Stocks or Options: Risk Choices and Compensation Design*

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Abstract

We analyze the impact of the existence of bad-tail risks on the decision to pay managers in stocks or in options and find that, contrary to conventional wisdom that options incent higher risk-taking, options are sometimes the superior vehicle for limiting managerial incentives to take bad-tail risks. Though options have not been the dominant form of compensation in the financial services industry in recent years, options are optimal under a condition similar to second-order stochastic dominance. While conventional options can incent the desired project choice under some dominance circumstances, collar-like options can do so in a wide variety of circumstances. Shareholder mistakes in choosing vehicles for executive compensation may have contributed to executives' incentives to take tail risks.

Keywords: Tail risk, Stock options, Moral hazard

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1 Introduction

Much public attention has been given to the role of executive compensation as one contributor to excessive financial-institution risk-taking that caused the recent financial crisis, and especially to incentives for employees to take bad-tail risks, meaning risks that result in very large losses with a small perceived probability. The pre-crisis attentiveness of boards of directors to risk-taking incentives is not clear. On the one hand, executives of very large financial services firms were compensated much more in stock than in options, and conventional wisdom has been that options incent greater risk-taking. On the other hand, there is little evidence, either anecdotal or in texts of proxy statements, that risk was a primary focus or expertise of compensation committees. It is possible that boards were inattentive to risk-taking incentives, especially since the literature has not previously focused on the varieties of bad-tail risks that major banks were revealed to have taken. In this paper, we show that under some circumstances, stock-based compensation is more likely to incent the taking of bad-tail risks than properly designed option-based compensation. We also show that compensation contracts that resemble collar options can disincentivize the taking of bad-tail risk in a fairly wide range of circumstances.

Existing literature has been attentive to some forms of risk-taking incentives: For example, Smith and Stulz (1985) argue that undiversified managers with high-delta compensation arrangements, such as pay in the form of stock, are likely to take too little risk, while those with high-vega arrangements, such as pay in the form of options, may take too much risk (a sampling of other relevant papers includes John and John 1993, Morck et al. 1988, and Jensen and Meckling 1976). Though the literature offers counterexamples, practitioners have come to view stock as preferable to stock options for purposes of providing appropriate risk-taking incentives. However, the literature has focused on project choice sets for managers that are relatively smooth in their risk characteristics. Projects posing tail risks can be qualitatively different from more ordinary projects, especially in the financial services industry.

We examine the nature of optimal compensation contracts that limit managerial incentives to take bad-tail risk. We use a simple principal-agent framework to model how bank shareholders structure compensation contracts with effort-averse bank managers in the presence of tail risk and when managerial choices are not observable. Our model departs from the standard moral hazard model (Holmstrom 1979) in three ways. First, we assume that the manager's compensation arrangement uses only stocks and stock options as vehicles for delivering pay. Second, we allow for tail risk in the manager's project choice set, that is, a project that may be somewhat more profitable than the ordinary project under ordinary circumstances but that has a small probability of resulting in disastrously large losses. The disaster outcome causes the tail risk project to have lower NPV overall than the ordinary project. Thus, all else equal, shareholders prefer that managers avoid tail risk.

Third, we assume that the manager receives a minimum positive amount of compensation in all states of the world. This captures not only limited liability of the manager and the fixed salary component of compensation (which tends to be quite small relative to bonuses for senior employees at major financial firms), but also the fact, observed during the crisis, that senior employees receive significant positive bonuses even in the wake of disastrous performance.[1]

We show that compensation in the form of conventional stock options (those with a single strike) is sometimes the optimal pay vehicle for deterring managers from taking bad-tail risk (while also incentivizing effort), in contrast to the aforementioned conventional wisdom that option-based compensation increases risk-taking incentives. Option-based compensation permits shareholders to establish a strike-price-like project outcome below which the manager's payoff is insensitive to the actual project outcome. Depending on the shape of the density functions for outcomes of the tail-risk and ordinary projects, the

[1] Though CEOs of many global universal banks ultimately refused any bonus for the 2008 pay year due to public pressure, their boards intended to grant bonuses until pressure was applied, and other employees did receive substantial bonuses, even employees in business units that imposed large losses on their firm.

effect of such insensitivity can be to increase the expected value to the manager more for the ordinary project than for the tail risk project, allowing shareholders to incent a value-maximizing choice using options that they cannot achieve using stock-based compensation.

The condition under which conventional options are the optimal vehicle resembles second-order stochastic dominance: In ordinary states of the world, the tail-risk project must dominate the ordinary project. This result is general for all distributions of ordinary-period project payoff. Intuition is easiest in the case of symmetric payoff distributions: The project with fatter ordinary tails has more mass of low returns cut off by the options strike, increasing its value to managers relative to the value of the project with thinner tails. Though it may seem counterintuitive that projects that pose bad-tail risks should have thinner tails in ordinary times, many strategies followed by financial services firms that blew up during the crisis did deliver modest excess returns with low volatility in the years prior to the crisis. Among the examples are: credit-arb ABCP vehicles that provided excess returns with only modest volatility for a number of years until virtually all imposed substantial funding-shock losses on sponsors during the crisis; highly rated securitized mortgage bonds; credit protection sold by monoline insurance companies on such bonds and on bank debt; and quant equity strategies used by some hedge funds to obtain high Sharpe ratios for many years until large losses were experienced in the fall of 2007.

Options can also incent choice of the ordinary project in the opposite case (when the ordinary risk project dominates) if structured as collars. The upper strike price is the key instrument in such situations: If it is set at an appropriate value above the mean payoff, it can be used to "remove" more expected value from the tail risk project than the ordinary risk project, thereby causing the manager to prefer the ordinary risk project (in the current draft we only prove this result for symmetric distributions of non-disaster project payoffs). Though collar options have rarely been used for executive compensation, examples exist of arrangements for some other bank employees that resemble collars: The incremental bonus per unit of incremental revenue flattens out or even is capped for revenue far above the employee's annual target.

Compensation arrangements must incentivize effort as well as the desired level of risk taking. We use a trade-off between avoiding tail risk and inducing managerial effort to determine the equilibrium option exercise price and the optimal proportion of firm value granted in equity or in option compensation. When conventional options are used, the same insensitivity of managerial payoffs to outcomes below the strike that has a differential impact on risk incentives also makes payoffs less sensitive to effort. To induce effort, the manager must be given a higher pay-for-performance incentive above the strike relative to that required with stock-based compensation (the manager must be given a larger share of the firm when options are used).

Based on this tension between deterring tail risk and motivating productive effort, we also characterize conditions under which stock compensation fails to promote prudent risk taking, making option compensation the preferred vehicle. By virtue of having only one measure (a piece rate) manipulatable in stock compensation, effort and risk decisions cannot be controlled separately as in options: there are complex feedback effects between effort and project selection.

Another prediction of our model is that more effective monitoring of risk choices by shareholders should be associated with a higher tendency by firms to compensate their executives in stock instead of options. The best monitoring policy is determined by a tradeoff between paying employees a lower wage and the cost of monitoring. When monitoring is sufficiently effective (that is, a high probability of detecting a choice of the tail risk project), stocks provide adequate incentives for managers to choose the ordinary, value-maximizing project while maintaining incentives to put forth effort. When monitoring is less effective, a piece-wise linear contract such as options is necessary to avoid tail risk, but the optimal strike value for the option is lower.

Overall, contrary to conventional wisdom in the literature to date, our results imply that properly designed options can be a powerful tool for boards of directors to use in deterring the taking of bad-tail risk by managers of financial services firms.

To the best of our knowledge, our paper is the first to consider the impact of incentives to take tail risk on the design of compensation contracts. However, the conventional wisdom that options induce more risk taking has been challenged by Carpenter (2000) and Russ (2004). They both show that the effect of option compensation on risk taking is ambiguous and depends on the manger's utility function. Our results are complementary in that we examine the impact of project choice sets rather than the impact of different utility functions.

This paper is also related to the literature on the role of options in optimal compensation contracts. Hemmer et al. (2000) examine the restrictions on outcome distributions and utility functions that add a convex component such as stock options to an optimal managerial contract. Recently, Kadan and Swinkels (2008) show that if nonviability is not a major issue or is out of managerial control, options always dominate stocks. Our results indicate that even without bankruptcy risk and restrictions on functional forms, introducing tail risk may result in a convexity of the optimal compensation contract.

We view our focus on the form of compensation in an environment with tail risk as an important first step toward development of a literature that addresses many issues recently in the public eye for which tail risk is relevant. Examples of subjects for future research that have recently been the subject of public debate include the term and vesting schedule for deferred compensation, the mix of fixed salary and bonus, and the mix of deferred versus upfront bonus payments.

The rest of this paper is organized as follows. Section 2 presents that model. Sections 3 and 4 show that stock compensation can be optimal without risk choice, but can induce a choice of tail risk when in the presence of tail risk and a state-independent minimum payoff to the manager. Section 5 studies the effects of monitoring on compensation design and discusses limited liability of shareholders and externality of executive risk taking. The Appendix contains a binary example (to illustrate the importance of risk-taking considerations in compensation design and to motivate our full model) and the proofs omitted from the text.

2 Model

A risk-neutral principal (shareholders) hires a risk-neutral agent (manager) for one period. The manager chooses a project and the stochastic payoff y is influenced by the manager's effort. We refer to y as the stock price hereafter.[2] The unobserved effort level of the manager, e, can take two values, low (l) and high (h), that is, $e \in \{l, h\}$, where $l < h$. The manager incurs disutility from exerting effort, denoted by the cost function $a(e)$, where high effort has a cost of $a(h) = c$, while low effort involves no cost: $a(l) = 0$.

The manager chooses one of two mutually exclusive projects: an ordinary risk project (o) or a tail risk project (t). The stock price of the firm when the ordinary risk project is chosen, denoted by $y_o \in [\underline{y}, \overline{y}]$, has density $f_o(\cdot|e)$ and cumulative distribution function $F_o(\cdot|e)$, where $e \in \{l, h\}$. If the manager chooses the tail risk project, with probability $(1 - p)$, a disaster payoff denoted by d occurs. With probability p, the payoff denoted by y'_t follows a distribution $g_t(\cdot|e)$, where $e \in \{l, h\}$. For simplicity, this payoff has the same support as y_o, i.e., $y'_t \in [\underline{y}, \overline{y}]$. To avoid carrying p through the notation, we define $y_t \in [\underline{y}, \overline{y}]$ to be the stock price excluding the disaster outcome: $y_t \sim f_t(y|e) \equiv g_t(\frac{y'_t}{p}|e)$. Thus, $E[y_t|e] = pE[y'_t|e]$. The cumulative distribution function associated with $f_t(\cdot|e)$ is $F_t(\cdot|e)$. We assume that $f_o(\cdot|e)$ and $f_t(\cdot|e)$ are continuous.

High effort by the manager induces a higher expected stock price than low effort. Effort has no impact on the probability of disaster or on the variance of continous payoff over the interval $[\underline{y}, \overline{y}]$, but it does change the expected value of the two projects: It moves the probability distribution function ($f_i(\cdot)$) of continuous payoff to the right by a constant amount κ, regardless of the risk choice ($\forall i \in \{o, t\}$). If the manager chooses the tail risk

[2]We use the term "stock price" because of the ubiquity of stock-based bonus plans. However, stocks are a claim on real firms' project cash flows after employee compensation is subtracted. In this paper, "stock price" refers to the project payoff before employee compensation is subtracted.

project, the disaster outcome d occurs with probability $(1-p)$ regardless of managerial effort. In particular, if the manager chooses the ordinary risk project, the distribution of y_o conditional on high effort is $f_o(y)$, and that conditional on low effort is $f_o(y+\kappa)$, where $\kappa > 0$. Excluding the bad tail, the stock price y_t follows $f_t(y)$ if the manager exerts high effort and follows $f_t(y+\kappa)$ if the manager exerts low effort.

For notational convenience, we denote the expected value of the ordinary risk project given high effort as μ_o, that is, $\mu_o = \int_{\underline{y}}^{\overline{y}} f_o(y)dy$. Similarly, the expected value of the tail risk project given high effort when excluding disaster is $\mu_t = \int_{\underline{y}}^{\overline{y}} f_t(y)dy$. The expected value of the tail risk project given high effort when including disaster is $\mu_t^d = \mu_t + (1-p)d$. The tail risk project has lower NPV ($\mu_t^d < \mu_o$).[3]

The manager is compensated as a function of the stock price, $w(y)$. We assume that in all states the manager has to be provided a non-negative minimum wage, denoted by $\underline{w}$, which reflects limited liability on the part of the manager and the observed tendency of firms to pay bonuses even after disastrous performance. Without a minimum wage constraint in the model, the principal could eliminate incentives to take tail risk by imposing sufficiently harsh penalties upon realizations of disasters.[4]

The net payoff to the principal is the terminal value of the firm less managerial compensation. Prudent risk taking (that is, a choice of the ordinary risk project) induces a higher expected payoff, but it may also induce a higher compensation cost to the principal. Thus, a priori it is not clear that the principal wants the manager to avoid tail risk. Here we assume that the difference in expected stock price $(\mu_o - \mu_t^d)$ is large enough that the principal always wants to motivate the manager to take ordinary risk. We also assume that κ is large enough that it is always in the best interest of the principal to induce high

[3]We restrict the project values in this way to make the problem interesting. If the tail risk project had lower expected value even without the disaster outcome, motivating the manager to avoid it would be relatively easy. If the ordinary risk project had lower expected value than the full tail risk project, the principal would prefer the tail risk project.

[4]Partnership-like models might achieve harsh penalties if partners have sufficient wealth at risk, but in a true partnership there is no separation between managers and shareholders.

effort. The objective of the manager is to maximize his utility by choosing an effort e and a project $R \in \{o, t\}$, subject to the contract he is offered. The manager is risk neutral, and his utility is of the form $U(e, y) = w(y) - a(e)$. The principal chooses $w(y)$ that minimizes the expected cost of inducing managerial effort and avoiding tail risk. Formally, the optimal contract solves

$$\min_{w(\cdot)} \quad E[w(y_o)|h]$$

subject to

$$h = \arg\max_{e \in \{l,h\}} E[w(y_o)] - a(e), \qquad \forall y_o \sim f_o(\cdot|e). \tag{IC_e}$$

$$E[U(h, y)] = E[w(y_o)|h] - a(h) \geq \bar{U}. \tag{PC}$$

$$w(y) \geq \underline{w}, \qquad \forall y. \tag{LL}$$

$$O = \arg\max_{R \in \{o,t\}} E[U(h, y)] \tag{IC_R}$$

The objective function is the expected cost for the principal to motivate effort and implement the ordinary risk project. The first constraint is the incentive constraint for the manager's choice of effort—here, we assume that the principal wants to induce high effort. The second is the participation constraint, where $\bar{U}$ is the manager's outside option. The third constraint is the minimum wage constraint for the manager: $\underline{w}$ is the minimum wage the manager must be provided in all states.

Though in general the participation constraint may or may not bind, as in the binary example. In this analysis we restrict our attention to the case where the limited liability constraint is tight enough that the participation constraint is always satisfied and does not bind — the manager's compensation inside the bank exceeds outside opportunities. Broadly similar results obtain when the participation constraint binds.

In addition to the aforementioned conventional constraints, when the risk choice is not observable, the principal faces another constraint (IC_R): the desired risk choice has to be voluntarily followed by the manager. Note that due to the interaction of multiple hidden actions (effort decision and risk choice), compensation off the equilibrium path as well as along the equilibrium path is used to deter joint deviation. Following Doepke and Townsend (2006), we attack this problem by specifying the expected wage that the manager can get when deviating from the recommended risk choice and making it no larger than the expected wage for the recommended risk choice. That is, in equilibrium, the expected wage conditional on taking ordinary risk is always greater than that conditional on taking on tail risk, regardless of the manager's effort choice when choosing the tail risk project. In particular,

$$E[U(h, y_o)] \geq E[U(h, y_t)], \tag{IC_{R1}}$$

$$E[U(h, y_o)] \geq E[U(l, y_t)]. \tag{IC_{R2}}$$

In this paper we analyze the case where compensation contracts can take the form of stocks or stock options. In particular, if options are used, $w(y)$ takes the form

$$w(y) = \underline{w} + \theta \max\{y - y^*, 0\},$$

where $\underline{w}$ is the minimum wage, $\theta \in [0, 1]$ is the proportion of the firm granted in options if exercised, and y^* is the exercise price on these options. If stocks are used, $w(y)$ can be expressed as

$$w(y) = \max\{\underline{w}, \theta y\},$$

which can be rewritten as

$$w(y) = \underline{w} + \theta \max\{y - \frac{\underline{w}}{\theta}, 0\}.$$

Note that our definition of stocks differs from the conventional definition because of the additional minimum-wage constraint.[5] There is an inherent kink at $(\underline{w}/\theta)$ in stock compensation. As the manager is guaranteed $\underline{w}$ in all states, when stock price falls below $\underline{w}/\theta$,

[5]Because the principal can freely choose the exercise price, the minimum wage constraint does not causes a deviation from the conventional form of option compensation. In particular, $w(y) = \max\{\underline{w}, \theta \max\{y - y^{**}, 0\}\}$ is equivalent to $w(y) = \underline{w} + \theta \max\{y - y^*, 0\}$ by setting $y^* = y^{**} + \frac{\underline{w}}{\theta}$.

compensation is independent of performance. This captures not only limited liability of the manager and the fixed salary component of compensation, but also the fact, observed during the crisis, that managers received significant positive bonuses even in the wake of disastrous performance.

3 Stocks versus options: the role of tail risk

We establish results in which only effort must be incentivized by first analyzing the model without project selection. We then develop the full model that focuses on project selection. Our goal is to show whether and how the existence of tail risk influences the compensation scheme. In particular, in Section 3.1 we derive the optimal stock compensation and show that stock compensation can be superior to options if only effort matters (because effort can be incentivized at a lower cost with stocks). Incorporating tail risk in Section 3.2, we show that the stock compensation that is optimal in the absence of project selection will induce the manager to take tail risk.

3.1 No project selection

We first consider an environment where the ordinary risk project is the only project available. The principal minimizes the expected cost of inducing high effort subject to the minimum wage constraint. Recall that high effort increases the expected payoff of the ordinary risk project by κ. Incentive compatibility requires the manager to exert high effort if (IC_e) is satisfied, which gives us

$$\theta\kappa \geq c \tag{1}$$

As in the standard principal-agent model, the principal and agent have a conflict of interest over variations in outcomes, and the optimal compensation contract consists of a proportion of output; the agent is thus always in the incentive region of the contract.

Lemma 1 *Suppose* $\underline{w} \leq \frac{c\underline{y}}{\kappa}$. *The optimal stock compensation in this case can be expressed as*

$$w(y) = \max\{\underline{w}, cy/\kappa\} = cy/\kappa.$$

The expected wage payment is thus $c\mu_o/\kappa$.

Proof: Because the expected wage is increasing in θ, if $\underline{w}$ is relatively small and the inherent kink in stock compensation is no greater than the lower bound of price distribution, i.e. $\underline{w}/\theta \leq \underline{y}$, inequality (1) must be binding:

$$\theta^* = \frac{c}{\kappa}.$$

Otherwise, a reduction of θ can reduce the principal's cost of implementing effort and thus will cause a contradiction. If $\underline{w} \leq \frac{c\underline{y}}{\kappa}$, the condition above is satisfied, i.e. $\theta^*\underline{y} \geq \underline{w}$. The expected wage payment in this case is

$$E[w|h] = \theta^* E[y_o|h] = c\mu_o/\kappa.$$

□

If options are used, in addition to a piece rate θ, the principal also chooses an optimal exercise price (y^*) to motivate desired managerial actions at the lowest cost. The manager's expected wage when compensated using options is

$$E[w] = \underline{w} + \theta[0 * F_o(y^*) + \int_{y^*}^{\overline{y}} (y - y^*) f_o(y) dy].$$

Imposing $y^* > \underline{y}$ and increasing y^* have two opposing effects on the cost to the principal of incentivizing effort, and the preferred vehicle depends on their relative strength. First, a higher y^* increases the outcome range over which only the minimum wage needs to be paid out, decreasing expected wage payments. Second, a higher piece rate θ is required to motivate managerial effort when y^* increases because of the reduced incentive region. This effect increases expected wages paid to the manager.[6] When increasing y^* increases the

[6]For presentation purposes, we delay the formal proof to Section 4.4.

contribution to the expected wage of the piece rate more than it reduces the expected wage by substituting the minimum wage for the piece rate, stocks are preferred. The following lemma illustrates the condition under which the manager's expected wage is non-decreasing in y^*, and therefore stocks dominate options if the manager does not choose among projects.

Lemma 2 *Stock compensation is superior to option compensation if the following condition is satisfied.*

$$[2 - F_o(y^* + \kappa)] \int_{y^*}^{\overline{y}} [1 - F_o(y)]dy \leq [2 - F_o(y^*)] \int_{y^*}^{\overline{y}} [1 - F_o(y + \kappa)]dy, \qquad \forall y^* \in [\underline{y}, \overline{y}].$$

Proof: See Appendix B.

3.2 Project selection with tail risk

We now turn to analysis of the use of stock compensation in the full model described in Section 2.

Stock compensation that was optimal in the restricted setting (in Section 3.1) may induce the manager to take tail risk.

Proposition 1 *Suppose that the condition specified in Lemma 1 is satisfied, and stock compensation described in Lemma 1 is used. The manager will choose the tail risk project if the following condition is satisfied:*

$$(1 - p)\underline{w} + c\mu_t/\kappa > c\mu_o/\kappa. \tag{2}$$

Proof: Recall that we assume the tail risk project generates a lower NPV: $(1-p)d+\mu_t < \mu_o$. For this to be satisfied together with inequality (2), we must have

$$\underline{w} > \theta^* d = cd/\kappa.$$

Thus the manager receives the minimum wage ($\underline{w}$) when the disaster outcome (d) occurs.[7] The expected wage the manager receives if he takes tail risk is

$$E[w_t(y)|h, T] = (1 - p)\underline{w} + \theta^* \mu_t.$$

[7]This condition is always satisfied if $\underline{w} > 0$ and $d = 0$.

The expected wage the manager receives if he takes ordinary risk is

$$E[w_o(y)|h, O] = \theta^* \mu_o,$$

where $\theta^* = c/\kappa$. As long as inequality (2) is satisfied, $E[w(y_t)|h, t] > E[w(y_o)|h, o]$. □

Since (IC_e) is just satisfied in equilibrium (i.e. $\theta^* = c/\kappa$), inequality (2) is that the expected value to the manager of the tail risk project is greater than that of the ordinary risk project. This occurs if the disaster payoff is sufficiently small relative to the minimum wage.

Proposition 1 shows that an optimal compensation scheme without taking into account risk taking incentives can provide the manager incentives to take tail risk. In the rest of this paper we assume inequality (2) holds. In the next section, we show that it may be infeasible to use stock compensation to promote prudent risk taking (that is, to incentivize a choice of the ordinary risk project).

4 Optimal compensation in the presence of tail risk

In this section, we mainly analyze the choice between stocks and conventional options with a single strike. We first analyze the effects of option compensation on risk taking and characterize conditions under which options can promote choice of the ordinary risk project. We discuss how risk characteristics of projects' payoffs influence the optimal exercise price. We show that, due to a trade-off between implementing managerial effort and avoiding tail risk, under certain conditions, it is infeasible to use stocks to avoid tail risk, whereas options work. This contrasts with prior research. In Section 4.2, we show that collar options enable incentivizing the ordinary risk project in a wider set of circumstances. Section 4.3 derives the optimal strike price for conventional options, while Section 4.4 derives the piece rate that incentivizes effort, and Section 4.5 shows circumstances under which stocks cannot be used to deter tail risk.

4.1 Do options imply tail risk?

While we are unaware of previous theoretical work linking tail risk to optimal compensation, the impact of compensation contracts on managerial risk attitudes has been studied. Beginning with Jensen and Meckling (1976), there has been a long-standing belief that options induce managers to seek greater risk. Carpenter (2000) suggests a more nuanced conclusion: she shows that when compensated with a call option, a risk-averse manager may moderate asset risk because the leverage inherent in his option magnifies his exposure to the asset volatility. Ross (2004) points out that effects of option compensation depend on the manager's utility function.

Our results reinforce theirs in that our model justifies the use of options as efficient compensation. In particular, we focus on the existence of tail risk, but our analysis does not depend on risk aversion of the manager (our results can be easily extended to a risk-averse manager, as shown in Appendix C).

When a minimum wage constraint exempts managers from bearing the full share of the cost of tail risk outcomes, and tail risk projects (assets) generate higher returns in most states of the world than ordinary risk projects, options may dominate stocks as a means of discouraging tail risk taking. By compensating managers based on their excess returns over the exercise price, options can effectively alter the relative expected compensation payoff from implementing ordinary risk projects versus tail risk ones and therefore alleviate distortions of incentives due to limited liability. We demonstrate the exact condition for options to be an effective compensation tool in controlling for risk-taking incentives.

Proposition 2 *For the constraint on the risk choice (IC_{R1}) to be satisfied using conventional option compensation,*

$$\int_{y^*}^{\overline{y}} F_o(y)dy \leq \int_{y^*}^{\overline{y}} F_t(y)dy$$

or equivalently,

$$\int_{y^*}^{\overline{y}} G_o(y)dy \geq \int_{y^*}^{\overline{y}} G_t(y)dy$$

must hold, where $y_o \in [\underline{y}, \overline{y}]$ *and* $y_t \in [\underline{y}, \overline{y}]$ *are the stock price generated by the ordinary risk project and tail risk project respectively;* $G_o(\cdot) = 1 - F_o(\cdot), G_t(\cdot) = 1 - F_t(\cdot)$*, and* $F_o(\cdot)$ *and* $F_t(\cdot)$ *are the cumulative distribution functions of stock prices associated with the ordinary risk and tail risk projects respectively; and* y^* *is the exercise price specified in the stock options.*[8]

Proof: Satisfying the constraint on the risk choice (IC_{R1}) requires

$$E[w(y_o)|y^*, h] \geq E[w(y_t)|y^*, h], \tag{3}$$

When options are used, we have

$$\begin{aligned} w(y) &= \underline{w} + \theta \max\{y - y^*, 0\} \\ &= (\underline{w} - \theta y^*) + \theta \max\{y, y^*\} \end{aligned}$$

Inequality (5) becomes equivalent to

$$E[y_o|y_o \geq y^*, h] \geq E[y_t|y_t \geq y^*, h]$$

We derive $E[y_i|y_i \geq y^*, h]$, where $i \in \{o, t\}$, in the following. High effort (h) is dropped for convenience.

$$\begin{aligned} & E[y_i|y_i \geq y^*] \\ = \; & y^* F_i(y^*) + \int_{y^*}^{\overline{y_i}} y f_i(y) dy \\ = \; & y^* F_i(y^*) + \Big[y F_i(y) \Big]_{y^*}^{\overline{y_i}} - \int_{y^*}^{\overline{y_i}} F_i(y) dy \\ = \; & y^* F_i(y^*) + \overline{y_i} F_i(\overline{y}) - y^* F_i(y^*) - \int_{y^*}^{\overline{y_i}} F_i(y) dy \\ = \; & \overline{y_i} - \int_{y^*}^{\overline{y_i}} F_i(y) dy \end{aligned}$$

[8] A condition analogous to this one is derived for unbounded distributions in Appendix D.

Alternatively, let $G_i(y) = 1 - F_i(y)$, and we have

$$\begin{aligned} & E[y_i | y_i \geq y^*] \\ = & \quad \overline{y_i} - \textstyle\int_{y^*}^{\overline{y_i}} [1 - G_i(y)] dy \\ = & \quad \overline{y_i} - [y]_{y^*}^{\overline{y_i}} + \textstyle\int_{y^*}^{\overline{y_i}} G_i(y) dy \\ = & \quad y^* + \textstyle\int_{y^*}^{\overline{y_i}} G_i(y) dy \end{aligned}$$

□

When the manager makes an unobserved effort decision and an unobserved risk choice, the compensation off the equilibrium path also needs to be considered to deter dual deviation from the recommended actions. That is, the other constraint on the risk choice (IC_{R2}) must be satisfied to ensure the desired level of risk taking.

Lemma 3 *If (IC_{R1}) and (IC_e) are met, (IC_{R2}) is also satisfied.*

Proof: As derived in the proof of Proposition 2,

$$E[y_i | y_i \geq y^*] = y^* + \int_{y^*}^{\overline{y_i}} G_i(y) dy.$$

Incentive compatibility on the effort decision (IC_e) requires

$$\theta \geq \frac{c}{\int_{y^*}^{\overline{y}} [G_o(y) - G_o(y + \kappa)] dy} = \theta_1,$$

and incentive compatibility on the risk choice (IC_{R2}) requires

$$\theta \geq \frac{c}{\int_{y^*}^{\overline{y}} [G_o(y) - G_t(y + \kappa)] dy} = \theta_2.$$

It can be shown that if $\int_{y^*}^{\overline{y}} G_o(y + \kappa) dy \geq \int_{y^*}^{\overline{y}} G_t(y + \kappa) dy$ is satisfied, then $\theta_1 \geq \theta_2$ and Lemma 3 follow.

Now we turn to showing that $\int_{y^*}^{\overline{y}} G_o(y + \kappa) dy \geq \int_{y^*}^{\overline{y}} G_t(y + \kappa) dy$ holds.

Recall that (IC_{R1}) requires

$$\int_{y^*}^{\overline{y}} G_o(y) dy \geq \int_{y^*}^{\overline{y}} G_t(y) dy.$$

It can be shown that $\Delta \equiv \int_{y^*}^{\overline{y}} G_o(y)dy - \int_{y^*}^{\overline{y}} G_t(y)dy$ is (weakly) increasing in y^*. It is convenient to delay the formal proof to Section 4.3 (Lemma 6). The inequality below therefore follows.

$$\int_{y^*+\kappa}^{\overline{y}} G_o(y)dy \geq \int_{y^*+\kappa}^{\overline{y}} G_t(y)dy$$

Using the following transformation

$$\int_{y^*+\kappa}^{\overline{y}} G_i(y)dy = \int_{y^*+\kappa}^{\overline{y}+\kappa} G_i(y)dy = \int_{y^*}^{\overline{y}} G_i(y+\kappa)dy, \quad \forall i \in \{o, t\}$$

we obtain

$$\int_{y^*}^{\overline{y}} G_o(y+\kappa)dy \geq \int_{y^*}^{\overline{y}} G_t(y+\kappa)dy.$$

Therefore, $\theta_1 \geq \theta_2$ □

Based on Proposition 2 and Lemma 3, we establish the necessary and sufficient condition for options to provide optimal risk-taking incentives.

Proposition 3 *A necessary and sufficient condition under which options can be used to deter tail risk project choice is*

$$\int_{y^*}^{\overline{y}} F_o(y)dy \leq \int_{y^*}^{\overline{y}} F_t(y)dy, \tag{4}$$

Thus, a sufficient condition for options to be optimal is the value generated from the tail risk project second-order stochastically dominates that from the ordinary risk project above the exercise price.

This condition allows us to think systematically about when investors might prefer options and when they might prefer stocks as a motivational tool. The fundamental problem in compensation design is that the principal has to reward productive effort, and thus better outcomes must be awarded with higher pay, such as in stocks. In the standard contracting problem stock compensation (a proportional sharing rule) is an effective means of motivation. The problem with stocks, however, is that when the principal cannot punish the manager sufficiently for realizations of disasters, tail risk projects appear attractive to

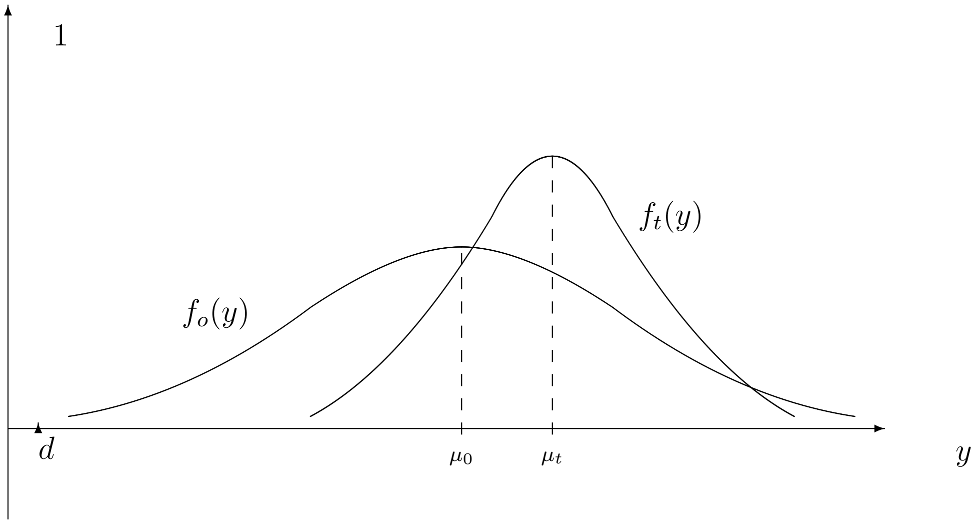


Figure 1: Stocks versus Options (PDF)

the manager as such projects generate higher gains in most states of the world (Figure 1). The principal can deal with the distortion by setting the option strike to trim off a larger portion of low price realizations of the ordinary risk project than the tail risk project, thus increasing the expected payoff to the manager of the ordinary risk project. The principal can use the piece rate (θ), which is the proportion of firm value granted in equity, to incentivize effort. However, conventional options achieve such a result only for a restricted set of project characteristics, illustrated in Proposition 3.

Proposition 3 characterizes some business strategies that contributed to the recent financial crisis. Among the examples are: credit-arb ABCP vehicles that provided excess returns with only modest volatility for a number of years until virtually all imposed substantial funding-shock losses on sponsors during the crisis; highly rated securitized mortgage bonds; monoline insurance companies writing of credit protection on such bonds and on bank debt; and quant equity strategies used by some hedge funds to obtain high Sharpe ratios for many years until large losses were experienced in the fall of 2007. When projects (or assets) that involve tail risk are featured as a safer project in normal times but with a drastic bad tail, and the minimum wage constraint exempts managers from bearing the losses of the bad tail, the tail risk project becomes an obvious choice if stock compensation is used. It is thus optimal for the principal to intentionally introduce convexity in the payment structure to promote the manager to tolerate the greater risk involved in the ordinary risk project in most states and avoid tail risk.

4.2 Collar Options

Many examples of tail risk projects that do not satisfy Proposition 3 come to mind, such as leveraged long-equity strategies. Compensation resembling collar options has the potential to deal with these. If collar options are used, compensation takes the following form:

$$w(y) = \underline{w} + \theta \min\{\max\{y - y^f, 0\}, y^c - y^f\},$$

where y^f is the strike price of the put option (the "floor") and y^c is the strike price of the call option (the "cap").

Similar to conventional option compensation, collar options make the managers' compensation insensitive to project payoffs below the strike price on the put. In addition, collar options restrict the upside above the strike price on the call. In the following, we establish the condition under which collar options can be used to deter tail risk.[9]

Lemma 4 *For (IC_{R1}) to be satisfied using collar options, it requires*

$$\int_{y^f}^{y^c} F_o(y)dy \leq \int_{y^f}^{y^c} F_t(y)dy.$$

Proof: To satisfy the constraint on the risk choice (IC_{R1}), it requires

$$E[w(y_o)|h] \geq E[w(y_t)|h], \tag{5}$$

When collar options are used, we have

$$w(y) = (\underline{w} - \theta y^*) + \theta \min\{\max\{y, y^f\}, y^c\}$$

[9] In the interest of paper length, we only derive the condition for (IC_{R1}) to be satisfied. As for (IC_{R2}) and (IC_e), conditions analogous to those in Section 4.1 follow.

Inequality (5) becomes equivalent to

$$E[y_o|y^f \leq y_o \leq y^c, h] \geq E[y_t|y^f \leq y_t \leq y^c, h]$$

We derive $E[y_i|y^f \leq y_i \leq y^c, h]$, where $i \in \{o, t\}$, in the following. High effort (h) is dropped for convenience.

$$\begin{aligned}
& E[y_i|y^f \leq y_i \leq y^c] \\
= \; & y^f F_i(y^f) + \textstyle\int_{y^f}^{y^f} y f_i(y) dy + y^c[1 - F_i(y^c)] \\
= \; & y^f F_i(y^f) + \Big[y F_i(y) \Big]_{y^f}^{y^c} - \textstyle\int_{y^f}^{y^c} F_i(y) dy + y^c[1 - F_i(y^c)] \\
= \; & y^f F_i(y^f) + y^c F_i(y^c) - y^f F_i(y^f) - \textstyle\int_{y^f}^{y^c} F_i(y) dy + y^c - y^c F_i(y^c) \\
= \; & y^c - \textstyle\int_{y^f}^{y^c} F_i(y) dy
\end{aligned}$$

□

By virtue of having an additional compensation tool to choose (i.e. the cap), compared to typical options, collar options provide more flexibility in finding a range of outcome where the above relationship holds, so that the desired project will be chosen. Thus, collar options can deter tail risk for a larger set of projects characteristics than options. When stock price distributions are symmetric, if $\mu_0 = \mu_t$, collar options can *always* be used to deter tail risk taking regardless of projects' risk characteristics.

Proposition 4 *If $\mu_o = \mu_t = \mu$, collar options can incent the choice of the ordinary risk project for all symmetric distributions of stock prices.*

Proof: If regular options can be used to promote prudent risk taking, that is, there exists a y^* such that the following holds:

$$\int_{y^*}^{\overline{y}} G_o(y) dy \geq \int_{y^*}^{\overline{y}} G_t(y) dy,$$

where $G_i(y) = 1 - F_i(y), \forall i \in \{o, t\}$. Then setting $y^f = y^*$ and $y^c = \overline{y}$ in the collar options will also promote prudent risk taking. That is, the collar is equivalent to a conventional option.

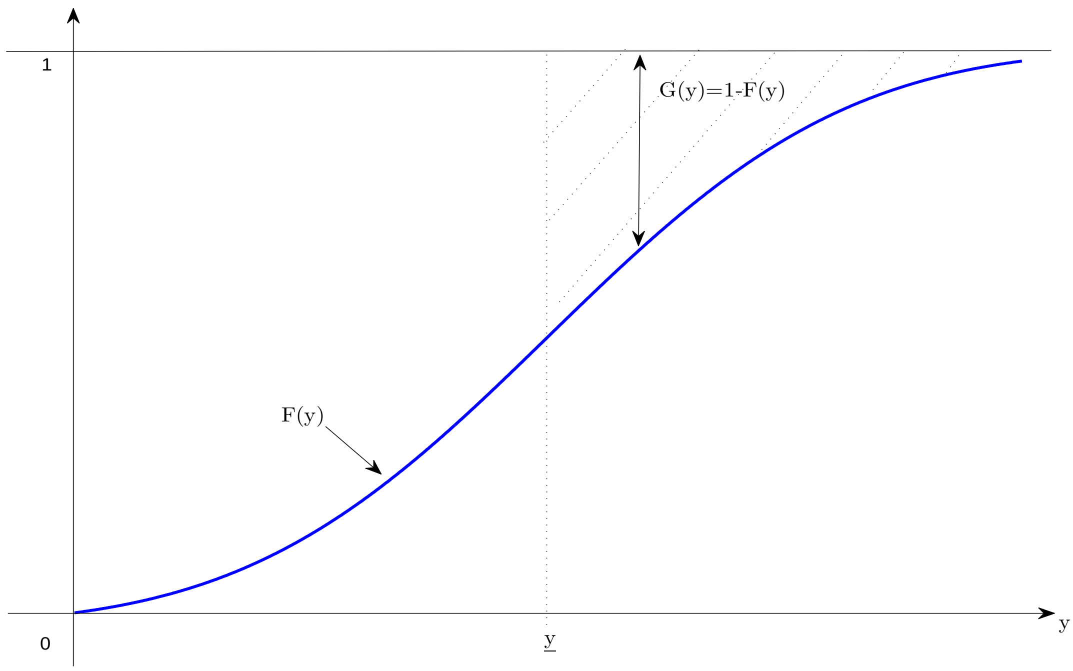


Figure 2: Stocks versus Options (CDF)

If regular options cannot be used to incent the choice of the ordinary risk project, that is, for all $y^* \in (\underline{y}, \overline{y})$, the following holds:

$$\int_{y^*}^{\overline{y}} G_o(y)dy < \int_{y^*}^{\overline{y}} G_t(y)dy.$$

Because the stock price distributions are symmetric, as shown in Figure 2, we have

$$\int_{y^*}^{\overline{y}} G_i(y)dy = \int_{\underline{y}}^{2\mu - y^*} F_i(y)dy, \quad \forall i \in \{o, t\}.$$

Thus, there always exists y^* such that the following holds:

$$\int_{\underline{y}}^{2\mu - y^*} F_o(y)dy < \int_{\underline{y}}^{2\mu - y^*} F_t(y)dy.$$

Then setting $y^f = \underline{y}$ and $y^c = y^*$ (determined by the cost-minimization problem with (IC_e)) will promote prudent risk taking. □

The intuition for the universality of collar options is the following. For symmetric price distributions, projects associated with a relatively greater mass in the lower tail also have a relatively greater mass in the higher tail. If the ordinary risk project has a greater mass at the low end than the tail risk project, we can deter tail rik by imposing an optimally chosen y^f to make low price realizations irrelevant for compensation. If the ordinary risk project has a smaller mass at the low end than the tail risk project, which means it also has a smaller mass at the high end of the price distribution, we can incentivize the choice of ordinary risk by removing high price realizations from the incentive region in compensation with y^c. This result suggests that it may be important to consider using compensation arrangements that share features of collar options, such as bonus schemes with caps and floors, to a greater extent in practice.

4.3 Equilibrium exercise price

In this section we analyze the exercise price specified in the optimal option compensation, that is, the cutoff project outcome below which the manager receives only the minimum wage. We focus on the regular options in the rest of the section. The results on collar options will be added once they are ready.

Changing the exercise price alters the distribution and hence the expected value of stock prices relevant for managerial compensation. We first demonstrate in the following lemma that the expected stock price that compensation is responsive to increases with the exercise price.

Lemma 5 *$E[y_i|y_i \geq y^*] > E[y_i]$, and $E[y_i|y_i \geq y^*]$ is increasing in y^*, $i \in \{o, t\}$.*

Proof: Following Proposition 4, we derive $E[y_i|y_i \geq y^*, h]$, where $i \in \{o, t\}$, in the following. High effort (h) is dropped for convenience.

$$
\begin{array}{rc}
 & E[y_i|y_i \geq y^*] \\
= & y^*F(y^*) + \int_{y^*}^{\overline{y_i}} yf_i(y_i)dy \\
= & \int_{\underline{y_i}}^{y^*} y^*f_i(y_i)dy + \int_{y^*}^{\overline{y_i}} yf_i(y_i)dy \\
> & \int_{\underline{y_i}}^{y^*} yf_i(y)dy + \int_{y^*}^{\overline{y_i}} yf_i(y_i)dy \\
= & \int_{\underline{y_i}}^{\overline{y_i}} yf_i(y)dy \\
= & E[y_i]
\end{array}
$$

From Proposition 4, we have

$$E[y_i|y_i \geq y^*] = \overline{y_i} - \int_{y^*}^{\overline{y_i}} F_i(y_i)dy, \qquad \forall i \in \{o, t\}.$$

which is increasing in y^*. □

The intuition is straightforward. Exercise price increases the expected stock price relevant for compensation by making compensation irresponsive to relatively low price realizations. For both projects, a higher exercise price thus induces a higher expected stock price that option compensation is responsive to.

How does the difference in option compensation between the two projects vary with the level of exercise price? Let Δ denote the difference in expected stock prices above exercise prices, that is, $\Delta \equiv E[y_o|y_o \geq y^*, h] - E[y_t|y_t \geq y^*, h]$. Thus, the option compensation differential is represented by $\theta\Delta$. We show below that under certain condition, Δ is weakly increasing in the exercise price.

Lemma 6 *Δ is weakly increasing in y^* if $F_t(y^*) \geq F_o(y^*)$.*

Proof: Following Proposition 4, we have

$$E[y_i|y_i \geq y^*] = y^*F(y^*) + \int_{y^*}^{\overline{y_i}} yf_i(y_i)dy.$$

We re-write Δ as follows.

$$\begin{aligned}
\Delta = &\quad y^*[F_o(y^*) - F_t(y^*)] + (\int_{y^*}^{\overline{y}} y f_o(y)dy - \int_{y^*}^{\overline{y}} y f_t(y)dy) \\
= &\quad (\int_{\underline{y}}^{y^*} y^* f_o(y)dy - \int_{\underline{y}}^{y^*} y^* f_t(y)dy) + (\int_{y^*}^{\overline{y}} y f_o(y)dy - \int_{y^*}^{\overline{y}} y f_t(y)dy) \\
= &\quad (\int_{\underline{y}}^{y^*} y^* f_o(y)dy + \int_{y^*}^{\overline{y}} y f_o(y)dy) - (\int_{\underline{y}}^{y^*} y^* f_t(y)dy + \int_{y^*}^{\overline{y}} y f_t(y)dy) \\
= &\quad [\mu_o + \int_{\underline{y}}^{y^*} (y^* - y) f_o(y)dy] - [\mu_t + \int_{\underline{y}}^{y^*} (y^* - y) f_t(y)dy] \\
= &\quad (\mu_o - \mu_t) + \int_{\underline{y}}^{y^*} (y^* - y) f_o(y)dy - \int_{\underline{y}}^{y^*} (y^* - y) f_t(y)dy \\
= &\quad (\mu_o - \mu_t) + \int_{\underline{y}}^{y^*} (y^* - y)[f_o(y) - f_t(y)]dy.
\end{aligned}$$

Integrating by parts, we obtain

$$\begin{aligned}
\Delta &= (\mu_o - \mu_t) + \Big[(y^* - y)(F_o(y) - F_t(y))\Big]_{\underline{y}}^{y^*} - \int_{\underline{y}}^{y^*} [F_o(y) - F_t(y)]dy \\
&= (\mu_o - \mu_t) - \int_{\underline{y}}^{y^*} [F_o(y) - F_t(y)]dy.
\end{aligned}$$

Taking derivatives of Δ with respect to y^*,

$$\frac{d\Delta}{dy^*} = F_t(y^*) - F_o(y^*) \geq 0$$

□

The intuition is that when the ordinary risk project has a greater mass at the low end (excluding disaster) than the tail risk project, the ordinary risk project benefits more from a truncation of stock price distribution that makes compensation immune to downside risk. Although the expected stock price relevant for compensation increases with the exercise price for both projects, it increases at an accelerated rate for ordinary risk project.

As Δ and θ are both (weakly) increasing in y^* (from Proposition 5 and Lemma 6), The exercise price in the optimal contract, denoted by $\underline{y}^*$, will be such that (IC_{R1}) is just satisfied with equality.

Lemma 7 *(IC_{R1}) is binding in the optimal contract:*

$$\int_{\underline{y}^*}^{\overline{y}} F_o(y)dy = \int_{\underline{y}^*}^{\overline{y}} F_t(y)dy,$$

if $F_t(y^*) \geq F_o(y^*)$.

Proof: Suppose that (IC_{R1}) is not binding. A reduction of $\underline{y}^*$ will still satisfy the constraint on risk choice and induce a reduction of θ^* and hence the value of the objective function — therefore, it causes a contradiction. □

When the ordinary risk project is comparatively riskier than the tail risk project in the sense of Proposition 3, a higher exercise price is required to make a sufficiently large proportion of low price realization irrelevant for compensation for the ordinary risk project to appear attractive to the manager. The economic significance of option compensation in controlling for risk taking incentives thus depends on the risk characteristics of both the type of projects banks want their employees to invest in and the type they want them to avoid.

4.4 Risk taking and moral hazard with respect to effort

Because a flat region below the exercise price in option compensation insulates managers from low payoffs, a higher powered compensation (pay-for-performance above the exercise price) is thus required to incentivize effort. The following proposition illustrates that the optimal pay-for-performance, namely, the share of the firm's equity granted, increases with the exercise price on options.

Proposition 5 *The optimal number of equity granted* ($\theta^{options}$) *increases with the exercise price* (y^*) *specified in stock options.*[10]

Proof: In the case of options, (IC_e) becomes

$$\theta^{options} E[y_o | y_o \geq y^*, h] - \theta^{options} E[y_o | y_o \geq y^*, l] \geq c,$$

[10] A proof for the case of unbounded distributions is provided in Appendix D.

which requires

$$\theta^{options} \geq \frac{c}{E[y_o|y_o \geq y^*, h] - E[y_o|y_o \geq y^*, l]} \tag{6}$$
$$\geq \frac{c}{\int_{y^*}^{\overline{y}} F_o(y+\kappa)dy - \int_{y^*}^{\overline{y}} F_o(y)dy}$$
$$= \frac{c}{\int_{y^*}^{\overline{y}} [F_o(y+\kappa) - F_o(y)]dy}$$

In the optimal contract in our setting, (IC_e) is binding. Suppose (IC_e) is not binding. Then a reduction of θ can satisfy the compatibility constraint on the risk choice (IC_{R1}) and (IC_{R2}), and decrease the value of the objective function — therefore, it causes a contradiction. Thus, (IC_e) is binding under the optimal contract:

$$\theta^{options} = \frac{c}{\int_{y^*}^{\overline{y}} [F_o(y+\kappa) - F_o(y)]dy}. \tag{7}$$

It is straightforward to see that $\theta^{options}$ is increasing in y^*. □

Stock options feature a nontrivial region where compensation does not vary with performance. The unresponsive region disproportionately benefits the low-effort manager, as it makes managerial compensation insensitive to values at the low end of stock price distribution. Additional incentives are therefore necessary above the exercise price, and additional shares of equity are required to motivate the desired level of effort. In the next subsection, we will use this tension between motivating effort and implementing prudent risk taking to derive conditions under which stocks fail to deter the manager from taking tail risk.

4.5 Can stocks promote prudent risk taking?

In this section we turn to analyze whether stock compensation can be used to incentivize choice of the ordinary risk project.

Incentive compatibility requires the manager to exert high effort if inequality (6) is satisfied, that is,

$$\theta \geq \frac{c}{\int_{\underline{w}/\theta}^{\overline{y}} [F_o(y+\kappa) - F_o(y)]dy},$$

It can be rewritten as

$$\theta \int_{\underline{w}/\theta}^{\overline{y}} [F_o(y+\kappa) - F_o(y)]dy \geq c \tag{8}$$

Because the left-hand side of this inequality is increasing in θ and the right-hand side is constant, inequality (8) determines a lower bound of θ that satisfies incentive compatibility for effort decision.

When the manager also makes a risk choice, it must be personally beneficial for the manager to take ordinary risk. Following Proposition 2, θ in the optimal stock compensation must also satisfy (IC_R)

$$\int_{\underline{w}/\theta}^{\overline{y}} F_o(y)dy \leq \int_{\underline{w}/\theta}^{\overline{y}} F_t(y)dy. \tag{9}$$

Lemma 5 shows the condition under which $\Delta \equiv \int_{y^*}^{\overline{y}} F_t(y)dy - \int_{y^*}^{\overline{y}} F_o(y)dy$ is weakly increasing in y^*. Therefore, inequality (9) determines an upper bound of θ that provides adequate risk taking incentives.

Indeed, the optimal θ is determined by a tradeoff between the negative effect of the piece rate on risk choice and its positive effect on effort, and thus depends on the magnitude of the two agency problems. Compared to the case of option compensation where incentive compatibility on effort choice (IC_e) and risk decision (IC_{R1}) and can be both binding in equilibrium, it is straightforward to see that there is likely to be slackness in incentive compatibility constraints when stocks are used, because the piece rate θ is the only measure the principal chooses. It is more expensive to compensate the manager using stocks than options.

Depending on κ and the distributional characteristics of projects available, it may be the case that there does not exist a θ that simultaneously satisfies both conditions. Based on this tension between motivating effort and implementing prudent risk taking, we derive conditions under which stock compensation is ineffective.

Proposition 6 *It is infeasible to use stocks to avoid tail risk if* $\theta_{max} < \theta_{min}$*, where* θ_{max}

and θ_{min} *are such that*

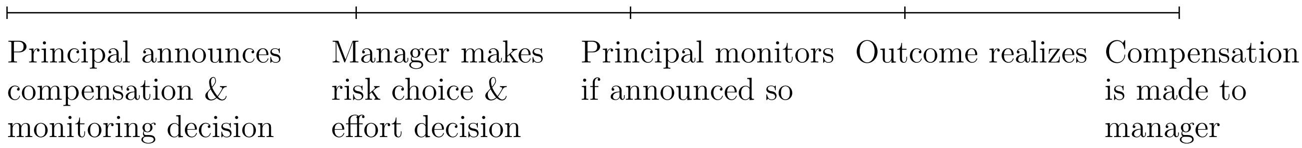
$$\int_{\underline{w}/\theta_{max}}^{\overline{y}} F_o(y)dy = \int_{\underline{w}/\theta_{max}}^{\overline{y}} F_t(y)dy,$$

$$\theta_{min} \int_{\underline{w}/\theta_{min}}^{\overline{y}} [F_o(y+\kappa) - F_o(y)]dy = c,$$

A sharp conflict arises between the desire of the principal to implement high effort and motivate prudent risk taking, as performance pay generates incentives not only to exert effort but also to take tail risk. In the case of option compensation, the principal can choose exercise price and pay for performance (above the exercise price) independently to achieve efficient effort and prudent risk taking. When pure stocks are used in compensation, however, the inherent kink ($\underline{w}/\theta$) below which the manager receives the minimum wage is also determined by pay for performance (θ) in stock compensation. Therefore any change of compensation that influences project selection also feeds back into effort decision. A low pay-performance sensitivity (θ) implies that an adequately large proportion of low price realizations are irrelevant for compensation for ordinary risk to appear attractive from the manager's viewpoint. However, the manager will exert high effort only if the compensation is sufficiently sensitive to performance to compensate for the cost of effort. The conflict makes it impossible to motivate effort while avoiding tail risk taking in this case. When the manager faces limited liability and hence does not bear the full cost of his risk taking, tail risk taking emerges under compensation contracts consisting pure stocks.

Proposition 6 gives a condition for stock compensation to incent tail risk taking. Note that it is more likely to be satisfied if managerial effort is more costly or less effective in improving project payoffs (i.e. a large c relative to κ), and if excluding the disaster outcome, the ordinary risk project is less riskier than the tail risk project in terms of mean-preserving spreads (i.e. a large y^* required). A greater severity of the risk taking issue calls for the inherent kink (caused by limited liability) in stock compensation to appear at a higher level, which is achieved by setting the piece rate small. However, a small piece rate fails to augment effort when managerial effort is not costly effective. In

Figure 3: Model Timeline



sum, the previous literature's justification of stock compensation as a motivational tool to align risk-taking incentives is warranted for firms in which managerial effort is costly and tail risk projects available are moderately attractive (in term of second-order stochastic dominance) than ordinary risk projects when excluding the bad tail.

5 Extensions

5.1 Monitoring of risk choices

In the aftermath of the financial crisis of 2008-2009, there are widespread concerns that compensation structures of financial firms have provided excessive risk-taking incentives. Responding to such concerns, firms are seeking to reform their pay packages, and regulators around the globe are moving toward setting standards for compensation arrangements in financial firms. In face of informational disadvantage and potential talent drain, some suggest that government intervention to ensure the adequacy of internal governance process would be sufficient in the financial sector. In particular, monitoring provides an alternative mechanism to incentive alignment for controlling agent activities. To gain some insight on the issue, we now turn to analyze the effect of monitoring on the choice between stocks and options.

Figure 3 describes the sequence of the events in the model. The principal announces the monitoring decision and managerial compensation structure in the beginning of the period: $m = 1$ denotes that the principal announces to monitor the manager's risk choice and $m = 0$ represents otherwise. To avoid the time-inconsistency problem, we assume that the principal is committed to her announced monitoring decision.[11] Monitoring involves a cost to the principal, that is, $C(m = 1) = c_m$, while no monitoring does not, $C(m = 0) = 0$. If the manager takes tail risk, it will be successfully detected by monitoring with probability $p_s \in [0, 1]$, and the manager will be punished with the minimum wage $\underline{w}$ in this case. With probability $(1 - p_s)$, tail risk taking is not detected and pay is according to the initial incentive compensation contract offered in the beginning of the period.

With monitoring, a compensation contract composed of stock grants can promote prudent risk taking. In particular, the "status quo" stock compensation (that is optimal without risk selection) motivates the manager to implement the ordinary risk project if monitoring is sufficiently effective.

Proposition 7 *If* $p_s \geq 1 - \left(\frac{\frac{c}{\kappa}\mu_o - \underline{w}}{\frac{c}{\kappa}\mu_t - p\underline{w}}\right)$, *the stock compensation described in Lemma 1 can be used to deter tail risk.*

Proof: With monitoring, managerial compensation conditional on taking ordinary risk under the "status quo" stock compensation illustrated in Lemma 1 is

$$E[w(y_o)|h, o] = \theta^* \mu_o = c\mu_o/\kappa.$$

$$E[w(y_t)|h, t] = p_s\underline{w} + (1 - p_s)[(1 - p)\underline{w} + c\mu_t/\kappa].$$

The manager will choose the ordinary risk project if and only if $E[w(y_o)|h, o] \geq E[w(y_t)|h, t]$, that is,

$$p_s \geq 1 - \left(\frac{\frac{c}{\kappa}\mu_o - \underline{w}}{\frac{c}{\kappa}\mu_t - p\underline{w}}\right).$$

□

A probabilistic detection essentially causes the pay-performance sensitivity for the tail risk project to be lower than the ordinary risk one, making the ordinary risk project more

[11] Without commitment, we would face the typical time-inconsistency problem: the principal always has an incentive to announce monitoring before the manager's risk choice but not to monitor afterwards.

attractive from the manager's perspective. With relatively effective monitoring, that is, a sufficiently large detection probability, tail risk can be avoided with the "status quo" stock compensation. When $p_s < 1 - \left(\frac{\frac{c}{\kappa}\mu_o - \underline{w}}{\frac{c}{\kappa}\mu_t - p\underline{w}}\right)$, option compensation is necessary to align incentives in risk choices, provided that the following condition holds.

Proposition 8 *A necessary and sufficient condition for options to be optimal in the presence of monitoring is*

$$\int_{y^*}^{\overline{y}} F_o(y)dy \leq p_s(\overline{y} - y^*) + (1 - p_s) \int_{y^*}^{\overline{y}} F_t(y)dy \tag{10}$$

Proof:

$$\begin{aligned} \underline{w} + E\theta \max\{y_o - y^*, 0\} \geq & \quad p_s\underline{w} + (1 - p_s)(\underline{w} + E\theta \max\{y_t - y^*, 0\}) \\ \underline{w} - \theta y^* + \theta E[y_o|y_o \geq y^*] \geq & \quad p_s\underline{w} + (1 - p_s)\left[\underline{w} - \theta y^* + \theta E[y_t|y_t \geq y^*]\right] \\ \theta[\overline{y} - \int_{y^*}^{\overline{y}} F_o(y)dy] \geq & \quad p_s\theta y^* + (1 - p_s)\theta[\overline{y} - \textstyle\int_{y^*}^{\overline{y}} F_t(y)dy] \\ \int_{y^*}^{\overline{y}} F_o(y)dy \leq & \quad p_s(\overline{y} - y^*) + (1 - p_s) \textstyle\int_{y^*}^{\overline{y}} F_t(y)dy. \end{aligned}$$

Alternatively writing in terms of $G_i(y) = 1 - F_i(y), i \in \{o, t\}$, we have

$$\begin{aligned} \underline{w} - \theta y^* + \theta E[y_o|y_o \geq y^*] \geq & \quad p_s\underline{w} + (1 - p_s)\left[\underline{w} - \theta y^* + \theta E[y_t|y_t \geq y^*]\right] \\ \theta[y^* + \int_{y^*}^{\overline{y}} G_o(y)dy] \geq & \quad p_s\theta y^* + (1 - p_s)\theta[y^* + \textstyle\int_{y^*}^{\overline{y}} G_t(y)dy] \\ \int_{y^*}^{\overline{y}} G_o(y)dy \geq & \quad (1 - p_s) \textstyle\int_{y^*}^{\overline{y}} G_t(y)dy. \end{aligned}$$

□

Since $p_s(\overline{y} - y^*) < p_s \int_{y^*}^{\overline{y}} F_t(y)dy$ (or in the alternative expression, $(1 - p_s) \int_{y^*}^{\overline{y}} G_t(y)dy < \int_{y^*}^{\overline{y}} G_t(y)dy$), compared to the condition in the absence of monitoring (condition 4), the condition becomes less restrictive when monitoring is possible and thus requires a lower level of exercise price. As the equilibrium share of options is an increasing function of exercise price (from Proposition 5), the optimal number of shares granted is also smaller with monitoring than otherwise.

Lemma 8 *Suppose that the condition specified in Lemma 6 holds. Then* $y_m^* \leq y^*, \theta_m^* \leq \theta^*$*, where* y_m^* *and* y^* *are equilibrium exercise prices with and without monitoring respectively;* θ_m^* *and* θ^* *are equilibrium shares of options with and without monitoring respectively.*

Proof: Let $\Delta' \equiv E[y_o|y_o \geq y^*, h] - (1-p_s)E[y_t|y_t \geq y^*, h] + p_s y^*$. Thus, the difference in expected managerial compensation between ordinary risk and tail risk is represented by $\theta_m \Delta'$, where θ_m is the shares of options granted. Recall that $\Delta \equiv E[y_o|y_o \geq y^*, h] - E[y_t|y_t \geq y^*, h]$. We have

$$\Delta' = \Delta + p_s E[y_t|y_t \geq y^*, h] + p_s y^*.$$

From Lemma 5, $E[y_t|y_t \geq y^*, h]$ is increasing in y^*. From Lemma 6, Δ is increasing in y^*. Δ' is thus also increasing in y^*.

Under the optimal contract, condition (10) is binding. Suppose that it is not binding, a reduction of exercise price y_m^* induces a reduction of the share of options θ_m^* and hence the objective function value — therefore, it causes a contradiction. Equilibrium exercise price y_m^* is thus such that condition (10) holds with equality, equivalently, $\Delta' = 0$. Equilibrium exercise price in the absence of monitoring technology (y^*) is such that condition (4) holds with equality, equivalently, $\Delta = 0$. Since $\Delta' \geq \Delta$ given y^*, in the equilibrium, $y_m^* \leq y^*$ holds. Because equilibrium share of options increases with exercise price, illustrated in Proposition 5, we have $\theta_m^* \leq \theta^*$. □

The following lemma shows that an increase in monitoring efficiency (p_s) induces a reduction of equilibrium exercise price (y_m^*). Because of the positive relationship between the equilibrium share of options and exercise price, expected managerial pay is strictly increasing in exercise price specified in option compensation. Increasing effectiveness of monitoring therefore reduces the level of compensation required to induce desired managerial actions.

Lemma 9 *If monitoring is more effective* (p_s *increases), equilibrium exercise price* (y_m^*) *will be lower.*

Proof: An increase in p_s raises Δ', therefore a smaller y^*_m is needed to satisfy $\Delta' = 0$. $\square$

When monitoring becomes more effective in detecting tail risk, expected compensation conditional on taking on ordinary risk remains constant while that conditional on taking on tail risk decreases, for a given level of exercise price. A lower exercise price is required to avoid tail risk, and expected managerial compensation is consequently lower than in the absence of monitoring.

Monitoring reduces managerial wage while maintaining the incentives to choose the ordinary risk project, but it is costly to conduct monitoring. It is optimal for shareholders to monitor if the reduction of expected managerial compensation overwhelms the cost incurred during monitoring.

Proposition 9 *Monitoring is implemented if* $\theta^* E[y_o|y_o \geq y^*] - \theta^*_m E[y_o|y_o \geq y^*_m] \geq c_m$.

Because more effective monitoring (a higher detection probability p_s) reduces expected wage payment, everything else constant, it is more costly effective to implement monitoring. Similarly, government subsidies for monitoring and inspections may reduce equilibrium exercise price in option compensation and decrease the number of equity granted in compensation contracts.

In a nutshell, a prediction of our model is that more effective monitoring should be associated with a higher tendency by firms to compensate their executives by stocks instead of options. When monitoring is effective (that is, a high detection probability), granting stocks can implement ordinary risk projects while maintaining incentives to put forth effort. When monitoring is not sufficiently effective in detecting tail risk, option compensation may be necessary to promote prudent risk taking, but the optimal exercise price is lower.

In an economy monitoring is conducted by regulatory authorities, as long as banks' executive pay arrangements are unconstrained, regulators should be stricter in their monitoring and direct regulation of banks' activities. As banks would optimally adjust their compensation contracts, monitoring should be an important instrument in the toolkit of financial regulators.

5.2 Discussion: externality and limited liability of shareholders

Many observers appear to believe that large losses at major banks impose negative externalities on society. In our main setup, such externalities do not arise because project payoffs are assumed to accrue entirely to shareholders. Moreover, all payoffs are positive, so whether shareholders have limited or unlimited liability is immaterial. Exploring the sources of such externalities, or modeling them in any detail, is well beyond the scope of this paper. However, we can make a few comments about the implications of their existence.

Suppose no negative externalities are produced by outcomes in the normal range, but the disaster outcome does produce such externalities. They might be represented by defining the social payoff in event of disaster to be $s = d - e$, where d is the portion of the total disaster payoff that accrues to shareholders and $-e$ is the loss of welfare that is absorbed by parties external to the firm (and external to our model). Naturally the shareholders will not take $-e$ into account when choosing the manager's contract. Even if they did, there would be no impact on the compensation choice in cases where the shareholders would prefer the ordinary project(throughout the paper, we assume that parameters and distributions are such that shareholders prefer the ordinary risk project, because in the absence of externalities, motivating the manager to choose the ordinary project is uninteresting if the shareholders do not prefer it). However, allowing externalities does open a wedge between preferences for the two projects of shareholders and the social planner. The wedge is material only for those project combinations where the planner prefers the ordinary risk project and the shareholders prefer the tail risk project. For such combinations, shareholders will choose stocks (because effort can be motivated more cheaply and the manager will choose the tail risk project as shareholders desire), whereas the planner would choose options in order to motivate choice of the ordinary risk project.

We speculate that the likelihood that the planner can reduce the incidence of externalities depends on the planner's power and information. If the planner has full information and the power to compel contract terms, then the planner can simply dictate that properly designed options be used. If the planner can only dictate the contract form, shareholders will simply set option terms to mimic stocks, so the planner can have no effect on the allocation. A planner that has power but does not know the project distributions cannot be sure of improving welfare by dictating that options be used, since the planner will not know how to set the terms of the option contract.

Overall, it seems unlikely that external costs of bank distress can be reduced by regulating the form of compensation contracts (stocks versus options). It is possible that a planner who compelled a reduction in the minimum wage might do some good, since the manager would then be exposed to more of the disaster outcome's downside, but for such an approach to be effective the shareholders would have to be compelled to include the cost of the externality in the measured payoff that determines the manager's pay, which might impose large information requirements on regulators. Moreover, such an approach would work in practice only if the manager's alternative wage were below the minimum wage chosen by the planner. Though beyond the scope of this paper, future research that takes a more detailed look at externalities and strategies to address them seems likely to be fruitful.

6 Conclusion

This paper analyzes the choice between stocks and options to control managers' incentives when risk choice is not observable. Our finding represents a disparity with theories that advocate pure stocks, and shows that option compensation is sometimes a superior remedy to the agency cost of tail risk. Tail risk project can appear attractive to the manager when the manager is protected by limited liability and is not fully financially exposed to the adverse consequences of bad tail. Option compensation implements prudent risk taking by cutting off sensitivity to more relatively low outcomes for the ordinary risk project than for the tail risk project, thereby increasing the NPV of the ordinary risk project that compensation is responsive to more than that of the tail risk project. Options are not preferred in all circumstances: distributions of project payoffs matter. We show that options are optimal when a condition analogous to second-order stochastic dominance is satisfied.

We use a trade-off between avoiding tail risk and inducing managerial effort to characterize conditions under which stock compensation induces tail risk taking and thus render option compensation necessary. By virtue of having only measure (a piece rate) manipulatable in stock compensation, effort and risk decision cannot be controlled separately as in options: there are complex feedback effects between effort and project selection. Prior research and conventional wisdom suggest that stock compensation is an effective and sufficient mechanism to deter managerial risk taking. This paper shows that stocks can be problematic given limited liability of managers, and can be inferior to option compensation in general.

In addition, we show that more effective monitoring should be associated with a higher tendency by firms to compensate their executives by stocks instead of options. When monitoring is sufficiently effective (that is, a high probability of detecting a choice of the tail risk project), stocks provide adequate incentives for managers to choose the ordinary, value-maximizing project while maintaining incentives to put forth effort. From the shareholders' perspective, arriving at the best monitoring policy is a question of trading off the benefits of paying employees a lower wage against the cost of conducting monitoring.

Our central result is that for firms with project selection involving tail risk, options can dominate pure stocks. Given that essentially all real-world stock-based compensation consists solely of stocks and options, this seems a highly relevant comparison. But one could ask the distinct question of an option-like compensation form arises when the contract space is general. It would be fruitful to study the optimal relative proportions of stocks and options when compensation comprises of these instruments and analyze the conditions under which option compensation is optimal in a general contract design setting.

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Appendix

A A binary example

In this section, we use a binary example to convey intuition about drivers in the continuous-state analysis that appears below.

A.1 No project choices

Consider an *ordinary risk project*, indexed by o. There are two possible outcomes, y_L and y_H, where $y_L < y_H$. Project outcome is influenced by the effort level (e) of the manager, which can be high, h, and low, ℓ. Exerting high effort imposes a cost $c > 0$ on the manager. Let $\pi_e^o \in (0, 1)$ be the probability that y_H occurs. Assume that $\pi_h^o > \pi_\ell^o$. The effort is not observable to the principal, and the principal designs a reward scheme that is a function of the outcome. Let w_s be the reward when y_s occurs, where $s \in \{L, H\}$.

The manager and the principal are both risk neutral, and the manager has an outside option that provides $\bar{U}$. Assuming that y_H is sufficiently large compared to y_L, the principal wants to induce the manager to put forth effort.

There are two additional constraints for the reward scheme. One is that the reward has to be monotonically increasing in output. As we will see, this constraint is not binding in this setting. The second is that there is a minimum non-negative reward $\underline{w}$ for w_s.

The principal solves the following optimal contract problem:

$$\min_{w_H, w_L} \quad (1 - \pi_h^o)w_L + \pi_h^o w_H,$$

subject to

$$\begin{aligned} w_H &\geq w_L, \\ w_L &\geq \underline{w}, \qquad\qquad (M1) \\ w_H &\geq \underline{w}, \end{aligned}$$

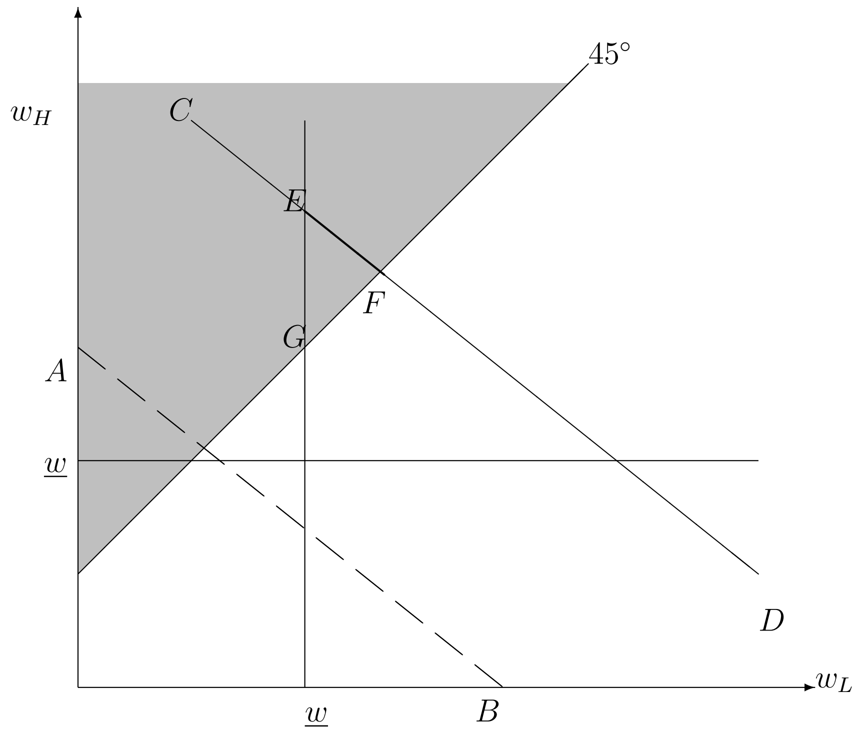
$$(1-\pi_h^o)w_L + \pi_h^o w_H - c \geq \bar{U}, \tag{PC}$$

and

$$(1-\pi_h^o)w_L + \pi_h^o w_H - c \geq (1-\pi_\ell^o)w_L + \pi_\ell^o w_H. \tag{IC}$$

(IC) implies that $w_H > w_L$, which implies that the first and the third constraints are not binding. Which of the rest of the three are binding depends on the parametric configurations. There are two different cases.

Figure 4: Without project choices



Case 1 $(\underline{w} + \pi_\ell^o c/(\pi_h^o - \pi_\ell^o) \geq \bar{U})$:

In this case, (PC) is not binding. The optimal solution is determined by binding $(M1)$ and (IC):

$$w_L = \underline{w}$$

and

$$w_H = \underline{w} + \frac{c}{\pi_h^o - \pi_\ell^o}.$$

The manager's expected payoff is $w + \pi_\ell^o c/(\pi_h^o - \pi_\ell^o)$.

Case 2 ($\underline{w} + \pi_\ell^o c/(\pi_h^o - \pi_\ell^o) < \bar{U}$):

In this case, (PC) is binding and any combination of w_L and w_H that satisfies (PC) with equality, and $(M1)$ and (IC) with inequality is an optimal solution. The manager's expected payoff is $\bar{U}$.

As shown in Figure 4, the shaded area above the 45° line GF represents the incentive compatible region defined by (IC). $(M1)$ requires that the solution be above (or on) the horizontal line at $\underline{w}$ and on the right of (or on) the vertical line at $\underline{w}$. (PC) is represented by the line AB if $(M1)$ is tighter than (PC) and denoted by the line CD otherwise. Line AB and CD share the same slope as the principal's indifference curve, closer to the origin being better (i.e. a lower objective function value). In Case 1, the point G denotes the optimal solution. In Case 2, any point on the segment EF can be one optimal solution to the principal's problem.

A.2 Project choices

Now suppose that there is a *tail risk project* that the manager can choose, in addition to the ordinary project. The tail risk project involves a possibility of having a disaster outcome, $y_d < y_L$, with the probability $(1-p) \in (0,1)$. Let w_d be the wage payment if y_d occurs. The probability of y_H outcome is $p\pi_e^t$ when the manager makes an effort e, where $\pi_e^t \in (0,1)$. Assume that the ordinary project is a better project in the sense that it yields a higher expected outcome than the tail risk project. That is,

$$(1-p)y_d + p(1-\pi_h^t)y_L + p\pi_h^t y_H < (1-\pi_h^o)y_L + \pi_h^o y_H$$

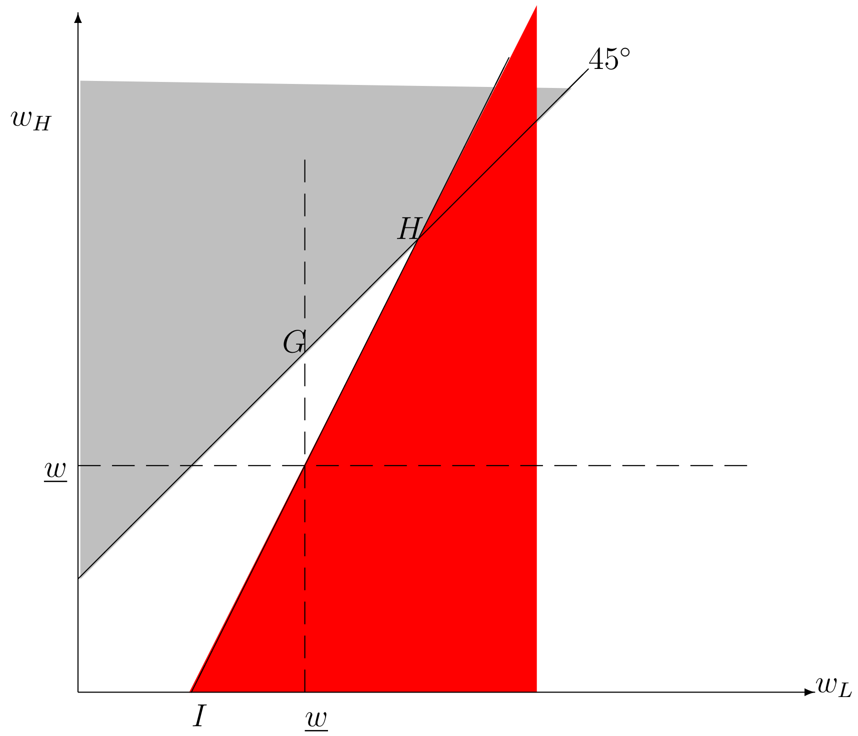
holds. In addition, assume that $p(\pi_h^t - \pi_\ell^t) \geq \pi_h^o - \pi_\ell^o$, so that the manager has an incentive to make an effort with the tail risk project if the ordinary risk project's (IC) is satisfied.

Consider Case 1 in the previous section. (Case 2 is less interesting.) Under the optimal payoff in the previous section (and $w_d = \underline{w}$ from the monotonicity constraint), the manager's payoff is

$$(1-p)\underline{w} + p(1-\pi_h^t)\underline{w} + p\pi_h^t\left(\underline{w} + \frac{c}{\pi_h^o - \pi_\ell^o}\right) - c = \underline{w} + \frac{\pi_h^t - \pi_h^o + \pi_\ell^o}{\pi_h^o - \pi_\ell^o}c.$$

This is larger than the manager's payoff when choosing the ordinary risk project if and only if $p\pi_h^t \geq \pi_h^o$. (Only the H event matters for the manager's incentive.) Then, when the principal cannot dictate the project choice, the manager will choose the tail risk project.

Figure 5: With project choices



Taking this into consideration, the investor has to design the reward with an additional constraint:

$$(1-p)w_d + p(1-\pi_h^t)w_L + p\pi_h^t w_H - c \leq (1-\pi_h^o)w_L + \pi_h^o w_H \qquad (PR)$$

When $\bar{U}$ is sufficiently low so that the participation constraint is not binding, clearly $w_d = \underline{w}$ has to be satisfied. Note that when $p\pi_h^t \geq \pi_h^o$ is satisfied, $p(1-\pi_h^t) < (1-\pi_h^o)$. This implies that y_L is relatively likely to happen with the ordinary risk project. The constraint (PR) can be rewritten as

$$w_H \leq \frac{1 - p + p\pi_h^t - \pi_h^o}{p\pi_h^t - \pi_h^o} w_L - \frac{(1-p)\underline{w}}{p\pi_h^t - \pi_h^o}.$$

Note that this equation holds with equality when $(w_L, w_H) = (\underline{w}, \underline{w})$ and the coefficient on w_L is larger than 1. It can be shown that at the optimal solution, (IC) and (PR) are binding.[12] The optimal reward satisfies $w_L > \underline{w}$ and $w_H = w_L + c/(\pi_h^o - \pi_\ell^o)$.

Compared to the previous section, the reward shifts up (w_L, w_H) in a parallel manner. The intuition is that since y_L is more likely to happen with the ordinary risk project, y_L event has to be rewarded so that the manager will choose the ordinary project. The fact that y_L became important is a result of having only two possible outcomes. As we will see in the continuous model presented in the next section, however, our key result is *not* an artifact of the simple contract space. If there are more than two possible outcomes, it is plausible that there is an event i that is not L and is more likely to happen with the ordinary risk project, so that the tail risk project looks better for the manager when the project choice is ignored in compensation design. Whichever event that is more likely to happen with the ordinary project is going to be rewarded when the project choice is involved. Introducing project choices into a standard moral hazard problem changes the optimal structure of compensation. A compensation arrangement that is optimal without tail risk choices may actually induce the choice of tail risk if it is available.

In Figure 5, the red area below the line IH represents the risk-taking incentive compatible region defined by (PR). As noted in Figure 4, the grey area above the line GH represents the effort-making incentive compatible region defined by (IC). Thus, the lowest point (H) at which the red area overlaps with the grey area is the optimal solution

[12] If either of (IC) and (PR) is not binding under the optimal contract, a reduction of w_L or w_H will still satisfy the incentive constraints and decrease the objective function value — it causes a contradiction.

when there is a project choice. The point H being higher than the point G indicates that incentive alignment becomes more expensive when there is risk choice.

B Optimality of stocks in the absence of project selection

Proof of Lemma 2: Following Proposition 5, incentive compatibility requires the manager to exert effort if

$$\theta \geq \frac{c}{\int_{y^*}^{\overline{y}}[F_o(y+\kappa) - F_o(y)]dy}.$$

To simplify algebraic expressions, we rewrite it in terms of $G_i(y) = 1 - F_i(y), i \in \{o, t\}$,

$$\theta \geq \frac{c}{\int_{y^*}^{\overline{y}}[G_o(y) - G_o(y+\kappa)]dy}.$$

The above inequality is binding in the optimal contract, otherwise a reduction of θ reduces the objective function value and thus causes a contradiction. Recall that following Proposition 2, the expected wage payment in the case of options can now be rewritten as follows.

$$\begin{aligned} E[w] &= \underline{w} - \theta y^* + \theta[y^* + \int_{y^*}^{\overline{y}} G_o(y)dy], \\ &= \underline{w} + \theta \int_{y^*}^{\overline{y}} G_o(y)dy, \\ &= \underline{w} + \frac{c\int_{y^*}^{\overline{y}} G_o(y)dy}{\int_{y^*}^{\overline{y}}[G_o(y) - G_o(y+\kappa)]dy}. \end{aligned}$$

Taking derivative of $E[w]$ with respect to y^*,

$$
\begin{aligned}
&dE[w]/dy^* \\
=&c\frac{d\left(\frac{\int_{y^*}^{\overline{y}} G_o(y)dy}{\int_{y^*}^{\overline{y}}[G_o(y)-G_o(y+\kappa)]dy}\right)}{dy^*}, \\
=&\frac{c\int_{y^*}^{\overline{y}}[G_o(y)-G_o(y+\kappa)]dy\frac{d\left(\int_{y^*}^{\overline{y}} G_o(y)dy\right)}{dy^*}}{\left(\int_{y^*}^{\overline{y}}[G_o(y)-G_o(y+\kappa)]dy\right)^2}-\frac{c\int_{y^*}^{\overline{y}} G_o(y)dy\frac{d\left(\int_{y^*}^{\overline{y}}[G_o(y)-G_o(y+\kappa)]dy\right)}{dy^*}}{\left(\int_{y^*}^{\overline{y}}[G_o(y)-G_o(y+\kappa)]dy\right)^2}.
\end{aligned}
$$

Since the denominator is positive, $E[w]$ is nondecreasing in y^* if the numerator is bounded above zero. Using

$$
\frac{d\left(\int_{y^*}^{\overline{y}}[G_o(y)-G_o(y+\kappa)]dy\right)}{dy^*}=F_o(y^*)-F_o(y^*+\kappa),
$$

and

$$
\frac{d\left(\int_{y^*}^{\overline{y}} G_o(y)dy\right)}{dy^*}=F_o(y^*)-2,
$$

we write the numerator as follows.

$$
\begin{aligned}
&[F_o(y^*)-2]\int_{y^*}^{\overline{y}}[F_o(y+\kappa)-F_o(y)]dy-[F_o(y^*)-F_o(y^*+\kappa)]\int_{y^*}^{\overline{y}} G_o(y)dy \\
=&[F_o(y^*)-2]\int_{y^*}^{\overline{y}}[G_o(y)-G_o(y+\kappa)]dy-F_o(y^*)\int_{y^*}^{\overline{y}} G_o(y)dy+F_o(y^*+\kappa)\int_{y^*}^{\overline{y}} G_o(y)dy \\
=&[F_o(y^*)-2]\int_{y^*}^{\overline{y}}[G_o(y)-G_o(y+\kappa)]dy-[F_o(y^*)-2]\int_{y^*}^{\overline{y}} G_o(y)dy+[F_o(y^*+\kappa)-2]\int_{y^*}^{\overline{y}} G_o(y)dy \\
=&[F_o(y^*+\kappa)-2]\int_{y^*}^{\overline{y}} G_o(y)dy-[F_o(y^*)-2]\int_{y^*}^{\overline{y}} G_o(y+\kappa)dy
\end{aligned}
$$

Therefore, the numerator is non-negative if

$$
[2-F_o(y^*+\kappa)]\int_{y^*}^{\overline{y}}[1-F_o(y)]dy\le[2-F_o(y^*)]\int_{y^*}^{\overline{y}}[1-F_o(y+\kappa)]dy,\qquad\forall y^*\in[\underline{y},\overline{y}].
$$

□

C The effect of risk aversion

It is informative to compare this result with the standard principal-agent result. In the standard principal-agent model, the principal and agent have a conflict of interest over variations in outcomes, and the optimal compensation contract consists of a base wage plus a piece rate; thereby the agent is thus always in the incentive region of the contract: if the agent is risk averse, the principal chooses to reduce the piece rate to minimize the agent's exposure to the variation; if the agent is risk neutral, there is no conflict of interest between the principal and the agent, and the contract with a piece rate of one hundred percent yields the first-best outcome. In the model in this paper, the agent is not averse to variations in income, but is shielded from bad tail outcomes due to the minimum wage constraint. A flat region irresponsive to low price realization effectively alters the relative value of project outcomes that managerial compensation is responsive to, and is thus introduced to control for managerial risk taking and align the interests of managers with shareholders.

Our key result can be extended to a risk averse manager. The manager's utility is then represented by $U(e, y) = u[w(y)] - a(e)$, where $u(\cdot)$ is strictly increasing and strictly concave. The condition for options to be an effective means of avoiding tail risk in this case is identical to that if the manager is risk neutral.

Proposition 10 *A necessary and sufficient condition under which options can be used to avoid tail risk taking when the manager is risk averse is identical to that when the manager is risk neutral:*

$$\int_{y^*}^{\overline{y}} F_o(y)dy \leq \int_{y^*}^{\overline{y}} F_t(y)dy,$$

Proof: Since $u(\cdot)$ is strictly increasing, (IC_{R1}) is reduced to

$$E[w(y_o)|y^*, h] \geq E[w(y_t)|y^*, h],$$

which leads to inequality (4), identical to the condition when the manager is risk neutral. Incentive compatibility requires the manager to exert effort if (IC_e) is met. Since $u(\cdot)$ is strictly concave, the manager is less motivated by compensation changes at different price levels, because higher wealth translates into a lower marginal utility of income. It is thus more costly to motivate a risk-averse manager to exert effort than a risk-neutral manager, that is,

$$\theta > \frac{c}{\int_{y^*}^{\overline{y}}[F_o(y+\kappa) - F_o(y)]}dy.$$

Analogous to the case of a risk neutral manager, the incentive compatibility constraint on risk choice off the equilibrium path (condition (IC_{R2})) is automatically satisfied when (IC_{R1}) and (IC_e) are both met. Therefore, inequality (4) remains the necessary and sufficient condition for option compensation to be an optimal motivational tool.

D The condition when distributions are unbounded

Proposition 11 *A necessary and sufficient condition for options to be optimal when the distributions of stock prices are unbounded is*

$$\int_{y^*}^{\infty} G_o(y)dy \geq \int_{y^*}^{\infty} G_t(y)dy, \tag{11}$$

where $y_o \in \{-\infty, \infty\}$ and $y_t \in \{-\infty, \infty\}$ are the stock price generated by the ordinary risk project and tail risk project respectively; $G_o(\cdot) = 1 - F_o(\cdot), G_t(\cdot) = 1 - F_t(\cdot)$, and $F_o(\cdot)$ and $F_t(\cdot)$ are the cumulative distribution functions of stock prices associated with the ordinary risk and tail risk projects respectively; y^* is the exercise price specified in the stock options.

Proof: If the payoffs follow an unbounded distribution, following the proof of Proposition 2,

$$\begin{aligned} & E[y|y \geq y^*] \\ & y^*F(y^*) + \textstyle\int_{y^*}^{\infty} yf(y)dy \\ = \quad & y^*F(y^*) + \Big[yF(y)\Big]_{y^*}^{\infty} - \textstyle\int_{y^*}^{\infty} F(y)dy \end{aligned}$$

Let $G(\cdot) = 1 - F(\cdot)$, and we have

$$\begin{aligned} & E[y|y \geq y^*] \\ = \quad & y^* F(y^*) + \left[y \Big(1 - G(y) \Big) \right]_{y^*}^{\infty} - \textstyle\int_{y^*}^{\infty} \Big(1 - G(y) \Big) dy \\ = \quad & y^* F(y^*) + [y]_{y^*}^{\infty} - [yG(y)]_{y^*}^{\infty} - [y]_{y^*}^{\infty} + \textstyle\int_{y^*}^{\infty} G(y) dr \\ = \quad & y^* F(y^*) + y^* G(y^*) - \lim_{\overline{y} \to \infty} [\overline{y} G(\overline{y})] + \textstyle\int_{y^*}^{\infty} G(y) dy \\ = \quad & y^* [1 - G(y^*)] + y^* G(y^*) - 0 + \textstyle\int_{y^*}^{\infty} G(y) dr \\ = \quad & y^* + \textstyle\int_{y^*}^{\infty} G(y) dy \end{aligned}$$

□

In figure 2, the dashed area represents the the integral specified in Proposition 3. As it is an integration from *above*, it does not directly imply stochastic dominance, which requires a comparison of integrals from *below*. However, when the distributions of the projects' outcomes are both symmetric (apart from the disaster outcome), this condition is equivalent to saying that the value generated from the tail risk project second-order stochastically dominates that from the ordinary risk project at the high end.

Lemma 10 *Proposition 5 can be extended to unbounded distributions.*

Proof: In the case of pure stock grants, (IC_e) is

$$\begin{aligned} \theta^{stocks} & \geq \frac{c}{E[y_o|h] - E[y_o|l]} \\ & \geq \frac{c}{\int_{-\infty}^{\infty} G_o(y|h) dy - \int_{-\infty}^{\infty} G_o(y|l) dy} \\ & = \frac{c}{\int_{-\infty}^{\infty} [G_o(y|h) dy - G_o(y|l)] dy} \end{aligned}$$

In the case of options, (IC_e) is

$$\begin{aligned} \theta^{options} & \geq \frac{c}{E[y_o|y_o \geq y^*, h] - E[y_o|y_o \geq y^*, l]} \\ & \geq \frac{c}{\int_{y^*}^{\infty} G_o(y|h) dy - \int_{y^*}^{\infty} G_o(y|l) dy} \\ & = \frac{c}{\int_{y^*}^{\infty} [G_o(y|h) dy - G_o(y|l)] dy} \end{aligned}$$

In the optimal contract in our setting, (IC_e) is binding:

$$\theta^{stocks} = \frac{c}{\int_{-\infty}^{\infty}[G_o(y|h)dy - G_o(y|l)]dy}, \tag{12}$$

$$\theta^{options} = \frac{c}{\int_{y^*}^{\infty}[G_o(y|h)dy - G_o(y|l)]dy}. \tag{13}$$

It is straightforward to see that $\theta^{stock} < \theta^{options}$. In addition, $\theta^{options}$ is increasing in y^*.□