Technical details for computing deflation probabilities with TIPS prices. By Pat Higgins

Summary of method:

1.) Following Sack (2000), assume that TIPS market participants discount TIPS coupon payments and principal repayments with fitted yields from a smoothed yield curve fitted to observed Treasury STRIPS yields. Also assume that TIPS market participants are risk neutral and value a TIPS security as the discounted present value of its expected coupon and principal payments.

2.) Assume that TIPS market participants expect that monthly trend seasonally adjusted annualized CPI inflation is an unknown constant where the constant has a $Normal(\bar{\pi}_t^*, \sigma^2_{\bar{\pi}_t^*})$ distribution. Even if $\sigma^2_{\bar{\pi}_t^*} = 0$ and $\bar{\pi}_t^*$ were known, there would be idiosyncratic uncertainty in future monthly inflation around the trend since monthly inflation has noise even when the trend is stable. We assume this idiosyncratic noise in annualized inflation has a $Normal(0, \sigma^2_{\pi})$ distribution with $\sigma_{\pi} = 3.5\%$ (approximately the standard deviation of annualized monthly inflation since 1994).

3.) Assume that trend inflation for an on-the-run 5-year TIPS and an off-the-run 10-year TIPS with maturity dates that are only separated by three months – e.g. the 5-year TIPS issued in April 2010 and due April 2015 and the 10-year TIPS issued in July 2005 and due July 2015 – have identical $Normal(\bar{\pi}_t^*, \sigma^2_{\bar{\pi}_t^*})$ distributions. Solve for the values of $\bar{\pi}_t^*$ and $\sigma^2_{\bar{\pi}_t^*}$ that imply the expected discounted coupon and principal payments of both the 5-year TIPS and the 10-year TIPS are equal to the observed prices. With $\bar{\pi}_t^*$ and $\sigma^2_{\bar{\pi}_t^*}$ in hand you can solve for the distribution of inflation or probability of deflation.

Detailed Explanation

Using STRIPS yields and TIPS prices to solve for a constant breakeven inflation rate

Sack (2000) outlines the following procedure to estimate breakeven inflation in the case where inflation is constant. Assume that the discounted present value of a zero-coupon bond paying \$1 in n years is worth $d_t^*(n)$ dollars. $d_t^*(n)$ can be computed from an n-year zero-coupon yield $y_t^{zero}(n)$ with the formula

$$d_t^*(n) = \exp(-y_t^{zero}(n)n)$$

Following Sack (2000), we estimate $y_t^{zero}(n)$ by fitting a smoothed yield curve to observed Treasury STRIPS yields. For the functional form of the smoothed yield curve, we use the Svensson (1994) extension of the Nelson and Siegel (1987) functional form. For coupon payments occuring in less than 3 months we use a cubic spline to interpolate our yield curve with the 4-week Treasury yield.

Assuming $\pi > 0$ is the constant annual inflation rate going forward [e.g. $\pi = .02$ is 2% inflation] the discounted present value of an N-year TIPS issued today that promises a real payment of \$100 with a coupon rate c [e.g. $c = .02$ denotes a 2% coupon rate] and semi-annual coupon payments is

$$P_t^*(N) = \sum_{n=1}^{2N} \frac{c}{2} 100(1+\pi)^{n/2} d_t^*(\frac{n}{2}) + 100(1+\pi)^{N/2} d_t^*(N)$$

This formula is used by Sack (2000) to compute breakeven inflation. If $\pi < 0$, then the $(1+\pi)^{N/2}$ term vanishes since the principal repayment cannot be less than \$100 and the discounted present value of the TIPS issue becomes.

$$P_t^*(N) = \sum_{n=1}^{2N} \frac{c}{2} 100(1+\pi)^{n/2} d_t^*(\frac{n}{2}) + 100 d_t^*(N)$$

Formula for reference CPI and index ratio for a TIPS security

Let $IR_{t+h,t}$ denote the gross inflation rate (or 1 plus the inflation rate) from time t to time $t+h$ in the reference CPI. The reference CPI is simply a daily interpolation of the monthly non-seasonally adjusted CPI. Explictly if time t is in month M and month M has M^{Days} in it and it is the t^{day}th of the month (e.g. if $t =$ September 28, 2010 then $M =$ September 2010, $M-1 =$ August 2010, $M^{Days} = 30$, $t^{day} = 28$) then

$$REFcpi_t = \frac{t^{day}-1}{M^{Days}} CPINSA(M-2) + \frac{M^{Days} - t^{day} + 1}{M^{Days}} CPINSA(M-3)$$

and $IR_{t+h,t} = \frac{REFcpi_{t+h}}{REFcpi_t}$. $IR_{t+h,t}$ is referred to as the index ratio for a TIPS that was issued at time t . For TIPS issues, the coupon and principal repayment dates always take place on the 15th day of a month.

Incorporating uncertainty in inflation into TIPS prices

Assuming risk neutrality and a stochastic inflation rate implies a TIPS expiring N years from now that was issued M years ago has a net present value of

$$P_t^{TIPS}(N) = \sum_{n=1}^{2(N+M)} \frac{c}{2} 100 E_t[(IR_{t-M+\frac{n}{2},t-M})] d_t^*(\frac{n}{2} - M) + 100 d_t^*(N) E_t[\max(1, IR_{t+N,t-M})]$$

For notational conveinance we define $d_t^*(x) = 0$ for $x \leq 0$ so that coupons that have already been paid are not assigned a positive value.

In the case of a zero-coupon TIPS, $P_t^{TIPS}(N) = 100d_t^*(N)E_t[\max(1, IR_{t+N,t-M})]$. In this case the zero-coupon TIPS pays \$100 real dollars if there is no gross deflation since the issue date and a larger real payment if there is gross deflation. Assume that monthly inflation for the seasonally-adjusted CPI measured in logarithims, is an iid random normal variable with the distribution

$$12\log(CPISA(M+h)/CPISA(M+h-1))\tilde{}Normal(\bar{\pi}_t, \sigma_\pi^2)$$

where $Normal(\bar{\pi}_t, \sigma_\pi^2)$ denotes a normal distribution with mean $\bar{\pi}_t$ and variance σ_π^2

For example using the mean and standard deviation of monthly (annualized) inflation for the seasonally adjusted CPI since 1994 corresponds to $\bar{\pi}_t = 2.4\%$, $\sigma_\pi = 3.5\%$. In what follows below, we assume that $\sigma_\pi = 3.5\%$. We interpret σ_π as the idiosyncratic uncertainty in monthly inflation that would still be present even if we knew what trend inflation was going forward. We also assume that TIPS market participants expect with certainty that the current seasonal adjustment factors CPI will continue to evolve as they have over the past 12 months [i.e. we do not incorporate uncertainty about the evolution of seasonality]. Then $IR_{t,t-M}$ is a known constant, so we can write $P_t^{TIPS}(N)$ given $\bar{\pi}_t$ as

$$(1)P_t^{TIPS}(N|\bar{\pi}_t) = \sum_{n=1}^{2(N+M)} \frac{c}{2}100(IR_{t,t-M})E_t[(IR_{t-M+\frac{n}{2},t})|\bar{\pi}_t]d_t^*(\frac{n}{2}-M)+100d_t^*(N)E_t[\max(1, IR_{t+N,t-M})|\bar{\pi}_t]$$

Gross deflation over the entire life of the TIPS occurs if $IR_{t+N,t-M} < 1$. We assume that $\bar{\pi}_t\tilde{}Normal(\bar{\pi}_t^*, \sigma_{\bar{\pi}_t^*}^2)$, i.e. TIPS market participants do not know what trend inflation is going forward, but they have a subjective probability distribution for it. We will use prices of the most recently issued 5-year TIPS and the off-the-run 10-year TIPS that matures 3 months after the on-the-run 5 year TIPS matures to derive what $\bar{\pi}_t^*$ and $\sigma_{\bar{\pi}_t^*}^2$ are.

Given our assumption about the evolution of inflation, the expected value of $P_t^{TIPS}(N|\bar{\pi}_t)$ is

$$(2)E_t[P_t^{TIPS}(N|\bar{\pi}_t)] = \int_{-\infty}^{\infty} P_t^{TIPS}(N|\bar{\pi}_t)\phi(\frac{\bar{\pi}_t - \bar{\pi}_t^*}{\sigma_{\bar{\pi}_t^*}})d\bar{\pi}_t$$

where ϕ denotes the standard normal pdf. The probability of gross deflation for the on-the-run 5-year TIPS that we assume was issued M years ago is

$$(3)E_t[\Pr(IR^{TIPS5yr}_{t+5-M,t-M} < 1|\bar{\pi}_t)] = \int_{-\infty}^{\infty} \Pr(IR^{TIPS5yr}_{t+5-M,t-M} < 1|\bar{\pi}_t)\phi(\frac{\bar{\pi}_t - \bar{\pi}^*_t}{\sigma_{\bar{\pi}^*_t}})d\bar{\pi}_t$$

We will discuss how to compute (1), (2) and (3) in the appendixes.

Under the assumption of risk-neutrality, we have $P^{TIPS}_t(N) = E_t[P^{TIPS}_t(N|\bar{\pi}_t)]$. For the 5-year TIPS maturing in April 2015 and the 10-year TIPS maturing in July 2015, we will assume that trend inflation has identical means and variances for both distributions, i.e. both $\bar{\pi}^{July2015}_t$ and $\bar{\pi}^{April2015}_t$ have identical distributions $Normal(\bar{\pi}^*_t, \sigma^2_{\bar{\pi}^*_t})$. Let the observed values of the April 2015 and July 2015 TIPS bond prices be $\bar{P}^{April15TIPS}$ and $\bar{P}^{July15TIPS}$ respectively. We solve for the values of $\bar{\pi}^*_t$ and $\sigma^2_{\bar{\pi}^*_t}$ that satisfy $P^{April15TIPS}_t(N) = \bar{P}^{April15TIPS}$ and $P^{July15TIPS}_t(N) = \bar{P}^{July15TIPS}$.

For each realization of $\bar{\pi}_t$, trend inflation is a constant so that the uncertainty of 1-month inflation in September 2010 is the same as the uncertainty in 1-month inflation in September 2014. This is somewhat unappealing as the uncertainty in monthly inflation should probably increase with the forecast horizon. We experimented with allowing trend inflation to evolve as a random walk process. I.e. $\bar{\pi}_{t+h+1} = \bar{\pi}_{t+h} + \eta_{t+h+1}$, where the standard deviation of η_{t+h+1} is assumed to be constant across forecast horizons and calibrated with historical CPI data. The deflation probabilities calculated in this manner were similar to those computed with the model we used, where $\eta_{t+h+1} \equiv 0$. We show a graph of this probability in figure 1 along with a lower bound on the probability of deflation proposed by Wright (2009) and a modified lower bound that adjusts for seasonality in the CPI.

Discussion of results

There are more than a few pitfalls in calculating a deflation probability as we have. For example, the 5-year TIPS maturing in April 2015 and the 10-year TIPS maturing in July 2015 do not have the same maturity dates, coupon rates, or coupon payment dates. This makes it challenging to distinguish the price difference in the securities due to these features and the price difference due to the different deflation safeguards.

Furthermore TIPS market participants may believe that the probability distribution for future inflation has "fatter tails" than the normal distribution. Figure 2 plots the cumulative distribution function for average CPI inflation over the next 5-years implied by our TIPS pricing model along with a second cumulative distribution function based on the historical forecast errors of a very simple forecasting model. The model is a variant of the Atkeson-Ohanian model that uses the current 12-month inflation rate for the core CPI as a forecast of the average inflation rate over the next 5 years using all forecasts since 1958 (the core CPI starts in 1957). The historical forecast errors are added to the current 12-month core CPI inflation rate (0.89%) to get a distribution for the 5-year inflation rate. The probability of inflation being below 3% is about 6% according to the emprical distribution and just slightly more than 1% according to the TIPS model. The deflation protection of the 5-year TIPS will turn out to be more valuable ex-post if inflation turns out to be -3% as opposed to, say, -0.5%. Hence it is possible to get a lower probability of deflation and have the TIPS prices be consistent with the preferences of a risk-neutral investor by, say, taking some of the probability mass assigned to an inflation outcome between -1% and 0% and assigning part of it to an inflation outcome above 0% and the other part to an inflation outcome below -1%. As a robustness check, we used a variant of an inflation model proposed by Stock and Watson, specifically, their unobserved components with stochastic volatility model to generate a probability distribution for inflation over the next 5-years. The stochastic volatility feature of the Stock and Watson (2007) model generates fatter tails for inflation than the normal distribution. This model says that there is about a 10% chance that average inflation over the next 5 years will be below 0. But the model also says that expected inflation is about 1.8%; if we change this expectation to the TIPS model expectation of 1.1% for October 4 2010, then the probability of deflation increases to 20%. In any case, these empirical checks suggest that the TIPS model may be overstating the probability of deflation at least somewhat. As shown in figure 1, the probability of deflation has been above 25% since June.

Our model could also be overstating the probability of deflation if investors are willing to pay a higher premium for the enhanced deflation safeguard of the on-the-run five-year TIPS than a risk neutral investor would. Nevertheless, concluding that TIPS market participants think there is virtually no probability of deflation would require some explanation as well. Figure 3 plots breakeven inflation rates for various TIPS issues using the same adjustment for the seasonality in the CPI as in Appendix B. The breakeven inflation rates for the five-year TIPS clearly lie above the breakeven rates for the 10-year TIPS. Without a positive probability of deflation, this occurrence would be difficult to explain unless there is a liquidity premium of the five-year TIPS over the 10-year TIPS. Before the recent financial crisis, when the probability of deflation was presumably low, there was little evidence of such a premium.

Appendix A: More on accounting for seasonality.

In March, April and May nonseasonally CPI inflation is higher than seasonally adjusted inflation. Since TIPS payments are indexed to the NSA CPI, the seasonal pattern in these 3 months will push up the value of the off-the-run 10-year TIPS maturing in July 2015 relative to the on-the-run TIPS maturing in April 2015. If TIPS market participants expected the monthly seasonally adjusted inflation rate to be a single constant for all forecast horizons, seasonal factors would cause the breakeven inflation rate for the July 2015 TIPS to be about 0.15 percentage points higher than the breakeven rate for the April 2015 TIPS. Hence, not adjusting for seasonality in the CPI will result in understating the probability of deflation since the value of the enhanced deflation safeguard of the on-the-run 5-year TIPS maturing in April 2015 will be offset somewhat by the more favorable seasonable factors of the off-the-run 10-year TIPS maturing in July 2015. In a comment on a paper Campbell, Shiller and Viceira (2009), Wright (2009) derives a simple formula for finding a lower bound on the probability of deflation. We will show to adjust this lower bound for seasonality in the CPI.

For now, we fix trend inflation $\bar{\pi}_t$ at any particular value. Suppose the latest reading for the monthly CPI we have is for month T (e.g. as of this writing on September 28th, 2010 the most recent CPI is for August 2010). Letting CPI_T^{NSA} denote the level (index reading) of the seasonally unadjusted CPI and letting CPI_T^{SA} denote the level of the seasonally adjusted CPI, for any horizon h months in the future, we can write

$$\frac{CPI_{T+h}^{NSA}}{CPI_T^{NSA}} = (\frac{CPI_{T+h}^{SA}}{CPI_T^{SA}})[(\frac{CPI_T^{SA}}{CPI_T^{NSA}})(\frac{CPI_{T+h}^{NSA}}{CPI_{T+h}^{SA}})]$$

or

$$\log(\frac{CPI_{T+h}^{NSA}}{CPI_T^{NSA}}) = \log(\frac{CPI_{T+h}^{SA}}{CPI_T^{SA}}) - \sum_{n=1}^{h} \Delta\log(\frac{CPI_{T+n}^{NSA}}{CPI_{T+n}^{SA}})$$

Our assumption that $12\log(P_{t+h}^{SA}/P_{t+h-1}^{SA})$ is iid $Normal(\bar{\pi}_t, \sigma_\pi^2)$ implies that $\log(\frac{CPI_{T+h}^{SA}}{CPI_T^{SA}})$ is normally distributed with mean $(\frac{h}{12})\bar{\pi}_t$and variance $h(\frac{\sigma_\pi}{12})^2$. Furthermore we assume that TIPS market participants forecast the following evolution of seasonality

$$\Delta\log(\frac{CPI_{T+n}^{NSA}}{CPI_{T+n}^{SA}}) = \Delta\log(\frac{CPI_{T-12+n}^{NSA}}{CPI_{T-12+n}^{SA}}) - ((\frac{1}{12})\sum_{m=0}^{11}\Delta\log(\frac{CPI_{T-m}^{NSA}}{CPI_{T-m}^{SA}})) \qquad \text{(for } n \leq 12\text{)}$$

$$\Delta\log(\frac{CPI_{T+n}^{NSA}}{CPI_{T+n}^{SA}}) = \Delta\log(\frac{CPI_{T+n-12}^{NSA}}{CPI_{T+n-12}^{SA}}) \qquad \text{(for } n > 12\text{)}$$

For each of the first 12 months out, TIPS market partipants expect the log change in the seasonal factor to equal whatever the change in the seasonal factor was 12 months before. There is also a small correction so that the forecasted average 12-month inflation rate for the SA and NSA CPI for 12, 24, 36,... months out are the identical. For example, the SA CPI increased 0.3% in July 2010 while the NSA CPI was unchanged. In each of July 2008 and July 2009 the SA CPI 1-month inflation rate was also 0.3 percentage points higher than the 1-month NSA CPI inflation rate. So it is assumed that TIPS market participants also expect that in July 2011 the 1-month SA CPI inflation rate will exceed the 1-month NSA CPI inflation rate by 0.3 percentage points. Thus, we can iteratively forecast

$$CumSA_{T+h} = \sum_{n=1}^{h}\Delta\log(\frac{CPI_{T+n}^{NSA}}{CPI_{T+n}^{SA}})$$

so that a given value $\bar{\pi}_t$ of trend inflation implies

$$\log(\frac{CPI^{NSA}_{T+h}}{CPI^{NSA}_{T}})\tilde{\ } Normal((\frac{h}{12})\bar{\pi}_t + CumSA_{T+h}, h(\frac{\sigma_\pi}{12})^2) \tag{A1}$$

Thus CPI^{NSA}_{T+h} is log-normally distributed.

Appendix B: More on calculating the expected value of TIPS returns.

As in the above Appendix A, we fix trend inflation $\bar{\pi}_t$ at any particular value. Suppose the latest reading for the monthly CPI we have is for month T (e.g. as of this writing on September 28th, 2010 the most recent CPI is for month $T =$ August 2010) and suppose we are interested in the expected value of the reference CPI for a coupon or principal repayment that will take place on the 15th day of month $T + h$. For example, for the 10-year July 2015 TIPS, the next coupon payment is on January 15, 2011, which is for month $T + 5$ in this example. The reference CPI for the coupon or principle repayment is

$$AvgCPI^{15th}_{T+h} = (\frac{14}{d_{T+h}})CPI^{NSA}_{T+h-2} + (\frac{d_{T+h} - 14}{d^{NSA}_{T+h}})CPI^{NSA}_{T+h-3}$$

where d_{T+h} is the number of days in month $T + h$ (e.g. $d_{T+h} = 31$ in January 2011) and CPI^{NSA}_n is the index level of the NSA CPI in month n. Then $AvgCPI^{15th}_{T+h}$ is the sum of two log-normal random variables (see Appendix A) and from (A1) its expected value is

$$\begin{aligned} E_T[AvgCPI^{15th}_{T+h}] \quad = \quad & CPI^{NSA}_T[\{(\frac{14}{d_{T+h}})\exp((\frac{h-2}{12})\bar{\pi}_t \\ & + CumSA_{T+h-2} + (\frac{h-2}{2})(\frac{\sigma_\pi}{12})^2)\} \\ & + \{(\frac{d_{T+h} - 14}{d_{T+h}})\exp((\frac{h-3}{12})\bar{\pi}_t \\ & + CumSA_{T+h-3} + (\frac{h-3}{2})(\frac{\sigma_\pi}{12})^2)\}] \end{aligned} \tag{B1}$$

This uses the fact that if $\log(X)\tilde{\ }Normal(\mu, \sigma^2)$, then $E[X] = e^{\mu + \frac{\sigma^2}{2}}$. We have ignored the case where $h = 1$ or $h = 2$. In these special cases

$$E_T[AvgCPI^{15th}_{T+h}] = (\frac{14}{d_{T+h}})CPI^{NSA}_{T-1} + (\frac{days_{T+h} - 14}{days_{T+h}})CPI^{NSA}_{T-2} \qquad \text{(for h=1)}$$

$$E_T[AvgCPI^{15th}_{T+h}] = (\frac{14}{d_{T+h}})CPI^{NSA}_{T} + (\frac{d_{T+h} - 14}{d_{T+h}})CPI^{NSA}_{T-1} \qquad \text{(for h=2)}$$

Looking back at equation (1), we can see how to derive the expected value of the coupon payments. The expected value of a coupon payment depended on $E_t[E_t[(IR_{t-M+\frac{n}{2},t})|\bar{\pi}_t]]$ where $IR_{t-M+\frac{n}{2},t} = \frac{REFcpi_{t-M+\frac{n}{2}}}{REFcpi_t}$ is the gross inflation rate in the reference CPI from time t to time $t - M + \frac{n}{2}$. But we can rewrite this term as

$$E_t[E_t[(IR_{t-M+\frac{n}{2},t})|\bar{\pi}_t]] = (\frac{1}{REFcpi_t})E_t[E_t[(AvgCPI^{15th}_{T+h})|\bar{\pi}_t]]$$

$E_t[(AvgCPI^{15th}_{T+h})|\bar{\pi}_t]$ can be read off from (B1), (B2) or (B3) depending on the value of h, and its expected value can be solved by integrating over the possible values of $\bar{\pi}_t$ since $\bar{\pi}_t\tilde{\ }Normal(\bar{\pi}^*_t, \sigma^2_{\bar{\pi}^*_t})$. Because of the deflation protection for the principal repayment for TIPS, we also need to know the entire distribution of $AvgCPI^{15th}_{T+h}$ in order to compute the expected value of the principal repayment taking place in month $T+h$ [we can safely assume $h > 2$]. We can write

$$AvgCPI^{15th}_{T+h} = (\frac{d_{T+h} - 14}{d_{T+h}})CPI^{NSA}_{T+h-3} + (\frac{14}{d_{T+h}})CPI^{NSA}_{T+h-2} = CPI^{NSA}_{T+h-3}[1 + (\frac{14}{d_{T+h}})(\exp(\pi^{CPI-NSA}_{T+h-2}) - 1)]$$

where $\pi^{CPI-NSA}_{T+h-2} = \log(\frac{CPI^{NSA}_{T+h-2}}{CPI^{NSA}_{T+h-3}})$. Thus

$$\begin{aligned}
\log(AvgCPI^{15th}_{T+h}) - \log(CPI^{NSA}_T) &= \log(CPI^{NSA}_{T+h-3}[1 + (\frac{14}{d_{T+h}})(\exp(\pi^{CPI-NSA}_{T+h-2}) - 1)]) - \log(CPI^{NSA}_T) \\
&= \log(\frac{CPI^{NSA}_{T+h-3}}{CPI^{NSA}_T}) + \log([1 + (\frac{14}{d_{T+h}})(\exp(\pi^{CPI-NSA}_{T+h-2}) - 1)]) \\
&\approx \log(\frac{CPI^{NSA}_{T+h-3}}{CPI^{NSA}_T}) + (\frac{14}{d_{T+h}})(\exp(\pi^{CPI-NSA}_{T+h-2}) - 1) \\
&\approx \log(\frac{CPI^{NSA}_{T+h-3}}{CPI^{NSA}_T}) + (\frac{14}{d_{T+h}})\pi^{CPI-NSA}_{T+h-2}
\end{aligned}$$

Note that $\log(\frac{CPI^{NSA}_{T+h-3}}{CPI^{NSA}_T})$ and $(\frac{14}{days_{T+h}})\pi^{CPI-NSA}_{T+h-2}$ are independent random normal variables with

$$\log(\frac{CPI^{NSA}_{T+h-3}}{CPI^{NSA}_T})\tilde{\ }Normal((\frac{h-3}{12})\bar{\pi}_t + CumSA_{T+h-3}, (h-3)(\frac{\sigma_\pi}{12})^2)$$

and

$$\pi_{T+h-2}^{CPI-NSA} \sim Normal((\frac{1}{12})\bar{\pi}_t + CumSA_{T+h-2} - CumSA_{T+h-3}, (\frac{\sigma_\pi}{12})^2)$$

[see A1 at end of Appendix A] . So $\log(AvgCPI_{T+h}^{15th})$ is approximately a linear combination of two independent random normal variables, so it is also approximately a random normal variable with a mean of

$$\begin{aligned} \mu_{\log AvgCPI|\bar{\pi}_t} &= \log(CPI_T^{NSA}) + (\frac{h-3}{12})\bar{\pi}_t + (\frac{14}{d_{T+h}})(\frac{1}{12})\bar{\pi}_t \\ &\quad + CumSA_{T+h-3} - (\frac{14}{d_{T+h}})[CumSA_{T+h-2} - CumSA_{T+h-3}] \end{aligned} \tag{B4}$$

and a variance of

$$\sigma^2_{\log AvgCPI|\bar{\pi}_t} = (h-3)(\frac{\sigma_\pi}{12})^2 + [(\frac{14}{d_{T+h}})(\frac{\sigma_\pi}{12})]^2 \tag{B5}$$

This implies that $IR_{t+N,t-M} = (\frac{1}{REFcpi_{t-M}})AvgCPI_{T+h}^{15th}$, is approximately a lognormal random variable. Then the $E_t[\max(1, IR_{t+N,t-M})|\bar{\pi}_t]$ term in equation (1) determining the value of the expected principal is

$$E_t[\max(1, IR_{t+N,t-M})|\bar{\pi}_t] = \int_{-\infty}^{REFcpi_{t-M}} f(x|\bar{\pi}_t)dx + (\frac{1}{REFcpi_{t-M}}) \int_{REFcpi_{t-M}}^{\infty} xf(x|\bar{\pi}_t)dx$$

where $f(x|\bar{\pi}_t)$ is the probability density function for $AvgCPI_{T+h}^{15th}$ which we know since $\log AvgCPI_{T+h}^{15th}$ is (approximately) normal with known mean and variance in (B4) and (B5). Explicitly

$$f(x|\bar{\pi}_t) = (\frac{1}{x\sqrt{2\pi\sigma^2_{\log AvgCPI|\bar{\pi}_t}}})\exp(-\frac{(\log x - \mu_{\log AvgCPI|\bar{\pi}_t})^2}{2\sigma^2_{\log AvgCPI|\bar{\pi}_t}}) \tag{B6}$$

The probability of gross deflation given $\bar{\pi}_t$ is then $\Pr[x < REFcpi_{t-M}] = \int_{-\infty}^{REFcpi_{t-M}} f(x|\bar{\pi}_t)dx$

Appendix C: Solving for the probability of deflation.

For any given value of $\bar{\pi}_t$, we can solve for $P_t^{TIPS}(N|\bar{\pi}_t)$ in equation (1) using the calculations in the above 2 appendixes. For any proposed values of $\bar{\pi}_t^*$ and $\sigma^2_{\bar{\pi}_t^*}$ where

$\bar{\pi}_t \tilde{} Normal(\bar{\pi}_t^*, \sigma^2_{\bar{\pi}_t^*})$, we approximate the expected value of $P_t^{TIPS}(N|\bar{\pi}_t)$ using the formula

$$\begin{aligned}
E_t[P_t^{TIPS}(N|\bar{\pi}_t)] &= \{\sum_{n=1}^{9999} P_t^{TIPS}(N|\bar{\pi}_t = -.50 + n/10^4)[\Phi(\frac{-.50 + (n+.5)/10^4 - \bar{\pi}_t^*}{\sigma_{\bar{\pi}_t^*}}) \\
& -\Phi(\frac{-.50 + (n-.5)/10^4 - \bar{\pi}_t^*}{\sigma_{\bar{\pi}_t^*}})]\} \\
+P_t^{TIPS}(N|\bar{\pi}_t &= -.50)\Phi(\frac{-.50 + .5/10^4 - \bar{\pi}_t^*}{\sigma_{\bar{\pi}_t^*}}) \\
+P_t^{TIPS}(N|\bar{\pi}_t &= .50)(1 - \Phi(\frac{.50 - .5/10^4 - \bar{\pi}_t^*}{\sigma_{\bar{\pi}_t^*}}))
\end{aligned}$$

where Φ is the cumulative distribution function for a standard normal variable. This is just numerical integration with grid points for trend inflation of $\bar{\pi}_t$ = -50%, -49.99%, -49.98%, ... , 49.99%, 50%.

Once we solve for the values of $\bar{\pi}_t^*$ and $\sigma^2_{\bar{\pi}_t^*}$ which equate $E_t[P_t^{April2015TIPS}(N|\bar{\pi}_t)]$ and $E_t[P_t^{July2015TIPS}(N|\bar{\pi}_t)]$ with their observed prices, we can solve for the probabilty of deflation using the formula.

$$\begin{aligned}
E_t[\Pr(IR_{t+5-M,t-M}^{TIPS5yr} &< 1|\bar{\pi}_t)] = \\
& \{\sum_{n=1}^{9999} \int_{-\infty}^{REFcpi_{t-M}} f(x|\bar{\pi}_t = -.50 + n/10^4)dx[\Phi(\frac{-.50 + (n+.5)/10^4 - \bar{\pi}_t^*}{\sigma_{\bar{\pi}_t^*}}) \\
& -\Phi(\frac{-.50 + (n-.5)/10^4 - \bar{\pi}_t^*}{\sigma_{\bar{\pi}_t^*}})] \\
+P_t^{TIPS}(N|\bar{\pi}_t &= -.50)\Phi(\frac{-.50 + .5/10^4 - \bar{\pi}_t^*}{\sigma_{\bar{\pi}_t^*}}) \\
+P_t^{TIPS}(N|\bar{\pi}_t &= .50)(1 - \Phi(\frac{.50 - .5/10^4 - \bar{\pi}_t^*}{\sigma_{\bar{\pi}_t^*}}))
\end{aligned}$$

where $f(x|\bar{\pi}_t)dx$ is the probability of deflation for a known value of $\bar{\pi}_t$ shown in (B6).

Appendix D: Adjusting Wright's model for seasonality in the CPI.

Wright derives the following formula for calculating a lower bound on the probability of deflation. In order to keep the notation consistent with the above sections, we modify his notation slightly. Then Wright's formula is

$$\Pr(IR^{TIPS5yr}_{t+5-M,t-M} < 1) = (\frac{r((5-M)+\frac{1}{8})}{\log(IR^{TIPS10yr}_{t+5-M+\frac{1}{4},t-M-5+\frac{1}{4}}/IR^{TIPS5yr}_{t+5-M,t-M})})$$

where r is the spread between the real yield on the off-the-run 10-year TIPS and on-the-run 5-year TIPS, M is the number of years ago that the on-the-run 5-year TIPS was issued [currently April 15, 2010], $(5-M)$ is the number of years until the on-the-run 5-year TIPS issue expires, and $(5-M)+\frac{1}{8}$ is the number of years until June 1 of the year that the 5-year TIPS matures. The $IR^{TIPS5yr}_{t+(5-M),t-M}$ term refers to the current index-ratio of the on-the-run 5-year TIPS and $IR^{TIPS10yr}_{t+5-M+\frac{1}{4},t-M-5+\frac{1}{4}}$ is the current index-ratio of the off-the-run 10-year TIPS that matures 3 months after the 5-year TIPS matures (that is why a $\frac{1}{4}$ term appears). The ratio of the index ratios in the denomanaitor is also the reference CPI on the day the 5-year TIPS was issued divided by the reference CPI on the day the 10-year TIPS was issued. Wright's formula can therefore be rewritten as

$$\Pr(IR^{TIPS5yr}_{t+5-M,t-M} < 1) = (\frac{rM^*}{\log(\mathrm{Re}\,fCPI_{IssueDate5yr}/\mathrm{Re}\,fCPI_{IssueDate5yr})})$$

where $M^* = (5-M)+\frac{1}{8}$. Our modification of Wright's formula will adjust r to r^{SA} by doing the following for both the 5-year and 10-year TIPS issues.

(A) Set $\sigma_{\bar{\pi}^*_t} = \sigma_\pi = 0$, so that there is no uncertainty in inflation, but continue to assume that non-seasonally adjusted CPI inflation varies from month to month according to the seasonal pattern described in Appendix B. Given a single TIPS price – either the 5-year TIPS or 10-year TIPS – we can then solve for $\bar{\pi}^*_t$ using the same formulas in appendix B.

(B) Using the value of $\bar{\pi}^*_t$ solved for in step (1), compute what the value of the TIPS issue would be if there were no seasonality in the CPI. This can be setting $CumSA_{T+h} = 0$ for $h > 0$, and plugging $\bar{\pi}^*_t$ into equation (1) to solve for the nominal TIPS price, i.e. what you would pay in actual dollars for the TIPS security.

(C) Convert the nominal TIPS price to a real TIPS price using the standard conversion formula provided at www.treasurydirect.gov/govt/apps/fip/news/TIPS&ZCBRevised.ppt (see slide 14).

(D) Compute the real yield from the real bond price.

r^{SA} is then calculated as the spread between the modified real yields of the off-the-run 10-year TIPS and the on-the-run 5-year TIPS calculated from the above four steps. The modified lower bound on the probability of deflation is then

$$\Pr(IR_{t+5-M,t-M}^{TIPS5yr} < 1)^{SA} = (\frac{r^{SA}M^{*}}{\log(\operatorname{Re}fCPI_{IssueDate5yr}/\operatorname{Re}fCPI_{IssueDate5yr})})$$

The exact probability calculated with the TIPS pricing model described above along with the Wright lower bound and seasonally adjusted Wright lower bound are shown in the figure below.

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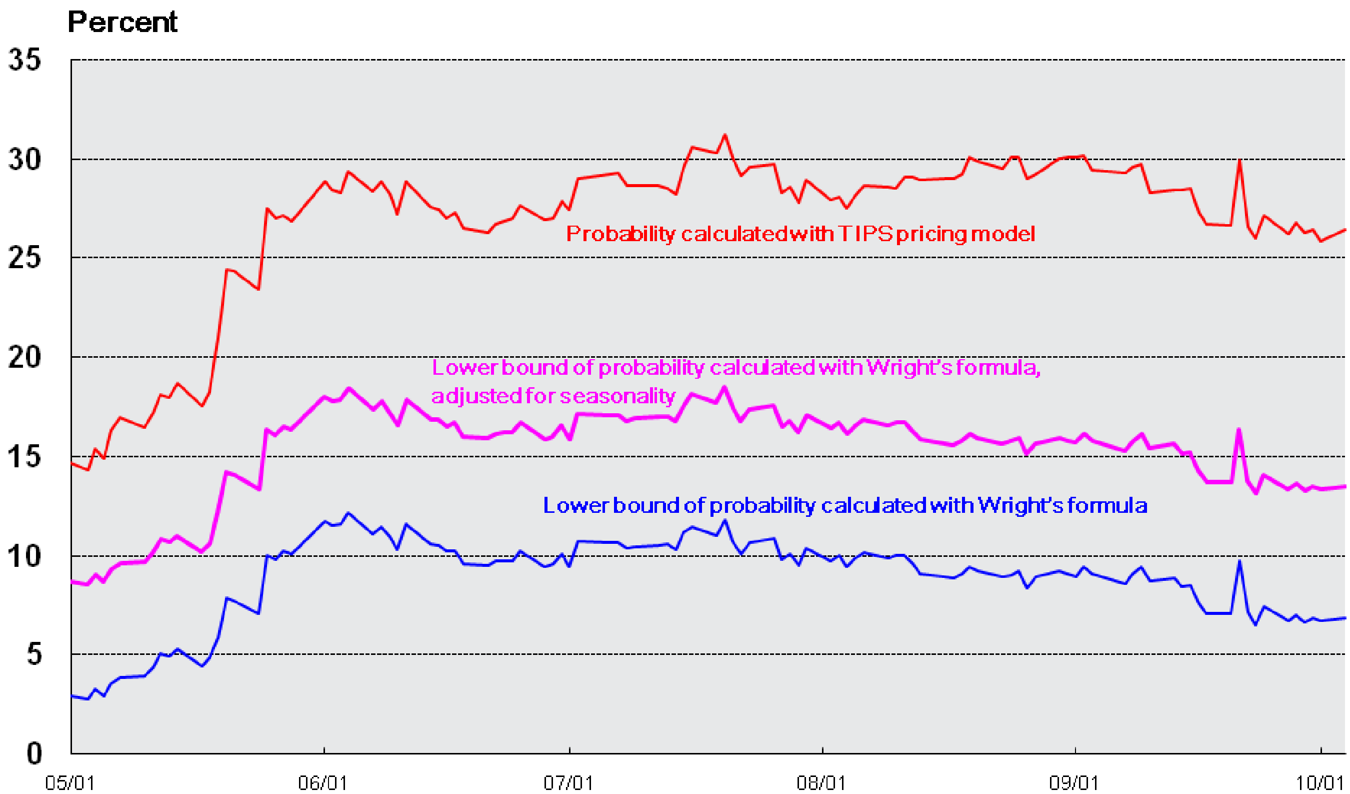
Figure 1: Probability that reference CPI on April 15, 2015 is below its April 15, 2010 level



Source: Author's calculations

Figure 2: Cumulative Distribution Function for 5-year Inflation Forecast

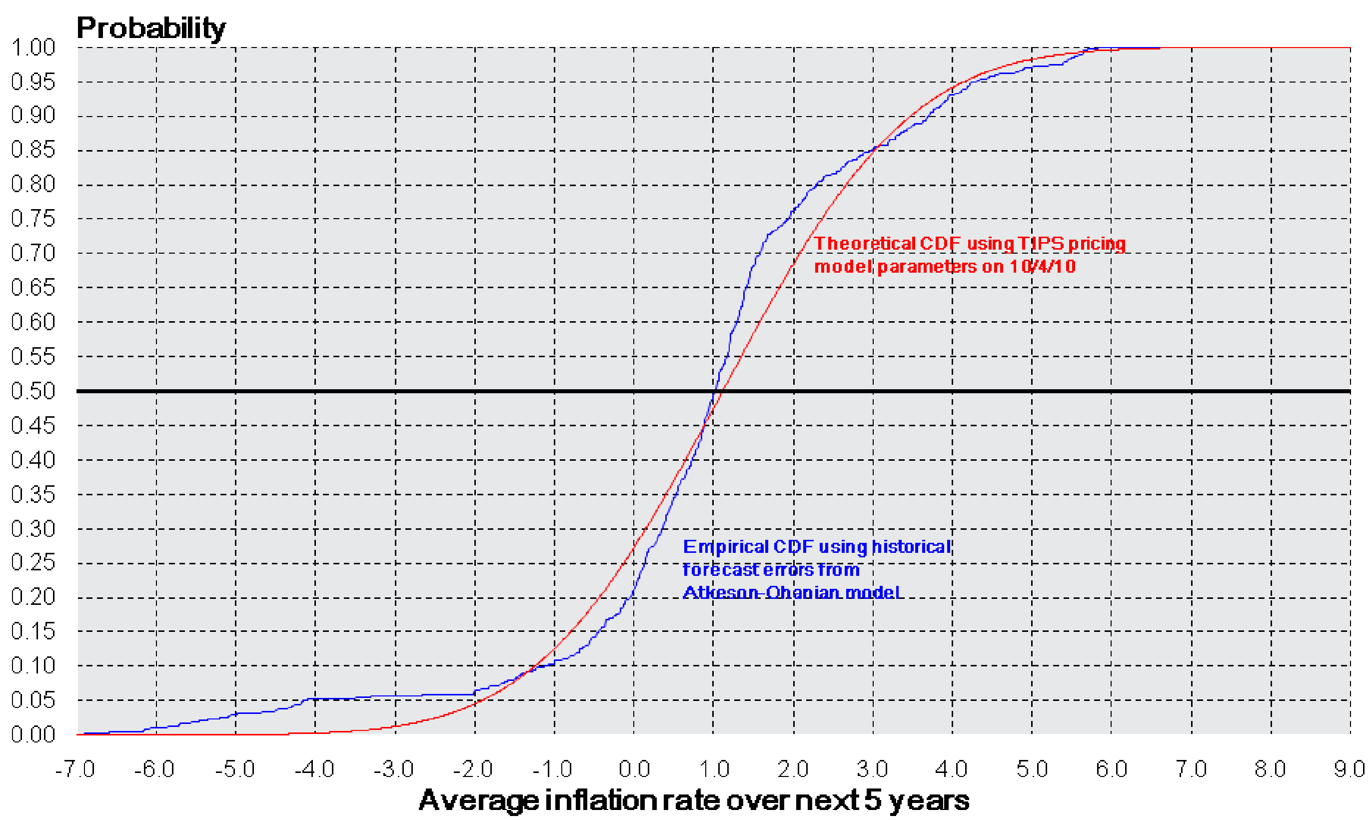


Figure 3: Breakeven inflation rates for various TIPS issues on October 4, 2010