Investment Dynamics with Natural Expectations

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Abstract

We study an investment model in which agents have the wrong beliefs about the dynamic properties of fundamentals. Specifically, we assume that agents under-estimate the rate of mean reversion. The model exhibits the following six properties. (1) Beliefs are excessively optimistic in good times and excessively pessimistic in bad times. (2) Asset prices are too volatile. (3) Excess returns are negatively autocorrelated. (4) High levels of corporate profits predict negative future excess returns. (5) Real economic activity is excessively volatile; the economy experiences amplified investment cycles. (6) Corporate profits are positively autocorrelated in the short-run and negatively autocorrelated in the medium run. The paper provides a formal model of animal spirits, amplified business cycles, and excess volatility.

PRELIMINARY AND INCOMPLETE

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1 Introduction

We study an economy in which agents have the wrong beliefs about the dynamic properties of fundamentals (cf. Friedman 1979). In particular, we assume that agents under-estimate the rate of mean reversion in fundamentals. We believe that this case is relevant for three inter-related reasons.

First, a large body of research, which we review below, reports evidence for extrapolation bias.

Second, there are several fundamental psychological biases that lead agents to underestimate mean reversion: e.g., representativeness and anchoring.

Third, agents tend to estimate and employ simple forecasting models that incorporate a small number of lags. When the true fundamentals follow hump-shaped dynamics, simple forecasting models underestimate the amount of mean reversion (Fuster, Laibson and Mendel 2010, Fuster, Hebert, and Laibson 2011).

Whatever the reason that agents underestimate mean reversion, an economy that features such a bias will exhibit the following six properties: (1) procyclical excess optimism, (2) excessively volatile asset prices, (3) negatively autocorrelated excess returns, (4) a negative relationship between current corporate profits and future excess returns, (5) amplified investment cycles, and (6) negatively autocorrelated corporate profits in the medium run. In summary, the model provides a formal theory of animal spirits, amplified business cycles, and excess volatility.

The argument of the paper is organized in the following way. Section 2 discusses the empirical and psychological motivations for our key assumption, as well as the related literature. Section 3 solves an investment 'q' model in which agents underestimate the degree of mean reversion in fundamentals. Section 4 calibrates the model. Section 5 discusses the key properties of the model and illustrates these properties by studying the impulse response functions associated with the model. Section 6 identifies directions for future research.

2 Evidence on and reasons for the underappreciation of mean reversion

2.1 Evidence for Extrapolation Bias

A large body of empirical research reports evidence for extrapolation bias. Lab experiments in which subjects are asked to forecast financial or other time series find that extrapolative expectations or "trend following" provide a good description of observed beliefs (De Bondt 1993; Hey 1994) and may be a driving force behind the bubbles that are observed in asset-market experiments (Haruvy, Lahav, and Noussair 2007; Hommes et al. 2008).[1]

In field data, a number of papers have argued that asset allocation choices are affected by extrapolation of recent price appreciation (Chevalier and Ellison 1997; Sirri and Tufano 1998; Benartzi 2001; Choi et al 2004, 2009; Benartzi and Thaler 2007; Chalmers and Reuter 2009; Previtero 2010; Malmendier and Nagel 2011). One could argue that biases in expectations do not matter for asset pricing because investors with biased beliefs hold relatively little wealth. However, Vissing-Jorgensen (2003) shows that at the peak of the market in 2000-01, even wealthy investors expected stock returns to remain high. Bacchetta, Mertens, and van Wincoop (2009) conduct a similar exercise and find that, in several asset markets, investors' expectational errors about future returns are predicted by the same variables that predict excess returns.[2]

One might alternatively think that the expectations held (and made public) by financial analysts are not biased. However, De Bondt and Thaler (1990) argue that security analysts overreact and make earnings-per-share forecasts that are too extreme.[3] Most of the studies

[1] On the other hand, Dwyer et al. (1993) finds that subjects' forecasts of a random walk (in which growth has no persistence) do not deviate systematically from the rational expectations forecast.

[2] For instance, a high dividend/price ratio is a strong predictor of high subsequent excess returns. Similarly, when 'cay' (Lettau and Ludvigson 2001) is high, excess returns also tend to be higher than anticipated. However, surveyed investors tend to expect low excess returns when 'cay' is high.

[3] Other studies instead find that analysts underreact. Easterwood and Nutt (1999) argue that analysts overreact to positive information but underreact to negative information. Lim (2001) argues that considering analysts' objective function can "rationalize" their biases.

in this literature look at relatively short-run forecasts, while our model mostly has implications for long-run forecasts. Bulkley and Harris (1997) study five-years earnings forecasts for about 500 U.S. companies and report that i) analysts appear to extrapolate past growth in earnings when forecasting future growth, even though there is pronounced negative serial correlation in earnings growth over five-year periods, and ii) analysts' forecasts and excess returns over the subsequent five years are significantly negatively correlated.[4] Chan, Karceski, and Lakonishok (2003) provide further evidence that there is little predictability of long-term earnings growth rates, but that investors and analysts behave as if recent growth rates were positive predictors of future growth.

A significant literature in behavioral finance has accumulated evidence on cross-sectional stock return patterns that are consistent with such biases in expectations having strong effects on prices: De Bondt and Thaler (1985, 1989) and Lakonishok, Shleifer, and Vishny (1994) are among the best-known examples of such work.[5] Baker and Wurgler (2007) document that empirical measures of investor sentiment predict cross-sectional return patterns and also aggregate returns.

Apart from stock markets, other asset markets may also be influenced by biased beliefs. For instance, Greenwood and Hanson (2010) document patterns in bond risk premia that can be explained by investors extrapolating recent returns or default rates. Periods of high returns on corporate bonds are followed by a decline in issuer quality and low or negative excess returns on corporate debt in a highly predictable manner. Also, biased (extrapolative) beliefs have been advanced as a key explanation behind the recent housing bubble as well as earlier boom-bust cycles (Abraham and Hendershott 1996, Muellbauer and Murphy 1997,

[4]La Porta (1996) finds a negative relation between analysts' long-term growth estimates and future one-year risk-adjusted returns. Bergman and Roychowdhury (2008) document a positive relation between the consumer confidence index (a proxy for market sentiment) and the error in long-horizon earnings estimates of financial analysts, consistent with the idea that when times are good, market participants may insufficiently adjust for subsequent mean reversion.

[5]More recently, Chen, Moise and Zhao (2009) argue that myopic extrapolation can also explain momentum, if investors completely miss the hump-shaped dynamics of firm-specific earnings shocks and simply treat current earnings shocks as permanent. They point out that apart from cognitive biases, the practice to price securities using earnings multiples can also contribute to this phenomenon.

Case and Shiller 2003, Gerardi et al. 2008, Goetzman, Peng and Yen 2009, Piazzesi and Schneider 2009, Glaeser, Gottlieb and Gyourko 2010).

2.2 Fundamental Psychological Biases

There are two important psychological biases that lead agents to underestimate mean reversion. The heuristic of representativeness (Kahneman and Tversky 1973; Tverksy and Kahneman 1974) describes the fact that people mistakenly believe that small samples are representative of population samples. This mistake leads agents to believe that recent observations are viewed as representative of the future. Thus representativeness leads agents to underestimate the degree of mean reversion (Kahneman and Tversky 1973).

The availability heuristic (Tversky and Kahneman 1973) also predicts that agents will under-estimate the strength of mean reversion. Availability leads people to overweight information that is easily accessible and salient. Hence, availability bias implies that people will believe that the future will look like the highly available present.

Some observors have argued that related biases play an important role in driving aggregate dynamics. For instance, Reinhart and Rogoff (2009) document how investors time and time again fall prey to the belief that "this time is different" and that this causes recurrent financial crises. Relatedly, Shiller (2005) points out the lure of "new era" stories and how they are associated with episodes of bubbles in asset markets. Barberis (2010) notes that over-extrapolation of past prices changes may have been an important psychological driving force during the run-up to the Great Recession.

2.3 Natural Expectations

Agents tend to estimate and employ simple forecasting models that incorporate a small number of lags. When the true fundamentals follow hump-shaped dynamics, simple forecasting models underestimate the amount of mean reversion (Fuster, Laibson and Mendel 2010, Fuster, Hebert, and Laibson 2011). The premise of this approach is that economic agents tend to make forecasts based on statistical or mental models that are reasonable given the data available to them, but "too simple" to fully capture the long-term dynamics of many economic time series.

For example, in Fuster, Hebert, and Laibson (2011), we study total capital income in the U.S. NIPA accounts. We find that the estimated level of long-run persistence of shocks is very sensitive the order of the model being estimated: Models with a small number of (high-frequency) lags generate estimates of persistence around one, while models with a large number of lags generate much lower estimates of persistence. For example, Figure 1 plots the associated impulse response functions for ARIMA(p,1,0) models with $p = 1, 10, 20, 30, 40$. For ARIMA(p,1,0) models with $p = 1$ and 10, the estimated magnitude of persistence is greater than or equal to one. For ARIMA(p,1,0) models with $p = 30$ and 40, the estimated level of persistence is less than or equal to 0.6. More generally, Fuster, Mendel, and Laibson (2010) show that several macroeconomic time series have persistence estimates that fall sharply with the order of the model being estimated.

This line of analysis implies that if the true data generating process is hump-shaped and if agents use simple models, then agents will have upwardly biased estimates of persistence – i.e., underappreciation of mean reversion.

2.4 Related Models

A variety of "behavioral" models have been proposed to explain stock return patterns, including DeLong et al. (1990), Barberis, Shleifer, and Vishny (1998), Daniel, Hirshleifer and Subrahmanyam (1998) and Hong and Stein (1999).Closely related are models in which investors continuously update their belief about future dividend growth or other parameters. This learning, which can be interpreted as behavioral or fully rational (similar to our model), generates predictability in returns as well as excess volatiliy. Among the best known papers in this literature are Barsky and DeLong (1993) and Timmermann (1993). While our approach is closely related to these earlier papers, most previous authors consider simpler setups (often partial equiliibrium valuation models without consumption) to illustrate the consequences of biased beliefs or learning, and do not study the interrelation between asset prices and other macroeconomic variables. Exceptions to this include papers by Cecchetti, Lam, and Mark (2000), Lansing (2006), Choi (2006) and Adam and Marcet (2010) who study consumption-based asset pricing models with distorted beliefs or misspecified models. Lansing (2009) studies a general equilibrium model with endogenous growth and capital adjustment costs, in which the Solow residual follows a stationary but highly persistent AR(1) process. However, agents misperceive the process as a random walk. This induces excess volatility in the stock market, as well as amplified investment and consumption cycles.[6]

More generally, a large literature in macroeconomics and finance, surveyed in Sargent (1993), Evans and Honkapohja (2001, 2009, 2011) and Pastor and Veronesi (2009) assumes that agents are rational (in the sense that they think like highly sophisticated statisticians) but have to learn the relevant parameters of the reduced form equations governing the economy over time (e.g., Friedman 1979). While many early papers in this literature focused on whether expectations would ultimately converge to the rational expectations equilibrium, more recent work has considered what happens if agents have misspecified models and/or downweight older data, and finds that this can generate additional volatility and persistence of shocks in asset prices and/or the economy (e.g. Friedman and Laibson 1989, Branch and Evans 2007, 2010; Hong, Stein, and Yu 2007; Huang, Liu and Zha 2009; Eusepi and Preston forthcoming). In these models, "misspecification" means that agents omit a relevant variable from their forecasting equation, while in our model, it means that they may not include enough lags of the variable they are trying to forecast.[7]

An alternative modeling approach, exemplified by the agent-based literature, assumes that agents (probabilistically) select among different forecasting models based on past per-

[6] Lansing also references a number of empirical papers that document a relationship between stock market mispricings and real investment.

[7] The downweighting of old data is often captured by assuming "constant gain" rather than "decreasing gain" (least squares) learning. Some papers, such as Marcet and Nicolini (2003), endogenize agents' choice between constant and decreasing gain based on recent prediction errors.

formance of the models (for example, LeBaron et al. 1999; Tesfatsion and Judd 2006; De Grauwe 2010; LeBaron 2010). An advantage of such models, which are usually analyzed computationally, is that they generate heterogeneity in beliefs, which our model omits. Such heterogeneity allows for instance the study of wealth dynamics and trading volume.[8]

3 Investment Model

We study a tractable version of the continuous-time q-model (e.g., Hayashi 1982). This is a partial equilibrium model in which agents/firms are assumed to be risk neutral and the risk free rate is fixed.

We first present the model assuming that agents have correct beliefs about the data generating process (DGP) for fundamentals. We then analyze the model's properties assuming that agents *believe* that they have the correct beliefs about the DGP but actually don't. We study the model in a deterministic setting, but this assumption is without loss of generality. Adding Brownian motion to the DGP won't change the impulse response functions that we report below.

3.1 Notation and definition of the problem for a rational agent

Let i index a fixed set of firms on the unit interval, $i \in [0, 1]$. Let $k(i, t)$ represent the level of firm i's capital stock at time$t$. It therefore follows that aggregate capital is given by

$$K(t) = \int_0^1 k(i, t)di.$$

Henceforth, we assume that all firms are identical and suppress the i index. Therefore, we can write

$$K(t) = k(t).$$

[8] See Hong and Stein (2007) for a discussion of models of disagreement in a finance context.

Let $\pi(K, X)$ represent the instantaneous flow of revenue per unit of capital, where X is an exogenous productivity measure. We make the standard assumption that greater (industry-wide) competition reduces the flow of revenue per unit of capital (holding all else equal). In other words,

$$\frac{\partial \pi(K, X)}{\partial K} < 0.$$

By definition, $k(t)\,\pi(K, X)$ is the instantaneous revenue flow realized by a firm with $k(t)$ units of capital. This multiplicative structure implies that individual firms have a constant returns to scale technology.

To keep the model analytically tractable, we assume

$$\pi(K, X) = 1 - K(t) + X(t).$$

We assume that the exogenous productivity parameter X mean reverts at at rate ϕ. Specifically,

$$dX(t) = -\phi X(t)$$

where ϕ is a constant.

Firms only have one decision to make: the flow of investment. Let $\dot{k} = I$, so I is firm-level investment. Since firms are identical and indexed on [0,1] it also follows that

$$\dot{K} = \int_0^1 I(i, t) di = I(t).$$

We assume that firms pay quadratic adjustment costs $C(I) = \frac{\alpha}{2} I^2$. We assume that firms also pay a (normalized) price of one for each unit of uninstalled capital. So the total instantaneous flow cost of a flow of I units of capital is $I + C(I)$.

Finally, ρ is the discount rate, which is also the (fixed) real interest rate, r. Hence, the objective function of a firm can be written:

$$\sup_{I(t)} E_0 \int_{t=0}^{\infty} \exp(-\rho t)\,[k(t)\,\pi(K(t),X(t)) - I(t) - C(I(t))]\,dt.$$

subject to the dynamic accumulation equation

$$\frac{dk(t)}{dt} = I(t).$$

3.2 Value Function, FOC, and q

The state variables for this optimization problem are k, K, and X. We include both k and K since these variables can deviate in principle, though they won't deviate in equilibrium. The continuous-time Bellman Equation is

$$\rho V(k,K,X) = \sup_I \left\{ (k\,\pi(K,X) - I - C(I)) + E\left[\frac{dV}{dt}\right] \right\}. \tag{1}$$

Expanding $\frac{dV}{dt}$,

$$E\left[\frac{dV}{dt}\right] = \frac{\partial V}{\partial k} I + \frac{\partial V}{\partial K} I - \frac{\partial V}{\partial X} \phi X. \tag{2}$$

Note that no second-order terms arise, since we are considering the deterministic case. The first order condition is the standard one:

$$1 + C'(I) = \frac{\partial V}{\partial k} \tag{3}$$

Hence, the marginal cost of acquiring and installing capital equals the marginal value of installed capital.

Alternatively, we can define the value function as the expected present value of the flow payoffs.

$$V(k(t),K(t),X(t)) = \sup_{I(s)} E\left[\int_{s=t}^{\infty} \exp(-\rho(s-t))\left[k(s)\pi(K(s),X(s)) - I(s) - C(I(s))\right] ds\right] \tag{4}$$

For now, assume the firm has correct expectations about the future. Following the standard treatment of this model, define $q(t)$ as the marginal present value of a unit of installed capital:

$$q(t) = E_t\left[\int_t^{\infty} \exp(-\rho(s-t))\pi(K(s),X(s))ds\right] \tag{5}$$

It follows that,

$$\frac{\partial V(k(t),K(t),X(t))}{\partial k(t)} = q(t) \tag{6}$$

To show this, note that

$$k(s) = k(t) + \int_t^s I(u)du \tag{7}$$

Substituting into the value function integral,

$$V(k,K,X) = \sup_{i(s)} E\left[\int_t^{\infty} \exp(-\rho(s-t))\left[\left(k(t) + \int_t^s I(u)du\right)\pi(K(s),X(s)) - I(s) - C(I(s))\right] ds\right] \tag{8}$$

Differentiating by $k(t)$, and applying the envelope theorem,

$$\frac{\partial V(k(s),K(s),X(s))}{\partial k(s)} = E\left[\int_{s=t}^{\infty} \exp(-\rho(s-t))\pi(K(s),X(s))ds\right] = q(s) \tag{9}$$

We can think of q as a value function, with a flow payoff of $\pi(K(t),X(t))$. Apply Leibniz's rule to show that

$$\rho q = \pi(K(t),X(t)) + E_t\left[\frac{dq}{dt}\right] \tag{10}$$

3.3 Solving the system

From our assumption about $C(i)$,

$$\begin{aligned} C'(I) &= \alpha I & (11) \\ C'^{-1}(y) &= \frac{y}{\alpha}. & (12) \end{aligned}$$

The firm's policy is

$$I = C'^{-1}(q-1) = \frac{1}{\alpha}(q-1) \tag{13}$$

Aggregate capital evolves as

$$\frac{dK}{dt} = I = \frac{1}{\alpha}(q-1) \tag{14}$$

We can now define a system of first-order differential equations. Define the state vector, z, for the differential equation system:

$$z = \begin{bmatrix} q \\ K \\ X \end{bmatrix} \tag{15}$$

The evolution of the system is:

$$\frac{dz(t)}{dt} = D + Bz(t) = \begin{bmatrix} -1 \\ -\frac{1}{\alpha} \\ 0 \end{bmatrix} + \begin{bmatrix} \rho & 1 & -1 \\ \frac{1}{\alpha} & 0 & 0 \\ 0 & 0 & -\phi \end{bmatrix} z(t) \tag{16}$$

Define the vector Q:

$$Q = \begin{bmatrix} -1 \\ \rho - 1 \\ 0 \end{bmatrix} \tag{17}$$

and note that,

$$BQ = D. \tag{18}$$

Solving for the expectation of $z(t)$, assuming B is invertible (which is a convergence assumption),

$$E_t[z(t+\tau)] = -Q + \exp(B\tau)C(t). \tag{19}$$

Hence, $-Q$ is the steady state vector for the state vector $z(t)$.

All that remains is to solve for the date-t forecasting "constant" $C(t)$. We know the initial conditions for K and X, but need one more condition. That condition is a transversality condition (finite q), and it will allow us to eliminate one of the eigenvalues of B. The characteristic equation for B is

$$-(\phi + \lambda)(\lambda^2 - \rho\lambda - 1) \tag{20}$$

The positive eigenvalue from the right term will be greater than ρ, and leads to infinite expected present value. Let V be the eigenvectors of B. Define a 2×3 matrix, L, as

$$L = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{21}$$

We define V to have the eigenvectors in the usual order, so that the first vector in V is the one associated with the largest eigenvalue (which is the one that should have zero weight). Define $C(t)$ as

$$C(t) = VL'A(t) \tag{22}$$

for some length 2 vector $A(t)$. The initial conditions for $z(t)$ satisfy

$$Lz(t) = -LQ + LVL'A(t) \tag{23}$$

Solving,

$$A(t) = (LVL')^{-1}L(z(t) + Q) \tag{24}$$

The constants are therefore

$$C(t) = M(z(t) + Q) = VL'(LVL')^{-1}L(z(t) + Q) \tag{25}$$

This constitutes a solution to the entire system.

$$z(t + \tau) = -Q + \exp(B\tau)M(z(t) + Q) \tag{26}$$

It is also useful to note that

$$MB^kM = B^kM \qquad \forall k \tag{27}$$

which we use in the subsequent subsection. This is true because M is "made from" the eigenvectors of B. We can also use M to determine how the vector $C(t)$, and therefore $q(t)$, evolve. Substituting for $\tau = 0$ into 26,

$$z(t) = -Q + M(z(t) + Q) \tag{28}$$

Taking the total derivative,

$$dz(t) = M\left[D + Bz(t)\right] \tag{29}$$

Finally, we can solve explicitly for $z(t + \tau)$

$$z(t + \tau) = -Q + \exp(MB\tau)M(z(t) + Q). \tag{30}$$

Note that this formulation is consistent with 26

3.4 When Agents Have the Wrong Beliefs

Until this point, we have characterized a model in which agents have correct beliefs about the DGP for X. We now study the case in which the agent has incorrect beliefs. Let $\hat{B}$

be the perceived DGP process, with associated eigenvectors $\hat{V}$ and related matrix $\hat{M}$. The initial condition problem is

$$z(t) = -Q + \hat{M}(z(t) + Q) \tag{31}$$

As in the previous section, we can differentiate and plug in the evolution of $z(t)$. Note that

$$dz(t) = \hat{M}\left[D + Bz(t)\right] \tag{32}$$

Again solving for $z(t+\tau)$,

$$z(t+\tau) = -Q + \exp(\hat{M}B\tau)\hat{M}(z(t) + Q) \tag{33}$$

Note that this equation simplifies to the no-mistakes solution if $\hat{M} = M$. This above equation fully describes the evolution of the system under the mistake policy.

3.5 Impulse Response Function

Assume for $t < t_0$,

$$z(t) = z_{ss} = \begin{bmatrix} 1 \\ 1-\rho \\ 0 \end{bmatrix} \tag{34}$$

Note that when $z(t) = z_{ss}$,

$$dz(t) = D + Bz_{ss} = 0 \tag{35}$$

Assume that

$$Lz_0 = z_{init} = \begin{bmatrix} 1-\rho \\ \lambda \end{bmatrix} \tag{36}$$

Note that z_{init} has only two elements– it does not include q. Then for all time $t \geq 0$,

$$z(t) = -Q + \exp\left(\hat{M}B\tau\right)\,\hat{M}\,\left[L'z_{init} + Q\right] \tag{37}$$

4 Illustrative Calibration

We now describe an illustrative calibration. The model has four free parameters: α, ρ, ϕ, and $\hat{\phi}$. The qualitative properties of the model are not affected by the specific calibration decisions that we discuss below. However, the calibration clarifies the quantitative predictions of the model.

The parameter that scales adjustment costs is set to $\alpha = 10/(1-\rho)$. With this calibration, a permanent 10% change in the steady state capital stock has a half-life of adjustment of slightly more than two years.

We set the annual risk-free rate to 5% per year: i.e., $\rho = 0.05$. Because of the way that we've scaled adjustment costs, ρ does not play an important role in driving the model's properties. Hence, we could choose any (plausible) value for ρ and our dynamics would effectively be unchanged.

We assume that the true differential equation for X is given by $\dot{X} = -0.25\,X$, so $\phi = 0.25$. However, agents perceive relatively little mean reversion: $\dot{X} = -.05\,X$, so $\hat{\phi} = 0.05$.

Finally, we study a productivity shock of $\Delta X = 0.10$. In the case of rational expectations, this would correspond to a temporary increase in the capital stock that would peak about 2% above the steady state capital stock (4 years after the initial impulse).

5 Simulation of the investment model

5.1 Impulse Response Functions

We first report a series of impulse response functions that characterize the behavior of the economy. For these figures we report the impulse response function for the first 20 years following the shock.In all of these figures we adopt the following conventions.

The blue line represents the equilibrium path that would arise if agents all had rational expectations (the case $\phi = \hat{\phi} = 0.25$).

The green line represents the equilibrium path that would arise if agents beliefs about the future dynamics of X were accurate ($\phi = \hat{\phi} = 0.05$). Motivated by our earlier work (Fuster, Hebert, and Laibson 2011), we call this case the "natural expectations forecast." This is the impulse response function that our agents (mistakenly) anticipate.

The red line represents the equilibrium path that actually does arise, given the mismatch between beliefs ($\hat{\phi} = 0.05$) and reality ($\phi = 0.25$). We call this case the "natural expectations path." This is the impulse response function that an outsider would observe. Of course, once one adds noise to the economy, it would be difficult to accurately estimate this impulse response function with a small data sample.

Figure 2 reports the impulse response for the productivity parameter X. In our illustrative calibration, the process decays at an annual rate of 25% (rational expectations). However, agents perceive that it decays at a rate of 5% (natural expectations forecast).

Figure 3 reports the impulse response function for q, the price of a unit of installed capital. Since investment is affine in q, this Figure also reports the impulse response function for investment. Under rational expectations, the price of capital should rise by 17% following the productivity impulse and then fall back to its steady state level with a small amount of overshooting on the way down. Under natural expectations, the price of capital will rise by 26% following the productivity impulse and then fall back to its steady state level with somewhat more overshooting on the way down. Hence, the natural expectations case exhibits two kinds of excess volatility.[9] The price rises far more in the first place and then overshoots more on the way back to the steady state. This overshooting arises because of the overhang of capital that needs to be decumulated as productivity falls. This capital overhang exists even when expectations are rational, however, the overhang is stronger in the natural expectations case, because agents under-estimate the degree of mean reversion in productivity (X) and therefore accumulate too much capital in the few years immediately following the impulse. Finally, note that all three plotted case will eventually return to a

[9] The classic papers on excess volatility in stock markets are LeRoy and Porter (1981) and Shiller (1981).

steady state value of 1.

Figure 4 reports the impulse response function for the instantaneous (annualized) excess returns (ignoring the "infinite" return when the initial impulse arrives). In the rational expectations case, which is *not* reported, there are no excess returns (the analogous line is everywhere equal to zero). In the natural expectations case, there is a long trail of negative excess returns. The magnitude of these excess returns is empirically plausible. The negative excess returns begin at an annualized rate of -4% and slowly decline in absolute magnitude. After ten years, the annualized excess return is -50 basis points.[10]

Figure 5 reports the impulse response function for the profitability of the corporate sector. Following the initial impulse profits jump up and then drift back down as (i) capital is accumulated, driving down industry profits,[11] and (ii) productivity itself, X, begins to mean-revert. In the rational expectations case, the convergence to the steady state level of profits is nearly monotonic, with only a modest degree of overshooting. In Figure 5 the rational expectations overshooting is nearly imperceptible. Hence, in the rational expectations case, profits are generally positively auto-correlated. In the natural expectations case, the overshooting is much more pronounced, since the capital overhang is much greater. The large degree of overshooting generates intermediate horizon negative auto-correlatoin in corporate profits.

Figure 6 reports the impulse response function for the level of aggregate capital. For the rational expectations case, capital follows a hump-shaped pattern that peaks about four years after the initial impulse. For the natural expectations case, capital also follows a hump-shaped pattern that peaks about four years after the initial impulse. However,

[10]See e.g. Fama and French (1988a) and Poterba and Summers (1988), and Cutler, Poterba and Summers (1991) for evidence on long-term mean reversion in stock prices and other asset prices. Other authors, such as Campbell and Shiller (1988ab, 2005) and Fama and French (1988b), study earnings and dividend yields as predictors of future returns. In Fuster, Hebert and Laibson (2011), we report that over the period 1929 to 2010, the correlation between excess returns of equity over the risk-free rate in year τ and cumulative excess returns from year $\tau + 2$ to year $\tau + 5$ was -0.22, while the correlation between the ratio of S&P price at the end of year τ and average earnings over years $\tau - 9$ to τ and excess returns from year $\tau + 2$ to year $\tau + 5$ was -0.38. That paper also gives an overview of statistical caveats that apply to these findings.

[11]Recall that the revenue per unit of capital function is assumed to be $1 + X - K$. As K rises, revenue per unit of capital falls.

in the natural expectations case, the amplitude of the hump is 1.5 times as large as the rational expectations case. The larger hump arises because of the mistaken belief that the productivity impulse will only slowly mean-revert, leading to a much larger accumulation of additional capital.

5.2 Dynamics in K-q space

It is also useful to summarize the economy's dynamics with a Figure in K-q space. This figure draws out some of the key properties of the economy.

To read the figure, start in the lower left-hand corner. That point is the steady state. After a shock arrives, the path jumps vertically. Specifically, the price q jumps when the initial news arrives (the stock K is not a jump variable). The jump in q is much greater for the natural expectations case than for the rational expectations case. After the jump, the dynamics take the economy in a loop that begins by moving to the southeast and eventually returns to the (original) steady state. This loop is anticipated to be quite large (and slow) in the natural expectations case (the forecast of the natural expectations agents is in green). The dynamics turn out to be quicker than anticipated because productivity turns out to mean-revert faster than anticipated. However, the path that is actually observed in equilibrium (in red) has a far larger loop than it would have had under rational expectations. Agents who under-estimate mean reversion accumulate too much capital and later come to regret it when the asset price (q) falls earlier and more than anticipated.

6 Conclusion

This paper examines a partial equilibrium investment problem in which agents underestimate the strength of mean reversion in fundamentals. This deviation from rational expectations engenders the following equilibrium properties: (1) procyclical excess optimism, (2) excessively volatile asset prices, (3) negatively autocorrelated excess returns, (4) a negative relationship between current corporate profits and future excess returns, (5) excessively volatile investment cycles, and (6) negatively autocorrelated corporate profits in the medium run. The analysis that we have described provides a parsimonious and psychologically plausible explanation for a wide range of puzzling empirical patterns. The model also generates a series of falsifiable predictions of some regularities that have not yet been empirically investigated. Future work should test these predictions.

Many macro variables may be affected by the mechanisms discussed in this paper: e.g., housing prices, residential investment, non-residential investment, inventory accumulation, international capital flows, bond markets, and commodity prices.[12] In any asset market, under-estimation of mean reversion will generate amplified cycles, overreaction, excess volatility, and asset returns that are negatively autocorrelated over the medium-run.

An natural follow-up question is how non-rational expectations and non-fundamental asset price movements affect optimal monetary policy. While the illustrative model in this paper is too simple to allow adequate analysis of the trade-offs involved, work by Dupor (2005) and Mertens (2010) makes progress on this important question.

[12] For some alternative approaches, see for example Lansing 2009, Hassan and Mertens 2010, Adam and Marcet 2010, LeBaron 2010, Burnside, Eichenbaum, and Rebelo 2011, and Piazzesi and Schneider 2011.

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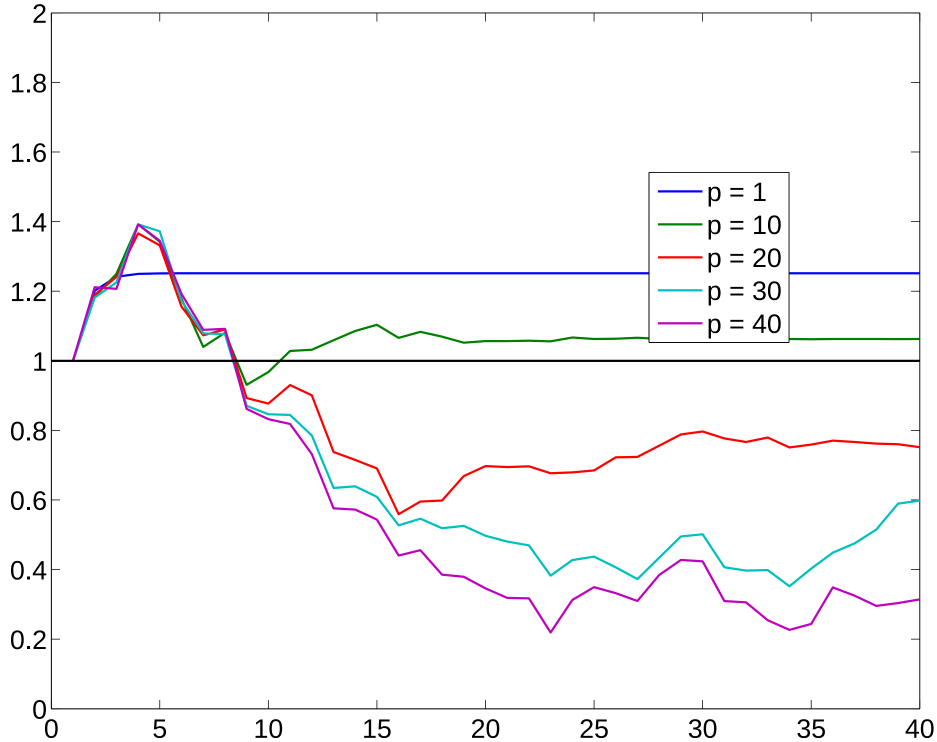
Figure 1:



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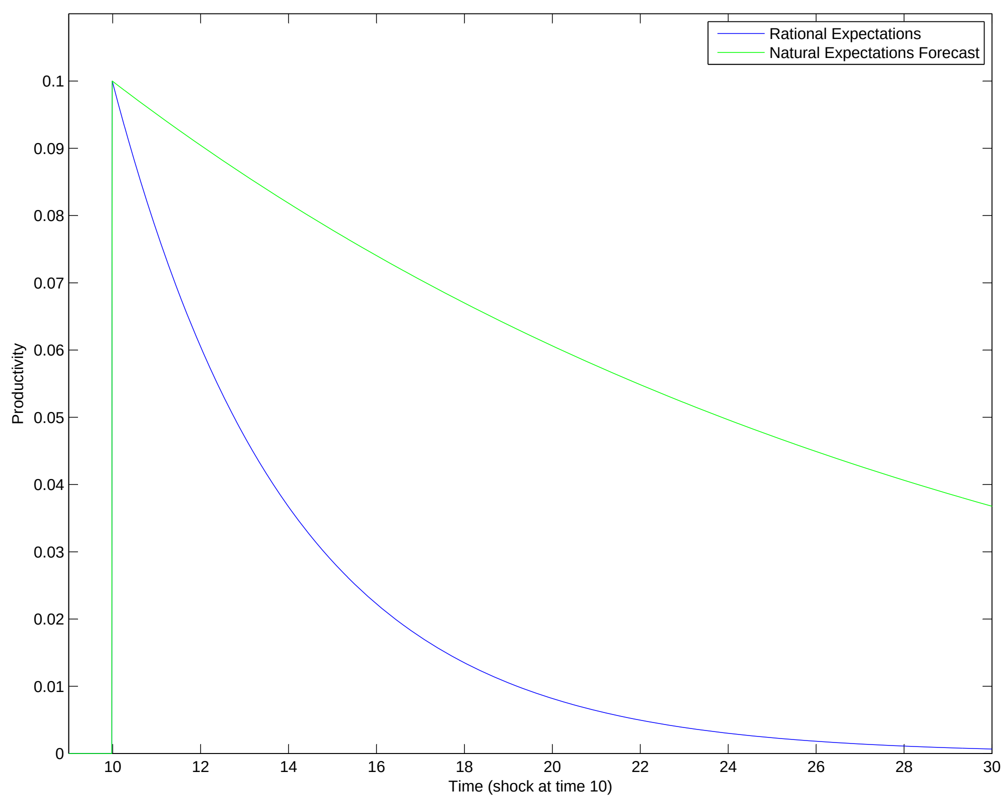

Rational Expectations
Natural Expectations Forecast
Productivity
0.1
0.09
0.08
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0.05
0.04
0.03
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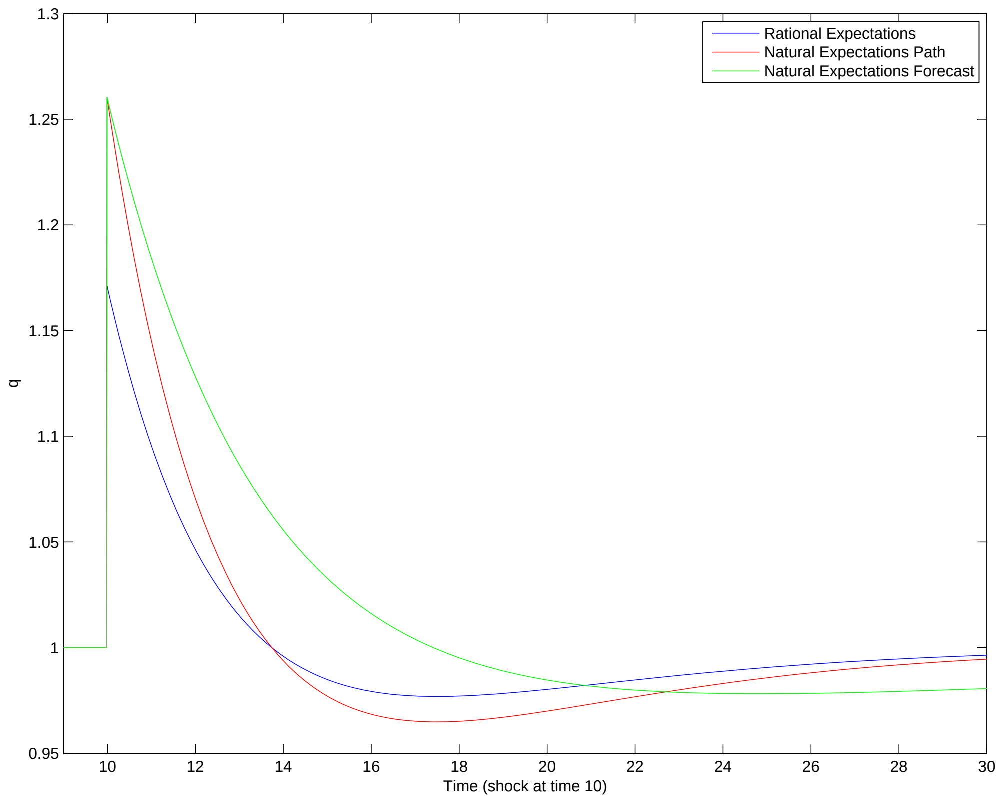
30
Time (shock at time 10)

Figure 3:


Rational Expectations
Natural Expectations Path
Natural Expectations Forecast
q
Time (shock at time 10)
1.3
1.25
1.2
1.15
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1
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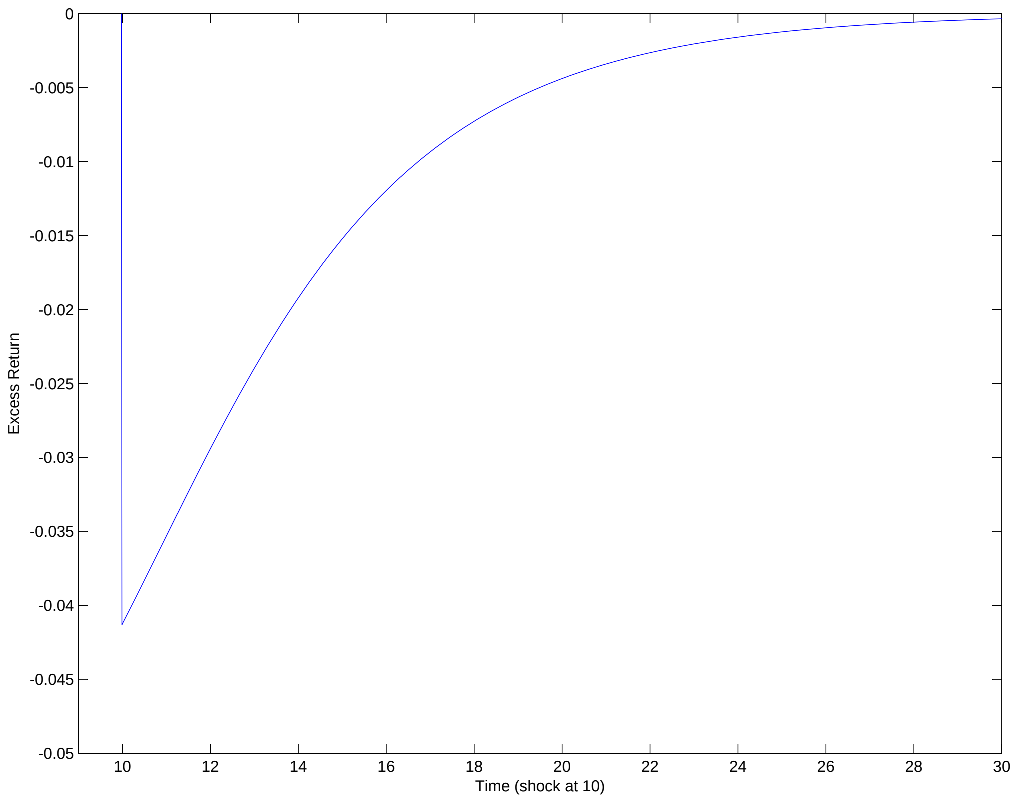
Figure 4:


Excess Return
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-0.005
-0.01
-0.015
-0.02
-0.025
-0.03
-0.035
-0.04
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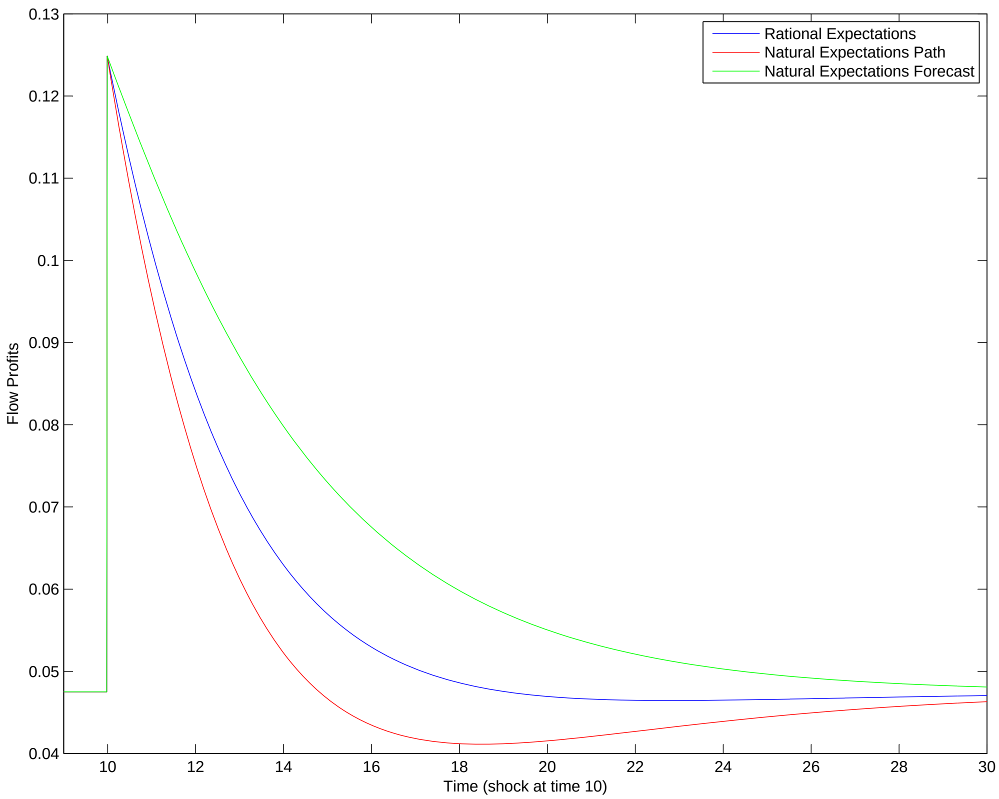
Time (shock at 10)

Figure 5:



Rational Expectations
Natural Expectations Path
Natural Expectations Forecast
Flow Profits
0.13
0.12
0.11
0.1
0.09
0.08
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0.06
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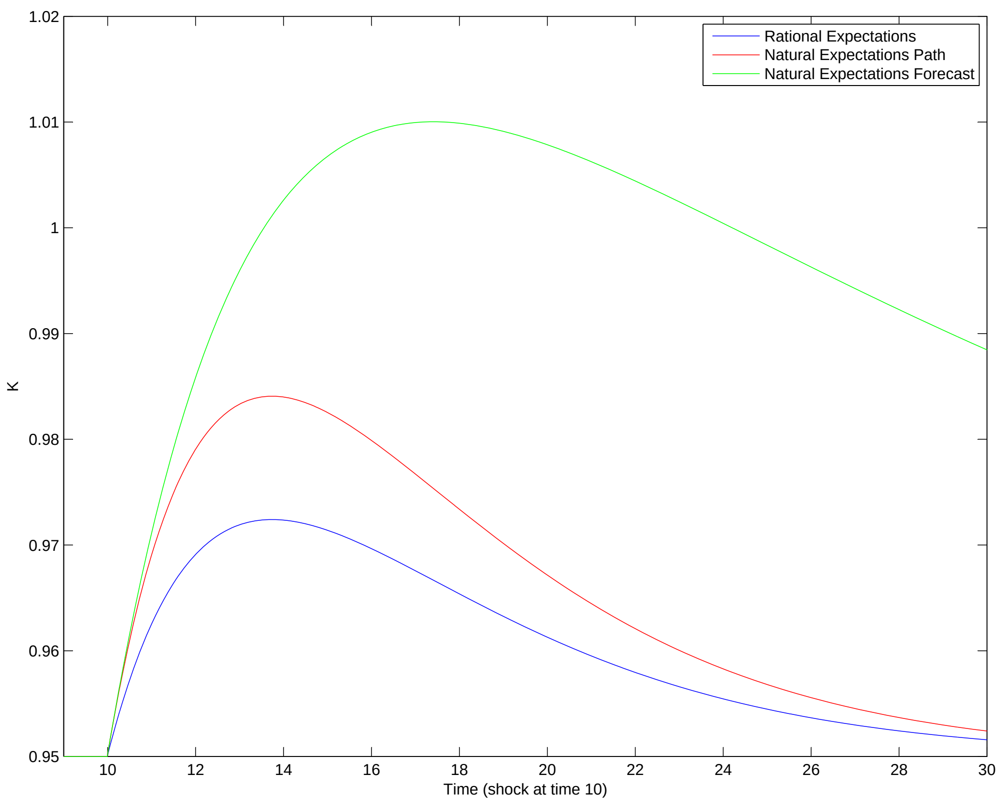
Time (shock at time 10)

Figure 6:



Rational Expectations
Natural Expectations Path
Natural Expectations Forecast
1.02
1.01
1
0.99
0.98
0.97
0.96
0.95
K
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22
24
26
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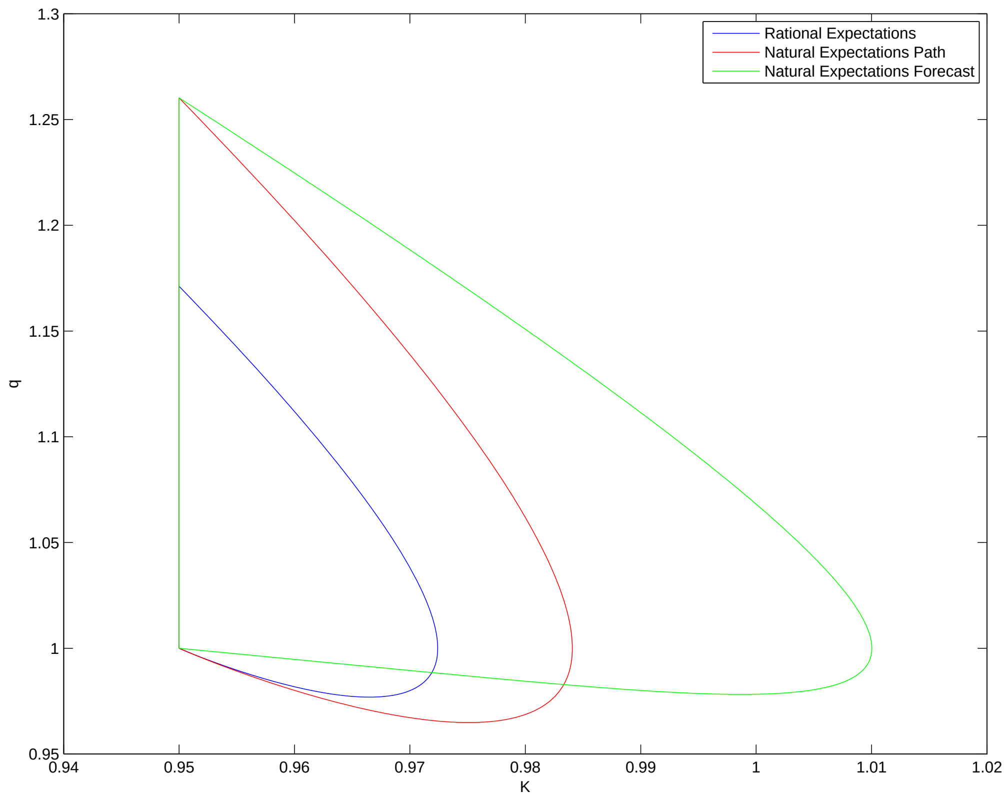
Time (shock at time 10)

Figure 7:



Rational Expectations
Natural Expectations Path
Natural Expectations Forecast
q
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1.3
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1.2
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1.1
1.05
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0.95
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