The role of expectations in inflation dynamics

Jeff Fuhrer

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Introduction

Once one assumes that prices are sticky—and "assume" is the appropriate word, for the deeper foundations that would motivate price-setters to hold nominal prices fixed for extended periods remain elusive in most applied models—expectations take on a prominent role in price modeling. The intuition is straightforward: If prices are expected to remain fixed for some period then the economic conditions that are expected to prevail during that period must be taken into account if prices are to be set optimally. To date, the bulk of research in inflation has used the rational expectations assumption to generate the required expectations (there are important and notable exceptions: See Roberts (1997), Nunes (2010), Adam *et al* (2010)).

The rational expectations assumption imposes significant restrictions on price-setting models with sticky prices, as has been well-documented. In particular, the canonical formulations of price-setting under the Calvo (1983) or Rotemberg (1982) or Taylor (1980) assumptions can imply a very flexible rate of inflation, in the sense that inflation can jump in response to shocks to marginal cost. Equivalently, any persistence in observed inflation must be attributed to the persistence of the driving process. The data for most (and perhaps all) of the postwar period is difficult to reconcile with such a model, as it appears that inflation behaves as if it both inherits the persistence of the driving process and adds some of its own, so-called "intrinsic" inflation persistence (Fuhrer 2006, 2011).

John Roberts' work in the 1990s suggested that it might be the characterization of expectations, rather than the mechanics of price-setting, that leads to such "sticky" inflation behavior. By substituting survey expectations (semi-annual observations from the Livingston Survey), Roberts found considerable success in explaining the behavior of inflation through the early 1990s. However, subsequent work on DSGE models largely continued with the rational expectations assumption, adopting instead a variety of augmentations to the Calvo *et al* model that allowed the models to replicate the key dynamic features of inflation, such as indexation (Christiano *et al* 2005), rule-of-thumb pricing (Gali and Gertler, 1999) or serial correlation in the shocks to inflation (Rotemberg 1997, Smets and Wouters 2003, Christiano et al 2005).

Some recent work returns to the role of survey expectations as a measure of the expectations employed by price-setters (see Nunes (2010), Del Negro and Eusepi (2010), and Adam and Padula (2010)). These authors arrive at somewhat contradictory conclusions about such expectations measures, with Nunes (2010) finding little empirical role for survey expectations, while Adam *et al* find survey expectations to be an important determinant of inflation for UK data.

This paper continues this strand of research, examining the role of survey expectations in the inflation process. It develops four principal findings: (1) Short-run inflation expectations appear to have a significant role in explaining US inflation over the past 20-25 years (this result is suggested but not fully explored in Fuhrer, Olivei and Tootell (2010)); (2) Long-run expectations generally do not appear to have a direct influence on US inflation over the same period, although they enter indirectly as a key determinant of the short-run expectations. The restrictions implied by "trend inflation" models of inflation are generally rejected in the data; (3) When nested in a model that allows expectations to be determined by a linear combination of rational expectations and survey expectations, the data strongly favor survey expectations, generally assigning a small and statistically insignificant role to rational expectations. This finding is robust across a variety of estimation methods; (4) As a partial explanation of these findings, several exercises suggest that the rational expectations of inflation generated by DSGE models with a New Keynesian Phillips curve differ significantly from those measured by the survey. In an important sense, the survey expectations reflect an expectations process that is not well-characterized by the rational expectations from simple DSGE models. Finally, the paper develops a first pass at a structural model that incorporates the features discussed above, and assesses its performance in explaining inflation in the postwar period.

Survey and rational expectations in reduced-form and structural models of inflation

This paper employs quarterly data for inflation, output, unemployment, real marginal cost (the conventional labor share measure), and survey expectations from the Survey of Professional Forecasters (SPF). The paper uses both short-term (four-quarter) and long-term (the ten-year average inflation rate) expectations from the SPF survey.[1] The ten-year expectations are taken to be

[1] Most of the empirical work in this area uses the SPF four-quarter inflation expectation as a proxy for the one-quarter-ahead inflation expectation in standard Phillips curves. The one-quarter-ahead SPF expectation, which is available at the Philadelphia Fed's website, would seem more natural. The estimates presented below have been duplicated using this expectational proxy, and the results are essentially the same.

a proxy for long-term or "trend" inflation expectations. In this paper we use the Board of Governor's estimate of the 10-year expectations as implemented in their FRB/US model.[2]

a. Reduced-form models

Ordinary least squares regressions can serve to motivate many of the key results in the paper. The regression takes the core inflation rate (here proxied by the consumer price index excluding food and energy) as the dependent variable, and regresses it on lagged inflation (two quarterly lags are included), the four-quarter SPF inflation expectation collected at time *t*, the long-run SPF inflation expectation dated time *t*, and the unemployment gap, where the NAIRU is as estimated by the CBO.[3] The sum of the coefficients on lagged inflation and the two expectations variables is constrained to one. The estimation period is 1982:Q1 to 2010:Q3, and the results are summarized in Table 1 below:

Table 1

Reduced-form Phillips curve estimates, OLS estimation

$$\pi_t = a\pi_{t-1} + b\pi_t^{SPF,4q} + c\pi_t^{10-year} + d\tilde{U}_t$$

Variable	Coefficient	Standard error
Lagged inflation (sum)	0.063	0.11
SPF 4-quarter expectation	0.62	0.24
10-year expectation (PTR)	0.32	0.12
Unemployment gap	-0.20	0.064
R^2=0.72.		

These unconstrained results suggest a strong and significant correlation between core inflation and the four-quarter expectation, little dependence on lagged inflation, and a modest correlation with the 10-year expectation. The unemployment gap enters quite significantly, with a coefficient that is sizable by recent standards.

But underlying this full-sample estimate lies an array of results for sub-samples within the thirty-year span. As figures 1 and 2 suggest, lagged inflation is strongly correlated with core inflation in the early part of the sample, and the four-quarter and 10-year expectations measures trade off in size and significance over the sample. The *p*-value for the unemployment gap lags, not shown, is typically below 0.05, although this is not the case early in the sample. These results suggest some combination of multicollinearity and sub-sample instability, at least in this reduced-form regression.[4]

[2] The model variable is "PTR." See the documentation at http://fweb.rsma.frb.gov/mq/frbus/ .

[3] We employ the four-quarter SPF expectation because others in the literature commonly use this as a proxy for next period's inflation. Results that use next quarter's expectation yield very similar results.

[4] The time-varying simple correlation between the two survey expectations measures is fairly steady at 0.9 across most of the sample, falling to 0.4 for the most recent fifteen years.

More progress can be made in making sense of this inflation data by imposing a modest amount of structure. To that end, we estimate Phillips curve regressions that impose two restrictions: (1) The sum of the coefficients on lagged inflation and the four-quarter survey expectation is constrained to one, as these measures normally serve as proxies for expected inflation in the next period, and (2) the long-run survey expectation is not included in that restriction, as we consider it more likely serves as a proxy for "trend inflation" in the sense of Cogley and Sbordone (2008). If that is a reasonable assumption, then the estimated coefficient(s) on this long-run trend inflation proxy should be zero. To see this, consider a trend inflation model that writes the Phillips curve in terms of inflation gap, or deviations of all inflation terms from trend inflation. Denoting the trend inflation proxy by $\overline{\pi}_t$, the Phillips curve as modified from Table 1 above takes the form

$$\begin{aligned} &\hat{\pi}_t = a\hat{\pi}_{t-1} + (1-a)\hat{\pi}_t^{SPF,4q} + d\tilde{U}_t \\ &\hat{\pi}_t \equiv \pi_t - \overline{\pi}_t ; \hat{\pi}_{t-1} \equiv \pi_{t-1} - \overline{\pi}_{t-1} ; \hat{\pi}_t^{SPF,4q} \equiv \pi_t^{SPF,4q} - \overline{\pi}_t \end{aligned} \qquad (1)$$

We can estimate this equation in unconstrained form, allowing us to test the restrictions implied by the trend inflation model:

$$\pi_t = a\pi_{t-1} + (1-a)\pi_t^{SPF,4q} + b_1\overline{\pi}_t + b_2\overline{\pi}_{t-1} + d\tilde{U}_t \qquad (2)$$

If the constraints are satisfied, the one should not be able to reject the constraint $b_1 + b_2 = 0$.[5]

Figures 3 and 4 display the results from estimating equation (2) on a rolling sample of 60-quarter windows from 1982 to the present . Here we obtain more consistent results across the sample. The four-quarter expectation is estimated extremely significantly with a coefficient that varies between 0.6 and 1 across the entire sample; lagged inflation is estimated with a modest coefficient that is marginally significant for part of the sample; and the 10-year expectation varies in sign across the sample, and in some cases would not reject the hypothesis that the sum of coefficients is zero, consistent with the trend inflation model. However, one can almost never reject the hypothesis that both lags are jointly insignificant, contributing nothing to this simple Phillips regression. Interestingly, the *p*-value for the unemployment gap rarely falls below 0.05, which could imply that the four-quarter expectation captures the dependence of inflation on a gap variable. We will return to the implications of this result below. The pattern will play out in tests of more structural versions of the inflation equation later in the paper.

[5] Rearranging the inflation gap equation (1), one can see that the coefficients on $\overline{\pi}_t$ and $\overline{\pi}_{t-1}$ are *a* and –*a* respectively.

These same results are replicated for a different measure of trend inflation (the Cogley-Sbordone filtered VAR estimate), for headline CPI, and for the GDP deflator. In addition, the timing of the four-quarter survey variable is shifted back one quarter, in part because the appropriate expectations date (*t* or *t-1*) is not uniform in the literature, and in part because one might worry about simultaneity bias in this simple regression.[6] The results are qualitatively similar in most all respects: In all cases, the 4-quarter SPF expectation enters significantly throughout the sample; the lags enter with at best modest significance and size; and the long-run inflation proxies are rarely if ever significant contributors to the regression. One exception is the GDP deflator, which shows a significant effect of the ten-year SPF expectation in the last 15 years or so of the sample. However, the coefficient estimates over this part of the sample center on -1, which is difficult to reconcile with any reasonable inflation model.

b. Models with explicit rational expectations

In Fuhrer, Olivei and Tootell (2010), we test the nested model that allows survey expectations, rational expectations, and lagged inflation to enter a NKPC, and examine the relative contributions of each to inflation dynamics for recent decades of US inflation data. Here we replicate the results of that paper, and augment the estimation with two GMM estimators to test robustness of the result to methodological differences.

The test equation is

$$\pi_t - \bar{\pi}_t = a(\pi_{t-1} - \bar{\pi}_{t-1}) + b(E_t\pi_{t+1} - \bar{\pi}_{t+1}) + (1-a-b)(\pi_t^{SPF,4q} - \bar{\pi}_{t+1}) + cs_t \tag{3}$$

where the inflation variables are as defined above, s is a proxy for real marginal cost (the labor share), which is more commonly used in rational expectations models of inflation, the unit sum constraint is again imposed across the variables that proxy for one-period-ahead expectations, and E_t denotes the model-consistent expectation. Following the example of the preceding section, we also examine less-constrained versions of the equation that allow the trend inflation variables to enter in an unconstrained fashion (as in the previous section), so as to allow testing of the restrictions imposed by the trend inflation model.[7]

[6] In fact, the SPF surveys are collected mid-quarter, at which point forecasters would have at most one month of CPI data (depending on the precise date). In any event, the maximum likelihood estimates presented later on, which take account of any simultaneity in the data, suggest that this should be of little concern in these simple OLS estimates.
[7] A set of results that use the unemployment gap are presented in table A.1 in the appendix. They are qualitatively similar to the results that employ real marginal cost. In all but the simple GMM estimator, the four-quarter expectation plays a dominant role as the inflation proxy. In the simple GMM case, the Stock-Yogo test for instrument relevance suggests that the instruments do a poor job spanning the relevant endogenous variables. The restrictions imposed by imposing

For the maximum likelihood estimates of equation 3, we include vector autoregressive equations that allow us to form rational expectations of inflation (and trend inflation) without imposing further structure on the model.[8] The trend inflation variable $\bar{\pi}_t$ is assumed to follow a random walk. For the GMM estimates, we use an instrument set that includes two lags each of the survey expectations variables, real marginal cost, core inflation, and the output gap as defined by the CBO's estimate of potential output. Note that we use two variants of GMM, a conventional iterated estimator with a standard weight matrix, and an "optimal instruments" GMM estimator (Fuhrer and Olivei 2004) that imposes more structure on the instruments that are chosen to form the unobserved expectations in the model. The latter is shown to have superior finite-sample properties, a characteristic shared with the maximum likelihood estimator and demonstrated for an inventory model application in Fuhrer, Moore and Schuh (1995).

Table 2 summarizes the estimation and test results for the three estimators, and includes the *p*-value for the test of the restrictions imposed by the trend inflation model.[9]

Table 2 Estimation results for equation 3, various estimators, 1990:Q1-2010:Q3			
Maximum likelihood **Trend inflation = SPF 10-year expectation**			
Coefficient	Estimate	Standard error[a]	*t*-statistic
Lagged inflation	0.25	0.10	2.5
Rational expectation	0.001	-	-
Survey expectation	0.75	0.25	3.0
Test of trend inflation restrictions			
Test of trend inflation restrictions: 0.44 Test for exclusion of 10-year expectation and RE from model: 0.44			
Maximum likelihood **Trend inflation = Cogley-Sbordone trend inflation measure**			
Coefficient	Estimate	Standard error[a]	*t*-statistic
Lagged inflation	0.36	0.11	3.47
Rational expectation	0.001	-	-

trend inflation, measured either by the 10-year expectation or the Cogley-Sbordone measure, are either rejected outright, or the trend variable can be excluded from the specification entirely without any damage to the specification. Files Phil_est_ml_U.m and Phil_est_gmm_U.m produce the parallel sets of results.

[8] The influence of the long-term expectation proxy on four-quarter inflation is extremely strong, whereas it enters insignificantly in the other VAR equations. As a consequence, we excluded the long-term expectation from all the reduced-form equations except for the four-quarter expectation. In addition, the four-quarter expectation enters insignificantly in the output gap and marginal cost equations, so it is excluded from these equations.

[9] The maximum likelihood estimates take the estimated coefficients in the VAR equations as fixed at their OLS estimates. This restriction is taken for convenience (to minimize the number of estimated parameters) and does not qualitatively affect the other estimated parameters in the model.

Survey expectation	0.73	0.17	3.81
Test of trend inflation restrictions			
Test of trend inflation restrictions: 0.016 Test for exclusion of Cogley-Sbordone trend inflation measure and RE from model: 0.99			
Robustness check: ML estimate including 1980s **Trend inflation = SPF 10-year expectations**			
Coefficient	Estimate	Standard error[a]	*t*-statistic
Lagged inflation	0.10	0.063	1.6
Rational expectation	0.029	0.14	0.22
Survey expectation	0.87	0.30	2.9
Optimal instruments GMM **Trend inflation imposed**			
Coefficient	Estimate	Standard error[a]	*t*-statistic
Lagged inflation	0.25	0.093	2.6
Rational expectation	0.11	0.21	0.54
Survey expectation	0.57	0.28	2.1
Conventional GMM, Trend inflation imposed			
Coefficient	Estimate	Standard error[a]	*t*-statistic
Lagged inflation	-0.043	0.13	-0.33
Rational expectation	0.77	0.33	2.3
Survey expectation	0.22	0.56	0.40
[a]BHHH standard errors			

The estimates tell a *nearly* consistent story: The exception is the conventional GMM estimate, which implies a much higher weight on rational expectations than the others, as has been found in previous papers. The Stock and Yogo (2005) test for weak instruments for this case returns a value of 1.15, well below the 5 percent critical values for the test. Thus the most likely explanation for the discrepancy between the simple GMM estimates and the others is that the instruments are weak, and cannot identify both lagged and expected inflation effects on current inflation. A similar set of results are reported in Fuhrer and Olivei (2004). We report the simple GMM estimate for completeness, but the bulk of the evidence points toward a prominent role for the survey expectations, and very little for the model-consistent or rational expectation.[10]

Overall, the estimates from these methods are broadly consistent with the simple OLS estimates presented above. They suggest that:

- The one-year expectation plays a prominent role in determining inflation in the US;
- The long-run expectations do not affect inflation directly, either as constrained along the lines of a trend inflation model, or unconstrained;

[10] This finding may reconcile our results—OLS, maximum likelihood, and optimal instruments estimator—with those of Nunes *et al*, who find a less significant role for survey expectations using a conventional GMM estimator.

- The restrictions imposed by the trend inflation model are rejected, either because the coefficient restrictions are directly rejected, or because one cannot reject the hypothesis that either trend inflation proxy should be dropped from the model;
- The rational expectations term develops a coefficient that is insignificantly different from zero in almost all cases.

Note that the GMM results echo those in Nunes *et al*, but these appear to be an artifact of the estimator. An alternative method of moments estimator produces results that conform more closely to the maximum likelihood and OLS estimates.

How different are the survey expectations from rational expectations?

The evidence in Table 2 above suggests that survey expectations dominate rational expectations in explaining consumer price inflation. This raises the question of what are the differences between the survey expectations and model-consistent expectations of inflation for comparable horizons. Are the survey expectations rational? How much do they deviate from model-consistent expectations of inflation? This section addresses these questions.

It is by now well-documented that survey expectations often fail tests of efficiency and unbiasedness (see Thomas (1999), Mehra (2002), Batchelor (1986), Bryan and Gavin (1986), and the references therein). A brief confirmatory exercise is summarized in Table A.2 in the appendix. As the table indicates, survey expectations are clearly not "rational" in this simple sense. Yet they enter as significant predictors in inflation equations. So despite their irrationality, it behooves us to determine how best to characterize the information contained in survey expectations.

In a recent paper, Del Negro and Eusepi (2010) examine the ability of a DSGE model to produce inflation expectations that match survey expectations. Using variants of fairly standard DSGE models and estimating key parameters that are well within the range of estimates produced in the extant literature, they find it quite difficult to mimic the behavior of survey expectations. This section expands on their work, by conducting two similar exercises:

1. Estimating the parameters of a simple DSGE model with a minimum distance estimator that uses the distance between the survey expectations and the rational expectations implied by the model as the estimation criterion. To enable this comparison, we augment the DSGE model with equations that define the model's implied 4-quarter and 40-quarter average inflation expectations; and

2. Using an unconstrained VAR to model the evolution of key macro variables. The VAR is augmented just as in the DSGE model, and we examine the distance between the VAR's implied 4-quarter and 40-quarter inflation expectations and the corresponding survey measures.

We begin with the more tightly-constrained DSGE model that incorporates Calvo pricing with price indexation, an output equation derived from the consumption Euler equation with habit formation, and a Taylor rule for the policy rate. The inflation equation allows for the presence of time-varying trend inflation as in Cogley and Sbordone (2008). Real marginal cost is linked to output and other model variables by a reduced-form equation. Thus the key model equations are

$$\begin{aligned}
&\pi_t - \bar{\pi}_t = \mu(\pi_{t-1} - \bar{\pi}_{t-1}) + (\beta - \mu)E_t(\pi_{t+1} - \bar{\pi}_{t+1}) + \gamma s_t + \varepsilon_t^{\pi} \\
&\tilde{y}_t = \mu_y \tilde{y}_{t-1} + (\beta - \mu_y)E_t \tilde{y}_{t+1} - \sigma(f_t - E_t \pi_{t+1} - \bar{\rho}) + \varepsilon_t^{y} \\
&f_t = \omega f_{t-1} + (1-\omega)[a_{\pi}(\pi_t - \bar{\pi}_t) + a_y \tilde{y}_t + \bar{\pi}_t + \bar{\rho}] + \varepsilon_t^{f} \\
&s_t = BZ_{t-1} + \varepsilon_t^{s}; Z_t = \begin{bmatrix} \pi_t \\ \tilde{y}_t \\ f_t \\ s_t \end{bmatrix} \\
&Z_t = AZ_{t-1} + e_t \\
&\pi_t^{4,e} \equiv 0.25E_t(\pi_{t+1} + \pi_{t+2} + \pi_{t+3} + \pi_{t+4}) \\
&\pi_t^{10y,e} \equiv E_t \frac{1}{1+D} \sum_{i=0}^{\infty} \left[\frac{D}{1+D}\right]^i \pi_{t+i} \\
&\bar{\pi}_t = \delta \bar{\pi}_{t-1} + \varepsilon_t^{\bar{\pi}}
\end{aligned} \tag{4}$$

The first exercise takes the estimated parameters from Del Negro and Eusepi that correspond to the key parameters in the DSGE model, and simulates the model to obtain model-implied inflation expectations. These are compared with the survey expectations for the four-quarter and ten-year average measures from the SPF. The results are displayed in figure 5. As the figure suggests, their estimated parameters achieve only modest success in replicating the short- and long-term inflation expectations measured by the surveys. The baseline parameters are summarized in Table 3.

Table 3
Parameter settings for Figures 5 and 6

Parameter	Baseline	Alternative
μ	0.22	0.7
γ	0.03	0.01

a_π	2.44	5
D	40	10

The next exercise considers alteration of some of the key parameters in the DSGE model (the "Alternative" column in Table 3). Figure 6 displays the model's implied values for short- and long-term inflation expectations for a variety of parameter settings. Raising the degree of indexation in the model improves the coherence between the model-implied one-year expectations and the survey measure in the early part of the sample, but not in the latter half. There is no parameter combination considered that leads to an implied 10-year expectation that closely matches the survey data. Recall that a time-varying trend inflation measure, which declines significantly in the first half of the sample, is incorporated into the model. Interestingly, shortening the duration of the 10-year model-implied expected inflation measure to only ten quarters, which is equivalent to front-weighting nearer-term expected inflation in this measure, achieves the best (although still only partial) success in matching the decline in 10-year survey expectations in the 1980s, but it is only partial.

Because the exercise above suggests that some parameter variations can improve the model's ability to match the survey data, we extend this exercise by estimating the model's parameters using a distance criterion that minimizes the difference between model-implied inflation expectations and survey expectations data.

$$C = \omega \sum (E_t \pi_{t+i}^{4} - \pi_{t+i}^{4,S})^2 + (1-\omega) \sum (E_t \pi_{t+i}^{10y} - \pi_{t+i}^{10y,S})^2 \quad (5)$$

We begin by setting the weights $[\omega, (1-\omega)]$ on the 4-quarter and 10-year expectations to one-half respectively.

Preliminary estimation attempts suggest that this estimation criterion does not contain enough information (in this sample) to estimate all eight of the key parameters.[11] As a consequence, we constrain γ to 0.007 and σ to 0.9. Alternative values for these parameters can significantly worsen the estimation criterion, but there appears little value in optimizing further with respect to these two parameters. The results of this exercise are summarized in Table 4 and Figure 7.

[11] The Hessian for this estimation problem has rank six, and a singular value decomposition of the Hessian suggests a strong dependence among γ, a_π and σ.

Table 4 **Estimates of DSGE model parameters using the minimum-distance criterion**					
Parameter	Optimized value 1982-2010	Standard error	Optimized value, 1990-2010	Standard error	Memo: Del Negro *et al* parameters, baseline parameters
μ	0.41	0.021	0.43	0.025	0.22
γ	0.007	-	0.007	-	0.03
ω	0.30	0.097	0.83	0.36	0.8
a_π	0.73	0.11	1.58	0.43	2.44
a_y	3.7	1.2	0.18	2.16	1.0
μ_y	0.99	0.12	0.89	0.46	0.5
σ	0.9	-	0.9	-	
D	14.7	0.000	10.0	0.094	40

The good news in these estimates is that the "fit" of the model-implied inflation expectations, shown in figure 7, is significantly improved over the estimates from Del Negro *et al.* The bad news is that the parameters required to achieve this fit, summarized in Table 4 above, while fairly precisely estimated, are in some cases difficult to reconcile with received wisdom about macroeconomic structure. For example, the smoothing parameter for the policy rule, ω, is estimated at 0.3, as compared to common estimates in the range of 0.7 and above. The weight on the habit parameter in the output equation, μ_y, is implausibly large at 0.99 (the upper limit of the feasible parameter set). The duration of the long-run inflation expectation is under four years, which is quite short for a model variable that is designed to match up with the ten-year average inflation expectation derived from the SPF survey.

As a first check on these results, we re-estimate the parameters from 1990 to the present. As the second set of columns in Table 4 suggests, this sample change suggests some important changes, and they center strongly on the monetary policy rule. The degree of interest rate smoothing and the relative emphasis on inflation versus output shift significantly in this more recent sample. This suggests that taking account of shifts in the systematic component of monetary policy will be important in understanding the behavior of the survey expectations (as in Fuhrer (1996)).

As a second check on this exercise, we take the estimated parameters in Table 4 and use them to simulate actual inflation over the sample period. Figure 8 compares simulated inflation from this exercise to the simulated values for inflation at the baseline parameters. As the figure indicates, the parameters required to match the model's expected inflation series with the survey expectations imply a vastly different trajectory of inflation than the baseline parameters, which track inflation reasonably well.

VAR-based expectations of inflation and survey expectations

The next exercise is a less constrained version of the previous—we take an unconstrained estimated VAR, append equations that define four-quarter and 10-year (duration) inflation expectations, and compute the VAR-implied inflation expectations. The VAR includes inflation, expressed as the deviation of the core inflation measure used above from Cogley and Sbordone's trend inflation measure, the output gap, the federal funds rate, real marginal cost, and the log change in the relative price of oil. The duration of the long-run inflation expectation is set to 10 years. Figure 9 displays the results for the sample 1982-2010. As indicated in the figure, the ability of the unconstrained model to replicate the shorter-term inflation expectations is not great, but better than the DSGE model at its baseline parameter values, and nearly as good as the DSGE model with parameters optimized to fit the expectations data.

The fit of the long-run expectations, on the other hand, is poor. Altering the duration of the (implied) long-run expectation to the value of 15 found in the DSGE optimization above improves the fit somewhat (not shown), but not dramatically. Taking a lesson on the importance of time-variation in the policy rule in the DSGE exercise, we re-estimate the VAR over the post-1989 period, hopefully controlling for the possibility of a shift in underlying monetary policy behavior. This exercise, not shown, results in at best a very modest improvement in the fit to the long-run survey expectation.

We draw some tentative conclusions from these exercises in matching constrained and unconstrained model-consistent inflation expectations to the survey measures:

1. One can compute a model-based inflation expectation that is fairly close to the four-quarter survey expectation. However, to do so, one needs to dramatically alter the parameters of a standard structural model in ways that cause it to behave quite poorly with respect to the other variables in the model;
2. One can achieve reasonably close fit as well with an unconstrained VAR, at least for the short-run expectation. This suggests that, with regard to the short-run expectation, the difficulty in replicating the survey data lies in the restrictions imposed by the structural model.

3. With regard to long-run expectations, it is difficult to match closely the long-run survey expectations with either constrained or unconstrained models. This is true even when allowing for a time-varying inflation trend that is taken as exogenous to the model. This difficulty represents a significant unresolved challenge.

Overall, this suggests that we have some way to go before we can model the kind of expectations that appear empirically relevant for the determination of inflation. The paper moves part way in this direction in the next and final section.

Endogenizing short-term expectations

We now revert to the somewhat reduced-form modeling techniques of the earlier sections of this paper to investigate key determinants of the short-term expectations that we have found to be of value in explaining inflation. As a starting point, consider the reduced-form regression linking the four-quarter, the 10-year expectations, lagged inflation and the unemployment gap. For notational convenience, the term π_{t-1} represents the sum of the four quarterly lags included in the regression, and the term $\tilde{U}_{t-1}$ represents the sum of the two quarterly lags of the unemployment gap included in the regression, and $\pi_t^{40,e}$ represents the current and single lag of the 10-year (40-quarter) SPF inflation expectation. We include the contemporaneous and lagged values of the latter in order to compare this specification with the next, which imposes that the four-quarter expectation is properly thought of as a deviation from this proxy for the trend in inflation. The constrained version of the equation may be written

$$\begin{aligned}\pi_t^{4,e} &= a\pi_{t-1} + (1-a)\pi_t^{40,e} - c\tilde{U}_{t-1} \\ \pi_t^{4,e} &= \pi_t^{40,e} + a(\pi_{t-1} - \pi_t^{40,e}) - c\tilde{U}_{t-1}\end{aligned} \tag{6}$$

In all cases, the sum of the estimated coefficients is significantly different from zero; obviously the test for the null that all the coefficients are jointly zero is rejected with overwhelming confidence.

Table 5
Estimates of reduced-form equation for four-quarter SPF expectations
$\pi_t^{4,e} = a\pi_{t-1} + b\pi_t^{40,e} - c\tilde{U}_{t-1}$

1. Unconstrained		
	Coefficient	***p*-value**
Lagged inflation (4 quarterly lags)	0.40	0.00
10-year SPF expectation (PTR)	0.40	0.00
Unemployment gap	-0.072	0.00

2. Constrained (deviations form, a+b=1)		
Lagged inflation (4 quarterly lags)	0.42	0.00
10-year SPF expectation (PTR)	0.58	0.00
Unemployment gap	-0.14	0.00
p-value for unit sum constraint: 0.000		

The next panel of the table considers the same regression with the constraint that the 10-year enter as a trend variable, that is that the coefficients on lagged and 10-year expected inflation sum to one. While this constraint boosts the contribution of the unemployment gap term, the unit sum constraint is rejected overwhelmingly.

Like the reduced-form Phillips curves estimated above, there is evidence of significant time variation in these simple relationships. Figures 10-13 summarize the evidence for the unconstrained and constrained versions of the regressions, using a rolling estimation window of 60 quarters. These figures suggest a less tidy picture for the four-quarter expectations than was obtained for the Phillips curves. While all three variables generally contribute to the "fit" of the short-run expectation, lagged inflation loses its significance toward the middle of the sample, and the unit sum constraint is often rejected. The unconstrained regressions show why the test tends to reject: The sum of the lags and the 10-year expectation fall significantly short of one in the first part of the sample, and significantly exceed one in the latter third of the sample. Interestingly, the unemployment gap is significant throughout, in contrast to the Phillips curve regressions above. This suggests that the four-quarter expectation influences inflation in part because it captures the dependence of inflation on the unemployment gap, an influence that was clear in the unconstrained Phillips curves presented above.

<u>A more structural model of inflation with survey expectations</u>

Take the evidence so far and construct a quasi-structural model of inflation with the following elements.

We begin with the survey-based New Keynesian Phillips curve, allowing for the presence of indexation

$$\begin{aligned} \pi_t &= a\pi_{t-1} + (1-a)\pi_t^{SPF,4} - b\tilde{U}_t \\ \pi_t &= a\pi_{t-1} + (1-a)S_t\pi_{t+1} - b\tilde{U}_t \end{aligned} \qquad (7)$$

where we have re-written the equation in the second line to introduce a "survey expectations" operator. Solving equation (7) for the unobserved expectation in period *t*+1 is straightforward under rational expectations. Endogenizing the survey expectations requires additional assumptions about how the survey expectations are formed. We will assume that, while clearly different from the rational expectations operator, the *S* operator maintains the following properties:

1. It can be iterated forward consistently;
2. It exhibits the simple property that the *t*-period expectation of *t* and previous periods' variables equals the realization of these variables.
3. In addition, we will assume that as the horizon of the expectation increases, the expectations formed by the *S* operator converge to those formed by the *E* operator. In the weak form, this assumes that survey expectations properly estimate the long-run equilibrium value of the variable. In the somewhat stronger form, this assumes that survey expectations capture the later stages of the reversion to equilibrium in the same way as rational expectations. As a practical matter, this assumption is also motivated by the limitations of the survey data set: Forecasts are available for the current and next four quarters, and for the current and next year.[12]

Taking these properties on board, one can then write the equation for the survey-based expectation of inflation next period as

$$S_t \pi_{t+1} = a S_t \pi_t - b \sum_{i=1}^{\infty} \omega_i S_t \tilde{U}_{t+i} \quad (8)$$

This expression is convenient, as we observe the SPF survey of future unemployment rates for four quarters beyond the survey date, and for the current year and the next year. If in addition we assume that $S_t \pi_t = \pi_t$, we can then approximate (8) by adding the rational expectation of unemployment gaps beyond the observable survey horizon:

$$S_t \pi_{t+1} = a \pi_t - b \sum_{i=1}^{8} \omega_i S_t \tilde{U}_{t+i} - b \sum_{i=9}^{\infty} \delta_i E_t \tilde{U}_{t+i} \qquad (9)$$

This assumes that the rational expectation of the unemployment gap for the horizon beyond the next several quarters is a reasonable proxy for the survey expectation. In many cases, this will be reasonable, as the unemployment gap will be expected to return to its equilibrium (zero) at this horizon. For many initial conditions, this approximation simply assumes that the survey expectations get the mean of the forecasted variable right. For cases in which the unemployment gap has been more dramatically perturbed from its long-run value, we assume that the survey and rational

[12] In the last several years of the dataset, the two-year-ahead forecast is published, but due to its very short span of availability, we cannot use it for this paper.

expectations converge at these longer horizons. This approximation is implemented by adding the auxiliary equation that defines the model's expectations of unemployment past the survey horizon as

$$\tilde{U}_t^+ = \delta \tilde{U}_{t+9} + (1-\delta) E_t \tilde{U}_{t+1}^+ \qquad (10)$$

and equation (9) becomes

$$S_t \pi_{t+1} = a\pi_t - b\sum_{i=1}^{8} \omega_i S_t \tilde{U}_{t+i} - bU_t^+ \qquad (11)$$

These assumptions do not completely "close" the model, as the determination of unemployment, required to form the model-consistent forecasts of unemployment that enter U_t^+ has not yet been specified. For this paper, we take an agnostic view on the determination of unemployment, modeling it with a reduced-form VAR equation in three observables (lags of the funds rate, inflation, and unemployment). The SPF unemployment gap forecasts are assumed to revert toward zero at a rate that is determined by a second-order autoregression.

In the empirical implementation of equation (11), we use quarterly values of the current-year and next-year forecast of the unemployment rate, from which the CBO's estimate of the NAIRU for the corresponding year is subtracted to form an output gap. In addition, in recognition of the important role played by trend inflation in the preliminary regressions reported in figures 10-13, we express the inflation variables as deviations from trend inflation.

$$S_t \pi_{t+1} = a\pi_t + (1-a)\overline{\pi}_t - b\sum_{i=0}^{1} \omega_i S_t \tilde{U}_{t+i}^Y - cU_t^+ \qquad (12)$$

Preliminary estimates suggest that it may be difficult to identify the ω_i independently, so we condense the equation to

$$S_t \pi_{t+1} = a\pi_t + (1-a)\overline{\pi}_t - b(S_t \tilde{U}_t^Y + S_t \tilde{U}_{t+1}^Y) + cU_t^+ \qquad (13)$$

For reference, the OLS estimate of this equation for the sample 1982-2010, excluding the model-consistent expectations of the unemployment gap, yields (HAC standard errors in parentheses)[13]

$$\underset{}{S_t \pi_{t+1}} = \underset{(0.049)}{0.22\pi_t} + \underset{(0.049)}{0.78\overline{\pi}_t} - \underset{(0.10)}{0.33\tilde{U}_t^Y} + \underset{(0.11)}{0.17\tilde{U}_{t+1}^Y}$$

The model comprises equations (7) and (13), the definition of U_t^+ from equation (10), and VAR equations to forecast the unemployment gap.[14] The sample begins in 1982:Q1, the first quarter

[13] The unit sum constraint is rejected over the full sample, as it is in the preliminary regressions above.

for which the expectations variables (accounting for lags) are available. We employ a 60-quarter estimation window, although the results are not particularly sensitive to the window size. Figure 14 presents rolling-sample ML estimates of the key equations (7) and (13), while figure 15 displays the *p*-values for the Wald tests of the null hypotheses that a model parameter (or two parameters jointly) are zero.[15] The test uses the BHHH estimate of the inverse of the covariance matrix of the parameters, which we denote Ω. Denoting the estimated and constrained parameter vectors by $\hat{\beta}$ and β^C, the test *W* is

$$W = (\hat{\beta} - \beta^C)'\Omega(\hat{\beta} - \beta^C)$$

The ML estimates for the survey expectations Phillips curve are reasonably stable across time. As in all the results presented above, the estimated coefficient on the one-year SPF expectation varies between 0.5 and 1, with an average value of about 0.7. This implies a modest coefficient of about one-third for lagged inflation in the Phillips curve. As indicated in figure 15, the *p*-value for the Wald test that its value is zero rejects overwhelmingly throughout the sample. The unemployment gap, the red line in figures 14 and 15, generally enters with the correct sign and quite significantly, except for the early part of the sample. The average coefficient across all rolling samples is -0.12.

Turning to the equation that explains the short-term inflation expectation, figure 14 shows that current inflation and the trend inflation proxy enter with roughly equal weights, although the coefficients fluctuate somewhat over the sample (recall that they are constrained to sum to one). The coefficient is estimated precisely, as indicated by the middle panel in figure 15. The influence of unemployment, the third panel in figure 14, is generally significant and of the expected sign. The average unemployment effect across all estimates, summing the SPF forecasts and the "model-consistent" forecasts, is -0.37. However, two caveats are worthy of note: First, the overall effect of unemployment (the sum of the SPF unemployment gap forecasts and the U^+ term) turns *positive* in several of the samples toward the end of the estimation period, which is difficult to interpret. Second, the influence of the SPF unemployment forecasts (not shown) varies in sign and significance over the estimation samples. This could reflect the presence of "speed limit" (change in

[14] These equations include two quarterly lags of the unemployment gap, the core inflation rate, and the federal funds rate.

[15] The parameters of the VAR equation and the second-order autoregressions are also estimated for each sample in the rolling window, jointly with the other structural parameters in the Phillips curve and the inflation expectations equation.

expected unemployment) effects on inflation expectations, but it also suggests that some caution should be exercised in interpreting the results.

Conclusions

As suggested in the introduction, these results support a renewed emphasis on non-rational expectations measures in inflation modeling. The reduced-form and moderately-constrained estimates suggest a strong role for survey expectations, particularly the one-year (although these results are replicated for one-quarter-ahead expectations as well). The estimated contribution of a structural model's rational expectation is almost always dominated by the contribution from survey expectations. The restrictions implied by the "trend inflation" model are almost uniformly rejected.

The attempt to articulate a model that endogenizes the survey expectations provides some interesting results. While long-run expectations may not enter the Phillips curve directly, they still serve as an anchor for the short-run survey expectations in all the estimates presented. A more careful modeling of the survey expectations that imposes more structural expectational assumptions is partly successful, as it finds *some* role for expected unemployment gaps, in a manner consistent with conventional theories. However, the empirical results in this regard are not uniform across all samples. The sign of the unemployment gap effect sometimes flips to positive, and the survey's unemployment forecasts do not always enter significantly.

While the results presented in this paper are neither conclusive nor free of problems, they do suggest that the use of survey expectations may present an important direction for inflation and DSGE modelers. While the simplicity of working with rational expectations is sacrificed to an extent, it is possible to use survey data on inflation expectations while maintaining a reasonable blend of theoretical and empirical rigor.

References

Adam, K. and M. Padula (2011), "Inflation Dynamics and Subjective Expectations in the United States," *Economic Inquiry*, Vol. 49, No. 1, January, 13–25.

Batchelor, Roy A. (1986), "Quantitative v. Qualitative Measures of Inflation Expectations," *Oxford Bulletin of Economics and Statistics*. **48**, pp. 99-119.

Bryan, Michael F. and William T. Gavin (1986), "Models of Inflation Expectations Formation: A Comparison of Household and Economist Forecasts," *Journal of Money, Credit, and Banking*, **18**, pp. 539-44.

Calvo G. (1983), "Staggered Prices in a Utility-Maximizing Framework," *Journal of Monetary Economics,* **12**, pp. 383 398.

Christiano L., Eichenbaum, M. and C. Evans (2005), "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy," *Journal of Political Economy,* **113** No. 1, pp. 1-45.

Cogley, T. and A. Sbordone (2009), "Trend Inflation, Indexation, and Inflation Persistence in the New Keynesian Phillips Curve," *American Economic Review,* 98:5, 2101–2126.
Del Negro, M. and S. Eusepi (2010), "Fitting Observed Inflation Expectations," Federal Reserve Bank of New York Staff Report no. 476, November**.**

Fuhrer, J. (2006), "Intrinsic and Inherited Inflation Persistence," *International Journal of Central Banking*, Vol. 2 No. 3, pp. 49-86.

Fuhrer, J. (2011), "Inflation Persistence," chapter 9 in Benjamin M. Friedman and Michael Woodford, editors: Handbook of Monetary Economics, Vol. 3A, The Netherlands: North-Holland, pp. 423-486.

Fuhrer, J., Moore, G. and S. Schuh (1995), "Estimating the linear-quadratic inventory model: Maximum likelihood versus generalized method of moments," *Journal of Monetary Economics* **35**, pp. 115-157.

Fuhrer, J. and G. Olivei (2004), "Estimating Forward-Looking Euler Equations with GMM and Maximum Likelihood Estimators: An Optimal Instruments Approach," in Models and Monetary Policy: Research in the Tradition of Dale Henderson, Richard Porter, and Peter Tinsley. Faust, J., Orphanides, A. and D. Reifschneider, eds., Board of Governors of the Federal Reserve System.

Fuhrer, J., Olivei, G. and G. Tootell (2010), "Inflation Dynamics When Inflation is Near Zero," presented at Federal Reserve Bank of Boston's annual economic conference, October 2010, forthcoming *Journal of Money, Credit and Banking.*

Galí, Jordi and Mark Gertler (1999). "Inflation dynamics: A structural econometric analysis," *Journal of Monetary Economics*, **44,** pp. 195-222.

Mehra, Yash (2002), "Survey Measures of Expected Inflation: Revisiting the Issues of Predictive Content and Rationality," Federal Reserve Bank of Richmond *Economic Quarterly,* Volume **88**/3, Summer, pp. 17-36.

Rotemberg, J. (1997), "Towards a compact, empirically-verified rational expectations model for monetary policy analysis: A comment," Carnegie-Rochester Conference Series on Public Policy **47**, 231-241, North-Holland.

Nunes, R. (2010), "Inflation Dynamics: The Role of Expectations," *Journal of Money, Credit and Banking,* Vol. 42, No. 6, 1161-1172.

Roberts, J. (1997), "Is Inflation Sticky?" *Journal of Monetary Economics* 39, 173-196.

Rotemberg, J. (1983), "Aggregate Consequences of Fixed Costs of Price Adjustment," *American Economic Review*, **73**, No. 3 (June), pp. 433-436.

Smets, F. and R. Wouters (2003), "An Estimated Dynamic Stochastic General Equilibrium Model of the Euro Area," *Journal of the European Economic* Association, Vol. 1, No. 5, Pages 1123-1175.

Stock, J. and M. Yogo (2005), "Testing for Weak Instruments in Linear IV Regression," ch. 5 in Donald W. K. Andrews (ed.), *Identification and Inference for Econometric Models*. New York: Cambridge University Press, 80-108.

Taylor, J. (1980), "Aggregate Dynamics and Staggered Contracts," *Journal of Political Economy*, **88**, No. 1 (Feb.), pp. 1-23.

Thomas, Lloyd B. Jr. (1999), "Survey Measures of Expected U.S. Inflation," *The Journal of Economic Perspectives*, Vol. **13**, No. 4 (Autumn), pp. 125-144.

Figure 1

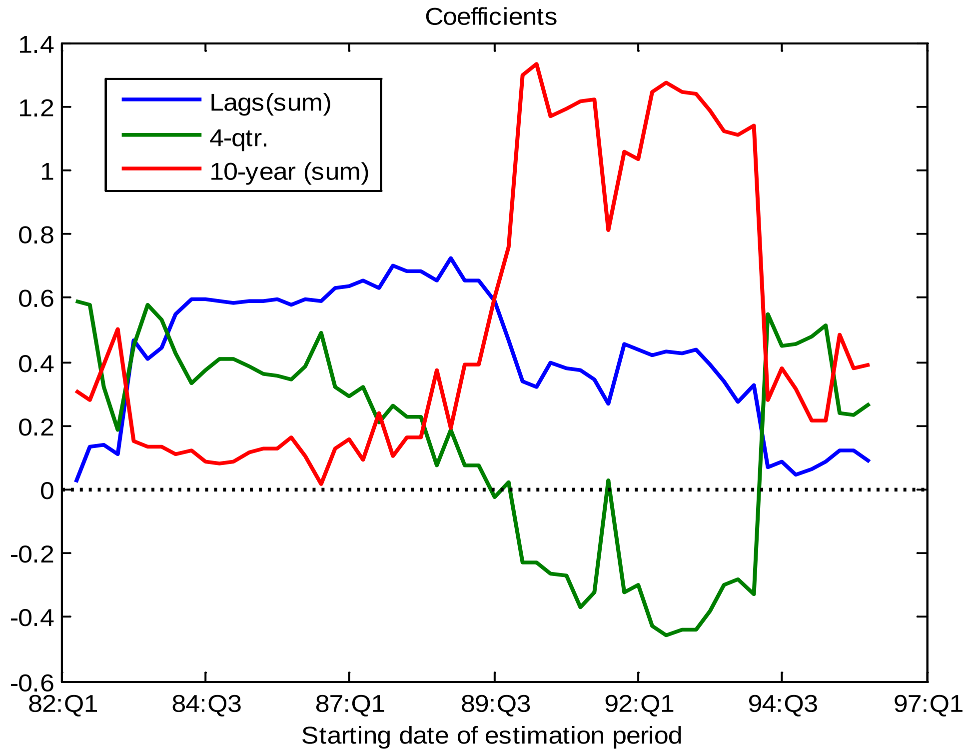


Figure 2

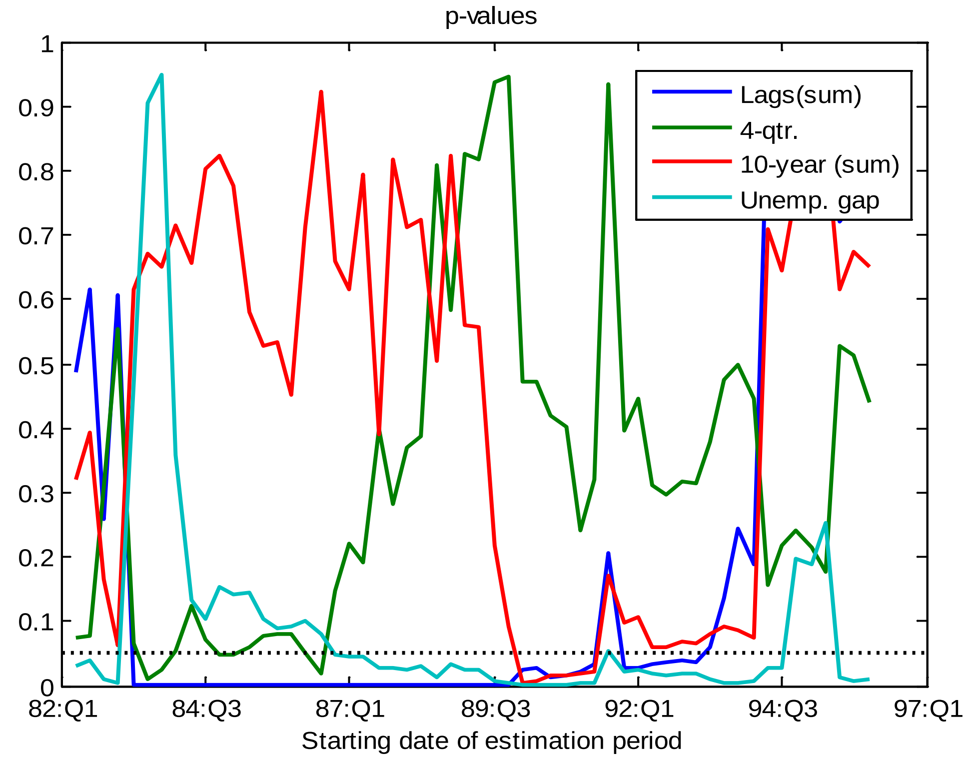


Figure 3

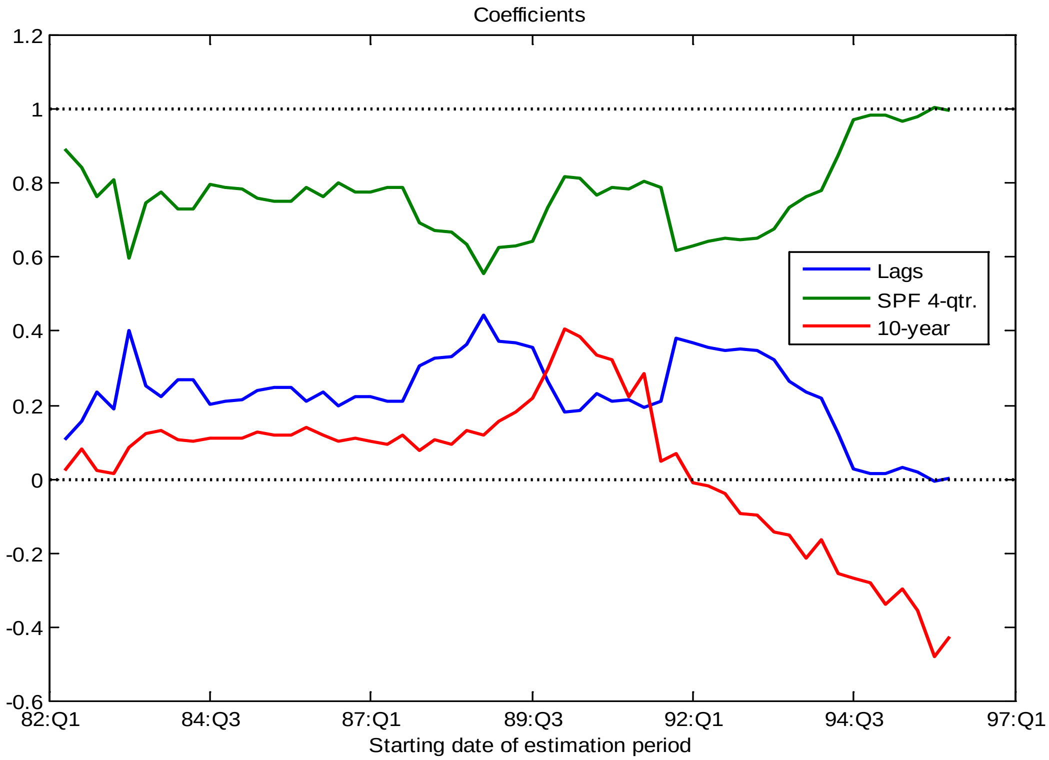


Figure 4

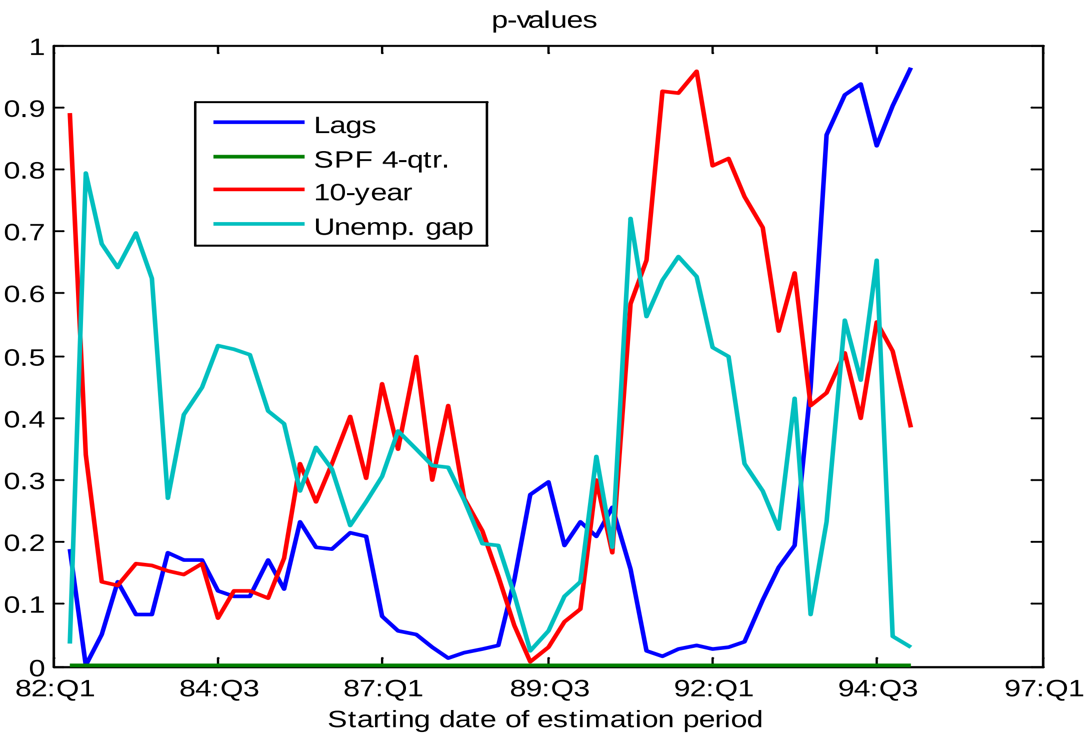


Figure 5

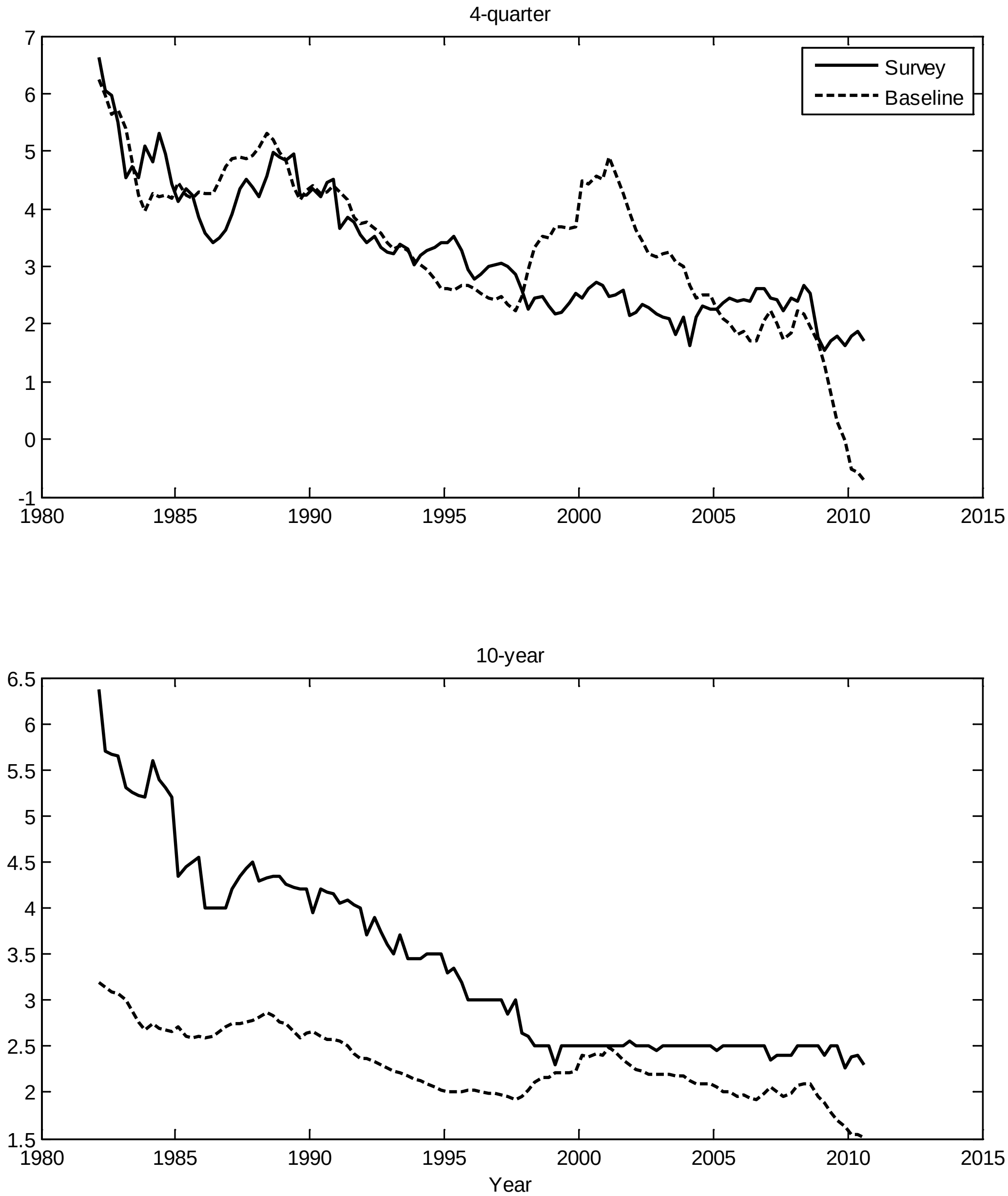

4-quarter
Survey
Baseline
10-year
Year
1980
1985
1990
1995
2000
2005
2010
2015

Figure 6

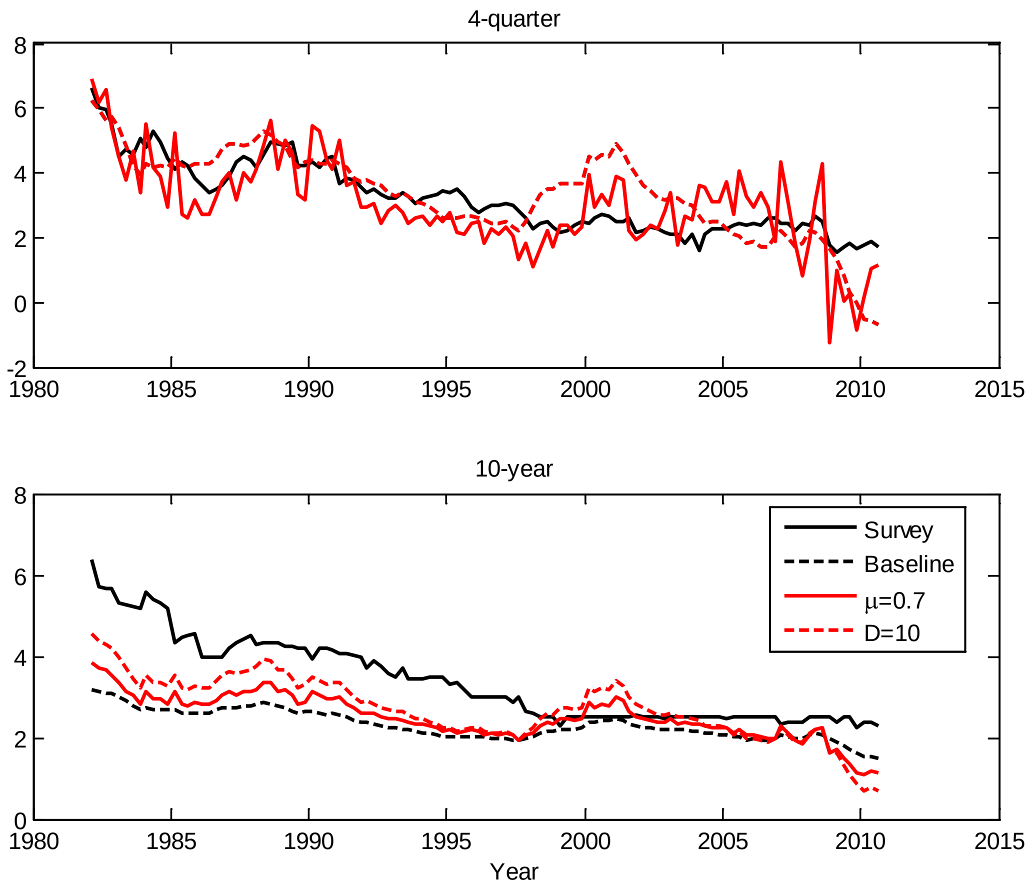

4-quarter
8
6
4
2
0
-2
1980
1985
1990
1995
2000
2005
2010
2015
10-year
8
6
4
2
0
1980
1985
1990
1995
2000
2005
2010
2015
Year
Survey
Baseline
μ=0.7
D=10

Figure 7

Fitted values of survey variables from minimum-distance estimation

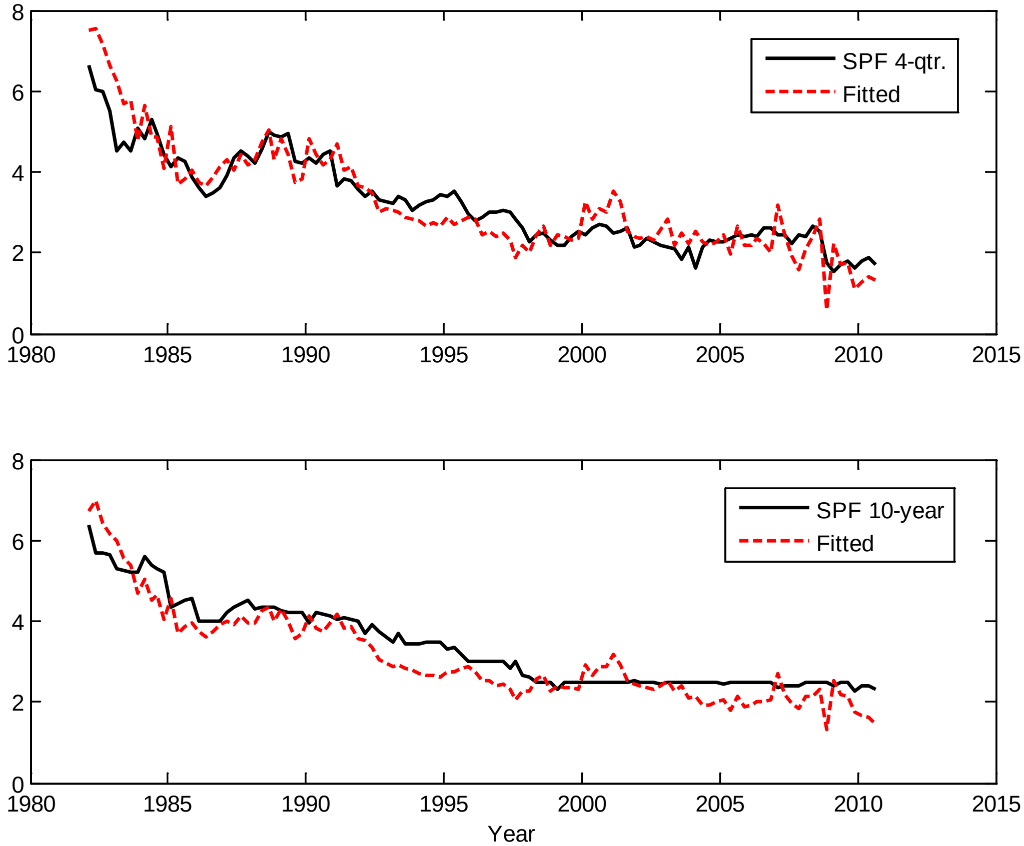


Figure 8

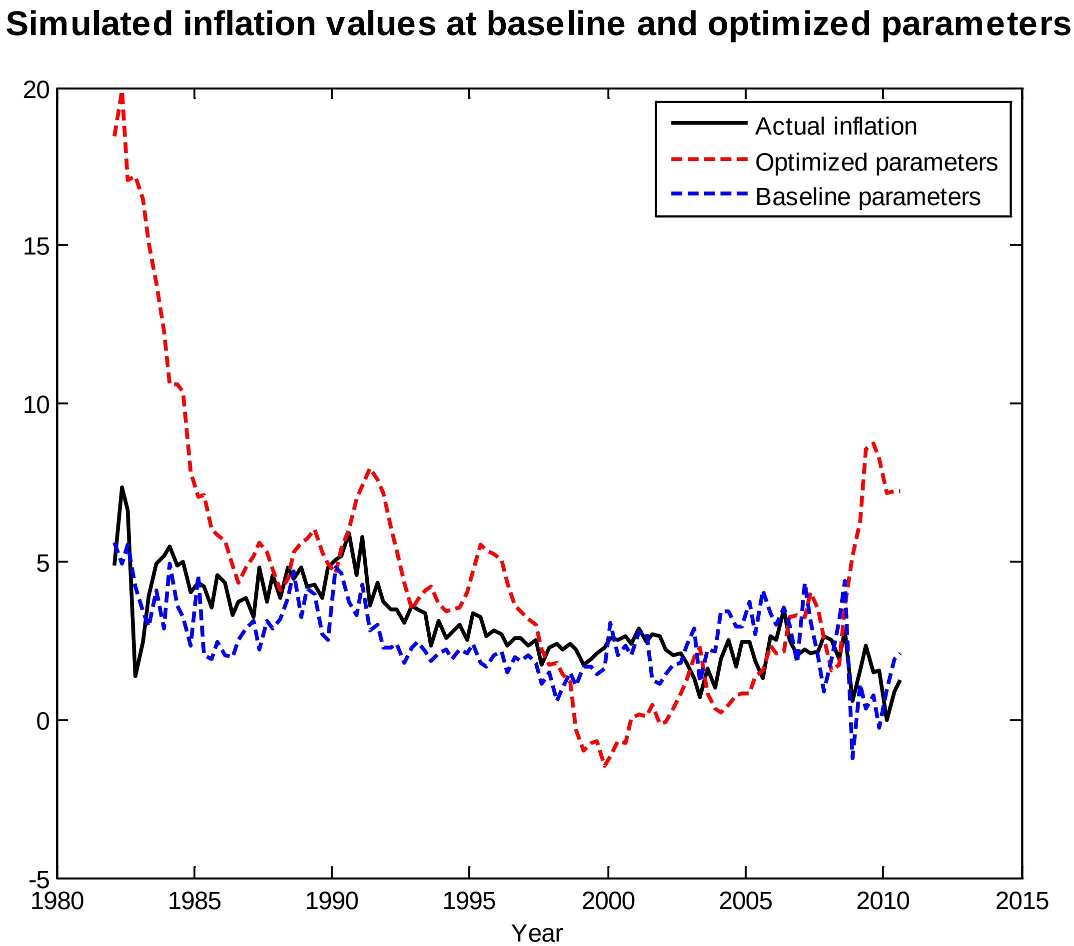

Simulated inflation values at baseline and optimized parameters
20
15
10
5
0
-5
1980
1985
1990
1995
2000
2005
2010
2015
Year
Actual inflation
Optimized parameters
Baseline parameters

Figure 9

VAR-implied inflation expectations

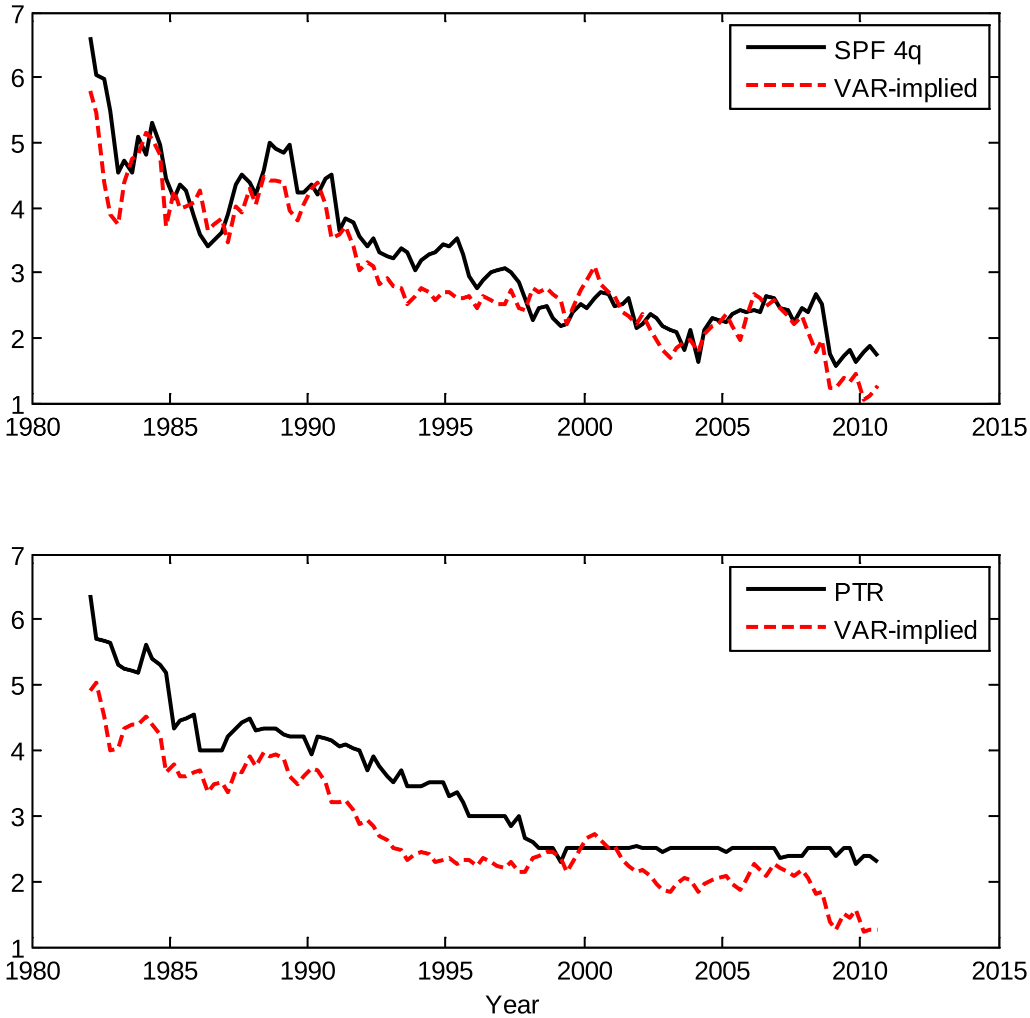


Figure 10

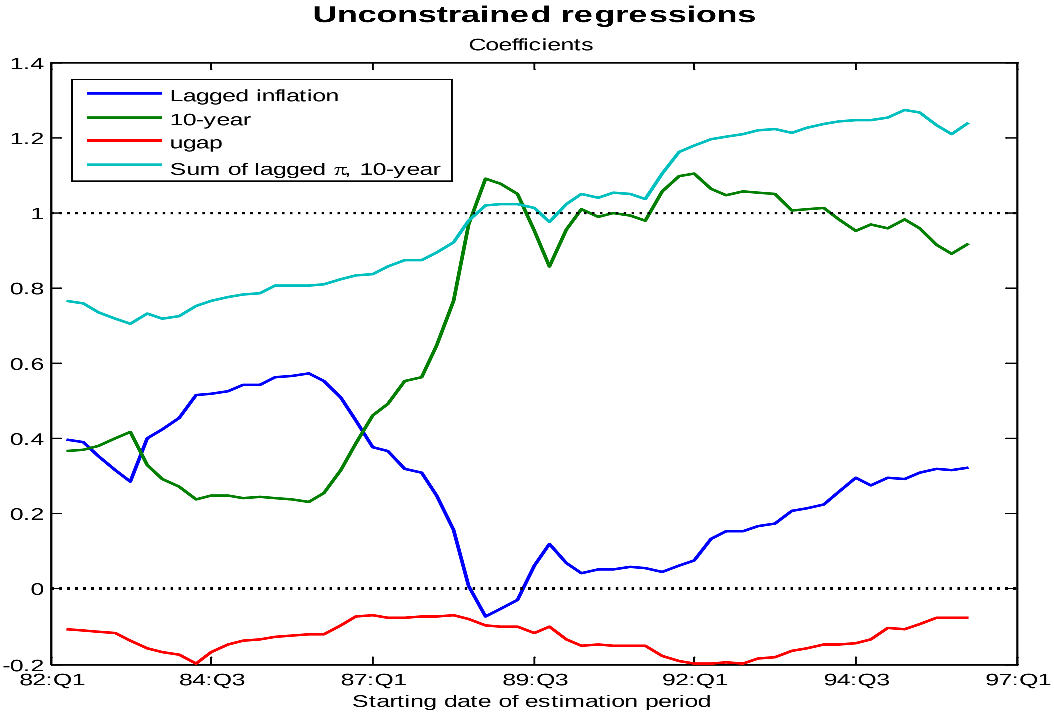


Figure 11

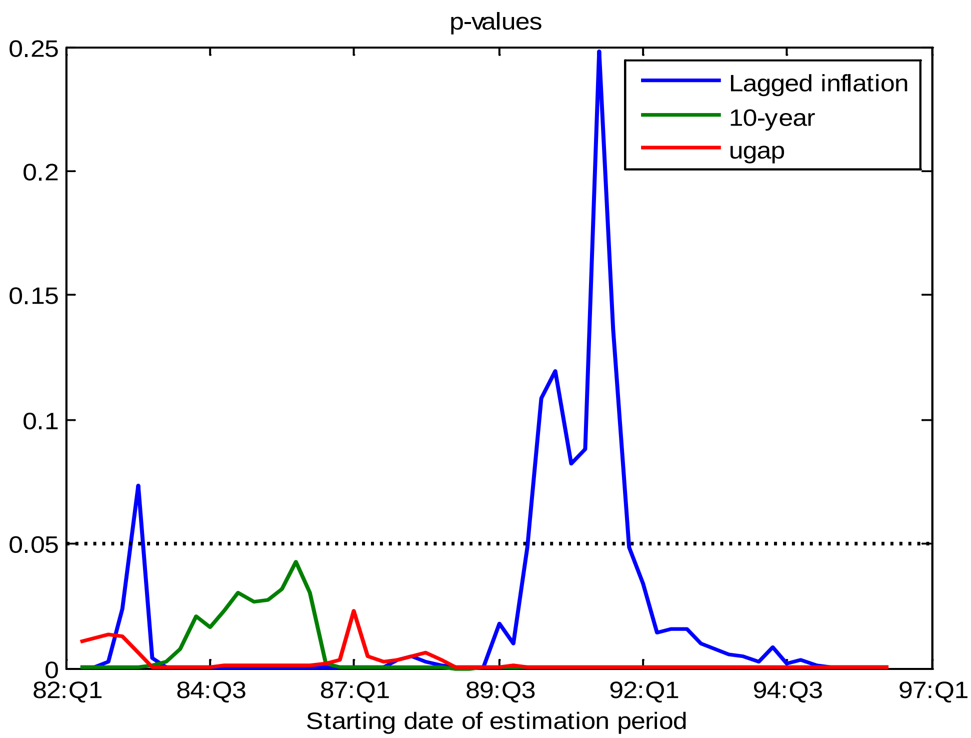


Figure 12

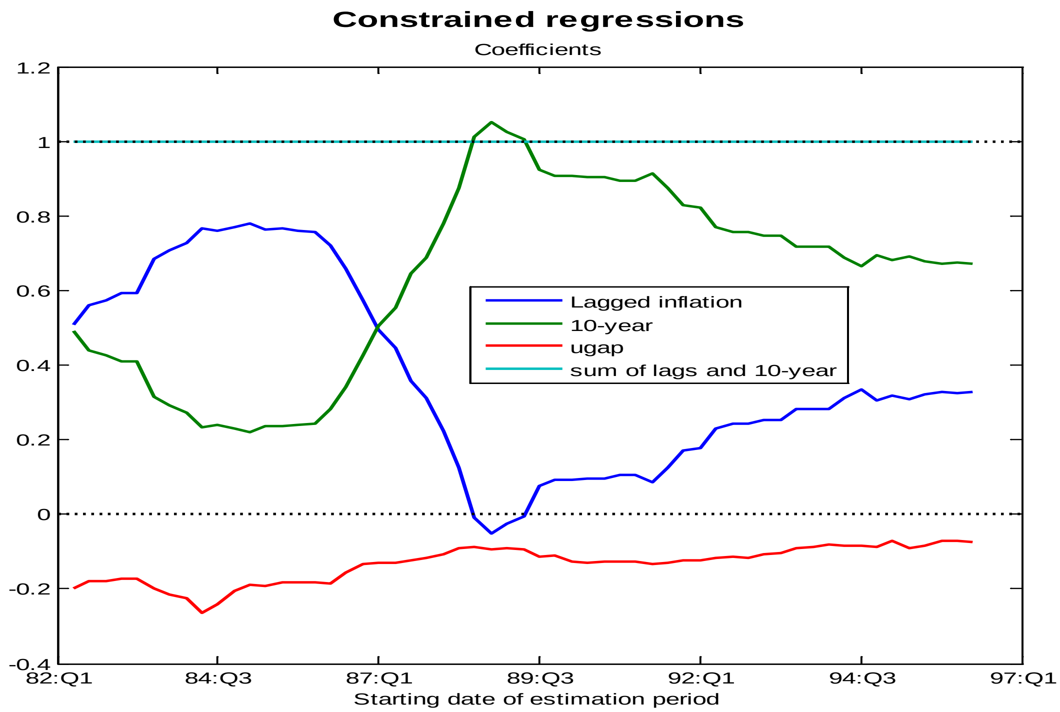


Figure 13

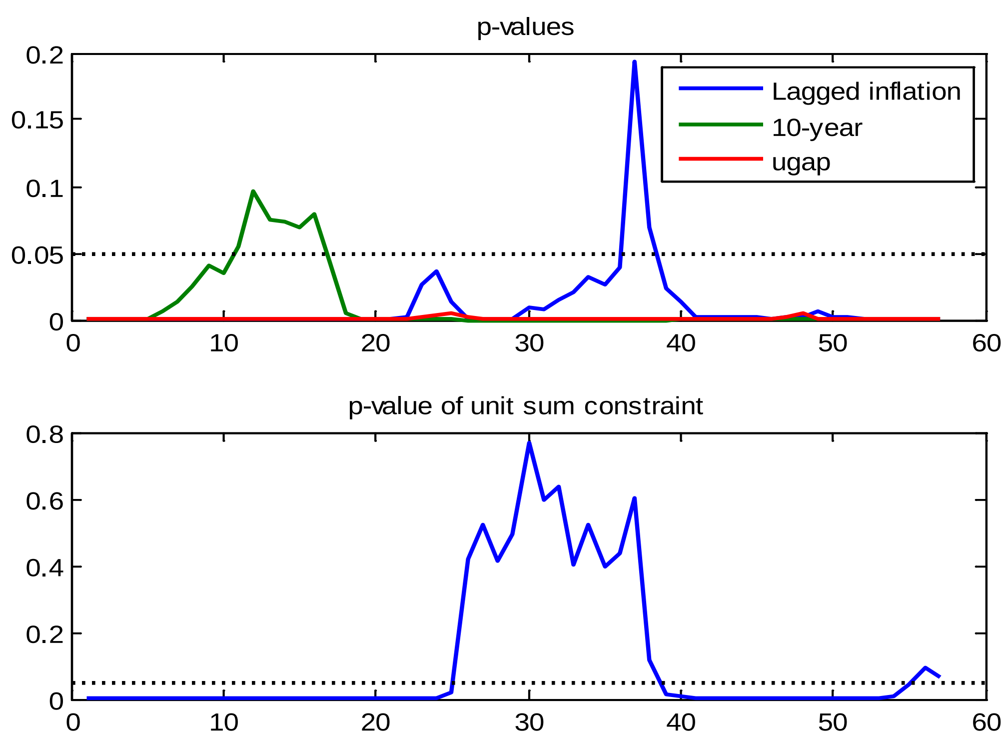


Figure 14

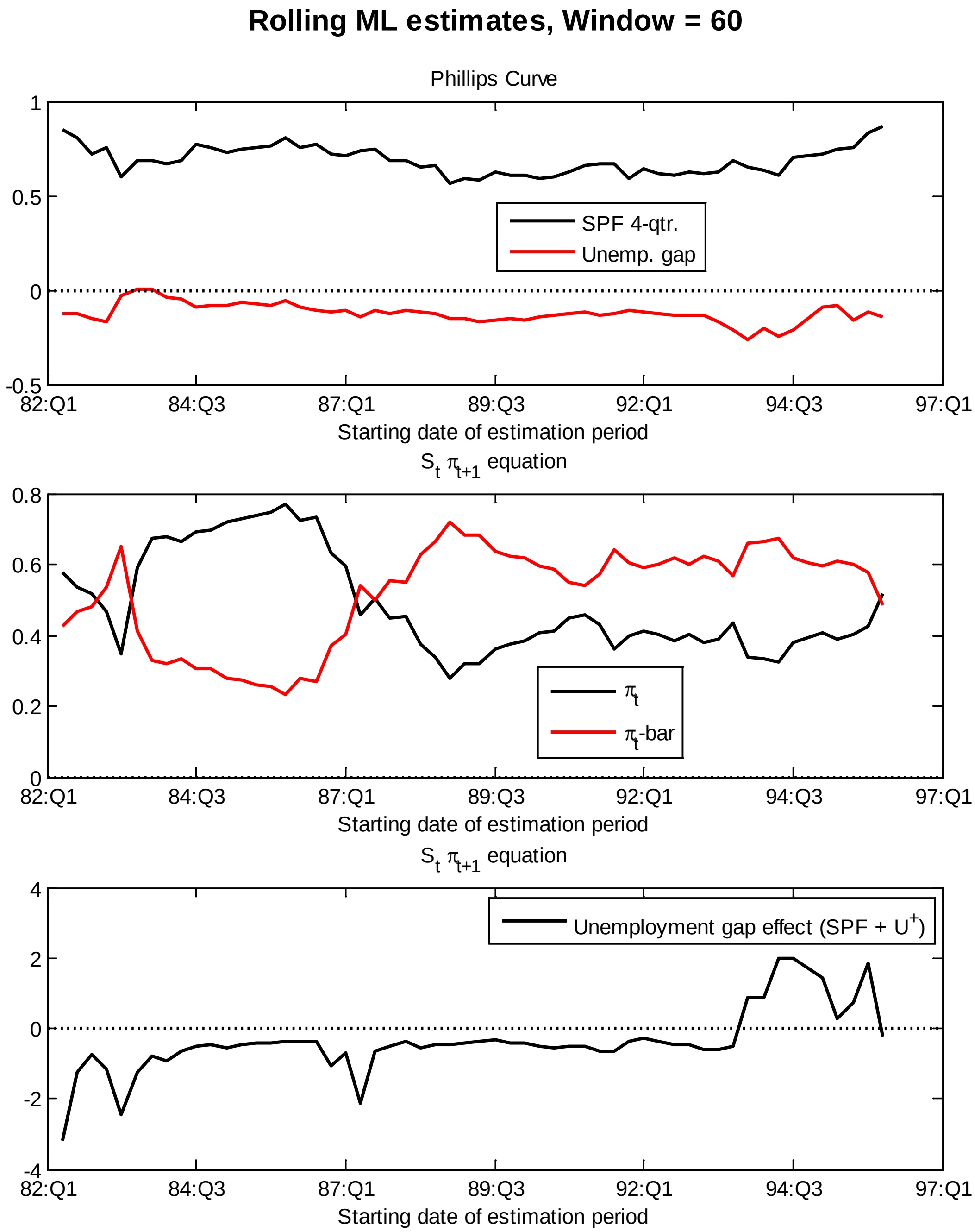
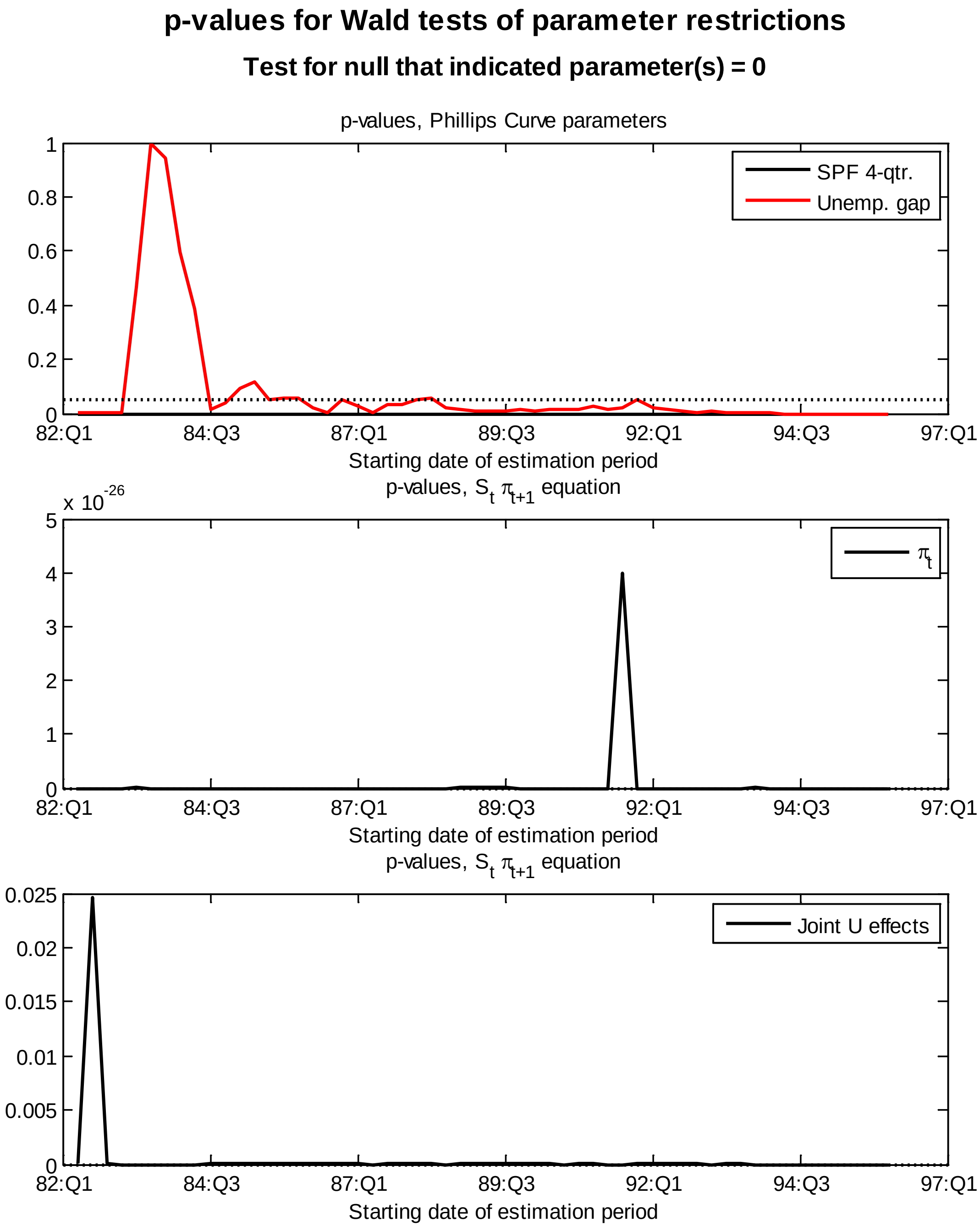

Rolling ML estimates, Window = 60
Phillips Curve
SPF 4-qtr.
Unemp. gap
1
0.5
0
-0.5
82:Q1
84:Q3
87:Q1
89:Q3
92:Q1
94:Q3
97:Q1
Starting date of estimation period
S_t π_{t+1} equation
π_t
π_t-bar
0.8
0.6
0.4
0.2
0
82:Q1
84:Q3
87:Q1
89:Q3
92:Q1
94:Q3
97:Q1
Starting date of estimation period
S_t π_{t+1} equation
Unemployment gap effect (SPF + U^+)
4
2
0
-2
-4
82:Q1
84:Q3
87:Q1
89:Q3
92:Q1
94:Q3
97:Q1
Starting date of estimation period

Figure 15



Appendix

Maximum likelihood estimates for Table 2, using the unemployment rate

Table A.1 Estimation results for equation 3, various estimators, 1990:Q1-2010:Q3 Unemployment rate substitutes for real marginal cost			
Maximum likelihood Trend inflation = SPF 10-year expectation			
Coefficient	Estimate	Standard error[a]	*t*-statistic
Lagged inflation	0.30	0.11	2.6
Rational expectation	0.00	-	-
Survey expectation	0.70	0.19	3.7
Test of trend inflation restrictions			
Test of trend inflation restrictions: 0.00 Test for exclusion of 10-year expectation and RE from model: 0.027			
Maximum likelihood Trend inflation = Cogley-Sbordone trend inflation measure			
Coefficient	Estimate	Standard error[a]	*t*-statistic
Lagged inflation	0.48	0.17	4.9
Rational expectation	0.00	0.094	0.01
Survey expectation	0.52	0.098	3.1
Test of trend inflation restrictions			
Test of trend inflation restrictions: 0.37 Test for exclusion of Cogley-Sbordone trend inflation measure and RE from model: 0.99			
Robustness check: ML estimate including 1980s Trend inflation = SPF 10-year expectations			
Coefficient	Estimate	Standard error[a]	*t*-statistic
Lagged inflation	0.022	0.071	0.31
Rational expectation	0.00	-	-
Survey expectation	0.98	0.25	3.9
Optimal instruments GMM Trend inflation imposed			
Coefficient	Estimate	Standard error[a]	*t*-statistic
Lagged inflation	0.80	0.080	9.9
Rational expectation	0.001	0.095	0.01
Survey expectation	0.98	0.30	3.2
Conventional GMM, Trend inflation imposed			
Coefficient	Estimate	Standard error[a]	*t*-statistic
Lagged inflation	-0.045	0.19	-.24
Rational expectation	0.96	0.42	2.3
Survey expectation	0.094	0.81	0.12
Stock-Yogo test for instrument relevance: 1.079 [a]BHHH standard errors **Phil_est_ml_U.m, Phil_est_gmm_U.m**			

Unbiasedness and efficiency tests

Conventional tests for unbiasedness and efficiency fail frequently for the SPF survey measures employed in this study. The test regressions employed for unbiasedness and efficiency, respectively, are

$$\begin{aligned} \pi_t &= a\pi_{t-1}^{SPF,t} + b; H_0 : a = 1, b = 0 \\ \pi_t - \pi_{t-1}^{SPF,t} &= BZ_{t-1}; H_0 : B = 0 \end{aligned} \tag{1.1}$$

where *Z* represents a set of variables known as of time *t-1* that might hold predictive power for the one-quarter forecast error made by the SPF. We include four quarterly lags of the civilian unemployment rate and both core and headline consumer price inflation. While the joint test for unbiasedness rejects for the second half of the sample for the core CPI, the test for the headline CPI fails to reject, perhaps owing to the extreme volatility of the headline series.

Table A.2
Unbiasedness and Efficiency Tests for SPF 1-quarter inflation forecast

Unbiasedness test, using core CPI 1-qtr. change					
Sample	*p*-value	Forecast	Std. err.	Intercept	*t*-statistic
1982:Q3-2010:Q2	0.57	0.95	0.12	0.086	0.26
1995:Q1-2010:Q2	0.001	0.64	0.18	0.66	1.4
2000:Q1-2010:Q2	0.044	0.60	0.27	0.74	1.2
Unbiasedness test, using headline CPI 1-qtr. change					
Sample	*p*-value	Forecast	Std. err.	Intercept	*t*-statistic
1982:Q3-2010:Q2	0.19	0.71	0.16	0.78	1.4
1995:Q1-2010:Q2	0.63	0.54	0.26	1.18	1.7
2000:Q1-2010:Q2	0.77	0.83	0.43	0.66	0.78

Efficiency tests, core CPI 1-qtr. change				
Sample	Unemployment	Inflation	Core inflation	Joint test
1982:Q3-2010:Q2	0.19	0.0018	0.0042	0.0083
1995:Q1-2010:Q2	0.083	0.0057	0.15	0.0075
2000:Q1-2010:Q2	0.078	0.037	0.24	0.030
Efficiency tests, headline CPI 1-qtr. change				
Sample	Unemployment	Inflation	Core inflation	Joint test
1982:Q3-2010:Q2	0.0021	0.095	0.57	0.0020
1995:Q1-2010:Q2	0.030	0.14	0.30	0.050
2000:Q1-2010:Q2	0.034	0.10	0.49	0.088

More compelling are the efficiency tests, displayed in the bottom panels of the table. The test uses CPI and unemployment data, which are largely unrevised over time, so as to avoid serious real-time data problems. It is not the case that the tests fails for every predictor for every sub-sample. However, the joint test for efficiency with respect to all the variables in the test rejects uniformly for the core CPI, and for all but the last decade for the headline CPI. While the overall efficiency test fails to reject at the five percent level, it is still the case that the headline CPI forecast is always inefficient with respect to the unemployment rate.

Alternative estimates of equation (9)

An alternative way to estimate equation (9) is to compute the U^+ term as the weighted forecasts from an unconstrained vector autoregression. Table A.3 below presents results from the following exercise:

- For a given estimation range, estimate the coefficients for a three-variable VAR in the core inflation rate, the unemployment gap, and the federal funds rate;
- Use these coefficients to compute the weighted sum of unemployment gap forecasts (U^+);
- Forming this weighted sum requires a value for δ in equation (6). We iterate over values of δ between 0 and 1;
- Now estimate equation (9) via OLS, using the estimated values of U^+ for given δ, and choose the value of δ that minimizes the sum of squared residuals for the regression.

Table A.3

Estimates of structural equation for inflation expectations, various sub-samples

Variable	Coeff.	Stand. Error	Coeff.	Stand. Error	Coeff.	Stand. Error	Coeff.	Stand. Error	Coeff.	Stand. Error
	1982:Q1-2010:Q3		1982:Q1-1996:Q4		1987:Q1-2001:Q4		1992:Q1-2006:Q4		1996:Q1-2010:Q3	
π_t	0.24	0.089	0.27	0.095	0.23	0.10	-0.064	0.058	-0.052	0.053
$\overline{\pi}_t$	0.76	0.11	0.73	0.15	0.77	0.19	1.06	0.075	1.05	0.053
$S_t\tilde{U}_t^Y$	-0.20	0.14	-0.61	0.17	-0.18	0.21	-0.26	0.12	-0.062	0.062
$S_t\tilde{U}_{t+1}^Y$	0.12	0.14	0.70	0.25	0.11	0.25	-0.070	0.11	-0.30	0.050
U^+	-0.25	0.32	-0.37	0.65	-0.18	0.32	0.52	0.25	0.72	0.16
δ	0.995		0.18		0.995		0.32		0.69	