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Technical Appendix: On Financing Retirement with an Aging Population*

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1. Introduction

This appendix provides additional details for our paper "On Financing Retirement with an Aging Population." Specifically, we provide more details on our data sources and construction of the model's national accounts and fixed asset tables, some balance sheet items, and sources underlying the demographic variables. We discuss in more detail the baseline parameterization and the methods used in computing the model equilibria. And, finally, we conduct a sensitivity analysis of the main results, providing details that are not in the main text. For those interested in trying their own experiments, we have also made the codes available at our website, www.minneapolisfed.org.

2. U.S. Data

Here, we describe the main sources of our data: the U.S. national income and product accounts, the fixed asset tables, several balance sheet items from the flow of funds, population statistics and projections, and employment and hours.

2.1. National Accounts and Fixed Assets

The primary source of data used in our model accounts is the *U.S. National Income and Product Accounts* (NIPA) and *Fixed Asset Tables* compiled by the Bureau of Economic Analysis. These data are published in their *Survey of Economic Business* (and online at www.bea.gov). For certain imputations that we make, we also rely on data from the *Flow of Funds Accounts of the United States* compiled by the Federal Reserve Board of Governors and the *Statistics of Income* compiled by the Internal Revenue Service (IRS).

In Table 1—which is an expanded version of Table 1 in the main text—we provide all of the details of how we revise the NIPA to conform with theory. The main source of the domestic income data is the NIPA, Table 1.10. We also note the specific line numbers. With labor income we include compensation of employees and 70 percent of proprietors' income. All other income is categorized as capital income, which is adjusted in two ways. First, we subtract taxes other than property tax from the NIPA measure of taxes on production and imports. Second, we impute capital services for consumer durables—which we treat as investment—and government capital. The imputed services are estimated to be 4 percent times the current-cost net stock of consumer durable goods and government fixed assets. These stocks are reported in the BEA's fixed asset tables. In addition, we need to include depreciation of consumer durables, which is reported in the flow of funds accounts. With these adjustments, capital income is the sum of corporate profits, part of proprietors' income, surplus on government enterprises, rents, net income, property taxes, depreciation of capital, and imputed capital services.

On the product side, revisions must also be made with regard to sales taxes and capital services. The sales taxes are assumed to be primarily taxes on personal consumption expenditures.[1] We assume pro rata shares when assessing how much of the taxes are on durables, nondurables, and services. We include nondurables and services with consumption and durable goods with tangible investment. Therefore, we subtract sales taxes from both product categories. The imputed capital services only affect our measure of consumption which combines personal and government consumption from NIPA.

In the model, we distinguish between businesses that pay corporate income taxes (sector 1) and those that do not (sector 2). Businesses that pay corporate income taxes are Schedule C corporations. The others are Schedule S corporations, regulated investment companies, real estate investment trusts, proprietors, partnerships, household businesses, and government businesses. The BEA does not break out income and product data for Schedule C corporations, but the IRS does report data from tax returns separately for Schedule C corporations in the *Statistics of Income.* We use these return data to estimate investment and capital of our sector 1. In particular, we use the ratio of depreciable assets for Schedule C and all other corporations to estimate the ratios of stocks and investments in the model. According to the IRS, 83.5 percent of corporate depreciable assets are owned by Schedule C corporations. If we decompose gross private domestic investment into corporate and noncorporate components and assign 83.5 percent of corporate investment to Schedule C corporations, then we estimate that 0.07 GNPs of investment is done by Schedule C corporations. The remaining investment, 0.144 times GNP, is the sum of gross private domestic investment for other private business plus consumer durable goods net of tax, nondefense government gross investment, and net foreign investment.

Fixed assets and other capital stocks used in our analysis are shown in Table 2. In addition to fixed assets and consumer durables reported by the BEA, we include inventories, land, and intangible assets. The source of data for inventories is NIPA; the source of data for land values is the flow of funds; and the source of data for intangible capital is McGrattan and Prescott (2010). As with investment, we decompose corporate capital stocks into those of all other corporations by assuming that the ratio of corporate stocks is equal to the ratio of depreciable assets reported in corporate tax returns. We then add together capital stocks of non–Schedule C corporations and

[1] Some taxes are assessed on purchases of goods and services that should in theory be subtracted from investment or government spending. Unfortunately, we do not have a breakdown by product category.

noncorporate businesses. This results in an estimate of 0.892 GNPs for tangible capital in Schedule C corporations and 3.262 GNPs for tangible capital in all other private businesses. Our estimate of 1.718 GNPs for the stock of intangible capital is based on our earlier work. We experimented with the share of this stock in our two sectors.

2.2. Balance Sheets

Table 3 lists balance sheet items that we reference in the paper, namely household net worth and government debt. The source of these data is the flow of funds accounts.

The first item is net worth of households, which also includes assets of nonprofit institutions. Households have tangible assets that averaged 1.82 times adjusted GNP over the period 2000–2009 and financial assets that averaged 3.14 times adjusted GNP. Subtracting liabilities of 0.85 GNPs implies a net worth of 4.1 GNPs over the same period.

The second item is end-of-period government debt, which averaged 0.526 GNPs over 2000–2009. Close to 70 percent of this debt is in the form of U.S. Treasury securities.

2.3. Population, Employment, and Hours

Using data from the U.S. Census, the Social Security Administration, NIPA, and the Bureau of Labor Statistics, we have estimates of population by age, survival probabilities, full-time equivalent employees, and annual hours of work. We summarize the relevant statistics for population, employment, and hours in Table 4 and in Figure 1.

According to U.S. Census estimates of the population, the annual growth rate in the population ages 16 to 64 over the decade 2000–2009 is a little over 1 percent per year, more specifically, 1.11 percent (see Table B-34 of the *Economic Report of the President,* 2012). The annual growth rate for the total population is slightly lower than 1 percent, more specifically 0.94 percent.

Survival probabilities are found in the period life tables used by the Social Security Administration. (See Bell and Miller, 2005, Table 6.) We take an average over males and females for the year 2010. This average is plotted in Figure 1.

Data on full-time equivalent (FTE) employees are found in the NIPA, Table 6.5. The number of FTE employees equals the number of employees on full-time schedules plus the number of employees on part-time schedules converted to a full-time basis. Over the period 2000–2009, the number of FTEs averaged 124 million.

The primary source of our annual hours of work series the U.S. Department of Labor, Bureau of Labor Statistics, *Employment and Earnings*. The raw data underlying the series are persons at work, aged 16 years and over, and average hours worked per week for persons at work. They are based on the Current Population Survey (CPS). Total hours for military are added using data on military personnel from the Department of Defense and an estimate of a 40-hour week. (See Prescott, Ueberfeldt, and Cociuba, 2005, for full details of the primary sources.)

3. Baseline Parameters

We now use the U.S. data described above to set parameters for the baseline economy.

The parameters that govern demographics are the population growth rate η, the number of years of working life J_R, and the survival probabilities $\{\sigma^j\}$. Based on the data presented in Table 4, we set $\eta = 1$ percent, $J_R = 43$, and the sequence σ^j equal to the values in Figure 1. These values imply that there are 3.39 workers per retiree.

The parameters that govern preferences and technologies are set so that the model national accounts and fixed asset tables are consistent with the revised accounts in Tables 1 and 2. To accomplish this, we added additional equilibrium conditions to the code for computing a balanced growth path. The additional conditions are as follows:

$$K'_{1T} = 0.892\,\mathrm{GNP} \tag{3.1}$$

$$K'_{2T} = 3.262\,\mathrm{GNP} \tag{3.2}$$

$$K'_{1I} = \omega\,1.718\,\mathrm{GNP} \tag{3.3}$$

$$K'_{2I} = (1-\omega)\ 1.718\,\mathrm{GNP} \tag{3.4}$$

$$wL = 0.587\,\mathrm{GNP} \tag{3.5}$$

$$L = L_1 + L_2 = 0.279, \tag{3.6}$$

where ω is a weight that we experiment with. The additional unknowns to be computed for the balanced growth path—in addition to the interest rate and one policy choice of the government that ensures budget balance—are the sectoral capital shares $(\theta_{1T}, \theta_{2T}, \theta_{1I}, \theta_{2I})$ and the preference parameters (α, β). The depreciation rates $(\delta_{1T}, \delta_{2T}, \delta_{1I}, \delta_{2I})$ can be pre-set so that the investment rates of the model match those of the United States. When computing these rates, we detrend the investments and stocks by dividing by population and technological growth. The technological growth rate is chosen to be 2 percent. That leaves only one technology parameter, namely θ_1. We arbitrarily set this parameter to 1/2 because we do not have Schedule C incomes and stocks broken out in the U.S. accounts. This is another parameter that we experiment with.

For baseline policy parameters, we need the defense and debt shares, tax rates, and transfers. The share of defense spending GNP, is set equal to 0.043, which is equivalent to the U.S. share from the NIPA shown in Table 1. The share of debt to GNP is 0.511, which is equivalent to the U.S. share from the flow of funds shown in Table 3, after dividing by the growth terms $(1+\gamma)(1+\eta)$. Note that these shares depend on endogenously determined GNP, which must, in equilibrium, satisfy the resource constraint. The tax rates for Schedule C corporations are 40 percent on profits and 20 percent on distributions, which are estimates based on federal and state tax data. The payroll tax rate is 15 percent, consistent with the current U.S. level. The tax rate for non–Schedule C businesses is more difficult to estimate because we include a lot of different entities in this category. We somewhat arbitrarily set it at 40 percent and experiment with the rate.

The final policies to be set are the tax rate on consumption and transfers. For the baseline economy, we assume τ^{ℓ} is a payroll tax and τ^{c} is a tax on labor income that gets deferred until retirement, as well as a tax on consumption purchases.[2] We also assume that workers and retirees in the aggregate receive different per capita transfers. Once we pick a tax rate on consumption and a ratio of transfers for workers and retirees, total transfers are determined because the government budget has to balance. For the baseline, we set the tax rate on consumption equal to 26.7 percent, which is the average of the TAXSIM rate for wage income, and we set the transfers so that the population of retirees receives 0.065 times GNP more than the population of workers. This number comes from adding together the expenditures for Social Security plus Medicare. (See Table 3.) With these two choices made, we find that equilibrium transfers, when aggregated, are equal to 0.369 times GNP, with 0.136 GNPs going to the population of retirees and 0.233 GNPs going to the population of workers.[3]

With total transfers to GNP equal to 0.369, the implied implicit taxes–that is, the difference

[2] Below we run an experiment assuming no tax deferral. In that case, τ^{ℓ} is much larger than τ^{c}.

[3] In the code, we set the per capita transfers for retirees and workers equal to 0.427 and 0.217, respectively. With 22.8 percent of the population retired, each receiving 0.427, the value of transfers going to retirees is 0.097 GNPs. With the remainder of the population working, each receiving 0.217, the value of transfers going to workers is 0.168 GNPs. In the baseline economy, TFP is normalized to 1 and GNP is equal to 0.718. Thus, the ratio of transfers to GNP is 0.136 for retirees and 0.233 for workers. When we change demographics, holding policy fixed, we assume that general transfers stay the same, that is, equal to 0.369 less 0.065 times GNP, and we assume that transfers for Social Security and Medicare increase with the number of retirees. With current demographics, 22.8 percent of the population are retired and with new demographics 32.4 percent are retired. Therefore, total transfers in the economy with new demographics and current policy are equal to 0.396 GNPs $(= [.369 + .065(.324/.228 - 1)]$ GNPs). When we change both demographics and policy, holding the per capita transfers at the baseline level, we have total transfers of 0.138 times GNP $(= 0.369 \times .718/1.007$, where 0.718 is the baseline level of GNP and 1.007 is the new level).

between the marginal tax rate and average tax rate times income–is equal to 0.111 times GNP. This estimate is derived by taking total transfers predicted by the model and subtracting the NIPA measure of transfers. The NIPA measure of transfers that we use is the sum of government nondefense expenditures and the usual NIPA transfers. Government nondefense expenditures are equal to 0.135 times GNP and the usual NIPA transfers are equal to 0.123 GNP. These sum to 0.258 GNPs, or 0.111 times GNP more than the model prediction. We view this estimate as reasonable because estimates of marginal tax rates for individuals are on the order of 25 to 30 percent, while average tax rates are on the order of 10 to 15 percent. If labor income is of the order of 60 percent of GNP, then 11 percent of GNP of implicit taxes is within the range of empirically reasonable estimates.

Table 5 is a summary of the baseline parameters (and is the same as Table 3 in the main text).

4. Computation

In this section, we provide details on computing equilibria for the balanced growth paths and then for the transitions.

4.1. Balanced Growth Paths

We have several codes at our website for computing balanced growth paths. They differ in the fixed-point method employed and in the choice of unknown variables. A fixed point is found for the equilibrium interest rate and for a residually determined variable, which is either common government transfers to households or the tax rate on consumption.

There are two fixed-point methods available: functional iteration and Newton-Raphson. The former simply updates the unknown variables iteratively as follows:

$$x^{k+1} = \omega x^k + (1-\omega)\left(x^k - r\left(x^k\right)\right), \tag{4.1}$$

where x^k is the kth iteration of the unknowns, $\omega \in [0,1]$ is a weighting parameter that aids convergence in many cases, and $r(x^k)$ are the first-order conditions that need to be satisfied by an appropriate choice of x^k. The Newton-Raphson method uses the following updating scheme:

$$x^{k+1} = x^k - \left[dr\left(x\right)/dx|_{x=x^k}\right]^{-1} r\left(x^k\right) \tag{4.2}$$

and, if necessary, a weighting parameter can be used to help with convergence:

$$x^{k+1} = \omega x^k + (1-\omega)\left(x^k - \left[dr\left(x\right)/dx|_{x=x^k}\right]^{-1} r\left(x^k\right)\right). \tag{4.3}$$

The two first-order conditions $r(x)$ that have to be satisfied by x are (1) the condition that sets the return on tangible capital equal to its marginal product and (2) the condition that imposes government budget balance. After manipulating all the other necessary conditions, we can write a step by step algorithm needed to evaluate $r(x)$, starting with a guess for x, as follows:[4]

- Set the interest rate i equal to the first element of x.
- Set the common transfer ζ equal to the second element of x.
- Use the fact that after-tax returns on capital net of depreciation are equated to the interest rate to get the four capital rental rates (that is, for the two types of capital in the two sectors),

$$\begin{aligned} r_{1T} &= i/\left(1-\tau_1^{\pi}\right) - \delta_{1T} \\ r_{2T} &= i/\left(1-\tau_2^{\pi}\right) - \delta_{2T} \\ r_{1I} &= i - \delta_{1I} \\ r_{2I} &= i - \delta_{2I}. \end{aligned}$$

- Use the capital share parameters to get estimates for the two ratios of sectoral labor inputs to total labor inputs,

$$\begin{aligned} L_1/L &= \theta_{1L}\theta_1/\left(\theta_{1L}\theta_1 + \theta_{2L}\theta_2\right) \\ L_2/L &= \theta_{2L}\theta_2/\left(\theta_{1L}\theta_1 + \theta_{2L}\theta_2\right), \end{aligned}$$

  where $\theta_{iL} = 1 - \theta_{iT} - \theta_{iI}$.

- Use the capital shares and capital rental rates to get estimates for ratios of capital stocks to compensation. Note that there are four ratios due to the fact that there are two types of capital and two sectors, that is,

$$\begin{aligned} K_{1T}/\left(wL_1\right) &= \theta_{1T}/\left(\theta_{1L}r_{1T}\right) \\ K_{2T}/\left(wL_2\right) &= \theta_{2T}/\left(\theta_{2L}r_{2T}\right) \\ K_{1I}/\left(wL_1\right) &= \theta_{1I}/\left(\theta_{1L}r_{1I}\right) \\ K_{2I}/\left(wL_2\right) &= \theta_{2I}/\left(\theta_{2L}r_{2I}\right). \end{aligned}$$

- Use the aggregate production function, sectoral labor ratios, and capital-compensation ratios to get an intermediate variable, call it z,

$$z = 2\{[K_{1T}/\left(wL_1\right)]^{\theta_{1T}}\,[K_{1I}/\left(wL_1\right)]^{\theta_{1I}}\,L_1/L\}^{\theta_1}\{[K_{2T}/\left(wL_2\right)]^{\theta_{2T}}\,[K_{2I}/\left(wL_2\right)]^{\theta_{2I}}\,L_2/L\}^{\theta_2}.$$

[4] In order to be precise, assume that the second element of x is the level of common transfers given to the households.

Note that the coefficient of 2 is used to normalize the ratios of outputs (found below) but can be changed without loss of generality.

- Use capital shares and the intermediate variable z to get an estimate of the wage rate:

$$w = \left[z\left(\theta_{1L}\theta_1 + \theta_{2L}\theta_2\right)\right]^{1/(1-(1-\theta_{1L})\theta_1-(1-\theta_{2L})\theta_2)}.$$

- Multiply the four ratios of capital stocks to compensation by the wage rate to get estimates of the capital-labor ratios, K_{1T}/L_1, K_{2T}/L_2, K_{1I}/L_1, and K_{2I}/L_2.

- Use the capital-labor ratios to construct ratios of intangible to tangible capital for the two sectors and the ratio of tangible capitals across the two sectors:

$$\begin{aligned}
K_{1I}/K_{1T} &= \left(K_{1I}/L_1\right)/\left(K_{1T}/L_1\right) \\
K_{2I}/K_{2T} &= \left(K_{2I}/L_2\right)/\left(K_{2T}/L_2\right) \\
K_{2T}/K_{1T} &= \left[\left(K_{2T}/L_2\right)/\left(K_{1T}/L_1\right)\right]\left[\left(L/L_2\right)/\left(L/L_1\right)\right].
\end{aligned}$$

- Multiply the intermediate variable z by the wage rate raised to a power to get the aggregate labor productivity, that is,

$$Y/L = zw^{(1-\theta_{1L})\theta_1+(1-\theta_{2L})\theta_2}.$$

- Solve the household dynamic programming problem—assuming the set of asset choices are $\{a_i\}$ which are equally-spaced points on $[0,\bar{a}]$. The steps are as follows:
  - For the terminal value function v_J, assume that the optimal next period assets and current labor supply are both 0 (that is, if $J_r < J$) and that determines the final level of consumption via the household budget constraint.
  - Working backwards from $j = J$ to $j = 1$, iteratively solve

$$v_j\left(a,s\right) = \max_{a',c,\ell}\{u\left(c,\ell\right) + \beta\sigma^j v_{j+1}\left(a',s'\right)\}$$

    subject to the budget constraints

$$a'\sigma^j = \left(1+i\right)a + \left(1-\tau^\ell\right)w\ell - \left(1+\tau^c\right)c + \psi^j.$$

    The programs find the maximum in a brute-force way, which is slow, but ensures that inequality constraints on asset holdings are enforced.
  - At each step $j = J-1,\ldots,1$, store the optimal decision functions.

- Use the probabilities of survival and the growth rate in the population to determine the fraction of people in each age group j, call this μ^j, where $\sum_j \mu^j = 1$.
- Add everything up by summing up optimal choices for consumption, labor, and asset holdings, weighted by the μ^j's. This implies values for total consumption C, total labor L, and total beginning-of-period assets A.
- Multiply the aggregate labor productivity Y/L by L to get total output, Y.
- Multiply the ratios of sectoral labor to total labor by L to get L_1 and L_2.
- Use the fact that assets are equal to business equity V plus government debt B in order to back out values for the capital stocks. In doing this, we need to remember that $B = \phi_B$ GNP and GNP is output less intangible investments. In other words, we have[5]

$$\begin{aligned} A &= V + B \\ &= V_1 + V_2 + \phi_B \text{GNP} \\ &= V_1 + V_2 + \phi_B \left(Y - X_{1I} - X_{2I}\right) \\ &= \left(1 - \tau_1^d\right) \left(K_{1T} + \left(1 - \tau_1^\pi\right) K_{2T}\right) + K_{2T} + \left(1 - \tau_2^d\right) K_{2I} + \phi_B \left(Y - X_{1I} - X_{2I}\right). \end{aligned}$$

  Also, note that on a balanced growth path, $X_{iI} = [(1+\gamma)(1+\eta) - 1 + \delta_{iI}]K_{iI}$. Using this fact plus the values for A and Y computed in the earlier steps we have:

$$\begin{aligned} K_{1T} = \left(A - \phi_B Y\right) / \{ &\left(1 - \tau_1^d\right) \left(1 + \left(1 - \tau_1^\pi\right) \left(K_{1I}/K_{1T}\right)\right) \\ &+ \left(K_{2T}/K_{1T}\right) + \left(1 - \tau_2^d\right) \left(K_{2I}/K_{2T}\right) \left(K_{2T}/K_{1T}\right) \\ &- \phi_B \left[(1+\gamma)(1+\eta) - 1 + \delta_{1I}\right] \left(K_{1I}/K_{1T}\right) \\ &- \phi_B \left[(1+\gamma)(1+\eta) - 1 + \delta_{2I}\right] \left(K_{2I}/K_{2T}\right) \left(K_{2T}/K_{1T}\right) \} \\ K_{1I} &= \left(K_{1I}/K_{1T}\right) K_{1T} \\ K_{2T} &= \left(K_{2T}/K_{1T}\right) K_{1T} \\ K_{2I} &= \left(K_{2I}/K_{2T}\right) K_{2T}. \end{aligned}$$

- Use the capital stocks, growth rates, and depreciation rates to compute the four investments

$$\begin{aligned} X_{1T} &= \left[(1+\gamma)(1+\eta) - 1 + \delta_{1T}\right] K_{1T} \\ X_{1I} &= \left[(1+\gamma)(1+\eta) - 1 + \delta_{1I}\right] K_{1I} \\ X_{2T} &= \left[(1+\gamma)(1+\eta) - 1 + \delta_{2T}\right] K_{2T} \\ X_{2I} &= \left[(1+\gamma)(1+\eta) - 1 + \delta_{2I}\right] K_{2I}. \end{aligned}$$

[5] Note that here we are equating beginning of period stocks.

- Use the capital stocks and labor inputs for the sectoral outputs and prices:

$$Y_i = K_{iT}^{\theta_{iT}} K_{iI}^{\theta_{iI}} L_i^{\theta_{iL}}$$
$$p_i = \theta_i Y / Y_i.$$

- Use output and the intangible investments to compute the NIPA analogues of GNP, accounting profits, and corporate dividends:

$$\text{GNP} = Y - X_{1I} - X_{2I}$$
$$\Pi_1 = p_1 Y_1 - wL_1 - \delta_{1T} K_{1T} - X_{1I}$$
$$D_1 = p_1 Y_1 - wL_1 - X_{1T} - X_{1I} - \tau_1^{\pi} \Pi_1$$
$$D_2 = p_2 Y_2 - wL_2 - \delta_{2T} K_{2T} - X_{2I}.$$

- Use GNP and age-dependent transfers to construct the variables relevant to the government budget constraint:

$$G = \phi_G \text{GNP}$$
$$B = \phi_B \text{GNP}$$
$$\Psi = \sum_j \mu^j \psi^j + \zeta.$$

- Construct the first-order conditions $r(x)$ as follows

$$r_1(x) = r_{1T} - \theta_{1T} \theta_1 Y / K_{1T}$$
$$r_2(x) = \Psi + G - \tau^{\ell} wL - \tau_1^d D_1 - \tau_2^d D_2 - \tau_1^{\pi} \Pi_1 - B' + (1+i) B - \tau^c C.$$

- Update x and check if the iterations have converged.

We add elements to $r(x)$ when we compute our initial baseline economy (with current demographics and current policy). Specifically, we add the constraints in (3.1)–(3.6) as residual equations in $r(x)$, and we add θ_{1T}, θ_{1I}, θ_{2T}, θ_{2I}, α, and β as unknowns in the vector x.

The Fortran programs for computing balanced growth equilibria are available at our website in the directory ./codes/balgrowth. The naming convention for the codes is bgxxyy.f90 with choices for xx and yy. The choices for xx, namely xx='tr' and xx='tc,' depend on whether the residual variable is the common transfer to households (tr) or the tax rate on consumption (tc). The choices for yy, namely yy='fi' and yy='nr,' depend on whether we employ a functional iteration update as in (4.1) or a Newton-Raphson update as in (4.2) or (4.3).

4.2. Transitions

For the dynamic case, we need to compute time paths ($t = 1, \ldots, T$) across different birth cohorts. To speed up the computations, we wrote the transition code to take advantage of parallel processors, assuming they are available. To simplify the code (called tran.f90), we assumed that T/n cohorts would be assigned to each processor, where n is the number of processors. For example, if $T = 240$ and $n = 48$ (as is true in our case), then there would be 5 cohorts per processor. For cohorts alive at $t = 1$, computation is done starting with the initial conditions of our baseline economy.

The core of the computation in transition is the same as for the balanced growth paths, namely solving the household problem. But, in this case, we are solving the household problem for each cohort and, therefore, have to keep track of all variables by age and time.

The iterations for finding x_t to solve $r(x_t)$ are also similar except for the fact that now we keep track of time series for the unknown variables, and we add another unknown. We add another unknown because we cannot write all other variables explicitly in terms of the interest rate and the residual variable for government budget balance. We add the wage rate as an unknown and we add a residual condition that equates the wage rate to the marginal product of labor. Doing this allows us to directly compute the capital-labor ratios and aggregate labor productivity.

Otherwise, the steps are the same as in the case of the balanced growth path computation.

5. Sensitivity Analysis

We conduct two types of experiments. First, we vary the parameters in Table 5 that are hard to estimate, holding all others fixed, and compare ratios of key statistics on the balanced growth paths. The point of this exercise is to show that the main conclusions are not sensitive to varying parameters within empirically plausible ranges. Second, we recompute the transition path from the baseline to an economy with new policies and new demographics, this time assuming a more traditional calibration of the model, one consistent with a capital-output ratio of 3 rather than 5.9. The point of this exercise is to show that this choice matters a lot. In the economy with a small capital-output ratio, our procedure of phasing in taxes and transfers in such a way as to make existing retirees at least as well off as they were in the old policy regime does not yield a welfare improvement for all birth-year cohorts.

5.1. Balanced Growth Paths with Alternative Parameters

The main results of our sensitivity analysis for balanced growth paths are summarized in Table 6.

The results in the column marked "Benchmark" are based on the economies reported in the main paper, which use the parameters in Table 5.

The next column, with heading "$\lambda = 0$," is the case in which annuity markets are shut down. The parameter λ is used as an indicator of whether these markets are open ($\lambda = 1$) or closed ($\lambda = 0$).[6] With the markets closed, we predict a larger impact of the change in policies, with GNP predicted to rise by 60 percent and the welfare gain equal to 28 percent.

If we assume taxes on labor cannot be deferred and set τ^ℓ equal to 40 percent and τ^c equal to 6 percent, we find results similar to those in the benchmark—which has τ^ℓ equal to 15 percent and τ^c equal to 27 percent. The national accounts and factor inputs change by similar amounts, and the welfare gain of the new policy is 24 percent rather than 20 percent.

Varying parameters related to sector income shares and intangible capital—which we cannot easily estimate with the data we have—we find almost no difference in the results. We lowered θ_1 and found almost no change in the results. We set the share of intangible capital in sector 2 equal to 0 while adjusting the share in sector 1 to keep the size of intangible capital fixed, and found almost no change in the results. Finally, we set the depreciation rates on intangible capital in the two sectors equal, adjusting the income shares to keep the size of intangible capital fixed, and again found almost no change in the results.

Lowering the tax on distributions from non–Schedule C corporations does make some difference, although we find a large welfare gain even when we use a relatively low estimate of 25 percent. The welfare gain is 15 percent in this case, rather than 20 percent, because the main policy change is the elimination of capital income taxes.

We tried other experiments that had almost no impact on the results. For example, another experiment that we tried was to use the life tables for 2050 in the new demographics. This did not make any difference once we changed the length of the work life to get a 2 to 1 ratio of workers to retirees. We also explored an alternative "new policy" with the ratio of general transfers to GNP set equal to the baseline ratio. Recall that the new policy discussed in the paper assumes the per capita transfers stay the same. If transfers relative to GNP stay the same, we need to increase

[6] In the code, we nest these two cases by setting the coefficient on next period assets equal to $1 - \lambda(1 - \sigma^j)$ in the individual budget constraints.

the tax on consumption to 35 percent (rather than 27 percent). The increased taxes and transfers roughly offset each other and the welfare gain remains at 20 percent.

5.2. Transition in a Small Capital-Output Economy

The second experiment explores the transitional dynamics in an economy parameterized in a more traditional way: with a capital-output ratio equal to 3. As we noted earlier, the point of the exercise is to see if we achieve a welfare-improving transition for all birth-year cohorts. We find that we do not.

More specifically, we redo the numerical exercise with a baseline economy that has one sector and one type of capital. This is achieved by setting $\theta_1 = 1$ and $\theta_{1I} = 0$. To generate a capital-output ratio of 3, we set the tangible capital share in sector 1 equal to 1/3, the depreciation rate of capital equal to 6 percent and the discount factor equal to 0.99. As is the tradition of the literature, we also abstract from taxes on distributions. The other tax rates are set as in our baseline economy: $\tau^{\pi} = 40$ percent, $\tau^{\ell} = 15$ percent, and $\tau^{c} = 27$ percent. The ratio of per capita transfers for a retiree relative to that of a worker is set at 2.11 and—with the government budget balance enforced—this choice implies an aggregate level of transfers equal to 0.304 times GNP: 0.065 times GNP for Social Security and Medicare as before and 0.239 times GNP for general transfers made to all individuals. The demographic assumptions are kept the same as in the baseline economy.

Figure 2 shows the transition from this new baseline economy with a capital-output ratio of 3 to an economy with payroll taxes, capital income taxes, and old-age transfers eliminated. The phase-in of policies is done exactly as before: we set transfers for cohorts alive at the time of the policy change in such a way as to make them at least as well off under the new policy, we set the payroll taxes equal to 0, and we gradually eliminate taxes on capital income. Notice that the cohorts entering the workforce subsequent to the change are worse off with the new policy. They are paying for the transfers to the retirees but not benefiting sufficiently from the tax reform.

It may well be possible to find transfer schemes among the existing cohorts that will make them all better off, especially since the retirees have positive gains, but it is much harder to do in this environment with a small stock of capital. As the figure shows, there is less wiggle room than we had before. In the end, the benefits to future cohorts are about 9 percent in terms of annual consumption, less than half of the 20 percent that we found for the economy with a large stock of productive capital.

References

Bell, Felicitie C., and Michael L. Miller. 2005. "Life Tables for the United States Social Security Area 1900–2100." Social Security Administration, Publication No. 11-11536.

Board of Governors. 1945–2012. *Flow of Funds Accounts of the United States*, Statistical Release Z.1 (Washington, DC: Board of Governors of the Federal Reserve System).

Economic Report of the President. 2012. (Washington, DC: U.S. Government Printing Office).

McGrattan, Ellen R., and Edward C. Prescott. 2010. "Technology Capital and the U.S. Current Account." *American Economic Review,* 100(4): 1493–1522.

Prescott, Edward C., Alexander Ueberfeldt, and Simona Cociuba. 2005. "U.S. Hours and Productivity Behavior Using CPS Hours Worked Data: 1959-I to 2005-II." Manuscript. Research Department, Federal Reserve Bank of Minneapolis.

U.S. Department of Commerce, Bureau of the Census. 2012. *Statistical Abstract of the United States* (Washington, DC: U.S. Government Printing Office).

U.S. Department of Commerce, Bureau of Economic Analysis. 1929–2012. National Income and Product Accounts of the United States, *Survey of Current Business* (Washington, DC: U.S. Government Printing Office).

U.S. Department of the Treasury, Internal Revenue Service. 1918–2012. *Statistics of Income* (Washington, DC: U.S. Government Printing Office).

Table 1. Revised National Income and Product Accounts, Averages Relative to Adjusted GNP, 2000–2009[a]

Total Adjusted Income	1.000
Labor Income	.587
Compensation of employees (NIPA 1.10, l. 2)	.534
Wages and salary accruals (NIPA 1.10, l. 3)	.435
Supplements to wages and salaries (NIPA 1.10, l. 8)	.099
70% of proprietors' income with IVA, CCadj (NIPA 1.10, l. 15)	.053
Capital Income	.413
Corporate profits with IVA and CCadj (NIPA 1.10, l. 17)	.072
30% of proprietors' income with IVA, CCadj (NIPA 1.10, l. 15)	.023
Rental income of persons with CCadj (NIPA 1.10, l. 16)	.016
Surplus on government enterprises (NIPA 1.10, l. 22)	.000
Net income, rest of world (NIPA 1.13, l. 60)	.007
Indirect business taxes	.072
Taxes on production and imports (NIPA 1.10, l. 9)	.068
Less: Subsidies (NIPA 1.10, l. 10)	.004
Business current transfer payments (NIPA 1.10, l. 14)	.007
Less: Sales tax	.042
Federal excise taxes (NIPA 3.5, l. 3)	.005
Federal customs duties (NIPA 3.5, l. 11)	.002
State and local sales taxes (NIPA 3.5, l. 14)	.030
Motor vehicle licenses (NIPA 3.5, l. 28)	.001
Severance taxes (NIPA 3.5, l. 29)	.001
Special assessments (NIPA 3.5, l. 30)	.001
Other taxes on production and imports (NIPA 3.5, l. 12,31)	.004
Consumption of fixed capital (NIPA 1.10, l. 23)	.117
Consumer durable depreciation (FOF F.10, l. 27)	.060
Statistical discrepancy (NIPA 1.10, l. 26)	−.004
Imputed capital services[c]	.037
Consumer durable services	.013
Government capital services	.024

See footnotes at the end of the table.

Table 1. Revised National Income and Product Accounts, Averages Relative to Adjusted GNP, 2000–2009[a] (Cont.)

Total Adjusted Product	1.000
Consumption	.743
Personal consumption expenditures (NIPA 1.1.5, l. 2)	.655
Less: Consumer durable goods (NIPA 1.1.5, l. 4)	.082
Less: Imputed sales tax, nondurables and services[b]	.035
Plus: Imputed capital services, durables[c]	.013
Government consumption expenditures, nondefense (NIPA 3.9.5, l. 17)	.110
Plus: Imputed capital services, government capital[c]	.024
Consumer durable depreciation (FOF F.10, l. 27)	.060
Tangible investment	.214
Gross private domestic investment[d] (NIPA 1.1.5, l. 7)	.149
Schedule C corporations	.070
Other private business	.079
Consumer durable goods (NIPA 1.1.5, l. 4)	.082
Less: Imputed sales tax, durables[b]	.005
Government gross investment, nondefense (NIPA 3.9.5)	.025
Net exports of goods and services (NIPA 1.1.5, l. 14)	−.043
Net income rest of world (NIPA 1.13, l. 60)	.007
Defense spending	.043
Government expenditures, national defense (NIPA 3.9.5, l. 11)	.043

Note: IVA, inventory valuation adjustment; CCadj, capital consumption adjustment; NIPA, national income and product accounts; FA, fixed assets; FOF, flow of funds.

[a] Expressions in parentheses are the data sources and table and line numbers.

[b] This category includes business transfers and excludes subsidies.

[c] Imputed capital services are equal to 4 percent times the current-cost net stock of government fixed assets and consumer durable goods.

[d] The corporate share of gross private domestic investment is 56.5 percent. To determine the share of Schedule C corporations, we assume that the ratio of investments for these corporations and all other corporations is the same as the ratio of their depreciable assets. Based on balance sheet data from the IRS corporate tax returns, this would imply that 83.5 percent of corporate investment is made by Schedule C corporations.

Table 2. Revised Fixed Asset Tables with Stocks End of Period, Averages Relative to Adjusted GNP, 2000–2009[a]

Tangible Capital	4.153
Fixed assets, private[b] (FA 1.1, l. 2)	2.192
Schedule C corporations	.673
Other private business	1.519
Fixed assets, government (FA 1.1, l. 2)	.595
Consumer durables (FA 1.1, l. 13)	.305
Inventories[b] (NIPA 5.7.5, l. 1)	.134
Schedule C corporations	.103
Other private business	.031
Land[b]	.928
Schedule C corporations	.116
Other private business	.812
Nonfinancial corporate (FOF B.102, l. 3,33,34)	.023
Nonfinancial noncorporate (FOF B.103, l. 3,33,34)	.306
Households and nonprofits (FOF B.100, l. 3,43,46)	.483
Intangible Capital	1.718
Plant-specific (McGrattan and Prescott, 2010)	1.198
Technology capital (McGrattan and Prescott, 2010)	.519
Total	5.871

Note: FA, fixed assets; FOF, flow of funds.

[a] Expressions in parentheses are the data sources and table and line numbers.

[b] The corporate shares of private fixed assets, inventories, and land are 36.8 percent, 92.1 percent, and 15.0 percent, respectively. In the case of inventories, we assume that 13 percent of farm inventories are corporate based on the ratio of corporate farmland and buildings relative to total corporate stocks reported in Table 828 of the *U.S. Statistical Abstract,* 2012. To determine the share of Schedule C corporations, we assume that the ratio of stocks for these corporations and all other corporations is the same as the ratio of their depreciable assets. Based on balance sheet data from the IRS corporate tax returns, this would imply that 83.5 percent of corporate capital is owned by Schedule C corporations.

TABLE 3. U.S. HOUSEHOLD NET WORTH AND GOVERNMENT DEBT AVERAGES RELATIVE TO ADJUSTED GNP, 2000–2009[a]

HOUSEHOLD NET WORTH, END OF PERIOD	4.100
Assets (FOF B.100, l. 1)	4.954
Tangible	1.817
Financial	3.138
Liabilities (FOF B.100, l. 31)	.854
GOVERNMENT DEBT, END OF PERIOD	.526
State and local municipal securities (FOF L.104, l. 20)	.165
Federal Treasury securities (FOF L.105, l. 20-22)	.359
Federal budget agency securities (FOF L.105, l. 23)	.002

Note: FOF, flow of funds.

[a] Expressions in parentheses are the data sources and table and line numbers.

Table 4. U.S. Population, Employment, and Hours Averages, 2000–2009[a]

Population in millions	
All ages (ERP B-34)	294
Ages 16 to 64 (ERP B-34)	193
Population growth (%)	
All ages (ERP B-34)	.94
Ages 16 to 64 (ERP B-34)	1.11
Full-time employees in millions (NIPA 6.5, l. 1)	124
Annual hours per population 16-64 (CPS, various)	1,452

Note: ERP, *Economic Report of the President*; NIPA, national income and product accounts; CPS, Current Population Survey.

[a] Expressions in parentheses are the data sources and table and line numbers. See Prescott et al. (2005) for the full details on primary sources.

Table 5. Parameters of the Economy Calibrated to U.S. Data

Demographic parameters	
Growth rate of population (η)	1%
Work life in years	43
Number of workers per retiree	3.39
Preference parameters	
Disutility of leisure (α)	1.297
Discount factor (β)	0.984
Technology parameters	
Growth rate of technology (γ)	2%
Income share, sector 1 (θ_1)	0.500
Capital shares	
Tangible capital, sector 1 (θ_{1T})	0.193
Intangible capital, sector 1 (θ_{1I})	0.189
Tangible capital, sector 2 (θ_{2T})	0.505
Intangible capital, sector 2 (θ_{2I})	0.059
Depreciation rates	
Tangible capital, sector 1 (δ_{1T})	0.051
Intangible capital, sector 1 (δ_{1I})	0.051
Tangible capital, sector 2 (δ_{2T})	0.015
Intangible capital, sector 2 (δ_{2I})	0.015

Table 5. Parameters of the Economy Calibrated to U.S. Data (Cont.)

Policy parameters	
Spending and debt shares	
Defense spending (ϕ_G)	0.043
Government debt (ϕ_B)	0.511
Tax rates	
Profits, sector 1 (τ_1^π)	0.400
Distributions, sector 1 (τ_1^d)	0.200
Distributions, sector 2 (τ_2^d)	0.400
Labor (τ^ℓ)	0.150
Consumption (τ^c)	0.267
Transfer ratio[a] (ψ_r/ψ_w)	1.968

[a] Each retiree and worker receives transfers equal to ψ_r and ψ_w, respectively.

Table 6. Balanced Growth Aggregate Statistics with New Demographics
Ratios of Per Capita Predictions for New vs. Current Policy[a]

	Experiments						
	Benchmark	$\lambda = 0$	$\tau^\ell = .4$	$\theta_1 = .45$	$\theta_{I2} = 0$	$\delta_{1I} = \delta_{2I}$	$\tau_2^d = .25$
GNP	1.51	1.60	1.53	1.54	1.52	1.51	1.37
Incomes							
Labor income	1.50	1.61	1.53	1.54	1.52	1.51	1.37
Capital income	1.50	1.60	1.53	1.54	1.52	1.51	1.37
Products							
Consumption	1.37	1.46	1.40	1.40	1.38	1.37	1.27
Defense spending	1.51	1.60	1.53	1.54	1.52	1.51	1.37
Tangible investment	2.11	2.19	2.17	2.16	2.11	2.12	1.75
Other expenditures							
Intangible investment	1.49	1.62	1.51	1.51	1.49	1.50	1.38
Government transfers	0.52	0.54	0.91	0.48	0.46	0.52	0.53
Factor Inputs							
Labor	1.20	1.10	1.09	1.08	1.08	1.07	1.17
Tangible capital	2.15	2.38	2.20	2.19	2.15	2.16	1.74
Intangible capital	1.49	1.62	1.51	1.51	1.49	1.50	1.38
Household Net Worth	2.29	2.50	2.32	2.30	2.30	2.29	1.87
Welfare Gain[b] (%)	20	28	24	21	19	20	15

[a] The $\lambda = 0$ experiment assumes there are no annuity markets; the $\tau^\ell = .4$ experiment assumes that taxes on labor include both payroll taxes and taxes on 1040 wage income and that taxes on consumption include only sales taxes with $\tau^c = .06$ based on estimates from NIPA; the $\theta_1 = .45$ experiment sets the share of income to Schedule C corporations lower than the 1/2 used in the baseline economy; the $\theta_{I2} = 0$ experiment is one with all intangible capital used in sector 1 and θ_{I1} increased to 0.265 in order to keep the size of the intangible capital stock the same as in the baseline economy; the $\delta_{1I} = \delta_{2I}$ experiment assumes that intangible capital depreciates at the same rate in the two sectors and, in order to keep the size of intangible capital the same as in the baseline economy, the value of θ_{I2} is set equal to 0.085; the $\tau_2^d = .25$ experiment has a lower tax on distributions in sector 2.

[b] This is the absolute gain, not the ratio of predicted gains.

Figure 1
One−year Survival Probabilities, 2010

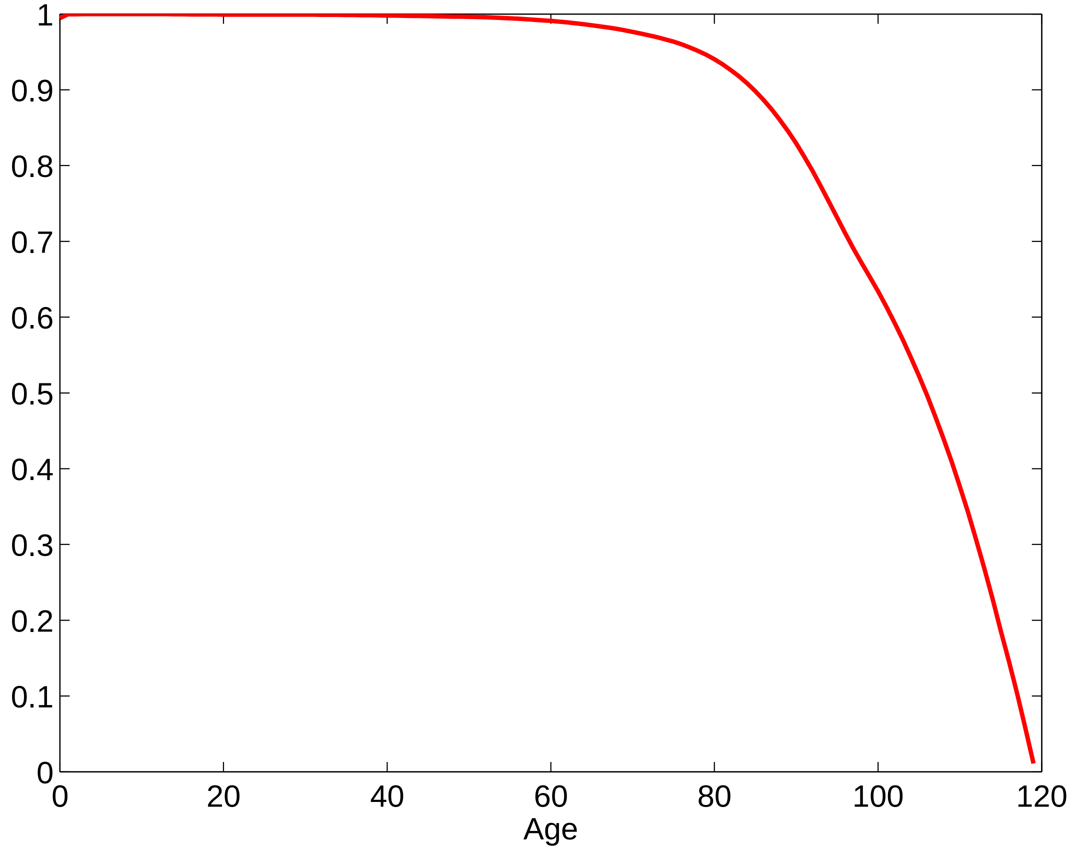

Figure 2
Percentage Welfare Gain of New Policy by Cohort Age
(Economy with Small Capital−Output Ratio)