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In Noise We Trust?
Optimal Monetary Policy with Random Targets*

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Abstract

Monetary policy in which the central bank commits to a *randomized* inflation target allows for potentially faster expectations convergence than with a fixed target. Randomization acts as a substitute for full credibility, enabling the central bank to transfer its intent to the economy more quickly. With a fixed target, individual occurrences of high inflation are regarded as confirming the low credibility of the institution. Under a random target, such events can be regarded as generated by the randomization process and thus harm credibility less on the margin. We thus emphasize the role of this mechanism in particular in transition environments.

Keywords: monetary policy; credibility; emerging economies

JEL codes: E52; E61; E42

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I. Introduction

While the past 30 years has seen remarkable progress is the conquest of inflation in most parts of the world, the specter of high, uncontrollable, or persistent inflation remains. Notable cases include cases as varied as Zimbabwe and Brazil. The former suffers from its leaders' reckless disregard of the economy and the latter from a long history of inflationary episodes and a concomitant fear of a rapid return. Our paper provides an alternate monetary policy that can accelerate the population expectations convergence, and thus transition to a new low-inflation more rapidly.

The intellectual backdrop to this is a long-standing debate on the appropriate "speed" of convergence in transition environments. The existing literature on gradual vs. "shock" approaches to transition draws either on credibility, or on trend or targeting approaches. Guillermo Calvo's (1983) model of stochastic price adjustments suggests that immediate disinflation can be costless, and Laurence Ball (1994) suggests that a credible and rapid disinflation can even prompt a boom. Thomas Sargent (1994) has suggested that four historic cases of hyperinflation were halted through *credible* institutional change.[1] While these go some distance in resolving new Keynesian concerns of wage and price friction in economic adjustment, they don't capture the credibility constraints at hand in transition environments.

Within the targeting literature, Svensson (1999b) argues that prior controversy on the role of inflation targeting in real variability has been resolved in that there is relative consensus on the inclusion of the output gap in the monetary policy loss function. This leads, in turn, to convergence rates being slower than would otherwise be the case. He points to a collection of research (Fischer 1996, King 1996, Taylor 1996 and Svensson 1996) that all emphasize loss

[1] Additional country studies are available in Sachs (1990).

function with relatively larger weight on the output gap than the gap between target and actual inflation. Svensson (1997) and Ball (1999) both show that concern with output gap stability translates into slower transitions. This solution, namely gradualism, arises in other contexts as well. Svensson (1999b) indicates that model uncertainty and interest rate smoothing both lead to solutions with increased convergence horizons. This discussion is essentially a synopsis of the field's opinion on optimal policy conditional on a given (consensus) loss function.

We re-ask the fundamental question about gradualism slightly differently by looking at the role of credibility in the expectations formation process. One can view the key problem as the ability of the population to discern the intent of the central bank. The crux of our argument that acceleration of expectations, and consequently of targets themselves, is a function of the population's response to the institutions lack of complete credibility. Without full credibility, adverse shocks or small errors on the part of the central bank during the transition period can be interpreted by the population as malign intent. This leads to increasingly slow convergence. In our opinion, this mechanism contributes to the finding of gradualism in the targeting literature. Fast convergence paths are punished with higher output gap variability in part because of the reaction of population; as much of monetary policy, it's about credibility.

Our method for exploring this issue is a stylized one. We take a transitional expectations framework (Taylor 1975) and generalize it to a stochastic environment. This is an environment with fully rational expectations; real impact can only occur during an expectations formation period at the onset of a new regime.[2] The population will form expectations of inflation in each period using Bayesian updating and the government will minimize a classic loss function. In this context, it is possible to create a mechanism which enables the population to "learn" faster.

[2] For reasons that will be apparent, a randomized policy makes little sense with this framework during a time period in which the economy is close to its optimal inflation level.

Using Randomization as a Substitute for Credibility

In our model, we find that randomization of monetary policy in transition cases provides a better response mechanism to the population's behavior, because it eliminates a portion of the information asymmetry problem. That is, randomization can act as a partial substitute for an increase in bank credibility. Through such a policy, the monetary authority can "encourage" rapid convergence without an increase in credibility or reputation. Consider the following example. Suppose a new government takes over monetary policy in a developing country with a "traditional" inflation rate of 40-50 percent a year. The new government "knows" that the optimal inflation level is 5 percent. However, since a 5 percent target would not be credible, the government would potentially have to announce a higher target or to implement a relatively slow convergence plan in an inflation targeting framework.[3] We show that if the government commits to a so-called "random target", the situation can change. The government would enact a policy whereby if it commits to implement the targeting decisions of a computer run by some independent third party.[4] The computer in each period would generate a random number based on a target around 5 percent, with some small variance.[5] Now consider the case when the public observes inflation in a given period that is relatively high compared to recent history. With a traditional fixed target, the public attributes this either to duplicity or to implementation errors on the part of the bank. Under the random target, individual aberrant observations can be obtained as part of the randomization and thus do not harm credibility.

[3] Good discussions, literature reviews and case examples are available in Bernanke et al 1999.
[4] The third party is used to illustrate the feasibility of a trustworthy random draw. The computer could even be placed within the offices of a "trusted" institution outside the country.
[5] In fact, the model will specify that the computer generate a random walk target, though for the sake of explanation, random is sufficient at this stage. Simulations at the conclusion of the paper reveal that the two provide similar convergence paths.

Notice as well that the government's commitment to a randomized policy is credible in of itself, because there is no *a priori* reason why the government is better off under the fixed target. In fact, we will specify below that the population is completely unaware of the target in each period. This further "ties the hands" of the central banks and highlights the credibility enhancement features of the method itself. Notice as well the distinction here from inflation targeting in that the stated commitment of the central bank is to a target that is itself moving, not to a band around a fixed target.

From this starting point, we compare two basic environments: a traditional "fixed" inflation target and a "random" one. Both of these will have targets close to the optimal inflation level and will depart from the inflation targeting literature in that we do not look for deterministic paths for the target; our focus is on the informational differences between fixed and random targets. Monetary policy randomization has potential application in at least three cases. Two of these cover the "high" inflation situation, hyperinflation and persistent inflation. In situations of hyperinflation, a faster convergence rate is clearly desirable, especially since most "losses" occur in the short run. The Japanese deflation-cum-liquidity trap case is also an example in which expectations are too sticky (this time downwards) and in which, for structural and credibility reasons, a change in inflationary expectations is needed where traditional monetary policy has failed (see Krugman 1998 for discussion).

We have focused our discussion on such "extreme" cases because we believe lack of credibility to be an important limitation in the availability of monetary policy options for such countries. While credibility can be built over time, these countries may find themselves too often in a "transitional" period, in which credibility building is impractical. Our solution prescribing inflation targeting with a random target rate seems to provide at least some improvement in the efficacy of monetary policy in such cases.

As mentioned, two situations are discussed: a "classic" one in which the government commits to a particular inflation target; and our proposal for a "randomized" target, in which the government pre-commits to a particular randomization of its inflation target. In either case, the target inflation rate is unknown to the public. We cover in Section II the rationale behind the ability of the government to obtain real gains through monetary policy, even under rational expectations, and we address the effect on expectations formation of increased uncertainly in inflation policy. In order to achieve this increased uncertainty, we use a "random target". Specifically, this refers to a random walk in the target inflation rate of the country's monetary authority.[6] Section III describes our model of monetary policy equilibrium in the case of a fixed monetary target. Subsequently, Section IV explains optimal monetary policy when the government chooses a random walk inflation target. The benefits of the randomized policy are discussed. We show simulated results for the random walk process, a white noise process and the standard fixed target in Section V and conclude in Section VI.

II. A transitional expectations framework

In order to evaluate the tradeoffs between a fixed and random targeting regime, we explore a transition expectations environment (Taylor 1975). In a world of full information and fully rational expectations, monetary policy has limited effect. Indeed, economic agents, given full information, can rationally determine the inflation that the government wants to achieve, and adjust their prices and wages accordingly. In Taylor's classic set up, he describes as a transitional

[6] In the empirical section, we also show results from a randomization that is based on a white noise process. For small values of the variance of this process, the results are in line with the random walk version.

period towards rational expectations, a period during which the central bank's target inflation rate is either not known or not credible. Under such conditions of asymmetric information, Taylor shows that monetary policy has a significant impact in the short run, that is, until expectations fully adjust to reflect the actual chosen rate.

Taylor uses an output-inflation targeting loss function (i.e. the loss is measured in squared deviations from some target output level and some target inflation rate, both of which are held constant across time). In particular, Taylor is concerned with the potential gains from generating some extra "unexpected" inflation. Indeed, suppose that for some reason (fiscal policy distortions, etc.) the current rate of inflation (which is fully known by the public and thus also expected by the public) is below the optimal rate. If the central bank unexpectedly moves to target the optimal rate, it will obtain some output gains in the transitional period until the population learns the new rate. The central bank can capitalize on creating "confusion" about the true inflation rate.

We begin by walking through the Taylor setup. First, we explain, as Taylor has, the logic behind the inefficacy of monetary policy under rational expectations. Suppose the relationship between inflation and unemployment can be shown as

$$\pi(t) = \phi[u(t)] + x(t),\ \phi'(\cdot) < 0,\ \phi(u^*) = 0, \quad u^* > 0. \tag{1}$$

Here $\pi(t)$ and $x(t)$ are the actual and expected rates of inflation, respectively, at each time t. Unemployment is denoted by u, and ϕ is some decreasing function. Taylor further assumes that the monetary authority has direct control over the inflation rate through control of the money supply,[7] so that $\pi(t)$ is effectively a choice variable. Long-run effects on employment are not

7 We later relax this assumption in order to add reality and as the mechanism for further uncertainty.

possible in (1), but short-run effects are. This is a well-known argument made by Phelps, who considers that inflationary expectations adapt according to the linear rule:

$$dx(t) = \beta\left[\pi(t) - x(t)\right]dt. \tag{2}$$

Where β is some constant and the government chooses an inflation target that maximizes:

$$\int_0^\infty e^{-\rho t} W(x,u)\,dt, \tag{3}$$

where $W(x,u)$ is the instantaneous social utility rate, u is the unemployment rate, and β and ρ are discount rates. The short-run effects mentioned above are obtained when $\pi(t)$ is systematically greater than $x(t)$. Rational expectations theory suggests that over a period of time, agents understand that their expectations based on (2) are biased and modify accordingly. Taylor explains that under rational expectations the formulation appears as:

$$x(t \mid t_0) = E\left[\pi(t) \mid I(t_0)\right]. \tag{4}$$

Here $I(t_0)$ represents information available at time t_0. This explains that under a deterministic policy with no uncertainty, $\phi\left[u(t)\right] = 0$ and $u(t) = u^*$ for all t (Taylor 1975).

III. Monetary policy commitment to a fixed target

From the above argument, it follows that some information asymmetry is necessary in order for the short-run Phillips curve to have an effect on output. We will thus model the monetary policy effects of central bank policies as an incomplete information game between the central bank and the public. The game is set in continuous time, so that stochastic random variables are used to model random disturbances. The public does not have knowledge of the government's methods or desires, but is able to learn from the government's prior actions. This is an abstraction that seems appropriate in a situation of low government credibility. Essentially it assumes zero credibility – any announcement is equivalent to none at all. The public derives all new information from the actions themselves. Taylor illustrates such a system: during the time period when old beliefs mix with new information, the rational expectations hypothesis no longer holds, and monetary policy has real and significant effects.

A. The Government (and Central Bank)

The government commits itself at time $t = 0$ to a target inflation rate for the economy, which we denote by μ. Once the target inflation rate is set, the economy generates an actual inflation rate $\pi(t)$. We assume that the government has limited control over the generated inflation rate. More precisely, $\pi(t)$ is modeled as a normally distributed random variable, where $E[\pi(t)] = \mu$ (thus, the government can control the average inflation rate it generates, but not the actual value). Furthermore, we assume that $\text{var}[\pi(t)] = \sigma$, where σ is a known constant depending on the characteristics of the interaction between the central bank and the economy.[8]

Let us define price level at each time t as $p(t)$, and then let $z(t) = \log[p(t)]$. When $\sigma =$ 0, the government has complete control over inflation, and by definition $\pi(t) = \mu(t)$. With $\sigma >$ 0, inflation is generated through the diffusion process:

$$dz(t) = \mu dt + \sigma dv, \tag{5}$$

where $v(t)$ is a Wiener process with a zero mean and unit variance.

8 A lower σ implies a more precise monetary policy. In particular, we assume that σ is the minimum possible variance, i.e. the government cannot improve the precision with which it generates inflation beyond this point.

Once inflation is generated, the public observes it and (as discussed below) forms some inflationary expectations $x(t) = E_{public}\left[\pi(t)\right]$. We assume that the government knows the process through which these expectations are formed, so the government can infer $x(t)$.

The government also has a negative payoff function or loss function $L(t)$ (see Svensson 1999a). At this point, we will not specify *L*; suffice it to say that *L* depends on two arguments, actual inflation $\pi(t)$ and some other target variable (unemployment, or real GDP, etc.) which depends again on inflation and on public inflationary expectations, $x(t)$. Since by choosing μ the government determines both $\pi(t)$ and $x(t)$, the government's goal is to choose a target μ that minimizes the expected value of the loss function. Indeed, both $\pi(t)$ and $x(t)$ are stochastic, and the government can control only moments of their distribution, not the actual realized values. In a full commitment regime, the government chooses a value μ that minimizes the (discounted) expected future losses, $\int_0^\infty e^{-\delta t} E\left[L(t)\right] dt$.

B. The Public

At each time *t*, the public observes the inflation rate $\pi(t)$ generated in the economy. The public, however, does not have a full knowledge of the government's target inflation rate μ. More precisely, consider that at time zero the public expects the government's inflation target to be $\mu(0) = \mu_0$, different from μ. In other words, $E_{public}\left[\mu(0)\right] = x(0) = \mu_0$. Also, the public belief about the accuracy of this initial guess is such that $\text{var}_{public}\left[\mu(0)\right] = \sigma_0$. We assume throughout that the government knows both μ_0 and σ_0. The fact that the initial public expectations are different from the actual government target may result, as Taylor emphasizes, from the fact that the initial basis for expectations of inflation setting is a combination of expectations from a new regime and information from an old regime. Moreover, the same situation can result from a lack of credibility of the "new" government. Such a construction captures the reality that an emerging market government's open commitment to a particularly "low" inflation level, μ, may not be entirely credible.

It follows from standard quadratic loss functions that the public's goal is to generate at each time $t > 0$ the most appropriate inflationary expectations. The public observes the actual inflation generated at each time t, and based on it and on its current inflationary expectations "infers" the most likely inflation target chosen by the government and adjusts its own expectations. Note that this method assumes that the public knows σ, but not μ. Under these conditions, it can be shown that the population's inflationary expectations are given by the following stochastic differential equation:

$$dx(t) = \frac{1}{t + \sigma^2 / \sigma_0^2} \left[dz(t) - x(t)\, dt \right]. \tag{6}$$

Given than this equation will have a stochastic solution, even under perfect information, the government will not be able to have deterministic knowledge of the expectations path $x(t)$. However, as will become apparent shortly, because we use quadratic loss functions, the government only needs to know the mean and the variance of the populations' expectations.

If we define:

$$\begin{aligned} \bar{x}(t) &= E_{government}\left[x(t) \right] \\ \tilde{x}(t) &= \mathrm{var}_{government}\left[x(t) \right] = E_{government}\left[x(t)^2 \right] - \bar{x}(t)^2, \end{aligned} \tag{7}$$

then, from the theory of linear stochastic differential equations (Arnold 1974) it follows that these deterministic variables are given by the following deterministic differential equations:

$$d\overline{x}(t)=\left[-\frac{1}{t+\sigma^2/\sigma_0^2}\overline{x}(t)+\frac{\mu}{t+\sigma^2/\sigma_0^2}\right]dt \text{, with } \overline{x}(0)=\mu_0 \text{, and} \tag{8}$$

$$d\tilde{x}(t)=\left[-\frac{2}{t+\sigma^2/\sigma_0^2}\tilde{x}(t)+\frac{\sigma^2}{\left(t+\sigma^2/\sigma_0^2\right)^2}\right]dt \text{, with } \tilde{x}(0)=\sigma_0^2 . \tag{9}$$

Solving the above equations for the mean and variance of the inflation expectations we get:

$$\overline{x}(t) = \mu + \frac{\mu_0 - \mu}{t + \sigma^2/\sigma_0^2} \text{, and} \tag{10}$$

$$\widetilde{x}(t) = \frac{\sigma^2}{t + \sigma^2/\sigma_0^2} . \tag{11}$$

From this knowledge of the expected path of people's expectations, the government can set μ appropriately to minimize some given loss function. The population, as implied above, simply attempts to have full understanding of the inflation rate in order to set prices and wages to match.

C. The loss function and the resulting Bayesian equilibrium

The most commonly used loss function is a variant of that suggested by Taylor (See also Kydland and Prescott (1977) and Barro and Gordon (1983)):

$$L(t)=\frac{1}{2}\left\{\lambda\left[y(t)-y^*\right]^2+\left[\pi(t)-\pi^*\right]^2\right\} \tag{12}$$

We use a variant of this, a generalization of the well accepted standard, where y can be output or other appropriate macro target variable such as unemployment. Our generalization is as follows. Begin with the form: $L(t)=A\left[x(t)-\pi(t)\right]^2+B\left[\pi(t)-\pi^*\right]^2$. The first term is the squared deviation from optimal unemployment. This deviation enters the function as a result of the loss created from "wrong" wage setting (this affects unemployment and thus output). Firms set wages to equal expected inflation in the next period (in our model this corresponds to $x(t)$). However, the ideal wage that would generate the ideal unemployment is the wage firms would choose if they had perfect knowledge about future inflation. But in our model, if firms had perfect information they would choose a wage determined by μ, as this is the true expected inflation. We can thus rewrite the above loss function as:

$$L(t) = A\left[x(t) - \mu\right]^2 + B\left[\pi(t) - \pi^*\right]^2. \qquad (13)$$

One can see that this is equivalent to equation 12 by writing $y(t) = y\left[x(t)\right]; y^* = y(t)$. Our structure requires simply that the mappings are one-to-one. Since the firms do not know (or do not believe) μ, they wrongly set wages to $x(t)$ instead. This term is weighted by a constant factor, A.[9] The second term is the deviation from an "optimal" inflation level, π^*. We have included this term as well, weighted by a constant B. Setting $A = 1/2; B = (1/2)\lambda$, and substituting the output equation mappings returns the standard loss function.

Thus, the expected loss, $EL(t)$ is given by:

$$EL(t) = A\left\{E\left[x(t)\right] - \mu\right\}^2 + B\left\{E\left[\pi(t)\right] - \pi^*\right\}^2 + \text{var}_A\left[x(t)\right] + \text{var}_B\left[\pi(t)\right]. \qquad (14)$$

Since $E\left[x(t)\right] = \overline{x}(t)$; $E\left[\pi(t)\right] = \mu$; $\text{var}\left[x(t)\right] = \tilde{x}(t)$ and $\text{var}\left[\pi(t)\right] = \sigma^2$, the expected loss function can be re-written as:

$$EL(t) = A\left[\overline{x}(t) - \mu^2 + B\mu - \pi^*\right]^2 + A\tilde{x}(t) + B\sigma^2. \qquad (15)$$

9 Under the general targeting framework described by Svensson (1999b), such a loss function corresponds to a combination of inflation targeting and inflationary expectations targeting, where the target inflation rate is π^*, and the target expectation is the actual policy choice μ.

In equilibrium, the government thus chooses μ so that $\left(\partial TL/\partial\mu\right)=0$, where $TL=\int_0^\infty e^{\delta t}EL\left(t\right)dt$.

Solving for the optimal μ, we get:

$$\mu^* = \frac{\mu_0\left(AI\right)+\pi^*\left(B/\delta\right)}{AI+B/\delta} \text{, where } I=\int_0^\infty \frac{e^{-\delta t}}{\left(t+\sigma^2/\sigma_0^2\right)^2}dt. \tag{16}$$

A Bayesian Nash Equilibrium exists in which the government chooses a target inflation rate as described above. The population does not know this rate, but its inflationary expectations converge to the above rate. Note that with discounting, the equilibrium target rate is not the socially optimal inflation rate. In particular, in a hyperinflationary situation, the government will choose a target rate that is higher than optimal (a lower rate would not be credible and would generate losses due to sticky expectations). With no discounting, the optimal policy is to choose the ideal inflation rate, and this is the same result as Taylor's similar model.

IV. Monetary policy commitment to a random target

We have seen from the above discussion that the government is constrained to choosing a sub-optimal target inflation rate in a situation in which its commitment is not credible. However, if we could manage to accelerate the rate at which the public changes its expectations, we would be able to draw closer to the socially optimal level of inflation. As we discussed in the introduction, the concept behind such an acceleration of expectations is the use of randomness as a proxy for credibility. The intuition derives from the fact that errors in implementation or adverse shocks under a traditional target can be misinterpreted by the population as changes in policy. These mistakes can result in reductions in credibility and long delays in the convergence to low inflation.

In this section, we model this possibility under the assumption that the random process for the target inflation rate follows a diffusion process. For completeness and as a reality check on the results, in the simulation section below, we also consider the case of a white-noise process.

A. The Government (and Central Bank)

As before, the government chooses at time $t = 0$ some target inflation rate μ. However, at each time, the government "randomizes" its target around μ. More precisely, the government chooses an initial target rate and specifies that it evolves according to a random walk with variance ω^2. Thus the target can now be written as $\mu(t)$, evolving over time. Now, inflation is generated according to the same rule as before, but it is more volatile, because both the government and the state of the economy add uncertainty regarding the final observed inflation rate level. We now have:

$$d\mu(t) = \omega dw \text{, and } \mu(0) = \mu \text{, and} \tag{17}$$

$$dz(t) = \mu(t)dt + \sigma dv. \tag{18}$$

Here, $v(t)$ and $w(t)$ are standard Wiener processes with a zero mean and unit variance, independent of each other.

B. The public

The public again attempts to "guess" the full-information expected inflation rate $\mu(t)$, based on its initial assumptions and on the observed inflation rate (an imperfect signal of the true target rate). The public's information set at time *t* consists of the actual inflation rate, and of the parameters σ and ω.

One can show that the population's inflationary expectations are given by the following stochastic differential equation:

$$dx(t) = \alpha \frac{Ce^{2\alpha t} - 1}{Ce^{2\alpha t} + 1}\left[dz(t) - x(t)dt\right] \text{, where } \alpha = \omega/\sigma \text{ and } C = \frac{\omega\sigma + \sigma_0^2}{\omega\sigma - \sigma_0^2}. \tag{19}$$

Again, since this equation will generate stochastic solutions, we look for the (deterministic) paths of the mean and variance of the inflationary expectations:

$$d\bar{x}(t) = \left[-\alpha \frac{Ce^{2\alpha t} - 1}{Ce^{2\alpha t} + 1}\bar{x}(t) + \mu(t)\alpha \frac{Ce^{2\alpha t} - 1}{Ce^{2\alpha t} + 1}\right]dt \text{, with } \bar{x}(0) = \mu_0 \text{, and} \tag{20}$$

$$d\tilde{x}(t) = \left[-2\alpha \frac{Ce^{2\alpha t} - 1}{Ce^{2\alpha t} + 1}\tilde{x}(t) + \alpha^2 \left(\frac{Ce^{2\alpha t} - 1}{Ce^{2\alpha t} + 1}\right)^2 \left(\sigma^2 + \omega^2\right)\right]dt \text{, with } \tilde{x}(0) = \sigma_0^2. \tag{21}$$

Solving the above equations for the mean and variance of the inflation expectations we get:

$$\bar{x}(t) = \mu(t) + [\mu_0 - \mu(t)] \frac{2\alpha}{(\alpha+\beta)e^{\alpha t} + (\alpha-\beta)e^{-\alpha t}} \text{ , where } \alpha = \frac{\omega}{\sigma} \text{ and } \beta = \frac{\sigma_0^2}{\sigma^2}\text{; and} \tag{22}$$

$$\tilde{x}(t) = \frac{\alpha}{2}(1+\alpha^2) + \frac{2\alpha^2(1-\alpha^2)\beta - \alpha(1+\alpha^2)(\alpha^2-\beta^2)(1+2\alpha t) - \alpha(1+\alpha^2)(\alpha-\beta)^2 e^{-2\alpha t}}{[(\alpha+\beta)e^{\alpha t} + (\alpha-\beta)e^{-\alpha t}]^2}. \tag{23}$$

C. The loss function and the Bayesian equilibrium

Again, intuitively, a fast convergence without the benefit of the monetary authority's full credibility is particularly useful in emerging market transition situations. The loss function is described by (13): $L(t) = A[x(t) - \mu(t)]^2 + B[\pi(t) - \pi^*]^2$, and the expected loss function by (14): $EL(t) = A\{E[x(t)] - \mu(t)\}^2 + B\{E[(\pi(t)] - \pi^*\}^2 + \text{var}_A[x(t)] + \text{var}_B[\pi(t)]$.

Here, $E[x(t)] = \bar{x}(t)$; $E[\pi(t)] = \mu(t)$; $\text{var}[x(t)] = \tilde{x}(t)$ and $\text{var}[\pi(t)] = \omega^2 + \sigma^2$ [10] It follows that the expected loss function can be re-written as:

$$EL(t) = A[\bar{x}(t) - \mu(t)]^2 + B[\mu(t) - \pi^*]^2 + A\tilde{x}(t) + B(\omega^2 + \sigma^2). \tag{24}$$

Now, both $\mu(0)$ and ω are choice variables, and the government thus chooses them so that

$$\frac{\partial TL}{\partial \mu} = \frac{\partial TL}{\partial \omega} = 0 \text{ , where } TL = \int_0^\infty e^{-\delta t} EL(t)\, dt. \tag{25}$$

For simplicity, let's denote the ratio of ω/σ by α. Since σ is a constant, choosing ω is the same as choosing α. Solving (25) for $\mu(t)$ while holding ω (and thus α.) temporarily constant we get the optimal target as:

$$\mu^* = \frac{\mu_0(AI) + \pi^*(B/\delta)}{AI + B/\delta}\text{, where } J \equiv \int_0^\infty \frac{4\alpha^2 e^{-\delta t}}{[(\alpha+\beta)e^{\alpha t} + (\alpha-\beta)e^{-\alpha t}]^2} dt,\ \alpha = \frac{\omega}{\sigma} \text{ and } \beta = \frac{\sigma_0^2}{\sigma^2}. \tag{26}$$

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[10] Note that this assumes a zero covariance between the random target Wiener process and that of the inflation process. There is no reason to assume otherwise, and the random process could in fact be constructed to follow such an assumption.

At this point, μ is a function of α. To solve the problem completely, we would need to determine the value of α > 0 for which the value of the total loss function is minimized. Lacking a closed form solution, we derive some intuitive properties of the optimal policy.

Our key result here is that expectations converge faster under the randomized target policy. This can be seen by inspection of equations (10) and (22). Recall that equation (10):

$$\bar{x}(t) = \mu + \frac{\mu_0 - \mu}{t + \sigma^2 / \sigma_0^2} \tag{10}$$

displays hyperbolic convergence to the target rate. Equation (22)

$$\bar{x}(t) = \mu(t) + \left[\mu_0 - \mu(t)\right] \frac{2\alpha}{(\alpha+\beta)e^{\alpha t} + (\alpha-\beta)e^{-\alpha t}} \tag{22}$$

shows exponential convergence. This result is apparently counterintuitive: by randomizing monetary policy, that is, creating further uncertainty for the public, the central bank allows the public converge faster to its true target rate.

The key to this apparent contradiction is that by randomizing the target, the central bank allows agents to condition their expectations on the existence of the randomization process. By doing so, responses to shocks are ameliorated and the public and converge to the true target more quickly. We have thus shown that convergence is faster in the random target case, indicating that the method may be useful in a context in which the central bank needs to overcome sticky expectations.

On the same grounds, it can be shown that $I(\sigma, \sigma_0, \delta) \geq J(\omega, \sigma, \sigma_0, \delta), \forall \omega > 0$. If we now define the optimal target described in (26) as $\mu^*(\alpha)$ and the optimal commitment target defined in (16) by μ^*_F, we notice that both $\mu^*(\alpha)$ and μ^*_F are weighted averages of μ_0 and π^*, with *J* and *I* as weights on the side of μ_0. But since *J* is lower than *I*, it follows that under random targeting a lower weight is put on the public's initial inflation guess. Consequently, the resulting optimal target is closer to the socially optimal π^* under random targeting than under fixed targeting.[11]

The final question is whether or not the total loss associated with implementing the optimal random target policy is lower than the loss under a fixed target, that is, whether or not $TL_{Random}\left[\mu^*(\omega^*)\right] \leq TL_{Fixed}\left(\mu^*_F\right)$. Since we cannot solve directly for ω^*, we cannot check directly whether this inequality holds in general (or for particular values of the parameters A, B, σ, σ_0, δ). However, the inequality seems intuitively possible. Indeed, consider that we start with a fixed target policy μ^*_F. If at this point we introduce some randomization of the policy (say a small ω), we can reduce the target rate to $\mu^*(\omega)$. *W*hat are the effects of this small change on the total loss function? First, there is the negative effect of generating more volatile inflation; and this effect is weighted by the factor *B*. There exist, however, two positive effects. On the one hand, we have brought the inflationary policy closer to the social optimum, which decreases the value of the loss (again scaled by the factor *B*). On the other hand, because randomization leads to faster convergence of the inflationary expectations, we have a second improvement in the loss function, scaled by the factor *A*. Whether the two positive effects dominate the negative effect or not will most likely depend on the parameters of the problem, but it is conceivable that in the case of "mild" randomization (when $\alpha = \omega/\sigma \cong 0$), the negative effect on the volatility of inflation will not be dominant.

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11 Note, incidentally, that this conclusion does not apply if there is no discounting of the future. If $\delta = 0$, both policies imply setting the target inflation rate to the optimal level π^*. It is easy to see why this is so: with no discounting of the future, the speed with which the convergence takes place is irrelevant compared with the level at which expectations converge. Thus, in order to prevent losses from some point *t* to infinity, expectations have to become arbitrarily close to π^*. This can only be achieved if the target inflation rate is set to π^* as well in both the fixed and the random scenarios.

Indeed, it is possible to show mathematically that this is the case under fairly un-restrictive conditions on the parameters of the problem. If the initial guess of the public is not accurate, so that $\sigma_0 > \sigma$ (with strict inequality), it can be shown that there exists a continuous range of small values of α (and thus ω) for which the total loss associated with an optimal randomized policy is *strictly* lower than the total loss associated with an optimal fixed target. This finding suggests that for situations characterized by a high degree of uncertainty, randomized targeting both decreases social losses and moves the economy closer to the optimal inflation rate.[12]

V. Some simulations and results

In this section we provide some basic illustrations of the theoretical results contained above. We simulate a policy game as described above with a 200 period path of inflation. In each period, given a set of initial parameters, we calculate expected inflation, mean squared error, and total loss from both the Taylor baseline and the two variations of the random policy (random walk and white noise). Our parameters are set as follows: recall that σ is the parameter on the inflation dispersion process, σ_0 is the public's initial expectation of inflation variance, ω is the government's choice of a dispersion parameter on the inflation target (or in the case of the white noise process, the variance), μ^* is defined in equation (26), and μ_0 is the public's initial guess of the government inflation target. We choose parameters as follows:

12 The case $\sigma_0 = \sigma$ is more difficult to treat mathematically. For this case, we have constructed total loss curves, graphing the total loss in the random targeting scenario as a function of the choice of α. The optimal α should be chosen as the point where these curves bottom; however, the shape of the curves depends highly on the other parameters of the problem. As a result,

TABLE I. Parameter values.

Parameter	Value
ω	0.1
σ	0.1
σ_0	0.01
μ^*	3%
μ_0	15%

Figure 1, below, shows a sample case that might be relevant for a high inflation country. In this circumstance, a change in government policy or other structural shift might lead the population to believe that the government has a target of about 15 percent. Note that the exercise is not sensitive to this choice. The figure shows a sample time path of inflation under the two cases. The fixed target and the initial value of the random one are set at 3 percent. The initial public guess of the target is 15 percent. One can see that under this parameterization, the random policy leads to a more rapid convergence to the target rate than the Taylor version. Figure 2 repeats the exercise for a white noise random process. It should be apparent that two achieve similar gains in convergence speed vis-à-vis the fixed version. An explicit comparison is available in Figure 3. The similarity of results comes in part from the relatively small value of omega. For larger values, the random walk process shows "divergence" after about 100 periods. The initial convergence is fast, but at the target itself starts to move, public expectations begin to oscillate as well. See Figure 4 for an illustration.

Figure 5 repeats the initial exercise given a government in a deflationary environment. Consider a public that expects an optimal inflation rate of -1 percent. A similar pattern of

depending on the particular values of the parameters, there may or may not exist values of α for which the total loss in random targeting is lower than the total loss in fixed targeting.

convergence can be seen. We can also consider the squared error and mean squared error for these cases. Figures 6 and 7 show the squared error and mean squared error respectively for the high inflation case (comparing the random walk and the fixed versions). Figures 8 and 9 show the same for the deflationary case.

Further exploration reveals a bit more about the properties of this system. The discussion above suggested that the presence of noise meant that implementation errors were not punished in the same way as in a fixed system. This allows the populations expectations to converge at a faster rate. We look at the convergence rate in a number of cases to explore the role of ω in determining the speed of expectations adjustment. Figure 10 compares the rate of convergence for the fixed target and the random walk target against the size of ω. Our metric for convergence is the number of periods to reach the target rate (with some small tolerance). One should be able to see that the rate of convergence for the random case, and thus its degree of advantage over the fixed case, is increasing (the number of periods to reach the target is falling) in the ω (and thus in α). Figure 11 shows the same set of rates comparing the fixed target to the random process generated by white noise. Again, the convergence is increasing in ω. Figure 12 compares only the random walk and white noise processes.

VI. Conclusions

Under rational expectations theory, central banks automatically achieve rapid convergence of public inflation beliefs to the actual rate of inflation. In many cases, this is not plausible monetary policy, given credibility constraints. If the public is ignorant of the parameters of the government's optimization problem, then the announcement of a new target will not be automatically credible. This may be particularly true in developing countries where, because of political instability, the central banks and/or government may minimize a private loss function that does not reflect the societal optimization. We find that in such situations, a government can achieve faster expectations convergence to an economy's optimal inflation rate through the use of a randomized inflation target.

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Appendix I:
Determination of the public's expected inflation path under a fixed inflation target

As indicated, the public wants to determine the true government inflation target μ. The public cannot observe μ directly, but instead observes a noisy signal (inflation) that is based on μ. The equation of the signal is: $d\left[\log\left(p\right)\right] = dz\left(t\right) = \mu dt + \sigma dv$.

The public thus observes $z\left(t\right)$, and must infer the value of μ. This is known as a filtering problem; it was solved in continuous time by Kalman and Bucy (Fleming and Rishel 1975, pp. 133-140). The public is assumed to start with an initial guess μ_0, and then to develop conditional expectations based on current knowledge and the observed signal. The expected value for μ, which we denote by *x(t)* follows the stochastic differential equation:

$$dx\left(t\right) = F\left(t\right)\left[dz\left(t\right) - x\left(t\right)dt\right], \text{ where } x\left(0\right) = E_{public}\left[\mu\left(0\right)\right] = \mu_0, \tag{A1}$$

and $F\left(t\right) = R\left(t\right)/\sigma^2$, where $R\left(t\right)$ follows the deterministic differential equation (known as the Ricatti equation):

$$dR\left(t\right) = -\frac{R\left(t\right)^2}{\sigma^2}dt \text{ with initial condition } R\left(0\right) = \sigma_0^2. \tag{A2}$$

Solving for *R*, we get: $R\left(t\right) = 1/\left(t/\sigma^2 + 1/\sigma^2\right)$, and thus $F\left(t\right) = 1/\left(t + \sigma^2/\sigma_0^2\right)$.

This implies that the public's expectations follow the stochastic differential equation:

$$dx\left(t\right) = \frac{1}{t + \sigma^2/\sigma_0^2}\left[dz\left(t\right) - x\left(t\right)dt\right]. \tag{A3}$$

Solving for *x* would generate a solution with a deterministic part and a stochastic part, because *x* must be a stochastic variable. Such a solution would not be particularly interesting, because the government is mostly interested in the expected mean and variance of the population's expectations. Let us define:

$$\begin{aligned} \bar{x}\left(t\right) &= E_{government}\left[x(t)\right] \\ \tilde{x}\left(t\right) &= \text{var}_{government}\left[x\left(t\right)\right] = E_{government}\left[x\left(t\right)^2\right] - \bar{x}\left(t\right)^2. \end{aligned} \tag{A4}$$

The government also knows μ, so it can rewrite (A3) as:

$$dx\left(t\right) = \frac{1}{t + \sigma^2/\sigma_0^2}\left[\mu dt + \sigma dv - x\left(t\right)dt\right], \text{ or:} \tag{A5}$$

$$dx(t) = \left[-\frac{1}{t + \sigma^2/\sigma_0^2}x(t) + \frac{\mu}{t + \sigma^2/\sigma_0^2}\right]dt + \frac{\sigma}{t + \sigma^2/\sigma_0^2}dv. \tag{A6}$$

Based on Arnold (1973, pp. 130), we know that the mean and variance of *x* follow the deterministic differential equations:

$$d\bar{x}\left(t\right) = \left[-\frac{1}{t + \sigma^2/\sigma_0^2}\bar{x}\left(t\right) + \frac{\mu}{t + \sigma^2/\sigma_0^2}\right]dt, \text{ with } \bar{x}\left(0\right) = \mu_0 \quad \text{, and} \tag{A7}$$

$$d\tilde{x}\left(t\right) = \left[-\frac{2}{t + \sigma^2/\sigma_0^2}\tilde{x}\left(t\right) + \frac{\sigma^2}{\left(t + \sigma^2/\sigma_0^2\right)^2}\right]dt, \text{ with } \tilde{x}\left(0\right) = \sigma_0^2. \tag{A8}$$

Solving the above equations for the mean and variance of the inflation expectations we get:

$$\bar{x}(t) = \mu + \frac{\mu_0 - \mu}{t + \sigma^2/\sigma_0^2} \text{ : the mean inflationary expectations; and} \tag{A9}$$

$$\tilde{x}(t)=\frac{\sigma^2}{t+\sigma^2/\sigma_0^2}\text{ : the variance of the inflationary expectations.}\tag{A10}$$

In equilibrium, the population forms rational expectations based on a Bayesian rule. These expectations are described by (A3). The government knows (A3), and thus derives the deterministic paths of the expectations' mean and variance, (A9) and (A10), respectively. Based on these paths, and on its own loss function, the government finally chooses the optimal commitment level μ^*.

Appendix II:

Determination of the optimal government commitment with a fixed target

As justified in the text, the expected loss associated with monetary policy is given by:

$$EL(t)=A\left[\bar{x}(t)-\mu(t)\right]^2+B\left[\mu(t)-\pi^*\right]^2+A\tilde{x}(t)+B\sigma^2 .\tag{A11}$$

The total loss is the integral of the (discounted) losses from the introduction of the policy to infinity:

$$TL=A(\mu-\mu_0)^2\int_0^\infty\frac{e^{-\delta t}}{\left(t+\sigma^2/\sigma_0^2\right)^2}dt+B\left[\mu-\pi^*\right]^2\int_0^\infty e^{-\delta t}dt+A\sigma^2\int_0^\infty\frac{e^{-\delta t}}{t+\sigma^2/\sigma_0^2}dt+B\sigma^2\int_0^\infty e^{-\delta t}dt .\tag{A12}$$

The government then chooses μ to minimize the total loss, by setting the first derivative of the total loss with respect to μ to zero. (It is clear from the below expression that the second derivative is positive, so this is indeed a minimum).

$$\frac{\partial TL}{\partial\mu}=2A(\mu-\mu_0)\int_0^\infty\frac{e^{-\delta t}}{\left(t+\sigma^2/\sigma_0^2\right)^2}dt+2B\left(\mu-\pi^*\right)/\delta .\tag{A13}$$

Let us define: $I=\int_0^\infty\left[e^{-\delta t}/\left(t+\sigma^2/\sigma_0^2\right)\right]dt$, which is a function of δ, σ, and σ_0, but not μ. Now, setting the first derivative of the total loss with respect to the target rate to zero, we get:

$$\partial TL/\partial\mu=2A(\mu-\mu_0)I+2B\left(\mu-\pi^*\right)/\delta=0\rightarrow\mu\left(AI+B/\delta\right)=\mu_0AI+\pi^*B/\delta .$$

Finally, we obtain:

$$\mu^*=\frac{\mu_0(AI)+\pi^*(B/\delta)}{AI+B/\delta}\text{ , where }I=\int_0^\infty\frac{e^{-\delta t}}{\left(t+\sigma^2/\sigma_0^2\right)^2}dt .\tag{A14}$$

Appendix III:

Determination of the public's expected inflation path under random targeting

As indicated in the text, the public wants to determine the true government inflation target μ. As this target varies over time, we will use $\mu(t)$ throughout this appendix. The public knows that $\mu(t)$ is generated randomly so that $d\mu(t)=\omega dw$, but cannot observe $\mu(t)$ directly. The public instead observes a noisy signal (inflation), which is based on $\mu(t)$. The equation of the signal is:

$$d\left[\log(p)\right]=dz(t)=\mu(t)dt+\sigma dv .\tag{A15}$$

The public thus observes *z(t)*, and must infer the value of *μ(t)*. The Kalman-Bucy solution for the filtering problem is described by the stochastic differential equation:

$$dx(t)=F(t)[dz(t)-x(t)dt]\text{, where }x(0)=E_{public}\left[\mu(0)\right]=\mu_0 .\tag{A16}$$

As before, $F(t)=R(t)/\sigma^2$, and *R(t)* follows the deterministic Ricatti equation:

$$dR(t)=\left[-\frac{R(t)^2}{\sigma^2}+\omega^2\right]dt \text{ with initial condition } R(0)=\sigma_0^2 . \tag{A17}$$

Solving for *R*, we get:

$$R(t)=\sigma\omega\frac{Ce^{2\alpha t}-1}{Ce^{2\alpha t}+1} \text{ , where } \alpha=\omega/\sigma \text{ and } C=\frac{\omega\sigma+\sigma_0^2}{\omega\sigma-\sigma_0^2}, \tag{A18}$$

and thus:

$$F(t)=\alpha\frac{Ce^{2\alpha t}-1}{Ce^{2\alpha t}+1} \text{ , where } \alpha=\omega/\sigma \text{ and } C=\frac{\omega\sigma+\sigma_0^2}{\omega\sigma-\sigma_0^2}. \tag{A19}$$

This implies that the public's expectations follow the stochastic differential equation:

$$dx(t)=\alpha\frac{Ce^{2\alpha t}-1}{Ce^{2\alpha t}+1}[dz(t)-x(t)dt]. \tag{A20}$$

Let us define as before:

$$\begin{aligned}\bar{x}(t)&=E_{government}\left[x(t)\right]\\ \tilde{x}(t)&=\mathrm{var}_{government}\left[x(t)\right]=E_{government}\left[x(t)^2-\bar{x}(t)\right].\end{aligned} \tag{A21}$$

Rewriting (A20) based on (A15) and the definition of $\mu(t)$, we obtain:

$$dx(t)=\alpha\frac{Ce^{2\alpha t}-1}{Ce^{2\alpha t}+1}\left[\mu(t)dt+\sigma dv-x(t)dt\right] \text{, or:} \tag{A22}$$

$$dx(t)=\left[-\alpha\frac{Ce^{2\alpha t}-1}{Ce^{2\alpha t}+1}x(t)+\mu\frac{Ce^{2\alpha t}-1}{Ce^{2\alpha t}+1}\right]dt+\sigma\alpha\frac{Ce^{2\alpha t}-1}{Ce^{2\alpha t}+1}dv \tag{A23}$$

Based on Arnold (1973) the mean and variance of *x* follow the deterministic differential equations:

$$d\bar{x}(t)=\left[-\alpha\frac{Ce^{2\alpha t}-1}{Ce^{2\alpha t}+1}\bar{x}(t)+\mu(t)\alpha\frac{Ce^{2\alpha t}-1}{Ce^{2\alpha t}+1}\right]dt \text{, with } \bar{x}(0)=\mu_0 \text{, and} \tag{A24}$$

$$d\tilde{x}(t)=\left[-2\alpha\frac{Ce^{2\alpha t}-1}{Ce^{2\alpha t}+1}\tilde{x}(t)+\alpha^2\left(\frac{Ce^{2\alpha t}-1}{Ce^{2\alpha t}+1}\right)^2\left(\sigma^2+\omega^2\right)\right]dt \text{, with } \tilde{x}(0)=\sigma_0^2 . \tag{A25}$$

Solving the above equations for the mean and variance of the inflation expectations we get:

$$\bar{x}(t)=\mu(t)+\left[\mu_0-\mu(t)\right]\frac{2\alpha}{(\alpha+\beta)e^{\alpha t}+(\alpha-\beta)e^{-\alpha t}} \text{ , where } \alpha=\frac{\omega}{\sigma} \text{ and } \beta=\frac{\sigma_0^2}{\sigma^2} \text{, and} \tag{A26}$$

$$\tilde{x}(t)=\frac{\alpha}{2}\left(1+\alpha^2\right)+\frac{2\alpha^2\left(1-\alpha^2\right)\beta-\alpha\left(1+\alpha^2\right)\left(\alpha^2-\beta^2\right)\left(1+2\alpha t\right)-\alpha\left(1+\alpha^2\right)\left(\alpha-\beta\right)^2e^{-2\alpha t}}{\left[(\alpha+\beta)e^{\alpha t}+(\alpha-\beta)e^{-\alpha t}\right]^2}. \tag{A27}$$

Appendix IV: Determination of the optimal government commitment with a random target

As before, the expected loss at each time *t* is given by:

$$EL(t)=A\left[\bar{x}(t)-\mu(t)\right]^2+B\left[\mu(t)-\pi^*\right]^2+A\tilde{x}(t)+B\left(\sigma^2+\omega^2\right). \tag{A28}$$

It follows that the total loss is

$$TL = A\left(\mu(t)-\mu_0\right)^2 \int_0^\infty \frac{4\sigma^2\omega^2 e^{-\delta t}}{\left[\left(\sigma\omega+\sigma_0^2\right)e^{\alpha t}+\left(\sigma\omega-\sigma_0^2\right)e^{-\alpha t}\right]^2} dt + B\left[\mu(t)-\pi^*\right]^2 \int_0^\infty e^{-\delta t} dt + A\int_0^\infty e^{-\delta t}\tilde{x}(t) dt + B\left(\sigma^2+\omega^2\right)\int_0^\infty e^{-\delta t} dt \quad \text{(A29)}$$

To optimize governmental policy, let us first determine the best monetary target under the temporary assumption that ω is constant (that is, not a choice variable). This is achieved by setting the derivative of the total loss function with respect to the target rate to zero:

$$\frac{\partial TL}{\partial \mu} = 2A\left[\mu(t)-\mu_0\right]\int_0^\infty \frac{\left(4\sigma^2\omega^2\right)e^{-\delta t}}{\left[\left(\sigma\omega+\sigma_0^2\right)e^{\alpha t}+\left(\sigma\omega-\sigma_0^2\right)e^{-\alpha t}\right]^2} dt + 2B\left[\mu(t)-\pi^*\right]\big/\delta \,.$$

(A30)

Let us define: $J(\omega) \equiv \int_0^\infty \left[\left(4\sigma^2\omega^2\right)e^{-\delta t}\right]\Big/\left[\left(\sigma\omega+\sigma_0^2\right)e^{\alpha t}+\left(\sigma\omega-\sigma_0^2\right)e^{\alpha t}\right]^2 dt$, which again does not depend on $\mu(t)$. Now, setting the first derivative of the total loss with respect to the target rate to zero, we get:

$$\begin{aligned}\partial TL/\partial \mu J(t) &\equiv \int_0^\infty \left[\left(4\sigma^2\omega^2\right)e^{-\delta t}\right]\Big/\left[\left(\sigma\omega+\sigma_0^2\right)e^{\alpha t}+\left(\sigma\omega-\sigma_0^2\right)e^{\alpha t}\right]^2 dt \\ &= 2A\left(\mu(t)-\mu_0\right)J + 2B\left[\mu(t)-\pi^*\right]\big/\delta = 0 \rightarrow \mu(t)\left(AJ+B/\delta\right) = \mu_0 AJ + \pi^* B/\delta\end{aligned}.$$

Finally, we obtain

$$\mu^*(\omega) = \frac{\mu_0\left[AJ(\omega)\right]+\pi^*\left(B/\delta\right)}{AJ(\omega)+B/\delta} \quad \text{,where} \quad \text{(A31)}$$

$$J \equiv \int_0^\infty \frac{4\alpha^2 e^{-\delta t}}{\left[(\alpha+\beta)e^{\alpha t}+(\alpha-\beta)e^{-\alpha t}\right]^2} dt,\ \alpha = \frac{\omega}{\sigma} \text{ and } \beta = \frac{\sigma_0^2}{\sigma^2}.$$

In equilibrium, the government should choose a target inflation rate described by (A31) for any particular (predetermined) level of randomization ω. Subsequently, the government should choose the particular level of randomization ω^* that minimizes the total loss function (A29). Due to the complexities of the equations involved, a closed solution for ω^* is not available.

Appendix V

Comparison of expectations convergence speed under fixed and random targets

We attempt to show that the (expected) convergence of the expectations towards the true target rate is faster under random targeting (A26) than under fixed targeting (A9). Let $f(t) = e^{\alpha t}, t \in \Re$ be an exponential function, with $\alpha = \omega/\sigma$ as above. Since f is convex, we have from Jensen's inequality:

$$\begin{aligned}&s\cdot f(t_1)+(1-s)\cdot f(t_2) \geq f\left[s\cdot t_1+(1-s)\cdot t_2\right],\\ &\forall t_1, t_2, s \in \Re\end{aligned} \quad \text{(A32)}$$

In particular, let us set: $t_1 = t$; $t_2 = -t$, and $s = (\varpi\omega+\sigma_0^2)/2\sigma\omega$. From the above inequality (A32), we get:

$$\frac{\sigma\omega+\sigma_0^2}{2\sigma\omega}e^{\alpha t} + \frac{\sigma\omega-\sigma_0^2}{2\sigma\omega}e^{-\alpha t} \geq e^{\alpha\frac{\sigma_0^2}{\sigma\omega}t} = e^{\frac{\sigma_0^2}{\sigma^2}t}. \quad \text{(A33)}$$

But for any value q, we know that $e^q \geq 1+q$. In particular:

$$e^{\frac{\sigma_0^2}{\sigma^2}t} \geq 1+\frac{\sigma_0^2}{\sigma^2}t = \frac{\sigma_0^2}{\sigma^2}\left(t+\frac{\sigma^2}{\sigma_0^2}\right). \quad \text{(A34)}$$

Based on the fact that σ corresponds to the maximum inflation generation precision, it follows that:

$$\sigma \le \sigma_0, \text{ and so we also have: } \frac{\sigma_0^2}{\sigma^2} \ge 1. \tag{A35}$$

Combining (A33), (A34) and (A35), we get:

$$\frac{\sigma\omega + \sigma_0^2}{2\sigma\omega} e^{\alpha t} + \frac{\sigma\omega - \sigma_0^2}{2\sigma\omega} e^{-\alpha t} \ge t + \frac{\sigma^2}{\sigma_0^2}. \tag{A36}$$

Inverting, we obtain:

$$\left(\frac{\sigma\omega + \sigma_0^2}{2\sigma\omega} e^{\alpha t} + \frac{\sigma\omega - \sigma_0^2}{2\sigma\omega} e^{-\alpha t} \right)^{-1} \le \left(t + \frac{\sigma^2}{\sigma_0^2} \right)^{-1}. \tag{A37}$$

Multiplying by $\mu_0 - \mu(t)$ we obtain:

$$\frac{2\sigma\omega\left(\mu_0 - \mu(t)\right)}{\left(\sigma\omega + \sigma_0^2\right)e^{\alpha t} + \left(\sigma\omega - \sigma_0^2\right)e^{-\alpha t}} \le \frac{\mu_0 - \mu(t)}{t + \dfrac{\sigma^2}{\sigma_0^2}}, \text{ iff } \mu(t) < \mu_0. \text{ And,} \tag{A38a}$$

$$\frac{2\sigma\omega\left[\mu_0 - \mu(t)\right]}{\left(\sigma\omega + \sigma_0^2\right)e^{\alpha t} + \left(\sigma\omega - \sigma_0^2\right)e^{-\alpha t}} \ge \frac{\mu_0 - \mu(t)}{t + \dfrac{\sigma^2}{\sigma_0^2}}, \text{ iff } \mu(t) > \mu_0. \tag{A38b}$$

The above equations, together with the equations (A26) and (A9), show that the convergence of $\bar{x}$ towards μ is always faster under random targeting than under fixed rate targeting. Moreover, if we compare (A14) and (A31), with (A37), we find that the weighting factor *I* that appears in the fixed rate optimization problem is always higher than the factor *J* that appears in the random rate optimization. Indeed, squaring (A37) and using the definitions of *I* and *J*, we obtain that $I \ge J(\omega), \forall \omega > 0$. In particular, this implies that the optimal target under randomization will be always closer to the socially desired level π^* than will the optimal rate under a fixed target policy.

Appendix VI

Comparison of total losses under fixed and random targets

We have shown that optimality in the fixed and randomized targeting cases requires the government to choose μ_F^* and α^*, μ_R^* (α^*) respectively. These choices generate respective values of the loss function of $TL*_F = TL(\mu_F *)$ and $TL*_F = TL\left[\mu_R *(\alpha)\right]$. Below, we establish a sufficient (but not necessary) condition for $TL_R *$ to be lower than $TL_F *$. For simplicity, calculations below are carried out using the symbols $\alpha = \omega / \sigma$ and $\beta = \sigma_0^2 / \sigma^2$.

Lemma 1: Under random targeting, $\lim_{\alpha \to 0} \tilde{x}(t) = \sigma^2 \left[\beta(1+t) \big/ (1+\beta t)^2 \right], \forall t > 0$.

Proof of Lemma 1*:* Starting with the formula for $\tilde{x}(t)$ in (A27), we apply the L'Hospital rule twice, since α is the only variable factor. The fact that the resulting limit exists and is finite proves the lemma (for clarity of exposition, actual calculations are skipped here).

Lemma 2*:* Let μ_F^* be the optimal inflation target chosen under fixed targeting, and $TL*_F = TL(\mu_F *)$ the total loss associated with it. Also, for a given $\alpha > 0$, let μ_R^* (α) be the optimal inflation target chosen under random targeting, and $TL_R *(\alpha) = TL\left[\mu_R *(\alpha)\right]$. Then, the following relation holds: $\lim_{\alpha \to 0} TL_R \left[\mu_R^*(\alpha)\right] \le TL_F^*$, with equality iff $\beta = 1$.

Proof of Lemma 2: We know that

$$TL_F - TL_R\left(\mu_R^*(\alpha)\right) =$$

$$A(\mu-\mu_0)^2\int_0^\infty \frac{e^{-\delta t}}{\left(t+\sigma^2/\sigma_0^2\right)^2}dt + B\left[\mu-\pi^*\right]^2\int_0^\infty e^{-\delta t}dt + A\sigma^2\int_0^\infty \frac{e^{-\delta t}}{t+\sigma^2/\sigma_0^2}dt + B\sigma^2\int_0^\infty e^{-\delta t}dt -$$

$$-A\left(\mu(t)-\mu_0\right)^2\int_0^\infty \frac{4\sigma^2\omega^2 e^{-\delta t}}{[\left[\left(\sigma\omega+\sigma_0^2\right)e^{\alpha t}+\left(\sigma\omega-\sigma_0^2\right)e^{-\alpha t}\right]^2}dt - B\left[\mu(t)-\pi^*\right]^2\int_0^\infty e^{-\delta t}dt - A\int_0^\infty e^{-\delta t}\tilde{x}(t)dt - B\left(\sigma^2+\omega^2\right)\int_0^\infty e^{-\delta t}dt$$

. (A39)

In the above equality, let us take $\alpha \to 0$, which also implies $\omega \to 0$. First of all, from the derivation in Appendix V (or simply applying the l'Hospital rule twice) we have that:

$$\lim_{\alpha\to 0} A(\mu-\mu_0)^2\int_0^\infty \frac{4\sigma^2\omega^2 e^{-\delta t}}{\left[\left(\sigma\omega+\sigma_0^2\right)e^{\alpha t}+\left(\sigma\omega-\sigma_0^2\right)e^{-\alpha t}\right]^2}dt = A(\mu-\mu_0)^2\int_0^\infty \left\{\lim_{\alpha\to 0}\frac{4\sigma^2\omega^2 e^{-\delta t}}{\left[\left(\sigma\omega+\sigma_0^2\right)e^{\alpha t}+\left(\sigma\omega-\sigma_0^2\right)e^{-\alpha t}\right]^2}\right\}dt =$$

$$= A(\mu-\mu_0)^2\int_0^\infty \frac{e^{-\delta t}}{\left(t+\sigma^2/\sigma_0^2\right)^2}dt. \quad \text{(A39a)}$$

Second, it is trivial to see that:

$$\lim_{\alpha\to 0} B\left(\sigma^2+\omega^2\right)\int_0^\infty e^{-\delta t}dt = B\left(\sigma^2+\lim_{\alpha\to 0}\omega^2\right)\int_0^\infty e^{-\delta t}dt = B\sigma^2\int_0^\infty e^{-\delta t}dt. \quad \text{(A40)}$$

Using the above (A39) and (A40) results, we obtain:

$$TL_F - \lim_{\alpha\to 0} TL_R\left[\mu_R^*(\alpha)\right] = A\sigma^2\int_0^\infty \frac{e^{-\delta t}}{t+\sigma^2/\sigma_0^2}dt - A\int_0^\infty e^{-\delta t}\left[\lim_{\alpha\to 0}\tilde{x}(t)\right]dt. \quad \text{(A41)}$$

Based on Lemma 1, we can simplify further:

$$TL_F - \lim_{\alpha\to 0} TL_R\left[\mu_R^*(\alpha)\right] = A\sigma^2\int_0^\infty e^{-\delta t}\left[\frac{1}{t+1/\beta} - \frac{\beta(1+t)}{(1+\beta t)^2}\right]dt = A\sigma^2\int_0^\infty e^{-\delta t}\left[\frac{\beta}{1+\beta t} - \frac{\beta(1+t)}{(1+\beta t)^2}\right]dt =$$

$$= A\sigma^2\int_0^\infty e^{-\delta t}\left[\frac{\beta+\beta^2 t-\beta-\beta t}{(1+\beta t)^2}\right]dt = A\sigma^2\int_0^\infty e^{-\delta t}\frac{\beta t(\beta-1)}{(1+\beta t)^2}dt. \quad \text{(A42)}$$

Since by construction $\beta \ge 1$, it follows that $A\sigma^2\int_0^\infty e^{-\delta t}\left[\beta t(\beta-1)\right]/(1+\beta t)^2\, dt \ge 0$, with equality iff $\beta = 1$.

Combining this with (A42), we reach the desired conclusion, namely, that:

$$\lim_{\alpha\to 0} TL_R\left[\mu_R^*(\alpha)\right] \le TL_F^*, \text{ with equality iff } \beta = 1. \quad \text{(A43)}$$

Lemma 3: With the same notation as above, $\lim_{\alpha\to\infty} TL_R\left[\mu_R^*(\alpha)\right] = \infty$.

Proof of Lemma 3: The result is derived trivially by passing α to infinity in (A28) and noticing that the first three terms are all positive and that the last one is infinite.

Theorem 1: If $\sigma_0 > \sigma$, that is, if $\beta > 1$, then $\exists\, \overline{\omega} > 0$ (and thus $\overline{\alpha} = \overline{\omega}/\sigma$), such that

$$TL_R\left[\mu_R^*(\alpha)\right] < TL_F \,\forall \alpha \in \left(0, \overline{\alpha}\right).$$

Proof of Theorem 1: Since $\beta > 1$, we know from Lemma 2 that $\lim_{\alpha \to 0} TL_R\left[\mu_R^*(\alpha)\right] < TL_F^*$, This implies that there exists a neighborhood of zero, say, $N = (0, 2\varepsilon)$ with $\varepsilon > 0$, such that:

$$TL_R\left[\mu_R^*(\alpha)\right] < TL_F^*, \forall \varepsilon \in N = (0, 2\varepsilon). \tag{A44}$$

In particular, it follows that:

$$\exists \varepsilon > 0 \therefore TL_R\left[\mu_R^*(\varepsilon)\right] < TL_F^*. \tag{A44a}$$

From Lemma 3, we know that $\lim_{\alpha \to \infty} TL_R\left[\mu_R^*(\alpha)\right] = \infty$. This implies that there exists a neighborhood of infinity, say $N' = (\delta/2, \infty)$ with $\delta > \varepsilon > 0$, such that:

$$TL_R\left[\mu_R^*(\alpha)\right] < TL_F^*, \forall \varepsilon \in N' = (\delta/2, \infty). \tag{A45}$$

In particular, it follows that:

$$\exists \delta > \varepsilon \therefore TL_R\left[\mu_R^*(\delta)\right] > TL_F^* \tag{A45a}$$

The expressions (A44a) and (A45a) show that the function $TL_R\left[\mu_R^*(\cdot)\right]$ takes values both below and above TL^*_F. But $TL_R\left[\mu_R^*(\alpha)\right]$ is a continuous function of α, and thus (A44a) and (A45a) guarantee that:

$$\exists \hat{\alpha} \in (\varepsilon, \delta) \therefore TL_R\left[\mu_R^*(\hat{\alpha})\right] = TL_F. \tag{A46}$$

Let us now define the set A as follows: $A = \{x > 0 \mid TL_R\left[\mu_R^*(x)\right] = TL_F^*$. Expression (A46) guarantees that the set A is non-empty. Let $\bar{\alpha} = \inf_x A$. We can easily see that $\bar{\alpha} > 0$. Indeed, if this were not the case, there would exist points x arbitrarily close to zero such that $TL_R\left[\mu_R^*(x)\right] = TL_F$. But this contradicts (A45), which says that there exists an entire neighborhood of zero with points for which the relation above is not true. We contend now that the $\bar{\alpha} > 0$ as defined above has the desired property: $TL_R\left[\mu_R^*(\alpha)\right] < TL_F^* \forall \varepsilon \in (0, \bar{\alpha})$. Indeed, suppose by contradiction that this were not the case, that is,

$$\exists \theta \in (0, \bar{\alpha}) \therefore TL_R\left[\mu_R^*(\theta)\right] > TL_F \tag{A47}$$

From (A45) we know that $TL_R\left[\mu_R^*(\cdot)\right]$ must take a value lower than TL^*_F in $(0, \bar{\alpha})$. If (A47) were true, then the function would also take a value greater than TL^*_F. By reason of continuity of the function $TL_R\left[\mu_R^*(\cdot)\right]$, we conclude that $\exists \gamma \in (0, \bar{\alpha}) \therefore TL_R\left[\mu_R^*(\gamma)\right] = TL_F^*$. It follows that $\gamma \in A$ and thus, by construction, $\gamma \geq \bar{\alpha} = \inf_x A$. But this contradicts the fact that by construction $\gamma \in (0, \bar{\alpha})$. It follows that (A47) cannot be true, and thus we conclude that:

$$\exists \bar{\alpha} > 0 \therefore TL_R\left[\mu_R^*(\alpha)\right] < TL_F^* \forall \in (0, \bar{\alpha}) \tag{A48}.$$

Figure 1: Inflation Vs. Time: High Inflation Case (Random Walk Target)

The below figure plots the time paths of expected inflation under the random walk target and under a 'traditional' fixed inflation target. The dotted line (pink) shows the path of inflation under the fixed target and the solid line (blue) under the random one. The fixed target and the initial value of the random walk target are both set to 3 percent. The public's initial guess in each case is set to 15 percent. The parameters for the calculation are as follows: $\omega = 0.1$, $\sigma = 0.1$, $\sigma_0 = 0.01$, $\mu^* = 3$, and $\mu_0 = 15$.

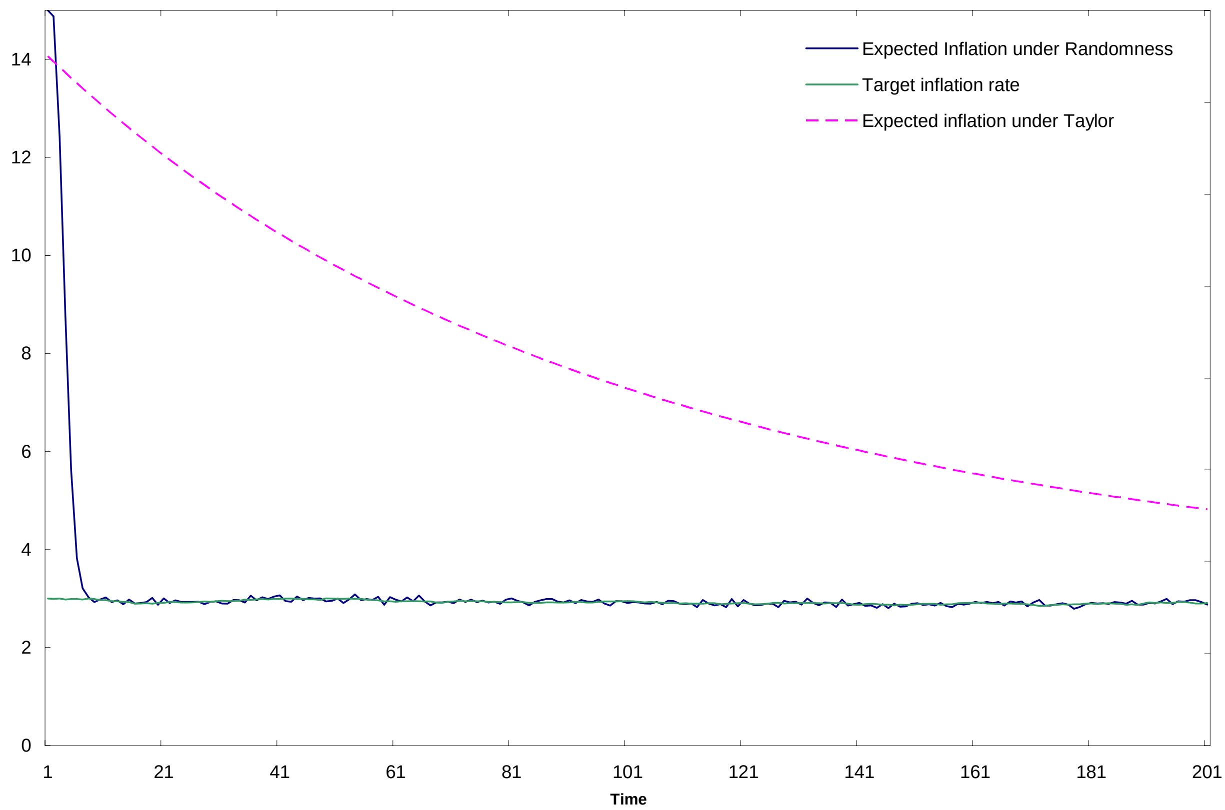

Figure 2: Inflation Vs. Time: with White Noise Random Process

The below figure plots the time paths of expected inflation under the white noise random target and under a 'traditional' fixed inflation target. The dotted line (pink) shows the path of inflation under the fixed target and the solid line (blue) under the random one. The fixed target and the initial value of the white noise target are both set to 3 percent. The public's initial guess in each case is set to 15 percent. The parameters for the calculation are as follows: ω = 0.1, σ = 0.1, σ_0 = 0.01, μ^* = 3, and μ_0 = 15.

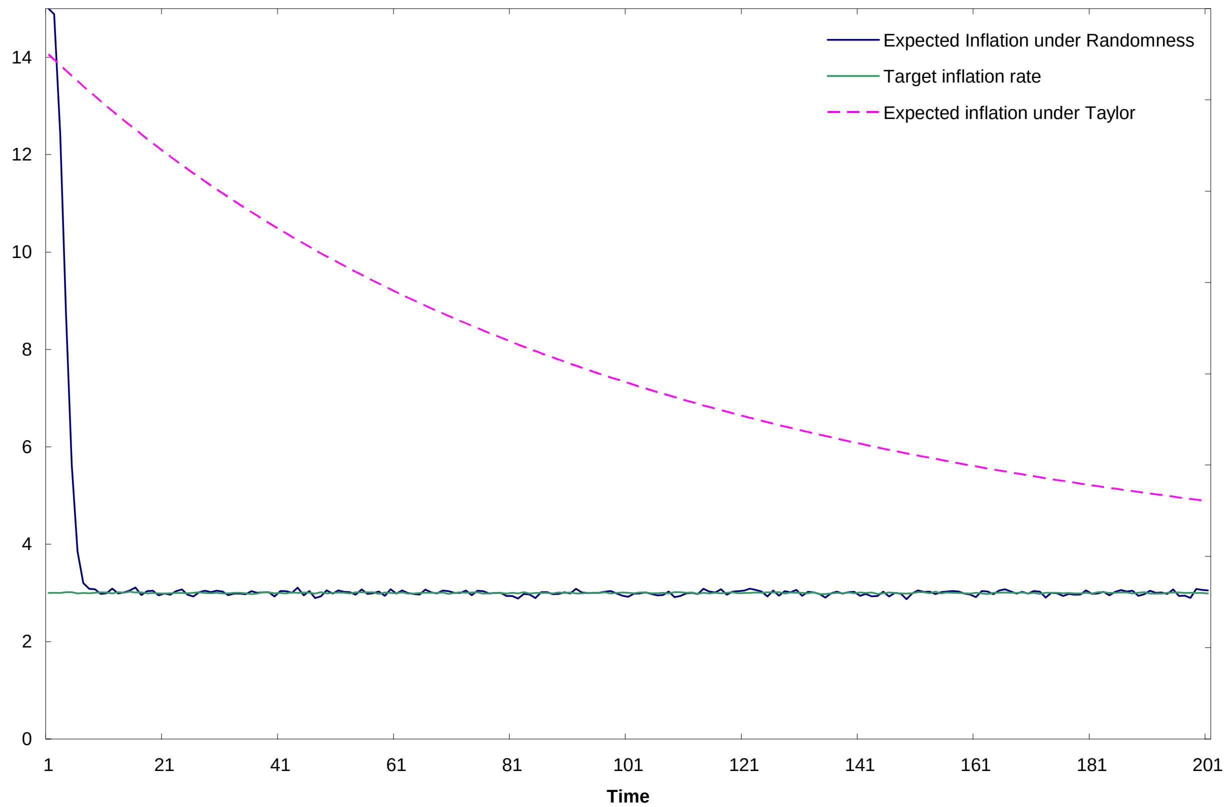

Figure 3: Inflation Vs. Time: Inflation, Comparing Random Processes

The below figure plots the time paths of expected inflation under the white noise random target and under a random walk inflation target. The dotted line (pink) shows the path of inflation under the white noise process and the solid line (blue) under the random walk one. The initial values of both are set to 3 percent. The public's initial guess in each case is set to 15 percent. The parameters for the calculation are as follows: ω = 0.1, σ = 0.1, σ_0 = 0.01, μ^* = 3, and μ_0 = 15.

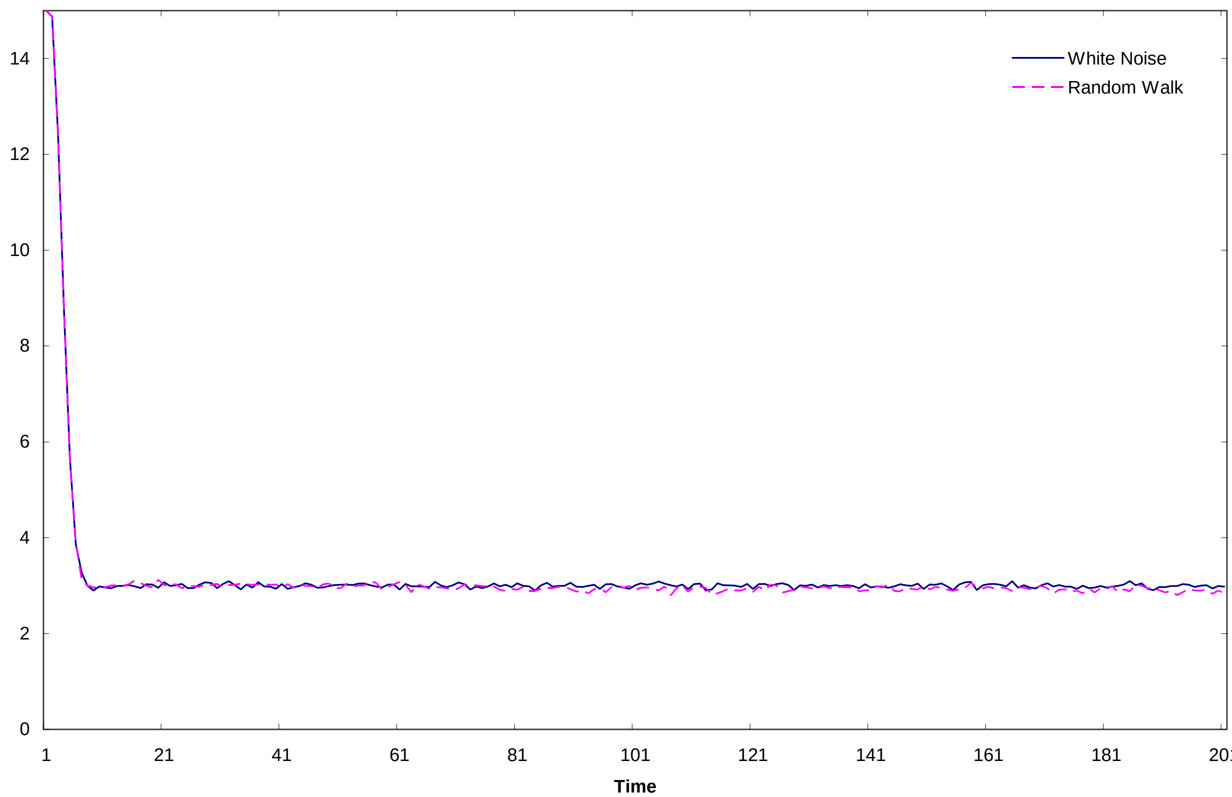

Figure 4: Inflation Vs. Time: Large Alpha

The below figure plots the time paths of expected inflation under the random walk target and under a 'traditional' fixed inflation target. This figure illustrates the role of a large "alpha". Recall that $\alpha = \omega/\sigma$. Thus since sigma is not a choice variable, the alpha variable reflects the government's choice of ω. The dotted line (pink) shows the path of inflation under the fixed target and the solid line (blue) under the random one. The fixed target and the initial value of the random walk target are both set to 3 percent. The public's initial guess in each case is set to 15 percent. The parameters for the calculation are as follows: ω = .19, σ = 0.1, σ_0 = 0.01, μ^* = 3, and μ_0 = 15. Note the variability in the target path of inflation (in addition to inflation itself) in the random walk case.

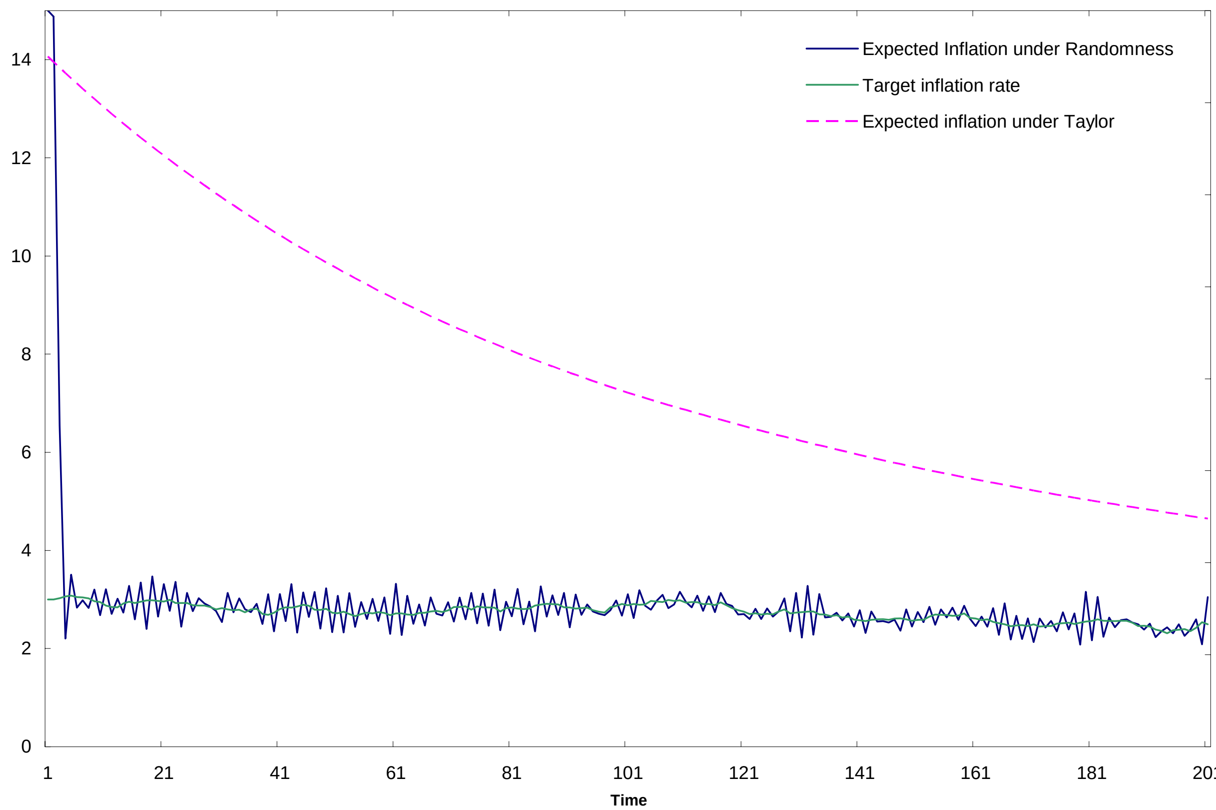

Figure 5: Inflation Vs. Time: Deflation Case

The below figure plots the time paths of expected inflation under the random walk target and under a 'traditional' fixed inflation target. The dotted line (pink) shows the path of inflation under the fixed target and the solid line (blue) under the random one. The fixed target and the initial value of the random walk target are both set to 3 percent. The public's initial guess in each case is set to minus one percent. The parameters for the calculation are as follows: ω = 0.1, σ = 0.1, σ_0 = 0.01, μ^* = 3, and μ_0 = -1.

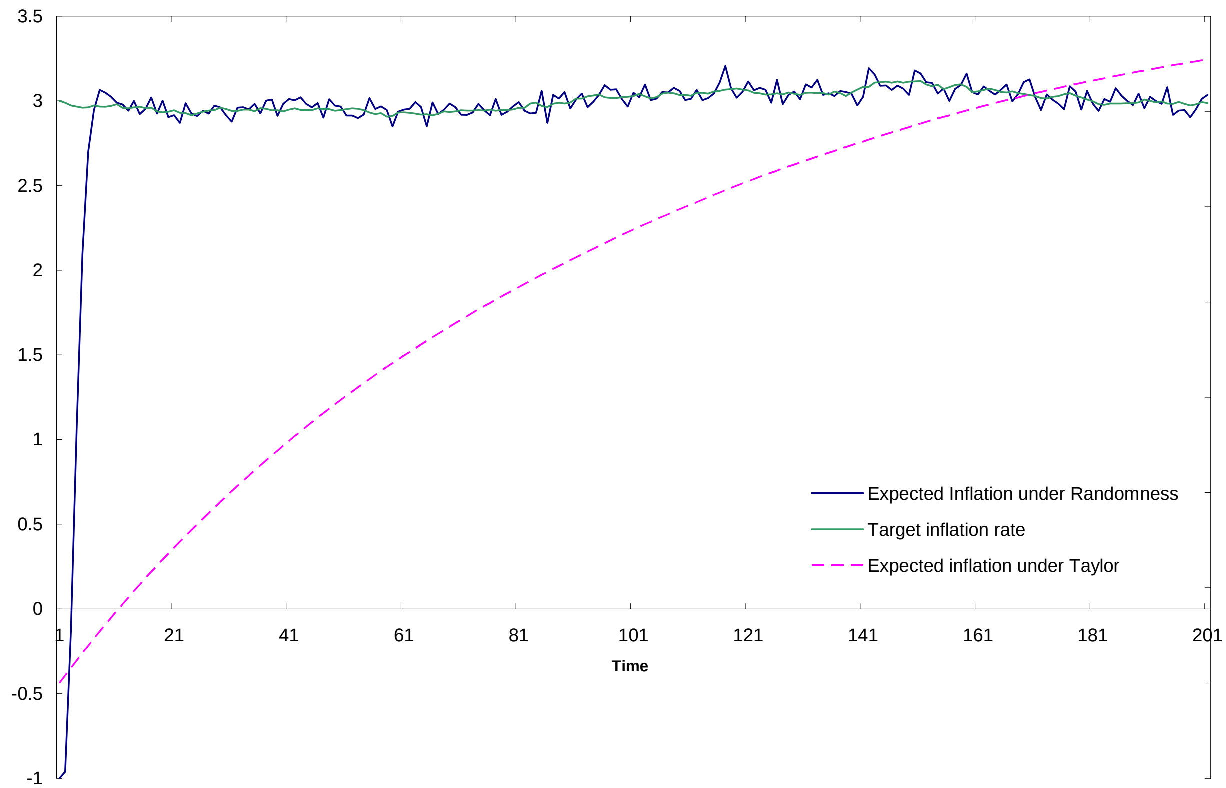

Figure 6: Squared Error for High Inflation Case

The below figure plots the time paths of the squared error in prediction under the random walk target and under a 'traditional' fixed inflation target. The dotted line (pink) shows the path of inflation under the fixed target and the solid line (blue) under the random one. The fixed target and the initial value of the random walk target are both set to 3 percent. The public's initial guess in each case is set to 15 percent. The parameters for the calculation are as follows: ω = 0.1, σ = 0.1, σ_0 = 0.01, μ^* = 3, and μ_0 = 15.

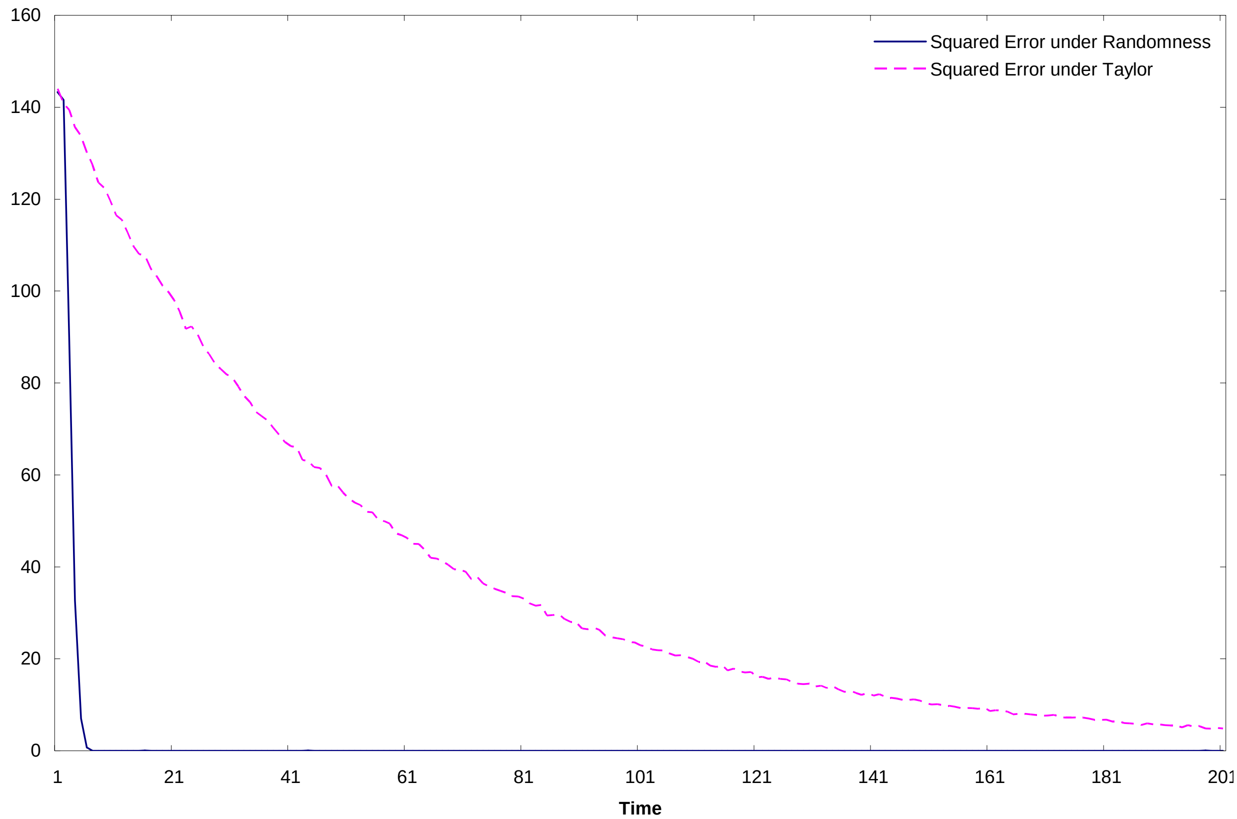

Figure 7: Mean Squared Error for High Inflation Case

The below figure plots the paths of the mean squared error, from time zero to present, in prediction under the random walk target and under a 'traditional' fixed inflation target. The dotted line (pink) shows the path of inflation under the fixed target and the solid line (blue) under the random one. The fixed target and the initial value of the random walk target are both set to 3 percent. The public's initial guess in each case is set to 15 percent. The parameters for the calculation are as follows: $\omega = 0.1$, $\sigma = 0.1$, $\sigma_0 = 0.01$, $\mu^* = 3$, and $\mu_0 = 15$.

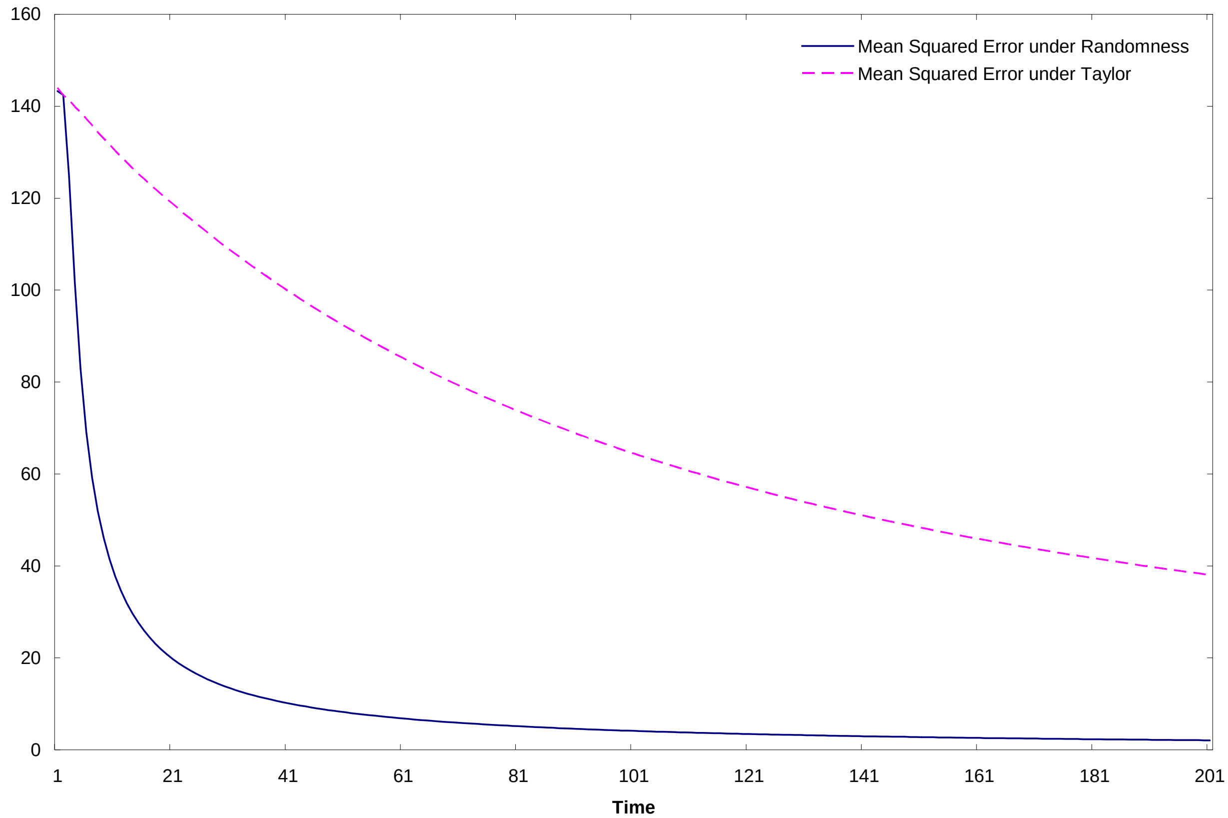

Figure 8: Squared Error for Deflation Case

The below figure plots the paths of the squared error in prediction under the random walk target and under a 'traditional' fixed inflation target. The dotted line (pink) shows the path of inflation under the fixed target and the solid line (blue) under the random one. The fixed target and the initial value of the random walk target are both set to 3 percent. The public's initial guess in each case is set to minus one percent. The parameters for the calculation are as follows: ω = 0.1, σ = 0.1, σ_0 = 0.01, μ^* = 3, and μ_0 = -1.

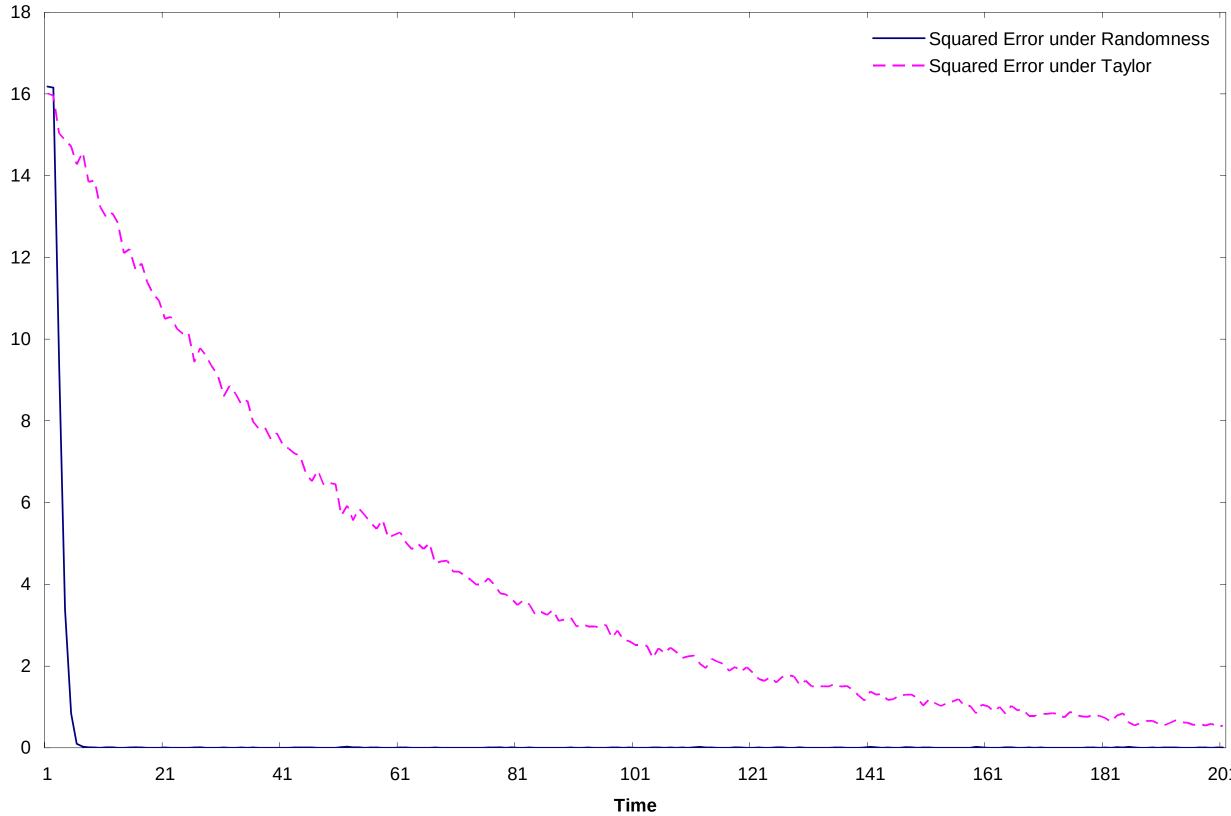

Figure 9: Mean Squared Error for Deflation Case

The below figure plots the paths of the mean squared error, from time zero to present, in prediction under the random walk target and under a 'traditional' fixed inflation target. The dotted line (pink) shows the path of inflation under the fixed target and the solid line (blue) under the random one. The fixed target and the initial value of the random walk target are both set to 3 percent. The public's initial guess in each case is set to minus one percent. The parameters for the calculation are as follows: $\omega = 0.1$, $\sigma = 0.1$, $\sigma_0 = 0.01$, $\mu^* = 3$, and $\mu_0 = -1$.

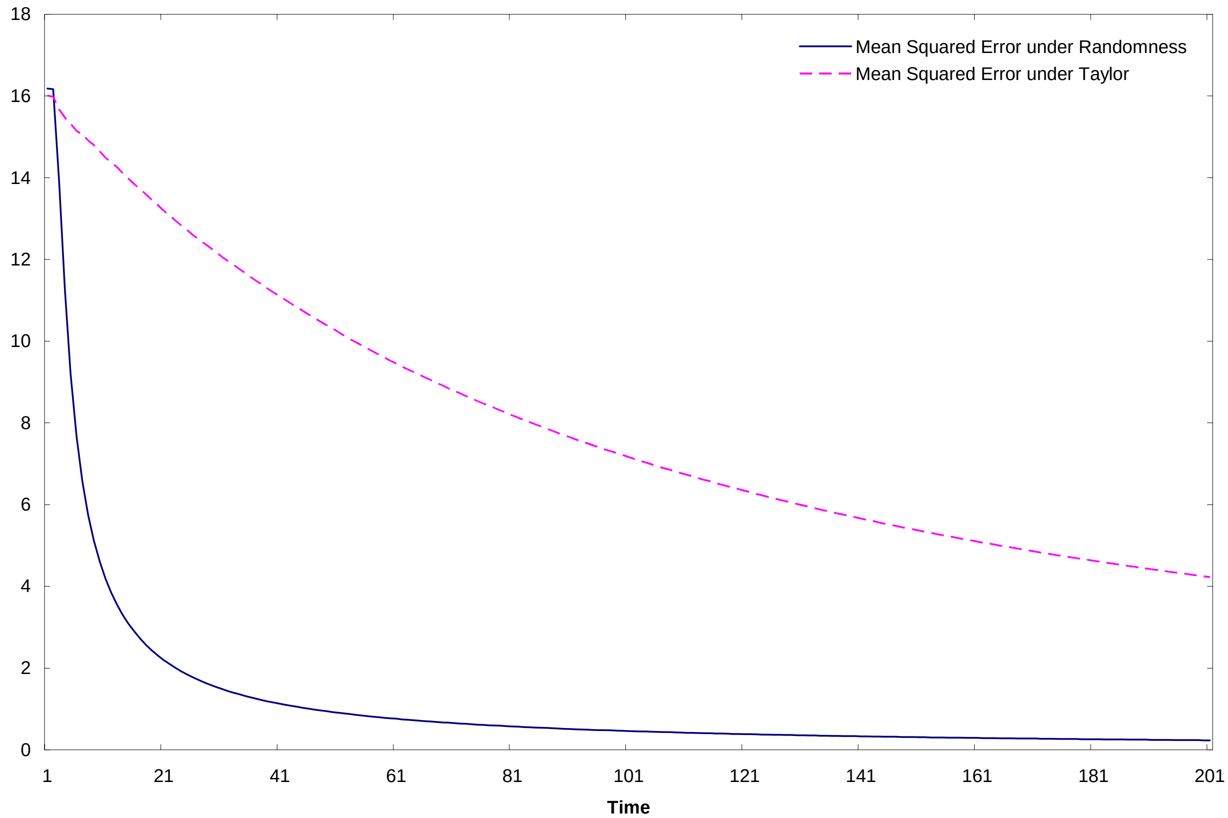

Figure 10: Convergence Rates Random Walk vs Fixed Target

The below figure plots the convergence rates of population expectations under the traditional 'fixed' target and the random walk target. We measure the number of periods required for the expectations path to reach the intended target (with a small permitted error of .05%). In each case, we compare the convergence rates with the value of ω. The dotted line (pink) shows the convergence rates under the fixed target and the solid line (blue) under the random one. The fixed target and the initial value of the random walk target are both set to 3 percent. The public's initial guess in each case is set to 15 percent. ω is incremented from 0.1 to 0.19, and the other parameters for the calculation are as follows:, $\sigma = 0.1$, $\sigma_0 = 0.01$, $\mu^* = 3$, and $\mu_0 = 15$.

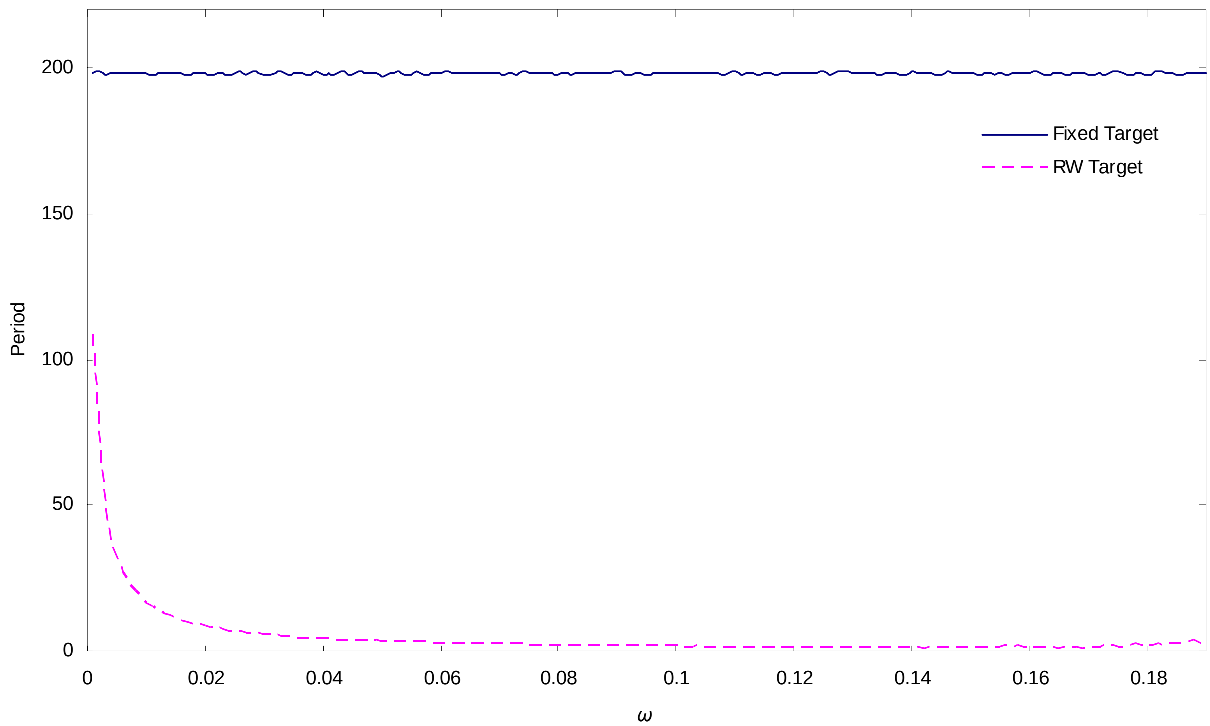

Figure 11: Convergence Rates White Noise vs Fixed Target

The below figure plots the convergence rates of population expectations under the traditional 'fixed' target and the white noise random target. We measure the number of periods required for the expectations path to reach the intended target (with a small permitted error of .05%). In each case, we compare the convergence rates with the value of ω. The dotted line (pink) shows the convergence rates under the fixed target and the solid line (yellow) under the white noise random one. The fixed target and the initial value of the white noise target are both set to 3 percent. The public's initial guess in each case is set to 15 percent. ω is incremented from 0.1 to 0.19, and the other parameters for the calculation are as follows:, σ = 0.1, σ_0 = 0.01, μ^* = 3, and μ_0 = 15.

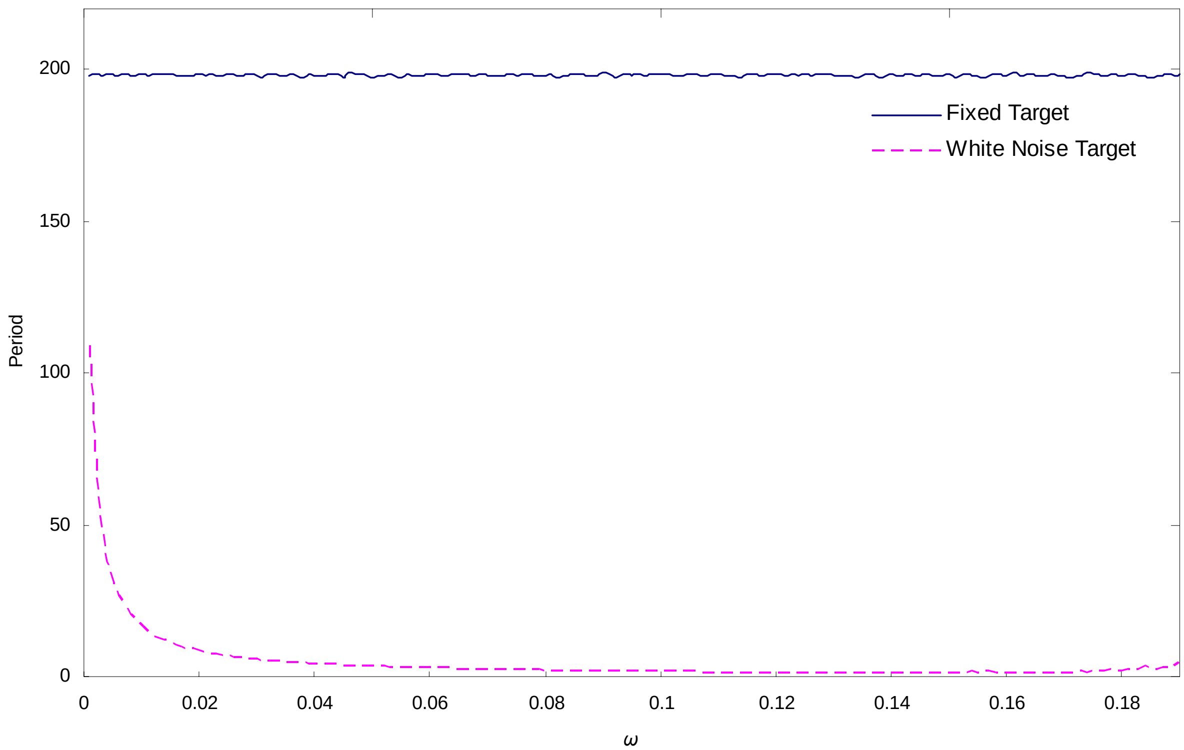

Figure 12: Convergence Rates White Noise vs Random Walk Target

The below figure plots the convergence rates of population expectations under the two studies forms of random targets, random walk and white noise. We measure the number of periods required for the expectations path to reach the intended target (with a small permitted error of .05%). In each case, we compare the convergence rates with the value of ω. The solid line (blue) shows the convergence rates under the random walk target and the dashed line (green) under the white noise one. The initial value of both targets is set to 3 percent. The public's initial guess in each case is set to 15 percent. ω is incremented from 0.1 to 0.19, and the other parameters for the calculation are as follows:, $\sigma = 0.1$, $\sigma_0 = 0.01$, $\mu^* = 3$, and $\mu_0 = 15$.