presents the model. Section 4 describes the baseline calibration and presents the main results. In Section 5, we examine the sensitivity of our baseline results to some key parameters of the model. In section 6, we use the model to conduct some simple policy analysis such as the effects of a trade liberalization and the impact of anti-offshoring legislation. Finally, Section 7 concludes.

2 The Model

We extend the textbook Diamond-Mortenson-Pissarides labor search model in three primary ways. First, we extend it to a two country setting and introduce a multinational firm residing in the Home country that engages in international production sharing. Second, we introduce a fixed cost of entry into the labor market, which has the implication that free entry does not drive the value of an unfilled vacancy to zero. Finally, we alter the intra-period timing of the model by introducing a sequential setup whereby the market for domestic jobs meets in the morning of each period and the market for offshore jobs meets in the evening. Taken together, these three ingredients allow us to capture, in a tractable manner, the idea that the ability of the multinational to shift production internationally alters its outside option in wage negotiations. It is through this outside option that we formalize the threat effect of offshoring on wages and labor market allocations.

2.1 Model Timing

There are two aspects regarding the timing of the model that require discussion. First, as in Davidson, Matusz, and Shevchenko (2008), and Rosen and Wasmer (2005), we differentiate between the short run, in which firm entry and the adjustment of the capital stock is impeded, and the long run, in which entry and capital are free to adjust in absence of frictions. Our notion of the short-run can be thought of as the transition between two long-run equilibria that may differ as a result of some large permanent shock or policy change. As will be made clear later on, the differentiation between the short and long run—and, in particular, the ability of firms to freely enter each labor market—is critical to thinking about the role of the threat effect on labor markets.

The second aspect of the model timing that requires further discussion is the intra-period timing summarized in [FIGURE X]. In our model there are three segmented labor markets: one market each for domestic jobs located in the Home and Foreign country, respectively, as well as one market for offshore jobs located in the Foreign country. Each labor market is characterized by search frictions whereby firms must pay a per period cost to post vacancies and households must expend time and effort in order to match with these open vacancies. Following the money search literature, we assume that each time period is broken up into two subperiods which we refer to as the morning and evening, respectively.⁶

In order to formalize the threat of offshoring, we assume that the market for domestic jobs meets in the morning while the market for offshored jobs meets in the evening. Moreover, we assume that a certain fraction, Ω , of domestic jobs are “offshorable” in the sense of Blinder (2007)—that is, only a fraction of domestic jobs exhibit characteristics that make them particularly susceptible to being easily relocated abroad. Thus, conditional on the job being offshorable, even if a domestic vacancy goes unfilled in the morning the multinational still has an opportunity to fill that open vacancy with a foreign worker later in that evening. This sequential markets setup, in conjunction with the fact that the fixed cost of entry implies that the value of an unfilled vacancy is not driven to zero, alters the multinational’s outside option when bargaining over the wage with domestic workers in the morning market.

⁶In xxx and yyy, a decentralized search market (in which money is essential for conducting goods transactions) meets in the morning, while a centralized market meets in the evening. This timing assumption is made for technical reasons; with quasi-linear utility, evening trade in the centralized market serves to kill the wealth distribution that arises due to trade in the decentralized morning market. Thus, the timing assumption is made in order to make the model more tractable. Our motivation for introducing a sequential market structure is similar: we want to formalize the threat of offshoring in the most tractable way possible.

2.2 Notation

In the notation that follows, subscript D's denote variables in either the Home or Foreign *domestic market*; subscript O's denote variables in the *offshore market* located in the Foreign country. Asterisks (*) denote variables that are physically located in the Foreign country, while the lack of an asterisk denotes variables that are physically located in the Home country. Finally, where applicable we differentiate short run variables with a hat, so that $\hat{w}_{D,t}$ is the short run wage in the domestic labor market in the Home country. Lack of a hat indicates a long run variable.

2.3 Production

Firms in the Home country (the North) are multinationals in the sense that they engage in international production sharing. The multinational operates plants in both countries, each of which produce an intermediate input using both local capital and labor. These intermediate inputs are then shipped back to the Home country and processed into a final good. This final good is, in turn, sold internationally. In contrast, the Foreign final good is processed using intermediate goods that are produced, also using both capital and labor, entirely in domestic plants. Thus, for tractability, we assume that offshoring activity in the model is North-South only.

2.3.1 The Multinational Firm

The multinational produces a final output good, denoted y_t , using intermediate goods produced both domestically, $y_{D,t}$, and abroad, $y_{O,t}^*$. The offshored intermediate good is potentially subject to an iceberg shipping cost, denoted Υ , so that, in terms of general notation, the technology for the production of the final good is given by $y_t = f(y_{D,t}, (1 - \Upsilon)y_{O,t}^*)$. Once the intermediate goods are combined, the final output is sold in perfectly competitive goods markets both at home and abroad.

At the intermediate goods level, regardless of where production takes place, plants must undergo a costly process for hiring labor in a frictional market. Once hired, labor is then matched with capital which is rented from domestic households in a frictionless capital market. Together, these two inputs are used to produce the intermediate good. Let intermediate goods produced at domestic and offshore plants, respectively, be denoted by:

$$y_{D,t} = z_{D,t}g(n_{D,t}, k_{D,t}); \quad y_{O,t}^* = z_{O,t}^*g(n_{O,t}^*, k_{O,t}^*) \quad (1)$$

where: $z_{D,t}$ and $z_{O,t}^*$ are technology shocks that can potentially differ across the multinational's domestic and foreign plants, respectively; $n_{D,t}$ and $n_{O,t}^*$ denote the stock of labor in domestic and offshored jobs; and $k_{D,t}$ and $k_{O,t}^*$ denote the capital stock for domestic and offshore plants.

In order to match a worker with capital, plants in either country must first create a position by paying an entry cost. The entry cost requires putting a stock of capital in place for the worker to use in production.⁷

Once a position is created and capital is in place, only then can a vacancy be posted so that a worker can be hired. In terms of notation, let $v_{D,t}$ and $v_{O,t}^*$ denote vacancies posted directly to the domestic and offshore labor markets, respectively. Let $k^f(\theta_{D,t})$ denote the probability that a vacancy posted by the multinational is matched with a worker in the domestic labor market. This probability depends on labor market tightness, which for the domestic labor market is defined as $\theta_{D,t} = v_{D,t}/s_{D,t}$ where $s_{D,t}$ is the total number of individuals searching for domestic jobs, as discussed in Section 2.4 below. Similarly, let $k^f(\theta_{O,t}^*)$ denote the probability that a vacancy posted by the multinational in the offshore labor market is matched with a Foreign worker.

The sequential nature of markets means that even if a vacancy that is posted directly to the domestic job market goes unfilled in the morning market, which happens with probability $1 - k^f(\theta_{D,t})$, the multinational still has an opportunity to fill that opening with a foreign worker in the evening, *provided the job is offshorable*. Recalling that Ω is the fraction of offshorable jobs, the total number of open vacancies in the offshore market, $\tilde{v}_{O,t}^*$, is the sum of vacancies posted directly in that market, $v_{O,t}^*$, and those that rolled over from the morning market, so that $\tilde{v}_{O,t}^* = v_{O,t}^* + \Omega(1 - k^f(\theta_{D,t}))v_{D,t}$. As will be made clear later, this link between the market for domestic and offshore jobs is critical for modeling the threat effect. Under the assumption that $\Omega = 0$ the intra period timing becomes irrelevant as the three labor markets are completely segmented from one another. The probability that a vacancy is filled in the offshore market is given by $k^f(\theta_{O,t}^*)$. This probability is a function of market tightness in the market for offshored jobs, defined as $\theta_{O,t}^* = \tilde{v}_{O,t}^*/s_{O,t}^*$ where $s_{O,t}^*$ is the number of individuals searching for offshore jobs, as will be discussed in greater detail in Section 2.4 below.

The resulting perceived laws of motion for the multinational's employment stock of domestic and offshore workers, respectively, are given by

$$n_{D,t} = (1 - \rho^x)(1 - \rho^n)n_{D,t-1} + v_{D,t}k^f(\theta_{D,t}) \quad (2)$$

$$n_{O,t}^* = (1 - \rho^{x*})(1 - \rho^{n*})n_{O,t-1}^* + \tilde{v}_{O,t}^*k^f(\theta_{O,t}^*) \quad (3)$$

These laws of motion simply say that employment at time t depends on the number of remaining jobs today plus the number of matches the firm expects to make by posting vacancies to the respective markets. The number of remaining domestic jobs today is equal to yesterday's end-of-period employment stock, $n_{D,t-1}$, net of the total number of jobs that are exogenously terminated at the beginning of period t . Job termination may occur as a result of an existing job becoming

⁷This aspect of the model builds on Rosen and Wasmer (2005) and Fujita and Ramey (2006).

obsolete, which occurs with probability ρ^x . Alternatively, even if a job remains operable, it may separate exogenously, which occurs with probability ρ^n . We require job separation along both margins: the first margin allows for a flow equilibrium in entry while the second allows for a flow equilibrium in employment conditional on entry. A similar set of notation applies to the probability of job termination due to obsolescence, ρ^{x*} , or separation, ρ^{n*} , for jobs in the offshore market.

As discussed in Fujita and Ramey (2007), a direct consequence of introducing the sunk cost of entry is that vacancies become a state variable.⁸ The associated laws of motion for vacancies posted domestically and abroad are given by:

$$v_{D,t} = (1 - \rho^x)\rho^n n_{D,t-1} + (1 - \rho^x)(1 - k^f(\theta_{D,t-1}))(1 - \Omega k^f(\theta_{O,t-1}^*))v_{D,t-1} + ne_{D,t} \quad (4)$$

$$v_{O,t}^* = (1 - \rho^{x*})\rho^{n*} n_{O,t-1}^* + (1 - \rho^{x*})(1 - k^f(\theta_{O,t-1}^*))v_{O,t-1}^* + ne_{O,t}^* \quad (5)$$

The stock of vacancies in a given market tomorrow is equal to newly opened vacancies resulting from non-obsolete jobs that have separated exogenously (which occurs with probability $(1 - \rho^x)\rho^n$ in the domestic market, for example) plus the sum of the stock of existing unfilled vacancies inherited from yesterday and newly created vacancies associated with entrants, denoted $ne_{D,t}$ and $ne_{O,t}^*$ for entrants into the domestic and offshore markets, respectively. Note that in equation (4) we also need to take into account the fact that, for domestic jobs that are offshorable, unfilled vacancies in the domestic market can potentially be filled in the evening.⁹

The multinational's optimization problem, therefore, is choose sequences of $k_{D,t}$, $k_{O,t}^*$, $n_{D,t}$, $n_{O,t}^*$, $v_{D,t}$, and $v_{O,t}^*$ to maximize discounted lifetime profits, defined as:

$$\Pi_t = \sum_{t=0}^{\infty} \beta^t \frac{\lambda_t}{\lambda_0} [f(y_{D,t}, (1 - \Upsilon)y_{O,t}^*) - w_{D,t}n_{D,t} - q_t w_{O,t}^* n_{O,t}^* - r_{D,t}^k k_{D,t} - q_t r_{O,t}^{k*} k_{O,t}^* - \gamma_D v_{D,t} - \gamma_O^* \tilde{v}_{O,t}^*] \quad (6)$$

subject to: the technologies for producing intermediate goods at home and abroad, given in equation (1); the laws of motion for domestic and offshore employment, given by equations (2) and (3), respectively; the laws of motion for domestic and foreign vacancies, equations (4) and (5); and the identity $\tilde{v}_{O,t}^* = v_{O,t}^* + \Omega(1 - k^f(\theta_{D,t}))v_{D,t}$.

In the multinational's profit function, once the cost of entry is paid and capital is put in place so that a new job opening is created, the firm must pay a per-period posting cost denoted by γ_D (γ_O^*)

⁸Fujita and Ramey (2007) introduced an exogenous fixed cost of vacancy creation to introduce persistence into vacancy postings over the business cycle in an effort to better fit the data. Our purposes for introducing (an endogenous) cost of entry is entirely different. In our paper, for the threat of offshoring to have any effect it must be the case that free entry does not drive the value of the vacancy to zero in the steady state. Thus, introducing this feature into the model serves a different purpose here than in Fujita and Ramey (2007).

⁹For jobs that are not offshorable, the probability that a vacancy goes unmatched in a given period is $(1 - k^f(\theta_{D,t}))$, while the same probability for jobs that are offshorable is given by $(1 - k^f(\theta_{D,t}))(1 - k^f(\theta_{O,t}^*))$. Weighting the two probabilities by $1 - \Omega$ and Ω , respectively, and adding resulting expressions gives $(1 - k^f(\theta_{D,t}))(1 - \Omega k^f(\theta_{O,t}^*))$, which appears in equation 4 weighted by the probability of non-obsolescence.

for vacancies posted domestically (abroad). Entry costs and vacancy posting costs in both markets are a drain on real resources in the Home country. The rental rates of domestic and offshore capital are given by $r_{D,t}^k$ and $r_{O,t}^{k*}$. Finally, all factor payments made in the offshore market are made in units of the foreign currency, so the multinational must internalize movements in the real exchange rate, q_t , when making its optimal offshoring decision.

Details of the solution are shown in Appendix A. Beginning with the multinational's optimal offshoring decision, the first order conditions for $v_{O,t}^*$ and $n_{O,t}^*$, respectively, are given by

$$\lambda_{O,t}^* = -\gamma_O^* + k^f(\theta_{O,t}^*)\mu_{O,t}^* + (1 - k^f(\theta_{O,t}^*))(1 - \rho^{*x})E_t[\Xi_{t+1|t}\lambda_{O,t+1}^*] \quad (7)$$

$$\mu_{O,t}^* = f_{n_{O,t}^*} - q_t w_{O,t}^* + E_t \left[\Xi_{t+1|t} \left((1 - \rho^{*x})\rho^{*n}\lambda_{O,t+1}^* + (1 - \rho^*)\mu_{O,t+1}^* \right) \right] \quad (8)$$

where: $\lambda_{O,t}^*$ is the multiplier on equation (5) and $\mu_{O,t}^*$ is the multiplier on equation (3).

The first equation says that the value of an unfilled vacancy in the offshore market is equal to its expected return net of the per period cost of posting a vacancy. The expected return on an open vacancy is equal to the probability that vacancy is filled today times the value of the resulting job plus the expected continuation value of the vacancy tomorrow, conditional on it not being filled today and not being rendered obsolete. The second equation says that the value of an additional offshore worker to the multinational is equal to the worker's marginal product net of the wage (paid in local currency) plus the expected continuation value of the job. The continuation value is the stream of additional marginal revenue brought in over the expected life of the match plus, in the event that the match breaks up, stream of benefit that comes from having an unfilled vacancy.

Importantly, both jobs and unfilled vacancies deliver a flow of value over time. This is a key difference between our setup and a more standard labor search model in which there is no fixed cost of entry. To make this point more explicit, note that in absence of the entry cost we would have $\lambda_{O,t}^* = 0$ which would imply $\mu_{O,t}^* = \gamma^*/k^f(\theta_{O,t}^*)$ by equation (7). Plugging this into equation (8) results in a job creation condition that arises in most standard general equilibrium labor search models. Thus, to the degree that the *offshore job creation condition* looks different from a standard search model, it is due to the non-zero continuation value of a vacancy.

Turning to the multinational's search activity in the domestic market, the first order conditions for $v_{D,t}$ and $n_{D,t}$, respectively, are given by

$$\lambda_{D,t} = -\gamma - \Omega(1 - k^f(\theta_{D,t}))\gamma_O^* + k^f(\theta_{D,t})\mu_{D,t} + \Omega(1 - k^f(\theta_{D,t}))k^f(\theta_{O,t}^*)\mu_{O,t}^* + (1 - k^f(\theta_{D,t}))(1 - \Omega k^f(\theta_{O,t}^*))(1 - \rho^x)E_t[\Xi_{t+1|t}\lambda_{D,t+1}] \quad (9)$$

$$\mu_{D,t} = f_{n_{D,t}} - w_{D,t} + (1 - \rho^x)E_t \left[\Xi_{t+1|t} \left(\rho^n \lambda_{D,t+1} + (1 - \rho^n) \mu_{D,t+1} \right) \right] \quad (10)$$

where: $\lambda_{D,t}$ is the multiplier on equation (4) and $\mu_{D,t}$ is the multiplier on equation (2).

The value of a vacancy in the domestic market differs from the value of a vacancy in the offshore market in one important way. The last term on the right side of the first line in equation (9) captures the idea that, to the degree that a job exhibits characteristics that make it offshoreable, the ability to fill a vacancy originally posted in domestic market with a foreigner changes the outside option of the firm. This outside option increases the value of an unfilled domestic vacancy and is the primary lever through which the threat of offshoring influences wages and labor market allocations in our model. Note that if $\Omega = 0$, so that no jobs are offshoreable, then the outside option disappears and equation (9) will look very similar to equation (7) above. Thus, to the degree that the *domestic job creation condition* looks different from a standard search model, it is due to both the continuation value of a vacancy as well as the sequential nature of labor markets.

Finally, the multinational's optimal capital demand equations are given by:

$$f_{k_D,t} = r_{D,t}^k \quad (11)$$

$$fk_{O,t}^* = qt r_{O,t}^{k^*} \quad (12)$$

2.3.2 The Foreign Firm

The final goods producing firm in the Foreign country uses only domestically-produced intermediate goods, $y_{D,t}^*$, to produce the final good, y_t^* . The intermediate good is produced using domestic labor and capital, so that $y_{D,t}^* = z_{D,t}^* g^*(n_{D,t}^*, k_{D,t}^*)$ and is assumed to be transformed unit-for-unit into the final good, so that $y_t^* = f(y_{D,t}^*) = y_{D,t}^*$.

The foreign firm's optimization problem is to choose sequences $k_{D,t}^*$, $n_{D,t}^*$, and $v_{D,t}^*$ to maximize discounted lifetime profits subject to the production technology and the laws of motion for both domestic employment and vacancies.

$$\Pi_t^* = \sum_{t=0}^{\infty} \beta^{*t} \frac{\lambda_t^*}{\lambda_0^*} \left[f(y_{D,t}^*) - w_{D,t}^* n_{D,t}^* - r_t^{k^*} k_{D,t}^* - \gamma_D^* v_{D,t}^* \right] \quad (13)$$

subject to:

$$y_{D,t}^* = z_{D,t}^* g(n_{D,t}^*, k_{D,t}^*) \quad (14)$$

$$n_{D,t}^* = (1 - \rho^{x^*})(1 - \rho^{n^*})n_{D,t-1}^* + v_{D,t}^* k^f(\theta_{D,t}^*) \quad (15)$$

$$v_{D,t}^* = (1 - \rho^{x^*})\rho^{n^*}n_{D,t-1}^* + (1 - \rho^{x^*})(1 - k^f(\theta_{D,t-1}^*))v_{D,t-1}^* + ne_{D,t}^* \quad (16)$$

where: $k^f(\theta_{D,t}^*)$ is the probability that a job posting will be matched with a Foreign worker in the domestic labor market; γ_D^* denotes the vacancy posting cost in the Foreign labor market; and ne_t^* is entry into the Foreign domestic market.

As shown in Appendix A, the firm's first order conditions for $v_{D,t}^*$ and $n_{D,t}^*$, respectively, are given by:

$$\lambda_{D,t}^* = -\gamma^* + k^f(\theta_{D,t}^*)\mu_{D,t}^* + (1 - k^f(\theta_{D,t}^*))E_t[\Xi_{t+1|t}^*(1 - \rho^{*o})\lambda_{D,t+1}^*] \quad (17)$$

and

$$\mu_{D,t}^* = f_{n_{D,t}^*} - w_{D,t}^* + E_t\left[\Xi_{t+1|t}^*\left((1 - \rho^{*o})\rho^{*n}\lambda_{D,t+1}^* + (1 - \rho^{*o})(1 - \rho^{*n})\mu_{D,t+1}^*\right)\right] \quad (18)$$

where $\mu_{D,t}^*$ is the multiplier on equation (15) and $\lambda_{D,t}^*$ is the multiplier on equation(4).

Equations (17) and (18) have similar interpretations as the multinational's first order conditions given by 9 and 10. However, note that the foreign firm does not search sequentially within the period, it only searches in the morning market for domestic workers.

Finally, the optimal capital accumulation equation is given by

$$f_{k_{D,t}^*} = r_{D,t}^{k^*} \quad (19)$$

2.4 Households

There is a continuum of identical households in both the Home and Foreign economies. The representative household in each country consists of a continuum of measure one of family members. During a given time period, each member of the household either works, is actively searching for a job, or is out of the labor force enjoying leisure. Individuals in the Home country search for jobs operated domestically by the Home multinational while individuals in the Foreign country optimally allocate search activity across two separate labor markets: one for jobs operated by Foreign firms producing domestically and one for jobs that have been offshored to the foreign plant by the Home multinational. We rule out on-the-job search and assume that total household income in each country is divided evenly amongst all individuals, so each individual within a country has the same consumption. This later assumption follows Andolfatto (1996) and Merz (1995) and is common in general equilibrium search-theoretic models of labor markets.

2.4.1 Home Households

Aggregate consumption in the Home country is measured by a composite consumption index that is a CES aggregate of both a domestic and foreign final good

$$c_t \equiv \left(\lambda^{\frac{1}{\zeta}} c_{H,t}^{\frac{\zeta-1}{\zeta}} + (1 - \lambda)^{\frac{1}{\zeta}} c_{F,t}^{\frac{\zeta-1}{\zeta}} \right)^{\frac{\zeta}{\zeta-1}} \quad (20)$$

where the parameter $\lambda \in (0, 1)$ governs the share of the Home final good in the composite consumption index and $\zeta > 0$ is the constant elasticity of substitution between the Home and Foreign final good, $c_{H,t}$ and $c_{F,t}$, respectively. There exists an identical consumption index with parameters

λ^* and ζ^* denoting Foreign aggregate consumption, c_t^* , which aggregates Foreign consumption of the Home and Foreign produced final goods, $c_{H,t}^*$ and $c_{F,t}^*$, respectively.

We normalize $p_{H,t} = 1$, so that all goods in the economy are valued in terms of the Home produced final good. With this normalization, the aggregate consumption-based price index in the Home country is given by

$$p_t \equiv \left(\lambda + (1 - \lambda) p_{F,t}^{(1-\zeta)} \right)^{1/(1-\zeta)} \quad (21)$$

where $p_{F,t}$ is the price of imports from the Foreign country relative to the price of domestically produced goods; equivalently, $p_{F,t}$ is the terms of trade for the Home country.

Demand functions for the Home and Foreign final consumption goods are given by

$$c_{H,t} = \lambda \left(\frac{1}{p_t} \right)^{-\zeta} c_t, \quad c_{F,t} = (1 - \lambda) \left(\frac{p_{F,t}}{p_t} \right)^{-\zeta} c_t \quad (22)$$

Workers in the Home country search only for jobs operated domestically by the multinational. In terms of notation, let $s_{D,t}$ denote the time spent searching to achieve the desired level of employment with the domestic firm, $n_{D,t}$, and let $k^w(\theta_{D,t})$ denote the probability that a searching individual will be matched in a domestic job. Finally, we define labor force participation as $lfp_t = (1 - k^w(\theta_{D,t}))s_{D,t} + n_{D,t}$. That is, participation is unsuccessful searchers (unemployed) plus those actively working in jobs (employed).¹⁰

The utility of the representative household is given by

$$E_0 \sum_{t=0}^{\infty} \beta^t [u(c_t) - h(lfp_t)] \quad (23)$$

We assume that households can purchase state-contingent bonds b_{t+1} that are traded internationally, so that asset markets are complete. The household chooses sequences of c_t , k_{t+1} , b_{t+1} , $s_{H,t}$, and $n_{H,t+1}$ to maximize lifetime utility subject to an infinite sequence of flow budget constraints and perceived laws of motion for domestic jobs:

$$p_t c_t + p_t (k_{D,t+1} - (1 - \delta) k_{D,t}) + \int p_{bt,t+1} b_{t+1} = w_t n_{D,t}^w + r_{D,t}^k p_t k_{D,t} + (1 - k^w(\theta_{D,t})) s_{D,t} \chi + b_t + d_t \quad (24)$$

$$n_{D,t}^w = (1 - \rho) n_{D,t-1}^w + s_{D,t} k^w(\theta_{D,t}) \quad (25)$$

where: $k_{D,t}$ is the domestic capital stock; δ is the rate of depreciation of the capital stock; $p_{bt,t+1}$ is the price of the state-contingent bond that pays one unit of the domestic consumption good in a particular state of nature at time $t+1$; w_t is the real wage paid to a worker in the Home country; $r_{D,t}^k$ is the real return on a unit of capital; χ is the unemployment benefit that accrues to individuals

¹⁰The timing of labor market activity allows for instantaneous matching. To avoid double counting, we need to net out successful searchers (ie, those that find jobs with probability $k^w(\theta_{D,t})$) from labor force participation. As in Arseneau and Chugh (2010), we use this timing convention for analytical convenience—in the case of this paper, it helps us to express the threat effect in a tractable way.

actively searching for employment; and, finally, d_t denotes the dividend paid to households by intermediate goods producing firms. For convenience, we have introduced the parameter $\rho = \rho^o + (1 - \rho^o)\rho^n$ to denote the total exogenous probability of job termination, inclusive of both job obsolescence and exogenous destruction.

As shown in Appendix B, the first order conditions on c_t and b_{t+1} can be manipulated into a standard consumption Euler equation

$$\frac{u'(c_t)}{p_t} = \beta E_t \left[\frac{1}{p_{bt,t+1}} \frac{u'(c_{t+1})}{p_{t+1}} \right] \quad (26)$$

which defines the one period ahead stochastic discount factor, $\Xi_{t+1|t} = \beta E_t \left[\frac{u'(c_{t+1})}{u'(c_t)} \frac{p_t}{p_{t+1}} \right]$.

Combining the first order conditions on c_t , b_{t+1} , and $k_{D,t+1}$ yields the standard no arbitrage condition between capital and bond holdings

$$\frac{1}{p_{bt,t+1}} = E_t \left[1 - \delta + r_{D,t+1}^k \right] \quad (27)$$

Finally, combining the first order conditions on $s_{D,t}$ and $n_{D,t}^w$ yields an optimal search condition in the labor market for domestic intermediate goods production

$$\frac{1-k^h(\theta_{D,t})}{k^h(\theta_{D,t})} \frac{h'(lfp_t) - \chi \frac{u'(c_t)}{p_t}}{\frac{u'(c_t)}{p_t}} = w_t - \frac{h'(lfp_t)}{\frac{u'(c_t)}{p_t}} + (1 - \rho) E_t \left[\Xi_{t+1|t} \frac{1-k^h(\theta_{D,t+1})}{k^h(\theta_{D,t+1})} \frac{h'(lfp_{t+1}) - \chi \frac{u'(c_{t+1})}{p_{t+1}}}{\frac{u'(c_{t+1})}{p_{t+1}}} \right] \quad (28)$$

The interpretation is standard. Optimal search on the part of the Home household equates the marginal utility of an additional unit of time spent searching net of the unemployment benefit to the expected gain of search. The expected gain is the wage net of the disutility of labor effort expended in the job plus the continuation value of entering into a long-lasting working relationship with a firm.¹¹

2.4.2 Foreign Households

The Foreign household solves a similar problem as the Home household, but—just as with the multinational—the Foreign household's problem involves optimally allocating search activity across *two segmented labor markets*. In addition, the Foreign household invests in two separate capital stocks for use in intermediate goods production by the domestic firm and the multinational, respectively.

In terms of notation, let $s_{D,t}^*$ denote search activity in the market for domestic jobs operated by the Foreign firm, and let $s_{O,t}^*$ denote search activity in the market for offshored jobs operated by the multinational. Similarly, let $k^w(\theta_{D,t}^*)$ and $k^w(\theta_{O,t}^*)$ denote the probability of successful search on the

¹¹The $1 - k^h(\theta_{D,t})$ term in the numerator of the right hand side of equation (28) and in the continuation value shows up due to the instantaneous timing assumption. See Appendix B for details

part of households in the market for domestic and offshored jobs, respectively. Define labor force participation in the Foreign country as $lfp_t^* = (1 - k^w(\theta_{D,t}^*))s_{D,t}^* + (1 - k^w(\theta_{O,t}^*))s_{O,t}^* + n_{D,t}^* + n_{O,t}^*$. Total unemployment is the sum of the measure of unsuccessful searchers in both markets; similarly, total employment is the sum of the measure of employed in both markets.

The Foreign household's problem is to choose sequences of c_t^* , b_{t+1}^* , $k_{D,t+1}^*$, $k_{O,t+1}^*$, $s_{O,t}^*$, $s_{D,t}^*$, $n_{O,t+1}^*$, and $n_{D,t+1}^*$ to maximize:

$$E_0 \sum_{t=0}^{\infty} \beta^{*t} [u^*(c_t^*) - h^*(lfp_t^*)] \quad (29)$$

subject to:

$$p_t^* c_t^* + k_{D,t+1}^* + k_{O,t+1}^* - (1 - \delta^*)(k_{D,t}^* + k_{O,t}^*) + \int p_{bt,t+1} b_{t+1}^* = w_{O,t}^* n_{O,t}^* + w_{D,t}^* n_{D,t}^* + r_{D,t}^* k_{D,t}^* + r_{O,t}^* k_{O,t}^* + ((1 - k^w(\theta_{D,t}^*))s_{D,t}^* + (1 - k^w(\theta_{O,t}^*))s_{O,t}^*)\chi^* + b_t^* + d_t^* \quad (30)$$

$$n_{D,t}^* = (1 - \rho^{*o})(1 - \rho^{*n})n_{D,t-1}^* + k^w(\theta_{D,t}^*)s_{D,t}^* \quad (31)$$

$$n_{O,t}^* = (1 - \rho^{*o})(1 - \rho^{*n})n_{O,t-1}^* + k^w(\theta_{O,t}^*)s_{O,t}^* \quad (32)$$

While the Foreign household and the multinational solve a similar problem in the sense that both allocate search activity across two segmented labor markets, the two problems differ in that we have shut down sequential search for the Foreign household. All search activity in the market for offshore jobs is directly allocated to that market. This assumption is made in order to simplify the model and is based on the idea that the threat of offshoring is more relevant to the demand side of the labor market.

Optimization on the part of the Foreign household yields an analogue to equation (26); two arbitrage conditions analogous to equation (27) that pin down the supply of the two capital stocks; and two optimal search conditions analogous to equation (28). Details are given in Appendix B.

2.5 Free Entry

In all three labor markets, free entry in the long run equilibrium drives the value of an unfilled vacancy to the creation cost, or the value of capital in place. Thus, the free entry conditions for the multinational into the domestic and offshore labor markets, respectively, are given by:

$$\mathbf{V}_{D,t} = r_{D,t}^k k_{D,t} \quad (33)$$

and

$$\mathbf{V}_{O,t}^* = qt r_{O,t}^{k^*} k_{O,t}^* \quad (34)$$

where: $\mathbf{V}_{D,t}$ ($\mathbf{V}_{O,t}^*$) is defined as the value to the Home multinational of an unfilled domestic (offshore) vacancy in the *long run equilibrium*.

Similarly, the free entry condition for the Foreign firm into the Foreign labor market is given by

$$\mathbf{V}_{D,t}^* = r_{D,t}^{k^*} k_{D,t}^* \quad (35)$$

where: $\mathbf{V}_{D,t}^*$ is the value to the Foreign firm of an unfilled vacancy in the domestic labor market in the *long run equilibrium*.

2.6 Matching Technology

Matches between unemployed individuals searching for jobs and firms searching to fill vacancies are formed according to a matching technology. There are three distinct labor markets in this model, each requiring its own matching function. All take a similar form.

Letting $m(s_{D,t}, v_{D,t})$ denote domestic matches formed in the Home country—that is, matches between the multinational and Home workers—the evolution of total domestic employment in the Home country is given by:

$$n_{D,t} = (1 - \rho^x)(1 - \rho^n)n_{D,t-1} + m(s_{D,t}, v_{D,t}) \quad (36)$$

Using similar notation, the evolution of foreign domestic matches is given by:

$$n_{D,t}^* = (1 - \rho^{x^*})(1 - \rho^{n^*})n_{D,t-1}^* + m(s_{D,t}^*, v_{D,t}^*) \quad (37)$$

Finally, the evolution of offshore matches is given by:

$$n_{O,t}^* = (1 - \rho^{x^*})(1 - \rho^{n^*})n_{O,t-1}^* + m(s_{O,t}^*, \tilde{v}_{O,t}^*), \quad (38)$$

Note that $\tilde{v}_{O,t}^* = v_{O,t}^* + \Omega(1 - k^f(\theta_{D,t}))v_{D,t}$ directly links the evolution of the domestic and offshore labor stock. When the multinational posts a vacancy in the domestic market, it influences market tightness at home, as one would expect. But, to the degree that jobs are offshorable, it also influences tightness in the offshore labor market abroad. Moreover, the foreign household will optimally reallocate search activity in response to this change in tightness in the market for offshore jobs. As a result, a vacancy posted by the multinational in the Home country can have an indirect influence on domestic labor markets in the Foreign country. In this sense, the offshorability of jobs links global labor markets together more tightly.

2.7 Wage Determination

The wage paid in any given job is determined in via Nash bargain between a matched worker and firm pair.¹² The equilibrium of the economy has a total of three wages: two paid by the

¹²We chose Nash bargaining as the wage determination mechanism because it is easy to work with and well understood. Clearly, there are other bargaining protocols we could investigate, but we leave that for future research.

multinational paid to domestic and offshore workers, respectively, and one paid by the Foreign firm to domestic workers. In what follows we present the solutions for the bargained wages in the short and long run, respectively, leaving the details of the solution to Appendix C.

2.7.1 The Short Run

In the short run, the number of firms and the amount of physical capital is assumed to be fixed. Beginning with the Home country, the short run wage paid by the multinational to domestic workers is given by:

$$\begin{aligned}\widehat{w}_{D,t} &= (1 - \eta) \frac{h'(lfp_t)}{u'(c_t)} + \eta f_{n_{D,t}} \\ &+ \eta \left(\gamma - k^f(\theta_{D,t}) \left(\widehat{\mathbf{J}}_{D,t} - (1 - \rho^o) E_t \left[\Xi_{t+1|t} \widehat{\mathbf{V}}_{D,t+1} \right] \right) \right) \\ &+ \eta \Omega (1 - k^f(\theta_{D,t})) \left(\gamma_o^* - k^f(\theta_{O,t}^*) \left(\widehat{\mathbf{J}}_{O,t}^* - (1 - \rho^o) E_t \left[\Xi_{t+1|t} \widehat{\mathbf{V}}_{D,t+1} \right] \right) \right)\end{aligned}\quad (39)$$

Generally speaking, the bargained wage is simply a weighted average of the worker and firm threat points in wage negotiations where the weight is given by the worker's bargaining power, η . In the interest of easing exposition, we leave a detailed intuitive discussion until later in a stand-alone Section 3. For now, we will simply say that the threat points in wage negotiations are driven by the value of the worker's and firm's respective outside options. From the multinational's point of view, the higher is the value of its outside option that comes from walking away from a match, the lower is the resulting bargained wage.

The short run wage paid to workers at domestic intermediate goods producing plants in the Foreign country is given by:

$$\begin{aligned}\widehat{w}_{D,t}^* &= (1 - \eta^*) \frac{h_t'^*}{u_t'^*} + \eta^* f_{n_{D,t}^*} \\ &+ \eta^* \left(\gamma_D^* - k^f(\theta_{D,t}^*) \left(\widehat{\mathbf{J}}_{D,t}^* - (1 - \rho^{*x}) E_t \left[\Xi_{t+1|t}^* \widehat{\mathbf{V}}_{D,t+1}^* \right] \right) \right)\end{aligned}\quad (40)$$

where: η^* is the bargaining power of Foreign workers. Equation 40 takes an identical form as equation 39 in the case in which $\Omega = 0$, so the intuition behind what drives the domestic wage in the Foreign country is similar to what drives the domestic wage in the Home country in this special case. As such, and again in the interest of ease of exposition, we leave a detailed intuitive discussion until Section 3.

Finally, the short run wage paid by the Home multinational to Foreign workers employed in offshored jobs is given by:

$$\begin{aligned}\widehat{w}_{O,t}^* &= (1 - \eta^*) \frac{h_t'^*}{u_t'^*} + \eta^* \frac{1}{q_t} f_{n_{O,t}^*} \\ &+ \eta^* \frac{1}{q_t} \left(\gamma_o^* - k^f(\theta_{O,t}^*) \left(\widehat{\mathbf{J}}_{O,t}^* - (1 - \rho^{*x}) E_t \left[\Xi_{t+1|t} \widehat{\mathbf{V}}_{O,t+1}^* \right] \right) \right)\end{aligned}\quad (41)$$

$$+\eta^* \frac{1}{q_t} (1 - \rho^{x*}) (1 - \rho^{n*}) E_t \left[\frac{\Xi_{t+1|t}^* q_t - \Xi_{t+1|t} q_{t+1}}{q_{t+1}} \left(\hat{\mathbf{J}}_{O,t+1}^* - \hat{\mathbf{V}}_{O,t+1}^* \right) \right]$$

There are two things worth pointing out about the offshore wage, each of which stem from the fact that bargaining is done internationally. First, the real exchange rate enters into the effective bargaining share. When the real exchange rate appreciates (q_t gets larger) the effective bargaining weight of the multinational, η/q_t , increases putting downward pressure on the negotiated wage. Second, the term in the third line captures the fact that the surplus split moves around dynamically in response to movements in both the real exchange rate as well as to differences in the stochastic discount factors of Home and Foreign households.

2.7.2 The Long Run

In the long run, capital is free to adjust and free entry into the labor market drives the value of an unfilled vacancy to the cost of capital. The long run wage paid to domestic workers in the Home country is given by:

$$w_D = (1 - \eta) \frac{h'}{u'} + \eta \left(f_{n_D} - (1 - \beta(1 - \rho^o)) r^k k_D \right) \quad (42)$$

where we have dropped the time subscripts because w_D is a long run steady state variable.

The long run wage paid by the multinational to offshore workers in the Foreign country is given by:

$$w_D^* = (1 - \eta^*) \frac{h'^*}{u'^*} + \eta^* \left(f_{n_D^*} - (1 - \beta(1 - \rho^{o*})) r^{k^*} k_D^* \right) \quad (43)$$

Finally, the long run wage paid by the multinational to offshore workers in the Foreign country is given by:

$$w_O^* = (1 - \eta^*) \frac{h'^*}{u'^*} + \eta^* \frac{1}{q} \left(f_{n_O^*} - (1 - \beta(1 - \rho^{o*})) q r^{k^*} k_O^* \right) \quad (44)$$

All three equations have a similar form in the steady state. As with the subsection above, we leave an intuitive discussion of the long run wage until Section 3 below.

2.8 Equilibrium

Taking as given the trade costs, Υ , a private sector equilibrium in *the long run* is made up of the endogenous processes $\{c_t, c_t^*, p_{bt,t+1}, p_{bt,t+1}^*, r_{D,t}^k, r_{D,t}^{k^*}, r_{O,t}^{k^*}, k_{D,t}, k_{D,t}^*, k_{O,t}^*, w_{D,t}, w_{D,t}^*, w_{O,t}^*, s_{D,t}, s_{D,t}^*, s_{O,t}^*, \theta_{D,t}, \theta_{D,t}^*, \theta_{O,t}^*, n_{D,t}, n_{D,t}^*, n_{O,t}^*, \mathbf{V}_{D,t}, \mathbf{V}_{D,t}^*, \mathbf{V}_{O,t}^*, \mathbf{J}_{D,t}, \mathbf{J}_{D,t}^*, \mathbf{J}_{O,t}^*, ne_{D,t}, ne_{D,t}^*, ne_{O,t}^*, \frac{1}{p_t}, \frac{p_{F,t}^*}{p_t^*}, q_t\}$ that satisfy:

The risk sharing arrangement

$$q_t = \frac{u'(c_t)}{u'^*(c_t^*)} \quad (45)$$

the definitions of the price indexes in the Home and Foreign country (2 equations); the Home Euler equation (26), and its Foreign counterpart (1 equation); the Home arbitrage condition given by equation (27) and its foreign counterparts (2 equations); optimal search behavior on the part of the Home household, represented by equation (28), and the Foreign counterparts (2 equations); optimal capital accumulation on the part of the Home firm, equations (11) and (12) and the Foreign counterpart equation 19; optimal search behavior for the Home firm, equations (7), (8), (10), (9) and their Foreign counterparts, equations (17) and (18); the long run wage equations, given by equations (43) through (44); the laws of motion for vacancies, given by equations (4), (5), and (16); the free entry conditions, given by equations (33), (35), and (34); and the laws of motion for employment, given by (36) through (38).

Finally, we have the resource constraints for each of the two countries, which are given below for the Home and Foreign country, respectively.

$$f(z_{D,t}g(n_{D,t}, k_{D,t}), (1 - \Upsilon)z_{O,t}^*g(n_{O,t}^*, k_{O,t}^*)) = \lambda \left(\frac{1}{p_t}\right)^{-\zeta} \left(c_t + \left(\frac{1}{q_t}\right)^{-\zeta} c_t^*\right) \quad (46)$$

$$+ k_{D,t+1} - (1 - \delta)k_{D,t} + \gamma_{D,t}v_{D,t} + \gamma_{O,t}^*v_{O,t}^* + \mathbf{V}_{D,t}ne_{D,t} + \mathbf{V}_{O,t}^*ne_{O,t}^*$$

$$f(z_{D,t}^*g(n_{D,t}^*, k_{D,t}^*)) = (1 - \lambda) \left(\frac{p_{F,t}^*}{p_t^*}\right)^{-\zeta} \left(q_t^{-\zeta}c_t + c_t^*\right) + k_{D,t+1} - (1 - \delta^*)k_{D,t}^* \quad (47)$$

$$+ k_{D,t+1}^* - (1 - \delta^*)k_{D,t}^* + \gamma_{D,t}^*v_{D,t}^* + \mathbf{V}_{D,t}^*ne_{D,t}^*$$

Note that the total cost of entry into each market shows up in the resource constraint. All told, the *long run equilibrium* is a system of 34 equations in 34 unknowns.

In contrast, in the *short run equilibrium* both entry and the physical capital stock are assumed to be constant at some initial long run equilibrium. Thus, we drop the capital demand equations, equations (11), (12), and 19, and the free entry conditions, equations (33), (35), and (34), from the system and replace the long run wage expressions with their short run counterparts given by equations (40) through (39). All told, the in the short run equilibrium, the system is 28 equations in 28 unknowns.

3 The Threat Effect

In this section, we offer an intuitive discussion about how the two key modeling mechanisms that we have introduced—the sequential nature of markets and entry cost—change the outside option of the multinational in wage negotiations both in the short and long run. In order to highlight how these two mechanisms operate both separately and together, note that we can shut down the

sequential nature of markets by assuming $\Omega = 0$ so that no jobs are offshorable. Alternatively, we can shut down the cost of entry into the Home domestic labor market so that under free entry the value of an open domestic vacancy is driven to zero, $\mathbf{V}_D = 0$, implying $\gamma = k^f(\theta_{D,t})\widehat{\mathbf{J}}_{D,t}$ as is standard in a typical labor search model.

The short run. For convenience, we restate the wage paid by the multinational to domestic workers in the short-run equilibrium.

$$\begin{aligned}\widehat{w}_{D,t} = & (1 - \eta)\frac{h'(lfp_t)}{u'(c_t)} + \eta f_{n_{D,t}} \\ & + \eta \left(\gamma - k^f(\theta_{D,t}) \left(\widehat{\mathbf{J}}_{D,t} - (1 - \rho^o) E_t \left[\Xi_{t+1|t} \widehat{\mathbf{V}}_{D,t+1} \right] \right) \right) \\ & + \eta \Omega (1 - k^f(\theta_{D,t})) \left(\gamma_o^* - k^f(\theta_{o,t}^*) \left(\widehat{\mathbf{J}}_{o,t}^* - (1 - \rho^o) E_t \left[\Xi_{t+1|t} \widehat{\mathbf{V}}_{D,t+1} \right] \right) \right)\end{aligned}$$

The worker's threat point is the marginal rate of substitution (MRS) between consumption and leisure—if the wage drops below the MRS, the worker is better off walking away from the match to enjoy leisure instead. The threat point of the multinational is the marginal product of domestic labor plus the *outside option* to the firm of walking away from the match. The multinational's outside option is critical to our main results and consists of two components: (1.) one that stems from the fact that an open vacancy has positive value independent of the threat effect; and (2.) one that stems directly from the threat effect.

In order to isolate the first component, consider the special case in which $\Omega = 0$, so that the threat of offshoring is shut down. Next, using equation 9 and the fact that $\lambda_{D,t} = \widehat{\mathbf{V}}_{D,t}$ we can rewrite the entire term in the second line as the contemporaneous value of an open vacancy net of its continuation value, $-\eta(\widehat{\mathbf{V}}_{D,t} - (1 - \rho^o) E_t[\Xi_{t+1|t} \widehat{\mathbf{V}}_{D,t+1}])$. Writing the expression this way makes it easy to see that the multinational can exercise its outside option by walking away from a match and, in doing so, it retains the value of the open vacancy. Importantly, this outside option can be either positive or negative in the short run equilibrium where entry is prevented from adjusting instantaneously. For example, when $\widehat{\mathbf{V}}_{D,t} > (1 - \rho^o) E_t[\Xi_{t+1|t} \widehat{\mathbf{V}}_{D,t+1}]$ the outside option is positive, putting downward pressure on the wage.

The second component of the multinational's outside option stems from the possibility of filling domestic vacancies with Foreign workers—that is, the ability of the multinational to offshore when $\Omega > 0$. We can isolate this by assuming no cost of entry into the Home domestic labor market, so that $\mathbf{V}_D = 0$. In this case, the multinational's outside option simplifies to a term directly related to entry into the offshore market, $\eta \Omega (1 - k^f(\theta_{D,t})) (\gamma_o^* - k^f(\theta_{o,t}^*) \widehat{\mathbf{J}}_{o,t}^*)$. Concentrating on the second term, it is clear that conditional on the open position being for an offshorable job (which occurs with probability Ω) and provided the vacancy for that particular position is not filled with a domestic worker in the morning market (which occurs with probability $1 - k^f(\theta_{D,t})$), then the ability of the multinational to fill that opening with a Foreign worker will have an influence on the short run

domestic wage.

In particular, impediments to entry imply that a higher valuation by the multinational of offshored workers lowers the domestic wage. Thus, in our model we articulate the threat of offshoring through the term $k^f(\theta_{o,t}^*)\widehat{\mathbf{J}}_{o,t}^*$. This is a key contribution of the paper. In addition to the value of offshored workers, the strength of the threat effect is governed by labor market tightness both at home (decreasing in $k^f(\theta_{d,t})$) and abroad (increasing in $k^f(\theta_{o,t}^*)$). Finally, it is important to point out that even though the threat of offshoring puts unambiguous downward pressure on domestic wages, impediments to entry imply that offshoring, more generally, can either increase or decrease the domestic wage depending on the sign of $\gamma_o^* - k^f(\theta_{o,t}^*)\widehat{\mathbf{J}}_{o,t}^*$ and the multinational's resulting incentives regarding entry.

The long run. In the long run, free entry drives the value of an open vacancy to the creation cost, so that $\mathbf{V}_D = r^k k_D$. For convenience, we restate the wage paid by the multinational to domestic workers in the long-run equilibrium.

$$w_D = (1 - \eta) \frac{h'}{u'} + \eta \left(f_{n_D} - (1 - \beta(1 - \rho^o)) r^k k_D \right)$$

In the long run, the positive value of an unfilled vacancy puts unambiguous downward pressure on the domestic wage, that is $(1 - \beta(1 - \rho^o)) > 0$. What is interesting about this result is that it obtains in the long run equilibrium regardless of whether or not jobs are offshorable (i.e., regardless of the value of Ω). In other words, the affect that the threat of offshoring has on domestic wages does not show up explicitly in the wage equation. Instead, it is embedded in the equilibrium allocations through free entry and the adjustment of the capital stock. Thus, the long run impact of the threat of offshoring is purely a quantitative question. Finally, it is useful to note that in absence of the cost of entry, so that $\mathbf{V}_D = 0$, the third term on the right hand side of the equals sign goes to zero and the wage collapses to $w_D = (1 - \eta) \frac{h'}{u'} + \eta f_{n_D}$, which is familiar from standard general equilibrium search models.

4 Quantitative Analysis

In this section, we derive a model-based estimate of the quantitative magnitude of the effect that the threat of offshoring has on global wages and labor market allocations. We begin with a description of the baseline parameterization and then present the main results.

4.1 Calibration

The parameter values used in the baseline model are summarized in Table 1. The Home country is calibrated to US data, where the existing labor search literature acts as a guide on parameter

values. For the Foreign country, we mostly use Mexican data to guide our calibration. Our strategy is to parameterize the foreign country so that its labor market is more rigid than the Home one. According to the OECD index of employment protection, this description would apply to the Mexican labor market relative to labor markets in the United States. In 2008, the OECD index ranked the US labor market as the most flexible of the 40 countries studied, with Mexico's labor market being ranked one of the most rigid.

Production. The functional form of the production function for the final good produced by the multinational is a CES aggregate of the domestic and offshored intermediate goods.

$$y_t = \left(\Gamma (y_{H,t})^\vartheta + (1 - \Gamma) (y_{H,t}^*)^\vartheta \right)^{\frac{1}{\vartheta}}$$

In contrast, Foreign final goods production is assumed to be linear, $y_t^* = z_{F,t}^* y_{F,t}^*$. For the multinational, we assume $\vartheta = 0$, so that production is a Cobb-Douglas aggregate of (imperfectly substitutable) domestic and offshored intermediate inputs. The share of domestically-produced inputs into final production of the multinational is set to $\Gamma = 0.99$, in line with the BEA's data on the sales of US multinationals' affiliates in Mexico back to their US parent companies as a ratio of the total sales of US parent companies.

Intermediate goods production is a Cobb-Douglas aggregate of capital and labor input for plants operated by the multinational (located domestically and abroad) and the Foreign firm, respectively.

$$y_{H,t} = z_{H,t} n_{H,t}^\alpha k_{H,t}^{1-\alpha} \quad y_{H,t}^* = z_{H,t}^* n_{H,t}^{*\alpha} k_{H,t}^{*1-\alpha} \quad y_{F,t}^* = z_{F,t}^* n_{F,t}^{*\alpha} k_{F,t}^{*1-\alpha}$$

Labor's share for the multinational is set to $\alpha = 0.7$, while intermediate goods production in the Foreign country is assumed to be more labor intensive, so that $\alpha^* = 0.85$. Note that we assume that plants operated by the multinational located in the Foreign country use capital more intensively than domestic plants operated by the Foreign firm.

With regard to technology, we assume that the level of aggregate technology is symmetric across the two countries, so that $z_H = z_H^* = z_F^* = 1$. This contrasts with much of the literature on offshoring in which technological differences are the primary source of offshoring activity. Nonetheless, we impose this assumption in order to highlight the role of labor market institutions in driving the (intensive) offshoring decision and, hence, the main results in the paper.

Capital Accumulation. The rate of depreciation for capital in both the Home and Foreign country is $\delta = \delta^* = 0.02$.

Preferences. The model is calibrated to quarterly data, so we set the subjective discount factor to $\beta = \beta^* = 0.99$, yielding an annual real interest rate of about 4 percent.

The functional form for instantaneous utility is standard

$$u(c_t, lfp_t) = \frac{1}{1-\sigma} c_t^{1-\sigma} - \frac{\kappa}{1+1/\iota} lfp_t^{1+1/\iota} \quad (48)$$

Table 1: Baseline Calibration

Home Country		Description	Foreign Country	
Parameter	Value		Value	Parameter
Production				
z	1	Steady state technology	1	z^*
ϑ	0	Elasticity of substitution between domestic and offshored labor		
Γ	0.90	Share of domestic intermediate good in final production		
α	0.70	Share of labor in intermediate goods production	0.85	α^*
Capital Accumulation				
δ	0.02	Depreciation rate for capital stock	0.02	δ^*
Preferences				
β	0.99	Discount factor	0.99	β^*
σ	2	Risk aversion	2	σ^*
ι	0.18	Elasticity of participation	0.18	ι^*
κ	18.6	Scale parameter for subutility of leisure	58.7	κ^*
ζ	0.5	Elasticity of substitution between Home and Foreign goods	0.5	ζ^*
λ	0.73	Share of domestically-produced goods in consumption basket	0.80	λ^*
Labor Market				
ξ	0.50	Elasticity of matching function	0.50	ξ^*
η	0.50	Worker's bargaining power	0.25	η^*
ρ^o	0.0075	Probability of job obsolescence	0.0075	ρ^{*o}
ρ^n	0.017635	Probability of job separation	0.017635	ρ^{*n}
ψ	0.56	Matching efficiency	0.40	$\psi_h^* = \psi_f^*$
γ_h	3.47	Vacancy posting cost in domestic labor market	5.45	γ_f^*
		Vacancy posting cost in offshored labor market	4.40	γ_h^*
χ	0.379	Unemployment benefit	0.183	χ^*
Trade Costs and Policy				
Υ	0	Iceberg cost		
$\tau_{H,t}^n$	0	Wage tax paid by multinational on domestic employees		
$\tau_{H,t}^{*n}$	0	Wage tax paid by multinational on foreign employees		
$\tau_{H,t}^v$	0	Vacancy tax paid by multinational on domestic job creation		
$\tau_{H,t}^{*v}$	0	Vacancy tax paid by multinational on foreign job creation		

where the risk aversion parameter is set to $\sigma = \sigma^* = 2$ for both the Home and Foreign household, consistent with much of the existing literature.

For the subutility function over participation, we introduce asymmetry to reflect differences in long run labor force participation rates observed across countries. We calibrate the Home country to US data; specifically, we set $\iota = 0.18$ following Arseneau and Chugh (2008) who showed that this value for the elasticity of labor force participation with respect to the real wage delivers participation dynamics over the business cycle that match the U.S. data. Similarly, the scale parameter is set to $\kappa = 18.6$ to deliver a steady-state participation rate of 66 percent in the US. For the Foreign

country, we maintain a symmetric elasticity of participation, $\iota^* = 0.18$, under the assumption that the business cycle dynamics of participation do not differ much across countries. However, we introduce asymmetry into the scale parameter in order to deliver a lower participation rate in the Foreign country than in the US. We set $\kappa^* = 58.7$ to deliver a steady-state participation rate of 59.2 percent, which is the average in annual Mexican data (1980 to 2008) taken from the World Bank World Development Indicators (WDI).

The elasticity of substitution between Home and Foreign goods in the final consumption basket is symmetric across countries and set to $\zeta = \zeta^* = 0.5$. With regard to the weights of domestic and foreign goods in the final consumption good, λ and λ^* are chosen so that the import to GDP ratio is 12 and 26 percent in the Home and Foreign country, respectively. These numbers correspond to the average share of imports in GDP for the US and Mexico (1980 to 2010), respectively, taken from Haver Analytics.

Labor Markets. For each of the segmented labor markets (one in the Home country and two in the Foreign country) we assume a Cobb-Douglas matching function of the following general form:

$$m(s_t, v_t) = \psi s_t^\xi v_t^{1-\xi}$$

For the Home country, the elasticity of matches with respect to unemployed job seekers is set to $\xi = 0.50$, which is in the midpoint of estimates typically used in the literature and is in line with results reported in Petrongolo and Pissarides (2001). Following much of the existing literature, we impose symmetry between the elasticity of the matching function and the Home worker's bargaining power, so that $\eta = 0.5$. The job obsolescence rate is set to $\rho^o = 0.0075$ and the separation rate is set to $\rho^n = 0.017635$. Together these probabilities imply that the total job separation rate $\rho = \rho^o + (1 - \rho^o)\rho^n = 0.025$, which is in line with Shimer (2005) who calculates the average duration of a job to be two-and-a-half years. Matching efficiency in the Home country, $\psi = 0.56$, is chosen so that the quarterly job-filling rate of a vacancy is 90 percent, in line with Andolfatto (1990). We set the cost of posting a vacancy to target a steady state level of market tightness in the home country of $\theta_{H,t} = 0.3$ which is a touch below the the measure obtained from JOLTS data. The resulting value is $\gamma_H = 3.47$. Finally, we calibrate the worker's outside option in the Home country to 40 percent of the wages of employed individuals in the Home household, implying a value of $\chi = 0.379$. The resulting implied aggregate unemployment rate for the Home country in our baseline calibration is roughly 6.5 percent.

For Foreign country, there is little in the way of data to guide us in calibrating the labor market of the countries to which the U.S. primarily offshores. In light of this our strategy is as follows. We impose cross-country symmetry in the matching elasticity parameter, so $\xi^* = \xi = 0.5$, the average duration of a job, so that $\rho^{*o} = \rho^o$, $\rho^{*n} = \rho^n$, and the job filling probabilities, so that $\gamma_F^* = 5.45$ and $\gamma_H^* = 4.40$ implying $k_F^* = k_H^* = 0.9$. We then introduce asymmetry aimed at capturing the

general perception that that the countries to which the US offshores have labor markets that are more frictional.

First, workers in the Foreign country are assumed to have *less bargaining power* in wage negotiations relative to US workers, so that $\eta^* = 0.25$. Next, we calibrate matching efficiency in the market for domestic and offshore jobs to hit an unemployment rate of 12 percent, the average level of Mexican unemployment using data from the WDI. The resulting values are $\psi_H^* = \psi_F^* = 0.40$. Finally, we assume that the US is more generous in its provision of unemployment benefits relative to a country such as Mexico. Accordingly we calibrate χ^* to a replacement rate of 20 percent of the wages of employed individuals. The resulting value is $\chi^* = 0.18$.

Trade Costs. We assume that there are no trade costs in the baseline calibration, so that $\tau = \tau^* = 0$. Section 6 examines how a change in trade costs interacts with the threat of offshoring.

4.2 Main Result

We measure the threat effect in two separate ways. First, in a *ceterus paribus* exercise, we simply decompose the expression for the short run wage to isolate the threat effect and measure it given the calibration in our baseline economy. Using the equation (39), we separate the domestic wage into three components: (1.) the core wage $(1 - \eta)\frac{h'_t}{w'_t} + \eta f_{n_d,t}$; (2.) the threat effect that arises due to the ability of the multinational to fill a domestic job with a foreign worker, $-\eta\Omega(1 - k^f(\theta_{D,t}))k^f(\theta_{O,t}^*)\mathbf{J}_{O,t}^*$; and (3.) everything else. Using this decomposition, we measure the *ceterus paribus* threat effect as a percentage of the long run steady state wage.

Table 2: Short run wage decomposition

	Home Wage	Core	Threat Effect	Residual
Level	1.419	1.423	-0.110	0.107
Percent	--	100.3	-7.8	7.5

The result is presented in Table 2 which shows that the ability of the multinational to fill a domestic vacancy with a foreign worker in the evening market depresses the domestic wage by nearly 8 percent relative to its long run steady state value. This suggests that the short run effect of the threat of offshoring on domestic wages is potentially large. However, summing the threat effect and the residual leaves only a small negative total effect on the domestic wage. The explanation is that free entry allows allocations to adjust in such a way that it mutes the threat effect in the long run equilibrium. This finding—that the threat effect is potentially large when there are barriers to firm entry but tends to be muted when entry is free to adjust frictionlessly—is a robust message that comes out of this modeling framework.

5 Sensitivity Analysis

6 Policy Experiments

6.1 Trade Liberalization

6.2 Anti-Offshoring Legislation

7 Conclusion

We developed a two-country labor search model to assess the role of the threat of offshoring for global wages and labor market allocations. Our model features a multinational firm in the Home country that operates both domestic and foreign production plants, so that the parent company can shift production from the domestic country to foreign affiliates. Foreign firms produce only domestically. Regardless of where it produces, each firm must hire labor in a frictional labor market; labor market frictions, in turn, give rise to an explicit role for bargaining in the wage formation process. We exploit this feature of the model to assess how the threat of offshoring influences wage formation and the resulting implications for global labor market allocations. To model the threat of offshoring we allow for a sequential bargaining problem in which bargaining over the wage in the market for domestic labor relationships takes place prior to bargaining over the wage in offshored jobs. In this sequential setup, multinational firms exploit the outside option of walking away from a match and instead shifting production across borders to influence the bargained wage.

Our main finding is that, in the short run, the threat of offshoring has a quantitatively large effect both on global wages as well as global labor market allocations. Specifically, we find that when the multinational exploits the threat of offshoring in wage negotiations bargained wages are depressed by as much as 8 percent in the source country. However, this appears to be largely a short-run effect. In the long run, we find that the quantitative magnitude of the impact of the threat of offshoring has on domestic wages is muted considerably when firm entry and the capital stock are allowed to adjust freely. In subsequent drafts of this paper we will look at transition dynamics to get a better handle on the quantitative importance of the short-run relative to the long-run effect.

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A Details of the Firm's Problem

A.1 Foreign Firm

The foreign firm chooses $n_{D,t}^*$, $v_{D,t}^*$, and $k_{D,t}^*$ to solve the following problem

$$\Pi_t^* = \sum_{t=0}^{\infty} \beta^t \frac{\lambda_t^*}{\lambda_0^*} [f(y_{D,t}^*) - w_{D,t}^* n_{D,t}^* - r_{D,t}^{*k} k_{D,t}^* - \gamma_D^* v_{D,t}^*]$$

subject to:

$$y_{D,t}^* = z_{D,t}^* g(n_{D,t}^*, k_{D,t}^*)$$

$$n_{D,t}^* = (1 - \rho^{*o})(1 - \rho^{*n}) n_{D,t-1}^* + v_{D,t}^* k^f(\theta_{D,t}^*)$$

$$v_{D,t}^* = (1 - \rho^{*o}) \rho^{*n} n_{D,t-1}^* + (1 - \rho^{*o})(1 - k^f(\theta_{D,t-1}^*)) v_{D,t-1}^* + n e_{D,t}^*$$

Let $\mu_{D,t}^*$ and $\lambda_{D,t+1}^*$ be the multipliers on the laws of motion for jobs and vacancies, respectively.

The first order conditions for are:

$$\lambda_{D,t}^* = -\gamma^* + k^f(\theta_{D,t}^*) \mu_{D,t}^* + (1 - k^f(\theta_{D,t}^*)) E_t[\Xi_{t+1}^* | t] (1 - \rho^{*o}) \lambda_{D,t+1}^*$$

$$\mu_{D,t}^* = f_{n_{D,t}^*} - w_{D,t}^* + E_t \left[\Xi_{t+1}^* | t \left((1 - \rho^{*o}) \rho^{*n} \lambda_{D,t+1}^* + (1 - \rho^{*o})(1 - \rho^{*n}) \mu_{D,t+1}^* \right) \right]$$

$$f_{k_{D,t}^*} = r_{D,t}^{*k}$$

A.2 Home Multinational

The Home multinational chooses $n_{D,t}$, $v_{D,t}$, $n_{O,t}^*$, $v_{O,t}^*$, $k_{D,t}$, and $k_{O,t}^*$ to solve the following problem

$$\Pi_t = \sum_{t=0}^{\infty} \beta^t \frac{\lambda_t}{\lambda_0} [f(y_{D,t}, (1 - \Upsilon) y_{O,t}^*) - w_{D,t} n_{D,t} - q_t w_{O,t}^* n_{O,t}^* - r_{D,t}^k k_{D,t} - q_t r_{O,t}^{k*} k_{O,t}^* - \gamma_D v_{D,t} - \gamma_O^* \tilde{v}_{O,t}^*]$$

subject to:

$$y_{D,t} = z_{D,t} g(n_{D,t}, k_{D,t})$$

$$y_{O,t}^* = z_{O,t}^* g(n_{O,t}^*, k_{O,t}^*)$$

$$n_{D,t} = (1 - \rho^o)(1 - \rho^n) n_{D,t-1} + k^f(\theta_{D,t}) v_{D,t}$$

$$n_{O,t}^* = (1 - \rho^{*o})(1 - \rho^{*n}) n_{O,t-1}^* + k^f(\theta_{O,t}^*) \tilde{v}_{O,t}^*$$

$$v_{D,t} = (1 - \rho^o)\rho^n n_{D,t-1} + (1 - k^f(\theta_{D,t-1}))(1 - \Omega^F k^f(\theta_{O,t-1}^*)) (1 - \rho^o)v_{D,t-1} + ne_{D,t}$$

$$v_{O,t}^* = (1 - \rho^{*o})\rho^{*n} n_{O,t-1}^* + (1 - k^f(\theta_{O,t-1}^*)) (1 - \rho^{*o})v_{O,t-1}^* + ne_{O,t}^*$$

$$\tilde{v}_{O,t}^* = v_{O,t}^* + \Omega^F (1 - k^f(\theta_{D,t}))v_{D,t}$$

Associate the multipliers $\mu_{D,t}$, and $\mu_{O,t}^*$ to the Home and offshored employment constraints, respectively, and the multipliers $\lambda_{D,t}$ and $\lambda_{O,t}^*$ to the Home and offshored vacancy constraints, respectively. The first-order conditions are

$$\begin{aligned} \lambda_{D,t} = & -\gamma_D + k^f(\theta_{D,t})\mu_{D,t} - \Omega^F (1 - k^f(\theta_{D,t}))(\gamma_O^* - k^f(\theta_{O,t}^*)\mu_{O,t}^*) \\ & + (1 - k^f(\theta_{D,t}))(1 - \Omega^F k^f(\theta_{O,t}^*)) (1 - \rho^o)E_t[\Xi_{t+1|t}\lambda_{D,t+1}] \end{aligned}$$

$$\mu_{D,t} = f_{n_{D,t}} - w_{D,t} + E_t \left[\Xi_{t+1|t} \left((1 - \rho^o)\rho\lambda_{D,t+1} + (1 - \rho^o)(1 - \rho^n)\mu_{D,t+1} \right) \right]$$

$$f_{k_{D,t}} = r_{D,t}^k$$

$$\lambda_{O,t}^* = -\gamma_O^* + k^f(\theta_{O,t}^*)\mu_{O,t}^* + (1 - k^f(\theta_{O,t}^*)) (1 - \rho^{*o})E_t[\Xi_{t+1|t}\lambda_{O,t+1}^*]$$

$$\mu_{O,t}^* = f_{n_{O,t}^*} - q_t w_{O,t}^* + E_t \left[\Xi_{t+1|t} \left((1 - \rho^{*o})\rho^{*n}\lambda_{O,t+1}^* + (1 - \rho^{*o})(1 - \rho^{*n})\mu_{O,t+1}^* \right) \right]$$

$$f_{k_{O,t}^*} = q_t r_{O,t}^{k^*}$$

B Details of the Household's Problem

B.1 Home Household

The household in the Home country searches in the domestic labor market for jobs operated by the Home multinational. The Foreign household's problem is to choose sequences of c_t , k_{t+1} , b_{t+1} , $s_{H,t}$, and $n_{H,t+1}$ to maximize:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[u(c_t) - h \left((1 - k^w(\theta_{D,t})) s_{D,t} + n_{D,t} \right) \right]$$

subject to:

$$p_t c_t + k_{t+1} - (1 - \delta) k_t + \int p_{bt,t+1} b_{t+1} = w_{H,t} n_{D,t} + r_t^k k_t + (1 - k^w(\theta_{D,t})) s_{D,t} \chi + b_t + d_t$$

$$n_{D,t} = (1 - \rho^o)(1 - \rho^n) n_{D,t-1} + k^w(\theta_{D,t}) s_{D,t}$$

Defining λ_t and μ_t as the multipliers on the budget constraint and the law of motion for employment, respectively, the first order conditions for c_t , k_{t+1} , b_{t+1} , $s_{D,t}$, and $n_{D,t}$, are:

$$u'(c_t) - p_t \lambda_t = 0$$

$$\lambda_t - \beta E_t (1 - \delta + r_{t+1}^k) \lambda_{t+1} = 0$$

$$p_{bt,t+1} \lambda_t - \beta E_t \lambda_{t+1} = 0$$

$$(1 - k^w(\theta_{D,t})) (-h'_t + \lambda_t \chi) + \mu_{D,t} k^w(\theta_{D,t}) = 0$$

$$-h'_t + \lambda_t w_{D,t} - \mu_{D,t} + \beta (1 - \rho^o)(1 - \rho^n) E_t \mu_{D,t+1} = 0$$

Combining the first order conditions on c_t and b_{t+1} gives the consumption Euler equation

$$\frac{u'(c_t)}{p_t} = \beta E_t \left[\frac{1}{p_{bt,t+1}} \frac{u'(c_{t+1})}{p_{t+1}} \right]$$

which defines the one period ahead stochastic discount factor, $E_t [\Xi_{t+1|t}] = \beta E_t \left[\frac{u'(c_{t+1})}{u'(c_t)} \frac{p_t}{p_{t+1}} \right]$.

Combining the first order conditions on k_{t+1} and b_{t+1} gives an arbitrage condition between bond and physical capital holdings

$$\frac{1}{p_{bt,t+1}} = E_t \left[1 - \delta + r_{t+1}^k \right]$$

Combining the first order conditions for $s_{D,t}$, and $n_{D,t}$ gives rise to a standard optimal search condition for domestic households

$$\frac{1 - k^w(\theta_{D,t})}{k^w(\theta_{D,t})} (h'_t - \lambda_t \chi) = \lambda_t w_{D,t} - h'_t + (1 - \rho^o)(1 - \rho^n) \beta E_t \left[\frac{1 - k^w(\theta_{D,t+1})}{k^w(\theta_{D,t+1})} (h'_{t+1} - \lambda_{t+1} \chi) \right]$$

B.2 Foreign Household

The household in the Foreign country searches in two differentiated labor markets: one for jobs operated by domestic firms and one for offshored jobs operated by the Home multinational. The Foreign household's problem is to choose sequences of c_t^* , k_{t+1}^* , b_{t+1}^* , $s_{o,t}^*$, $s_{d,t}^*$, $n_{o,t+1}^*$, and $n_{d,t+1}^*$ to maximize:

$$E_0 \sum_{t=0}^{\infty} \beta^{*t} \left[u^*(c_t^*) - h^*((1 - k^w(\theta_{d,t}^*)) (1 - \Omega^W) s_{d,t}^* + (1 - k^w(\theta_{o,t}^*)) \tilde{s}_{o,t}^* + n_{d,t}^* + n_{o,t}^*) \right]$$

subject to:

$$p_t^* c_t^* + k_{d,t+1}^* + k_{o,t+1}^* - (1 - \delta^*)(k_{d,t}^* + k_{o,t}^*) + \int p_{bt,t+1} b_{t+1}^* = w_{o,t}^* n_{o,t}^* + w_{d,t}^* n_{d,t}^* + r_{d,t}^* k_{d,t}^* + r_{o,t}^* k_{o,t}^* + ((1 - k^w(\theta_{d,t}^*)) (1 - \Omega^W) s_{d,t}^* + (1 - k^w(\theta_{o,t}^*)) \tilde{s}_{o,t}^*) \chi^* + b_t^* + d_t^*$$

$$n_{d,t}^* = (1 - \rho^{*o})(1 - \rho^{*n}) n_{d,t-1}^* + k^w(\theta_{d,t}^*) s_{d,t}^*$$

$$n_{o,t}^* = (1 - \rho^{*o})(1 - \rho^{*n}) n_{o,t-1}^* + k^w(\theta_{o,t}^*) \tilde{s}_{o,t}^*$$

$$\tilde{s}_{o,t}^* = s_{o,t}^* + \Omega^W (1 - k^w(\theta_{d,t}^*)) s_{d,t}^*$$

Defining λ_t^* and μ_t^* as the multipliers on the budget constraint and the law of motion for employment, respectively, the first order conditions for c_t^* , k_{t+1}^* , b_{t+1}^* , $s_{d,t}^*$, $s_{o,t}^*$, $n_{d,t}^*$, and $n_{o,t}^*$, are:

$$u'^*(c_t^*) - p_t^* \lambda_t^* = 0$$

$$\lambda_t^* - \beta^* E_t (1 - \delta^* + r_{d,t+1}^{k*}) \lambda_{t+1}^* = 0$$

$$\lambda_t^* - \beta^* E_t (1 - \delta^* + r_{o,t+1}^{k*}) \lambda_{t+1}^* = 0$$

$$p_{bt,t+1}^* \lambda_t^* - \beta^* E_t \lambda_{t+1}^* = 0$$

$$(1 - k^w(\theta_{d,t}^*)) (1 - \Omega^W k^w(\theta_{o,t}^*)) (-h_t^{*'} + \lambda_t^* \chi^*) + k^w(\theta_{d,t}^*) \mu_{d,t}^* + k^w(\theta_{o,t}^*) \Omega^W (1 - k^w(\theta_{d,t}^*)) \mu_{o,t}^* = 0$$

$$(1 - k^w(\theta_{o,t}^*)) (-h_t^{*'} + \lambda_t^* \chi^*) + k^w(\theta_{o,t}^*) \mu_{o,t}^* = 0$$

$$-h_t^{*'} + \lambda_t^* w_{d,t}^* - \mu_{d,t}^* + \beta (1 - \rho^{*o})(1 - \rho^{*n}) E_t \mu_{d,t+1}^* = 0$$

$$-h_t^{*'} + \lambda_t^* w_{o,t}^* - \mu_{o,t}^* + \beta(1 - \rho^{*o})(1 - \rho^{*n})E_t \mu_{o,t+1}^* = 0$$

Combining the first order conditions on c_t^* and b_{t+1}^* gives the consumption Euler equation

$$\frac{u'^*(c_t^*)}{p_t^*} = \beta E_t \left[\frac{1}{p_{bt,t+1}^*} \frac{u'^*(c_{t+1}^*)}{p_{t+1}^*} \right]$$

which defines the one period ahead stochastic discount factor, $E_t [\Xi_{t+1|t}^*] = \beta E_t \left[\frac{u'^*(c_{t+1}^*)}{u'^*(c_t^*)} \frac{p_t^*}{p_{t+1}^*} \right]$.

Combining the first order conditions on k_{t+1}^* and b_{t+1}^* gives an arbitrage condition between bond and physical capital holdings

$$\frac{1}{p_{bt,t+1}^*} = E_t \left[1 - \delta^* + r_{D,t+1}^{*k} \right]$$

$$\frac{1}{p_{bt,t+1}^*} = E_t \left[1 - \delta^* + r_{O,t+1}^{*k} \right]$$

C Wage Bargaining

The wage is determined via bargaining between workers and firms over the total surplus of a match, which is defined as

$$\left(\mathbf{W}_{i,t} - \mathbf{U}_{i,t}\right)^\eta \left(\mathbf{J}_{i,t} - \mathbf{V}_{i,t}\right)^{1-\eta}$$

for $i \in (\mathbb{D}, \mathbb{O})$ in either the Home or Foreign country, depending on whether it is a Home or Foreign domestic match or an offshore international match. In what follows, we first derive the definitions of the value functions for workers and firms and then, using these value functions, solve for the resulting Nash wage given by the above generalized sharing rule for each of the three labor markets.

C.1 Value Functions

C.1.1 Households

For the Home household, define $\mathbf{V}(n_{\mathbb{D},t-1})$ as the value function associated with the optimal plan that solves the household's problem.

The envelope condition is $\mathbf{V}_{n_{\mathbb{D}}}(n_{\mathbb{D},t-1}) = (1 - \rho^o)(1 - \rho^n)\mu_{\mathbb{D},t}$ where $\mu_{\mathbb{D},t}$ is the Lagrangian multiplier associated with the constraint on the time t perceived law of motion for employment. From the first order condition on $n_{\mathbb{D},t}$ we can express the envelope condition as

$$\frac{\mathbf{V}_{n_{\mathbb{D}}}(n_{\mathbb{D},t-1})}{(1 - \rho^o)(1 - \rho^n)} = \lambda_t w_{\mathbb{D},t} - h'_t + (1 - \rho^o)(1 - \rho^n)\beta E_t \left[\frac{\mathbf{V}_{n_{\mathbb{D}}}(n_{\mathbb{D},t+1})}{(1 - \rho^o)(1 - \rho^n)} \right]$$

Define $\mathbf{W}_{\mathbb{D},t}$ as

$$\begin{aligned} \mathbf{W}_{\mathbb{D},t} &\equiv \frac{\mathbf{V}_{n_{\mathbb{D}}}(n_{\mathbb{D},t-1})}{\lambda_t(1 - \rho^o)(1 - \rho^n)} = w_{\mathbb{D},t} - \frac{h'_t}{\lambda_t} + (1 - \rho^o)(1 - \rho^n)\beta E_t \left[\Xi_{t+1|t} \frac{\mathbf{V}_{n_{\mathbb{D}}}(n_{\mathbb{D},t+1})}{\lambda_{t+1}(1 - \rho^o)(1 - \rho^n)} \right] \\ &= w_{\mathbb{D},t} - \frac{h'_t}{\lambda_t} + (1 - \rho^o)(1 - \rho^n)\beta E_t \left[\Xi_{t+1|t} \mathbf{W}_{\mathbb{D},t+1} \right] \end{aligned}$$

For the Foreign household, define $\mathbf{V}^*(n_{\mathbb{D},t-1}^*, n_{\mathbb{O},t-1}^*)$ as the value function associated with the optimal plan that solves the household problem.

The envelope condition with respect to domestic employment in the Foreign country is $\mathbf{V}_{n_{\mathbb{D}}^*}(n_{\mathbb{D},t-1}^*, n_{\mathbb{O},t-1}^*) = (1 - \rho^{*o})(1 - \rho^{*n})\mu_{\mathbb{D},t}^*$ where $\mu_{\mathbb{D},t}^*$ is the Lagrangian multiplier associated with the constraint on the time t perceived law of motion for employment for domestic jobs. From the first order condition on $n_{\mathbb{D},t}^*$ we can express the envelope condition as

$$\frac{\mathbf{V}_{n_{\mathbb{D}}^*}(n_{\mathbb{D},t-1}^*, n_{\mathbb{O},t-1}^*)}{(1 - \rho^{*o})(1 - \rho^{*n})} = \lambda_t^* w_{\mathbb{D},t}^* - h_t^{*'} + (1 - \rho^{*o})(1 - \rho^{*n})\beta E_t \left[\frac{\mathbf{V}_{n_{\mathbb{D}}^*}(n_{\mathbb{F},t}^*, n_{\mathbb{O},t}^*)}{(1 - \rho^{*o})(1 - \rho^{*n})} \right]$$

Define $\mathbf{W}_{D,t}^*$ as

$$\begin{aligned}\mathbf{W}_{D,t}^* &\equiv \frac{\mathbf{V}_{n_D^*}(n_{D,t-1}^*, n_{O,t-1}^*)}{\lambda_t^*(1-\rho^{*o})(1-\rho^{*n})} = w_{D,t}^* - \frac{h_t^{*'}}{\lambda_t^*} + (1-\rho^{*o})(1-\rho^{*n})E_t \left[\Lambda_{t+1|t}^* \frac{\mathbf{V}_{n_D^*}(n_{D,t+1}^*, n_{O,t+1}^*)}{\lambda_{t+1}^*(1-\rho^{*o})(1-\rho^{*n})} \right] \\ &= w_{D,t}^* - \frac{h_t^{*'}}{\lambda_t^*} + (1-\rho^{*o})(1-\rho^{*n})\beta E_t \left[\Lambda_{t+1|t}^* \mathbf{W}_{D,t+1}^* \right]\end{aligned}$$

The envelope condition with respect to offshore employment is $\mathbf{V}_{n_O^*}(n_{D,t-1}^*, n_{O,t-1}^*) = (1-\rho^{*o})(1-\rho^{*n})\mu_{O,t}^*$ where $\mu_{O,t}^*$ is the Lagrangian multiplier associated with the constraint on the time t perceived law of motion for employment for offshore jobs. From the first order condition on $n_{O,t}^*$ we can express the envelope condition as

$$\frac{\mathbf{V}_{n_O^*}(n_{D,t-1}^*, n_{O,t-1}^*)}{(1-\rho^{*o})(1-\rho^{*n})} = \lambda_t^* w_{O,t}^* - h_t^{*'} + (1-\rho^{*o})(1-\rho^{*n})E_t \left[\frac{\mathbf{V}_{n_O^*}(n_{D,t}^*, n_{O,t}^*)}{(1-\rho^{*o})(1-\rho^{*n})} \right]$$

Define $\mathbf{W}_{O,t}^*$ as

$$\begin{aligned}\mathbf{W}_{O,t}^* &\equiv \frac{\mathbf{V}_{n_O^*}(n_{D,t-1}^*, n_{O,t-1}^*)}{\lambda_t^*(1-\rho^{*o})(1-\rho^{*n})} = w_{O,t}^* - \frac{h_t^{*'}}{\lambda_t^*} + (1-\rho^{*o})(1-\rho^{*n})E_t \left[\Lambda_{t+1|t}^* \frac{\mathbf{V}_{n_O^*}(n_{D,t+1}^*, n_{O,t+1}^*)}{\lambda_{t+1}^*(1-\rho^{*o})(1-\rho^{*n})} \right] \\ &= w_{O,t}^* - \frac{h_t^{*'}}{\lambda_t^*} + (1-\rho^{*o})(1-\rho^{*n})E_t \left[\Lambda_{t+1|t}^* \mathbf{W}_{O,t+1}^* \right]\end{aligned}$$

Finally, note that free entry into the labor force drives the value of search to zero in all markets across all countries, so that

$$\mathbf{U}_{D,t} = \mathbf{U}_{D,t}^* = \mathbf{U}_{O,t}^* = 0$$

C.1.2 Firms

For the Foreign firm, define $\mathbf{V}^*(n_{D,t-1}^*, v_{D,t-1}^*)$ as the value function associated with the optimal plan that solves the firms problem.

The envelope condition with respect to domestic employment is $\mathbf{V}_{n_D^*}(n_{D,t-1}^*, v_{D,t-1}^*) = (1-\rho^{*o})(1-\rho^{*n})\mu_{D,t}^*$ where $\mu_{D,t}^*$ is the Lagrangian multiplier associated with the constraint on the time t perceived law of motion for employment for domestic jobs. From the first order condition on $n_{D,t}^*$ we can express the envelope condition as

$$\frac{\mathbf{V}_{n_D^*}(n_{D,t-1}^*, v_{D,t-1}^*)}{(1-\rho^{*o})(1-\rho^{*n})} = f_{n_{D,t}^*} - w_{D,t}^* + (1-\rho^{*o})E_t \left[\Xi_{t+1|t}^* \left(\rho^{*n} \lambda_{D,t+1}^* + (1-\rho^{*n}) \frac{\mathbf{V}_{n_D^*}(n_{D,t}^*, v_{D,t}^*)}{(1-\rho^{*o})(1-\rho^{*n})} \right) \right]$$

The envelope condition with respect to domestic vacancy postings is $\mathbf{V}_{v_D^*}(n_{D,t-1}^*, v_{D,t-1}^*) = (1-\rho^{*o})(1-k^f(\theta_{D,t-1}^*))\lambda_{D,t}^*$ where $\lambda_{D,t}^*$ is the Lagrangian multiplier associated with the constraint on the time t perceived law of motion for vacancies for domestic jobs. From the first order condition

on $v_{D,t}^*$ we can express the envelope condition as

$$\begin{aligned} \frac{\mathbf{V}_{v_D^*}(n_{D,t-1}^*, v_{D,t-1}^*)}{(1-\rho^{*o})(1-k^f(\theta_{D,t-1}^*))} &= -\gamma_D^* + (1-\rho^{*o})(1-k^f(\theta_{D,t}^*))E_t \left[\Xi_{t+1|t}^* \frac{\mathbf{V}_{v_D^*}(n_{D,t}^*, v_{D,t}^*)}{(1-\rho^{*o})(1-k^f(\theta_{D,t}^*))} \right] \\ &\quad + k^f(\theta_{D,t}^*) \frac{\mathbf{V}_{n_D^*}(n_{D,t-1}^*, v_{D,t-1}^*)}{(1-\rho^{*o})(1-\rho^{*n})} \end{aligned}$$

Define $\mathbf{J}_{D,t}^* \equiv \frac{\mathbf{V}_{n_D^*}(n_{D,t-1}^*, v_{D,t-1}^*)}{(1-\rho^{*o})(1-\rho^{*n})}$ and $\mathbf{V}_{D,t}^* \equiv \frac{\mathbf{V}_{v_D^*}(n_{D,t-1}^*, v_{D,t-1}^*)}{(1-\rho^{*o})(1-k^f(\theta_{D,t-1}^*))}$. We can re-write the above two expressions in terms of value equations as

$$\mathbf{J}_{D,t}^* = f_{n_{D,t}^*} - w_{D,t}^* + (1-\rho^{*o})E_t \left\{ \Xi_{t+1|t}^* \left(\rho^{*n} \mathbf{V}_{D,t+1}^* + (1-\rho^{*n}) \mathbf{J}_{D,t+1}^* \right) \right\}$$

and

$$\begin{aligned} \mathbf{V}_{D,t}^* &= -\gamma_D^* + k^f(\theta_{D,t}^*) \mathbf{J}_{D,t}^* + (1-\rho^{*o})(1-k^f(\theta_{D,t}^*))E_t \left\{ \Xi_{t+1|t}^* \mathbf{V}_{D,t}^* \right\} \\ &= r_{D,t}^{k^*} k_{D,t}^{k^*} \end{aligned}$$

Note that in the above equation the first line represents the value of an open vacancy in the short run, that is, prior to imposing free entry, while the second line represents the value of an open vacancy after imposing free entry.

For the Home multinational, define $\mathbf{V}(n_{D,t-1}, v_{D,t-1}, n_{O,t-1}^*, v_{O,t-1}^*)$ as the value function associated with the optimal plan that solves the firms problem.

The envelope condition with respect to domestic employment is $\mathbf{V}_{n_D}(n_{D,t-1}, v_{D,t-1}, n_{O,t-1}^*, v_{O,t-1}^*) = (1-\rho^o)(1-\rho^n)\mu_{D,t}$ where $\mu_{D,t}$ is the Lagrangian multiplier associated with the constraint on the time t perceived law of motion for employment for domestic jobs. From the first order condition on $n_{D,t}$ we can express the envelope condition as

$$\begin{aligned} \frac{\mathbf{V}_{n_D}(n_{D,t-1}, v_{D,t-1}, n_{O,t-1}^*, v_{O,t-1}^*)}{(1-\rho^o)(1-\rho^n)} &= f_{n_{D,t}} - w_{D,t} \\ + E_t \left[\Xi_{t+1|t} \left((1-\rho^o)\rho^n \lambda_{D,t+1} + (1-\rho^o)(1-\rho^n) \frac{\mathbf{V}_{n_D}(n_{D,t}, v_{D,t}, n_{O,t}^*, v_{O,t}^*)}{(1-\rho^o)(1-\rho^n)} \right) \right] \end{aligned}$$

The envelope condition with respect to offshore employment is $\mathbf{V}_{n_O^*}(n_{D,t-1}, v_{D,t-1}, n_{O,t-1}^*, v_{O,t-1}^*) = (1-\rho^{*o})(1-\rho^{*n})\mu_{O,t}^*$ where $\mu_{O,t}^*$ is the Lagrangian multiplier associated with the constraint on the time t perceived law of motion for employment for offshore jobs. From the first order condition on $n_{O,t}^*$ we can express the envelope condition as

$$\begin{aligned} \frac{\mathbf{V}_{n_O^*}(n_{D,t-1}, v_{D,t-1}, n_{O,t-1}^*, v_{O,t-1}^*)}{(1-\rho^{*o})(1-\rho^{*n})} &= f_{n_{O,t}^*} - q_t w_{O,t}^* \\ + E_t \left[\Xi_{t+1|t} \left((1-\rho^{*o})\rho^{*n} \lambda_{O,t+1}^* + (1-\rho^{*o})(1-\rho^{*n}) \frac{\mathbf{V}_{n_O^*}(n_{D,t}, v_{D,t}, n_{O,t}^*, v_{O,t}^*)}{(1-\rho^{*o})(1-\rho^{*n})} \right) \right] \end{aligned}$$

The envelope condition with respect to domestic vacancy postings is $\mathbf{V}_{v_D}(n_{D,t-1}, v_{D,t-1}, n_{O,t-1}^*, v_{O,t-1}^*) = (1-k^f(\theta_{D,t-1}))(1-\Omega^F k^f(\theta_{O,t-1}^*))(1-\rho^o)\lambda_{D,t}$ where $\lambda_{D,t}$ is the Lagrangian multiplier

associated with the constraint on the time t perceived law of motion for vacancies for domestic jobs. From the first order condition on $v_{D,t}$ we can express the envelope condition as

$$\begin{aligned} \frac{\mathbf{V}_{v_D}(n_{D,t-1}, v_{D,t-1}, n_{O,t-1}^*, v_{O,t-1}^*)}{(1-k^f(\theta_{D,t-1}))(1-\Omega^F k^f(\theta_{O,t-1}^*)) (1-\rho^o)} &= -\gamma + k^f(\theta_{D,t}) \mu_{D,t} \\ &+ \Omega^F (1 - k^f(\theta_{D,t})) \left(-\gamma_o^* + k^f(\theta_{O,t}^*) \mu_{O,t}^* \right) \\ &+ (1 - k^f(\theta_{D,t})) (1 - \Omega^F k^f(\theta_{O,t}^*)) (1 - \rho^o) E_t \left[\Lambda_{t+1|t} \frac{\mathbf{V}_{v_D}(n_{D,t}, v_{D,t}, n_{O,t}^*, v_{O,t}^*)}{(1-k^f(\theta_{D,t}))(1-\Omega^F k^f(\theta_{O,t}^*)) (1-\rho^o)} \right] \end{aligned}$$

The envelope condition with respect to offshore vacancy postings is $\mathbf{V}_{v_o^*}(n_{D,t-1}, v_{D,t-1}, n_{O,t-1}^*, v_{O,t-1}^*) = (1 - k^f(\theta_{O,t-1}^*)) (1 - \rho^{*o}) \lambda_{O,t}^*$ where $\lambda_{O,t}^*$ is the Lagrangian multiplier associated with the constraint on the time t perceived law of motion for vacancies for offshored jobs. From the first order condition on $v_{O,t}^*$ we can express the envelope condition as

$$\begin{aligned} \frac{\mathbf{V}_{v_o^*}(n_{D,t-1}, v_{D,t-1}, n_{O,t-1}^*, v_{O,t-1}^*)}{(1-k^f(\theta_{O,t-1}^*)) (1-\rho^{*o})} &= \gamma_o^* + k^f(\theta_{O,t}^*) \mu_{O,t}^* \\ &+ (1 - k^f(\theta_{O,t}^*)) (1 - \rho^{*o}) E_t \left[\Xi_{t+1|t} \frac{\mathbf{V}_{v_o^*}(n_{H,t}, v_{H,t}, n_{O,t}^*, v_{O,t}^*)}{(1-k^f(\theta_{O,t}^*)) (1-\rho^{*o})} \right] \end{aligned}$$

Define $\mathbf{J}_{D,t} \equiv \frac{\mathbf{V}_{n_D}(n_{D,t-1}, v_{D,t-1}, n_{O,t-1}^*, v_{O,t-1}^*)}{(1-\rho^o)(1-\rho^n)}$, $\mathbf{V}_{D,t} \equiv \frac{\mathbf{V}_{v_D}(n_{D,t-1}, v_{D,t-1}, n_{O,t-1}^*, v_{O,t-1}^*)}{(1-k^f(\theta_{D,t-1}))(1-\Omega^F k^f(\theta_{O,t-1}^*)) (1-\rho^o)}$, $\mathbf{J}_{O,t}^* \equiv \frac{\mathbf{V}_{n_o^*}(n_{D,t-1}, v_{D,t-1}, n_{O,t-1}^*, v_{O,t-1}^*)}{(1-\rho^{*o})(1-\rho^{*n})}$, and $\mathbf{V}_{O,t}^* \equiv \frac{\mathbf{V}_{v_o^*}(n_{D,t-1}, v_{D,t-1}, n_{O,t-1}^*, v_{O,t-1}^*)}{(1-k^f(\theta_{O,t-1}^*)) (1-\Omega^F k^f(\theta_{O,t-1}^*)) (1-\rho^{*o})}$. We can re-write the above expressions in terms of value equations as

$$\mathbf{J}_{D,t} = f_{n_{D,t}} - w_{D,t} + (1 - \rho^o) E_t \left[\Xi_{t+1|t} \left(\rho^n \mathbf{V}_{D,t+1} + (1 - \rho^n) \mathbf{J}_{D,t+1} \right) \right]$$

$$\begin{aligned} \mathbf{V}_{D,t} &= -\gamma + k^f(\theta_{D,t}) \mathbf{J}_{D,t} \\ &+ \Omega^F (1 - k^f(\theta_{D,t})) \left(-\gamma_o^* + k^f(\theta_{O,t}^*) \mathbf{J}_{O,t}^* \right) \\ &+ (1 - k^f(\theta_{D,t})) (1 - \Omega^F k^f(\theta_{O,t}^*)) (1 - \rho^o) E_t \left\{ \Xi_{t+1|t} \mathbf{V}_{D,t+1} \right\} \\ &= r_{D,t}^k k_{D,t}^k \end{aligned}$$

$$\mathbf{J}_{O,t}^* = f_{n_{O,t}^*} - q_t w_{O,t}^* + (1 - \rho^{*o}) E_t \left[\Xi_{t+1|t} \left(\rho^{*n} \mathbf{V}_{O,t+1}^* + (1 - \rho^{*n}) \mathbf{J}_{O,t+1}^* \right) \right]$$

$$\begin{aligned} \mathbf{V}_{O,t}^* &= -\gamma_o^* + k^f(\theta_{O,t}^*) \mathbf{J}_{O,t}^* + (1 - k^f(\theta_{O,t}^*)) (1 - \rho^{*o}) E_t \left\{ \Xi_{t+1|t} \mathbf{V}_{O,t+1}^* \right\} \\ &= q_t r_{O,t}^{k^*} k_{O,t}^{k^*} \end{aligned}$$

Where, again, the first equality in the expressions for $\mathbf{V}_{D,t}$ and $\mathbf{V}_{O,t}^*$, respectively, is the short-run wage that obtains prior to imposing free entry and the second equality in each expression is the wage that obtains after imposing free entry.

C.2 Bargained Wages

The bargained wage is that which solves the following generalized Nash sharing rule for each of the three respective labor markets

$$\eta \left[\frac{\partial \mathbf{W}_{1,t}}{w_{i,t}} - \frac{\partial \mathbf{U}_{1,t}}{w_{i,t}} \right] (\mathbf{J}_{1,t} - \mathbf{V}_{1,t}) + (1 - \eta) \left[\frac{\partial \mathbf{J}_{1,t}}{w_{i,t}} - \frac{\partial \mathbf{V}_{1,t}}{w_{i,t}} \right] (\mathbf{W}_{1,t} - \mathbf{U}_{1,t}) = 0$$

C.2.1 Domestic Jobs with the Home Multinational Firm

The sharing rule reduces to

$$(\mathbf{W}_{D,t} - \mathbf{U}_{D,t}) = \frac{\eta}{1 - \eta} (\mathbf{J}_{D,t} - \mathbf{V}_{D,t})$$

We begin by solving the wage in the short run, that is prior to imposing free entry. Substitute in equations xx, xx, xx, and xx, for the definitions of yy, yy, yy, and yy, respectively.

In the short run, the stock of physical capital and the number of firms is assumed to be fixed.

$$\begin{aligned} w_{D,t} &= (1 - \eta) \frac{h'(lfp_t)}{u'(c_t)} + \eta f_{n_{D,t}} \\ &\quad + \eta \left(\gamma - k^f(\theta_{D,t}) \left(\mathbf{J}_{D,t} - (1 - \rho^o) E_t \left[\Xi_{t+1|t} \mathbf{V}_{D,t+1} \right] \right) \right) \\ &\quad + \eta \Omega (1 - k^f(\theta_{D,t})) \left(\gamma^* - k^f(\theta_{O,t}^*) \left(\mathbf{J}_{O,t}^* - (1 - \rho^o) E_t \left[\Xi_{t+1|t} \mathbf{V}_{D,t+1} \right] \right) \right) \end{aligned}$$

The Long run home wage is given by

$$w_{D,t} = (1 - \eta) \frac{h'_t}{u'_t} + \eta \left(f_{n_{D,t}} - r_t^k k_{D,t} + (1 - \rho^o) E_t \left[\Xi_{t+1|t} r_{t+1}^k k_{D,t+1} \right] \right)$$

C.2.2 Offshore Jobs with the Home Multinational Firm

The sharing rule reduces to

$$(\mathbf{W}_{O,t}^* - \mathbf{U}_{O,t}^*) = \frac{\eta^*}{1 - \eta^*} \frac{1}{q_t} (\mathbf{J}_{O,t}^* - \mathbf{V}_{O,t}^*)$$

We begin by solving the wage in the short run, that is prior to imposing free entry. Substitute in equations xx, xx, xx, and xx, for the definitions of yy, yy, yy, and yy, respectively.

$$\begin{aligned} w_{O,t}^* &= (1 - \eta^*) \frac{h_t^*}{u_t^*} + \eta^* \frac{1}{q_t} f_{n_{H,t}^*} \\ &\quad + \eta^* \frac{1}{q_t} \left(\gamma_o^* - k^f(\theta_{O,t}^*) \left(\mathbf{J}_{O,t}^* - (1 - \rho^{*o}) E_t \left[\Xi_{t+1|t} \mathbf{V}_{O,t+1}^* \right] \right) \right) \\ &\quad + \eta^* \frac{1}{q_t} (1 - \rho^{*o}) (1 - \rho^{n*}) E_t \left[\frac{\Xi_{t+1|t}^* q_t - \Xi_{t+1|t}^* q_{t+1}}{q_{t+1}} \left(\mathbf{J}_{O,t+1}^* - \mathbf{V}_{O,t+1}^* \right) \right] \end{aligned}$$

The offshore wage is given by

$$w_{o,t}^* = (1 - \eta^*) \frac{h_t^*}{u_t^*} + \eta^* \frac{1}{q_t} \left(f_{n_o,t}^* - q_t r_t^{k^*} k_{o,t}^* + (1 - \rho^{o*}) E_t \left[\Xi_{t+1|t}^* q_{t+1} r_{t+1}^{k^*} k_{o,t+1}^* \right] \right) - \eta^* \frac{1}{q_t} (1 - \rho^{o*}) E_t \left[\frac{\Xi_{t+1|t}^* q_t - \Xi_{t+1|t}^* q_{t+1}}{q_{t+1}} \left((1 - \rho^{n*}) \mathbf{J}_{o,t+1}^* + \rho^{n*} q_{t+1} r_{t+1}^{k^*} k_{o,t+1}^* \right) \right]$$

C.2.3 Domestic Jobs with the Foreign Firm

The sharing rule reduces to

$$\left(\mathbf{W}_{D,t}^* - \mathbf{U}_{D,t}^* \right) = \frac{\eta^*}{1 - \eta^*} \left(\mathbf{J}_{D,t}^* - \mathbf{V}_{D,t}^* \right)$$

We begin by solving the wage in the short run, that is prior to imposing free entry. Substitute in equations xx, xx, xx, and xx, for the definitions of yy, yy, yy, and yy, respectively.

$$w_{D,t}^* = (1 - \eta^*) \frac{h_t^*}{u_t^*} + \eta^* f_{n_D,t}^* + \eta^* \left(\gamma_D^* - k^f(\theta_{D,t}^*) \left(\mathbf{J}_{D,t}^* - (1 - \rho^{*o}) E_t \left[\Xi_{t+1|t}^* \mathbf{V}_{D,t+1}^* \right] \right) \right)$$

The foreign wage is given by

$$w_{D,t}^* = (1 - \eta^*) \frac{h_t^*}{u_t^*} + \eta^* \left(f_{n_D,t}^* - r_t^{k^*} k_{D,t}^* + (1 - \rho^{o*}) E_t \left[\Xi_{t+1|t}^* r_{t+1}^{k^*} k_{D,t+1}^* \right] \right)$$

