Trade in Intermediate Inputs and Business Cycle Comovement

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First Draft: October 2010
This Draft: June 2011

Abstract

In data, bilateral trade is strongly correlated with bilateral GDP comovement. This paper examines whether trade in intermediate inputs explains this empirical fact. I integrate input trade into a many country, multi-sector model and calibrate the model to data on bilateral input-output linkages. With estimated productivity shocks, the model generates an aggregate trade-comovement correlation 30-40% as large as in data. This moderate aggregate correlation emerges because the model matches observed correlations of goods production well, but fails to match services correlations. With independent shocks across countries, the model accounts for one-quarter of the trade-comovement relationship for gross output of goods. However, because shocks are transmitted through input linkages, they synchronize gross output, not value added. Moreover, contrary to conventional wisdom, input complementarity does not reconcile model and data. Finally, using simulated data, I argue that caution is needed in interpreting trade-comovement regressions that include proxies for vertical linkages.

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1 Introduction

A large empirical literature suggests that international trade transmits shocks and synchronizes economic activity across borders. For example, bilateral trade is strongly (and robustly) correlated with bilateral GDP comovement.[1] Though standard international real business cycle (IRBC) models predict a positive correlation between trade and comovement, they cannot replicate the quantitative magnitude of the empirical correlation. For example, Kose and Yi (2006) show that the change in bilateral comovement generated by an exogenous change in bilateral trade intensity is at most one-tenth the size of the partial correlation between trade and comovement observed in the data. They have dubbed this the "trade comovement puzzle."

In addressing this puzzle, recent empirical work has turned attention to the role of intermediate goods trade as a conduit for shocks. For example, Ng (2010) documents that proxies for bilateral production fragmentation predict bilateral GDP correlations, while Di Giovanni and Levchenko (2010) document that bilateral trade is more important in explaining output comovement for home and foreign sectors that use each other as intermediates. Further, Burstein, Kurz, and Tesar (2008) show that countries that intensively engage in intra-firm trade with United States multinational parents display higher manufacturing output correlations with the U.S.[2]

This focus on input trade is potentially important, since intermediate inputs account for roughly two-thirds of international trade. To examine the role of input trade in shock propagation, I develop a many country, multi-sector extension of the standard international real business cycles model with trade in both intermediate and final goods. I then calibrate the model to data on bilateral final and intermediate goods trade flows for 22 countries and a composite rest-of-the-world region, and simulate model responses to sector-specific productivity shocks. Using simulated data, I assess the ability of the model with intermediates to explain observed bilateral output correlations, highlighting the role of input trade in driving comovement.

In the model, input trade transmits shocks across borders independent of, and in addition to, standard IRBC transmission mechanisms. In the cannonical model, idiosyncratic shocks generate output comovement by inducing comovement in factor supplies. Specifically, a positive shock in the home country raises home output and depreciates home's terms of

[1] See, for example, Frankel and Rose (1998), Imbs (2004), Baxter and Kouparitsas (2005), Kose and Yi (2005), Calderón, Chong, and Stein (2007), Inklaar, Jong-A-Pin, and Haan (2008), Di Giovanni and Levchenko (2010), and Ng (2010).

[2] In a related vein, Bergin, Feenstra, and Hanson (2009) find that Mexican export assembly (maquiladora) industries are twice as volatile as their US counterparts, suggesting possibly strong transmission of US shocks to Mexico via production sharing linkages.

trade, which induces increased factor supply and hence output abroad.[3] This mechanism continues to operate in the augmented model with intermediate inputs. However, with traded intermediates, productivity shocks are passed downstream through the production chain directly.[4] One implication of this is that input linkages generate comovement in gross output even if factor supply is exogenous, which in turn implies that comovement in gross output may be delinked from comovement in real value added. Thus, the production chain puts significant additional structure to how shocks are transmitted.

To evaluate these channels quantitatively, I calibrate the model to data on bilateral final and intermediate goods trade. Following Johnson and Noguera (2010), I use data from national input-output tables combined with data on bilateral trade to construct a synthetic global input-output framework. This framework describes how individual sectors in each country source intermediate goods from both home and bilateral import sources, as well as how each country sources final goods. This data has several advantageous features for calibration of international macro models. First, the framework respects national accounts definitions of final and intermediate goods, and therefore is consistent with standard macro aggregates. Second, the framework explicitly accounts for the "double counting" problem in gross trade statistics, wherein the gross exports exceeds the value added content of exports. These features provide for a more realistic calibration of openness and bilateral linkages than has been previously possible in the literature.

Proceeding to the numerical analysis, I first simulate the model using an estimated productivity process in which shocks are allowed to be correlated across countries, as in the data. This model generates an aggregate trade-comovement correlation 30-40% the size of the observed correlation. Disaggregating this result, the model generates strong cross-country correlations for goods, but not for services. For example, a trade-comovement regression for gross output of goods returns a coefficient roughly 3/4 the size of the correlation in the data, as compared to a correlation for services that is insignificantly different from zero. The aggregate trade-comovement coefficient then lies between these extremes, which implies that generating higher aggregate comovement from the model requires modifying the model in ways that raise the correlation of services.

These initial results represent an upper bound on the role of trade in propagating shocks, as they they confound the effects of idiosyncratic shock propagation with the correlation of

[3]Several recent papers strenthen this mechanism by lowering the short run elasticity of substitution between home and foreign goods, for example by introducing durable goods (Engel and Wang (2011)) or search and matching frictions (Drozad and Nosal (2008)).

[4]Productivity shocks travel unidirectionally downstream when intermediate goods are aggregated in a Cobb-Douglas fashion, the case considered in the benchmark model below. More generally, productivity shocks travel both downstream to input users and upstream to input suppliers.

shocks across countries themselves. To isolate the propagation mechanism, I simulate the model again using shocks that are uncorrelated across countries. In these simulations, the trade-comovement correlation falls substantially for real value added, both in the aggregate and at the sector level. This implies that the correlation of shocks across countries is primarily responsible for value added comovement.

Interestingly however, there is significant propagation of idiosyncratic shocks for gross output. For gross output, idiosyncratic shocks account for roughly one-quarter of the trade-comovement correlation in the data. This discrepancy between the comovement in real value added versus intermediate goods points to the role of intermediates in the model. Specifically, gross output in the model is a composite of real value added and intermediate inputs. Therefore, gross output can be correlated across countries either because real value added is correlated, or because intermediate use is correlated. In the model, comovement following idiosyncratic shocks is primarily due to comovement in intermediate use. This is because intermediate trade is the primary conduit through which shocks travel in the model.

Using this framework, I explore whether complementarity of intermediates amplifies comovement. I introduce complementarity in two different ways: first making intermediates complements among themselves, and second making intermediates complementary with non-produced factor inputs (i.e., capital and labor). Contrary to conventional wisdom, complementarity fails to narrow the gap between the model and data in both cases. Complementarity within the input bundle raises output comovement dramatically, but does not amplify real value added comovement. Complementarity between intermediates and factor inputs constrains fluctuations in demand for intermediates, thereby lowering comovement in gross output.

Finally, one advantage to simulating a many country model is that I generate an entire data set similar to those used in empirical work. To exploit this, I use my simulated data to examine whether trade-comovement regressions that control for 'vertical linkages' or cross-border 'fragmentation' are capable of cleanly identifying the role of intermediates in generating comovement. I argue that coefficients on proxies for production sharing in trade-comovement regressions are difficult to interpret, as they appear to be correlated with omitted shocks driving output correlations.

In addition to the empirical work cited above, this paper is related to a number of recent attempts to incorporate production sharing into business cycle models. The closest antecedent to the model developed below is a two-country, two-sector IRBC model with intermediates by Ambler, Cardia, and Zimmerman (2002).[5] This paper is distinguished

[5] Both Ambler et al. and this paper are also related to Cole and Obstfeld (1991) who write down a two country model with intermediate linkages and full depreciation of capital in the spirit of Long and Plosser

from Amber et al. in both scope and focus. Whereas Amber et al. focus on a stylized two country case, I calibrate and simulate a many country model to match data on bilateral production sharing relations. Further, I hone the empirical focus toward understanding the trade-comovement puzzle, in contrast to the focus on general business cycle properties of the model in Ambler et al. Lastly, my exposition and analysis of the basic mechanisms underlying international comovement differs substantially from Ambler et al.[6]

This paper is also related in spirit to recent models by Burstein, Kurz, and Tesar (2008) and Arkolakis and Ramanarayan (2009). Burstein, Kurz, and Tesar (2008) specify a two sector IRBC model in which the production sharing sector has a lower elasticity of substitution between home and foreign goods than the non-production sharing sector, which effectively lowers the aggregate elasticity of substitution and raises comovement.[7] Arkolakis and Ramanarayan (2009) adopt a multi-stage production function, an approach that is significantly different and less tractable in a multi-country setting than the approach in this paper.

More broadly, the basic structure of the model in this paper has important characteristics in common with models of sectoral linkages within the domestic economy, such as those analyzed by Long and Plosser (1983), Horvath (1998, 2000), Dupor (1999), Shea (2002), Carvalho (2008), or Foerster, Sarte, and Watson (2011). These papers provide many insights into the role input-linkages play in translating idiosyncratic shocks into aggregate fluctuations that could be applied to understanding regional business cycles using the framework and data introduced below. However, there is an important difference to keep in mind. Within the domestic economy, factors may be reallocated across sectors following a shock, whereas factors are comparatively immobile across countries in the international framework considered below.

Finally, in simulating a international macro model with more than two heterogeneous countries, the paper is also related to work by Zimmerman (1997), Kose and Yi (2006), Ishise (2009, 2010), and Juvenal and Monteiro (2010). These papers emphasize that third-country effects may be important in driving bilateral correlations, effects that are picked up in my many country framework. None feature trade in inputs, however.

(1983). This seems to be an under-appreciated contribution of their paper.

[6]Ambler et al. devote attention to analyzing the role of investment frictions in their framework and explaining the differences between their empirical findings and those of Long and Plosser (1983) by appealing to different assumptions regarding capital depreciation.

[7]In contrast to the model in this paper, the performance of the Burstein et al. model is identical regardless of whether they assume that goods cross borders only once or whether there is back-and-forth shipment of goods across the border associated with production sharing.

2 Mechanics of Output Comovement

I begin by articulating a stylized static model that isolates some key features of the full dynamic model. The general formulation of the static model combines international trade in both final and intermediate goods with endogenous factor supply. This framework nests two separate channels for transmitting shocks across borders and generating output comovement. To develop intuition, I compare two polar opposite cases of the framework that clearly separate the two channels.

In the first case, I assume that there is no trade in intermediate goods. This case corresponds to the static version of the standard multi-good international real business cycle model, in which comovement is driven by endogenous factor supply.[8] In the second case, I assume that there is no trade in final goods and that factor supply is exogenous. This case isolates the role of intermediate goods linkages in generating output comovement, and highlights an important distinction between comovement in gross output versus value added.

2.1 A Benchmark Model

Consider a static world economy with many countries ($i, j \in \{1, \ldots, N\}$). Country i produces a single tradable Armington differentiated good using labor L_i and composite intermediate good X_i, which is a CES aggregate of intermediate goods produced by different source countries. The aggregate production function is Cobb-Douglas in the domestic factor and the composite intermediate:

$$
\begin{aligned}
&Q_i = Z_i \left(X_i\right)^{\theta} L_i^{1-\theta} \\
&\text{with} \quad X_i = \left(\sum_j \omega_{ji}^{x} X_{ji}^{\rho}\right)^{1/\rho},
\end{aligned} \tag{1}
$$

where X_i is a CES aggregate of intermediate inputs produced in j and shipped to i (with technology weights ω_{ji}^{x}), θ is the intermediate input share in production, and Z_i is exogenous productivity.

Each country is populated by a representative consumer. The consumer is endowed with

[8] See Backus, Kehoe, and Kydland (1994) or Baxter (1995).

labor that it supplies to firms and consumes final goods. The consumer has preferences:

$$U_i(C_i, L_i) = \log(C_i) - \frac{\chi\epsilon}{1+\epsilon} L_i^{(1+\epsilon)/\epsilon} \\ \text{with} \quad C_i = \left(\sum_j \omega_{ji}^c C_{ji}^\gamma\right)^{1/\gamma}, \tag{2}$$

where C_i is a CES aggregate of final goods produced in j and shipped to i (with preference weights ω_{ji}^c), χ measures the disutility of working, and ϵ is the Frisch elasticity of labor supply.

For simplicity, I assume there exists a social planner.[9] The planner maximizes a social welfare function that is the weighted sum of utility of consumers from each country: $\sum_i \mu_i U_i(C_i, L_i)$, where μ_i is the welfare weight assigned to the consumer in country i. The social planner is constrained by the following adding-up condition for output from each country: $Q_i = \sum_j C_{ij} + X_{ij}$. This states that output in each country equals the sum of shipments of final and intermediate goods from country i to all destinations j.

The social planners problem is then to choose $\{\{C_{ji}, X_{ji}\}_{\forall j}, L_i\}_{\forall i}$ to solve:

$$\max \quad \sum_i \mu_i \left[\log(C_i) - \frac{\chi\epsilon}{1+\epsilon} L_i^{(1+\epsilon)/\epsilon}\right] \\ \text{s.t.} \quad Q_i = Z_i (X_i)^\theta L_i^{1-\theta} \\ \text{and} \quad Q_i = \sum_j C_{ij} + X_{ij}, \tag{3}$$

where C_i and X_i are defined above.

The production structure here differs in an important way from the standard IRBC framework.[10] The standard framework does not admit multi-stage, vertically specialized production processes in which imports are used to produce exports. In contrast, the production function and resource constraints above represent a multi-stage production process with an effectively infinite number of production stages, where value is added at each stage in a decreasing geometric sequence. Because production requires both domestic and imported intermediates, gross trade will be a multiple over actual value exchanged between countries,

[9] I elect to have a social planner here for expositional simplicity. No result depends on this assumption. Moreover, in Section 2.3, I discuss the mechanics of the model in a case with Cobb-Douglas preferences and technologies, which implies that perfect risk sharing obtains through terms of trade effects even without the existence of a social planner.

[10] Some semantic confusion may arise in comparing these frameworks. Starting at least with Backus, Kehoe, and Kydland (1994), IRBC models typically talk about trade in "intermediate goods," which are aggregated to produce a "composite final good." Despite this nomenclature, trade in these models should be thought of as trade in quasi-final goods, wherein each good crosses an international border only once.

as goods cross borders many times throughout the production process. In contrast, the domestic value added content of exports is equal to one in the standard framework.

2.2 Case One: No Intermediate Goods Trade

To mimic the IRBC framework, I assume here that there are no intermediate goods in the model, setting $\theta = 0$, which necessarily eliminates trade in intermediates.[11] In this event, the production function is linear in labor: $Q_i = Z_i L_i$. As such, if productivity innovations are independent across countries, output in country i is correlated with output in country j only if factor supplies L_i and L_j co-move.

To understand when these factor supplies co-move, we can turn to the first-order conditions for the social planners problem in this case. Using the first-order condition for labor, we can write factor supply in country i as:

$$L_i = \left(\frac{\lambda_i Z_i}{\chi \mu_i}\right)^{\epsilon}, \tag{4}$$

where λ_i is the shadow price of output in country i. Labor supply here is increasing in productivity and the shadow price of output in country i, as both raise the marginal revenue product of labor. Using the production function, then output can be written as:

$$Q_i = Z_i^{1+\epsilon} \lambda_i^{\epsilon} (\chi \mu_i)^{-\epsilon}. \tag{5}$$

Given a productivity innovation in country i, the resulting change in output is given by:

$$\hat{Q}_i = (1 + \epsilon)\hat{Z}_i + \epsilon \hat{\lambda}_i. \tag{6}$$

Obviously, the shadow price of output λ_i itself depends on productivity, but this formulation is instructive because it highlights three channels for understanding the effect of productivity on output. First, a productivity shock directly raises output. Second, a productivity shock raises the amount of labor supplied, holding the output price fixed. Third, a productivity shock will tend to drive down the shadow price of output (λ_i), which will attenuate the amount by which labor supply (and hence output) rises.

In this formulation, a productivity shock spills across borders via relative prices. As productivity rises in country i, the relative price of output in country i falls, equivalently the relative price of output in country j rises. As the relative price of output in country j rises,

[11] A natural alternative assumption would be that each country uses only its own good as an intermediate. This yields similar results to assuming that there are no intermediates in the model.

this induces the representative consumer in j to supply more labor, which raises country j's output. Thus, output in country i rises due to the direct effect of productivity on output and the indirect effect of productivity in raising labor supply, while output in country j rises because terms of trade movements raise the return to supplying labor.

In this version of model, endogenous factor supply is the basic mechanism that drives comovement, as in IRBC models more generally.[12] The strength with which productivity shocks spill across borders then depends on: (a) how responsive relative prices are to the underlying shocks; (b) the elasticity of factor supply. In the extreme, when labor supply is inelastic and productivity shocks are independent across countries, there is no output comovement across countries.

2.3 Case Two: No Final Goods Trade, Exogenous Factor Supply

Traded intermediate goods serve to synchronize output movements across countries, independent of the standard endogenous factor supply mechanism discussed above. To illustrate this point, I consider a second case of the general framework in which I shut down endogenous factor supply entirely and assume labor supply is exogenous, set to $\bar{L}_i$ in country i. Further, I assume there is no trade in final goods to focus attention on intermediate goods linkages. This can be thought of as a restriction that $\omega^c_{ji} = 0 \; \forall j \neq i$ and $\omega^c_{ii} = 1$. Then output from each country is allocated across uses to satisfy: $Q_i = C_{ii} + \sum_j X_{ij}$.

This model can be solved in a general case to relate output in each country to productivity shocks in all other countries via intermediate goods linkages. To develop intuition regarding how comovement depends on the input sourcing structure, I focus here on an analytically tractable special case and relegate the general model and detailed algebra to Appendix A. Specifically, I assume that the the intermediate goods aggregator takes a Cobb Douglas form. The production function is then:

$$\begin{aligned} Q_i &= Z_i X_i^{\theta} \bar{L}_i^{1-\theta} \\ \text{with} \quad X_i &= \prod_j \left(X_{ji}\right)^{\theta_{ji}/\theta}, \end{aligned} \tag{7}$$

with $\sum_j \theta_{ji} = \theta$.

With this set-up, one can show that the proportional change in output following produc-

[12] With capital, factor supply continues to play an important role. However, the "resource shifting effect" whereby agents reallocate capital to the country with the positive productivity shock and falls in other countries attenuates output comovement. Specifically, resource shifting induces a negative correlation in capital across countries which offsets the positive correlation in labor supply across countries that arises due to terms of trade effects. See Kose and Yi (2006) for additional analysis of these issues.

tivity innovations is given by:

$$\hat{Q} = \Theta' \hat{Q} + \hat{Z}. \tag{8}$$

The Θ matrix is a global bilateral input-output matrix that summarizes flows of intermediate goods across countries, with elements θ_{ij} equal to the share of expenditure on intermediates that j directly purchases from i as a fraction of the value of output in country j. Rearranging this equation, I write the change in log output as a reduced form function of productivity innovations:

$$\hat{Q} = [I - \Theta']^{-1} \hat{Z}. \tag{9}$$

The matrix $[I - \Theta']^{-1}$ provides a set of weights that indicate how production in country i responds to productivity shocks in country j. The weights can be interpreted as the total cost share of intermediates from j in production in country i, taking into account both direct and indirect purchases of inputs from j. These cost shares reflect global production sharing relationships. This is intuitive, since a positive productivity shock in country k benefits countries that use country k goods as inputs. This is true whether they use k goods directly or whether they rely on country k goods indirectly, in the sense that they source intermediates from some third country that itself relies heavily on inputs from country k. This has the implication that output will be correlated for country i and country j when they have similar overall sourcing patterns.[13] I discuss this intuition for a three country version of the model at greater length in Appendix B.

2.3.1 Gross Output versus Value Added

Thus far, I have implicitly focused the discussion of comovement via intermediate goods linkages on comovement in gross output. This is because there is an important distinction between gross output and value added in models with intermediate goods that does not arise in standard IRBC models without intermediates. To make this distinction explicit, I rewrite the production function in equation (1) as:

$$\begin{aligned} Q_i &= V_i^{1-\theta} X_i^{\theta} \\ \text{with} \quad V_i &\equiv Z_i^{\frac{1}{1-\theta}} L_i. \end{aligned} \tag{10}$$

The quantity V_i is real value added. Real value added in this framework is a sub-function of gross output, which itself a composite of productivity and factor inputs (labor). Gross output then is a composite, homogeneous of degree one, function of real value added and

[13] There are two distinct elements to differences in sourcing patterns. First, the overall level of trade will differ across countries. Second, conditional on overall openness, bilateral trade patterns also differ.

intermediate goods.[14] This set-up implies that real value added can be computed using the "double-deflation" method, the current best practice in sector-level national accounts. Under double deflation, nominal output and nominal input purchases for each sector are deflated via their own price indices. Real value added growth is then equal to: $\hat{V}_i = \frac{1}{(1-\theta)}\left(\hat{Q}_i - \theta \hat{X}_i\right)$, where $\hat{Q}_i$ and $\hat{X}_i$ are directly measured in the national accounts.[15]

One important implication of this distinction between real value added and gross output is that output comoves across countries for two reasons. First, real value added may comove across countries. Second, input use may comove across countries. In this section with exogenous factor supply, value added comoves across countries if and only if productivity shocks are correlated across countries. On the other hand, gross output can comove across countries even if productivity shocks are uncorrelated if input use is correlated. Intermediate goods linkages imply that input use will in fact be correlated, most intensely so for countries that either have strong bilateral production sharing linkages or are exposed to common shocks originating in an input supplier to both countries.

With endogenous factor supply, the logic is obviously more complicated, as one layers this mechanism on top of the standard IRBC transmission of shocks via relative prices and factor supply. However, distinguishing output and value added comovement in this special case yields important intuition regarding mechanics that I will exploit below.

2.3.2 Consumption Comovement

One final point to note about this simple model is that consumption (alternatively, real income or expenditure) comoves across countries, even if real value added does not.[16] To see this, note first that $\hat{C} = -\hat{\lambda}$. This says that nominal consumption expenditure is constant in each country, consistent with perfect risk sharing. Further, manipulating market clearing and first order conditions, one can show that $\hat{Q} = -\hat{\lambda}$. This means the nominal value of gross output is also constant in each country (i.e., relative quantities are proportional to relative relative prices), as is standard in models with Cobb-Douglas preferences/technologies.

Putting these together naturally implies: $\hat{C} = \hat{Q}$. Thus, consumption inherits the comovement properties of gross output, such that consumption comoves across countries following idiosyncratic shocks. This consumption comovement occurs despite the fact that real value added does not comove following idiosyncratic shocks in this simple model. To under-

[14] To generalize the definition of real value added, consider a general production function (supressing country subscripts and time indexation): $Q = f(K, L, X)$. Then if the production function is weakly separable in capital and labor, it can be rewritten as: $Q = f(h(K, L), X)$. The sub-function $h(K, L)$ is then "real value added."

[15] Of course, input shares θ are also measured in national accounts.

[16] See Appendix A for the detailed algebra underlying this argument.

stand this disconnect, note that consumption is drawn from the stream of gross output, not the stream of real value added. A productivity shock abroad increases the supply intermediates used in production, which increases gross output even if it is combined with a constant level of domestic real value added.

One straightforward implication of this is that one needs to be careful to match real GDP measured on the production side in the data to real value added in the model, not real consumption or expenditure. A more important point is that input trade may synchronize consumption across countries even it does not synchronize real value added, a point that has been overlooked in the existing literature.

3 Dynamic Many Country, Multi-Sector Sector Model

The full model extends the benchmark model in a number of directions. First, the full model includes both transmission channels discussed above: endogenous factor supply (capital and labor) and intermediate goods linkages. The model admits both trade in final and intermediate goods, as well as dynamic adjustment of capital. Second, the full model includes multiple sectors. Disaggregating the model is important because sectors differ substantially in both overall openness and integration into cross-border production chains. In specifying equilibrium in the full model, I need to take a stand on financial market structure. In what follows, I focus on the case of financial autarky (equivalently, balanced trade) on the grounds that financial autarky has been shown to generate terms of trade movements and cross-country correlations that align more closely with data.[17]

3.1 Production

Consider a multi-period world economy with many countries ($i, j \in \{1, \ldots, N\}$). Country i produces a tradable differentiated good in sector s using capital $K_{it}(s)$, labor $L_{it}(s)$, and composite intermediate good $X_{it}(s)$, which is an aggregate of intermediate goods produced by different source countries. The aggregate production function is Cobb-Douglas in the domestic factor and the composite intermediate:

$$\begin{aligned} &Q_{it}(s) = Z_{it}(s) K_{it}(s)^{\alpha_i(s)} X_{it}(s)^{\theta_i(s)} L_{it}(s)^{1-\alpha_i(s)-\theta_i(s)} \\ &\text{with} \quad X_{it}(s) = X_i(\ldots, X_{jit}(s', s), \ldots; s) \end{aligned} \tag{11}$$

[17]For example, see Heathcote and Perri (2002) and Kose and Yi (2006). Financial autarky tends to deliver stronger comovement because it shuts down "resource-shifting" effects where in capital is reallocated toward countries with positive productivity shocks.

where $X_i(\cdot; s)$ is an aggregator of intermediate inputs for sector s in country i, $X_{jit}(s', s)$ is the quantity of intermediate goods from sector s' in country j used by sector s in country i, $\{\theta_i(s), \alpha_i(s)\}$ are the intermediate input and capital shares in production for sector s and country i, and $Z_{it}(s)$ is exogenous sector-specific productivity.

Output is produced under conditions of perfect competition. A representative firm in country i, sector s takes the prices for it's output and inputs as given, and the firm rents capital and hires labor to solve:

$$\begin{aligned} \max \quad & p_{it}(s)Q_{it}(s) - w_{it}L_{it}(s) - r_{it}K_{it}(s) - \sum_{j=1}^{N}\sum_{s'=1}^{S} p_{jt}(s')X_{jit}(s', s) \\ \text{s.t.} \quad & L_{it}(s), K_{it}(s), X_{jit}(s', s) \geq 0 \end{aligned} \tag{12}$$

where $p_{it}(s)$ denotes the price of output, w_{it} is the wage, r_{it} is the rental rate for capital, and the production function for $Q_{it}(s)$ is given above by (11).

Labor, capital, and intermediate goods choices for production in country i satisfy:

$$\alpha_i(s)p_{it}(s)Q_{it}(s) = r_{it}K_{it}(s) \tag{13}$$

$$\left(\frac{\theta_i(s)p_{it}(s)Q_{it}(s)}{X_{it}(s)}\right)\frac{\partial X_{it}(s)}{\partial X_{jit}(s', s)} = p_{jt}(s') \tag{14}$$

$$(1 - \alpha_i(s) - \theta_i(s))p_{it}(s)Q_{it}(s) = w_{it}L_{it}(s). \tag{15}$$

Output is used as an intermediate good in production and to produce a composite final good for consumption and investment. Within each sector, perfectly competitive firms aggregate final goods from all sources to form a sector-level composite using production function: $F_{it}(s) = F_i(\ldots, F_{jit}(s), \ldots; s)$. These sector composites are then aggregated to form an aggregate final good via a Cobb-Douglas technology: $F_{it} = \prod_s F_{it}(s)^{\gamma_i(s)}$, where $\gamma_i(s)$ is the expenditure share on final goods of type s in country i. Note that I assume that there is no value added at this stage to be consistent with the accounting conventions in my input-output data which records the value of retail and distribution services as production of a separate services sector.

A representative final goods firms maximizes:

$$\max \quad p^f_{it}F_{it} - \sum_{j=1}^{N}\sum_{s=1}^{S} p_{jt}(s)F_{jit}(s), \tag{16}$$

where p^f_{it} is the price of the composite final good and F_{it} is defined above. Purchases of individual final goods F_{jit} for aggregation into the final good satisfy:

$$\left(\frac{\gamma_i(s)p^f_{it}F_{it}}{F_{it}(s)}\right)\frac{\partial F_{it}(s)}{\partial F_{jit}(s)} = p_{jt}(s). \tag{17}$$

Aggregate final goods are used for consumption and investment: $F_{it} = C_{it} + I_{it}$.[18] Gross output equals total purchases used as intermediates and to produce final composite goods: $Q_{it}(s) = \sum_{j=1}^{N}\sum_{s'=1}^{S} F_{ijt}(s) + X_{ijt}(s,s')$.

3.2 Consumption and Labor Supply

Each country is populated by a representative consumer. The consumer is endowed with labor (with time endowment normalized to one) that it supplies to firms and consumes final goods. The representative consumer also owns the capital stock in her country and makes investment decisions. The capital stock evolves according to: $K_{it+1} = I_{it} + (1-\delta)K_{it}$, where $K_{it} = \sum_{s=1}^{S} K_{it}(s)$. Under financial autarky (balanced trade), expenditure on final goods must equal income in each period for the consumer: $p^f_{it}F_{it} = w_{it}L_{it} + r_{it}K_{it}$, where $L_{it} = \sum_{s=1}^{S} L_{it}(s)$.

The consumer chooses $\{C_{it}, L_{it}, K_{it+1}\}$ to solve:

$$\begin{aligned} \max \quad & E_0 \sum_{t=0}^{\infty} \beta^t U_i(C_{it}, L_{it}) \\ & \text{s.t.} \quad p^f_{it}(C_{it} + I_{it}) = w_{it}L_{it} + r_{it}K_{it} \\ & \text{and} \quad K_{it+1} = I_{it} + (1-\delta)K_{it}. \end{aligned} \tag{18}$$

The Euler equation and first-order condition for labor supply are then:

$$\frac{\partial U_i(C_{it}, L_{it})}{\partial C_{it}} = \beta E_t\left[\frac{\partial U_i(C_{it+1}, L_{it+1})}{\partial C_{it+1}}\left(\frac{r_{it+1}}{p^f_{it+1}} + (1-\delta)\right)\right] \tag{19}$$

$$\frac{\partial U_i(C_{it}, L_{it})}{\partial L_{it}} = \frac{\partial U_i(C_{it}, L_{it})}{\partial C_{it}}\frac{w_{it}}{p^f_{it}}. \tag{20}$$

3.3 Equilibrium

Given a stochastic process for productivity, an equilibrium in the model is a collection of quantities $\{C_{it}, F_{it}\}$ for each country, $\{Q_{it}(s), K_{it}(s), L_{it}(s), \{F_{jit}(s)\}_j, \{X_{jit}(s', s)\}_{j,s'}\}_{i,s}$ for

[18] Note that this assumption implies that the aggregator is the same for consumption goods and investment goods. This assumption could be relaxed.

each country-sector, and prices $\{r_{it}, w_{it}, p^f_{it}, \{p_{it}(s)\}_s\}_i$. These must satisfy the producers' first order conditions (13)-(15) and (17) and the consumer's Euler equation (19) and first-order condition for labor supply (20). They must also satisfy market clearing conditions $Q_{it}(s) = \sum_j \sum_{s'} F_{ijt}(s) + X_{ijt}(s, s')$ and $F_{it} = C_{it} + K_{it+1} - (1-\delta)K_{it}$, the budget constraint $p^f_{it}F_{it} = w_{it}L_{it} + r_{it}K_{it}$, and the production function (11). The equilibrium conditions are collected explicitly in Appendix C.

3.4 Calibration

3.4.1 Functional Forms

To calibrate the model, I need to specify functional forms for preferences, the final goods aggregator, and the intermediate goods aggregator. I assume that preferences are given by: $U_i(C_{it}, L_{it}) = \log(C_{it}) - \frac{\chi\epsilon}{1+\epsilon} L_{it}^{(1+\epsilon)/\epsilon}$. Further, I assume that the final goods are produced via a CES production function: $F_{it}(s) = \left(\sum_j \omega^f_{ji}(s) F_{jit}(s)^\rho\right)^{1/\rho}$, where $\{\omega^f_{ji}(s)\}$ and ρ are parameters to be calibrated.

In the benchmark calibration, I assume that the intermediate goods aggregator is Cobb-Douglas: $X_{it}(s) = \prod_j \prod_{s'} \left(X_{jit}(s', s)\right)^{\theta_{ji}(s',s)/\theta_i(s)}$, where $\{\theta_{ji}(s', s)\}$ are parameters be calibrated. If the elasticity of substitution between final goods is greater than one, this Cobb-Douglas assumption implies that the elasticity of substitution within intermediates is lower than that between final goods. This is consistent with existing work such as Burstein, Kurz, and Tesar (2008) or Jones (2011), among others, who argue that the scope for substitution across intermediate goods is lower than for final goods. I discuss modifications of the production function that modify the degree of complementarity of intermediates among themselves or with value added in Section 4.3.

3.4.2 Technology and Preferences

With these assumptions, I need values for the following parameters: $\{\beta, \epsilon\}$ for preferences and $\{\alpha_i(s), \theta_i(s), \{\theta_{ji}(s', s)\}, \rho, \{\omega^f_{ji}(s)\}, \delta\}$ for the technology.[19] I set $\rho = .33$, $\delta = .1$, $\beta = .96$, and $\epsilon = 4$ based on standard values in the literature.[20] I calibrate the remaining parameters using the GTAP 7.1 Data Base, which contains benchmark production, input-output and trade

[19]Note, some parameters are not needed to simulate the model. For example, χ governs the level of labor supplied in the steady state, but model dynamics are independent of this value due to the constant elasticity of labor supply.

[20]On the Frisch elasticity, see King and Rebelo (1999) or Chetty, Guren, Manoli, and Weber (2011). While a Frisch elasticity of 4 is required to generate fluctuations in hours worked similar to data in the standard RBC model, it has been criticized as too high relative to micro estimates. In unreported results, I have examined the results of lowering the Frisch elasticity of labor supply to 1, and the performance of the model is both qualitatively and quantitatively similar.

data for 2004. Due to limitations on the availability of time series data on gross production and productivity data (see below), I extract country level data for 22 countries from GTAP, covering approximately 80% of world GDP, and aggregate the remaining countries to form a composite "rest-of-the world" region.

The GTAP data allow me to match data for output and value added in each country for two composite sectors, defined as "goods" (including agriculture, natural resources, and manufacturing) and "services." I calculate the intermediate goods share of output in each country and sector $\theta_i(s)$. The median intermediate share for goods producing sectors is 0.65 for my country sample, while the corresponding share for services is 0.46. Then, I calculate the capital share in gross output as $\alpha_i(s) = (1/3) * (1 - \theta_i(s))$, equal to an assumed capital share in value added (1/3) times the value added to output ratio $(1 - \alpha_i(s))$.

A key part of the calibration is accurate data on bilateral intermediate and final goods flows. I construct these flows by combining input-output tables with data on bilateral trade (both from GTAP), as in Johnson and Noguera (2010).[21] Bilateral intermediate and final goods shipments then serve as data targets for $\{\theta_{ji}(s', s)\}$ and $\{\omega^f_{ji}(s)\}$. See Appendix D for details on the source data, the procedure for constructing bilateral final and intermediate goods shipments, and further calibration details.

In the data, trade is unbalanced. Therefore, in calibrating the model, I allow steady state trade to be unbalanced as well to recover 'true' preference and technology parameters. I then solve for dynamics in the model by linearizing around this unbalanced steady state, assuming that trade imbalances are constant.[22] The linearized equilibrium conditions are included in Appendix C.

3.4.3 Productivity

To estimate stochastic processes for productivity, I use sectoral productivity data from the Groningen Growth and Development Centre's EU KLEMS and 10-Sector databases. Because data on TFP is not available for many countries over long periods of time, I follow the literature and estimate the productivity process using data on labor productivity.[23] I take

[21] Similar approaches have been used by Daudin, Rifflart, and Schweisguth (forthcoming), Koopman, Powers, Wang, and Wei (2010), and Trefler and Zhu (2010).

[22] An alternative approach would be to calibrate the model to the unbalanced steady state, then solve for and linearize around the corresponding balanced trade equilibrium. In practice, the differences in behavior of the model linearized around balanced steady state versus imbalanced steady states are second order.

[23] The main data constraint is that estimates of sector level capital stocks and/or labor quality are difficult to obtain. Though motivated by data constraints, using labor productivity in place of TFP implicitly assumes that capital and/or labor quality dynamics do not drive variation in labor productivity at business cycle frequencies. This assumption is common in the aggregate IRBC literature: see Backus, Kehoe, and Kydland (1992), Heathcote and Perri (2002), or Kose and Yi (2006) for example. Examining countries in the Groningen data for which both TFP and labor productivity growth rates are available for specific periods,

sectoral labor productivity growth for 19 OECD countries over the period 1970-2007 from the EU KLEMS data, where labor productivity growth is computed as the difference between real value added growth and growth in hours worked for each sector.[24] I turn to the 10-Sector data to compute productivity growth rates for three large emerging markets – Brazil, India, and Mexico – over the same period. Productivity in this data is measured as the difference between real value added growth less growth in the number of workers employed.

For each country and sector, I estimate univariate, trend stationary productivity process. Suppressing constants and time trends, the estimating equation is:

$$\log LP^{VA}_{it}(s) = \lambda_i(s) \log LP^{VA}_{it-1}(s) + \epsilon_{it}(s), \tag{21}$$

where $LP^{VA}_{it}(s)$ is the level labor productivity (measured using value added) and $\lambda_i(s)$ is the persistence parameter.[25] The correlation of productivity shocks $\epsilon_{it}(s)$ is unrestricted. To compute this correlation, I estimate equation 21 for each country and sector separately, recover regression residuals $\hat{\epsilon}_{it}(s)$, and then construct the covariance matrix of the shocks as: $\Sigma \equiv \frac{1}{T}\sum_t \hat{\epsilon}_t \hat{\epsilon}'_t$.[26] To simulate the model, I need to convert the covariance matrix Σ, constructed using residuals from estimation of the process for productivity measured using real value added, into an equivalent covariance matrix for shocks to productivity measured on a gross output basis. The adjustment multiplies each residual by the ratio of value added to output: $\hat{\hat{\epsilon}}_{it}(s) \equiv (1 - \theta_i(s))\hat{\epsilon}_{it}(s)$. See Appendix D for details.

In the simulations below, I will use this covariance matrix in two ways. One set of simulations will allow shocks to be correlated across countries, with correlations determined by the estimated covariance matrix. This is the standard approach in the literature. The shortcoming of this approach is that comovement in this set of simulations is driven both by

the year-on-year growth rates of TFP and labor productivity are roughly proportional, which suggests this assumption is innocuous.

[24]Countries include Australia, Austria, Belgium, Canada, Denmark, Spain, Finland, France, Germany, Greece, Ireland, Italy, Japan, Korea, Netherlands, Portugal, Sweden, United Kingdom, and the United States. I omit most Central and Eastern European countries in the data with short time series starting in the mid-1990s.

[25]In a modest departure from the existing literature, I restrict cross-country spillovers to be equal to zero and further assume that there are no spillovers across sectors withing a country. I restrict cross-country spillovers as a matter of necessity. With N countries and 2 sectors, there are too many unrestricted spillover parameters to estimate given the relatively short length of the time series available. I have experimented with estimation of cross-sector spillovers within countries. Point estimates for cross-sector spillovers are generally unstable across countries and imprecisely estimated (often indistinguishable from zero).

[26]For three of the forty-four country-sector pairs, the estimated persistence parameters exceed one. Examination of the data indicates that this is due to breaks in the trend for these country-sector time series. For these countries, I estimate productivity processes assuming that each experiences only aggregate productivity shocks (i.e., productivity growth in goods and services is equal to aggregate productivity growth). These three countries are Italy, India, and Mexico.

transmission of shocks across countries via trade linkages and the direct correlation of the underlying shocks themselves.

To more cleanly identify the trade transmission mechanism, I will also simulate the model under the (counterfactual) assumption that shocks are uncorrelated across countries. To parameterize this counterfactual scenario, I zero out the "off-diagonal" elements of the covariance matrix.[27] Specifically, I impose $\text{cov}(Z_{it}(s), Z_{jt}(s')) = 0$ for all $i \neq j$. This allows shocks to be correlated across sectors within countries, but uncorrelated for any cross-country sector pairs. While this eliminates cross-country correlations in shocks, it should be noted that $\text{cov}(Z_{it}(s), Z_{it}(s'))$ is an upper bound to the size of the truly independent productivity shocks.[28] This implies that simulated shocks using this method will be somewhat too large relative to the truly idiosyncratic shocks that countries face. Thus, one should interpret simulation results using these idiosyncratic shocks as an upper bound on the ability of the model to generate comovement from true (correctly measured) idiosyncratic country shocks.

One last detail regarding the simulation is that I include a composite rest-of-the-world region in the simulations, but do not have directly measured productivity data for this composite region. Therefore, I assume that productivity shocks in the rest-of-the-world are uncorrelated with productivity shocks to countries in my sample.[29] I parameterize the persistence, variance, and cross-sector correlations of the shocks to this region based on median values in the data.

4 Results

I begin by examining the model's ability to replicate the aggregate trade-comovement relationship with estimated productivity shocks. In this baseline analysis, I allow productivity shocks to be correlated across countries, as in the data. To isolate the role of trade in propagation of shocks, I turn to simulations with "orthogonalized" productivity shocks. Here I focus on contrasting the performance of the model for gross output versus value added, and examine whether introducing stronger complementarity for intermediate goods into the production function strengthens propagation. Finally, I explore whether augmented trade-comovement regressions with vertical linkages isolate the causal influence of input linkages on comovement.

[27] This approach is adapted from Horvath (1998).

[28] For example, suppose that there are global shocks and i.i.d. country shocks. Then $\text{cov}(Z_{it}(s), Z_{it}(s'))$ is equal to the sum of the variance of the global shock plus the variance of the idiosyncratic country shock, and hence an upper bound on the variance of the idiosyncratic shock.

[29] This assumption will likely bias downward the trade-comovement correlation in the model with correlated shocks, since in reality the rest-of-the-world productivity is likely positively correlated with most in-sample countries.

4.1 Trade-Comovement Correlations: Model vs. Data

To compare the model and data, I compute the correlation of year-on-year aggregate growth rates of gross output or real value added for each country pair. I also compute sector-level correlations across countries for three sector pairs: goods-goods, goods-services, and services-services.[30] Correlations in the model are computed as averages over 500 replications of 35 years each, roughly the same period over which correlations are computed in the data.

For aggregate output and value added, model-based correlations are positively related to data-based correlations, though the fit is imperfect. A regression line of best fit for correlations of real value added in model versus data is $\rho_{ij}(\text{data}) = .26 + .46\rho_{ij}(\text{model})$ with standard error on the slope of .08 and $R^2 = .14$. The positive intercept indicates that the model generally under predicts the average correlation in the data, which is quite reasonable given that there are other shocks not included in the model (e.g., demand shocks) that may be positively correlated across countries.[31]

To evaluate the aggregate trade-comovement relationship directly, I regress bilateral correlations in the data and model on bilateral trade intensity. Aggregate bilateral trade intensity is defined as: $\log\left(\frac{EX_{ij}+EX_{ji}}{GDP_i+GDP_j}\right)$, computed for the benchmark 2004 year in my data.[32] Table 1 and Table 2 contain results for gross output and real value added, respectively.[33] Panel A contains results from data, while Panel B contains results from the baseline model with correlated shocks.

Looking at the first column of Panel A in the tables, aggregate comovement is positively correlated with log bilateral trade intensity in the data. Further, comparing Tables 1 and 2, the quantitative magnitude of this relationship is similar for both gross output and real value added. Turning to model simulations in Panel B, the aggregate trade-comovement correlation is weaker, but evidently positive. Regression coefficients in the simulated data are roughly 30-40% as large as those in the actual data. Thus, while the model does not

[30]Note that for each country pair, there are two possible cross-sector (goods-services) correlations. In the analysis, I pool these correlations, so that the correlation of goods in country i with services in country j is treated the same as the correlation of services in country i with goods in country j.

[31]One possible candidate for these omitted shocks would be monetary shocks. Indeed, examining the model's fit for EU-pairs versus non-EU pairs (or Eurozone versus non-Eurozone), the model does a better job explaining variation in bilateral correlations for non-EU pairs than among EU-pairs. While the model does not fit EU-pairs in the aggregate, I show below that it does fit EU-pairs well for the goods sector. This is indirect evidence that demand shocks could be an important driver of services correlations observed in the data that cannot be explained by the model.

[32]Because trade shares are stable over time, results are not sensitive as to whether one computes bilateral trade intensity using trade data single year or averages bilateral trade over time prior to computing the metric. The basic results also hold if the level, rather than log, of bilateral trade intensity is used.

[33]Gross output correlations are computed using the Groningen EU KLEMS database, which implies that I cannot calculate correlations for pairs involving Brazil, India, and Mexico. Therefore, I also omit them in calculating gross output correlations in the model.

explain the aggregate trade-comovement correlation entirely, it accounts for a significant share of it.

To understand this result, I turn to sector-level correlations for output and value added. Figure 1 plots bilateral sector-level correlations in the data and model with correlated shocks for gross output of goods and services separately. The upper panel contains the data for each country's goods sector paired with a bilateral foreign goods sector, and the lower panel contains the same for services. The results are striking: the model with correlated shocks does a good job predicting gross output correlations for goods, but does a weak job for services. The correlation of model and data-based correlations is .47 for goods, and only .15 for services. Cross-sector pairs are in between with a correlation of .26.[34] This basic dichotomy – the model fits relatively well for goods and poorly for services – is borne out no matter whether one looks at gross output or real value added.

Not surprisingly, the good model fit for goods and poor fit for services manifests itself in trade-comovement regressions. Tables 1 and 2 reports regression coefficients for each sector pairing – goods-goods, services-services, and goods-services (cross) – separately. In these sector level regressions, log bilateral trade intensity between sector s in country i and sector s' in country j is defined as: $\log(\frac{EX_{ij}(s)+EX_{ji}(s')}{GDP_i+GDP_j})$.[35]

Looking at results for both gross output and real value added, trade predicts comovement for all sector pairs in the data. While trade somewhat better job predicting comovement for goods-goods sector pairs, regression coefficients are positive, significant, and large for all sector pairs. Turning to the model with correlated shocks, the model generates significant trade-comovement correlations only for goods-goods pairs. For goods-goods pairs, the coefficient on trade is roughly 3/4 the size of the correlation in the data for gross output and 1/2 for real value added.[36] In contrast, the model generates markedly weaker correlations for other sector pairings. For services-services pairs, trade is only weakly and insignificantly correlated with comovement for both gross output and real value added.

These results help us understand the origins of the aggregate trade-comovement correlation. The model with correlated shocks yields a strong relationship between trade and comovement for goods, but not for services. The aggregate trade-comovement coefficient then lies between these extremes, pulled toward zero by the model's inability to explain

[34] A regression line for goods is $\rho^g_{ij}(\text{data}) = .29 + .48\rho^g_{ij}(\text{model})$ with standard error on the slope of .07 and $R^2 = .22$, while for services the line of best fit is $\rho^s_{ij}(\text{data}) = .23 + .16\rho^s_{ij}(\text{model})$ with standard error on the slope of .07 and $R^2 = .02$. For cross sector pairs, $\rho^c_{ij}(\text{data}) = .24 + .27\rho^c_{ij}(\text{model})$ with standard error on the slope of .05 and $R^2 = .07$.

[35] This definition follows di Giovanni and Levchenko (2010). One could alternatively define bilateral trade intensity using sector-to-sector shipments $EX_{ij}(s, s')$.

[36] If all 231 country pairs are included in the simulated data regression, the coefficient rises to .072 for gross output.

services sector correlations. To raise the model implied trade-comovement correlation would require introducing elements that raise the correlation of services sectors across countries. Put differently, neither the measured correlation of services productivity shocks across countries, nor the transmission of idiosyncratic shocks through trade, is strong enough to generate a large aggregate correlation of trade with aggregate output comovement in this model.

4.2 Propagation of Idiosyncratic Shocks via Trade

The model-based trade-comovement correlations reported above represent an upper bound on the role of trade in generating comovement. Specifically, the trade-comovement regressions confound two possible reasons why trade predicts comovement. Bilateral trade can predict comovement either because it propagates shocks across border, or because it is a proxy for another force that generates comovement. Of principal concern, countries that trade more may have more correlated underlying productivity shocks.

To focus on pure propagation of idiosyncratic shocks, I turn to simulated data from the model with uncorrelated shocks. Panel C of Tables 1 and 2 report trade-comovement regressions for these simulations. In the first column, the aggregate trade-comovement correlation declines substantially once one removes common shocks from the productivity process. This decline is particularly pronounced for real value added in Table 2, where the coefficient is roughly one-fifth the size of the coefficient in the model with correlated shocks and only one-twentieth the size of the coefficient in the data.

The inability of the model here to generate a sizable correlation between trade and comovement when shocks are uncorrelated is the analog to the Kose and Yi (2006) puzzle in my framework. In a three-country IRBC model, Kose and Yi vary bilateral trade-intensity exogenously by manipulating trade costs, holding the correlation of shocks across countries constant. Then comparing value added correlations across equilibria with different trade costs, they compute a trade-comovement quasi-regression coefficient that is at most 1/10th the size of the coefficient in the data, a similar order of magnitude to the coefficients here. Given that Kose-Yi examine a model without intermediate goods trade, this leads to the conclusion that input trade does not "solve" the trade-comovement puzzle, at least in this standard class of models. Despite the introduction of input trade into the IRBC model, trade does not propagate shocks strongly enough to generate much comovement in aggregate GDP. This implies that the positive coefficient on bilateral trade in data and the model with correlated shocks arises because bilateral trade intensity proxies for the correlation of shocks themselves.

An important caveat, however, is that there is significant propagation of idiosyncratic shocks for gross output. The trade-comovement correlation in the model with uncorrelated shocks is roughly 60% of the correlation in the model with correlated shocks. Thus, there is an important discrepancy between the model's ability to generate comovement in gross output versus comovement in real value added. This result deserves separate attention, as it highlights the role that intermediates play in this framework.

The discrepancy between the propagation mechanism for gross output versus real value added is mostly clearly illustrated by examining cross-country correlations for goods production, so I focus on this sub-set of the data. For gross output of goods, propagation of independent shocks explains roughly one-third of the observed comovement in the data. Figure 2 plots actual gross output correlations for goods against those predicted by the model with uncorrelated shocks. There is a clear positive relationship, particularly among EU country pairs. The U.S.-Canada outlier is particularly instructive. The predicted correlation is roughly .23, while the actual correlation in the data is near .75, roughly a ratio of three to one. More generally, this magnitude is consistent with the overall spread in the data. Focusing on EU-pairs, predicted correlations vary in the range $(0, .15)$ while actual correlations lie in the range $(.25, .75)$, so the ratio of the ranges is roughly .5/.15 or three to one.[37]

These relationships are borne out in looking at the trade-comovement regressions for goods trade in Panel C of Table 1, where the coefficient generated by the model with uncorrelated shocks is one-third the size of the coefficient in the model with correlated shocks and one-quarter of that in the data. Thus, while two-thirds of the goods trade-comovement relationship for gross output is due to correlated shocks in the model, one-third is explained by the propagation of uncorrelated shocks across countries. At the same time, the model generates much weaker comovement in real value added, even for goods-goods sector pairs. One can see this by comparing the trade-comovement regression for goods in Panel C of Table 2 to those in Table 1, where the coefficient for value added is near zero (just more than 1/10 that for gross output).

To explain why, one needs to look at how correlations in gross output are related to real value added in the model. I plot the correlation of gross output against the correlation for real value added for goods sector-pairs in Figure 3. The top panel depicts the relationship in the data (which is matched by the model with correlated shocks), while the lower panel depicts this relationship in the model with uncorrelated shocks. Whereas correlations for value added and gross output track each other closely in the data, there are large differences between the two in the model. First, dispersion in correlations of real value added across

[37] In this comparison, I relate changes in comovement across pairs to changes in predicted model correlations for EU pairs. This obviously ignores the fact that the model grossly underestimates the median correlation. The median ratio of the model correlation with uncorrelated shocks to the actual correlation is $\approx 10\%$ for EU pairs.

country is much smaller than the variance of correlations in gross output. Second, the correlation of gross output is typically larger (sometimes much larger) than the correlation of real value added for individual country pairs.

These discrepancies shed light on the role of intermediate goods in the model. Recall from the discussion in previous sections that gross output is a composite of real value added and intermediate inputs, as in Equation (D3). The correlation of gross output can then be decomposed into a weighted sum of the correlation of real value added across countries, the correlation of input use across countries, and the cross-correlation of real value added and input use:

$$\rho_{ij}(Q) = w_{ij}^{vv}\rho_{ij}(V) + w_{ij}^{xx}\rho_{ij}(X) + w_{ij}^{vx}\rho_{ij}(V,X) + w_{ij}^{xv}\rho_{ij}(V,X), \tag{22}$$

where $w_{ij}^{vv}, w_{ij}^{xx}, w_{ij}^{vx}, w_{ij}^{xv}$ are the appropriate weighting terms for each correlation, themselves functions of the Cobb-Douglas share parameters and standard deviations of gross output, real value added, and input use. To provide a visual sense of how these correlations aggregate, I plot the correlations $\rho_{ij}(V)$ and $\rho_{ij}(X)$ for select country pairs in Figure 4. As is evident, the correlation in input use across countries dwarfs the correlation in real value added. Further, the correlation of output lies somewhere in between, near the simple average of these two correlations.[38] Thus, the correlation of gross output is high because intermediate use is highly correlated, not because value added is highly correlated.

The fact that intermediate use is highly correlated is direct evidence that productivity shocks are being forcefully transmitted through cross-border production chains in the model. Because the share of intermediates in gross output for goods is roughly 2/3, this translates into significant output comovement. On the other hand, value added comovement is not high. Recall that one reason value added comoves in the model is that factor supply responds to relative prices. The low comovement of real value added indicates this channel is relatively weak in the model. To raise comovement in value added, one would need to strengthen this channel. In particular, the model would need to be adapted to translate the relatively strong comovement in intermediate use into stronger comovement in value added. With this motivation, I turn to analyzing whether input complementarity amplifies comovement.

[38]In the simulated data, the weights on each term are approximately equal (roughly 1/4) and the typical cross-correlation ($\rho_{ij}(V,X)$ or $\rho_{ij}(V,X)$) is relatively close to $\rho_{ij}(Q)$, lying between the extremes of $\rho_{ij}(V)$ and $\rho_{ij}(X)$. Hence, the simple average of $\rho_{ij}(V)$ and $\rho_{ij}(X)$ approximates $\rho_{ij}(Q)$ quite well.

4.3 Complementarity and Comovement

A recent strain of thought holds that disruptions in input-sourcing produce large output losses because inputs are complements in production. This argument surfaces in Burstein, Kurz, and Tesar (2008), Jones (2011), Di Giovanni and Levchenko (2010), or news coverage of the economic repercussions of the 2011 earthquake and tsunami in Japan for global supply chains. This is intuitively plausible, as negative supply shocks in a particular country or sector should be particular painful to upstream input users who have limited ability to substitute toward using inputs from alternative suppliers, or toward using non-produced factors of production (i.e., capital and labor) more intensively.

Building on these ideas, there are two distinct ways to introduce limited substitution for intermediates into the production function used in previous sections. First, inputs may complements to each other. In this instance, complementaries among inputs could be symmetric, or complementaries could vary among subsets of inputs (e.g., home and foreign inputs could be complements, while foreign inputs are substitutable among themselves). Second, inputs may be complementary to other factors of production. Put differently, inputs may be complementary to value added. To my knowledge, there is scant evidence as to which form of complementarity is more important empirically, particularly, particularly in contexts with imported intermediates. Therefore, I consider both types of complementarity in turn.

To generalize the set-up used above, I now assume that the production function is given by:

$$\begin{aligned} Q_{it}(s) &= Z_{it}(s)\left(\theta_i(s)V_{it}(s)^{\sigma} + (1-\theta_i(s))X_{it}(s)^{\sigma}\right)^{1/\sigma} \\ \text{with} \quad X_{it}(s) &= \left(\sum_j \sum_{s'} \omega_i^x(s',s) X_{jit}(s',s)^{\eta}\right)^{1/\eta} \\ \text{with} \quad V_{it}(s) &= K_{it}(s)^{\phi} L_{it}(s)^{1-\phi}, \end{aligned} \tag{23}$$

where $V_{it}(s)$ now denotes a Cobb-Douglas composite domestic factor, composed of capital and labor, and ϕ denotes the capital share in this composite. The elasticity parameter σ controls the substitution possibilities between intermediates and factor inputs, while η governs the (symmetric) substitution among intermediates. Finally, $\theta_i(s)$ here is redefined as a share parameter that can be chosen to match the intermediate input share in gross output, and $\omega_i^x(s', s)$ are share parameters that are calibrated to match bilateral intermediate goods flows.

There is scant evidence in the IRBC literature that guides calibration of complementarity in the production function. Perhaps this is unsurprising, as nearly all IRBC models ignore input trade and model production and consumption in value added terms. To illustrate the consequences of complementarity, I simulate the model for two extreme cases. In the first case, I assume the production function is Cobb-Douglas (setting σ effectively to zero) and the intermediate goods aggregator is near-Leontief (setting $\eta = -19$, corresponding to an elasticity of substitution equal to .05). In the second case, I assume the intermediate goods aggregator is Cobb-Douglas (setting η effectively to zero) and the production function is near-Leontief in $V_{it}(s)$ and $X_{it}(s)$ (setting $\sigma = -19$).[39]

Using this set-up, I re-simulate the model with uncorrelated productivity shocks as in previous sections and run trade-comovement regressions in this new simulated data. I present the results for sector-level correlations for the goods-goods sector pairing in Table 3. The column labeled "benchmark" repeats results from previous tables for reference. The column labeled "Complements w/in X" presents results for the simulation with $\sigma = 0$ and $\eta = -19$, while the column labeled "Complements b/n V & X" presents results for the simulation with $\sigma = -19$ and $\eta = 0$.

The results point to problems with the conventional view that complementarity is important in explaining comovement. Introducing complementarity among intermediates (column 2) does substantially strengthen the propagation of shocks for gross output. In fact, the model here generates a trade-comovement coefficient that exceeds the coefficient in data. However, even with this extreme comovement in output, the model does not generate much comovement in real value added. This implies that even extremely strong transmission of shocks through input linkages fails to generate enough comovement in factor supplies across countries to replicate real value added correlations.

In contrast, complementarity between inputs and factors fails on both counts: it neither generates comovement in gross output, nor real value added. In particular, the trade-comovement correlation for gross output is even lower than in the benchmark model. What is going on here? When agents are unable to substitute between factor inputs (V) and intermediate inputs (X), the less responsive input effectively constrains fluctuations in demand for the other input. In the model, factor input supply is fairly inelastic. This dampens fluctuations in input use, which weakens the transmission of shocks through intermediate linkages and lowers comovement in gross output.

These results seem to run counter to the received wisdom regarding the role of intermediates in propagation of shocks. In particular, they seem to contradict simulation evidence in Burstein, Kurz, and Tesar (2008) that suggests complementary intermediates are important for understanding the trade-comovement relationship. The results in this paper and their

[39] Strong complementary between factors and intermediates is common within the static computable general equilibrium trade literature, where Leontief production functions have been commonly employed. See Kehoe and Kehoe (1994), for example.

work are, in fact, less contradictory than they first seem. The key difference is that Burstein et al. specify complementarity in terms of value added, whereas I specify complementarity in terms of gross output. In their model, the "production-sharing" (vertically integrated) sector features a low elasticity of substitution between home and foreign value added. As in the standard IRBC model, this low elasticity amplifies comovement, because low elasticities imply volatile relative prices and strong transmission through the channel discussed in Section 2.2. I do not directly assume home and foreign value added are complementary, but rather embed complementarity into the production function for gross output. One way of reading my results is that complementarity of this form is not sufficient to induce the complementarity between home and foreign value added needed to replicate observed comovement.

4.4 Vertical Linkages in Trade-Comovement Regressions

In previous sections, I have used simple bivariate trade-comovement regressions to compare model and data. Several recent papers have attempted to isolate the role of intermediates in explaining comovement using more sophisticated specifications. Specifically, Di Giovanni and Levchenko (2010) and Ng (2010) both construct proxies for bilateral vertical linkages by combining trade and input-output data, and look at the partial effect of these linkages controlling for overall bilateral trade intensity. Further, Di Giovanni and Levchenko also estimate sector-level regressions with sector-pair and/or country-pair fixed effects to control for common shocks across countries. It is an open question whether these augmented trade-comovement regressions with vertical linkages can be interpreted as evidence of a causal relationship between vertical linkages and output comovement. I therefore explore this question using my simulated data.

Because Di Giovanni and Levchenko (2010) examine sector-level data, it is straightforward to map their empirical exercise to my framework and therefore I focus on their work. Di Giovanni and Levchenko attack the identification problem by estimating trade-comovement regressions at the sector level, pooling across sectors, and adding fixed effects to absorb particular unobservable shocks. Specifically, they construct a metric of bilateral vertical linkages at the sector level to capture the intensity with which exports from sector s in country i are used as intermediates by sector s' in country j (and vice versa). This takes the form: $\left[\text{IO}(s,s') \times \text{Exports}_{ij}(s) + \text{IO}(s',s) \times \text{Exports}_{ji}(s')\right]$, where $\text{IO}(s,s')$ is a measure of input-output linkages between sectors s and s' taken from a single country's input-output table and $\text{Exports}_{ij}(s) = \log\left(\frac{EX_{ij}(s)}{GDP_i+GDP_j}\right)$ is the log of exports from i to j in sector s normalized by the sum of value added in the source and destination countries.[40]

Then, Di Giovanni and Levchenko estimate the following regression:

$$\begin{aligned}\rho_{ij}(s,s') =& \alpha + \beta \text{Trade}_{ij}(s,s') \\ &+ \gamma \left[\text{IO}(s,s') \times \text{Exports}_{ij}(s) + \text{IO}(s',s) \times \text{Exports}_{ji}(s') \right] \\ &+ \text{FE} + \epsilon_{ij}(s,s'),\end{aligned} \tag{24}$$

where $\text{Trade}_{ij}(s,s') \equiv \log\left(\frac{EX_{ij}(s)+EX_{ji}(s')}{GDP_i+GDP_j}\right)$ and FE denotes fixed effects that vary by specification. One specification includes sector-pair fixed effects, while a second specification includes sector-pair fixed effects and country-pair fixed effects. These fixed effects are introduced to address concerns about omitted common shocks. The sector pair effects control for worldwide sector-specific shocks (possibly correlated across sectors) that hit all countries simultaneously. The country pair fixed effects control for aggregate shocks that may be correlated across countries, but hit all sectors symmetrically within each country.[41]

I report the results of running these regressions in my data in Table 4. Focusing on results for gross output, the regression results in the actual data are generally consistent with those reported in Di-Giovanni and Levchenko. Both bilateral trade and vertical linkages (Trade $\times$ IO) are positively correlated with bilateral sector-level comovement. Vertical linkages are significant in both specifications, while trade intensity is not significant when country fixed are included (though the t-stat of 1.47 is sizable).[42]

Examining results in the model with correlated shocks, vertical linkages remain significant and the coefficient magnitudes are the same or larger than those found in the data. Turning to the model with uncorrelated shocks, however, the magnitude of the coefficient on vertical linkages drops significantly, explaining at most 1/5 of the magnitude of the coefficients in the data. Further, looking at real value added, the model with correlated shocks continues to generate coefficients on vertical linkages similar to those in the data, though smaller in magnitude. However, the sign on vertical linkages actually flips sign in regressions in the simulated data with uncorrelated shocks.

Recall that the fixed effects are intended to control for correlated shocks driving correlations in the data. If these fixed effects adequately control for these shocks, one should expect that regression results in the model with uncorrelated shocks to be similar to those

[40] I use the direct input-requirements $\text{IO}(s,s')$ the U.S. to proxy for cross-sector input links. Di Giovanni and Levchenko also use input links for a single country.

[41] Ng (2010) embeds a vertical linkages metric into an aggregate trade-comovement regression, and therefore cannot use pair fixed effects to absorb common shocks. Instead, he includes other possible determinants of correlations (e.g., financial openness, output composition, etc.) directly as control variables in the regression.

[42] One point to note is that my country sample is much smaller than Di Giovanni and Levchenko (2010), so lower significance levels may be expected.

in the data (alternatively, the model with correlated shocks). Given that they are not, this suggests that vertical linkages proxies in the data may themselves be picking up shocks that vary by country-pair and sector-pair that the fixed effects cannot absorb. As such, these regressions are of dubious value as evidence that vertical linkages play an important role in propagation of shocks.

5 Conclusion

This paper uses a multi-sector, many country extension of the IRBC model with trade in both final and intermediate goods to dissect the trade-comovement puzzle. Using the model, I attempt to refine our understanding of the trade comovement puzzle along several dimensions.

First, input trade does not resolve the aggregate trade-comovement puzzle in a straightforward manner. That said, input trade does appear to play a role in explaining the relatively good fit of the model for the gross output of goods. Surprisingly, however, transmission of shocks through intermediate input channels does not generate strong comovement in value added. Moreover, complementarity for intermediates within the production function does not resolve this problem, or strengthen the trade transmission mechanism.

Second, and more generally, the aggregate trade-comovement correlation in the model is induced by correlation of shocks across countries. In this, the low aggregate correlation of trade with comovement in the model is due to the low correlation of shocks to services productivity across countries. This suggests that closing the gap between model and data will require expanding the set of shocks considered beyond productivity to include shocks that synchronize services more forcefully.

Third, trade-comovement regressions are difficult to interpret because it is not generally possible to control unobservable common shocks. This is true for plain-vanilla specifications, as well as augmented specifications that include proxies for vertical linkages and/or employ sector-level data with fixed effects. Model simulations with uncorrelated shocks suggest that the "causal" role of bilateral trade and/or vertical linkages is much smaller than suggested by raw regression estimates.

Despite these generally pessimistic results regarding the role of intermediates in generating comovement, there is promising evidence here that the introduction of intermediates into macro models alters the role of trade as a conduit of shocks. While intermediates in this benchmark IRBC framework do not replicate observed value added comovement, this may speak to the shortcomings of the IRBC framework rather than to the role of inputs per se. That is, there may be an important role for intermediates in models that capture the micro-structure of trading relationships more accurately. For example, the bulk of intermediates are traded within multinational firms, and this concentration of input trade among largest firms in the economy may mean shocks to intermediate suppliers are passed to aggregates. It is also true that many traded intermediates are tailored to a specific input purchaser – e.g., screens for the iPad. This specificity at the firm-product level is difficult to capture in the type of aggregate model developed in this paper. More careful consideration of these microeconomic features of input trade would be useful in future work.

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Table 1: Trade-Comovement Regressions for Real Gross Output: Data and Model

Panel A: Data

	Aggregate	Goods	Services	Cross
Log Bilateral Trade	0.093***	0.090***	0.071***	0.071***
	(0.012)	(0.012)	(0.012)	(0.008)
N	171	171	171	342
R-sq	0.25	0.23	0.15	0.16

Panel B: Model with Correlated Shocks

	Aggregate	Goods	Services	Cross
Log Bilateral Trade	0.039**	0.066***	0.017	0.032***
	(0.016)	(0.014)	(0.017)	(0.009)
N	171	171	171	342
R-sq	0.04	0.13	0.01	0.04

Panel C: Model with Uncorrelated Shocks

	Aggregate	Goods	Services	Cross
Log Bilateral Trade	0.023***	0.023***	0.011***	0.017***
	(0.002)	(0.002)	(0.001)	(0.001)
N	171	171	171	342
R-sq	0.52	0.55	0.33	0.48

Robust standard errors in parentheses. Significance levels: * $p < .1$, ** $p < .05$, *** $p < .01$. Constants included in all regressions. Log bilateral trade for aggregate regression: $\log\left(\frac{EX_{ij}+EX_{ji}}{GDP_i+GDP_j}\right)$. Log bilateral trade for sector-level regressions: $\log(\frac{EX_{ij}(s)+EX_{ji}(s')}{GDP_i+GDP_j})$.

Table 2: Trade-Comovement Regressions for Real Value Added: Data and Model

Panel A: Data

	Aggregate	Goods	Services	Cross
Log Bilateral Trade	0.104***	0.098***	0.054***	0.055***
	(0.014)	(0.012)	(0.011)	(0.009)
N	231	231	231	462
R-sq	0.18	0.22	0.07	0.08

Panel B: Model with Correlated Shocks

	Aggregate	Goods	Services	Cross
Log Bilateral Trade	0.032**	0.054***	0.014	0.023***
	(0.013)	(0.013)	(0.014)	(0.008)
N	231	231	231	462
R-sq	0.03	0.08	0.01	0.02

Panel C: Model with Uncorrelated Shocks

	Aggregate	Goods	Services	Cross
Log Bilateral Trade	0.006***	0.003***	0.002***	0.004***
	(0.001)	(0.001)	(0.001)	(0.000)
N	231	231	231	462
R-sq	0.28	0.13	0.09	0.20

Robust standard errors in parentheses. Significance levels: * $p < .1$, ** $p < .05$, *** $p < .01$. Constants included in all regressions. Log bilateral trade for aggregate regression: $\log\left(\frac{EX_{ij}+EX_{ji}}{GDP_i+GDP_j}\right)$. Log bilateral trade for sector-level regressions: $\log(\frac{EX_{ij}(s)+EX_{ji}(s')}{GDP_i+GDP_j})$.

Table 3: Trade-Comovement Regressions with Complementarity: Results for Goods-Goods Sector Pair in Model with Uncorrelated Shocks

Panel A: Gross Output

	Benchmark	Complements w/in X	Complements b/n V & X
Log Bilateral Trade	0.023***	0.137***	0.005***
	(0.002)	(0.017)	(0.001)
N	171	231	231
R-sq	0.52	0.23	0.23

Panel B: Real Value Added

	Benchmark	Complements w/in X	Complements b/n V & X
Log Bilateral Trade	0.006***	0.002	0.002***
	(0.001)	(0.003)	(0.000)
N	231	231	231
R-sq	0.28	0.00	0.05

Robust standard errors in parentheses. Significance levels: * $p < .1$, ** $p < .05$, *** $p < .01$. Constants included in all regressions. Log Bilateral Trade: $\log(\frac{EX_{ij}(s)+EX_{ji}(s')}{GDP_i+GDP_j})$. The column labeled "Complements w/in X" presents results for the simulation with $\sigma = 0$ and $\eta = -19$, while the column labeled "Complements b/n V & X" presents results for the simulation with $\sigma = -19$ and $\eta = 0$. See the text for details.

Table 4: Disaggregate Trade-Comovement Regressions with "Vertical Linkages"

Panel A: Gross Output

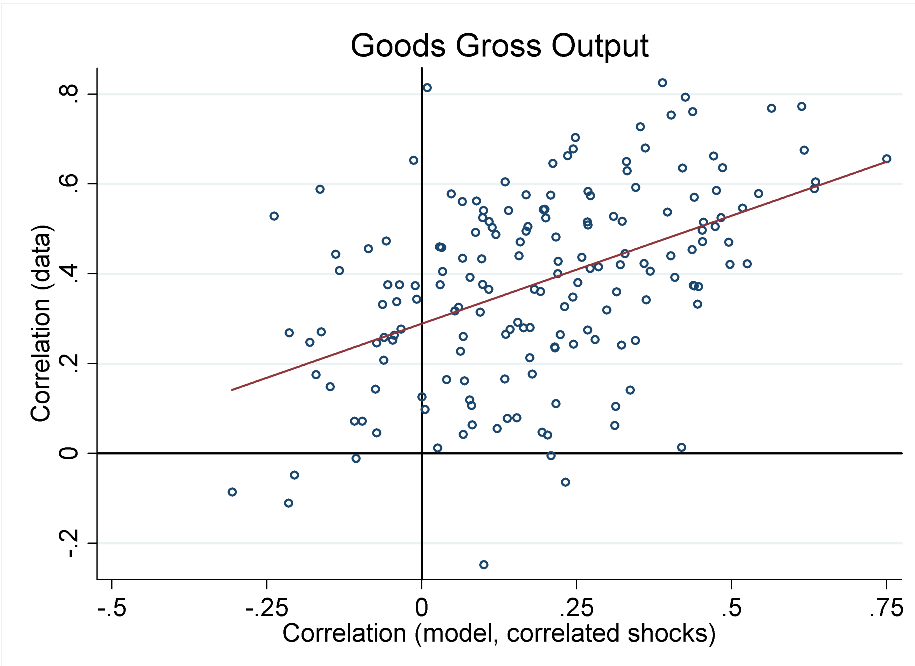
	Sector-Pair Fixed Effects			Sector-Pair & Country-Pair Fixed Effects		
	Data	Model (corr. shocks)	Model (uncorr. shocks)	Data	Model (corr. shocks)	Model (uncorr. shocks)
Log Bilateral Trade	0.046***	0.008	0.014***	0.025	0.019	0.004***
	(0.013)	(0.015)	(0.002)	(0.017)	(0.017)	(0.001)
Trade x IO	0.059**	0.058**	0.007	0.036**	0.054***	0.008***
	(0.024)	(0.028)	(0.005)	(0.018)	(0.018)	(0.002)
N	684	684	684	684	684	684
R-sq	0.23	0.09	0.49	0.68	0.70	0.92

Panel B: Real Value Added

	Sector-Pair Fixed Effects			Sector-Pair & Country-Pair Fixed Effects		
	Data	Model (corr. shocks)	Model (uncorr. shocks)	Data	Model (corr. shocks)	Model (uncorr. shocks)
Log Bilateral Trade	0.021	0.006	0.004***	-0.031**	0.002	0.002***
	(0.014)	(0.013)	(0.001)	(0.013)	(0.016)	(0.001)
Trade x IO	0.088***	0.044*	-0.001	0.075***	0.047**	-0.002*
	(0.024)	(0.024)	(0.001)	(0.016)	(0.019)	(0.001)
N	924	924	924	924	924	924
R-sq	0.13	0.04	0.17	0.70	0.58	0.66

Robust standard errors in parentheses. Significance levels: * $p < .1$, ** $p < .05$, *** $p < .01$. Constants included in all regressions. Log Bilateral Trade: $\text{Trade}_{ij}(s,s') \equiv \log\left(\frac{EX_{ij}(s)+EX_{ji}(s')}{GDP_i+GDP_j}\right)$. Trade x IO: $\left[\text{IO}(s,s') \times \text{Exports}_{ij}(s) + \text{IO}(s',s) \times \text{Exports}_{ji}(s')\right]$, where $\text{IO}(s,s')$ is a measure of input-output linkages between sectors s and s' and $\text{Exports}_{ij}(s) = \log\left(\frac{EX_{ij}(s)}{GDP_i+GDP_j}\right)$. See the text for details.

Figure 1: Correlations in Data vs. Correlations in Model with Correlated Shocks



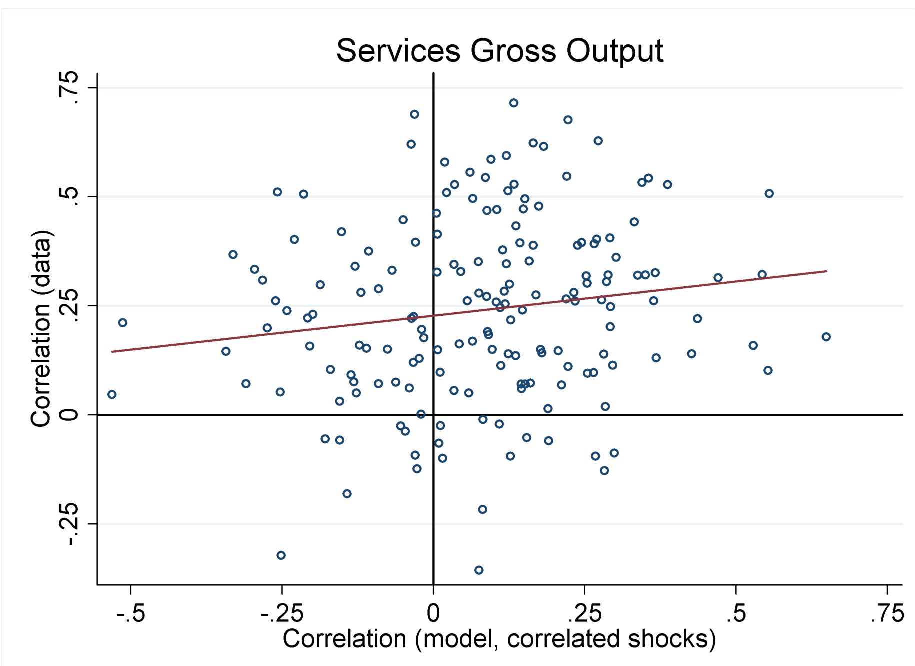


Figure 2: Correlations in Data vs. Correlations in Model with Uncorrelated Shocks

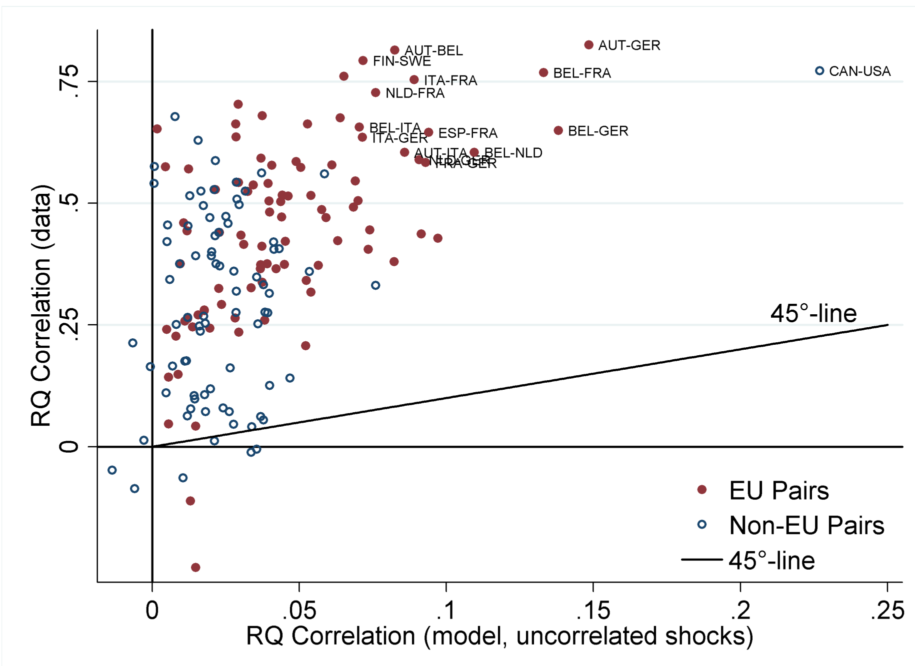
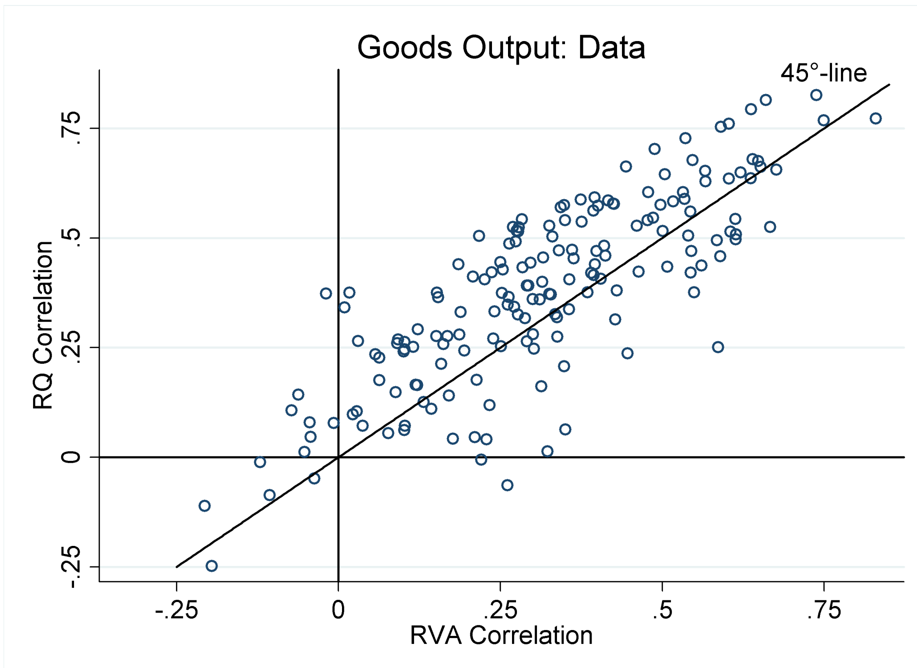


Figure 3: Correlations in Gross Output vs. Correlations in Value Added: Model and Data



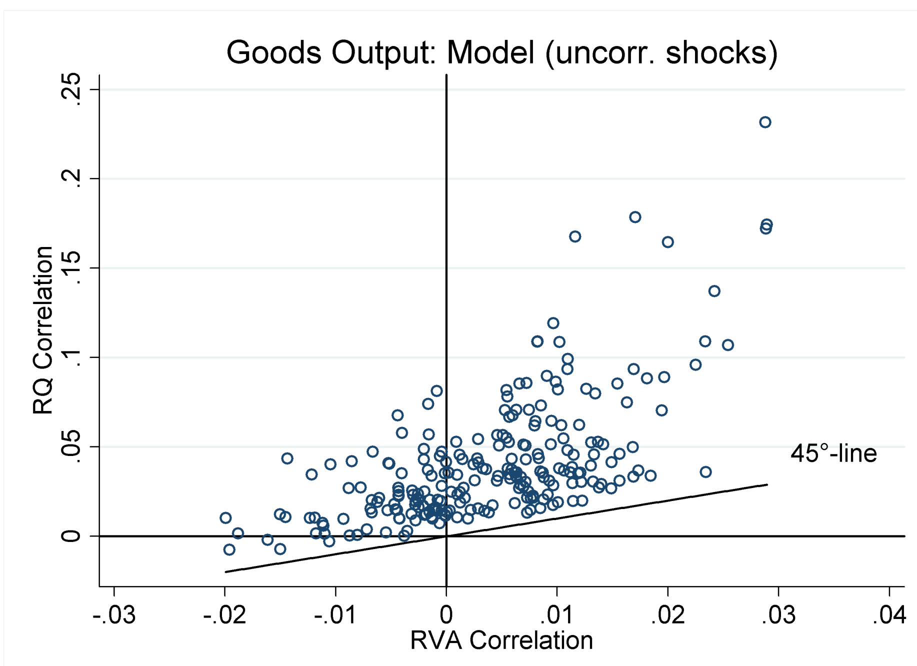
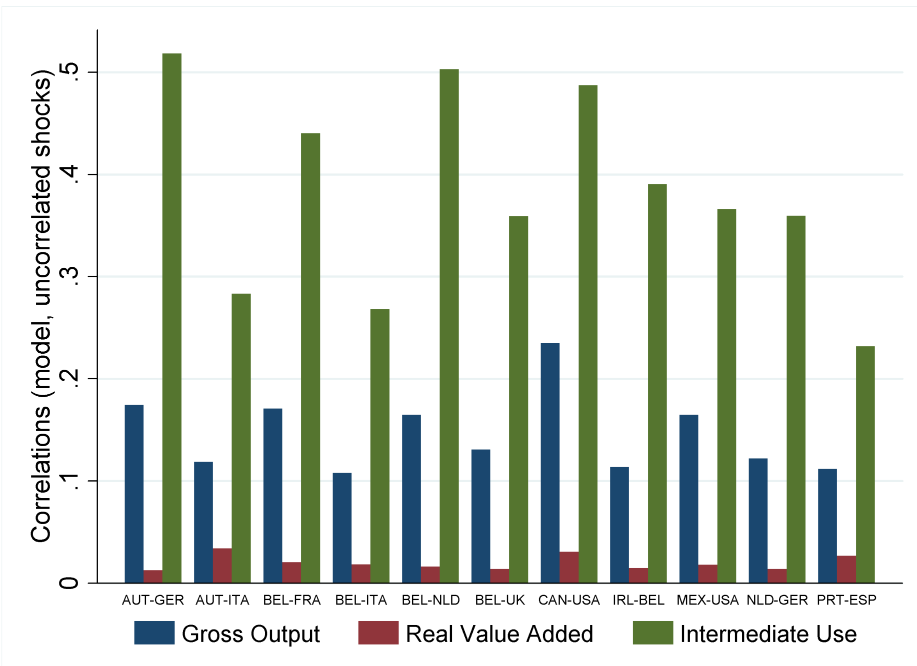


Figure 4: Correlations of Goods Output and Components in Model with Uncorrelated Shocks



Appendix A [Not For Publication]

Building on the set-up introduced in Sections 2.1 and 2.3, the social planner maximizes $\sum_i \mu_i \log(C_i)$ by choosing $\{C_{ii}, \{X_{ji}\}_{\forall j}\}_{\forall i}$. The first order conditions are then:

$$\frac{\mu_i}{C_{ii}} = \lambda_i \tag{A1}$$

$$\lambda_i \theta Q_i X_i^{-\rho} \omega_{ji}^x X_{ji}^{\rho-1} = \lambda_j. \tag{A2}$$

These first order conditions along with the technology and resource constraints can be linearized around the equilibrium to analyze the effects of a productivity shock. It is convenient to stack the equilibrium conditions to generate the following system:

$$\hat{C} = -\hat{\lambda} \tag{A3}$$

$$\hat{\mathbb{X}} = \frac{1}{1-\rho}\left[M_\lambda \hat{\lambda} + M_Q \hat{Q} + M_Q \hat{X}\right] \tag{A4}$$

$$\hat{X} = W\hat{\mathbb{X}} \tag{A5}$$

$$\hat{Q} = S_X \hat{\mathbb{X}} + S_C \hat{C} \tag{A6}$$

$$\hat{Q} = \hat{Z} + \theta \hat{X}, \tag{A7}$$

where $\hat{\mathbb{X}} = [\hat{X}_{11}, \hat{X}_{12}, \ldots, \hat{X}_{1N}, \hat{X}_{21}, \hat{X}_{22}, \ldots]'$ is an $(N^2 \times 1)$ vector and $\{\hat{C}, \hat{\lambda}, \hat{Q}, \hat{X}, \hat{Z}\}$ are $(N \times 1)$ vectors of the underlying variables for each country. The matrices are defined as follows:

$$M_\lambda \equiv \mathbf{1}_{N\times 1} \otimes I_{N\times N} - I_{N\times N} \otimes \mathbf{1}_{N\times 1},$$
$$M_Q \equiv \mathbf{1}_{N\times 1} \otimes I_{N\times N},$$
$$W \equiv [diag(w_1), diag(w_2), \ldots] \quad \text{with} \quad w_i = [w_{i1}, \cdots, w_{iN}] \quad \text{and} \quad w_{ij} \equiv \frac{\omega_{ij}^x X_{ij}^\rho}{X_j^\rho},$$
$$S_X \equiv \begin{pmatrix} s_1^x & \mathbf{0} & \cdots \\ \mathbf{0} & s_2^x & \cdots \\ \vdots & \cdots & \ddots \end{pmatrix} \quad \text{with} \quad s_i^x = [s_{i1}^x, \cdots, s_{iN}^x] \quad \text{and} \quad s_{ij}^x = \frac{X_{ij}}{Q_i},$$
$$S_C \equiv [diag(s^c)] \quad \text{with} \quad s_i^c = \frac{C_{ii}}{Q_i}.$$

Equations (A3)-(A7) can be solved to yield a reduced form expression that relates output in each country to productivity shocks in all other countries via intermediate goods linkages. Rather than analyze this general case, I turn to an analytically elegant Cobb-Douglas case in Section 2.3. Using the production function in equation (7), equation (A4) is replaced by:

$$\hat{\mathbb{X}} = M_\lambda \hat{\lambda} + M_Q \hat{Q}. \tag{A8}$$

Then derivation of equation (9) proceeds in two steps. First, the production function in equation (A7) can be re-written as:

$$\hat{Q} = \hat{Z} + \theta W M_{\lambda} \hat{\lambda} + \theta W M_{Q} \hat{Q}, \tag{A9}$$

using (A4) and (A5). Second, the market clearing and first order conditions can be used to solve for prices as a function of quantities. Specifically, using (A4) and (A3), the market clearing constraint in (A6) can be rewritten as:

$$\hat{Q} = \left(S_X M_{\lambda} - S_C\right) \hat{\lambda} + S_X M_Q \hat{Q}. \tag{A10}$$

This equation reduces by noting that $\left(I - S_x M_Q\right)^{-1} \left(S_X M_{\lambda} - S_C\right) = -I$. So then, $\hat{Q} = -\hat{\lambda}$. This result that relative prices are proportional to relative quantities is a familiar result from Cobb-Douglas models.

Substituting $\hat{Q} = -\hat{\lambda}$ into (A9) then yields

$$\hat{Q} = \Theta^{'} \hat{Q} + \hat{Z}, \tag{A11}$$

because $\Theta^{'} = \theta W \left(M_Q - M_{\lambda}\right)$. Manipulation of this equation then completes the derivation of equation (9).

Appendix B [Not For Publication]

To make the ideas in Section 2.3 concrete, consider a simple three country version of the Cobb-Douglas version of the model above in which there are no domestic intermediates ($\theta_{ii} = 0$). Then the solution for the vector of output growth takes the form:

$$\hat{Q} = [I - \Theta^{'}]^{-1} \hat{Z} \quad \text{with} \quad \Theta = \begin{pmatrix} 0 & \theta_{12} & \theta_{13} \\ \theta_{21} & 0 & \theta_{23} \\ \theta_{31} & \theta_{32} & 0 \end{pmatrix}. \tag{B1}$$

This solution can be rewritten as:

$$\hat{Q} = M \begin{pmatrix} 1 - \theta_{32}\theta_{23} & \theta_{21} + \theta_{23}\theta_{31} & \theta_{31} + \theta_{32}\theta_{21} \\ \theta_{12} + \theta_{13}\theta_{32} & 1 - \theta_{31}\theta_{13} & \theta_{32} + \theta_{31}\theta_{12} \\ \theta_{13} + \theta_{12}\theta_{23} & \theta_{23} + \theta_{21}\theta_{13} & 1 - \theta_{21}\theta_{12} \end{pmatrix} \hat{Z}, \tag{B2}$$

where $M = \frac{1}{\det [I - \Theta']^{-1}}$. There are two points to note regarding this solution.

First, for each country, the loadings on foreign country shocks are a function both of parameters associated with bilateral trade as well as trade with third countries. For example, the impact of a productivity innovation in country 2 on country 1's output is: $M(\theta_{21} + \theta_{23}\theta_{31})\hat{z}_2$. This effect is a function of both the intensity with which country 1 sources intermediates from country 2 (θ_{21}) and the compound term $\theta_{23}\theta_{32}$. This compound term picks up the indirect effect of country 2 productivity shocks operating via country 1's sourcing intermediates from country 3. Specifically, a shock in country 2 raises the supply of the country 2 intermediate good. This benefits country 1 directly because it uses this intermediate in production, but also benefits it indirectly because it uses intermediates from country 3 and country 3 intermediates are in turn produced using country 2 goods. Thus, the structure of the entire production chain matters, not just bilateral input sourcing patterns.[43]

Second, there is multiplier effect that controls the magnitude of effect of shocks on each country. To see this clearly, I re-write output growth for country 1 as:

$$\hat{Q}_1 = M_1 \left[\hat{Z}_1 + \left(\frac{\theta_{21} + \theta_{23}\theta_{31}}{1 - \theta_{32}\theta_{23}} \right) \hat{Z}_2 + \left(\frac{\theta_{31} + \theta_{32}\theta_{21}}{1 - \theta_{32}\theta_{23}} \right) \hat{Z}_3 \right], \tag{B3}$$

where I define $M_1 = \frac{1-\theta_{32}\theta_{23}}{\det [I-\Theta']^{-1}}$ to be country 1's multiplier. M_1 summarizes how much country 1 output increases with shocks to its own productivity and is generally greater than one. Thus, the sensitivity of output to shocks for different countries can be decomposed into a country specific effect M_i and a vector of weights on different shocks that varies across countries.

Appendix C [Not For Publication]

The equilibrium conditions for the model in Section 3 are collected here. The first order conditions for the consumer and production problems are given by:

$$\frac{\partial U_i(C_{it}, L_{it})}{\partial C_{it}} = \beta E_t \left[\frac{\partial U_i(C_{it+1}, L_{it+1})}{\partial C_{it+1}} \left(\frac{r_{it+1}}{p^f_{it+1}} + (1-\delta) \right) \right] \tag{C1}$$

$$\frac{\partial U_i(C_{it}, L_{it})}{\partial L_{it}} = \frac{\partial U_i(C_{it}, L_{it})}{\partial C_{it}} \frac{w_{it}}{p^f_{it}} \tag{C2}$$

$$\gamma_i(s) p^f_{it} F_{it} = p^f_{it}(s) F_{it}(s) \tag{C3}$$

$$\frac{\partial F_{it}(s)}{\partial F_{jit}(s)} = \frac{p_{jt}(s)}{p^f_{it}(s)} \tag{C4}$$

$$\alpha_i(s) p_{it}(s) Q_{it}(s) = r_{it} K_{it}(s) \tag{C5}$$

$$\theta_i(s) p_{it}(s) Q_{it}(s) = p^x_{it}(s) X_{it}(s) \tag{C6}$$

$$\frac{\partial X_{it}(s)}{\partial X_{jit}(s', s)} = \frac{p_{jt}(s')}{p^x_{it}(s)} \tag{C7}$$

$$(1 - \alpha_i(s) - \theta_i(s)) p_{it}(s) Q_{it}(s) = w_{it} L_{it}(s). \tag{C8}$$

[43] In a concrete example, the U.S. benefits from productivity increases in China not only because it imports from China, but also because the U.S. sources intermediates from Japan and Japan uses Chinese goods as inputs in production.

The market clearing conditions are given by:

$$Q_{it}(s) = \sum_{j=1}^{N} \sum_{s'=1}^{S} F_{ijt}(s) + X_{ijt}(s, s') \tag{C9}$$

$$F_{it} = C_{it} + K_{it+1} - (1 - \delta)K_{it} \tag{C10}$$

$$p_{it}^{f} F_{it} = w_{it} L_{it} + r_{it} K_{it} + T_i \tag{C11}$$

$$K_{it} = \sum_{s=1}^{S} K_{it}(s) \tag{C12}$$

$$L_{it} = \sum_{s=1}^{S} L_{it}(s). \tag{C13}$$

And remaining production functions and composite aggregators are given by:

$$Q_{it}(s) = Z_{it}(s) K_{it}(s)^{\alpha_i(s)} X_{it}(s)^{\theta_i(s)} L_{it}(s)^{1-\alpha_i(s)-\theta_i(s)} \tag{C14}$$

$$X_{it}(s) = X_i(\ldots, X_{jit}(s', s), \ldots; s) \tag{C15}$$

$$F_{it}(s) = F_i(\ldots, F_{jit}(s), \ldots; s) \tag{C16}$$

$$F_{it} = \prod_{s} F_{it}(s)^{\gamma_i(s)}. \tag{C17}$$

These equations represent $7N + 8(S \times N) + 6N^2$ equations (minus one after choosing a numeraire) in the same number of unknowns. The unknowns include $\{C_{it}, F_{it}, K_{it}, L_{it}\}$ for each country, $\{Q_{it}(s), K_{it}(s), X_{it}(s), L_{it}(s), F_{it}(s), \{F_{jit}(s)\}_j, \{X_{jit}(s', s)\}_{j,s'}\}_{i,s}$ for each country-sector, and prices $\{r_{it}, w_{it}, p_{it}^{f}, \{p_{it}^{f}(s), p_{it}^{x}(s), p_{it}(s)\}_s\}_i$.[44] The $\{T_i\}$ terms in the budget constraint represent time-invariant transfers across countries, which allow me to fit the model to a steady state with unbalanced trade. As discussed in the calibration section, I linearize around this unbalanced trade equilibrium assuming that trade imbalances are constant.

With the functional forms for preferences and production functions given in Section 3.4.1, these equilibrium can be linearized as follows. The linearized and stacked first order

[44] The equilibrium here can be easily reduced to the equilibrium defined in the main text via substitution.

conditions are given by:

$$0 = E_t\left[\hat{C}_t - \hat{C}_{t+1} + \beta(1-\delta)\hat{r}_{t+1}\right] \tag{C18}$$

$$0 = \frac{1}{\epsilon}\hat{L}_t + \hat{C}_t - \hat{w}_t + \hat{p}^f_t \tag{C19}$$

$$0 = \hat{p}^f_t + \hat{F}_t - \hat{p}^f_t(s) - \hat{F}_t(s) \tag{C20}$$

$$0 = \hat{\mathbb{F}}_t(s) + \frac{1}{1-\rho}M_i\hat{p}_t(s) - \frac{1}{1-\rho}M_j\hat{p}^f_t(s) - M_j\hat{F}_t(s) \tag{C21}$$

$$0 = \hat{p}_t(s) + \hat{Q}_t(s) - \hat{r}_t - \hat{K}_t(s) \tag{C22}$$

$$0 = \hat{p}_t(s) + \hat{Q}_t(s) - \hat{p}^x_t(s) - \hat{X}_t(s) \tag{C23}$$

$$0 = \hat{\mathbb{X}}(s', s) - M_j\hat{p}_t(s) + M_i\hat{p}_t(s') - M_j\hat{Q}_t(s) \tag{C24}$$

$$0 = \hat{p}_t(s) + \hat{Q}_t(s) - \hat{w}_t - \hat{L}_t(s), \tag{C25}$$

where $M_i \equiv I_{N\times N} \otimes \mathbf{1}_{N\times 1}$ and $M_j \equiv \mathbf{1}_{N\times 1} \otimes I_{N\times N}$. Here $\{r_t, w_t, p^f_t, p^f_t(s), p^x_t(s), p_t(s)\}$ and $\{C_t, L_t, F_t, Q_t(s), X_t(s), K_t(s), L_t(s), F_t(s)\}$ are vectors of prices and quantities, with element i equal to the relevant variable for country i. The vector $\mathbb{F}_t(s)$ is a N^2 dimensional vector that records final goods shipments for sector s, while $\mathbb{X}_t(s', s)$ is a N^2 dimensional vector that records intermediates goods flows from sector s' to sector s:

$$\hat{\mathbb{F}}_t(s) = [\hat{F}_{11t}(s), \hat{F}_{12t}(s), \ldots, \hat{F}_{1Nt}(s), \hat{F}_{21t}(s), \hat{F}_{22t}(s), \ldots]'$$
$$\hat{\mathbb{X}}_t(s', s) = [\hat{X}_{11t}(s', s), \hat{X}_{12t}(s', s), \ldots, \hat{X}_{1Nt}(s', s), \hat{X}_{21t}(s', s), \hat{X}_{22t}(s', s), \ldots]'$$

The stacked and linearized market clearing conditions are given by:

$$0 = \hat{Q}_t(s) - S_F(s)\hat{\mathbb{F}}_t(s) - \sum_{s'} S_X(s, s')\hat{\mathbb{X}}(s, s') \tag{C26}$$

$$0 = \hat{F}_t - diag\left(\frac{\bar{C}_i}{\bar{F}_i}\right)\hat{C}_t - diag\left(\frac{\bar{K}_i}{\bar{F}_i}\right)\hat{K}_{t+1} + diag\left(\frac{\bar{K}_i(1-\delta)}{\bar{F}_i}\right)\hat{K}_t \tag{C27}$$

$$0 = \hat{p}^f_t + \hat{F}_t - diag\left(\frac{\bar{w}_i\bar{L}_i}{\bar{p}^f_i\bar{F}_i}\right)(\hat{w}_t + \hat{L}_t) - diag\left(\frac{\bar{r}_i\bar{K}_i}{\bar{p}^f_i\bar{F}_i}\right)(\hat{r}_t + \hat{K}_t) \tag{C28}$$

$$0 = \hat{K}_t - \sum_{s=1}^{S} diag\left(\frac{\bar{K}_i(s)}{\bar{K}_i}\right)\hat{K}_t(s) \tag{C29}$$

$$0 = \hat{L}_t - \sum_{s=1}^{S} diag\left(\frac{\bar{L}_i(s)}{\bar{L}_i}\right)\hat{L}_t(s). \tag{C30}$$

The bar notation denotes steady state values. The matrices $S_F(s)$ and $S_X(s, s')$ collect the share of output allocated to final and intermediate use in destinations as follows:

$$S_F(s) \equiv \begin{pmatrix} s_1^f(s) & \mathbf{0} & \cdots \\ \mathbf{0} & s_2^f(s) & \cdots \\ \vdots & \cdots & \ddots \end{pmatrix} \quad \text{and} \quad S_X(s,s') \equiv \begin{pmatrix} s_1^x(s,s') & \mathbf{0} & \cdots \\ \mathbf{0} & s_2^x(s,s') & \cdots \\ \vdots & \cdots & \ddots \end{pmatrix}$$

$$\text{with} \quad s_i^f(s) = [s_{i1}^f(s), \cdots, s_{iN}^f(s)], \quad s_{ij}^f(s) = \frac{F_{ij}(s)}{Q_i(s)},$$

$$s_i^x(s,s') = [s_{i1}^x(s,s'), \cdots, s_{iN}^x(s,s')], \quad s_{ij}^x(s,s') = \frac{X_{ij}(s,s')}{Q_i(s)}.$$

Finally, the stacked and linearized production functions and aggregators are given by:

$$0=\hat{Q}_t(s)-\hat{Z}_t(s)-diag(\alpha_i(s))\hat{K}_t(s)-diag(\theta_i(s))\hat{X}_t(s)-diag(1-\alpha_i(s)-\theta_i(s))\hat{L}_t(s) \quad \text{(C31)}$$

$$0 = \hat{X}_t(s) - \sum_{s'} W_X(s',s)\hat{\mathbb{X}}_t(s',s) \quad \text{(C32)}$$

$$0 = \hat{F}_t(s) - W_F(s)\hat{\mathbb{F}}_t(s) \quad \text{(C33)}$$

$$0 = \hat{F}_t - \sum_{s} diag(\gamma_i(s))\hat{F}_t(s). \quad \text{(C34)}$$

The matrices $W_F(s)$ and $W_X(s',s)$ are sourcing shares for final and intermediate goods given by:

$$W_F(s) \equiv [diag(w_1^f(s)), diag(w_2^f(s)), \ldots]$$

$$\text{and} \quad W_X(s',s) \equiv [diag(w_1^x(s',s)), diag(w_2^x(s',s)), \ldots]$$

$$\text{with} \quad w_i^f(s) = [w_{i1}^f(s), \cdots, w_{iN}^f(s)], \quad w_{ij}^f(s) \equiv \frac{\omega_{ij}^f(s)F_{ij}(s)^\rho}{F_j(s)^\rho},$$

$$\text{and} \quad w_i^x(s',s) = [\theta_{i1}(s',s), \cdots, \theta_{iN}(s',s)].$$

To compute the dynamics, one needs to modify these conditions to reflect the choice of numeraire. Further, to reduce the computational burden, I manually substitute out for final and intermediate goods shipments $\{\{F_{jit}(s)\}_j, \{X_{jit}(s',s)\}_{j,s'}\}_{i,s}$, thereby reducing the size of the system by $6N^2$ (3174 with 23 countries) elements. Obviously other manual substitutions further reduce the dimensionality, but eliminating unknowns that increase in the square of the number of countries is most important. I use Harald Uhlig's "Toolkit for Analyzing Nonlinear Dynamic Stochastic Models" in MATLAB to compute solutions to this system.[45].

[45] See Uhlig (1999) or http://www2.wiwi.hu-berlin.de/institute/wpol/html/toolkit.htm

Appendix D [Not For Publication]

D.1 Final and Intermediate Goods Data and Calibration

The GTAP 7.1 Database is assembled by the Global Trade Analysis Project at Purdue University based on three main sources: (1) World Bank and IMF macroeconomic and Balance of Payments statistics; (2) United Nations Commodity Trade Statistics (Comtrade) Database; and (3) input-output tables from national statistical sources. To reconcile data from these different sources, GTAP researchers adjust the input-output tables to be consistent with international data sources. The data set includes internally consistent bilateral trade statistics combined with domestic and import input-output tables for 94 countries plus 19 composite regions covering 57 sectors in 2004.

From the GTAP database, I extract disaggregate input use tables for domestic input purchases and imported inputs. I then use bilateral trade data to split imported input use across bilateral partners, assuming that input purchases from each source are proportional to bilateral trade shares within a given sector. I split final goods imports across source countries using trade shares in a similar way. This yields bilateral final and intermediate goods shipments for 57 sectors. I then aggregate data on sectoral production, trade, final and intermediate shipments across sectors to form two composite sectors, defined as "goods" (including agriculture, natural resources, and manufacturing) and "services."

This data is the main input to calibrating $\{\theta_{ji}(s', s)\}$ and $\{\omega^f_{ji}(s)\}$. According to the producer's first order condition (14), $\theta_{ji}(s', s)$ is the ratio of expenditure on inputs from country j to gross output:

$$\theta_{ji}(s', s) = \frac{p_j(s')X_{ji}(s', s)}{p_i(s)Q_i(s)}. \tag{D1}$$

To calibrate $\{\omega^f_{ji}(s)\}$, note that the final goods producer's first order condition (17) can be rewritten in share form as:

$$\frac{p_i(s)F_{ij}(s)}{p^f_j F_j} = \gamma_j(s)\omega^f_{ij}(s)\left(\frac{p_i(s)}{p^f_j(s)}\right)^{-\rho/(1-\rho)}, \tag{D2}$$

where $\frac{p_i(s)F_{ij}(s)}{p^f_j F_j}$ is the share of final goods of sector s sourced from country i in total final goods expenditure in j. The share of final expenditure on goods of sector s – $\gamma_j(s)$ – can be computed directly in the data. Then, choosing quantity units so that the price of gross output and the final goods are equal to one in the steady state, $\{\omega^f_{ij}\}$ can computed by combining these expenditure shares.

D.2 Productivity Adjustment

To understand the productivity adjustment, recall the discussion in Section 2.3.1 about distinguishing gross output from real value added. At the sector level, gross output is a composite of real value added and intermediate inputs:

$$Q_{it}(s) = V_{it}(s)^{1-\theta_i(s)} X_{it}(s)^{\theta_i(s)}$$
$$\text{with} \quad V_{it}(s) \equiv Z_{it}^{\frac{1}{1-\theta_i(s)}} K_{it}(s)^{\frac{\alpha_i(s)}{1-\theta_i(s)}} L_{it}(s)^{\frac{1-\alpha_i(s)-\theta_i(s)}{1-\theta_i(s)}} \tag{D3}$$

Then, TFP measured using gross output is $\widehat{TFP}_{it}^{Q}(s) = \hat{Z}_{it}(s)$, while TFP measured using real value added is $\widehat{TFP}_{it}^{V}(s) = \frac{1}{1-\theta_i(s)} \hat{Z}_{it}(s)$. The two TFP measures are related by $\widehat{TFP}_{it}^{Q}(s) = (1-\theta_i(s))\widehat{TFP}_{it}^{V}(s)$, so shocks to productivity measured using value added will be larger than the corresponding shocks measured using gross output. This explains the need to adjust Σ and means that the correct covariance matrix for simulation is $\widetilde{\Sigma} = \frac{1}{T}\sum_t \hat{\tilde{\epsilon}}_t \hat{\tilde{\epsilon}}_t'$, where $\hat{\tilde{\epsilon}}_{it}(s) \equiv (1-\theta_i(s))\hat{\epsilon}_{it}(s)$ as in the main text. The persistence parameter $\lambda_i(s)$ obtained in estimation of (21) can be directly used in simulations, as it does not depend on which definition of productivity is used in the estimation.